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AN ALGORITHM FOR THE CALCULATION OF 3-D ICRF FIELDS
IN TOKAMAK GEOMETRY

By

D.N. Smithe, P.L. Colestock, R.J. Kashuba, and T. Kammash

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An Algorithm for the Calculation of 3-D ICRF Fields
in Tokamak Geometry

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Abstract

A computational scheme is developed which permits tractable calculation of three-dimensional full-wave solutions to the Maxwell-Vlasov equations under typical Ion Cyclotron Range of Frequencies (ICRF) experimental conditions. The method is unique in that power deposition to the plasma is determined via the anti-Hermitian part of a truncated warm-plasma dielectric operator, rather than as the result of an assumed phenomenological collision frequency. The resulting computer code allows arbitrary variation of density, temperature, magnetic field, and minority concentration in the poloidal plane by performing a convolution of poloidal modes to produce a coupled system of differential equations in the radial variable. By assuming no inhomogeneity along the toroidal axis, an inverse transform over $k_{\parallel}$ is performed to yield the full three-dimensional field solutions. The application of the code to TFTR-like plasmas shows a mild resonance structure in antenna loading related to the changing number of wavelengths between antenna and the resonance layer.

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1. INTRODUCTION

The basic goal behind ICRF modeling is the quantitative prediction of antenna performance and power deposition in the plasma over a wide range of plasma heating conditions. Present-day obstacles to this goal can generally be divided into two broad categories, those associated with deficiencies in the linearized Maxwell-Vlasov wave propagation theory, and those caused by restrictions of computational feasibility.

The second of these two obstacles, computational feasibility, represents a formidable problem due to the widely varying coupled spatial scales of the wave propagation in toroidal geometry. The model described in this paper is fully two-dimensional (poloidal plane) with an ignorable third dimension (toroidal axis), such that the three-dimensional boundary conditions are satisfied by repetitive solution of a two-dimensional boundary-value problem, followed by superposition, that is, inverse Fourier transform. This computational procedure is economical in terms of both memory space and computation time and is well suited for present-day computers. The extension to a fully three-dimensional (3-D) problem, or to a self-consistent two-dimensional (2-D) problem with poloidal field (three-dimensional in flux coordinates), may prove to be difficult and costly, but perhaps not impossible with the advent of next-generation supercomputers.

In addition to the considerations pertaining to computer capabilities, uncertainties also exist associated with the Maxwell-Vlasov theory in nonuniform media. The inclusion of two-dimensional effects raises several questions concerning the proper form of the wave equations when inhomogeneities, especially in the magnetic field (cyclotron frequency), are present. Ambiguities arise when the Fourier-Laplace representation of the radio-frequency (rf) conductivity is used, which is strictly valid only in uniform media, i.e., the plasma parameters cannot be position dependent while the equations are expressed in the transformed space. The WKB approximation, which does provide a mathematically valid extension of the theory to weak gradient conditions, becomes suspect near the cyclotron resonance because the coefficients of the homogeneous theory vary rapidly with respect to wavelength. Unfortunately, it is precisely this region that determines energy transfer to the plasma, so that the so-called full-wave or complete-wave solutions which are unattainable from the WKB approximation are required.

One escape from this dilemma has been the use of the cold-plasma approximation with a simple momentum collision term (increase mass by a complex factor $1+iv/\omega$ and related schemes) [1-4]. However, such an approach ignores entirely the physics associated with resonant absorption and energy transfer through the warm-plasma Bernstein branch [5-18]. Moreover, collision frequencies for typical thermonuclear conditions are in the range $v/\omega = 2 \times 10^{-4}$. Collision frequencies used in the collisional cold-plasma models generally are 100 to 1000 times this number. Thus, one must recognize the collision frequency of these models as simply an adjustable parameter whose purpose is to imitate the noncollisional warm-plasma response near resonance. A feature of our model, which will be discussed in Sec. III, is the replacement of the collisional parameter by the actual warm-plasma dispersion functions as the source of nonhermiticity in the dielectric kernel.

A second weak point in the theory of ICRF is the lack of an explicit three-dimensional mode conversion theory. Most warm-plasma one-dimensional models employ some form of the second-order finite Larmor radius expansion of the homogeneous dielectric, with the theoretical emphasis being placed on the manner in which one orders the spatial derivatives of the fields and plasma parameter variations. The inevitable result is an increase in the order of the differential equation that must be solved, generally from second order to fourth order, leading to a significant increase in computational difficulty, especially in the two-dimensional case [1-3,19].

One method of coping with the higher-order equations is to restrict them to the resonance region only and to couple the solutions in that region to the simple cold-plasma solutions outside the resonance zone. This approach, employing the one-dimensional mode-conversion equation of Swanson [9,10,12], has been studied by Phillips *et al.* [20] and Brambilla *et al.* [16]. The technique may become invalid with increasing temperature as the resonance region widens, thereby permitting diffraction within the region. Further difficulty may come from matching solutions between regions, especially with regard to boundary conditions on the additional mode. An alternate approach has been suggested by Cairns and Lashmore-Davies [17,18], who claim that the one-dimensional (1-D) fourth-order mode conversion equation can be contracted into two second-order equations. Whether such a procedure can be accomplished in three dimensions is uncertain. The model presented in this paper will form

precisely such a contraction to maintain a second-order equation, although it is implemented by way of heuristic arguments based on results of 1-D mode conversion studies compared with a numerically constructed wave operator.

In Sec. II we outline the derivation of the mode conversion equations including finite Larmor radius effects which are commonly employed in mode conversion studies. Section III introduces the model equations, which result from approximation and contraction of the higher-order wave equation presented in Sec. II. The computational formulation of these model equations for cylindrical geometry is the subject of Sec. IV. Finally, Sec. V offers results from computer studies involving the model. A density parameter study is investigated in detail, in which antenna loading resonances caused by geometry-related mode structure are observed.

2. WAVE PROPAGATION IN AN INHOMOGENEOUS PLASMA

The dielectric response in the case of a nonuniform plasma involves an integral operator, strictly speaking, a convolution over wavenumbers or position. Moreover, the rf conductivity operator is based on the unperturbed orbits in the nonuniform fields as well as on the spatial nonuniformities of the distribution function. In the literature, the formal wave equations have been derived either in variational form [15,21] for the electric field in Fourier space, or by direct integration of the characteristics of the linearized Maxwell-Vlasov equation using the electric field in position space [16]. To obtain a tractable set of wave propagation equations, the conductivity operator is usually expanded in the small Larmor radius limit as well as ordered with respect to the assumed small spatially dependent particle drifts. The inclusion of first-order Larmor radius effects in any of the commonly employed orderings leads to the appearance of a Bernstein wave which can couple to the cold-plasma waves through the well-known mode conversion process.

The variational approach of Ref. 15 results in the following wave equation, which we write in mixed Fourier/position notation:

$$\nabla \times \nabla \times \underline{E}(\underline{r}) - k_0^2 \int d^3k e^{i\mathbf{k} \cdot \underline{r}} \underline{K}(\underline{r}, \underline{k}) \cdot \underline{E}(\underline{k}) = i\omega \mu_0 \underline{J}_f(\underline{r}) \quad (1)$$

$\underline{J}_f(\underline{r})$ is the free current (antenna and wall) and k_0 is the vacuum wavenumber. All quantities are assumed to vary as $e^{-i\omega t}$. The dielectric kernel $\underline{K}(\underline{r}, \underline{k})$ is calculated from the familiar integration along the unperturbed orbits (in nonuniform fields), and its explicit position dependence results from the different plasma parameter values at the particles' endpoints, as well as the changing parameter values along the unperturbed trajectories. Equation (1) is a very convenient form since the homogeneous limit $\underline{K}(\underline{k})$ and cold-plasma limit $\underline{K}(\underline{r})$ are easily retrieved. Moreover, the finite Larmor radius expansion is readily incorporated in the variational formalism using the familiar uniform plasma form of the dielectric kernel for fixed $k_\perp$:

$$\underline{K}(\underline{r}, \underline{k}) = \underline{K}^{(0)}(\underline{r}) + \underline{K}^{(1)}(\underline{r}) k_\perp^2 \quad (2)$$

When substituted into Eq. (1), the position dependent coefficients can be taken outside the integral over $\underline{k}$, so that the correspondence $k_\perp^2 \rightarrow -v_\perp^2$ is unambiguous. This results in the differential equation

$$\nabla \times \nabla \times \underline{E}(\underline{r}) - k_0^2 \underline{K}^{(0)}(\underline{r}) \cdot \underline{E}(\underline{r}) + k_0^2 \underline{K}^{(1)}(\underline{r}) \cdot \nabla_\perp^2 \underline{E}(\underline{r}) = i\omega\mu_c \underline{J}_f(\underline{r}) \quad (3)$$

A more careful treatment of the dielectric kernel by several authors [10,13,15] shows that the kernel of Eq. (1) becomes valid with the addition of a term of order $\rho_L \nabla B/B$, which is due to a correction of the cyclotron frequency to the particle's guiding center. This term may be written as:

$$\begin{bmatrix} 1 & \pm i & 0 \\ \pm i & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} (-i\nabla_\perp \cdot \underline{K}^{(1)}(\underline{r})) \cdot \underline{k}_\perp \quad (4)$$

where the top sign is chosen for the term of $\underline{K}^{(1)}$ involving $Z[(\omega-2\Omega)/|k_\parallel|v_{th}]$ and the bottom sign for the term involving $Z[(\omega+2\Omega)/|k_\parallel|v_{th}]$. Further discussion of this odd order in the $\underline{k}_\perp$ term appears in the next section.

3. MODEL EQUATIONS

We turn now to the subject of transforming the theory of the last section to a feasible calculation. Several simplifications and approximations will be

made, based on the propagation characteristics specific to the ICRF and on the need to maintain a reasonably tractable (and programmable) problem. The primary objective will be a computer code which can provide high-resolution full-field solutions of wave Eq. (1), such that local power absorption by the plasma can be examined. Wave propagation characteristics and global antenna loading properties are also of interest. The full-field solution approach, as opposed to a ray tracing model, for example, also permits the analysis of small, low density systems.

The second goal of the model will be the use of the warm-plasma kernel, such that plasma temperature and temperature profile effects can be studied. As is evident from the previous section, collisions have been completely ignored. As an aside, it should be mentioned that the addition of collision frequencies to account for various interparticle collisions would be an easily implementable modification [22].

The first simplification of wave Eq. (1) involves the assumption of a cylindrical coordinate system, with plasma parameters uniform along the axial direction. The magnetic field is also assumed to be axial, with no shear, and to have gradients in the poloidal plane only. While no such simplification is necessary, the resulting system will still describe the essential character of the fast magnetosonic wave propagation and will greatly simplify the analysis. The extension of the method to flux coordinates with sheared magnetic fields will be left for a later work. With such a geometry the K_{zz} element of the dielectric kernel is order m_i/m_e greater in magnitude than the purely transverse elements in the ICRF frequency range. Such a term forces E_z , the parallel electric field, to be order m_e/m_i smaller than E_1 . Thus, from this point on, E_z shall be neglected. This approximation has been widely used by others [14-18,20], and it appears to be quite sound, especially in the case of the fast magnetosonic wave propagating in a plasma with no shear. An exception to this rule has been noted by Colestock and Kashuba [15] for the weakly damped Bernstein-wave branch.

The dielectric kernel has now been effectively reduced to the upper left 2×2 matrix involving the purely transverse elements. In addition, the removal of z dependence in the kernel allows the related convolution over k_z to be removed from Eq. (1). Thus, the wave equation separates into a series of two-dimensional problems governed by

$$\begin{aligned}
 & (\nabla_1 + i k_z \hat{z}) \times (\nabla_1 + i k_z \hat{z}) \times \underline{E}(\underline{r}_1, k_z) = k_0^2 \int d^2 k_\perp e^{i \underline{k}_\perp \cdot \underline{r}_1} \underline{K}(\underline{r}_1, \underline{k}_\perp, k_z) \cdot \underline{E}(\underline{K}) \\
 & = i \omega \mu_0 \underline{J}_F(\underline{r}_1, k_z) \quad . \quad (5)
 \end{aligned}$$

Further progress can be made by the use of the finite Larmor radius expansion. The kernel, $\underline{K}(\underline{r}_1, \underline{k}_\perp, k_z)$, is to be expanded in terms of the parameter $\lambda = k_\perp^2 v_{th}^2 / 2\Omega^2$. An estimate of the magnitude of $k_\perp$ for fast-wave heating can be found from the Alfvén velocity according to $k_\perp^2 \approx \omega^2 / v_A^2$, where $v_A^2 = B_0^2 / \mu_0 \rho_m$. Taking $\omega \approx 2\Omega$, one finds:

$$\lambda \approx 10 \frac{n(10^{14} \text{ cm}^{-3}) T(\text{keV})}{B^2(\text{kGauss})^2} \quad (6)$$

or roughly twice the plasma beta. For conditions on TFTR, λ is of order 1/10 so that the expansion is valid, but perhaps not overwhelmingly so. It seems generally acceptable for the fast wave, however, its reliability for the Bernstein wave, which may have considerably shorter wavelength, is questionable.

Let us proceed now to formulate the elements of the dielectric kernel. By assuming that ion species #1 sees a second harmonic resonance in the plasma, while the ion species normally in minority sees its fundamental frequency, the usual uniform plasma dielectric tensor can be expanded as follows:

$$\underline{K}(\underline{k}) = \begin{bmatrix} K_\perp & iK_x \\ -iK_x & K_\perp \end{bmatrix} \quad \begin{aligned} K_\perp &= K_\perp^{(0)} + n_\perp^2 K_\perp^{(1)} \\ K_x &= K_x^{(0)} + n_\perp^2 K_x^{(1)} \end{aligned}, \quad n_\perp^2 = k_\perp^2 / k_0^2, \quad (7)$$

where

$$K_\perp^{(0)} = 1 + \frac{\omega_{pe}^2}{\omega^2} + \sum_{\text{ions}} \frac{\omega_{pi}^2}{2\omega |k_z| v_{thi}} \left[Z\left(\frac{\omega - \Omega_i}{|k_z| v_{thi}}\right) + Z\left(\frac{\omega + \Omega_i}{|k_z| v_{thi}}\right) \right],$$

$$K_\perp^{(1)} = \frac{k_0^2 v_{thi} \omega_{pi}^2}{4\omega |k_z| \Omega_i^2} \left[Z\left(\frac{\omega - 2\Omega_i}{|k_z| v_{thi}}\right) + Z\left(\frac{\omega + 2\Omega_i}{|k_z| v_{thi}}\right) \right],$$

$$K_x^{(0)} = -\frac{\omega_{pe}^2}{\omega \Omega_e} + \sum_{\text{ions}} \frac{\omega_{pi}^2}{2\omega |k_z| v_{thi}} \left[Z\left(\frac{\omega - \Omega_i}{|k_z| v_{thi}}\right) - Z\left(\frac{\omega + \Omega_i}{|k_z| v_{thi}}\right) \right],$$

and

$$K_x^{(1)} = \frac{k_o^2 v_{th1} \omega_{p1}^2}{4\omega |k_z| n_1^2} \left[Z\left(\frac{\omega - 2\Omega_1}{|k_z| v_{th1}}\right) - Z\left(\frac{\omega + 2\Omega_1}{|k_z| v_{th1}}\right) \right].$$

Z is the plasma dispersion function. The nonuniform plasma dielectric kernel, $\underline{K}(\underline{r}_\perp, \underline{k}_\perp, k_z)$, will contain additional variation due to (a) guiding center correction to the cyclotron frequency (the so-called odd-order derivative terms), (b) ∇B and curvature drifts, and (c) nonuniform distribution function effects (essentially the pressure drift). The net effect of the drifts is negligible, given their small magnitudes; however, the odd-order derivative terms are formally of the same order as the first order λ terms, and they are known to be important to global energy conservation [12,13,15,16,19].

On the other hand, one-dimensional mode conversion studies in the ICRF have shown that the fast wave scattering coefficients with and without the odd-order term show remarkable agreement [15]. There is perhaps a good reason for this. The odd-order derivative term is order $k_\perp \rho_L / (k_\parallel L_B) \approx \rho_L / L_B$, where L_B is the characteristic scale length of the magnetic inhomogeneity. This quantity is of the same order as λ if we assume $1/(k_\parallel L_B)$ is the first order similar to $k_\perp \rho_L$. However, the typical gradient in large tokamaks is relatively weak, so that $\rho_L / L_B \approx 1/100$, or only 10% of the first order λ term. This may explain its seeming insignificance in terms of the actual calculations. For this model, the odd-order derivative terms will be ignored. In this simplified case, the fourth-order wave equation may be written as

$$\nabla_\perp \times \nabla_\perp \times \underline{E} = k_o^2 \left(\underline{K}^{(0)} - n_z^2 \underline{I} \right) \cdot \underline{E} - \underline{K}^{(1)} \cdot \nabla_\perp^2 \underline{E} + i\omega \mu_o \underline{J}_F. \quad (8)$$

Equation (8) would be a legitimate starting point for a propagation code, since it is, in principle, well-posed and solvable numerically. However, certain pathologies still remain, namely, that Eq. (8) is a fourth-order system in the radial derivative because of the $\nabla_\perp^2 \underline{E}$ term on the right-hand side. In regions where the dielectric coefficients are slowly varying, a local dispersion relation can be formed to find the respective wavenumbers for a WKB analysis. Two distinct roots appear, given asymptotically by

$$n_{\perp}^2 = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A} \approx \frac{(K_{\perp}^{(0)} - n_z^2) - K_x^{(0)2}}{K_{\perp}^{(0)} - n_z^2}, - \frac{K_{\perp}^{(0)} - n_z^2}{K_{\perp}^{(1)}}, \quad (9)$$

where

$$\begin{aligned} A &= K_{\perp}^{(1)} - K_{\perp}^{(1)2} + K_x^{(1)2}, \\ B &= (K_{\perp}^{(0)} - n_z^2) (1 - 2K_{\perp}^{(1)}) + 2K_x^{(0)} K_x^{(1)}, \\ C &= -(K_{\perp}^{(0)} - n_z^2)^2 + K_x^{(0)2}. \end{aligned} \quad (10)$$

These roots correspond to the warm-plasma analog of the fast magnetosonic wave and to the Bernstein wave. Under most circumstances the Bernstein root from Eq. (9) is large in the asymptotic region, corresponding to a very short wavelength (on the order of ρ_L). Furthermore, the Bernstein root of the fourth-order dispersion relation usually indicates strong damping at the second harmonic resonance and is subject to strong Landau damping. For these reasons, it is likely that the power which mode converts to the Bernstein branch propagates only a short distance before being damped. This being the case, then it may not be necessary to maintain in the solution of Eq. (8) as wide a spectrum in $k_{\perp}$ as would be indicated from typical values of Eq. (9). Such a "contraction" of the required spectrum would be a tremendous advantage in terms of programming considerations.

For our model, we are motivated by this argument to construct a "contracted" wave equation which is only second order and which eliminates the large $k_{\perp}$ spectrum of the Bernstein branch in the asymptotic region. An essential element of the contraction process is the wave operator, which must contain the correct characteristics of fast-wave propagation in the cold-plasma limit as well as preserve the character of the cyclotron resonance-mode conversion zone. We assert that if a second-order operator can be constructed that accurately represents the fast-wave scattering coefficients (reflection and transmission), then it may be used to find the global fast-wave solution without the introduction of a phenomenological collision frequency to remove the singularity at resonance.

Following the heuristic arguments made in the lossless mode conversion analysis by Cairns and Lashmore-Davies [17,18], we note that the mode

conversion tunneling parameter, which determines the fast-wave scattering coefficients, depends only on the width of the mode conversion layer, as shown in Fig. 1. We may construct a second-order operator which preserves this distance by simply dropping the Bernstein wave branch, without altering the fast wave branch. Such a procedure does not have a simple analytic representation, but is trivially accomplished numerically. In addition, it is a simple matter to include damping, although the correspondence of the scattering coefficients to those of the fully lossy mode conversion process has yet to be established. We shall not attempt a proof of that correspondence in the manner of Ref. 17 here, rather we present the results of a direct numerical integration of this second-order operator in comparison to the results of the full fourth-order mode conversion operator [15].

The scattering coefficients for the second-order operator may be found by integrating along the fast-wave branch with the boundary conditions associated with either high-field incidence, a wave of unity amplitude approaching from the high-field side with an outgoing wave on the low-field side, or low-field incidence, a wave approaching from the low-field side resulting in both a transmitted and a reflected wave. A scalar form of the resulting second-order differential equation may be written simply as

$$\frac{d^2}{dx^2} \psi + n_1^2(x) \psi = 0, \quad (11)$$

where n_1^2 is the numerically extracted WKB index of refraction as described above. If we denote the transfer matrix T as the transformation which carries the vector (ψ, ψ') from, say, the high-field side to the low-field side, then the scattering coefficients are given by

Low-Field Incidence

$$R_L = \frac{T_{12}}{T_{22}},$$

$$T_L = \frac{1}{T_{22}},$$

High-Field Incidence

$$R_H = -\frac{T_{21}}{T_{22}},$$

$$T_H = T_{11} - \frac{T_{21}T_{12}}{T_{22}}. \quad (12)$$

Evaluating these coefficients for parameters of experimental relevance and comparing them to the corresponding coefficients from the full fourth-order wave equations reveals a surprisingly good agreement, even with the

inclusion of damping, as shown in Fig. 2. This agreement persists over a wide range of densities and $k_{\parallel}$ values, covering the full range of the tunneling parameter expected in experimental conditions. We suggest that this numerical proof, although nonrigorous, is sufficient to qualify the contracted second-order operator as an adequate representation of the mode conversion zone.

It is worthwhile to note, that while the fast wave may be represented by this technique, the Bernstein-wave fields are, of course, lost in the contraction. The result is that the anti-Hermitian part of the dielectric tensor represents not only true damping but also conversion to the Bernstein wave. The details of the damping among the various loss mechanisms are also lost, and they cannot directly be determined from this calculation. Nonetheless, the accuracy of the fast-wave scattering coefficients ensures that the correct amount of power is lost from the fast wave, by whatever means. It is also noteworthy that Eq. (11) can be readily generalized to a two-dimensional operator, owing to the absence of spatial gradient terms.

Hence, we postulate a wave equation for the fast wave only by selecting the fast-wave root from the fourth-order dispersion relation, that is, letting

$$v^2(\underline{r}_1, k_z) = n_1^2 \Big|_{\text{Local Fast Wave}} \quad (13)$$

and, in the right-hand side of wave Eq. (8), make the substitution

$$v_1^2 \underline{E} + -k_z^2 v^2 \underline{E} . \quad (14)$$

In practice, one may define the "fast-wave dielectric tensor" $K_{FW}(\underline{r}_1, k_z)$ as

$$K_{FW} = K^{(0)} - n_z^2 \underline{I} + v^2 K^{(1)} , \quad (15)$$

so that the final form of the wave equation becomes the second-order equation

$$\nabla_1 \times \nabla_1 \times \underline{E} = k_0^2 K_{FW} \cdot \underline{E} + i\omega\mu_0 \underline{J}_f \quad (16)$$

whose local dispersion relation then yields the same fast-wave roots as the fourth-order system.

In this model, the energy conservation law (Poynting's Theorem) can be formulated by making the correspondence of $\underline{K}_{FW} \cdot \underline{E}$ to the electric displacement, $\underline{D}$. Then, the general plasma impedance formula is

$$Z_d^* = \frac{-i\omega\epsilon_0}{|I|^2} \int_{A_1} r dr d\theta \int_{-\infty}^{+\infty} dk_z (\underline{E}^* \cdot \underline{K}_{FW} \cdot \underline{E} - \frac{\mu_0}{\epsilon_0} |H_z|^2) \quad (17)$$

The free current is zero, and reflection at the vessel wall prevents energy from leaving the system. Note that the quantity

$$P_d = \frac{\omega\epsilon_0}{|I|^2} \int_{-\infty}^{+\infty} dK_z \text{Im}(\underline{E}^* \cdot \underline{K}_{FW} \cdot \underline{E}) \quad (18)$$

can be interpreted as the z-integrated local power deposition.

4. COMPUTATIONAL FORMULATION

A computer code has been written which solves Eq. (16) and which diagnoses power absorption via Eqs. (17) and (18). In this section we discuss the techniques used by the code, as well as the application of boundary conditions and selection of the proper dispersion equation root.

Separating the wave Eq. (16) into Faraday's and Ampere's Law again, we find

$$\nabla_{\perp} \times \underline{E}(\underline{r}_1, k_z) = i\omega\mu_0 \hat{z} H_z(\underline{r}_1, k_z) \quad (19)$$

and

$$\nabla_{\perp} \times \hat{z} H_z(\underline{r}_1, k_z) = -i\omega\epsilon_0 \underline{K}_{FW}(\underline{r}_1, k_z) \cdot \underline{E}(\underline{r}_1, k_z) + \underline{J}_F(\underline{r}_1, k_z) \quad (20)$$

Since E_z is negligible, these equations can be converted into two coupled differential equations for E_{θ} and H_z and an explicit equation for E_r . Taking $J_F = 0$ one finds, after some manipulation,

$$\frac{\partial}{\partial r} r E_{\theta} = \frac{\partial}{\partial \theta} \left(-i \frac{K_x}{K_1} E_{\theta} + \frac{1}{K_1} \frac{1}{r} \frac{\partial}{\partial \theta} \frac{i\omega\mu_0}{k_0^2} H_z \right) + K_0^2 r \frac{i\omega\mu_0}{k_0^2} H_z ,$$

$$\frac{\partial}{\partial r} H_z = -\frac{k_0^2}{i\omega\mu_0} v^2 E_\theta + i \frac{K_x}{K_\perp} \frac{1}{r} \frac{\partial}{\partial \theta} H_z, \text{ and}$$

$$E_r = -i \frac{K_x}{K_\perp} E_\theta + \frac{1}{K_\perp} \frac{1}{r} \frac{\partial}{\partial \theta} \frac{i\omega\mu_0}{k_0^2} H_z. \quad (21)$$

Here $K_\perp$ and K_x are elements of $\underline{K}_{FW}$, and v^2 is the fast-wave root used to find $\underline{K}_{FW}$. Note the relationship

$$v^2 = K_\perp - \frac{K_x^2}{K_\perp}. \quad (22)$$

Next we proceed to expand in a poloidal Fourier series. Defining the following Fourier coefficients, we have

$$\begin{aligned} rE_\theta &= \sum_{m=-\infty}^{+\infty} F_m e^{im\theta}, & \frac{i\omega\mu_0}{k_0^2} H_z &= \sum_{m=-\infty}^{+\infty} H_m e^{im\theta} \\ \frac{1}{K_\perp} &= \sum_{m=-\infty}^{+\infty} d_m e^{im\theta}, & -i \frac{K_x}{K_\perp} &= \sum_{m=-\infty}^{+\infty} c_m e^{im\theta}, & v^2 &= \sum_{m=-\infty}^{+\infty} f_m e^{im\theta}. \end{aligned} \quad (23)$$

F_m , H_m , d_m , c_m , and f_m are all functions of r and k_z . We substitute these forms into Eq. (21) and isolate each poloidal component to give the following infinite system:

$$\frac{\partial F_m}{\partial r} = k_0^2 r H_m + \frac{im}{r} \sum_n c_{m-n} F_n - \frac{m}{r} \sum_n n d_{m-n} H_n$$

and

$$\frac{\partial H_m}{\partial r} = -\frac{1}{r} \sum_n f_{m-n} F_n - \frac{i}{r} \sum_n n c_{m-n} H_n. \quad (24)$$

This is easily put into matrix form, the final form solved numerically. The above system is truncated at a finite value of m . This value need not be too large because field variation is usually a weak function of θ . We have found that for cases of experimental interest, good accuracy ($\approx 1\%$) may be obtained by keeping poloidal mode numbers up to only $m = \pm 6$.

Three boundaries are present: the origin, the antenna, and the conducting vacuum vessel wall. The origin in cylindrical coordinates requires careful consideration to be able to avoid numerical error as r approaches

zero. From fundamental considerations of the Fourier transform one expects the scalar quantities d_m , c_m , and f_m to behave near the origin like $r^{|m|} + O(r^{|m|+2})$. This conclusion and the finiteness requirement are then enough to demonstrate that, near the origin, the fields F_m and H_m also behave like $r^{|m|}$, with the exception of F_0 , which behaves like r^2 . Assuming these behaviors in Equation (24), one discovers the following systems relating F_m 's and H_m 's near zero:

$$F_0 = \frac{1}{2} k_0^2 H_0 r^2 ,$$

$$(1 - ic_0)F_N = -Nd_0 H_N + \sum_{n=1}^{N-1} (ic_{N-n} F_n - nd_{N-n} H_n) ,$$

$$(1 + ic_0)F_{-N} = -Nd_0 H_{-N} + \sum_{n=1}^{N-1} (-ic_{-N+n} F_{-n} - nd_{-N+n} H_{-n}) , \quad N > 0 , \quad (25)$$

or equivalently:

$$N(1 + ic_0)H_N = -f_0 H_N - \sum_{n=1}^{N-1} (f_{N-n} F_n + inC_{N-n} H_n) ,$$

$$N(1 - ic_0)H_{-N} = -f_0 H_{-N} - \sum_{n=1}^{N-1} (f_{-N+n} F_{-n} - inC_{-N+n} H_{-n}) , \quad N > 0 . \quad (26)$$

The above cascaded systems can be used to define a matrix D , which states the boundary condition in column vector form:

$$\underline{F}_m(r, k_z) = \underline{D}(r, k_z) \cdot \underline{H}_m(r, k_z) \quad \text{for small } r . \quad (27)$$

In practice, the fields are never explicitly calculated at the origin in the code, but the computation is started at a small r using the above relation as a starting condition. In order to estimate how far to step off the origin, the variation scale length can be estimated to be roughly $2/k_0 |f_0|^{1/2}$.

The antenna boundary condition assumes a current density of form

$$\underline{J}_f(\underline{r}) = -IK(\theta) p(z) \delta(r - r_{\text{ant}}) \hat{\theta} . \quad (28)$$

This is a sheet current of strength I at radius r_{ant} , with arbitrary θ and z profiles. The value of $K(\theta)$ at the current feedpoints must be unity, the area under $p(z)$ must be unity. Transforms of K and p are required. Let

$$K_m = \frac{1}{2\pi} \int_0^{2\pi} d\theta K(\theta) e^{-im\theta}$$

and

$$p(k_z) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} dz p(z) e^{-ik_z z} \quad (29)$$

The H_z field must jump by an amount $IK(\theta)p(z)$ at the antenna radius. Thus, the antenna boundary condition is (take $I = 1$)

$$H_m(r_{\text{ant}}^+, k_z) = H_m(r_{\text{ant}}^-, k_z) + \frac{i\omega\mu_0}{k_0} K_m p(k_z)$$

and

$$F_m(r_{\text{ant}}^+, k_z) = F_m(r_{\text{ant}}^-, k_z) \quad (30)$$

The conducting outer shell boundary condition is simplest; namely E_θ must vanish:

$$F_m(r_{\text{wall}}, k_z) = 0 \quad (31)$$

Summarizing Eqs. (26), (30), and (31), there are two homogeneous conditions, each on half the variable quantities, at either end of the radial interval and one inhomogeneous condition at r_{ant} .

The radial integration problem is ideally suited for a generalized linear shooting technique. Let k_z be fixed. First, one integrates a series of independent initial value problems from origin to outer wall, assuming no jump condition at r_{ant} . At the origin, each integration of this series has the first coefficient of some H_m set to unity with the rest of the H_m zero and E_m coefficients given by Eq. (26). Next, one integrates an initial value problem from r_{ant} to the wall, this time with the jump condition and with zero fields inside the antenna radius. Then by simple matrix inversion a linear combination of all these solutions is found which satisfies the boundary condition in Eq. (31). Since in this procedure one essentially solves a multitude of initial value problems, a variable step size integrator can be used, rather than finite difference. This provides very good accuracy with the ability to estimate and control the error tolerances during the actual

run. In addition, since the knowledge of the fields is required only at the given radius being calculated during the shooting method process, the required memory storage is much less than with comparable finite difference [2] or finite element [1,22] approaches, even with arbitrarily fine radial resolution.

A solution as described above must be done for many values of k_z , so the procedure is repeated 30 to 40 times in a single run. Some simplification is possible using the symmetry relation $\underline{\epsilon}_{FW}(k_z) = \underline{\epsilon}_{FW}(-k_z)$ and, thus

$$\begin{bmatrix} \underline{F}_m(r, -k_z) \\ \underline{H}_m(r, -k_z) \end{bmatrix} = \frac{p(-k_z)}{p(k_z)} \begin{bmatrix} \underline{F}_m(r, k_z) \\ \underline{H}_m(r, k_z) \end{bmatrix}. \quad (32)$$

Another point is that for large k_z the value of $k_\perp$ becomes largely imaginary, giving solutions which essentially fail to propagate in far enough to reach the resonance. Thus, by monitoring the loading spectrum values, which decrease exponentially for large k_z , one can estimate where to stop the k_z integrations. Care must also be taken so that solutions in this exponential regime are resolvable to the error tolerances of the integrator, which is usually taken to be around 10^{-4} or 10^{-5} .

Considerable effort has gone into an efficient and accurate way to provide a smoothly position dependent dielectric tensor $\underline{\epsilon}_{FW}$. Prior to the integrations of the linear shooting technique, the code first calculates, transforms [via a fast Fourier transform (FFT) routine], and stores the quantities $d_m(r)$, $c_m(r)$, and $f_m(r)$ on a fairly fine mesh (75 radial intervals, 72 angular intervals). Then, when the quantities are required at arbitrary radius by the integrator, the values are interpolated from the grid using centered four-point interpolation, except near the origin where the interpolation incorporates the $r^{|m|}$ behavior.

It is also very important, when calculating the dielectric tensor elements, to solve for the correct root of the local dispersion relation, a particularly delicate task in the resonance region. First, the appropriate root is found at the outer most edge of the plasma. It is easily found, since the fast wave root is orders of magnitude smaller than the Bernstein root at that point. Then the code runs through the angular mesh at constant radius,

always picking the root closest to the previous root (continuity). When the angle comes around to 2π , the code steps in one radial mesh and repeats the process. Thus, root selection is determined by spiraling in toward the origin, demanding the computational equivalent of continuity.

Finally, it is necessary to discuss the convergence of the method with regard to the poloidal Fourier series and subsequent truncation. If one assumes that the radial integration leads to negligible error and that all quantities and their derivatives are well behaved, then the convergence in increasing number of modes kept ($-M \leq m \leq +M$) should be geometric for large M . In practice, the required number of modes depends to a great extent on the prevailing physical conditions, including resonance zone thickness, location of resonance with respect to the coordinate origin, and the poloidal spectrum launched by the antenna. In fact, if the latter is not analytic, then the convergence may be algebraic rather than geometric.

Figure 3 shows the calculated antenna loading values of two identical plasmas for increasing poloidal mode range M . In Fig. 3a, the launched spectrum is gaussian

$$K(\theta) = \frac{2}{\sqrt{\pi}} \sum_n e^{-4(\theta - 2\pi n)^2 / \theta_0^2} \rightarrow K_m = \frac{\theta_0}{2\pi} e^{-m^2 \theta_0^2 / 16}, \quad (33)$$

with approximate poloidal extent of $\theta_0 = \pi/2$ (quarter-turn antenna). Beyond $M = 4$, the loading appears to converge geometrically to 19.1Ω (Ohms), with the difference between the value and this final value being roughly proportional to K_m . In Fig. 3b, the launched spectrum is a discontinuous step function

$$K(\theta) = \begin{cases} 1, & |\theta| < \theta_0/2 \\ 0, & |\theta| > \theta_0/2 \end{cases} \rightarrow K_m = \frac{\sin(m\theta_0/2)}{m\pi}. \quad (34)$$

Again, past $M = 4$, the loading appears to oscillate and converge slowly (algebraically) around a value near 22.7Ω . The oscillations are of length $M = 4$, which is intuitively expected from a quarter-turn antenna. However, the difference between the values and final value are proportionally a much smaller fraction of K_m than for the gaussian spectrum. Thus, even in this

case where the high mode number part of the launched spectrum is significant, the antenna loading is still dominated by the lower modes. Notably, we see that the oscillations beyond $M = 4$ change the loading value by less than 5%.

5. RESULTS

In this section, we present results from a computer code which uses the theoretical and computational methods described in the previous sections. Highlighted is a series of runs in which all parameters are maintained constant except peak plasma density, which is varied from 1 to $20 \times 10^{13} \text{ cm}^{-3}$. Also presented are runs typical of various operating conditions which may be of interest, for example, pure second harmonic heating, inside antenna launch, and different locations of the cyclotron resonance.

A typical run, with peak density of $3.3 \times 10^{13} \text{ cm}^{-3}$, is shown in Fig. 4. Figures 4a and 4b show contours of the real parts of E_θ and E_r in the poloidal plane at the z location of the antenna. The cyclotron resonance is at 0.32 meters to the right of the origin (towards outside of tokamak). There are roughly three wavelengths visible between the wall and resonance, with very little power transmitted through to the high-field side of the tokamak. There is excellent focusing, primarily due to antenna shape and the radial density profile. Figure 4c shows contours of z -integrated local power deposition. It is peaked almost exactly where the E contours vanish and is raised slightly from being centered on the midplane ($\theta = 0$). We observe that the strongest absorption occurs at the mode conversion resonance which is a distance $0.75 R (n_H/n_D)^{1/4}$ ($\approx 11 \text{ cm}$) to the left (inside) of the resonance. Figure 4d shows the real part of the loading versus parallel wavenumber curve. It is peaked at $k_z = 2.2 \text{ m}^{-1}$. All low-field launch spectra have a peak such as this, which is generally not located at zero wavenumber. The rapidly decreasing power absorption at large wavenumbers is evident on this plot, as it is for all runs. Figure 4e shows the power deposition at the midplane, viewed from above the torus. We note the contours are peaked to either side of the antenna, a result of the loading spectrum being maximum at nonzero wavenumber. The plasma loading of this case was 22.51Ω .

Shown in Fig. 5 are the fast-wave roots $n_{\perp}^2$, real and imaginary parts, for the above sample at three different values of k_z . Figures 5a and 5b show contours in the low- k_z limit ($k_z = 1 \text{ m}^{-1}$), where the energy loss is primarily associated with mode conversion. At $k_z = 10 \text{ m}^{-1}$, Figs. 5c and 5d, the imaginary part shows a small amount of direct cyclotron damping, as evidenced by the contours which extend to the resonance location. The overall damping is decreased in magnitude though, and the perturbation on the real part from mode conversion is less. Figures 5e and 5f show $n_{\perp}^2$ at $k_z = 15 \text{ m}^{-1}$, the largest value for which the loading was calculated. The fast-wave cutoff is clearly evident, thus the wave reflects before reaching resonance, yielding little energy deposition to the plasma. The imaginary part shows cyclotron damping at large k_z in the lower temperature region near the wall, which can create, in some circumstances, a numerical trouble spot and which may be physically invalid because of the neglect of shear.

5.1 Density variation

The density profile of all runs in the density scan was parabolic in radius, with edge density at 5% the central peak density. The temperature profile was parabolic squared and had an edge temperature of 1% the peak temperature. For all runs the peak temperature was 5 keV. The usual 1/R behavior of magnetic field was used with axis strength of 40 kG. A single quarter-turn antenna with 10 cm width was employed, and flat current profiles in both θ and z were assumed. The hydrogen minority was taken to be 5% ion fraction. The ICRF wave frequency was 55 MHz.

A total of 48 such runs were done with varying plasma density. The antenna loading vs. density curve for these runs are shown in Fig. 6. Up to about $1.7 \times 10^{14} \text{ cm}^{-3}$ the graph shows a series of regularly spaced peaks superimposed on a general linear increase in loading. Comparisons with the E_0 plots show that each loading peak is associated with an increment of one additional half-wavelength between wall and resonance layer. Thus, these peaks are simple geometry-related resonances analogous to the normal modes of a cavity. An empirical fit to the density at each of the loading resonances gives good agreement to the following relationship:

$$n_{\text{peak}} = N^2 \times 0.367 \times 10^{13} \text{ cm}^{-3} \quad (35)$$

where n_{peak} = Density at loading peak and N = Number of wavelengths visible on E_r plot. Such a fit is physically reasonable since the wavelength for the fast wave should go as the reciprocal square root of density.

The general linear increase of the loading between resonances is more difficult to explain. An estimate of the trend is

$$R_V \approx n_V \times 2.25 \times 10^{-13} \text{ } \Omega \text{ cm}^3 \quad (37)$$

where R_V = Loading at valley location and n_V = density at valley location. The height of the resonances appears to vary considerably, however, this is undoubtedly due to the difficulty in resolving them cleanly because of their sharpness.

Along with each of the loading resonances are very regular patterns of variation in the E-field contours and the loading spectra. Examples of this behavior for the loading peak at $1.1 \times 10^{14} \text{ cm}^{-3}$ are shown in Figs. 7-10. At the loading peak, Fig. 8, the E_θ contours are centered about the vessel's midplane, the energy deposition appears predominantly in the mode conversion zone, and the spectral loading is noticeably peaked at zero wavenumber. This indicates a situation where the fast magnetosonic wave is propagating nearly perpendicular to the field, i.e., the extraordinary wave, such that the reflected portion of the wave reinforces itself, resulting in an integral number of wavelengths between wall and cyclotron resonance layer. High-field amplitudes are expected, and this is what is observed.

At a lower density, about halfway up the loading peak, Fig. 7, the E_θ contours are no longer symmetrical, instead they are pushed above the symmetry plane. This figure betrays a fundamental asymmetry of the system when the propagating wave is right elliptically polarized as is the fast wave. Specifically, equal change in the phases of a wave front launched above the symmetry plane and a wave front launched below the symmetry plane will result in different propagation phase shifts and different reflections at the cyclotron resonance layer. Hence the skewed E_θ contours. This effect is strongest for the $k_z = 0$ part of the spectrum, which is shown by the depressed spectral loading for low wavenumber as compared to Fig. 8. For a density slightly beyond the loading peak, Fig. 9, the situation is similar, except that the asymmetry is reversed.

Finally, Fig. 10 shows the situation when the density is in a loading valley. Here the E_0 contours are symmetric again, but the extraordinary wave ($k_z = 0$) is strongly interfering, as is evident from the low spectral loading for zero wavenumber. The power deposition contours also show less significant mode conversion, as a result of the lower field amplitudes at this point. Note that the degree to which the asymmetry appears and to which the spectral loading is depressed is a good indication that there is significant reflection from the cyclotron/mode conversion layer. An estimate of the fraction of reflected and absorbed powers is the relative height and width of the loading resonances in comparison to the loading valleys. If there is little reflection and near pure absorption, then one would expect geometrical influences to be less significant, and a smooth loading versus density curve should result. On the other hand, if there is almost no absorption, then the geometrical factors should produce very sharp pronounced loading resonances. Judging from the actual curve in Fig. 6, it appears that both reflection and absorption are significant in the parameter regimes of this density scan.

Lastly, a word about the rise in loading beyond the density of $1.7 \times 10^{14} \text{ cm}^{-3}$. This seems to result from a defocusing of the waves inside the mode conversion layer at k_z near 8 m^{-1} , since the power deposition profiles show two islands of strong absorption about three fourths the minor radius out from the origin (see Fig. 11). Unfortunately, the solutions for these cases had appreciable $m = \pm 6$ components, the limit of our angular resolution, and the loading is therefore not trustworthy in a quantitative sense.

5.2 Low temperature and temperature variation results

In Fig. 12, we show a case with the relatively low peak temperature of 500 eV, at a density of $6 \times 10^{13} \text{ cm}^{-3}$, and remaining parameters identical to those of the last section. The most obvious effect of the low temperatures is a much thinner mode conversion zone, as is readily apparent on the power deposition contours.

However, the thinner mode conversion zones also produce a subtler effect, namely, the sharpening of all geometry-related resonance structure. As a result, the spectral loading is no longer smooth, but instead shows dramatic resonances which pinpoint the k_z values in which constructive interference occurs on the mode conversion layer. As verification, we note that the first

peak of Fig. 12c occurs in precisely the same location as the maximum for the higher temperature case of the previous section. Thus, as the temperature is raised these resonance shorten and widen until they become indiscernible from the background. Accompanying the sharper spectral loading patterns are changes in the midplane deposition contours, as shown in Fig. 12d.

Despite this remarkable transformation of the spectral loading, we find that the global antenna loading (which is simply the area under the spectral loading curve) does not seem to depend significantly on temperature. In fact, over the range 200 eV to 1800 eV, the global loading changed by less than 10% (see Fig. 13). This property is analogous to the findings of Ref. 1, in which the global loading was independent of the value chosen for the phenomenological collision frequency.

5.3 Pure deuterium plasma and minority concentration results

Figure 14 shows a run in which there was no minority hydrogen present, density was 10^{14} cm^{-3} , temperature was 5 keV, and remaining parameters were unchanged. The E_0 plot shows significant transmission, in contrast to the preceding minority cases. The spectral loading shows a considerable amount of structure which we clearly failed to resolve adequately; in addition, the E_0 plot shows noticeable $m = \pm 6$ variations. Therefore, this particular run, while useful for illustrative purposes, is suspect from a numerical viewpoint.

Changes in minority concentration were found, like temperature variations, to have little effect on the global loading value. Figure 15 shows a series of runs at 10^{14} cm^{-3} and 5 keV where minority concentration is varied. Global loading is altered by about 20% through the range 0-8%, although, from study of the associated E_0 and spectral loading plots, it was likely that much of this was due to the accompanying small changes in the fast-wave's wavelength. In any case, it seems obvious that temperature and minority concentration variations are overshadowed by the geometry effects detailed in the density scan section.

5.4 Inside launch results

An alternate launching scheme for fast-wave heating is to place the antenna on the inside (high-field side) of the torus. Mode conversion and energy transfer to the plasma are altered because of the reversed order in

which a wave encounters the mode conversion resonance and cutoff. Transmission should be identical to the low-field launch [15] (i.e., very little in TFTR-like conditions), while reflection is reduced in favor of mode conversion.

Figure 16 shows an inside launch at $n = 10^{14} \text{ cm}^{-3}$, $T = 5 \text{ keV}$, $\eta_H = 5\%$, and a frequency of 65 MHz. The field patterns appear much like the nonresonant background runs of the outside launch density scan, however, the spectral loading is invariably peaked at zero wavenumber, indicating strong mode conversion. Indeed, the power deposition contours outline the mode conversion layer, and there is no sign of direct cyclotron damping to the right of this layer as with the outside launch. This is indicative of a situation where there is little reflection and, thus, no constructive interference to create a resonance.

Also notice that the loading spectrum drops off at much lower k_z than for the low-field launch, a simple result of the fast-wave cutoff which occurs inside the device on the low-field side at lower k_z (see, for example, Figs. 5c and 5e).

5.5 Focusing

Probably the area which permits the greatest flexibility towards utilizing the strong geometrical influences described in this paper is the location, shape, and design of the launching antenna. Strong localization of the absorption can be achieved where wavefronts converge in the cylindrical geometry. On the other hand, greater spatial spread in heating can be obtained by avoiding such convergence. As the wavelength to minor radius ratio decreases, refraction and focusing of the waves play an ever more important role. We have noticed that wavefronts from the simple circular arc antennas of this model converge noticeably towards the low-field side, a simple consequence of Snell's law. (A wavefront bends toward the region of high n_1 , see Fig. 5.) Evidence of this is clearly seen in Fig. 17, a low-field launch at 65 MHz which is defocused, and Fig. 18, a high-field launch at 55 MHz which is focused.

6. CONCLUSION

In this paper, we have described a three-dimensional ICRF wave propagation and heating model which encompasses the warm-plasma wave physics associated with the second harmonic resonance and mode conversion in a manner that remains within the grasp of present-day computational capabilities. This has been accomplished primarily via contraction of a fourth-order (in radial coordinate) equation of finite Larmor origin to a second-order equation which preserves the effective tunneling parameters.

The method of this paper was selected for its excellent error control and estimation qualities, which were necessary to resolve and quantify the many resonance-like situations. The method's major drawback was its rapid cost escalation with increasing poloidal mode range, which is formally order M^3 , while the memory requirements go as M^2 . Difficulties occurred when the mode conversion zone was placed exactly at the coordinate origin, when localized power deposition occurred near the walls, and when transmission allowed significant field structure on the low-field side; all three involve circumstances when poloidal variation was important.

The most interesting finding of our work is the confirmation of strong interaction between antenna and mode conversion zone through three-dimensional normal mode-like field behavior which occurs during low-field launch where reflection is possible. For fusion temperatures, the k_z influence over these modes is weak, hence, they are not cavity modes in the usual sense (perhaps better described as propagation in a lossy waveguide). Yet the axial propagation effects are important because of mode conversion's $k_\parallel$ dependence, thus, noticeable resonances result when density (i.e., wavelength) is varied (unlike a lossy waveguide). The simultaneous coupling of reflection and two-dimensional interference, a predominantly perpendicular propagation effect, to the parallel propagation dependence of the major loss mechanism, is a fundamental aspect absent from previous one-dimensional studies.

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FIGURE CAPTIONS

- Fig. 1 Dispersion relation roots evaluated locally for the case of weak mode conversion near the proton fundamental cyclotron frequency in a D-H plasma. The plasma magnetic field is assumed to vary in the $\hat{x}$ direction as $B = B_0/(1 + x/R_0)$ with $R_0 = 3$ m, $B_T = 50$ kG, $f = 75$ MHz, $n_e = 5 \times 10^{13}$ cm $^{-3}$, $k_{\parallel} = 5$ m $^{-1}$ (const.), $\eta_H = 5\%$, $T_e = T_d = T_H = 5$ keV. The solid and long dash lines represent the real and imaginary parts of the fast-wave root, respectively, while the alternating dash and short dash lines represent the real and imaginary parts of the Bernstein-wave root, respectively. The distance d represents the spacing of the roots and is a measure of the strength of mode conversion. A large value of d implies weak tunneling from the low-field side along with strong mode conversion from the high-field side.
- Fig. 2 Comparison of the fast-wave scattering coefficients (transmitted power T and reflected power R_L from low field incidence) for the truncated mode conversion operator (dashed) with those of the full fourth-order mode conversion solutions (solid) from Ref. [15].
- Fig. 3 (a) Calculated loading (antenna resistance) values for a Gaussian antenna spectrum as a function of maximum poloidal mode number retained, and (b) similar calculation for the spectrum from a step function. The convergence of the calculation, as noted by the approach to a final limiting value of the loading, is much more rapid in the case of a Gaussian spectrum.
- Fig. 4 (a) Real part of E_{θ} and (b) E_r in the plane of the antenna for the case of 5% hydrogen in a deuterium plasma. Density $n_{eo} = 3.3 \times 10^{13}$ cm $^{-3}$; the resonance layer is 0.32 meters to the low-field side of the axis; $R_0 = 3$ m, $a = 1$ m; (c) corresponding power deposition contours; (d) the antenna loading as a function of k_z ; and (e) the loading on the torus midplane. Absorption is strong in this case and leads to strongly localized absorption, both toroidally and poloidally.

Fig. 5 Contour plots of the index of refraction (n_1^2) for the preceding case evaluated locally over the cross section for three different k_z values: (a) $R_e(n_1^2)$, $k_z = 1 \text{ m}^{-1}$; (b) $\text{Im}(n_1^2)$, $k_z = 1 \text{ m}^{-1}$; (c) $R_e(n_1^2)$, $k_z = 10 \text{ m}^{-1}$; (d) $\text{Im}(n_1^2)$, $k_z = 10 \text{ m}^{-1}$; and (e) $R_e(n_1^2)$, $k_z = 15 \text{ m}^{-1}$; (f) $\text{Im}(n_1^2)$, $k_z = 15 \text{ m}^{-1}$. Note the appearance of a sharp peak in n_1^2 at the plasma edge at high k_z and at the mode conversion layer.

Fig. 6 Antenna loading versus density constructed from a series of runs keeping all parameters fixed as in Fig. 4 above. A series of peaks in the loading occur, corresponding to partially standing waves between the mode conversion zone and the wall.

Fig. 7 (a) $R_e(E_\theta)$ contours for the case of low-field excitation with a $\pm 45^\circ$ loop antenna at a density slightly below a loading peak (cf Fig. 6); (b) corresponding power deposition contours; and (c) corresponding antenna loading versus parallel wavenumber k_z . All parameters but density are kept constant in Figs. 7-10.

Fig. 8 (a) $R_e(E_\theta)$ contours, (b) power deposition contours, and (c) corresponding antenna loading versus wavenumber for a case on a loading peak. The field pattern is symmetrized relative to the off-resonant cases, and the depression of loading at low k_z is minimized.

Fig. 9 (a) $R_e(E_\theta)$ contours, (b) power deposition contours, and (c) corresponding antenna loading versus wavenumber for a case slightly above a loading peak. The fields show the same asymmetry as in Fig. 7 but reversed with respect to the midplane.

Fig. 10 (a) $R_e(E_\theta)$ contours, (b) power deposition contours, and (c) corresponding antenna loading versus wavenumber for a case in a loading valley. The fields are symmetric but are peaked away from the midplane while the low- k_z values indicate the largest decrease of these several cases.

Fig. 11 (a) $R_e(E_\theta)$ contours, (b) power deposition contours, and (c) antenna loading versus wavenumber for a high density case. A marked increase in the loading is accompanied by strong absorption islands near the plasma edge. The accuracy of the calculation is suspect, however, due to the appearance of an $m = \pm 6$ component.

Fig. 12 (a) $R_e(E_\theta)$ contours, (b) power deposition contours, (c) antenna loading versus wavenumber, and (d) toroidal projection of the power deposition contours for a low temperature case $T_d = T_H = 500$ eV at $n_{e0} = 6 \times 10^{13}$ cm $^{-3}$. The absorption zone becomes characteristically thinner, and the loading peaks are larger relative to the background. The absorption, even at low temperature, is localized both radially and toroidally.

Fig. 13 Variation of the antenna loading with temperature. The loading is observed to be relatively insensitive to temperature, even though the width and extent of the absorption layer varies markedly.

Fig. 14 (a) $R_e(E_\theta)$ contours, (b) power deposition contours, and (c) antenna loading versus wavenumber for a pure second harmonic deuterium case, $n_H = 0$, $n_{e0} = 10^{14}$ cm $^{-3}$, $T_d = T_e = 5$ keV. The transmission through the absorption zone is larger relative to previous cases giving rise to more structure in the loading.

Fig. 15 Variation of antenna loading with minority hydrogen concentration. The sensitivity of the global loading value on this parameter is weak, even though the reflectivity of the mode conversion at low- k_z values varies strongly (Ref. [15]).

Fig. 16 (a) $R_e(E_\theta)$ contours, (b) power deposition contours, and (c) antenna loading versus wavenumber for the case of inside launch. Density $n_{e0} = 10^{14}$ cm $^{-3}$, $T_d = T_H = 5$ keV, $n_H = 5\%$. The antenna loading is invariably peaked at zero wavenumber, indicating the lack of a reflected wave from the mode conversion zone.

Fig. 17 Effects of focusing. (a) $R_e(E_\theta)$ contours and (b) power deposition contours for a low-field launch case. Some defocusing of the power deposition is observed due to the effect of the toroidal field gradient.

Fig. 18 (a) $R_e(E_\theta)$ contours and (b) power deposition contours for a high-field launch case. Power deposition is strongly focused relative to the low-field launch case.

Fig. 1

86X1308

Fig. 2(a)

Fig. 2(b)

86X1309

Fig. 3(a)

Fig. 3(b)

Fig. 4(a)

Fig. 4(b)

Fig. 4(c)

Fig. 4(d)

Fig. 4(e)

Fig. 5(a)

Fig. 5(b)

Fig. 5(c)

Fig. 5(d)

Fig. 5(e)

Fig. 5(f)

Fig. 6

Fig. 7(a)

Fig. 7(b)

Fig. 7(c)

Fig. 8(a)

Fig. 8(b)

Fig. 8(c)

Fig. 9(a)

Fig. 9(b)

Fig. 9(c)

Fig. 10(a)

Fig. 10(b)

Fig. 10(c)

Fig. 11(a)

Fig. 11(b)

Fig. 11(c)

Fig. 12(a)

Fig. 12(b)

#86X1376

Fig. 12(c)

Fig. 12(d)

Fig. 13

Fig. 14(a)

Fig. 14(b)

Fig. 14(c)

#86X1371

Fig. 15

Fig. 16(a)

Fig. 16(b)

Fig. 16(c)

Fig. 17(a)

Fig. 17(b)

Fig. 18(a)

Fig. 18(2)

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