CONF-890403--1

IDEAL LOW-n AND MERCIER MODE STABILITY BOUNDARIES FOR $\ell = 2$ TORSATRONS*

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CONF-890403--1

DE89 010837

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ABSTRACT. We studied the relationship between the stability properties of ideal low-n internal modes and the three-dimensional (3-D) ideal Mercier criterion for $\ell = 2$ torsatron configurations. For the low-n stability studies, we used the stellarator expansion as implemented in the FAR code. The 3-D Mercier criterion was applied to equilibria calculated with the VMEC code. We found that (1) low-n modes with singular surfaces lying in a Mercier region are, in general, unstable and (2) the critical beta given by the Mercier criterion agrees well with the critical beta for the lowest-n unstable mode. This is verified even in the case of global $n = 1$ modes. Therefore, the 3-D Mercier criterion is a useful guide in mapping the ideal stability beta limits for these torsatron configurations.

INTRODUCTION

Gruber et al. [1] and Merkel et al. [2] compared the unstable regions and equilibrium beta limits for low-n Mercier modes and high-n ballooning modes for a helically symmetric equilibrium. We compared stability limits for ideal low-n modes and Mercier modes for more general equilibria using different techniques.

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*Research sponsored by the Office of Fusion Energy, U.S. Department of Energy, under contract DE-AC05-84OR21400 with Martin Marietta Energy Systems, Inc.

CONFIGURATIONS UNDER STUDY

The vacuum magnetic flux surfaces were generated from a magnetic field produced by currents in axisymmetric coils and $\ell = 2$ helical coils having the winding law $\phi = \phi_0 + \Theta/M$, where ϕ and Θ are the geometric toroidal and poloidal angles. We studied torsatron configurations with different average plasma minor radius a_p and coil minor radius a_c, different numbers of field periods M, and different values of the rotational transform at the axis ι_0 and at the boundary ι_a. These configurations had the following parameters:

M	A_p	p_c	ι_0	ι_a
10	7.6	1.27	0.45	0.95
12	7.8	1.44	0.33	0.98
14	9.2	1.31	0.52	1.45
19	10.0	1.38	0.50	2.3

Here A_p is the aspect ratio of the configuration and $p_c = a_c M/R_0 \ell$ is the coil pitch parameter, with R_0 the major radius.

NUMERICAL METHODS

2-D equilibrium and low-n modes

For the two-dimensional (2-D) equilibria and low-n stability, we used the stellarator expansion as implemented in the FAR code [3]. In this code, a perturbed potential Ξ is expanded as

$$\Xi(\rho,\theta,\zeta,n) = \sum_{mn} \xi_{mn}(\rho)\cos(m\theta + n\zeta) + \sum_{mn} \tau_{mn}(\rho)\sin(m\theta + n\zeta)$$

where n and m denote the toroidal and poloidal mode numbers. In this initial value code, a component with helicity q $(= 1/\iota = m/n)$ is initially perturbed. In this study, we considered n between 1 and 6. We used 800 radial grid points in the numerical calculation, with convergence studies using 200 and 400 points. The growth rate is interpolated to zero radial grid spacing. The higher the value of n, the more localized the mode and the more resolution needed.

3-D equilibrium and Mercier modes

For Mercier modes, only equilibrium quantities are needed. For this part of the study we used the VMEC code [4]. Closed flux surfaces were assumed and expressed in the inverse coordinate representation:

$$R = \sum_{mn} R_{mn}(s)\cos(m\alpha - n\phi)$$

$$Z = \sum_{mn} Z_{mn}(s)\sin(m\alpha - n\phi)$$

Here α is a poloidal-like angle and the flux label s is proportional to the toroidal magnetic flux. In calculating the VMEC equilibrium we used the set of modes $n = (-3M, -2M, -M, M, 2M, 3M)$ and $m = (0,1,2,3,4,5,6)$. A radial grid of 61 points is considered.

The Mercier criterion is a necessary condition for stability that must be evaluated on each flux surface. The ideal stability analysis is based on the energy principle. We write the condition for stability à la Bauer, Betancourt, and Garabedian [5] as $D_M = D_S + D_W + D_I + D_G > 0$, where

$$D_S = \frac{(\chi''\Phi')^2}{4} \quad \text{shear}$$

$$D_W = \left\langle \frac{gB^2}{g^{ss}} \right\rangle P'V'' - (P')^2 \left\langle \frac{g}{B^2} \right\rangle \left\langle \frac{B^2 g}{g^{ss}} \right\rangle \quad \text{well}$$

$$D_I = \left\langle \frac{gB^2}{g^{ss}} \left(\chi'' I' - \chi''\Phi' \frac{\mathbf{J}\cdot\mathbf{B}}{B^2} \right) \right\rangle \quad \text{net currents}$$

$$D_G = -\left\langle \frac{(\mathbf{J}\cdot\mathbf{B})^2}{B^2} \frac{g}{g^{ss}} \right\rangle \left\langle \frac{B^2 g}{g^{ss}} \right\rangle + \left\langle \frac{g\,\mathbf{J}\cdot\mathbf{B}}{g^{ss}} \right\rangle^2 \quad \text{geodesic curvature}$$

with g the Jacobian. The prime indicates the derivative with respect to the flux label s, V' is the magnetic well, I is the toroidal net current enclosed in the flux surface, Φ is the toroidal flux, and χ is the poloidal flux; $g^{ss} = \nabla s \cdot \nabla s$, and the angle brackets indicate the operation $\langle(x)\rangle = \int\int d\alpha \; d\phi \; (x)$.

NUMERICAL RESULTS OF THE STABILITY ANALYSIS

Our results on stability for low-n and Mercier modes for the configurations with $M = 10$, 14, and 19 are presented in Figs. 1–3, which shows the unstable regions and the positions at which the dominant component of the low-n modes is localized (thick lines). The helicities of the most dominant modes are also indicated. Figure 1 shows the results for two flux conserving $M = 10$ configurations, one unshifted and the other with an inward shift in magnetic axis of 2.5% of the major radius. Figure 2 shows the flux-conserving and zero-current modes of operation for the $M = 14$ configuration. Figure 3 shows the unstable regions for the zero-current $M = 19$ torsatron.

CONCLUSIONS

The Mercier criterion usually yields a more pessimistic stability beta limit. However, the unstable low-n modes map out the radial boundaries of the Mercier unstable region quite closely. There is good agreement between the 3-D low-n calculations made using a 2-D equilibrium and the 3-D Mercier stability calculations for the full 3-D equilibrium, even for $\ell = 2$ torsatrons with fairly low aspect ratio. Therefore, the similarity in stability with respect to low-n perturbations and Mercier modes that was found for helically symmetric configurations by Gruber, Merkel, and others still holds for the more general $\ell = 2$ configurations studied here. The practical application of our calculations is that we can perform a stability analysis for the ideal pressure-driven modes by studying either the low-n or the Mercier modes.

ACKNOWLEDGEMENTS

We are indebted to S. P. Hirshman for allowing us to use his VMEC code.

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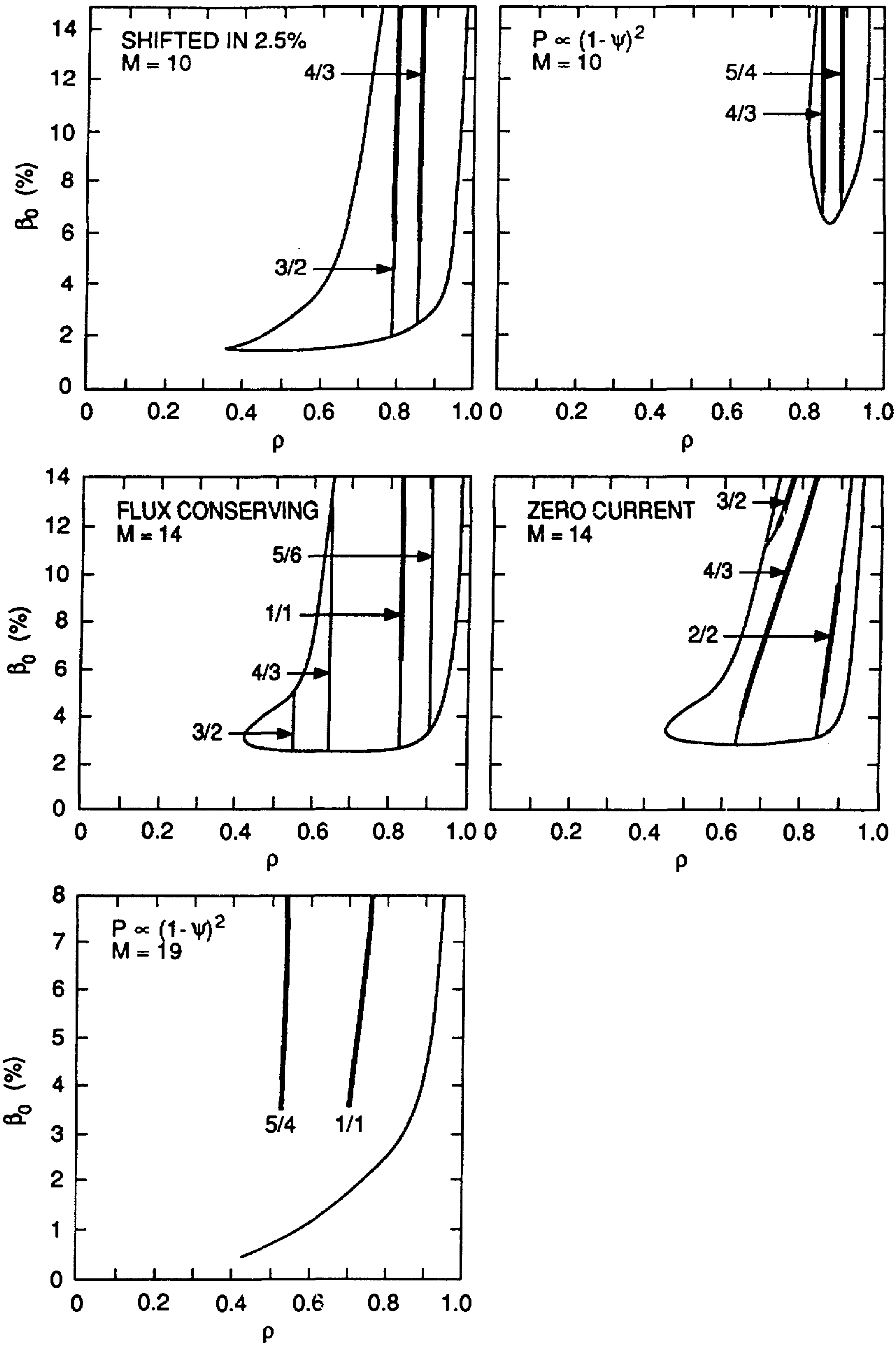

SHIFTED IN 2.5%
M = 10
4/3
3/2
β_0 (%)
ρ
P ∝ (1- ψ)^2
M = 10
5/4
4/3
ρ
FLUX CONSERVING
M = 14
5/6
1/1
4/3
3/2
β_0 (%)
ρ
ZERO CURRENT
M = 14
3/2
4/3
2/2
ρ
P ∝ (1- ψ)^2
M = 19
5/4
1/1
β_0 (%)
ρ