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**TERRAIN EFFECTS IN RESISTIVITY AND
MAGNETOTELLURIC SURVEYS**

By
H. Truman Holcombe

December 1982

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**The University of New Mexico
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AND

MAGNETOTELLURIC SURVEYS

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TERRAIN EFFECTS IN RESISTIVITY AND
MAGNETOTELLURIC SURVEYS

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TERRAIN EFFECTS IN RESISTIVITY AND MAGNETOTELLURIC SURVEYS

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The effects of topography in electrical and electromagnetic surface studies have not been adequately considered in field data analysis. In an effort to improve the state of the art in interpretation of resistivity and magnetotelluric data, the objectives of this study were therefore:

1. To develop computer algorithms which calculate the electrical resistivity response of earth models with arbitrary three-dimensional topography.
2. To develop computer algorithms which calculate the telluric and magnetotelluric response of earth models with arbitrary three-dimensional topography.
3. To provide validity and accuracy checks for the algorithms.
4. To provide case studies of the applications of the algorithms to the interpretation of existing resistivity and magnetotelluric field data.

While these objectives have not been entirely met, significant progress has been made, especially in the interpretation of direct current resistivity data.

A three-dimensional finite element computer algorithm which can accommodate arbitrarily complex topography and subsurface structure, has been developed to model the resistivity response of the earth. The algorithm has undergone extensive evaluation and is believed to provide accurate results for realistic earth models. Testing included comparison to scale model measurements, analytically calculated solutions, and results calculated numerically by other independent means.

Computer modeling experiments have demonstrated that it is possible to remove the effect of topography on resistivity data under conditions where such effects dominate the response. This can be done without resorting to lengthy and costly trial and error computer modeling. After correction, the data can be interpreted with confidence that the anomalies are due only to subsurface structure.

The results of case studies on resistivity field data measured in high relief topography indicate the following:

1. Pre- or post-survey three-dimensional computer analyses of topographic effects are worthwhile and can be done for a small fraction of the survey cost.
2. Three-dimensional terrain corrections can significantly improve the quality of subsurface interpretation in

terrain geometries which cannot be modeled with a two-dimensional algorithm.

3. Terrain effects on collinear electrode arrays can be minimized by alignment of the arrays parallel to the strike of the topography (that is, along streamlines and ridgelines).

A three-dimensional isoparametric finite element algorithm for modeling the magnetotelluric response of the earth has been written. However, it is computationally too inefficient to allow proper evaluation and testing within the currently available time and resources. The theoretical development is presented with the hope that such resources will be available in the future.

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INTRODUCTION

Direct Current Resistivity Methods

Direct current earth resistivity measurements have been effectively used by geophysicists for more than 50 years. Since the resistivity of naturally occurring materials varies over many orders of magnitude, resistivity measurements potentially provide a sensitive probing of the earth's subsurface. A principal weakness in the method is the inability to interpret the data in terms of complex three-dimensional subsurface resistivity structures and irregular terrain frequently present in the measuring environment.

Resistivity data are obtained by measuring the electric potential difference, ΔV , between a pair of grounded electrodes due to a current, I , injected into the earth via another pair of electrodes. The data are expressed as apparent resistivity, ρ_a , calculated by

$$\rho_a = K \frac{\Delta V}{I}$$

where K is a constant factor determined from the electrode geometry, using the assumption that the earth is a flat homogeneous half-space. Until the development of fast digital computers in the late 1960's, most interpretations were done by comparisons to simple structures from which predicted apparent resistivity data could be calculated analytically. These included horizontally layered

structures, buried conducting cylinders and spheroids, vertical faults, vertical dikes, planar dipping interfaces, and hemi-cylindrical and hemi-spheroidal sinks. A comprehensive history and bibliography of such techniques was given by Van Nostrand and Cook (1966).

Direct calculation of an earth resistivity model from measured apparent resistivity data did not become practical until quite recently. Although the horizontally layered resistivity inversion was first formulated by Slichter (1933), the technique was not widely used until easily implemented numerical methods were developed by Ghosh (1971), Inman (1975), Zohdy (1975), and Oldenburg (1978). Two-dimensional inversion has been developed (Pelton et al., 1978, and Tripp et al., 1979), but is not yet in widespread use. The only three-dimensional inversion thus far is a numerical ridge regression inversion developed by Petrick et al. (1981) using the alpha center solution formulated by Stefanescu (1950).

Numerical forward solutions (calculation of predicted apparent resistivity data from predetermined earth resistivity structures) have been successfully implemented by numerous authors over the past 10 years. An excellent summary and bibliography of this work was recently published by Ward (1980). Much of the work has been employed to calculate the electric potential distribution on the surface of a half-space, with arbitrary two-dimensional subsurface resistivity structure, due to a three-dimensional

point source of electric current. Two-dimensional structures are defined as bodies of arbitrary cross-section which are uniform along strike. Two authors (Coggon, 1971, and Fox et al., 1980) have applied the finite element method to model two-dimensional topography. Several three-dimensional modeling algorithms have been developed for resistivity structure beneath a flat earth surface.

Hohmann (1975) used an integral equation method, Pridmore et al. (1981) applied the finite element method, and Dey and Morrison (1979) described a finite difference approach.

The study presented here is one of the first efforts to model topography in three dimensions. The importance of such a study was best stated by Grant and West (1965, p. 441):

Probably no other group of geophysical methods is so adversely affected by topography and near-surface conditions as the d-c resistivity methods . . . the forms of the land surface may have a strong influence on shaping the equipotential surfaces even under uniform conditions, introducing local gradients which may simulate or confuse real anomalies.

It is inherently a difficult problem since the electric potential field around a current source or array of sources is a complex function of both topographic and subsurface effects. The logical approach to a better understanding of terrain effects is to include the topography in the modeling process. However, Fox et al. (1980) have shown that a terrain correction procedure can be applied, using a two-dimensional resistivity modeling program, to reduce the effect of the topography in dipole-dipole surveys. An

analogous procedure has been developed here for three-dimensional topography. It is, in effect, a numerical calculation of the correct geometric factors for an earth model with topography on its upper surface.

In order to improve the state of the art in the interpretation of resistivity data, the objectives of this study were therefore:

1. To develop computer algorithms which calculate the electrical resistivity response (using various electrode arrays) of earth models with arbitrary three-dimensional topography.
2. To provide satisfactory validity and accuracy checks for the algorithms.
3. To provide case studies of the application of the algorithms in interpreting existing resistivity field data.

These objectives have been met.

A practical technique for modeling the resistivity response of arbitrary three-dimensional earth models with terrain has been devised by adapting the finite element technique described by Pridmore et al. (1981). His method calculated the response of arbitrary subsurface structure beneath a flat horizontal earth-air interface. The new technique includes the terrain by replacing the near-surface layers of regular hexahedral elements with contiguous distorted isoparametric elements. Distortion of

regularly shaped elements is accomplished mathematically by a coordinate transformation. The term "isoparametric" refers to the order of the polynomial form (that is, linear, quadratic, etc.) used to relate the transformed to the untransformed coordinates. If it is the same order as the polynomial used to approximate the variation of the parameter of interest within a finite element, then the distorted element is called "isoparametric." The terms "subparametric" and "superparametric" are used for transformations of lower or higher order, respectively. This element distortion is illustrated in Figure 1 where the dashed box represents the original undistorted geometry and the solid-line grid is the distorted isoparametric element geometry. The geometry change is accomplished by using a co-ordinate transformation which translates the coordinates of the corners of the distorted elements (which are the locations of the finite element nodes) vertically downward an appropriate distance. The heavy solid line is an arbitrarily determined horizon which separates the distorted elements from the undistorted ones.

This paper contains details of the theoretical development, numerical implementation, and testing and verification procedures employed. Also included is a description of the computer modeling experiments performed to verify the terrain correction procedure, and a terrain case study done for resistivity data measured at the Los

Figure 1. Illustration of how terrain is incorporated in earth models for the resistivity modeling problem. The dashed box is the boundary of the original undistorted geometry. The solid-line grid represents the distorted isoparametric element geometry. The heavy solid line is the horizon separating the distorted elements above from the undistorted ones below.

Alamos National Laboratory Hot Dry Rock Geothermal Site,
Fenton Hill, New Mexico.

Magnetotelluric Methods

A newer geophysical measurement which is widely used is the magnetotelluric method. Data are obtained by measuring at the surface of the earth the total magnetic and electric fields due to naturally occurring electromagnetic waves. These are analyzed in the frequency domain with the intention of determining the subsurface resistivity

structure at the measurement site. Its conception is generally attributed to Tichonov (1950) and Cagniard (1953) although many other authors have contributed. Vozoff (1972) gave a comprehensive description of the methodology of the measurement and interpretation favored at the time. A brief description and related bibliography, including recent developments, is given by Ward (1980). The quality of the data measurement has improved dramatically in the last few years, primarily due to application of the remote reference technique developed by Gamble et al. (1979). The problem that remains is the multidimensional interpretation, which relies heavily on numerical modeling techniques.

Current practices produce "sounding curves" of the tensor apparent resistivities versus frequency components of electromagnetic waves incident on the earth. Although the anisotropy of earth resistivity (especially on the resolution scales which may be many tens of kilometers at the longer wavelength sampled) is recognized, analyses have routinely been done using horizontally layered earth models. However recent case studies by Wannamaker et al. (1980) have indicated that layered earth and many two-dimensional models may be inadequate for interpretation in typical earth environments. A two-dimensional finite element modeling program developed by Rijo (1977) and a three-dimensional integral equation program developed by Hohmann (1975) were used in the case studies.

Very little numerical modeling has been done which includes topography as part of the model. Ku et al. (1973) and Swift (1971) used a technique based on a transmission line analogy to model simple two-dimensional topographic features. No correlation or verification of these models with independent results has been published. In the three-dimensional terrain problem in magnetotellurics, the character and magnitude of the effects are virtually unknown. Therefore, we are also faced with the task of developing a satisfactory method for independently verifying any model results.

As in the resistivity case, the objectives of this study were:

1. To develop computer algorithms which calculate the telluric and magnetotelluric response of earth models with the three-dimensional topography.
2. To provide satisfactory validity and accuracy checks for the algorithms.
3. To provide case studies of the application of the algorithms in interpreting existing magnetotelluric field data.

It was not possible to meet these objectives, because the computer algorithm devised was not efficient enough to allow proper evaluation and testing within reasonable time and resource constraints. Nevertheless the formulation of the problem is considered an important step in the ultimate

solution to the problem. It is presented here with the hope that more efficient solution methods and additional resources will be available in the future.

The method is again an adaptation of the finite element methods of Rijo (1977) and Pridmore et al. (1981) and is similar to that done for the resistivity problem. The geometric differences are (1) an air layer is incorporated in the model with appropriate element distortions so that it is contiguous with the earth model in Figure 1, and (2) the model is continued outward in all directions and graded into a horizontally layered structure. Contained in this paper are the details of the theoretical development and a description of the preliminary testing.

NUMERICAL CALCULATIONS OF RESISTIVITY RESPONSE

Theoretical Development

The theoretical resistivity development follows that of Pridmore et al. (1981) for the most part. It is expanded here to incorporate isoparametric finite elements near the surface of an earth model. Isoparametric elements can be distorted to incorporate an irregular surface without loss of generality.

The domain equation is the steady-state equation of continuity of current density, that is,

$$\vec{\nabla} \cdot \vec{J}_c = 0 \quad (1)$$

where $\vec{J}_c$ is the conduction current density within an arbitrary volume of the earth with no current sources. For this problem we assume that the conductivity of the earth, σ , is isotropic and a function of position only. We can therefore substitute Ohm's law ($\vec{J}_c = \sigma \vec{E}$) into equation (1)

$$\vec{\nabla} \cdot \sigma \vec{E} = 0$$

where $\vec{E}$ is the electric field vector. Noting that $\vec{E} = -\vec{\nabla}\phi$,

$$-\vec{\nabla} \cdot \sigma \vec{\nabla}\phi = 0 \quad (2)$$

where ϕ is the scalar electric potential. When the volume of earth under investigation contains sources of diverging current, then

$$-\vec{\nabla} \cdot \sigma \vec{\nabla}\phi = \vec{\nabla} \cdot \vec{J}_s \quad (3)$$

where $\vec{J}_s$ is the current density due to the sources.

The domain for the finite element numerical solution is a model where topography is superimposed upon an otherwise rectangular prism shaped volume of earth. This model is divided into hexahedral or distorted hexahedral elements as shown in Figure 1. The distorted elements are mapped onto a regular hexahedral array by a vertical co-ordinate transformation.

The boundary conditions applied to the finite element formulation of equation (3) are as follows:

1. At the earth-air interface, the homogeneous Neumann condition is used, that is

$$\frac{\partial \phi}{\partial \eta} = 0 \quad (4)$$

where η is a spatial co-ordinate normal to the interface. This is the usual condition for resistivity problems and is equivalent to constraining current to flow along the interface with no normal component.

2. At the subsurface external boundaries an inhomogeneous Dirichlet boundary condition is used, that is

$$\phi_b \neq 0 \quad (5)$$

where ϕ_b is some calculated value of the electrical potential at the bounding nodes. Justification for use of this simple condition, in lieu of one more sophisticated, and the method of its calculation are discussed later.

As indicated, the technique chosen to solve the problem is the finite element method. It has a long

history of success in a wide variety of applications (Huebner, 1975, pp. 13-14) and its validity for the classic Poisson problem defined by equation (3) is well established.

Within each hexahedral finite element (see Figure 2) the true solution is approximated by a trial function of the form

$$\phi_t(u,v,w) = \sum_{i=1}^8 N_i(u,v,w) \phi_i \quad (6)$$

where the ϕ_i 's are the nodal values of the electric potential and the N_i 's are shape functions defined as

$$N_i(u,v,w) = \frac{1}{8a^2b^2c^2} (a^2 + uu_i)(b^2 + vv_i)(c^2 + ww_i) \quad (7)$$

Here u , v , and w are Cartesian co-ordinates in the transformed, undistorted finite element space, u_i , v_i , and w_i are the nodal values, and $2a$, $2b$, and $2c$ are the u , v , and w dimensions of the element. This type of element is an isoparametric tri-linear element of the serendipity class (Huebner, 1975, pp. 179-191). The variational method is chosen for the derivation of the finite element matrix, although other finite element methods (such as the Ritz method or Galerkin's method) will result in an identical formulation.

If the equation which the true solution satisfies is known, then the trial solution given in equation (6) may be forced to satisfy it. In this case, we may write equation (3) for the n th finite element in operator form

$$L \phi = f \quad (8)$$

where L is the Laplacian operator $[-\vec{\nabla} \cdot \sigma \vec{\nabla}]$ and $f=0$ or

Figure 2. Co-ordinate configuration and node numbering scheme for hexahedral finite element.

$f = \vec{\nabla} \cdot \vec{J}_s$ depending on whether or not the n th element contains a source. The minimum theorem (see Mikhlin and Smolitskiy, 1967, pp. 147-155) states that the solution of (8) minimizes the functional, expressed in inner product form,

$$F_n(\phi) = \langle L\phi, \phi \rangle - 2\langle \phi, f \rangle \quad (9)$$

provided the following conditions are met:

1. The operator L is self-adjoint and positive definite. This has been proven by Pridmore (1978) and others.
2. ϕ and f are elements of the same Hilbert space. In this case, they must be members of the sub-space of all

functions which have continuous first derivatives within the nth element.

The inner product notation in equation (9) is defined by

$$\langle a, b \rangle = \int_{V_n} (a \cdot b) dV_n$$

which is the volume integral over the nth element of the dot product of functions a and b. Therefore, for the nth element, $F_n(\phi)$ is minimum if the first variation is zero, that is

$$\left. \frac{\partial F_n(\phi)}{\partial \phi} \right|_{\phi_1} = \left. \frac{\partial F_n(\phi)}{\partial \phi} \right|_{\phi_2} = \dots = \left. \frac{\partial F_n(\phi)}{\partial \phi} \right|_{\phi_s} = 0 \quad (10)$$

where $\phi_1 \dots \phi_s$ are the nodal values of the potential in the nth element. Equation (10) may be evaluated as follows. Integrating the first term in equation (9) by parts, one gets

$$\langle -\vec{\nabla} \cdot \sigma_n \vec{\nabla} \phi, \phi \rangle = \langle \sigma_n \vec{\nabla} \phi, \vec{\nabla} \phi \rangle - \int_S \sigma_n \phi \frac{\partial \phi}{\partial \eta} dS \quad (11)$$

where the surface integral is done over a closed volume containing the domain which we define to be the nth finite element. $\frac{\partial \phi}{\partial \eta}$ is the partial derivative of ϕ normal to the bounding plane. Therefore

$$\langle -\vec{\nabla} \cdot \sigma \vec{\nabla} \phi, \phi \rangle = \sum_n \int_{V_n} \sigma_n (\vec{\nabla} \phi)^2 dV_n - \sum_n \int_{S_n} \sigma_n \phi \frac{\partial \phi}{\partial \eta} dS_n \quad (12)$$

Continuity of current within the conducting model requires that the internal surface integral components of adjoining elements cancel one another. At the external surface where

the homogeneous Neumann boundary condition applies, the component of the surface integral is zero by definition. Further, at the boundaries where the known inhomogeneous Dirichlet condition applies, the surface integral is, by definition, constant and can therefore play no part in determining the minimum of the functional. Therefore, the surface integral terms need not be considered, so that,

$$F_n(\phi) = \int_{V_n} [\sigma_n (\vec{\nabla} \phi)^2 - 2 \phi \vec{\nabla} \cdot \vec{J}_s(u, v, w)] dV_n \quad (13)$$

The first variation of the functional at node 1, therefore, becomes

$$\begin{aligned} \left. \frac{\partial F_n(\phi)}{\partial \phi} \right|_{\phi_1} = & -2 \int_{V_n} N_1 \vec{\nabla} \cdot \vec{J}_s(u, v, w) dV_n \\ & + 2 \sigma_n \int_{V_n} \left[\frac{\partial N_1}{\partial u} \left(\frac{\partial N_1}{\partial u} + \dots + \frac{\partial N_s}{\partial u} \right) + \frac{\partial N_1}{\partial v} \left(\frac{\partial N_1}{\partial v} + \dots + \frac{\partial N_s}{\partial v} \right) \right. \\ & \left. + \frac{\partial N_1}{\partial w} \left(\frac{\partial N_1}{\partial w} + \dots + \frac{\partial N_s}{\partial w} \right) \right] (\phi_1 + \dots + \phi_s) dV_n \end{aligned} \quad (14)$$

where σ_n is the isotropic conductivity within the homogeneous nth element, V_n is the volume of the element, and $\vec{J}_s(u, v, w)$ is the current density within the undistorted element.

Topography is incorporated into the earth model by mapping the regular hexahedral elements onto a continuous array of irregularly shaped elements as shown in Figure 1. This is done in the usual fashion, so that equation (14)

$$\begin{aligned} \left. \frac{\partial F_n(\phi)}{\partial \phi} \right|_{\phi_1} = & -2 \int_{V_n} N_1 \vec{\nabla} \cdot \vec{J}_s(u, v, w) (\det J_{uvw}) dV_n \\ & + 2 \sigma_n \int_{V_n} \left[\frac{\partial N_1}{\partial u} \left(\frac{\partial N_1}{\partial u} + \dots + \frac{\partial N_s}{\partial u} \right) + \frac{\partial N_1}{\partial v} \left(\frac{\partial N_1}{\partial v} + \dots + \frac{\partial N_s}{\partial v} \right) \right. \\ & \left. + \frac{\partial N_1}{\partial w} \left(\frac{\partial N_1}{\partial w} + \dots + \frac{\partial N_s}{\partial w} \right) \right] (\phi_1 + \dots + \phi_s) (\det J_{uvw}) dV_n \end{aligned} \quad (15)$$

where

$$\det J_{uvw} = \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} & \frac{\partial x}{\partial w} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} & \frac{\partial y}{\partial w} \\ \frac{\partial z}{\partial u} & \frac{\partial z}{\partial v} & \frac{\partial z}{\partial w} \end{vmatrix} \quad (16)$$

In this case we use a linear transformation in the vertical direction only, that is, the node locations in the hexahedral elements are translated vertically. Therefore

$$\det J_{uvw} = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \frac{\partial z}{\partial w} \end{vmatrix} \quad (17)$$

In order to facilitate integration, the shape function defined for the trial solution is also used for making the transformation, that is

$$z = \sum_{i=1}^8 N_i(u,v,w) z_i \quad (18)$$

where the z_i 's are the vertical co-ordinates of the nodes in the distorted element.

The evaluation of the integrals in equations (14) and (15) are analytic because of our choice of linear functions for both the trial solution and the co-ordinate transformation. The steps of this evaluation are given in Appendix A. The result is an 8 x 8 matrix equation of the form

$$\begin{bmatrix} a_{11} & a_{12} & \dots & a_{18} \\ a_{21} & a_{22} & & \vdots \\ \vdots & & \ddots & \vdots \\ a_{81} & \dots & \dots & a_{88} \end{bmatrix} \cdot \begin{bmatrix} \phi_1 \\ \phi_2 \\ \vdots \\ \phi_8 \end{bmatrix} = \begin{bmatrix} s_1 \\ s_2 \\ \vdots \\ s_8 \end{bmatrix} \quad (19)$$

where the coefficient matrix $[a_{ij}]$ is symmetric and usually referred to as the element matrix. For the simple hexahedral element case (equation 14), there are eight independent coefficients. In the distorted element case (equation 15), there are 18. Normally, the finite element matrix equations are assembled into a global matrix and then solved by one of the matrix equation solution techniques commonly used. Because of the very large number of elements (sometimes 50,000 or more) required for a realistic earth model, the conventional Gauss elimination or various matrix decomposition solutions require more core memory than is usually available on even the largest computers. This is because all of the non-zero coefficients in the matrix, which may be of order 50,000 or greater, must be stored in the solution process. Therefore, the results given here made use of a successive node iterative solution technique, which solves only the matrix equations for elements adjacent to a given node at any particular time. This allows a reduction in the memory required, but greatly increases the running time. The solution method chosen is usually called the successive point over-relaxation (SPOR) method. It is described in

great detail in many texts (e.g., Ames, 1977, pp. 119-125) so only a brief overview is presented here.

An initial guess at the solution vector $\vec{\phi}$ is substituted into an $m \times m$ system of equations

$$\begin{bmatrix} a_{11} & a_{12} & \dots & a_{1m} \\ a_{21} & a_{22} & & . \\ \vdots & & \ddots & \vdots \\ a_{m1} & \dots & a_{mm} \end{bmatrix} \cdot \begin{bmatrix} \phi_1 \\ \phi_2 \\ \vdots \\ \phi_m \end{bmatrix} = \begin{bmatrix} s_1 \\ s_2 \\ \vdots \\ s_m \end{bmatrix} \quad (20)$$

and successive iterations proceed in the following manner.

In iteration n , the i th row

$$\sum_{j=1}^m a_{ij} \phi_j = \sum_{j=1}^m s_j \quad (21)$$

is solved for the $n+1$ value of ϕ_k for $1 \leq k \leq m$, yielding

$$\phi_k^{n+1} = \left\{ \left[\frac{\sum_{j=1}^m s_j - \sum_{j=1}^{k-1} a_{ij} \phi_j^n - \sum_{j=k+1}^m a_{ij} \phi_j^n}{a_{ik}} \right] - \phi_k^n \right\} W_n + \phi_k^n \quad (22)$$

where the ϕ_j^n are the values of ϕ_j calculated in iteration n and W_n is a factor called the acceleration parameter. If $1 < W_n < 2$, this process is called over-relaxation. As might be expected, the rate of convergence is very much dependent on what value of W_n is selected. The term inside the large brackets, $\{ \}$, is called the residual. Its behavior in successive iterations determines whether or not the process is converging to a solution but not necessarily the rate of convergence. For very large systems of equations the process uses excessive computer time, so it is important to determine an optimum value for W_n which gives a maximum rate of convergence.

When the matrix has consistent ordering (property "A" in Ames, 1977, p. 120), an optimum value of W_n may be determined theoretically. Unfortunately, the finite element formulation used here does not have this property. However, Carre (1961) and Pridmore (1978) have suggested that W_n should be the maximum value for which the successive residuals calculated do not exhibit an oscillatory behavior. Therefore, the following technique has been used in this application. An initial value is chosen which has experimentally been determined to be less than but close to the optimum. The maximum residual is tested after each iteration, and, if it is less than that of the previous iteration, W_n is incremented upward by a predetermined amount. If it is greater, W_n is incremented downward. The initial value is typically about 1.5, the upward increment is 0.005, and the downward increment is 0.05.

Another important consideration is the choice of an optimum method of assembly of a local matrix equation (from the appropriate element matrices derived in Appendix A) for the SPOR process. Desai and Abel (1971, p. 105) and others have stated that empirical evidence suggests that a nodal assemblage is best when using an iterative scheme for the solution. The nodal assembly process is discussed in numerous texts (e.g., Norrie and de Vries, 1978, p. 179) but a brief description of what has been implemented in this application is included here for continuity.

Each node in the finite element array is included in eight adjacent elements (Figure 3a) if it is an interior node, and four adjacent elements if it is an exterior node (Figure 3b). A nodal assemblage is accomplished by grouping into a single row of the nodal matrix the eight (or four) rows in the respective adjacent element matrices in which the coefficient for the node is contained in the diagonal position. In the final analysis, each row of the nodal matrix yields an interpolation of the nodal potential from each of the surrounding 26 nodal values during each iteration. It is therefore equivalent computationally to a 27-point finite difference approximation.

In many approximate solution techniques, such as the finite difference approach of Dey and Morrison (1976), one can use a point current source in the formulation, that is

$$\vec{\nabla} \cdot \vec{J}_s = I \delta(\vec{r} - \vec{r}_0)$$

where $\delta(\vec{r} - \vec{r}_0)$ is the three-dimensional Dirac delta function, and I is the magnitude of a point current source at position $\vec{r}_0$. However, the variational method used here does not allow this form, since the Dirac delta function does not have a continuous first derivative, which is required by the minimum theorem. In general one needs a functional form for $\vec{\nabla} \cdot \vec{J}_s$ that meets the criteria for the minimum theorem and approximates as closely as possible the current flow from a point source. However in this case a simpler way of calculating the source term was employed and

Figure 3a. Interior node (A) in a hexahedral finite element array.

Figure 3b. Exterior node (B) in a hexahedral finite element array.

found empirically to provide accurate calculated responses in most cases. The procedure is to evaluate the source integral in equation (15) using the Dirac delta function form, and then divide this result equally among all eight nodes in the finite element containing the source. The details of this are given in Appendix A.

The boundary conditions, equations (4) and (5), are easily incorporated into the computer model. The homogeneous Neumann condition is a natural boundary condition of the Poisson problem. It is satisfied automatically in the finite element problem when the mesh is terminated (Norrie and de Vries, 1978, p. 188). The inhomogeneous Dirichlet condition is not a natural boundary condition, but the finite element formulation derived is still valid when the condition is imposed (Mikhlin and Smolitskiy, 1967, p. 163). It is satisfied by simply not performing the nodal calculation on the appropriate bounding nodes, so that the original potential values remain the same throughout the iteration process.

The inhomogeneous Dirichlet boundaries used are clearly incorrect from a theoretical point of view, since we are imposing an assumed value of the electric potential at some fixed, predetermined distance from the sources. Probably the best boundary condition for models of finite dimension is the asymptotic condition used by Dey and Morrison (1979),

$$\frac{\partial \phi}{\partial \eta} + \frac{\phi \cos \theta}{r_s} = 0 \quad (23)$$

where θ is the angle between the radial vector from the source, r_s , and the outward normal spatial co-ordinate, η . This condition imposed the asymptotic $\frac{1}{r}$ behavior of the potential at large distances from the source on the bounding planes of the earth model. Implementing this condition requires either (1) using a different finite element formulation at the boundaries which explicitly includes equation (23) (see Pridmore, 1978), or (2) appropriately adjusting the potential at the bounding nodes during the iteration process (see Norrie and de Vries, 1978, p. 213). Unfortunately, procedure (1) requires a considerable programming effort, which has not been attempted, and procedure (2) results in a significant increase in the number of iterations required to arrive at a good solution.

Instead, a simple electric potential calculation was contrived which was intended to provide (1) a reasonable initial guess at the array of nodal electric potential values (which is required for the SPOR solution process), and (2) values for the electric potential at the bounding nodes at the lateral and bottom surfaces of the model where the inhomogeneous Dirichlet boundary condition is applied. The bounding potentials are intended to be close enough to the true value, in a given model, so that there are no significant errors due to incorrect boundary conditions in the apparent resistivities determined from potentials calculated by the SPOR process in the interior of the

model. The calculation makes use of a Dar Zarrouk construction (Maillet, 1947, and Orellana, 1963) and the empirical behavior of the electric potential in topography.

For a single point source, the potential is given by

$$\phi_b(x, y, z) = \frac{I}{2\pi r_s} (DZ)(V_t) \quad (24)$$

where

$$DZ = \left(\frac{T_{n-1}}{S_{n-1}} + \frac{(z-z_n)}{\rho_n} \right)^{\frac{1}{2}} \quad n > 1 \quad (25)$$

approximates the potential, using the Dar Zarrouk construction in a horizontally layered earth. For n layers of resistivity ρ_i , $i = 1, \dots, n$

$$\begin{aligned} DZ &= \left(\frac{T_1}{S_1} \right)^{\frac{1}{2}} & 0 < r_s \leq (T_1 S_1)^{\frac{1}{2}} \\ &= \left(\frac{T_1}{S_1} + \frac{(z-z_2)}{\rho_2} \right)^{\frac{1}{2}} & (T_1 S_1)^{\frac{1}{2}} < r_s \leq (T_2 S_2)^{\frac{1}{2}} \\ &\vdots \end{aligned}$$

The term

$$V_t = 1 + \frac{\epsilon_s - e}{r_s} \quad (26)$$

approximates the change in potential due to the difference in elevation between the source and the location where the potential is calculated. The terms used in equations (24), (25), and (26) are defined as follows:

$h_1, 2, 3 \dots$ = horizontal layer thicknesses (from the surface downward)

$\rho_1, 2, 3 \dots$ = isotropic resistivity of the layers

$$T_n = h_1 \rho_1 + h_2 \rho_2 + \dots + h_n \rho_n$$

$$S_n = (h_1/\rho_1) + (h_2/\rho_2) + \dots + (h_n/\rho_n)$$

r_s = straight line separation distance between source
and point where potential is calculated

e_s = surface elevation of source point

e = surface elevation at point where potential is
calculated

z = depth to point where potential is calculated

z_n = depth to top of nth layer

Maillet (1947) showed that an isolated n-layered conductive column of square cross-section (with unit cross-sectional area) could be replaced with a single homogeneous column of resistivity $(T_n/S_n)^{\frac{1}{2}}$ and thickness $(T_n S_n)^{\frac{1}{2}}$, as far as any measured response outside the column was concerned. T_n and S_n are called the transverse unit resistance and the longitudinal unit conductance, respectively, for the layered square column. Further, he showed empirically that the potential response calculated from equation (24) will generally yield a useful approximation to the true electric sounding curve calculated for a flat layered earth. Figure 4 is an example of the Dar Zarrouk response plotted on top of a calculated Schlumberger response for a layered earth model.

The Dar Zarrouk construction (see Figure 4) approximates the surface potential in a segmented fashion. Each segment is the calculated response of a two-layer model.

The lower layer is assumed to be homogeneous, with resistivity ρ_n for the n th segment, and the upper layer is transversely anisotropic with resistivity properties T_n and S_n . Zohdy (1965) gives a detailed discussion of the relationship of the Dar Zarrouk construction to the true response. The application here extends the calculation to three dimensions by simply assuming that the radial symmetry of the construction on the surface is preserved throughout the model, that is, the calculated potentials are constant over any hemisphere symmetric about a single current source. This is clearly incorrect in a layered model, but the errors are thought to be insignificant because in the final solution they would affect only the boundary condition imposed at the edges of the model, which are typically far away from current electrodes and significant three-dimensional structures.

The term V_t is based on the asymptotic behavior of the potential far removed from a source near a simple ramp-slope (see Figure 5). It should be noted that equation (24) reduces to the simple homogeneous half-space solution when that condition is imposed.

Another important consideration is the determination of when the solution has converged adequately to be considered valid. Of equal concern is the practical problem of trying to minimize the number of iterations required to solve the very large matrix equation required for this problem. A straightforward "point of diminishing

Figure 4. Comparison of a Dar Zarrouk construction with a Schlumberger sounding for the resistivity configuration shown (from Orellana, 1963).

Figure 5. Illustration of asymptotic behavior of electric potential far removed from a unit point source near a ramp slope. The empirically determined relationship is given by $\phi_{rs} \approx \phi_h \left(1 + \frac{e_s - e}{r_s}\right)$.

returns" argument is used to determine the optimum point to terminate the iteration process. The following threshold for the maximum residual (equation 22) was adopted.

$$W_{max} \leq (\phi_{max}/10000) \quad (27)$$

where ϕ_{max} is the maximum value of the surface electric potential at the node nearest the source. The maximum residuals typically are calculated at or near the sources, and the rest of the residuals are generally proportional to the nodal electric potentials. Therefore, if this condition is met, it would require a large number of additional iterations (perhaps 100 or more) to make a significant change in the calculated surface potentials.

Since an isoparametric element matrix contains 18 independent coefficients compared to eight for the regular hexahedral one, the storage requirements for the global coefficient array is reduced if one can use as many undistorted elements as possible. Therefore, both types of elements are included in the computer program. A horizon is determined below which all the elements are undistorted (Figure 1), and included in the input to the program. The minimum number of layers of isoparametric elements needed to adequately approximate the terrain is included in the model.

It was still found that the main memory of the CDC 7600 used for the initial study was too small to contain both the potential array and the coefficient array for the models to be calculated. It was found by trial and error

that up to 50,000 elements are required in some cases. It was also determined that storing the coefficient array on a mass storage device and recalling it as needed was inefficient and costly. The least expensive method on the CDC 7600 is to recalculate the coefficients as needed in the iteration process. Therefore, the coefficient array is calculated and stored for a layer of elements at a time. Testing with small arrays indicated that this method cost only about twice as much as storing all of the arrays. Using mass storage cost more than ten times as much. A practical limit on the CDC 7600 is about 100,000 elements using the recalculation method.

If the nodal calculations are performed in the same order in each iteration, the result is an initial asymmetry in the over-relaxation potentials which slows the iteration process. Therefore, an alternating direction iterative process was adopted. First, nodal iteration proceeds row by row in each layer starting from an upper corner and finishing on the diagonally opposite lower corner. The next iteration begins where the first stopped and proceeds row by row and layer by layer in precisely the opposite direction. The iterations are alternated in this manner through completion of the solution. This method has been successfully employed by other authors in similar problems (e.g., Gunn, 1964).

A typical computer model takes 15 to 30 iterations to meet the convergence criteria. However, this is

dependent on the topographic relief and the complexity and magnitude of subsurface resistivity structure. A typical run cost \$15 to \$20 per source array on Lawrence Berkeley Laboratory's CDC 7600 when making use of low priority rates. The cost per run is doubled for regular priority runs.

Testing and Verification

As a preliminary test of the validity of the finite element programs, a model approximating a buried sphere in a half-space was calculated. A response calculated analytically by Snyder and Merkel (1973) was chosen for the test (Figure 6). The finite element response is shown in Figure 8. Figure 7 gives the systematics for the pole-quadrupole calculation (Figure 8). In this case a body made up of an assemblage of cubes approximates the buried sphere (see inset in Figure 8). The isoparametric element feature of the program was not used in this case. The boundary condition was simply the response of a homogeneous half-space. The model response met the convergence criterion stated previously after 24 iterations.

The response shown is clearly very close to the analytic solution. The four lobe contours near the source in Figure 8 are an artifact of the finite difference approximation of the total field used for the apparent resistivity calculation. The slightly greater anomaly magnitude in the finite element calculation is probably

Figure 6. Map of analytically calculated pole-dipole total field apparent resistivity response over a conductive sphere buried in a homogeneous half-space. Half-space resistivity is 100 ohm-m. Sphere resistivity is 20 ohm-m. Dotted line indicates the position of the sphere and an asterisk denotes the source location. Depth to the center of the sphere is one diameter (from Snyder and Merkel, 1973).

due to the slightly greater volume of the assemblage of cubes. While the ability to calculate the effects of terrain or complex subsurface structures is not tested, the fundamental validity of the finite element methods for this type of problem is demonstrated.

Figure 7. Pole-quadrupole measurement systematics used for the finite element computer study.

Another important characteristic of the finite element calculations is their sensitivity to the correctness of the inhomogeneous Dirichlet boundary conditions applied to the subsurface boundaries. Figures 9a and 9b show the response calculated from the finite element program for a single source in a homogeneous half-space. Curves A, B, and C are the electric potential (Figure 9a)

Figure 8. Map of pole-quadrupole response for the same buried sphere model used for Figure 6, calculated from the finite element program. Inset shows actual body used to approximate sphere.

and the pole-quadrupole total field apparent resistivity (Figure 9b) along a line radially away from the source. Curve B was calculated using the correct (homogeneous half-space) boundary condition. Curve A was calculated for boundary potential values of twice the correct value. Curve C was for one-half the correct value. The finite element array used was a uniform $21 \times 21 \times 16$ elements with an element dimension of $50 \times 50 \times 50$ meters. This is

Figure 9a. Plot of electric potential versus separation distance along a straight line radially away from a single source on a homogeneous half-space. Source strength is 10 amperes. Half-space resistivity is 100 ohm-meters. Curve B was calculated using the correct boundary condition, which agrees precisely with the correct analytic solution. Curves A and C were calculated using twice and one-half, respectively, of the correct bounding potentials.

Figure 9b. Plot of pole-quadrupole total field apparent resistivity versus separation distance along a straight line radially away from a single source on a homogeneous half-space. Half-space resistivity is 100 ohm-meters. Curves A, B, and C correspond to potential curves above.

typical of a minimum array size needed for a realistic model.

The errors in total field apparent resistivity (Figure 9b, curves A and C) are significantly less than the errors in electric potential at the boundaries. While the errors in the potential are greater at the boundaries, they do persist throughout the entire model. This illustrates one of the weaknesses in the method chosen for implementing the boundary conditions. It is very important that the boundary conditions be approximately correct and/or that the model boundaries be extended as far as possible away from the source position.

As a test of the accuracy of using the Dar Zarrouk construction described earlier as a primary model, a simple three layer test model was run (Figure 10). This is the Schlumberger sounding response for the three layer model shown. The solid line is the true response calculated from a Stefanescu integral solution.

The circles show the response calculated using a simple homogeneous half-space (100 ohm-m) as a boundary condition. The triangles show the response using the Dar Zarrouk construction described. Both calculations were terminated after 100 iterations, and both responses were well within the convergence criterion. However, the calculated residual at iteration 100 for the primary half-space response were an order of magnitude larger than that for the primary Dar Zarrouk response.

Figure 10. Calculated Schlumberger sounding for the simple three layer model shown. Solid line is response calculated from Stefanescu integral. Triangles and circles show the responses calculated from the finite element program.

Therefore, the primary model using the Dar Zarrouk construction yields satisfactory accuracy for the layered model response calculated, and is superior to the homogeneous half-space. However, subsequent testing with layered models different from that in Figure 10 has produced some significantly inaccurate responses near the boundaries of the model. This has been observed when the bottom layer (half-space) resistivity is substantially greater than the

top layer resistivity. This suggests that the assumption made in the previous section (i.e., that the errors, introduced by assuming a radial symmetry in the electric potentials at the model boundaries, are insignificant) is inappropriate in some cases. The best solution to the problem would be to use the linear filter method to evaluate the integral solution (derived using the image method) for the electric potential in a layered half-space. This has been done by Ghosh (1971), Anderson (1979), and others for the electric potential on the surface. However, the subsurface solution requires evaluation of a pair of such integrals, within each layer, corresponding to an upgoing and downgoing wave. It is believed that such an approach would provide an accurate calculation for a starting solution and boundary condition in the three-dimensional algorithms, thereby eliminating the observed difficulty. Implementation of this modification is in progress. The existing algorithm will, however, calculate reasonably accurate responses at nodes more than five or six element spacings away from the boundaries.

The following example was used as a test case for a true three-dimensional terrain feature.

The bipole-quadrupole response of a hemispherical sink in a homogeneous half-space may be calculated directly using the image method. Van Nostrand and Cook (1966, pp. 211-215) and Jakosky et al. (1942) have derived the spherical harmonic solutions for a filled hemispherical

sink with surface electrodes outside the sink (Figure 11). The potentials in this case are given by

$$\phi_p = \frac{I\rho}{2\pi} \left\{ \frac{1}{R} + \sum_{n=0}^{\infty} \frac{2nr^{2n+1}}{(2n+1+k)(r_0r)^{n+1}} P_n(\cos \theta) \right\} \quad (28)$$

for a surface point, P, outside the sink, and

$$\phi_p = \frac{I\rho}{2\pi} \left\{ \frac{1+k}{r_0} \sum_{n=0}^{\infty} \frac{2n+1}{(2n+1+k)} \left(\frac{r}{r_0}\right)^n P_n(\cos \theta) \right\} \quad (29)$$

for a surface point inside the sink. All parameters in equations (28) and (29) are defined in Figure 12 except $P_n(\cos \theta)$ which is the Legendre polynomial of order n and argument $\cos \theta$. If the sink is filled with air, reflection factor $k = 1$. We can, therefore, do an analytic calculation for a true three-dimensional terrain feature. Further, it is an extreme case which is atypical, but provides a rigorous test for the finite element programs.

A separate computer program calculates surface potentials using equations (28) and (29) and then calculates the bipole-quadrupole response in the same manner as the finite element code. Figure 12 illustrates the systematics for the bipole-quadrupole calculation. Figure 13 is a contour map of such a calculated response over a hemispherical sink with a 500 meter radius in a homogeneous half-space of resistivity 100 ohm-m. Figure 14 is the response calculated by the finite element code over an identical configuration.

As in any iterative solution process, it is difficult to determine the radius of convergence in evaluating

Figure 11. Map view of hemispherical sink showing parameters used in spherical harmonic solution.

the infinite sums in equations (28) and (29). However, successive terms in the sum are tested for convergence, and summing continues until successive terms are not quite small enough to cause underflow errors in 64-bit computer precision. For this reason, it is believed that the response calculated is exact for all practical purposes. The solutions do not converge near the sources or the spherical polar co-ordinate origin in the center of the sink. These are, of course, singularities in the solution.

As might be expected, the most extreme terrain effects are near the edge of the sink, but there are also significant effects (i.e., greater than 10 percent) on the

Figure 12. Bipole-quadrupole systematics used for finite element computer study.

flat surface around the sink up to several hundred meters away from the edges.

The finite element response (Figure 14) does not match the analytic solution very well where the terrain effects are extreme. It is believed that this is an artifact of the limitations inherent in the intra-element linear approximations used for the electric potential. Since such extreme terrain is seldom encountered in an actual field survey, it is believed that the inherent errors in the finite element approximation are not significant in terms of correctly predicting the qualitative

Figure 13. Bipole-quadrupole total field apparent resistivity response calculated from spherical harmonic solutions over a hemispherical topographic sink in a homogeneous half-space. Half-space resistivity is 100 ohm-m. Contour interval is 10 ohm-m. Asterisks indicate the location of sources. Dashed circle is the edge of the topographic sink.

characteristics of terrain effects. However, magnitudes of the resistivity anomaly may not be exact. The complex structure of the anomalies are somewhat surprising for such a simple terrain model. Undoubtedly, some of the fine structure near the rim of the sink is due to measurement geometry effects, that is, effects due to the use of finite length quadrupole arrays for approximation of the electric field. The measurement grid was evenly spaced when projected onto a map view, but actual slope distances were

Figure 14. Bipole-quadrupole total field apparent resistivity response calculated from finite element program over a hemispherical topographic sink in a homogeneous half-space. Model parameters are identical to those in Figure 13.

used for the calculation. However, the anomaly structure is still difficult to explain intuitively in terms of current channeling or other phenomena, as is done for example by Snyder and Merkel (1973) and Dorn (1978) for the buried sphere model. This is perhaps an argument for including terrain in the analysis from the beginning, rather than trying to separate it from subsurface effects.

The dipole-dipole response for a traverse tangent to the hemispherical sink was also calculated using both

the spherical harmonic and finite element solutions. The electrode geometry is shown in Figure 15. The results, in conventional apparent resistivity pseudosection form, are shown in Figures 16a and 16b. In the pseudosections, the apparent resistivities are plotted at points where lines (making a 45 degree angle with the surface) from points midway between each electrode pair intersect. For example, the number 104 in the upper left-hand corner of Figure 16b indicates that the apparent resistivity calculated for potential measurements at points 5 and 6 due to current electrodes at points 3 and 4 is 104 ohm-meters. In contrast to the total field modeling, the dipole-dipole response calculations agreed very well. The terrain effects are very significant in this case. Clearly, any attempt to model the results using an assumed flat surface two- and three-dimensional analysis would result in erroneous interpretation of high and low resistivity subsurface anomalies. The low apparent resistivities in the lower center of the pseudosection and the flanking high value are very similar to the response calculated by Fox et al. (1980) for a traverse across a two-dimensional valley (Figure 17). The only difference in the character of the two responses is the high anomaly near the surface in the center of the hemispherical sink pseudosection. The agreement suggests that two-dimensional analyses may be useful for very short strike length features.

Figure 15. Map view of dipole-dipole geometry for traverse tangent to hemispherical sink.

As a further validity test for the finite element program, a reproduction of a scale model result was attempted (Figure 18). This is the same model used by Fox et al. (1980) for verifying the two-dimensional triangular finite element program used in their analyses. The result is shown in Figure 19a and 19b. The response calculated is very similar in character to the scale model results, although the anomalous apparent resistivity magnitudes are not quite as large. This is probably due again to the

Figure 18. Comparison between finite element and scale model results for a two-dimensional ridge (from Fox et al., 1980). Scale model measurements were done by Hallof (1970).

hypothesis, an experiment was conducted using the finite element program. First the pole-quadrupole total field apparent resistivity response was calculated for the cosinusoidal depression topography shown in Figure 20 over a homogeneous half-space. By taking a simple ratio of the calculated resistivities to the half-space value, an array of surface terrain correction factors were calculated (Figure 22). They were applied to the pole-quadrupole response for the cosinusoidal depression over a buried sphere model shown in Figure 20. The position of the depression was deliberately chosen so that it disturbed the characteristic banana-shaped buried sphere anomaly

Figure 19a. Scale model results for two-dimensional ridge (same as in Figure 18). Electrode spacing is 250 meters, half-space resistivity is 250 ohm-meters, and the contour interval is 100 ohm-meters.

Figure 19b. Results for two-dimensional ridge calculated from finite element program. Electrode spacing, half-space resistivity, and contour interval are the same as for Figure 19a.

Figure 20. Map view of topography showing a cosinusoidal depression over the buried sphere model shown in Figures 6 and 8. Contour interval is 25 m.

(Figure 8). The calculated response is shown in Figure 21. The corrected response is shown in Figure 23, which can be compared directly to Figure 8. The corrected response is virtually identical to that calculated using a flat earth model.

Because of the proximity of the two features (cosinusoidal depression and buried sphere) and the relatively high resistivity contrast (five to one) between the half-space and the buried sphere, it was concluded that such a terrain correction procedure is very worthwhile and can effectively remove terrain anomalies from data for many realistic survey sites. This would allow interpretation of the data using conventional flat-earth modeling schemes with more confidence that the results were due to subsurface resistivity structures.

Caution should be used in applying this procedure, however, since a thorough evaluation of the extent of the coupling of topographic and subsurface effects has not been done. It should be viewed as the first order correction, although it is almost certain to improve the subsurface interpretation.

An additional test was performed using this model to determine how well the finite element solution met the reciprocity condition. The reciprocity theorem states that measured electric potential due to a point current source will be the same if the potential measurement position and the current injection point are reversed. Figure 24a is a

Figure 21. Calculated pole-quadrupole response for the cosinusoidal depression over buried sphere model in Figure 20. Half-space resistivity is 100 ohm-m. Sphere resistivity is 20 ohm-m. Center of sphere is one diameter below flat surface. Contour interval is 10 ohm-m.

Figure 22. Calculated pole-quadrupole correction factors for the cosinusoidal depression shown in Figure 20. Contour interval is 0.1.

Figure 23. Terrain corrected pole-quadrupole response for the cosinusoidal depression over buried sphere model (compare to Figure 8). Dashed line is location of boundary of Figure 8. Contour interval is 2 ohm-m.

plot of the response for a homogeneous half-space and Figure 24b is for the model shown in Figure 20. The responses plotted are for a traverse along the surface bisecting the buried sphere and the cosinusoidal topographic feature. A source was positioned at 100 meter intervals along each traverse, and the total field apparent resistivity (calculated using the pole-quadrupole method) at the original source position (asterisks) was plotted on top of the response calculated for the original source position (solid line).

The only significant differences occurred when the source was positioned on the slopes of the cosinusoidal depression at distances of 450 and 750 meters from the original source. The reason for this effect is thought to be the result of applying the rather simplistic source distribution scheme (described previously) to an element that is highly distorted. However, it is not considered a serious problem, since it can be avoided by simply not positioning current sources on steep slopes.

Figure 24a. Results of reciprocity test on homogeneous half-space model using finite element algorithm. Half-space resistivity is 100 ohm-meters. Traverse is radially away from original source.

Figure 24b. Results of reciprocity test on model in Figure 20. Traverse bisects buried sphere and cosinusoidal depression.

THREE-DIMENSIONAL TERRAIN CASE STUDY- RESISTIVITY SURVEYS

The results of an initial case study are presented here, even though the terrain effects were determined to be insignificant in the final interpretation of the data. This conclusion was important in itself and it also provided guidelines from which the importance of terrain effects in other field situations could be predicted.

The University of New Mexico Department of Geology, in cooperation with Los Alamos National Laboratory, has performed numerous resistivity measurements near the Hot Dry Rock Geothermal Test Site at Fenton Hill, New Mexico. These include several bipole-dipole total field apparent resistivity reconnaissance surveys, three intersecting deep dipole-dipole soundings, numerous shallow Schlumberger soundings, and a single deep Schlumberger sounding. Figure 25 shows the survey site location on the west boundary of the Valles Caldera in northern New Mexico. Figure 26 shows the survey electrode configurations near the Hot Dry Rock Test Site marked GT-2 (for Geothermal Test Well No. 2) near the center of the map. For the purpose of the analysis, two data sets were chosen.

One was the bipole-dipole total field apparent resistivity data measured from the current source pair located on the electrode array marked T-NW R-SE in Figure 26. These were the only areal data where the potential

Figure 25. Location of resistivity study area at Los Alamos National Laboratory Hot Dry Rock Geothermal Site in New Mexico.

Figure 26. Location map showing dipole-dipole lines and bipole transmitters in the immediate vicinity of the LANL Geothermal Test Well GT-2. Area inside dashed square is test area for three-dimensional resistivity case study.

measurement sites were close enough to the current sources to allow both current and potential electrodes to be contained in a single model. The area considered for terrain effects is inside the dashed square on Figure 26. Figure 27 is a three-dimensional view of the topography from a point southwest of the area. Figure 28 is a topographic contour plot of the same area. Figure 29 is a map view contour plot of the uncorrected total field apparent resistivity data within the area.

The other data considered were the dipole-dipole measurements on the 500 meter spacing array T-NW R-SE. This was selected because it traverses the most extreme topography in the area. Figure 31 is the measured dipole-dipole data in pseudosection form.

A 125 meter map grid was chosen for the finite element array. The resulting isoparametric element surface approximating the topography is shown in Figure 27. The total array size used was $55 \times 55 \times 16 = 48,350$ elements. It was used for both the dipole-dipole pseudosection and areal bipole-dipole terrain effect calculations. The 125 meter grid was considered appropriate, since it approximates the 50 to 100 meter potential electrode spacing used for the areal survey, and allows several elements between each dipole-dipole electrode.

Figure 30 is a contour plot of the normalized predicted total field apparent resistivity terrain response calculated by the three-dimensional computer code for the

Figure 27. Three-dimensional surface map of topography inside dashed square on Figure 26. This is also the isoparametric finite element surface used to calculate the terrain effects.

model in Figure 27. The bipole-quadrupole formulation (see Figure 12) was used, rather than the bipole-dipole used for the field data. It can be shown that they are equivalent within a few percent in the area of data coverage. It is clear that the terrain effects are rather insignificant for this data set. They are typically less than 10 percent, while there is an order of magnitude variation in the data values (Figure 29) reflecting even higher resistivity contrasts in the subsurface.

Figure 28. Topographic contour map of area inside dashed square on Figure 26. Contour interval is 20 meters. Asterisks indicate source positions for total field apparent resistivity study.

Figure 29. Map view of uncorrected bipole-dipole total field apparent resistivity data within dashed square on Figure 26. Apparent resistivities (in ohm-meters) are plotted at locations of the potential measurement arrays. Contour interval is 10 ohm-meters. Asterisks indicate the current electrode positions.

Figure 30. Normalized total field apparent resistivity response calculated for a homogeneous half-space with the terrain in Figure 27 on the upper surface. Contour interval is 0.02. Asterisks indicate the current electrode positions.

LASL GT-2 DIPOLE-DIPOLE PSEUDOSECTION
SPACING=500m, ρ_0 IN OHM-M, C.I.=10 OHM-M

Figure 31. Dipole-dipole field data for line T-NW R-SE on Figure 26. Contour interval is 10 ohm-meters.

Figure 32 shows the normalized dipole-dipole terrain response for the T-NW R-SE calculated from the terrain model in Figure 27. Figure 33 is the terrain corrected pseudosection derived by dividing the appropriate values on Figure 31 by those on Figure 32. This is analogous to the method used by Fox et al. (1980). It should be noted that the terrain effect calculation was not done using the usual collinear dipole-dipole formula. The apparent resistivities were calculated with the general formula for collinear electrode pairs, using the electrode separation distances measured along the irregular surface. This best approximates the actual procedure used for surveying the array and reducing the data. Further, it is considered appropriate for most field procedures.

Figure 32. Normalized dipole-dipole apparent resistivity pseudosection calculated for a homogeneous half-space with the terrain in Figure 27 on the upper surface. Contour interval is 0.02. The indicated electrode locations are shown in Figure 26.

The dipole-dipole terrain effects are larger than those in the total field apparent calculations and cannot be considered insignificant. Further, the terrain response is complex enough that it would be extremely difficult to predict intuitively.

My original intent was to perform a subsurface model study on the corrected and uncorrected data sets. However, the difficulty and expense of performing the numerous trial-and-error solutions required is considered prohibitive for the three-dimensional finite element

Figure 33. Terrain corrected dipole-dipole pseudosection for line T-NW R-SE. Contour interval is 10 ohm-meters.

algorithm. A two-dimensional analysis was attempted, but it was found that certain characteristics of the pseudosections could not be matched. Specifically, the variation of the apparent resistivity down the diagonal from electrode pair 4-5 to the southeast on Figures 31 and 33 presented the most difficulty. The isolated high resistivity value in position 4-5 8-9 is the main problem that was not resolved with the terrain correction procedure. This could be due to three-dimensional effects in the subsurface, which is certainly possible since the area is known to be structurally complex with warm, conductive water in the permeable and fractured formations. A measurement error is also possible, but there is no evidence of this in the records.

Figure 34 is the result of an attempt to match the response of the uncorrected pseudosection (Figure 31) by a

Figure 34. Model geoelectric section, and calculated data for flat earth two-dimensional interpretation of dipole-dipole measurement on line T-NW R-SE.

trial-and-error application of the algorithm by Dey and Morrison (1976). The fit is poor, but it is clear that it matches the corrected pseudosection (Figure 33) about as well. Further modeling was not considered worthwhile because of the unresolved difficulty with the data set. The original analysis given the data set in 1978 (see Appendix C) is considered to be as comprehensive as can be justified.

NUMERICAL CALCULATIONS OF MAGNETO- TELLURIC RESPONSE

This development, as in the finite element resistivity development, follows that of Pridmore et al. (1981). The spatial field parameter variation is handled by isoparametric finite elements in the manner described previously for the resistivity problem. However, in this case the boundary value problem is for frequency domain electromagnetic fields. Consequently, the electric and magnetic field parameters, $\vec{E}$ and $\vec{H}$, are complex valued vectors. In the frequency domain, Maxwell's equations are

$$-\vec{\nabla} \times \vec{E} = \hat{z} \vec{H} + \vec{M}_s \quad (30)$$

$$-\vec{\nabla} \times \vec{H} = \hat{y} \vec{E} + \vec{J}_s \quad (31)$$

where $\hat{z} = i\omega\mu$, $\hat{y} = \sigma + i\omega\epsilon$, ω is the angular frequency, μ is the magnetic permeability, ϵ is the dielectric permittivity, and σ is the conductivity. $\vec{M}_s$ and $\vec{J}_s$ are impressed magnetic and electric sources. Since only one type of source is present for a given problem, we can eliminate either $\vec{E}$ or $\vec{H}$ and express the equations in operator form ($Lu = f$), that is

$$[\vec{\nabla} \times \frac{\vec{\nabla} \times}{\hat{y}} + \hat{z}] \vec{H} = \vec{M}_s \quad (32)$$

$$[\vec{\nabla} \times \frac{\vec{\nabla} \times}{\hat{z}} + \hat{y}] \vec{E} = \vec{J}_s \quad (33)$$

These are the domain equations which have to be solved for the electromagnetic boundary value problem.

Ideally, the electromagnetic field vectors would obey the following interelement boundary conditions: (1) tangential $\vec{E}$ and tangential $\vec{H}$ are continuous, and (2) normal component of total current density $\vec{J}$, and normal component of $\vec{E}$ are continuous. Further, where $\hat{y}$ and $\hat{z}$ are discontinuous, the normal components of $\vec{E}$ and $\vec{H}$ are discontinuous by the ratio of the discontinuities in the quantities $\hat{y}$ and $\hat{z}$, respectively. It is assumed that μ is a real constant and σ is isotropic and a function of position only, in which case the $\vec{H}$ fields are continuous, and the components of $\vec{E}$ normal to the discontinuity in conductivity are discontinuous.

In contrast to the resistivity development, in which the unknown parameter vector is the total electric potential, the most appropriate solution method for the electromagnetic problem makes use of a secondary field formulation as described, for example, by Coggon (1971). This formulation states the problem as

$$[\hat{A}_{ij}] \cdot (\hat{H}_{ip} + \hat{H}_{is}) = 0 \quad (34)$$

where $[A_{ij}]$ is the complex valued finite element matrix to be derived, $\hat{H}_{ip}$ is some known complex primary field parameter vector, and $\hat{H}_{is}$ is the unknown secondary field vector to be calculated. In this case, $\hat{H}_{ip}$ is determined from a layered earth solution calculated in the usual fashion (see Ward, 1967, pp. 117-124) from the complex plane-wave impedance. The external boundary condition on the secondary field parameter is therefore the vector

homogeneous Dirichlet condition. The earth model must also include an air layer since, in this case, the plane wave propagates through the air as well as the earth. The lateral and vertical dimensions of the model are assumed to be great enough that the contribution of scattered fields from any terrain and/or closed subsurface bodies included in the model are negligible at all bounding surfaces.

Several techniques have been used by other authors to derive a suitable functional for the finite element solution of the frequency domain electromagnetic problem. Reddy et al. (1977) used the Galerkin weighted residual method. Coggon (1971) derived a functional based on the minimization of energy in the electromagnetic fields. Pridmore et al. (1981) used the minimum theorem approach of Stakgold (1968, pp. 355-357), because of the relative ease of incorporating the boundary conditions in the resulting element matrix. These techniques do not satisfy the interelement boundary condition for discontinuity of the component of the $\vec{E}$ field normal to the boundary, as described above. For the conditions assumed, the discontinuity in normal $\vec{E}$ is equivalent to a discontinuity in the first derivative of an appropriate component of the $\vec{H}$ field as determined by equation (30). The solution techniques employed by the above authors make use of a linear basis function, to approximate each component of the $\vec{H}$ field ($\vec{H} = \hat{H}_x \vec{i} + \hat{H}_y \vec{j} + \hat{H}_z \vec{k}$) or $\vec{E}$ field ($\vec{E} = \hat{E}_x \vec{i} + \hat{E}_y \vec{j} + \hat{E}_z \vec{k}$), which can only have one value at a boundary where

there is a step change in conductivity. The solutions therefore smooth through the described discontinuities. For the general case, two values of each component are required (one for each side of the boundary) to accommodate the step change. The assumption made is that the errors introduced by this approximation are negligible for cases where (1) the interelement variation in conductivity is small or (2) the contrasting boundaries are buried. Guidelines defining where the inherent errors become significant have not been adequately developed. However, it is clear that this type of solution method cannot handle large lateral contrasts in conductivity at the earth-air interface, which includes extreme topography. Unfortunately, the approach used in this development also has the same limitations.

The derivation presented here makes use of the functional defined by the minimum theorem as did Pridmore et al. (1981) and Lee et al. (1981) in the finite element portion of their hybrid formulation. However, it includes the coordinate transformation required to allow the elements to conform to topography present in the earth-air interface.

The minimum theorem states that the minimum of the functional

$$F_n(\vec{H}) = \int_{V_n} \left[\frac{(\vec{\nabla} \times \vec{H}) \cdot (\vec{\nabla} \times \vec{H})}{\hat{y}} + \hat{z} \vec{H} \cdot \vec{H} + 2 \vec{H} \cdot \vec{M}_s \right] dV_n \quad (35)$$

provides the solution in $\vec{H}$ for the nth finite element. However, the minimum theorem is not strictly valid in this case since the operator defined by equation (32) is not a positive definite form. Nevertheless, Pridmore (1978) has proven that the solution to

$$\frac{\partial F_n(\vec{H})}{\partial \vec{H}} = 0 \quad (36)$$

is a stationary point and produces apparently correct results.

For the magnetotelluric problem there are no impressed sources (e.g., $\vec{H} \cdot \vec{M}_s = 0$). Therefore, the magnetic field functional in expanded integral form is

$$\begin{aligned} F_n(\vec{H}) = \int_{V_n} \left\{ \frac{2}{\hat{y}} \left[\left(\frac{\partial \hat{H}_z}{\partial y} - \frac{\partial \hat{H}_y}{\partial z} \right)^2 + \left(\frac{\partial \hat{H}_x}{\partial z} - \frac{\partial \hat{H}_z}{\partial x} \right)^2 \right. \right. \\ \left. \left. + \left(\frac{\partial \hat{H}_y}{\partial x} - \frac{\partial \hat{H}_x}{\partial y} \right)^2 \right] + 2\hat{z}(\hat{H}_x^2 + \hat{H}_y^2 + \hat{H}_z^2) \right\} dV_n \end{aligned} \quad (37)$$

for the nth finite element. Topography is incorporated in the same fashion as the resistivity problem, so that the functional becomes

$$\begin{aligned} F_n(\vec{H}) = \int_{V_n} \left\{ \frac{2}{\hat{y}} \left[\left(\frac{\partial \hat{H}_w}{\partial v} - \frac{\partial \hat{H}_v}{\partial w} \right)^2 + \left(\frac{\partial \hat{H}_u}{\partial w} - \frac{\partial \hat{H}_w}{\partial u} \right)^2 \right. \right. \\ \left. \left. + \left(\frac{\partial \hat{H}_v}{\partial u} - \frac{\partial \hat{H}_u}{\partial v} \right)^2 \right] + 2\hat{z}(\hat{H}_u^2 + \hat{H}_v^2 + \hat{H}_w^2) \right\} (\det J_{uvw}) dV_n \end{aligned} \quad (38)$$

In this case trial functions are of the form

$$\begin{aligned} \hat{H}_u^t &= \sum_{i=1}^g N_i(u, v, w) \hat{H}_{ui} \\ \hat{H}_v^t &= \sum_{i=1}^g N_i(u, v, w) \hat{H}_{vi} \\ \hat{H}_w^t &= \sum_{i=1}^g N_i(u, v, w) \hat{H}_{wi} \end{aligned} \quad (39)$$

where the linear shape functions, $N_i(u, v, w)$, are given by equation (7). The functional is stationary when

$$\left. \frac{\partial F_n(\vec{H})}{\partial \vec{H}} \right|_{\hat{H}_{x1}} = \left. \frac{\partial F_n(\vec{H})}{\partial \vec{H}} \right|_{\hat{H}_{y1}} = \left. \frac{\partial F_n(\vec{H})}{\partial \vec{H}} \right|_{\hat{H}_{z1}} = \dots = \left. \frac{\partial F_n(\vec{H})}{\partial \vec{H}} \right|_{\hat{H}_{x8}} = \left. \frac{\partial F_n(\vec{H})}{\partial \vec{H}} \right|_{\hat{H}_{y8}} = \left. \frac{\partial F_n(\vec{H})}{\partial \vec{H}} \right|_{\hat{H}_{z8}} \quad (40)$$

where, for example,

$$\begin{aligned} \left. \frac{\partial F_n(\vec{H})}{\partial \vec{H}} \right|_{\hat{H}_{x1}} = \int_{V_n} \left\{ \frac{2}{\gamma} \left[\frac{\partial N_1}{\partial w} \left(\frac{\partial N_1}{\partial w} \hat{H}_{u1} + \dots + \frac{\partial N_8}{\partial w} \hat{H}_{u8} \right) \right. \right. \\ \left. \left. + \frac{\partial N_1}{\partial v} \left(\frac{\partial N_1}{\partial v} \hat{H}_{u1} + \dots + \frac{\partial N_8}{\partial v} \hat{H}_{u8} \right) - 2 \frac{\partial N_1}{\partial w} \left(\frac{\partial N_1}{\partial u} \hat{H}_{v1} + \dots + \frac{\partial N_8}{\partial u} \hat{H}_{v8} \right) \right. \right. \\ \left. \left. - 2 \frac{\partial N_1}{\partial w} \left(\frac{\partial N_1}{\partial u} \hat{H}_{w1} + \dots + \frac{\partial N_8}{\partial u} \hat{H}_{w8} \right) \right] + 2 \hat{z} N_1 (N_1 \hat{H}_{u1} + \dots + N_8 \hat{H}_{u8}) \right\} \frac{\partial z}{\partial w} dV_n \end{aligned} \quad (41)$$

is the equation for the x-component of the magnetic field at node 1 in the nth finite element. Appendix B is the derivation of the finite element matrix for an equation system of the form of equation (41). The result is an element matrix with $24 \times 24 = 576$ complex valued coefficients.

If one uses the same matrix solution technique as that for the resistivity problem, the number of arithmetic operations required per iteration is 15 to 20 times greater to calculate the response for a single discrete frequency. The computing required for a complete magnetotelluric solution is therefore at least 100 times greater for a model with the same number of elements. Nevertheless, an algorithm was constructed which calculated the magnetic field for an arbitrary earth model with topography. The solution method was similar to that used for the resistivity problem but contained the following specific

differences: (1) the element matrix coefficients were calculated in a subroutine from the polynomials in summation form in Appendix B (rather than explicitly writing them as in the resistivity algorithm), and (2) a nodal assembly, analogous to that done in the resistivity problem, was performed on the 3×3 complex submatrices indicated in Figure B1 (the real number arithmetic operations defined by equations 21 and 22 in the resistivity algorithm therefore became complex matrix operations in the magnetic field algorithm).

Preliminary test runs on relatively small (less than 2500 elements) arrays had the following results. Over-relaxation was found to be unstable for this problem. The optimum starting value for the acceleration parameter was found to be about 0.5, which agrees with the findings of Pridmore (1978). The algorithm produced accurate results for the trivial, homogeneous half-space case and appeared to produce sensible results for a simple three-dimensional terrain feature, which was a cosinusoidal depression similar to that shown in Figure 20.

However, the overwhelming memory and running time requirements led to the conclusion that proper verification and evaluation of the algorithm using the iterative solution method described were far beyond the available time and resource constraints. An additional motivating factor is that there is no viable technique for testing the validity of the calculated magnetotelluric response for a

three-dimensional terrain feature. The exercise was therefore terminated at this point.

Although it was not verified, it is believed that the H-field formulation described herein will be subject to all of the theoretical and computational difficulties described in some detail by Pridmore et al. (1981). With the exception of the isoparametric element distortion described previously, the finite element matrix equation derivation, the implementation of boundary conditions, and the numerical solution methodology were essentially the same.

CONCLUSIONS

Resistivity Methods

The three-dimensional computer algorithm described has proven to be a powerful tool for resistivity interpretation. It provides an unmatched flexibility in handling complex model geometries. There have been no convergence problems with the iterative solution presently employed. The over-relaxation process is performed using potential values which remain fixed on the bounding nodes during successive iterations. The initial choice of these fixed values using the Dar Zarrouk construction is a source of absolute error in the final result. Another potential error in the method is the necessary distribution of a point current source over all eight nodes defining the element containing the source. Defining model boundaries well beyond the region of interest and defining small finite elements in the source regions have successfully diminished the boundary condition and source problems, respectively. Valid results have been confirmed by extensive comparisons to scale model measurements, analytically calculated solutions, and other independent numerical results.

The difficulty described by Pridmore et al. (1981) with the numerical solution of models containing long, thin elements has not been observed. Dimension ratios (ratio of

largest to smallest dimension of an element) of up to 50 have been tested. The use of large dimension ratios is an important consideration since it allows the model discretization to distances well-removed from regions of interest. This minimizes the possible errors due to the final boundary conditions as described above.

Computer modeling tests have illustrated some expected effects of terrain on resistivity results. In an extension of known two-dimensional effects, the apparent resistivity is anomalously high over concave upward surfaces due to current focusing and anomalously low over convex upward regions due to current dispersion. Model studies (not presented herein) of collinear electrode configurations over predominantly two-dimensional surfaces have verified a much more pronounced terrain effect when the arrays are positioned perpendicular to topography. For example, in one proprietary case study (sponsored by the United Nations Office of Technical Development), the terrain effects for a dipole-dipole survey perpendicular to a deep sinuous canyon were more than ten times the effects for arrays emplaced along the canyon. This study evaluated terrain effects in data measured along deep wadies on the eastern shore of the Dead Sea in Jordan. Here the terrain geometry was sufficiently complex that a three-dimensional analysis was essential. Further the terrain relief was more extreme (over 600 meters vertical in one six square kilometer area evaluated) than at the Fenton Hill site.

Although the terrain response has been demonstrated to be effectively uncoupled from the surface response for two-dimensional conditions (Fox et al., 1980), it is somewhat surprising that such an approximation also remains valid for realistic three-dimensional cases. This enables the removal, or stripping, of undesirable effects due to topography on resistivity surveys through the use of simple multiplicative correction factors. Such a correction method works quite well even when the terrain response is large with respect to the subsurface target response. Our experience has included instances where the terrain correction has significantly altered the subsurface interpretation. However, in the New Mexico case study presented, the terrain response is considered secondary compared to the complex subsurface effects. Even so, the recognition of this condition is an important step in interpretation of the subsurface resistivity distribution alone.

Theoretical simulations and experience with actual resistivity data obtained in areas of suspected terrain contamination have resulted in the following recommended procedure for handling such situations:

1. Compare the calculated response of the actual topography to the responses of anticipated subsurface targets.
(Two-dimensional analysis may be adequate; presurvey

- studies may prove valuable in locating optimum survey sites.)
2. Apply terrain correction factors to field data if necessary.
 3. Analyze corrected data as if it were the response of a flat earth.

One should be cautious about arbitrarily applying any guidelines relating terrain effects to the terrain geometry alone. We have observed specific cases where the effects of extreme topography were negligible, and that of moderate topography very significant to the subsurface interpretation.

The algorithm has recently been adapted to include frequency domain induced polarization calculations. This is more costly because of the multiple model solutions required, but initial testing has produced satisfactory results. The algorithm can also accommodate a model calculation with buried current sources. It is fairly efficient when used on one of the new generation large mainframe computers such as the CRAY or CDC CYBER 200 series. The cost for a 50,000 element model solution is about \$15 per source configuration on a commercially available CRAY-1/S computer. This can vary somewhat with the complexity and conductivity contrasts within the model. The computing costs for the terrain effect evaluation and correction procedure described is therefore only a small

fraction of the cost of a survey. In fact, it is significantly less than the man-hour costs required to implement it. However, detailed three-dimensional trial-and-error modeling of the subsurface is considered prohibitively difficult and expensive at the present time. The computer algorithm, however, is certainly up to the task if one wants to attempt it. The value of the algorithm therefore lies in the terrain correction procedure described, and testing of hypotheses regarding the effect of assumed three-dimensional subsurface resistivity structures on surface resistivity data. At the present time, it is the only algorithm which can calculate the effect of three-dimensional terrain and subsurface structures for earth models sufficiently complex to approximate actual field environments.

Magnetotelluric Methods

Although it was not practical to complete a useful algorithm, the insights gained point toward two possible courses which may lead to a satisfactory and reasonably efficient algorithm to calculate magnetotelluric responses for earth models with arbitrarily complex topography and subsurface structure.

One course is to develop and/or implement an existing direct matrix solution algorithm on one of the new generation large mainframe computers. As indicated in the discussion of the resistivity solution method, an

intermediate step would be to develop a suitable global matrix assembly routine which would result in the smallest possible bandwidth. Such an algorithm has the potential to reduce the computer time required by many orders of magnitude. Because of the very large amount of computer memory required for a direct solution (perhaps 30 to 40 million words, see Pridmore, 1978) a system with both a vectorizer and a large scale virtual memory capability is required. One such existing system is the Control Data Corporation CYBER 205.

The other course is to resolve the described inter-element boundary condition problem. One method is to explicitly include the correct boundary condition in the finite element matrix equations. This has been done for existing two-dimensional (K. H. Lee, personal communication, 1981) and three-dimensional (I. K. Reddy, personal communication, 1981) finite element algorithms, which do not include the isoparametric feature derived in this report. It should be noted that this process significantly increases the size of the solution vector. For example, in the general case it requires calculation of two values of the field components at each node where there is an interelement conductivity contrast. However, it is believed that a finite element formulation, containing both the isoparametric co-ordinate transformation and the described boundary condition contrivance, is the best approach to a solution to the problem. Further development

along these lines is highly recommended and is being pursued.

APPENDICES

APPENDIX A

DERIVATION OF ISOPARAMETRIC FINITE ELEMENT MATRIX FOR RESISTIVITY MODELING

The finite element matrix is derived from equations of the form (see equation 15 in the text)

$$\begin{aligned} \left. \frac{\partial F_n(\phi)}{\partial \phi} \right|_{\phi_i} = & -2 \int_{V_n} N_i (\vec{\nabla} \cdot \vec{J}_s) (\det J_{uvw}) dV_n \\ & + 2 \sigma_n \int_{V_n} \left[\frac{\partial N_i}{\partial u} \left(\frac{\partial N_1}{\partial u} + \dots + \frac{\partial N_8}{\partial u} \right) + \frac{\partial N_i}{\partial v} \left(\frac{\partial N_1}{\partial v} + \dots + \frac{\partial N_8}{\partial v} \right) \right. \\ & \left. + \frac{\partial N_i}{\partial w} \left(\frac{\partial N_1}{\partial w} + \dots + \frac{\partial N_8}{\partial w} \right) \right] (\phi_1 + \dots + \phi_8) (\det J_{uvw}) dV_n \end{aligned} \quad (A1)$$

where the shape functions are defined by

$$N_i(u, v, w) = \frac{1}{8 a^2 b^2 c^2} (a^2 + u u_i) (b^2 + v v_i) (c^2 + w w_i) \quad (A2)$$

The terms in equation (A2) are given in Figure 2 in the text. Since the coordinate transformation is in the vertical direction only

$$\det J_{uvw} = \frac{\partial z}{\partial w} \quad (A3)$$

In this case, we also use the shape function as the basis function for the transformation, that is

$$Z = \sum_{i=1}^8 N_i(u, v, w) Z_i \quad (A4)$$

where Z_i are the nodal values of Z . Therefore

$$\frac{\partial Z}{\partial w} = \frac{1}{8 a^2 b^2 c^2} \sum_{i=1}^8 (a^2 + u u_i) (b^2 + v v_i) w_i Z_i \quad (A5)$$

Note that this provides a coordinate transformation which is exact for linear elements. Since all the terms under the integrals in equation (A1) are in polynomial form, the integration may be carried out analytically.

Equation (A1) can be expressed as

$$\left. \frac{\partial F_n(\phi)}{\partial \phi} \right|_{\phi_i} = 2 \sigma_n \left\{ \sum_{i=1}^8 \phi_i \sum_{j=1}^8 (B_{ij} + C_{ij} + D_{ij}) \right\} - 2 \int_{V_n} N_i (\vec{\nabla} \cdot \vec{J}_s) \frac{\partial z}{\partial w} dV_n \quad (A6)$$

where B_{ij} , C_{ij} , and D_{ij} have the forms

$$B_{ij} = \int_{V_n} \frac{\partial N_i}{\partial u} \frac{\partial N_j}{\partial u} \frac{\partial z}{\partial w} dV_n \quad (A7)$$

$$C_{ij} = \int_{V_n} \frac{\partial N_i}{\partial v} \frac{\partial N_j}{\partial v} \frac{\partial z}{\partial w} dV_n \quad (A8)$$

$$D_{ij} = \int_{V_n} \frac{\partial N_i}{\partial w} \frac{\partial N_j}{\partial w} \frac{\partial z}{\partial w} dV_n \quad (A9)$$

and the source term on the right-hand side of equation (A6) is handled in the following manner. A point source at the surface (coordinates u_0, v_0, w_0) in element n is assumed, so that

$$\vec{\nabla} \cdot \vec{J}_s = I \delta$$

where I is the magnitude of the current source, and δ is the three-dimensional Dirac delta function at point u_0, v_0, w_0 . Therefore

$$-2 \int_{V_n} N_i I \delta_{uvw} \frac{\partial z}{\partial w} dV_n = -4 I \frac{z_i}{w_i} \quad (A10)$$

This source term is then divided equally among each of the eight nodes in element n as indicated in equation (A11) below. Each element matrix consists of eight equations (one for each node) in the form of (A6), that is

AD1	A12	A13	A14	A15	A16	A17	A18
A12	AD2	A23	A13	A16	A26	A27	A17
A13	A23	AD3	A34	A17	A27	A37	A38
A14	A13	A34	AD4	A18	A17	A38	A48
A15	A16	A17	A18	AD1	A12	A13	A14
A16	A26	A27	A17	A12	AD2	A23	A13
A17	A27	A37	A38	A13	A23	AD3	A34
A18	A17	A38	A48	A14	A13	A34	AD4

Figure A1. Finite element coefficient matrix for resistivity problem. Coefficients are given in Table A1.

$$2\sigma_n \cdot \begin{bmatrix} A_{11} & A_{12} & \dots & A_{18} \\ A_{21} & A_{22} & & \vdots \\ \vdots & & \ddots & \vdots \\ A_{81} & \dots & \dots & A_{88} \end{bmatrix} \cdot \begin{bmatrix} \phi_1 \\ \phi_2 \\ \vdots \\ \phi_8 \end{bmatrix} = \begin{bmatrix} -\frac{1}{2}I\frac{z_1}{w_1} \\ -\frac{1}{2}I\frac{z_1}{w_1} \\ \vdots \\ -\frac{1}{2}I\frac{z_8}{w_8} \end{bmatrix} \quad (A11)$$

for a point current source in the nth element, where

$$A_{ij} = B_{ij} + C_{ij} + D_{ij} \quad (A12)$$

Evaluation of each integral B_{ij} , C_{ij} , or D_{ij} is quite laborious, requiring integration of a sum of eight cubic polynomials with 64 terms each. The resulting 8 x 8 coefficient matrix has 18 independent terms, each of which is a linear combination of three of 14 independent polynomials. The final result is given in Figure A1 and Table A1.

Table A1. Table of coefficients for resistivity finite element matrix

<u>Independent Coefficients</u>	<u>Independent Polynomials</u>
AD1 = $\alpha_{11} + \beta_{11} + \gamma_{11}$	$\alpha_{11} = (b/24a) (-Z_1 - Z_2 - Z_3/3 - Z_4/3 + Z_5 + Z_6 + Z_7/3 + Z_8/3)$
AD2 = $\alpha_{11} + \beta_{22} + \gamma_{22}$	$\alpha_{13} = (b/72a) (-Z_1 - Z_2 - Z_3 - Z_4 + Z_5 + Z_6 + Z_7 + Z_8)$
AD3 = $\alpha_{33} + \beta_{22} + \gamma_{33}$	$\alpha_{33} = (b/24a) (-Z_1/3 - Z_2/3 - Z_3 - Z_4 + Z_5/3 + Z_6/3 + Z_7 + Z_8)$
AD4 = $\alpha_{33} + \beta_{11} + \gamma_{33}$	$\beta_{11} = (a/24b) (-Z_1 - Z_2/3 - Z_3/3 - Z_4 + Z_5 + Z_6/3 + Z_7/3 + Z_8)$
A12 = $-\alpha_{11} + \beta_{12} + \gamma_{13}$	$\beta_{12} = (a/72b) (-Z_1 - Z_2 - Z_3 - Z_4 + Z_5 + Z_6 + Z_7 + Z_8)$
A13 = $-\alpha_{13} - \beta_{12} + \gamma_{13}$	$\beta_{22} = (a/24b) (-Z_1/3 - Z_2 - Z_3 - Z_4/3 + Z_5/3 + Z_6 + Z_7 + Z_8/3)$
A14 = $\alpha_{13} - \beta_{11} + \gamma_{14}$	$\gamma_{11} = (ab/16c^2) (-Z_1 - Z_2/3 - Z_3/9 - Z_4/3 + Z_5 + Z_6/3 + Z_7/9 + Z_8/3)$
A15 = $\alpha_{11}/2 + \beta_{11}/2 - \gamma_{11}$	$\gamma_{12} = (ab/48c^2) (-Z_1 - Z_2 - Z_3/3 - Z_4/3 + Z_5 + Z_6 + Z_7/3 + Z_8/3)$
A16 = $-\alpha_{11}/2 + \beta_{12}/2 - \gamma_{12}$	$\gamma_{13} = (ab/144c^2) (-Z_1 - Z_2 - Z_3 - Z_4 + Z_5 + Z_6 + Z_7 + Z_8)$
A17 = $-\alpha_{13}/2 - \beta_{12}/2 - \gamma_{13}$	$\gamma_{14} = (ab/48c^2) (-Z_1 - Z_2/3 - Z_3/3 - Z_4 + Z_5 + Z_6/3 + Z_7/3 + Z_8)$
A18 = $\alpha_{13}/2 - \beta_{11}/2 - \gamma_{14}$	$\gamma_{22} = (ab/16c^2) (-Z_1/3 - Z_2 - Z_3/3 - Z_4/9 + Z_5/3 + Z_6 + Z_7/3 + Z_8/9)$
A23 = $\alpha_{13} - \beta_{22} - \gamma_{23}$	$\gamma_{23} = (ab/48c^2) (-Z_1/3 - Z_2 - Z_3 - Z_4/3 + Z_5/3 + Z_6 + Z_7 + Z_8/3)$
A26 = $\alpha_{11}/2 + \beta_{22}/2 - \gamma_{22}$	$\gamma_{33} = (ab/16c^2) (-Z_1/3 - Z_2/9 - Z_3/3 - Z_4 + Z_5/3 + Z_6/9 + Z_7/3 + Z_8)$
A27 = $\alpha_{13}/2 - \beta_{22}/2 - \gamma_{33}$	$\gamma_{34} = (ab/48c^2) (-Z_1/3 - Z_2/3 - Z_3 - Z_4 + Z_5/3 + Z_6/3 + Z_7 + Z_8)$
A34 = $-\alpha_{33} + \beta_{12} + \gamma_{34}$	
A37 = $\alpha_{33}/2 + \beta_{22}/2 - \gamma_{33}$	
A38 = $-\alpha_{33}/2 + \beta_{12}/2 - \gamma_{34}$	
A48 = $\alpha_{33}/2 + \beta_{11}/2 - \gamma_{33}$	

*2a, 2b, and 2c are the u, v, and w dimensions of the undistorted finite element. Z1, Z2, . . . Z8 are the z-coordinate of nodes 1 through 8 (see Figure 2 in the text) of the distorted element with respect to the origin at the center of the element.

APPENDIX B

DERIVATION OF ISOPARAMETRIC FINITE ELEMENT MATRIX FOR MAGNETOTELLURIC MODELING

The finite element matrix is derived from equations of the form (see equation 41 in the text)

$$\begin{aligned} \frac{\partial F_n(\vec{H})}{\partial \vec{H}} \bigg|_{\vec{H}_n} = & \int_{V_n} \left\{ \frac{2}{\hat{y}} \left[\frac{\partial N_i}{\partial w} \left(\frac{\partial N_i}{\partial w} \hat{H}_{ui} + \dots + \frac{\partial N_g}{\partial w} \hat{H}_{ug} \right) \right. \right. \\ & + \frac{\partial N_i}{\partial v} \left(\frac{\partial N_i}{\partial v} \hat{H}_{vi} + \dots + \frac{\partial N_g}{\partial v} \hat{H}_{vg} \right) - 2 \frac{\partial N_i}{\partial v} \left(\frac{\partial N_i}{\partial u} \hat{H}_{vi} + \dots + \frac{\partial N_g}{\partial u} \hat{H}_{vg} \right) \\ & \left. \left. - 2 \frac{\partial N_i}{\partial w} \left(\frac{\partial N_i}{\partial u} \hat{H}_{wi} + \dots + \frac{\partial N_g}{\partial u} \hat{H}_{wg} \right) \right] + 2 \hat{z} \left[N_i (N_i \hat{H}_{ui} + \dots + N_g \hat{H}_{ug}) \right] \right\} \frac{z}{\partial w} dV_n = 0 \end{aligned} \quad (B1)$$

The N_i are the same linear shape functions defined for the resistivity problem (see Appendix A). The coordinate transformation (e.g., $\frac{\partial z}{\partial w}$) is also the same. There is no source term in equation (B1) since we are solving the secondary field problem. Equation (B1) can be expressed as

$$\frac{\partial F_n(\vec{H})}{\partial \vec{H}} \bigg|_{\vec{H}_n} = \sum_{i=1}^g \hat{H}_{ui} \sum_{j=1}^g \hat{O}_{ij} + \sum_{i=1}^g \hat{H}_{vi} \sum_{j=1}^g \hat{P}_{ij} + \sum_{i=1}^g \hat{H}_{wi} \sum_{j=1}^g \hat{Q}_{ij} = 0 \quad (B2)$$

where $\hat{O}_{ij}$, $\hat{P}_{ij}$, and $\hat{Q}_{ij}$ are of the form

$$\begin{aligned} \hat{O}_{ij} &= \int_{V_n} \left\{ \frac{2}{\hat{y}} \left(\frac{\partial N_i}{\partial w} \frac{\partial N_j}{\partial w} + \frac{\partial N_i}{\partial v} \frac{\partial N_j}{\partial v} \right) + 2 \hat{z} N_i N_j \right\} \frac{\partial z}{\partial w} dV_n \\ \hat{P}_{ij} &= -\frac{4}{\hat{y}} \int_{V_n} \frac{\partial N_i}{\partial v} \frac{\partial N_j}{\partial u} \frac{\partial z}{\partial w} dV_n \\ \hat{Q}_{ij} &= -\frac{4}{\hat{y}} \int_{V_n} \frac{\partial N_i}{\partial w} \frac{\partial N_j}{\partial u} \frac{\partial z}{\partial w} dV_n \end{aligned} \quad (B3)$$

Similarly

$$\left. \frac{\partial F_n(\vec{H})}{\partial \vec{H}} \right|_{\hat{H}_{yi}} = \sum_{i=1}^g \hat{H}_{ui} \sum_{j=1}^g \hat{R}_{ij} + \sum_{i=1}^g \hat{H}_{vi} \sum_{j=1}^g \hat{S}_{ij} + \sum_{i=1}^g \hat{H}_{wi} \sum_{j=1}^g \hat{P}_{ij} = 0 \quad (\text{B4})$$

where

$$\hat{R}_{ij} = \int_{V_n} \left\{ \frac{2}{\hat{y}} \left(\frac{\partial N_i}{\partial u} \frac{\partial N_j}{\partial u} + \frac{\partial N_i}{\partial w} \frac{\partial N_j}{\partial w} \right) + 2 \hat{z} N_i N_j \right\} \frac{\partial z}{\partial w} dV_n$$

$$\hat{S}_{ij} = -\frac{4}{\hat{y}} \int_{V_n} \frac{\partial N_i}{\partial v} \frac{\partial N_j}{\partial w} \frac{\partial z}{\partial w} dV_n \quad (\text{B5})$$

and

$$\left. \frac{\partial F_n(\vec{H})}{\partial \vec{H}} \right|_{\hat{H}_{zi}} = \sum_{i=1}^g \hat{H}_{ui} \sum_{j=1}^g \hat{T}_{ij} + \sum_{i=1}^g \hat{H}_{vi} \sum_{j=1}^g \hat{S}_{ij} + \sum_{i=1}^g \hat{H}_{wi} \sum_{j=1}^g \hat{Q}_{ij} = 0 \quad (\text{B6})$$

where

$$\hat{T}_{ij} = \int_{V_n} \left\{ \frac{2}{\hat{y}} \left(\frac{\partial N_i}{\partial v} \frac{\partial N_j}{\partial v} + \frac{\partial N_i}{\partial u} \frac{\partial N_j}{\partial u} \right) + 2 \hat{z} N_i N_j \right\} \frac{\partial z}{\partial w} dV_n \quad (\text{B7})$$

Evaluation of integrals $\hat{O}_{ij}$, $\hat{P}_{ij}$, $\hat{Q}_{ij}$, $\hat{R}_{ij}$, $\hat{S}_{ij}$ are $\hat{T}_{ij}$ are very laborious, but straightforward because of the polynomial form of the shape function, N_i . The final results are given in Figure B1 and Table B1.

A11	A12	A13	A14	A15	A16	A17	A14	A14	A110	A111	A111	A113	A114	A13	A116	A117	A16	A119	A120	A14	A122	A114	A12
A11	A22	A12	A110	A12	A15	A17	A12	A15	A14	A13	A12	A113	A117	A114	A122	A15	A120	A119	A15	A117	A16	A13	A113
A31	A12	A13	A34	A15	A16	A37	A15	A15	A34	A12	A12	A313	A114	A13	A316	A117	A16	A319	A120	A14	A316	A114	A12
A14	A15	A16	A11	A15	A16	A110	A14	A14	A17	A111	A111	A116	A114	A13	A113	A117	A16	A122	A120	A14	A119	A13	A113
A110	A12	A15	A11	A13	A15	A14	A13	A15	A14	A13	A15	A17	A11	A11	A113	A16	A120	A116	A117	A120	A119	A15	A114
A34	A15	A16	A31	A15	A16	A34	A15	A15	A37	A12	A12	A34	A114	A13	A313	A120	A16	A316	A120	A15	A316	A114	A12
A17	A14	A14	A110	A14	A14	A11	A12	A16	A14	A15	A13	A119	A117	A12	A122	A114	A15	A113	A114	A16	A116	A117	A13
A17	A12	A15	A14	A13	A15	A11	A16	A12	A110	A15	A15	A119	A12	A117	A116	A13	A114	A113	A13	A114	A122	A114	A117
A37	A15	A15	A34	A15	A15	A31	A12	A16	A34	A15	A13	A319	A117	A12	A316	A113	A15	A313	A113	A16	A316	A117	A13
A110	A111	A111	A17	A111	A111	A14	A15	A13	A11	A15	A13	A122	A117	A12	A119	A12	A114	A116	A114	A16	A113	A117	A13
A14	A13	A12	A14	A13	A15	A110	A15	A15	A11	A16	A15	A116	A13	A117	A119	A12	A114	A122	A12	A114	A113	A13	A117
A34	A12	A12	A37	A12	A12	A34	A15	A13	A31	A15	A13	A316	A117	A12	A319	A114	A15	A316	A114	A16	A313	A117	A13
A113	A114	A13	A116	A114	A13	A119	A117	A12	A122	A117	A12	A11	A12	A16	A14	A15	A13	A17	A14	A12	A110	A111	A15
A113	A117	A114	A17	A11	A11	A119	A12	A116	A11	A13	A117	A11	A16	A12	A110	A15	A15	A17	A15	A15	A14	A16	A12
A313	A114	A13	A34	A114	A13	A319	A117	A12	A316	A117	A12	A31	A12	A16	A34	A15	A13	A37	A15	A12	A34	A12	A15
A116	A117	A16	A113	A117	A16	A122	A114	A15	A119	A12	A114	A14	A15	A13	A11	A15	A13	A110	A15	A12	A17	A12	A15
A122	A15	A120	A113	A16	A120	A116	A13	A114	A119	A12	A114	A110	A15	A15	A11	A16	A15	A14	A16	A15	A17	A15	A12
A316	A117	A16	A313	A120	A16	A316	A113	A15	A319	A114	A15	A34	A15	A13	A31	A15	A13	A34	A15	A12	A37	A12	A15
A119	A120	A14	A122	A120	A14	A113	A114	A16	A116	A114	A16	A17	A14	A12	A110	A15	A12	A11	A114	A13	A116	A117	A16
A119	A15	A117	A116	A117	A120	A113	A13	A114	A122	A12	A114	A17	A15	A15	A14	A16	A15	A11	A13	A12	A110	A111	A15
A319	A120	A14	A316	A120	A15	A313	A113	A16	A316	A114	A16	A37	A15	A12	A34	A15	A12	A31	A12	A13	A34	A15	A16
A122	A114	A12	A119	A13	A113	A116	A117	A13	A113	A117	A13	A110	A111	A15	A17	A12	A15	A116	A117	A16	A11	A15	A16
A116	A13	A113	A119	A15	A114	A122	A114	A117	A113	A13	A117	A14	A16	A12	A17	A15	A12	A110	A111	A15	A11	A13	A15
A316	A114	A12	A316	A114	A12	A316	A117	A13	A313	A117	A13	A34	A12	A15	A37	A12	A15	A34	A15	A16	A31	A15	A16

Figure B1. Finite element coefficient matrix for magneto-telluric problem. Coefficients are given in Table B1.

Table B1. Table of coefficients for magnetotelluric finite element matrix

<u>Coefficient</u>	<u>Integral</u>
A11	$\hat{O}_{11}$
A12	$\hat{P}_{11}$
A13	$\hat{Q}_{11}$
A14	$\hat{O}_{12}$
A15	$\hat{P}_{12}$
A16	$\hat{Q}_{12}$
A17	$\hat{O}_{13}$
A110	$\hat{O}_{14}$
A111	$\hat{P}_{14}$
A113	$\hat{O}_{15}$
A114	$\hat{P}_{15}$
A116	$\hat{O}_{16}$
A117	$\hat{P}_{16}$
A119	$\hat{O}_{17}$
A120	$\hat{P}_{17}$
A122	$\hat{O}_{18}$
A22	$\hat{S}_{11}$
A31	$\hat{T}_{11}$
A34	$\hat{T}_{12}$
A37	$\hat{T}_{13}$
A313	$\hat{T}_{15}$
A316	$\hat{T}_{16}$
A319	$\hat{T}_{17}$

APPENDIX C

PRELIMINARY RESISTIVITY CASE STUDY

Computer Modeling of Dipole-Dipole and Schlumberger Sounding Data at the LASL Hot Dry Rock Geothermal Site, Fenton Hill, New Mexico

This preliminary study was done in 1978 and makes use of the two-dimensional resistivity modeling computer algorithm developed by Dey and Morrison (1976). The assumptions for this interpretation made are that (1) the earth-air interface is flat, and (2) that all subsurface resistivity structure illustrated extend to infinity along the strike direction.

Data from all three dipole-dipole lines were initially modeled using the flat earth two-dimensional computer program (Dey and Morrison, 1976). The objectives were as follows:

1. To systematically produce model two-dimensional geoelectric sections which are consistent with one another and match surface data as well as possible, using only the surface resistivity data.
2. To determine how well the model depths to electrical basement are resolved.

Previous studies of this type have been done by numerous authors, for example, Beyer (1977) at Grass Valley, Nevada. Prior analyses of part of the data were

LASL GT-2 DIPOLE-DIPOLE PSEUDOSECTION
 $X=500m$, P_0 IN OHM-M, $C.I.=10$ OHM-M

Figure C1. Dipole-dipole field data for line T-N R-S (from Kintzinger and Jiracek, 1976).

reported by Jiracek (1975), Jiracek et al. (1976), and Kintzinger and Jiracek (1976). These included flat earth two-dimensional model studies from two of the lines (T-N R-S and T-W R-E in Figure 26) which are reproduced in Figure C1, C2, C3, and C4.

As a starting point for the two-dimensional models, a simple layered model was derived from the deep Schlumberger data shown as open circles in Figure C5. First a one-dimensional numerical inversion was performed. The calculated layer depths and resistivity values were then adjusted by trial and error to fit the cell grid to be used

Figure C2. Model geoelectric section and calculated data for flat earth two-dimensional interpretation for line T-N R-S (from Kintzinger and Jiracek, 1976).

Figure C3. Dipole-dipole field data for line T-W R-E (from Kintzinger and Jiracek, 1976).

for the two-dimensional models. The resulting adjusted model and calculated response are also shown in Figure C5. The layered model was then modified by inserting two-dimensional bodies having different resistivities in an attempt to fit the dipole-dipole data sets. The deep layers (below 750 m) obtained in the one-dimensional model were left undisturbed because repeated trial and error has shown that the deep resolution of this resistivity structure is poor. Changes in the computed model show up as subtle, widely distributed changes in the calculated

Figure C4. Model geoelectric section and calculated data for flat earth two-dimensional interpretation for line T-W R-E (from Kintzinger and Jiracek, 1976).

Figure C5. Adjusted one-dimensional numerical inversion results for Schlumberger sounding measured on line T-NW R-SE.

surface data. Such changes are unresolved in actual field data.

Matching the surface data exactly is very difficult, since each change affects the current distribution in the entire model in complex ways. The models presented are the result of no more than 15 successive attempts.

Figure C6 shows the results of an attempted refinement of the T-N R-S model shown in Figure 22. Obviously the data are not matched exactly in either case. However, these results are typical of data fits obtained by other

Figure C6. Model geoelectric section and calculated data for flat earth two-dimensional interpretation of dipole-dipole measurements on line T-N R-S (results of this study).

authors (e.g., Beyer, 1977). The difference between the geoelectric sections in Figure C2 and C6 are mostly due to the constraint imposed by leaving undisturbed in Figure C6 the deep layers derived from the one-dimensional analysis. The only improvement in the recent model is that the resistive anomaly on the lower right of the calculated pseudosection has closed contours, as does the actual data. This is due to a more conductive zone (10 ohm-m) added to the far right of the geoelectric section.

Figure C7 shows the results of an attempted refinement of the TW-RE model in Figure C4. Again the differences in the geoelectric sections are due to different constraints. In this case there is no obvious improvement in the data fit.

The only previously unpublished dipole-dipole data (Figure C8) were taken along the T-NW R-SE line as was the Schlumberger sounding data used for the one-dimensional analysis. As a check for consistency, the two-dimensional modeling program was used to generate Schlumberger sounding data from the geoelectric section in Figure C9, to see if it matched the actual data. The comparison is given in Figure C10 where a significant disparity in the observed and calculated results is obvious. In an attempt to explain this mismatch some additional models were run. Relatively small conductive bodies were added to the geoelectric section (Figure C9) at the surface at the GT-2/EE-1 site. The rest of the geoelectric section was

Figure C7. Model geoelectric section and calculated data for flat earth two-dimensional interpretation of dipole-dipole measurements on line T-W R-E (results of this study).

CALCULATED DIPOLE-DIPOLE PSEUDOSECTION
SPACING = 500 m, ρ_s IN OHM-M, C.I. = 10 OHM-M

Figure C9. Model geoelectric section and calculated data for flat earth two-dimensional interpretation of dipole-dipole measurement on line T-NW R-SE.

Figure C10. Comparison of Schlumberger sounding data to calculated data from two-dimensional geoelectric section in Figure C9.

drilling fluid were located near geothermal well GT-2 (Figure 26). Isolated low resistivity zones due to local saturations may have caused biasing of the Schlumberger sounding data.

Because of the obvious complexity of subsurface resistivity structure, there is no satisfactory way to determine how accurately trial and error modeling can predict actual depths to a desired horizon. No technique currently exists to provide geometric parameter correlation information for two-dimensional resistivity models. However, an attempt is made here to predict the sensitivity of the model depth to electrical basement (the 100 ohm-m layer

Case A: Conductive body is 250 m wide by 62.5 m deep.

Case B: Conductive body is 500 m wide by 62.5 m deep.

Figure C11. Biasing of calculated Schlumberger sounding data due to local conductive bodies near the current sources.

shown at 1250 meters depth in all the models) suggested by the surface data. The following assumptions are made:

1. The accuracy of measurement of apparent resistivity data is $\pm 5\%$.
2. The resistivity structure of the overburden is that given in Figure C9.

Two additional models were run, one with the 30 to 100 ohm-meter interface at 1000 meters depth and another at 1500 meters depth. The rest of the geoelectric section is unchanged from Figure C9. The Schlumberger results are shown in Figure C12. Calculated data for the 1000 meter depth fall slightly outside the 5% uncertainty for both Schlumberger and dipole-dipole. However, all data for the 1500 meter depth falls within the uncertainty. From the assumptions made, therefore, the interface appears to be greater than about 1000 meters.

A similar study done on the 10 to 30 ohm-meter interface at 750 meters suggests that it falls between about 650 and 850 meters for a 5% measurement uncertainty. Note that these predictions are meaningful only for the resistivity structure given in Figure C9.

The following general correlations can be made between the new model sections (Figure C6, C7, and C9) and what is known from test well logging, seismic studies, and surface geology (see Figure C13, C14, and C15). The high resistivity zones (100 ohm-meters or greater) in the

Figure C12. Calculated Schlumberger sounding results showing sensitivity of depth to basement in Figure C5. Solid line is data calculated from geoelectric section, dashed lines represent a $\pm 5\%$ uncertainty. Open triangles (Δ) are calculated response for basement (30 to 100 ohm-meter interface) at 1000 meter depth. Open circles (O) are calculated response for basement at 1500 meters.

upper 200 meters corresponds approximately to the recent surface volcanics (the Bandelier and Paliza Canyon formations). The intermediate resistivity zones (about 30 ohm-meters) between about 200 and 500 meters depth correspond to the local Permian section (Abo formation). The low resistivity zone, 10 ohm-meters between 550 and 750 meters depth corresponds approximately to the Pennsylvanian section (Madera and Sandia formations). The 30 to 10

Figure C13. Interpreted east-west geologic section through the resistivity case study area (reproduced from Purtymun, 1973).

ohm-meters interface at 500 meters depth also corresponds approximately to the local permanent water table depth, so the coincidence with the Pennsylvanian section may be fortuitous.

The 10 to 30 ohm-meters interface at 750 meters corresponds with the depth to Precambrian basement. Given the assumptions made the dipole-dipole measurements are consistent with the interface being between 650 and 850 meters. The actual depth at GT-2 and EE-1 is 730 meters.

Figure C14. (cont.)

Figure C14. (cont.)

Figure C14. (cont.)

Figure C15. Map of case study area showing location of faults determined from surface mapping (Smith et al., 1970), seismic data (Kintzinger et al., 1978) and resistivity interpretation. Solid lines are faults determined from surface and seismic data. Dashed lines are faults inferred from seismic data. Dash-dot (— · — · —) line is fault inferred from resistivity data.

The Pennsylvanian section is made up of marine sediments and the PreCambrian of granitic rocks. The model apparent resistivities are much lower than one would expect if these formations were dry, contiguous units. Core samples in the PreCambrian show evidence of fracturing and hydrothermal alteration. Therefore it is reasonable to expect that the bulk apparent resistivities (on a scale of 10 to perhaps hundreds of meters) would be very low in the presence of abundant warm, mineralogical water.

The interface at 1250 meters, although poorly resolved, is probably due to the reduction in fracture porosity indicated by the well logs.

One can make the following specific correlations for the T-W R-E line (Figure C7). A low resistivity zone, which extends upward near the surface about one kilometer east of GT-2, coincides with a previously mapped north-south trending fault. A similar feature in the T-NW R-SE section (Figure C9) immediately northwest of GT-2 coincides with east-west trending fault features indicated by the seismic study. A third such feature southwest of GT-2 coincides with the north wall of Lake Fork Canyon, where no fault has previously been mapped. Further the high resistivity surface layer apparently becomes much thicker to the southeast. The relative elevations of outcrops of Permian rocks in San Antonio Canyon and San Diego Canyon are also consistent with a significant dropdown of subsurface formations to the southeast. The electric and

surface data are therefore consistent with a significant covered fault trending approximately east-west and coincident with Lake Fork Canyon.

Another major feature of the geoelectric section, e.g., large near surface high resistivity anomalies, coincide with the relatively flat bottom of both Lake Fork Canyon (T-NW R-SE line) and San Antonio Canyon (T-W R-E line). These correspond to no known geologic feature. A third feature, e.g., low-resistivity anomalies at intermediate depths, is evident east of San Antonio Canyon (T-W R-E line) and southwest of Lake Fork Canyon (T-N R-S line). The former may be a subsurface manifestation of the ring fracture, since it is under the resurgent domes immediately east of San Antonio Canyon. The latter corresponds to no known feature.

If one accepts the assumptions made in the analyses presented, the following statements about the utility of such a study can be made.

1. The technique may be useful for locating near surface geologic structure, if that structure is approximately perpendicular to the electrode array line.
2. It may also be useful in determining depths to electric basement, although in this case such a depth estimate is poorly resolved.

One must use caution, however, when evaluating two-dimensional interpretations. The three geoelectric

sections presented were fairly consistent with one another, despite the obvious three-dimensional subsurface resistivity structure. The degree to which three-dimensional structure affects a two-dimensional analysis is not fully understood at this time.

APPENDIX D

COMPUTER PROGRAM DOCUMENTATION AND LISTINGS

Instructions for Use of FORTRAN Algorithm RES3T

This program calculates the DC resistivity responses for a three-dimensional earth model with arbitrary topography and subsurface resistivity structure. It uses the finite element solution method described in detail in the previous sections. It can calculate responses for a variety of electrode configurations. There are two principal arrays. One is an areal total field apparent resistivity array for either a surface or subsurface source or pair of sources. The other is a collinear surface array which calculates responses for the dipole-dipole configuration.

The total field apparent resistivities are calculated using the bipole-quadrupole formulation (Figure 12 in the text) for a source pair, or a pole-quadrupole formulation (Figure 7 in the text) for a single source. The grid spacing is arbitrary. The separation distances used for the total electric field calculation are the slope distances determined from a projection of a regularly spaced map grid onto the actual surface.

The calculated dipole-dipole data are presented in pseudosection form. They are plotted at a point making a 45 degree angle with the centers of the source and receiver

pairs. The general form of the apparent resistivity formula is used (rather than that for a fixed spacing array) and, when topography is included, the slope distances are used for the calculation. The location of the sources and receiving pairs are arbitrary as long as they coincide with element locations. The code is limited to 7 source positions (6 source pairs for dipole-dipole) and 12 receiving dipoles. This represents a practical maximum for a reasonably sized three-dimensional model ($55 \times 55 \times 21 = 63,525$ nodes). The manner in which the data are entered into the program allows arbitrary grid spacing. The following guidelines are recommended for the best calculated response accuracy:

1. The grid spacing (horizontal and vertical) should be uniform in the area containing the source(s) and significant model structure. The total field apparent resistivity calculation assumes a uniform horizontal grid spacing.
2. The grid spacing should be graded with increasing increments near the edges of the model. However, the change in any element dimension for adjacent elements should not exceed a factor of two. The maximum ratio of largest to smallest element dimension should not exceed 50.
3. The spacing should be chosen so that (a) it is less than the potential electrode spacing used for total field

apparent resistivity measurements, or (b) there are at least four elements between elements containing electrodes in the dipole-dipole array.

Because of the nature of the boundary conditions employed (described in detail in the text), there is a chance for errors in the calculated responses. It is therefore important that the largest practical model be used to ensure that possibly incorrect boundary values are imposed as far away from the area of interest as possible.

Topography is introduced into the program by an array of surface elevations at specified node locations. Subsurface structure is included by (1) defining a horizontally layered primary model, and (2) superimposing closed rectangular prism bodies to approximate additional structure.

The actual form of the input data is as follows:

Line 1: (FORMAT (20A4)) TITLE

TITLE = arbitrary 80 character alphanumeric title

Line 2: (FORMAT (6I3)) NTOP, NDD, NTF, NBP, NIT

NTOP = 1 topography is included in model
= 0 no topography (flat earth)

NDD = 1 dipole-dipole geometry
= 0 geometry other than dipole-dipole

NTF = 1 total field apparent resistivity geometry
= 0 geometry other than total field apparent resistivity

NBP = 1 bipole-quadrupole geometry
= 0 pole-quadrupole geometry

NIT = 1 maximum number of matrix solution iterations allowed

Line 3: (FORMAT (6I3)) NX, NY, NZ, NLAYR, NZON, NLAYER

NX = number of nodes in x direction (longest dimensions of model for collinear arrays)
 NY = number of nodes in y direction
 NZ = number of nodes in z direction
 NLAYR = number of layers in primary model (up to five)
 NZON = number of closed rectangular prism bodies, including the layers in the primary model, to be added to the model
 NLAYER = number of element layers in which topography is to be included by distorting the elements (should be enough so that the depth to the bottom layer is at least twice the maximum relief in the elevation array)

Next NLAYR line (FORMAT (3F8.0)) (TL(I), RHOL(I), I=1, NLAYR)

TL(I) = layer thicknesses in meters from top to bottom (must be consistent with element boundaries)
 RHOL(I) = layer resistivities in ohm-meters from top to bottom

NOTE: If primary model is homogeneous half-space, use NLAYR=1, TL(1) = some thickness equal or greater than the maximum value of Z(K) below.

Next line (only needed if NTF = 1 and NBP = 1)
 (FORMAT (6I3)) NYS1, NXS1, NYS2, NXS2

NYS1 = y coordinate of current source one
 NXS1 = x coordinate of current source one
 NYS2 = y coordinate of current source two
 NXS2 = x coordinate of current source two

Next line (only needed if NTF = 1 and NBP = 0)
 (FORMAT (6I3)) NYS1, NXS1

NYS1 = y coordinate of single current source
 NXS1 = x coordinate of single current source

Next two lines (only needed if NDD = 1)
 (FORMAT (7 (I3, 1X, I3, 2X)) (NYR(J), NXR(J), J = 1, 13)

NYR(J) = array of y coordinates of receiving electrodes (NYR(4) through NYR(10) must be same as NYS(1) through NYS(7))

NXR(J) = array of x coordinates of receiving electrodes (NXR(4) through NXR(10) must be same as NXS(1) through NXS(7))

Next NZ/8 lines: (FORMAT (8F10.0)) (Z(K), K = 1, NZ)

Z(K) = array of z coordinates (in meters) of element nodes in undistorted model

Next NX/8 lines: (FORMAT (8F10.0)) (X(J), J = 1, NX)

X(J) = array of x coordinates (in meters) of element nodes

Next NY/8 lines: (FORMAT (8F10.0)) (Y(I), I = 1, NY)

Y(I) = array of y coordinates (in meters) of element nodes

Next $\frac{NX \cdot NY}{15}$ lines (only needed if NTOP = 1):

(FORMAT (1X, 15F5.0)) ((ELEV(I,J), I = 1, NY),
 J = 1, NX)

ELEV(I,J) = array of surface elevations in meters corresponding to map view surface node locations

Next NZON lines (FORMAT (6I3), 2F4.0)) NXB, NXN, NYB, NYN,
 NZB, NZN, S

NXB, NXN = nodal x coordinates for opposite corners of closed rectangular prism bodies to be included in model. NXN must be greater than NXB, first NLAYR sets must be 1, NX-1.

NYB, NYN = nodal y coordinates for opposite corners of closed rectangular prism bodies to be included in model. NYN must be greater than NYB. First NLAYR sets must be 1, NY-1.

NZB, NZN = nodal z coordinates for opposite corners of closed rectangular prism bodies to be included in model. First NLAYR sets must correspond to layered primary model. NZN must be greater than NZB.

S = resistivity of closed rectangular prism bodies corresponding to coordinates above. When bodies are superimposed, value stored is the one most recently read in.

PROGRAM RES3T - FORTRAN LISTING

```
PROGRAM RES3T(INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT)
```

```
PROGRAM RES3T
```

```
AUTHOR: H. T. HOLCOMBE
```

```
SEPTEMBER, 1982
```

```

* THIS PROGRAM CALCULATES THE DC APPARENT RESISTIVITY RESPONSE FOR THREE-
* DIMENSIONAL EARTH MODELS WITH ARBITRARY TOPOGRAPHY AND SUBSURFACE
* RESISTIVITY STRUCTURE. IT USES THE FINITE ELEMENT SOLUTION METHOD.
* IT CAN CALCULATE RESPONSES FOR A VARIETY OF ELECTRODE CONFIGURA-
* TIONS. THERE ARE TWO PRINCIPAL ARRAYS. ONE IS AN AREAL TOTAL FIELD
* APPARENT RESISTIVITY ARRAY FOR A PAIR OF SURFACE SOURCES. THE
* OTHER IS A COLLINEAR SURFACE ARRAY WHICH CALCULATES RESPONSES FOR
* THE DIPOLE-DIPOLE CONFIGURATION.

```

```
THE FORM OF THE INPUT DATA IS AS FOLLOWS:
```

```

LINE 1: FORMAT(20A4) TITLE
        TITLE = ARBITRARY 80 CHARACTER ALPHANUMERIC TITLE

```

```

LINE 2: FORMAT(6I3) NTOP,NDD,NTF,NBP,NIT
        NTOP=1 TOPOGRAPHY IS INCLUDED IN MODEL
             =2 NO TOPOGRAPHY (FLAT EARTH)
        NDD=1 DIPOLE-DIPOLE GEOMETRY
             =0 GEOMETRY OTHER THAN DIPOLE-DIPOLE
        NTF=1 TOTAL FIELD APPARENT RESISTIVITY GEOMETRY
             =0 GEOMETRY OTHER THAN TOTAL FIELD
        NBP=1 BIPOLE-QUADRUPOLE TOTAL FIELD APPARENT RESISTIVITY
             =0 POLE-QUADRUPOLE TOTAL FIELD APPARENT RESISTIVITY
        NIT=MAXIMUM NO. OF RELAXATION ITERATIONS ALLOWED

```

```

LINE 3: FORMAT(6I3) NX,NY,NZ,NLAYR,NZON,NLAYER
        NX = NUMBER OF NODES IN X DIRECTION (LONGEST DIMENSION OF
             MODEL FOR COLLINEAR ARRAYS)
        NY = NUMBER OF NODES IN Y DIRECTION
        NZ = NUMBER OF NODES IN Z DIRECTION
        NLAYR = NUMBER OF LAYERS IN PRIMARY MODEL (UP TO FIVE)
        NZON = NUMBER OF CLOSED RECTANGULAR PRISM BODIES, INCLUDING
             THE LAYERS IN THE PRIMARY MODEL, TO BE ADDED TO THE
             MODEL
        NLAYER = NUMBER OF ELEMENT LAYERS IN WHICH TOPOGRAPHY IS TO
             BE INCLUDED BY DISTORTING THE ELEMENTS (SHOULD BE
             ENOUGH SO THAT THE DEPTH TO THE BOTTOM LAYER IS AT
             LEAST TWICE THE MAXIMUM RELIEF IN THE ELEVATION
             ARRAY

```

```

NEXT NLAYR LINES: FORMAT(3F8.0) (TL(I),RHOL(I),I=1,NLAYR)
      TL(I) = LAYER THICKNESSES IN METERS FROM TOP TO BOTTOM
              (MUST BE CONSISTENT WITH ELEMENT BOUNDARIES)
      RHOL(I) = LAYER RESISTIVITIES IN OHM-METERS FROM TOP TO
              BOTTOM
NOTE: IF PRIMARY MODEL IS HOMOGENOUS HALF-SPACE, USE NLAYR=1,
      TL(1)=SOME THICKNESS EQUAL TO OR GREATER THAN THE
      MAXIMUM VALUE OF Z(K) BELOW.

NEXT LINE (ONLY NEEDED IF NTF=1 AND NBP=1)
FORMAT(6I3) NYS1,NXS1,NYS2,NXS2
      NYS1= Y COORDINATE OF CURRENT SOURCE ONE
      NXS1= X COORDINATE OF CURRENT SOURCE ONE
      NYS2= Y COORDINATE OF CURRENT SOURCE TWO
      NXS2= X COORDINATE OF CURRENT SOURCE TWO

NEXT LINE (ONLY NEEDED IF NTF=1 AND NBP=0)
FORMAT(6I3) NYS1,NXS1
      NYS1= Y COORDINATE OF SINGLE CURRENT SOURCE
      NXS1= X COORDINATE OF SINGLE CURRENT SOURCE

NEXT LINE (ONLY NEEDED IF NED=1)
FORMAT(7(I3,1X,I3,2X)) (NYS(J),NXS(J),J=1,7)

      NYS(J)= ARRAY OF Y COORDINATES OF CURRENT ELECTRODES
      NXS(J)= ARRAY OF X COORDINATES OF CURRENT ELECTRODES

NEXT TWO LINES (ONLY NEEDED IF NED=1)
FORMAT(7(I3,1X,I3,2X)) (NYR(J),NXR(J),J=1,13)

      NYR(J)= ARRAY OF Y COORDINATES OF RECEIVING ELECTRODES
              (NYR(4) THROUGH NYR(10) MUST BE SAME AS NYS(1)
              THROUGH NYS(7))
      NXR(J)= ARRAY OF X COORDINATES OF RECEIVING ELECTRODES
              (NXR(4) THROUGH NXR(10) MUST BE SAME AS NXS(1)
              THROUGH NXS(7))

NEXT NZ/8 LINES: FORMAT(8F10.0) (Z(K),K=1,NZ)
      Z(K)= ARRAY OF Z COORDINATES (IN METERS) OF ELEMENT NODES
            IN UNDISTORTED MODEL

NEXT NX/8 LINES: FORMAT(8F10.0) (X(J),J=1,NX)
      X(J)= ARRAY OF X COORDINATES (IN METERS) OF ELEMENT NODES

NEXT NY/8 LINES: FORMAT(8F10.0) (Y(I),I=1,NY)
      Y(I)= ARRAY OF Y COORDINATE (IN METERS) OF ELEMENT NODES

NEXT (NX*NY)/15 LINES (ONLY NEEDED IF NTOP=1):
FORMAT(1X,15F5.0) ((ELEV(I,J),I=1,NY),J=1,NX)

      ELEV(I,J)= ARRAY OF SURFACE ELEVATIONS IN METERS CORRESPOND-
              ING TO MAP VIEW SURFACE NODE LOCATIONS

NEXT NZON LINES: FORMAT(6I3,2F4.0) NXB,NXN,NYB,NYN,NZB,NZN,S

```

* NXB, NXN= NODAL X COORDINATES FOR OPPOSITE CORNERS OF CLOSED *
 * RECTANGULAR PRISM BODIES TO BE INCLUDED IN MODEL. *
 * NXN MUST BE GREATER THAN NXB, FIRST NLAYR SETS *
 * MUST BE 1, NX-1. *
 * NYB, NYN= NODAL Y COORDINATES FOR OPPOSITE CORNERS OF CLOSED *
 * RECTANGULAR PRISM BODIES TO BE INCLUDED IN MODEL. *
 * NYN MUST BE GREATER THAN NYB, FIRST NLAYR SETS *
 * MUST BE 1, NY-1. *
 * NZB, NZN= NODAL Z COORDINATES FOR OPPOSITE CORNERS OF CLOSED *
 * RECTANGULAR PRISM BODIES TO BE INCLUDED IN MODEL. *
 * FIRST NLAYR SETS MUST CORRESPOND TO LAYERED PRIMARY *
 * MODEL. NZN MUST BE GREATER THAN NZB. *
 * S= RESISTIVITY OF CLOSED RECTANGULAR PRISM BODIES CORRES- *
 * PONDING TO COORDINATES ABOVE. WHEN BODIES ARE SUPERIMPOSED *
 * VALUE STORED IS ONE MOST RECENTLY READ IN. *

COMMON ARRAYS: BLANK,ARRA1,ARRA2,ARRA3

* BLANK - CONTAINS COEFFICIENT ARRAYS FOR UNDISTORTED (FLAT *
 * EARTH) MODEL *
 * ARRA1 - CONTAINS PRIMARY (PH(I,J,K)) AND SECONDARY (PHS(I,J,K)) *
 * ELECTRIC POTENTIAL ARRAYS. THIS IS LARGEST COMMON BLOCK. *
 * ARRA2 - CONTAINS ARRAY OF RESISTIVITIES FOR EACH ELEMENT *
 * ARRA3 - CONTAINS ADDITIONAL COEFFICIENT ARRAYS FOR DISTORTED *
 * MODEL *

TO INCREASE OR DECREASE MAXIMUM DIMENSION OF FINITE ELEMENT MODEL
MAKE THE FOLLOWING CHANGES:

* MAIN PROGRAM - ADJUST I AND J DIMENSION OF ALL COEFFICIENT ARRAYS *
 * IN BLANK COMMON AND ARRA3. ADJUST I, J AND K DIMENSION OF ARRAYS IN *
 * ARRA2 AND ARRA3. ADJUST DIMENSIONS FOR ARRAYS X, Y, Z, ELEV, ELEV1, *
 * TFR, AND PHA. NOTE THAT I, J, K CORRESPONDS TO Y, X, Z DIMENSIONS OF *
 * FINITE ELEMENT MODEL. *

* SUBROUTINE TFRGEN - ADJUST DIMENSIONS OF ARRAYS PHA, X, Y, AND *
 * ELEV TO CORRESPOND TO ARRAY DIMENSION IN MAIN PROGRAM. *

* SUBROUTINE FESOL - ADJUST COMMON BLOCK DIMENSIONS TO MATCH MAIN *
 * PROGRAM. *

* SUBROUTINE DIPDIP - NO CHANGE *

```

COMMON//AD1(55,55,2),A12(55,55,2),A13(55,55,2),
1A14(55,55,2),A15(55,55,2),A16(55,55,2),A17(55,55,2),
1A18(55,55,2)
COMMON/ARRA1/PH(55,55,21),PHS(55,55,21)
COMMON/ARRA2/RHO(54,54,20)
COMMON/ARRA3/A02(55,55,2),A03(55,55,2),A04(55,55,2),
1A23(55,55,2),A26(55,55,2),A27(55,55,2),A34(55,55,2),
2A37(55,55,2),A38(55,55,2),A48(55,55,2)
DIMENSION X(55),Y(55),Z(21),ELEV(55,55),ELEV1(55,55)
1,TFR(54,54),TL(5),RHOL(3),TRES(5),LCOV(3),PHA(55,55)

```

```

2,NYS(7),NXS(7),NYR(13),VXR(13),SDR(12),VR(6,13),
3VP(7,13)
REAL TITLE(20)
PI=3.1415925

C
C
C   READ IN INPUT DATA

READ(5,511) TITLE
WRITE(6,512) TITLE
512 FORMAT(//,20X,20A4,/)
511 FORMAT(20A4)
READ(5,10) NTOP,NDD,NTF,NBP,NIT
READ(5,10) NX,NY,NZ,NLAYR,VZON,NLAYER
READ(5,11) (TL(I),R4OL(I),I=1,NLAYR)
11 FORMAT(2F8.0)
IF(NTF.EQ.1) GO TO 121
GO TO 131
121 IF(NBP.NE.1) GO TO 123
READ(5,10) NYS1,NXS1,NYS2,NXS2
GO TO 131
123 READ(5,10) NYS1,NXS1
NYS2=NYS1
NXS2=NXS1
131 IF(NDD.EQ.1) GO TO 151
GO TO 161
151 READ(5,811) (NYS(J),NXS(J),J=1,7)
811 FORMAT(7(I3,1X,13,2X))
READ(5,811) (NYR(J),NKR(J),J=1,13)
161 READ(5,30) (Z(K),K=1,NZ)
READ(5,30) (X(J),J=1,NX)
READ(5,30) (Y(I),I=1,NY)
20 FORMAT(4I3,3F4.0)
30 FORMAT(8F10.3)
40 FORMAT(1X,13F3.0)
45 FORMAT(//,10X,*ELEVATION ARRAY (METERS)=)
10 FORMAT(6I3)
451 FORMAT(//,5X,*LEFT SIDE*,/)
452 FORMAT(//,5X,*RIGHT SIDE*,/)
ZREF=Z(NLAYER)
NYH=(NY+1)/2
NYHA=1
DXY=Y(NYH+1)-Y(NYH)
IF(NYH.GT.25) NYHA=NYH-25
NYH3=NY
IF(NYH.GT.25) NYH3=NYH+25
IF(NTOP.NE.1) GO TO 469
READ(5,44) ((ELEV(I,J),I=1,NY),J=1,NX)
ELMAX=0.0
DO 414 J=1,NX
DO 414 I=1,NY
414 IF(ELEV(I,J).GT.ELMAX) ELMAX=ELEV(I,J)
WRITE(6,45)
WRITE(6,461)
DO 3 J=1,NX
3 WRITE(6,43) (ELEV(I,J),I=NYHA,NYH)
WRITE(6,462)
DO 4 J=1,NX
4 WRITE(6,43) (ELEV(I,J),I=NYH,NYH3)
GO TO 515
469 DO 313 J=1,NX

```

```

      DO 313 I=1,NY
313  ELFV(I,J)=1000.
      ELMAX=1000.
      43  FORMAT(/,1X,25F5.0)
515  RESMAX=0.0
      DO 5 N=1,NZON
      READ(5,31) NXB,NXN,NYB,NYN,NZB,NZN,S
      IF(V.EQ.1) CUR1=20.*(DKY/S)
      DO 7 K=NZB,NZN
      DO 5 J=NXB,NXN
      DO 5 I=NYB,NYN
      RHO(I,J,K)=S
      5  CONTINUE
      5  CONTINUE
      7  CONTINUE
      9  CONTINUE
      CUR2=0.0
      IF(NBP.EQ.1) CUR2=-CUR1
      P+MX=0.
      31  FORMAT(6I3,2F4.0)
      RHOA=RHO(1,1,1)
      TRF(1)=TL(1)*RHOL(1)
      LCON(1)=TL(1)/RHOL(1)
      IF(NLAYR.EQ.2.1) GO TO 555
      DO 55 I=2,NLAYR
      TRF(I)=(TRF(I-1))+ (TL(I)*RHOL(I))
      LCON(I)=(LCON(I-1))+ (TL(I)/RHOL(I))
      55  CONTINUE
355  DO 1100 ID=1,7
      IF(MOD.EQ.1.JR.NPD.EQ.1) GO TO 551
      GO TO 561
551  NYS1=NYS(ID)
      VXS1=NXS(ID)
      NYS2=NYS1
      VXS2=NXS1
561  XS1=(X(NXS1)+X(NXS1+1))/2.
      XS2=(X(NXS2)+X(NXS2+1))/2.
      YS1=(Y(NYS1)+Y(NYS1+1))/2.
      YS2=(Y(NYS2)+Y(NYS2+1))/2.
      ELAV1=(ELEV(NYS1,VXS1)+ELEV(NYS1+1,NXS1)+ELEV(NYS1,VXS1+1)+
      1 ELEV(NYS1+1,NXS1+1))/4.
      ELAV2=(ELEV(NYS2,NXS2)+ELEV(NYS2+1,NXS2)+ELEV(NYS2,VXS2+1)+
      1 ELEV(NYS1+1,NXS1+1))/4.
      ZS1=ELMAX-ELAV1
      ZS2=ELMAX-ELAV2
C
C  CALCULATE INITIAL POTENTIAL ARRAY
C
33  DO 110 K=1,NZ
      DO 100 J=1,NX
      DO 90 I=1,NY
      85  IF(K.GE.NLAYER) GOTO 87
      DZ=(ZREF-(ELMAX-ELEV(I,J)))/ZREF
      ZT=(ELMAX-ELEV(I,J))/(DZ+Z(K))
      ZA1=ABS(ZS1-ZT)
      ZA2=ABS(ZS2-ZT)
      GOTO 89
      87  ZA1=ABS(Z(K)-ZS1)
      ZA2=ABS(Z(K)-ZS2)
      89  XA12=ABS(XS1-X(J))

```

```

XA14=ABS(XS2-X(J))
YA12=ABS(YS1-Y(I))
YA14=ABS(YS2-Y(I))
R1=((XA12**2.0)+(YA12**2.0)+((ZA1)**2.0))**.5
R2=((XA14**2.0)+(YA14**2.0)+((ZA2)**2.0))**.5
TRFA=0.0
DZAR1=(TRES(NLAYR)/LCON(NLAYR))**.5
DZAR2=DZAR1
DO 57 I=1,NLAYR
TREB=TREA+TRES(I)
IF(R1.GE.TREA.AND.R1.LT.TREB) DZAR1=(TRES(I)/LCON(I))**.5
IF(R2.GE.TREA.AND.R2.LT.TREB) DZAR2=(TRES(I)/LCON(I))**.5
TREA=TRES
57 CONTINUE
VT1=1.0+((ELAV1-ELEV(I,J))/R1)
VT2=1.0+((ELAV2-ELEV(I,J))/R2)
PH1=(CUR1-DZAR1*VT1)/(2.0*PI*R1)
PH2=(CUR2-DZAR2*VT2)/(2.0*PI*R2)
PH(I,J,K)=(PH1-PH2)
PHM=ABS(PH(I,J,K))
IF(PHM.GT.PHMX) PHMX=PHM
PHS(I,J,K)=0.0
90 CONTINUE
100 CONTINUE
110 CONTINUE
RESTEST=PHMX/10000.
WRITE(6,71)
IF(NDD.EQ.1) WRITE(6,7117) VXS1,NYS1
7117 FORMAT(1H0,1)X,*,SOURCE POSITION AT ELEMENT COORDINATES X=*,
1I4,*,Y=*,I4)
WRITE(6,461)
DO 1034 J=1,NX
1034 WRITE(6,1111) (PH(I,J,1),I=NY4A,NYH)
WRITE(6,452)
DO 1035 J=1,NX
1035 WRITE(6,1111) (PH(I,J,1),I=NYH,NYHB)
1111 FORMAT(/,1X,25F5.2)
3243 FORMAT(1X,15.3)
71 FORMAT(/,1)X,*,INITIAL POTENTIAL ARRAY (VOLTS)*
NXF=NX-1
NYF=NY-1
VZF=NZ-1
VC=1
VRES=0
RESP=100.0
JT=1.5
REST4=10.0
IDIR=0
DO 1000 N=1,NIT
4=1
NK=0
900 NK=NK+1
DO 902 J=1,NX
DO 901 I=1,NY
ELEV(I,J)=ELEV(I,J)
901 CONTINUE
902 CONTINUE

```

```

C
C   CALCULATE COEFFICIENT ARRAYS
C

```

```

C=(Z(NK+1)-Z(NK))/2.0
DO 200 J=1,NXE
A=(X(J+1)-X(J))/2.0
DO 190 I=1,NYE
3=(Y(I+1)-Y(I))/2.0
IF(NK.LT.NLAYER) GO TO 190
B1=(2.0+B*C)/(9.0+A)
B2=B1/2.0
33=32/2.0
C1=(2.0+A*C)/(9.0+B)
C2=C1/2.0
C3=C2/2.0
D1=(2.0+A*B)/(9.0+C)
D2=D1/2.0
D3=D2/2.0
RHOS=2.0/RHO(I,J,NK)
A01(I,J,K)=(B1+C1+D1)*RHOS
A12(I,J,K)=(-B1+C2+D2)*RHOS
A14(I,J,K)=(32-C1+D2)*RHOS
A13(I,J,K)=(-32-C2+D3)*RHOS
A15(I,J,K)=(32+C2-D1)*RHOS
A16(I,J,K)=(-32+C3-D2)*RHOS
A17(I,J,K)=(-B3-C3-D3)*RHOS
A18(I,J,K)=(33-C2-D2)*RHOS
GO TO 190
190 JZ1=((ZREF-(ELMAX-ELEV(I,J)))/ZREF)*2.*C
DZ2=((ZREF-(ELMAX-ELEV(I,J+1)))/ZREF)*2.*C
DZ3=((ZREF-(ELMAX-ELEV(I+1,J+1)))/ZREF)*2.*C
DZ4=((ZREF-(ELMAX-ELEV(I+1,J)))/ZREF)*2.*C
Z1=-JZ1/2.0
Z2=Z1+(ELEV(I,J)-ELEV(I,J+1))
Z3=Z1-(ELEV(I+1,J+1)-ELEV(I,J))
Z4=Z1-(ELEV(I+1,J)-ELEV(I,J))
Z5=Z1+DZ1
Z6=Z2+DZ2
Z7=Z3+DZ3
Z8=Z4+DZ4
ELEV(I,J)=ELEV(I,J)-DZ1
IF(I.EQ.NYE) ELEV(I+1,J)=ELEV(I+1,J)-DZ4
IF(J.EQ.NXE) ELEV(I,J+1)=ELEV(I,J+1)-DZ2
IF(I.EQ.NYE.AND.J.EQ.NXE) ELEV(I+1,J+1)=ELEV(I+1,J+1)-
1DZ3
AA11=(R/(24.0*A))*(-Z1-Z2-(Z3/3.0)-(Z4/3.0)+Z5+Z6+(Z7/3.0)+
1(Z8/3.0))
BB11=(A/(24.0*B))*(-Z1-(Z2/3.0)-(Z3/3.0)-Z4+Z5+(Z6/3.0)+
1(Z7/3.0)+Z8)
CC11=((A*B)/(16.0*C*C))*(-Z1-(Z2/3.0)-(Z3/3.0)-(Z4/3.0)
1+Z5+(Z6/3.0)+(Z7/3.0)+(Z8/3.0))
BB22=(A/(24.0*B))*(-Z1/3.0-Z2-Z3-(Z4/3.0)+(Z5/3.0)
1+Z4+Z7+(Z8/3.0))
CC22=((A*B)/(16.0*C*C))*(-(Z1/3.0)-Z2-(Z3/3.0)-(Z4/3.0)+
1(Z5/3.0)+Z6+(Z7/3.0)+(Z8/3.0))
AA33=(B/(24.0*A))*(-(Z1/3.0)-(Z2/3.0)-Z3-Z4+(Z5/3.0)+
1(Z6/3.0)+Z7+Z8)
CC33=((A*B)/(16.0*C*C))*(-(Z1/3.0)-(Z2/3.0)-(Z3/3.0)-
1Z4+(Z5/3.0)+(Z6/3.0)+(Z7/3.0)+Z8)
BB12=(A/(72.0*B))*(-Z1-Z2-Z3-Z4+Z5+Z6+Z7+Z8)
CC12=((A*B)/(48.0*C*C))*(-Z1-Z2-(Z3/3.0)-(Z4/3.0)+Z5+
1Z6+(Z7/3.0)+(Z8/3.0))
AA13=-(R/(72.0*A))*(Z1+Z2+Z3+Z4-Z5-Z6-Z7-Z8)

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```

CC13=((A*B)/(144.0+C*C))*(-Z1-Z2-Z3-Z4+Z5+Z6+Z7+Z8)
CC14=((A*B)/(48.0+C*C))*(-Z1-(Z2/3.0)-(Z3/3.0)-Z4+Z5+
1(Z6/3.0)+(Z7/3.0)+Z8)
CC23=((A*B)/(48.0+C*C))*(-Z1/3.0-Z2-Z3-(Z4/3.0)+
1(Z5/3.0)+Z6+Z7+(Z8/3.0))
CC34=((A*B)/(48.0+C*C))*(-Z1/3.0-(Z2/3.0)-Z3-Z4+
1(Z5/3.0)+(Z6/3.0)+Z7+Z8)
432 RHOS=2.0/R4D(I,J,VK)
AD1(I,J,K)=(AA11+BB11+CC11)*RHOS
AD2(I,J,K)=(AA11+BB22+CC22)*RHOS
AD3(I,J,K)=(AA33+BB22+CC33)*RHOS
AD4(I,J,K)=(AA33+BB11+CC33)*RHOS
A12(I,J,K)=(-AA11+BB12+CC12)*RHOS
A13(I,J,K)=(-AA13+BB12+CC13)*RHOS
A14(I,J,K)=(AA13+BB11+CC14)*RHOS
A15(I,J,K)=((AA11/2.0)+(BB11/2.0)-CC11)*RHOS
A16(I,J,K)=(-(AA11/2.0)+(BB12/2.0)-CC12)*RHOS
A17(I,J,K)=(-(AA13/2.0)+(BB12/2.0)-CC13)*RHOS
A18(I,J,K)=((AA13/2.0)-(BB11/2.0)-CC14)*RHOS
A23(I,J,K)=(AA13+BB22+CC23)*RHOS
A26(I,J,K)=((AA11/2.0)+(BB22/2.0)-CC22)*RHOS
A27(I,J,K)=((AA13/2.0)-(BB22/2.0)-CC23)*RHOS
A34(I,J,K)=(-AA33+BB12+CC34)*RHOS
A37(I,J,K)=((AA33/2.0)+(BB22/2.0)-CC33)*RHOS
A38(I,J,K)=(-(AA33/2.0)+(BB12/2.0)-CC34)*RHOS
A48(I,J,K)=((AA33/2.0)+(BB11/2.0)-CC33)*RHOS
190 CONTINUE
200 CONTINUE
C
C
C      SOLVE NODAL MATRIX EQUATIONS FOR NEW VALUES OF POTENTIALS
DO 300 JJ=2,NXF
DO 290 II=2,NYE
IF(IDIR.EQ.0) GO TO 201
IF(IDIR.EQ.1) GO TO 202
201 I=II
J=JJ
CALL FESOL(I,J,K,NK,NLAYER,NXS1,NXS2,NYS1,NYS2,
1RESMAX,IY,JX,KZ,WT,CUR1,CUR2,ELAV1,ELAV2,ELMAX,ZREF)
GO TO 290
202 I=NY-II+1
J=NX-JJ+1
CALL FESOL(I,J,K,NK,NLAYER,NXS1,NXS2,NYS1,NYS2,
1RESMAX,IY,JX,KZ,WT,CUR1,CUR2,ELAV1,ELAV2,ELMAX,ZREF)
290 CONTINUE
300 CONTINUE
IF(IDIR.EQ.0) GO TO 301
IDIR=0
GO TO 302
301 IDIR=1
302 CONTINUE
IF(VK.FO.1) GOTO 500
IF(NK.EQ.NZE) GOTO 700
DO 530 J=1,NXF
DO 590 I=1,NYE
AD1(I,J,1)=AD1(I,J,2)
AD2(I,J,1)=AD2(I,J,2)
AD3(I,J,1)=AD3(I,J,2)
AD4(I,J,1)=AD4(I,J,2)
A12(I,J,1)=A12(I,J,2)

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A14(I,J,1)=A14(I,J,2)
A13(I,J,1)=A13(I,J,2)
A15(I,J,1)=A15(I,J,2)
A16(I,J,1)=A16(I,J,2)
A17(I,J,1)=A17(I,J,2)
A18(I,J,1)=A18(I,J,2)
A23(I,J,1)=A23(I,J,2)
A26(I,J,1)=A26(I,J,2)
A27(I,J,1)=A27(I,J,2)
A34(I,J,1)=A34(I,J,2)
A37(I,J,1)=A37(I,J,2)
A38(I,J,1)=A38(I,J,2)
A48(I,J,1)=A48(I,J,2)
590 CONTINUE
600 CONTINUE
500 K=2
    GOTD 900
700 CONTINUE
    WRITE(6,320) N,RESMAX,IY,JX,KZ,WT
C
C    CALCULATE NEW ACCELERATION PARAMETER
C
    IF(RESMAX.LT.RESTEST) GOTD 1001
    IF(RESMAX.GT.RESP) WT=WT-0.05
    IF(RESMAX.LT.RESP) WT=WT+0.005
    RESP=RESMAX
    VRES=0
    RESMAX=0.0
320  FORMAT(5X,*,ITERATION=*,I3,2X,*,RESIDUAL=*,F10.4,
12X,*,NODE=*,I3,2X,*,ACCELERATION PARAMETER=*,F5.2)
50  FORMAT(//,10X,*,FINAL POTENTIAL ARRAY (VOLTS)*
51  FORMAT(//,10X,*,TOTAL FIELD APPARENT RESISTIVITY*)
1000 CONTINUE
1001 DO 1003 J=1,NX
    DO 1002 I=1,NY
        PHA(I,J)=PH(I,J,1)+PHS(I,J,1)
1002 CONTINUE
1003 CONTINUE
C
C    WRITE FINAL POTENTIAL ARRAY
C
    WRITE(6,50)
    IF(NDD.EQ.1) WRITE(6,7117) NX,SY,NYS1
    WRITE(6,461)
    DO 1134 J=1,NX
1134  WRITE(6,1111) (PHA(I,J),I=NYHA,NYH)
        WRITE(6,462)
    DO 1135 J=1,NX
1135  WRITE(6,1111) (PHA(I,J),I=NYH,NYHB)
        IF(NMIS.EQ.1.OR.NTF.EQ.1) GOTD 1200
        IF(NJD.EQ.1) GOTD 1250
        GOTD 1200
1250  DO 1400 J=1,13
        NXP=NXP(J)
        NYP=NYP(J)
1400  VP(ID,J)=(PHA(NYP,NXP)+PHA(NYP+1,NXP)+PHA(NYP,NXP+1)+
1344(NYP+1,NXP+1))/4.0
1130 CONTINUE
C
C    CALCULATE DIPOL-DIPOLE APPARENT RESISTIVITIES

```

```

C
DO 1440 K=1,5
  CJ=C+1
  DO 1440 J=1,13
1440 VR(K,J)=VP(KD,J)-VP(KD-1,J)
  DO 1500 I=1,12
    VX1=VXR(I)
    VX2=VXR(I+1)
    VY1=VYR(I)
    NY2=NYR(I+1)
    XAA=ABS(X(NX1)-X(NX2))
    YAA=ABS(Y(NY1)-Y(NY2))
    ZAA=ABS(ELEV(NY1,NX1)-ELEV(NY2,NX2))
1500 SDR(I)=((XAA+XAA)+(YAA+YAA)+(ZAA+ZAA))**.5
    CALL DIPDIP(SDR,VR,CUR1,TITLE)
    GO TO 9111
1203 CUR=CUR1
C
C   CALCULATE AND WRITE TOTAL FIELD APPARENT RESISTIVITY ARRAYS
C

```

```

NSS=2
IF(CUR2.EQ.0.0) NSS=1
CALL TFAGEN(PHA,X,Y,NX,NY,ELEV,TFAR,XS1,XS2,YS1,YS2,
1VXE,NYE,CUR,NSS)
VYHA=NYHA+1
NYHB=NYHB-1
WRITE(6,51)
WRITE(6,461)
DO 1412 J=2,NXE
1412 WRITE(6,1112) (TFAR(I,J),I=VYHA,NYH)
WRITE(6,462)
DO 1413 J=2,NYE
1413 WRITE(6,1112) (TFAR(I,J),I=VY4,NYHB)
1112 FORMAT(/,1X,25F5.1)
9111 CONTINUE
STOP
END
SUBROUTINE TFAGEN(PHA,X,Y,NX,NY,ELEV,TFAR,XS1,XS2,
1YS1,YS2,NXE,NYE,CUR,NSS)

```

```

*
*   SUBROUTINE TFAGEN CALCULATES THE TOTAL FIELD APPARENT RESISTIVITY
*   FOR EACH SURFACE NODE FROM THE ARRAY OF SURFACE ELECTRIC POTENTIALS
*

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*****
DIMENSION PHA(55,55),X(55),Y(55),ELEV(55,55),
1TFAR(54,54)
NXA=NX/2
PI=3.1415926
RTH=2.2*(X(NXA)-X(NXA-1))
DO 710 J=2,NXE
DO 590 I=2,NYE
RP=((X(J)**2.0)+(Y(I)**2.0))**.5
XABSS=ABS(XS1-XS2)
YABSS=ABS(YS1-YS2)
SEPS=((XABSS**2.0)+(YABSS**2.0))**.5
XABS1=ABS(XS1-X(J))
YABS1=ABS(YS1-Y(I))
SEP1=((XABS1**2.0)+(YABS1**2.0))**.5
IF (SEP1.LT.RTH) GO TO 575

```

```

XABS2=ABS(XS2-X(J))
YABS2=ABS(YS2-Y(I))
SEP2=((XABS2**2.0)+(YABS2**2.0))**.5
IF (SEP2.LT.RTH) GO TO 675
P1=((SEPI**2.0)+(SEP2**2.0)-(SEPS**2.0))/(2.0*SEPI*SEP2)
P1L=-1.0
IF (P1.GT.1.0) P1=1.0
IF (P1.LT.P1L) P1=P1L
465 ELAE=ABS(ELEV(I+1,J)-ELEV(I,J))
ELAM=ABS(ELEV(I,J)-ELEV(I-1,J))
ELBS=ABS(ELEV(I,J+1)-ELEV(I,J))
DYE=Y(I+1)-Y(I)
DYW=Y(I)-Y(I-1)
ELBN=ABS(ELEV(I,J)-ELEV(I,J-1))
JSE=((DYE**2.0)+(ELAE**2.0))**.5
JGW=((DYW**2.0)+(ELAM**2.0))**.5
DXN=X(J)-X(J-1)
DXS=X(J+1)-X(J)
JG=JGE+DGW
DJN=((DXN**2.0)+(ELBN**2.0))**.5
DJS=((DXS**2.0)+(ELBS**2.0))**.5
DJ=DJN+DJS
E1=ABS((PHA(I+1,J)-PHA(I-1,J))/JG)
E2=ABS((PHA(I,J+1)-PHA(I,J-1))/DJ)
ES=((E1**2.0)+(E2**2.0))**.5
IF (NSS.EQ.1) GO TO 676
DENK=((SEPI**(-4.0))+(SEP2**(-4.0))-
1(2.0)*(SEPI**(-2.0))*(SEP2**(-2.0))*P1)
IF (DENK.LT.0.0) DENK=-DENK
DENK=DENK**0.5
IF (DENK.EQ.0.0) GO TO 675
DENK=(2.0**P1)/DENK
TFAR(I,J)=(DENK*ES)/CUR
GO TO 690
675 TFAR(I,J)=0.0
GO TO 690
676 TFAR(I,J)=(2.0**P1*(SEPI*SEP1)*ES)/CUR
690 CONTINUE
710 CONTINUE
RETURN
END

```

```

SUBROUTINE FESOL(I,J,K,VK,NLAYER,NXS1,NXS2,NYS1,NYS2,
1RESMAX,IY,JK,Z,WT,CUR1,CUR2,ELAV1,ELAV2,ELMAX,ZREF)

```

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* SUBROUTINE FESOL DOES THE MATRIX EQUATION SOLUTION FOR THE NODAL
* ASSEMBLAGE AT EACH FINITE ELEMENT NODE.

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*****
COMMON/AD1(55,55,2),A12(55,55,2),A13(55,55,2),
1A14(55,55,2),A15(55,55,2),A16(55,55,2),A17(55,55,2),
1A18(55,55,2)
COMMON/ARRA1/PH(55,55,21),P4S(55,55,21)
COMMON/ARRA2/R47(54,34,20)
COMMON/ARRA3/AD2(55,55,2),A23(55,55,2),A24(55,55,2),
1A23(55,55,2),A26(55,55,2),A27(55,55,2),A34(55,55,2),
2A37(55,55,2),A38(55,55,2),A48(55,55,2)
S=0.
WTF1=((ZREF-(ELMAX-ELAV1))/ZREF)
WTF2=((ZREF-(ELMAX-ELAV2))/ZREF)

```

```

IF(NK.LE.2) GOTO 210
IF(NK.GT.NLAYER) GOTO 255
GOTO 220
211 IF(I.EQ.NYS1.OR.I.EQ.(NYS1+1)) GO TO 211
IF(I.EQ.NYS2.OR.I.EQ.(NYS2+1)) GO TO 212
GO TO 215
211 IF(J.EQ.NXS1.OR.J.EQ.(NXS1+1)) S=.25*CUR1*WTE1
GO TO 215
212 IF(J.EQ.NXS2.OR.J.EQ.(NXS2+1)) S=-.25*CUR2*WTE2
215 IF(NK.EQ.1) GO TO 225
221 AA=(A17(I-1,J-1,K-1)*(PHS(I-1,J-1,NK-1)+PH(I-1,J-1,NK-1))) +
1(A27(I-1,J-1,K-1)*(PHS(I-1,J,NK-1)+PH(I-1,J,NK-1))) +
1(A37(I-1,J-1,K-1)*(PHS(I,J,NK-1)+PH(I,J,NK-1))) +
1(A38(I-1,J-1,K-1)*(PHS(I,J-1,NK-1)+PH(I,J-1,NK-1))) +
1(A13(I-1,J-1,K-1)*(PHS(I-1,J-1,NK)+PH(I-1,J-1,NK))) +
1(A23(I-1,J-1,K-1)*(PHS(I-1,J,NK)+PH(I-1,J,NK))) +
1(A34(I-1,J-1,K-1)*(PHS(I,J-1,NK)+PH(I,J-1,NK)))
AG=(A18(I-1,J,K-1)*(PHS(I-1,J,NK-1)+PH(I-1,J,NK-1))) +
1(A17(I-1,J,K-1)*(PHS(I-1,J+1,NK-1)+PH(I-1,J+1,NK-1))) +
1(A38(I-1,J,K-1)*(PHS(I,J+1,NK-1)+PH(I,J+1,NK-1))) +
1(A48(I-1,J,K-1)*(PHS(I,J,NK-1)+PH(I,J,NK-1))) +
1(A14(I-1,J,K-1)*(PHS(I-1,J,NK)+PH(I-1,J,NK))) +
1(A13(I-1,J,K-1)*(PHS(I-1,J+1,NK)+PH(I-1,J+1,NK))) +
1(A39(I-1,J,K-1)*(PHS(I,J+1,NK)+PH(I,J+1,NK)))
AC=(A15(I,J,K-1)*(PHS(I,J,NK-1)+PH(I,J,NK-1))) +
1(A16(I,J,K-1)*(PHS(I,J+1,NK-1)+PH(I,J+1,NK-1))) +
1(A17(I,J,K-1)*(PHS(I+1,J+1,NK-1)+PH(I+1,J+1,NK-1))) +
1(A18(I,J,K-1)*(PHS(I+1,J,NK-1)+PH(I+1,J,NK-1))) +
1(A12(I,J,K-1)*(PHS(I,J+1,NK)+PH(I,J+1,NK))) +
1(A13(I,J,K-1)*(PHS(I+1,J+1,NK)+PH(I+1,J+1,NK))) +
1(A14(I,J,K-1)*(PHS(I+1,J,NK)+PH(I+1,J,NK)))
ADA=(A16(I,J-1,K-1)*(PHS(I,J-1,NK-1)+PH(I,J-1,NK-1))) +
1(A26(I,J-1,K-1)*(PHS(I,J,NK-1)+PH(I,J,NK-1))) +
1(A27(I,J-1,K-1)*(PHS(I+1,J,NK-1)+PH(I+1,J,NK-1))) +
1(A17(I,J-1,K-1)*(PHS(I+1,J-1,NK-1)+PH(I+1,J-1,NK-1))) +
1(A12(I,J-1,K-1)*(PHS(I,J-1,NK)+PH(I,J-1,NK))) +
1(A23(I,J-1,K-1)*(PHS(I+1,J,NK)+PH(I+1,J,NK))) +
1(A13(I,J-1,K-1)*(PHS(I+1,J-1,NK)+PH(I+1,J-1,NK)))
225 IF(NK.EQ.NLAYER) GOTO 257
AE=(A13(I-1,J-1,K)*(PHS(I-1,J-1,NK)+PH(I-1,J-1,NK))) +
1(A23(I-1,J-1,K)*(PHS(I-1,J,NK)+PH(I-1,J,NK))) +
1(A34(I-1,J-1,K)*(PHS(I,J-1,NK)+PH(I,J-1,NK))) +
1(A17(I-1,J-1,K)*(PHS(I-1,J-1,NK+1)+PH(I-1,J-1,NK+1))) +
1(A27(I-1,J-1,K)*(PHS(I-1,J,NK+1)+PH(I-1,J,NK+1))) +
1(A37(I-1,J-1,K)*(PHS(I,J,NK+1)+PH(I,J,NK+1))) +
1(A38(I-1,J-1,K)*(PHS(I,J-1,NK+1)+PH(I,J-1,NK+1)))
AF=(A14(I-1,J,K)*(PHS(I-1,J,NK)+PH(I-1,J,NK))) +
1(A13(I-1,J,K)*(PHS(I-1,J+1,NK)+PH(I-1,J+1,NK))) +
1(A34(I-1,J,K)*(PHS(I,J+1,NK)+PH(I,J+1,NK))) +
1(A18(I-1,J,K)*(PHS(I-1,J,NK+1)+PH(I-1,J,NK+1))) +
1(A17(I-1,J,K)*(PHS(I-1,J+1,NK+1)+PH(I-1,J+1,NK+1))) +
1(A39(I-1,J,K)*(PHS(I,J+1,NK+1)+PH(I,J+1,NK+1))) +
1(A49(I-1,J,K)*(PHS(I,J,NK+1)+PH(I,J,NK+1)))
AG=(A12(I,J,K)*(PHS(I,J+1,NK)+PH(I,J+1,NK))) +
1(A13(I,J,K)*(PHS(I+1,J+1,NK)+PH(I+1,J+1,NK))) +
1(A14(I,J,K)*(PHS(I+1,J,NK)+PH(I+1,J,NK))) +
1(A15(I,J,K)*(PHS(I,J,NK+1)+PH(I,J,NK+1))) +
1(A15(I,J,K)*(PHS(I,J+1,NK+1)+PH(I,J+1,NK+1))) +
1(A17(I,J,K)*(PHS(I+1,J+1,NK+1)+PH(I+1,J+1,NK+1))) +
1(A19(I,J,K)*(PHS(I+1,J,NK+1)+PH(I+1,J,NK+1)))

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    AM=(A12(I,J-1,K)*(PHS(I,J-1,NK)+PH(I,J-1,NK)))+
    1(A23(I,J-1,K)*(PHS(I+1,J,NK)+PH(I+1,J,NK)))+
    1(A13(I,J-1,K)*(PHS(I+1,J-1,NK)+PH(I+1,J-1,NK)))+
    1(A15(I,J-1,K)*(PHS(I,J-1,NK+1)+PH(I,J-1,NK+1)))+
    1(A25(I,J-1,K)*(PHS(I,J,NK+1)+PH(I,J,NK+1)))+
    1(A27(I,J-1,K)*(PHS(I+1,J,NK+1)+PH(I+1,J,NK+1)))+
    1(A17(I,J-1,K)*(PHS(I+1,J-1,NK+1)+PH(I+1,J-1,NK+1)))
    GOTD 254
253 AA=(A17(I-1,J-1,K-1)*(PHS(I-1,J-1,NK-1)+PH(I-1,J-1,NK-1)))+
    1(A18(I-1,J-1,K-1)*(PHS(I-1,J,NK-1)+PH(I-1,J,NK-1)))+
    1(A15(I-1,J-1,K-1)*(PHS(I,J,NK-1)+PH(I,J,NK-1)))+
    1(A16(I-1,J-1,K-1)*(PHS(I,J-1,NK-1)+PH(I,J-1,NK-1)))+
    1(A13(I-1,J-1,K-1)*(PHS(I-1,J-1,NK)+PH(I-1,J-1,NK)))+
    1(A14(I-1,J-1,K-1)*(PHS(I-1,J,NK)+PH(I-1,J,NK)))+
    1(A12(I-1,J-1,K-1)*(PHS(I,J-1,NK)+PH(I,J-1,NK)))
    AB=(A18(I-1,J,K-1)*(PHS(I-1,J,NK-1)+PH(I-1,J,NK-1)))+
    1(A17(I-1,J,K-1)*(PHS(I-1,J+1,NK-1)+PH(I-1,J+1,NK-1)))+
    1(A16(I-1,J,K-1)*(PHS(I,J+1,NK-1)+PH(I,J+1,NK-1)))+
    1(A15(I-1,J,K-1)*(PHS(I,J,NK-1)+PH(I,J,NK-1)))+
    1(A14(I-1,J,K-1)*(PHS(I-1,J,NK)+PH(I-1,J,NK)))+
    1(A13(I-1,J,K-1)*(PHS(I-1,J+1,NK)+PH(I-1,J+1,NK)))+
    1(A12(I-1,J,K-1)*(PHS(I,J+1,NK)+PH(I,J+1,NK)))
    AC=(A15(I,J,K-1)*(PHS(I,J,NK-1)+PH(I,J,NK-1)))+
    1(A16(I,J,K-1)*(PHS(I,J+1,NK-1)+PH(I,J+1,NK-1)))+
    1(A17(I,J,K-1)*(PHS(I+1,J+1,NK-1)+PH(I+1,J+1,NK-1)))+
    1(A18(I,J,K-1)*(PHS(I+1,J,NK-1)+PH(I+1,J,NK-1)))+
    1(A12(I,J,K-1)*(PHS(I,J+1,NK)+PH(I,J+1,NK)))+
    1(A13(I,J,K-1)*(PHS(I+1,J+1,NK)+PH(I+1,J+1,NK)))+
    1(A14(I,J,K-1)*(PHS(I+1,J,NK)+PH(I+1,J,NK)))
    ADA=(A16(I,J-1,K-1)*(PHS(I,J-1,NK-1)+PH(I,J-1,NK-1)))+
    1(A15(I,J-1,K-1)*(PHS(I,J,NK-1)+PH(I,J,NK-1)))+
    1(A13(I,J-1,K-1)*(PHS(I+1,J,NK-1)+PH(I+1,J,NK-1)))+
    1(A17(I,J-1,K-1)*(PHS(I+1,J-1,NK-1)+PH(I+1,J-1,NK-1)))+
    1(A12(I,J-1,K-1)*(PHS(I,J-1,NK)+PH(I,J-1,NK)))+
    1(A14(I,J-1,K-1)*(PHS(I+1,J,NK)+PH(I+1,J,NK)))+
    1(A13(I,J-1,K-1)*(PHS(I+1,J-1,NK)+PH(I+1,J-1,NK)))
257 AE=(A13(I-1,J-1,K)*(PHS(I-1,J-1,NK)+PH(I-1,J-1,NK)))+
    1(A14(I-1,J-1,K)*(PHS(I-1,J,NK)+PH(I-1,J,NK)))+
    1(A12(I-1,J-1,K)*(PHS(I,J-1,NK)+PH(I,J-1,NK)))+
    1(A17(I-1,J-1,K)*(PHS(I-1,J-1,NK+1)+PH(I-1,J-1,NK+1)))+
    1(A13(I-1,J-1,K)*(PHS(I-1,J,NK+1)+PH(I-1,J,NK+1)))+
    1(A15(I-1,J-1,K)*(PHS(I,J,NK+1)+PH(I,J,NK+1)))+
    1(A15(I-1,J-1,K)*(PHS(I,J-1,NK+1)+PH(I,J-1,NK+1)))
    AF=(A14(I-1,J,K)*(PHS(I-1,J,NK)+PH(I-1,J,NK)))+
    1(A13(I-1,J,K)*(PHS(I-1,J+1,NK)+PH(I-1,J+1,NK)))+
    1(A12(I-1,J,K)*(PHS(I,J+1,NK)+PH(I,J+1,NK)))+
    1(A18(I-1,J,K)*(PHS(I-1,J,NK+1)+PH(I-1,J,NK+1)))+
    1(A17(I-1,J,K)*(PHS(I-1,J+1,NK+1)+PH(I-1,J+1,NK+1)))+
    1(A16(I-1,J,K)*(PHS(I,J+1,NK+1)+PH(I,J+1,NK+1)))+
    1(A15(I-1,J,K)*(PHS(I,J,NK+1)+PH(I,J,NK+1)))
    AG=(A17(I,J,K)*(PHS(I,J+1,NK)+PH(I,J+1,NK)))+
    1(A13(I,J,K)*(PHS(I+1,J+1,NK)+PH(I+1,J+1,NK)))+
    1(A14(I,J,K)*(PHS(I+1,J,NK)+PH(I+1,J,NK)))+
    1(A15(I,J,K)*(PHS(I,J,NK+1)+PH(I,J,NK+1)))+
    1(A15(I,J,K)*(PHS(I,J+1,NK+1)+PH(I,J+1,NK+1)))+
    1(A17(I,J,K)*(PHS(I+1,J+1,NK+1)+PH(I+1,J+1,NK+1)))+
    1(A18(I,J,K)*(PHS(I+1,J,NK+1)+PH(I+1,J,NK+1)))
    AH=(A12(I,J-1,K)*(PHS(I,J-1,NK)+PH(I,J-1,NK)))+
    1(A14(I,J-1,K)*(PHS(I+1,J,NK)+PH(I+1,J,NK)))+
    1(A13(I,J-1,K)*(PHS(I+1,J-1,NK)+PH(I+1,J-1,NK)))+

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1(A16(I,J-1,K)*(PHS(I,J-1,NK+1)+PH(I,J-1,NK+1)))+
1(A15(I,J-1,K)*(PHS(I,J,NK+1)+PH(I,J,NK+1)))+
1(A13(I,J-1,K)*(PHS(I+1,J,NK+1)+PH(I+1,J,NK+1)))+
1(A17(I,J-1,K)*(PHS(I+1,J-1,NK+1)+PH(I+1,J-1,NK+1))
254 IF (NK.EQ.1) RES=(((-AE+AF+AG+AH)+S)/(AD3(I-1,J-1,K)+
1AD4(I-1,J,K)+AD1(I,J,K)+AD2(I,J-1,K)))-PH(I,J,NK))-PHS(I,J,NK)
2542 IF(NK.NE.1) GO TO 253
GO TO 258
259 IF(NK.GE.NLAYER) GOTO 252
GO TO 253
252 AD3(I-1,J-1,K)=AD1(I-1,J-1,K)
AD4(I-1,J,K)=AD1(I-1,J,K)
AD2(I,J-1,K)=AD1(I,J-1,K)
IF(NK.EQ.NLAYER) GO TO 253
AD3(I-1,J-1,K-1)=AD1(I-1,J-1,K-1)
AD4(I-1,J,K-1)=AD1(I-1,J,K-1)
AD2(I,J-1,K-1)=AD1(I,J-1,K-1)
253 RES=(((-AA+AB+AC+ADA+AE+AG+AH+AF)+S)/(AD3(I-1,J-1,K-1)+
1AD4(I-1,J,K-1)+AD1(I,J,K-1)+AD2(I,J-1,K-1)+
1AD3(I-1,J-1,K)+AD4(I-1,J,K)+AD1(I,J,K)+AD2(I,J-1,K))
1-PH(I,J,NK))-PHS(I,J,NK)
259 RESA=ABS(RES)
999 PHS(I,J,NK)=(RES*WT)+PHS(I,J,NK)
260 IF(RESA.GT.RESMAX) GO TO 251
RETURN
251 RESMAX=RESA
IX=I
JX=J
KZ=NK
RETURN
END
SUBROUTINE DIPDIP(SD,V,I,TITLE)
*****
*
* SUBROUTINE DIPDIP CALCULATES THE DIPOLE-DIPOLE APPARENT RESISTIVITIES
* AND PRINTS THEM OUT IN PSEUDOSECTION FORM.
*
*****
INTEGER APRES1(8),APRES2(9),APRES3(8),APRES4(7),APRES5(6),APRES6(4)
1)
REAL V(6,13),SD(12),I,TITLE(20)
DOUBLE PRECISION LINE(20),X
DATA X/64X...../
PI=3.141593
WRITE(6,991)
991 FORMAT(1H1,10X,'DIPOLE-DIPOLE=///)
WRITE(6,60) I
60 FORMAT(10X,'INPUT CURRENT IN PEAK TO PEAK AMPS =',F7.1)
50 FORMAT(F4.0)
51 FORMAT(12F6.1)
52 FORMAT(8F10.5)
WRITE(6,55) TITLE
55 FORMAT(///19X,20A4,///)
DO 100 J=2,11
K=J+1
IF (J.EQ.4.OR.J.EQ.5) GOTO 100
D2=SD(J+1)
IF (J.GT.5) GOTO 110
D1=SD(J+2)
N=J-1

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      D3=SD(J)
      GOTO 140
110   D1=SD(J-2)
      V=J-3
      D2=SD(J-1)
      D3=SD(J)
140   DELTAV=ABS(V(N,J)-V(N,K))
      D=ABS((1/(D1+D2))-1/D2-1/(D1+D2+D3)+1/(D2+D3))
      IF(J.LT.4)L=J-1
      IF(J.GT.5)L=J-3
      APRES1(L)=2*PI*DELTAV/I/D
150   CONTINUE
      DO 200 J=1,12
      K=J+1
      IF (J.EQ.4.OR.J.EQ.5) GOTO 200
      IF (J.EQ.6) GOTO 200
      D3=SD(J)
      IF (J.GT.5) GOTO 210
      V=J
      D1=SD(J+3)
      D2=SD(J+1)+SD(J+2)
      GOTO 240
210   D1=SD(J-3)
      D2=SD(J-1)+SD(J-2)
      V=J-6
240   DELTAV=ABS(V(N,J)-V(N,K))
      D=ABS((1/(D1+D2))-1/D2-1/(D1+D2+D3)+1/(D2+D3))
      IF(J.LT.4)L=J
      IF(J.GT.5)L=J-3
      APRES2(L)=2*PI*DELTAV/I/D
250   CONTINUE
      DO 310 J=1,12
      K=J+1
      IF (J.EQ.4.OR.J.EQ.5) GOTO 300
      IF (J.EQ.6.OR.J.EQ.7) GOTO 300
      D3=SD(J)
      IF (J.GT.5) GOTO 310
      N=J+1
      D1=SD(J+4)
      D2=SD(J+1)+SD(J+2)+SD(J+3)
      GOTO 340
310   D1=SD(J-4)
      D2=SD(J-1)+SD(J-2)+SD(J-3)
      N=J-7
340   DELTAV=ABS(V(N,J)-V(N,K))
      D=ABS((1/(D1+D2))-1/D2-1/(D1+D2+D3)+1/(D2+D3))
      IF(J.LT.4)L=J
      IF(J.GT.5)L=J-4
      APRES3(L)=2*PI*DELTAV/I/D
350   CONTINUE
      DO 410 J=1,12
      K=J+1
      IF (J.GE.4) GOTO 410
      GOTO 420
410   IF (J.LE.8) GOTO 400
420   D3=SD(J)
      IF (J.GT.5) GOTO 430
      V=J+2
      D1=SD(J+5)
      D2=SD(J+1)+SD(J+2)+SD(J+3)+SD(J+4)

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      GOTO 440
430  V=J-9
      D1=SD(J-5)
      D2=SD(J-1)+SD(J-2)+SD(J-3)+SD(J-4)
440  DELTAV=ABS(V(N,J)-V(N,K))
      D=ABS((1/(D1+D2))-1/D2-1/(D1+D2+D3)+1/(D2+D3)))
      IF(J.LT.4)L=J
      IF(J.GT.5)L=J-5
      APRES4(L)=2*PI*DELTAV/I/D
450  CONTINUE
      DO 500 J=1,12
      K=J+1
      IF (J.GE.4) GOTO 510
      GOTO 520
510  IF (J.LE.9) GOTO 500
520  D3=SD(J)
      IF (J.GT.5) GOTO 530
      V=J+3
      D1=SD(J+6)
      D2=SD(J+1)+SD(J+2)+SD(J+3)+SD(J+4)+SD(J+5)
      GOTO 540
530  V=J-9
      D1=SD(J-6)
      D2=SD(J-1)+SD(J-2)+SD(J-3)+SD(J-4)+SD(J-5)
540  DELTAV=ABS(V(N,J)-V(N,K))
      D=ABS((1/(D1+D2))-1/D2-1/(D1+D2+D3)+1/(D2+D3)))
      IF(J.LT.4)L=J
      IF(J.GT.5)L=J-6
      APRES5(L)=2*PI*DELTAV/I/D
550  CONTINUE
      DO 600 J=1,12
      K=J+1
      IF (J.GT.2) GOTO 610
      GOTO 620
610  IF (J.LT.11) GOTO 600
620  D3=SD(J)
      IF (J.GT.5) GOTO 630
      V=J+4
      D1=SD(J+7)
      D2=SD(J+1)+SD(J+2)+SD(J+3)+SD(J+4)+SD(J+5)+SD(J+6)
      GOTO 640
630  V=J-10
      D1=SD(J-10)
      D2=SD(J-1)+SD(J-2)+SD(J-3)+SD(J-4)+SD(J-5)+SD(J-6)
640  DELTAV=ABS(V(N,J)-V(N,K))
      D=ABS((1/(D1+D2))-1/D2-1/(D1+D2+D3)+1/(D2+D3)))
      IF(J.LE.3)L=J
      IF(J.GT.5)L=J-9
      APRES6(L)=2*PI*DELTAV/I/D
650  CONTINUE
      WRITE (6,63)(APRES1(M),M=1,8)
      WRITE (6,64)(APRES2(M),M=1,9)
      WRITE (6,65)(APRES3(M),M=1,9)
      WRITE (6,65)(APRES4(M),M=1,7)
      WRITE (6,67)(APRES5(M),M=1,6)
      WRITE (6,63)(APRES6(M),M=1,4)
      WRITE(6,991)
63  FORMAT(1X,M=1,8X,9I6)
64  FORMAT(/,1X,M=2,5X,9I6)
65  FORMAT(/,1X,M=3,8X,9I6)

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66  FORMAT(/,1X,*N=4*,11X,7I5)
67  FORMAT(/,1X,*N=5*,14X,5I5)
68  FORMAT(/,1X,*N=6*,17X,2I5,5X,2I6)
    RETJRN
    END
```

LISTING OF INPUT DATA AND OUTPUT FOR TEST OF POLE-QUADRUPOLE
TOTAL FIELD APPARENT RESISTIVITY CALCULATION.

POLE-QUADRUPOLE RESPONSE FOR HOMOGENEOUS HALF-SPACE

0 3 1 0 3
55 55 16 1 1 11

3000. 100.

27 27

0.	25.	125.	250.	375.	500.	625.	750.
875.	1000.	1125.	1250.	1500.	2000.	2500.	3000.
0.	500.	1000.	1250.	1500.	1625.	1750.	1875.
2000.	2125.	2250.	2375.	2500.	2625.	2750.	2875.
3000.	3125.	3250.	3375.	3500.	3625.	3750.	3875.
4000.	4125.	4250.	4375.	4500.	4625.	4750.	4875.
5000.	5125.	5250.	5375.	5500.	5625.	5750.	5875.
6000.	6125.	6250.	6375.	6500.	6625.	6750.	6875.
7000.	7125.	7250.	7375.	7500.	7625.	7750.	7875.
0.	500.	1000.	1250.	1500.	1625.	1750.	1875.
2000.	2125.	2250.	2375.	2500.	2625.	2750.	2875.
3000.	3125.	3250.	3375.	3500.	3625.	3750.	3875.
4000.	4125.	4250.	4375.	4500.	4625.	4750.	4875.
5000.	5125.	5250.	5375.	5500.	5625.	5750.	5875.
6000.	6125.	6250.	6375.	6500.	6625.	6750.	6875.
7000.	7125.	7250.	7375.	7500.	7625.	7750.	7875.

1 34 1 54 1 15100.

POLY-QUADRUPOLE RESPONSE FOR HOMOGENEOUS HALF-SPACE

INITIAL POTENTIAL ARRAY (VOLTS)

LEFT SIDE

0.07	0.08	0.08	0.08	0.08	0.08	0.08	0.08	0.08	0.09	0.09	0.09	0.09	0.09	0.09	0.09	0.09	0.09	0.09	0.09	0.09	0.09	0.09	0.09	0.09
0.08	0.09	0.09	0.09	0.09	0.09	0.09	0.09	0.09	0.09	0.09	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10
0.08	0.09	0.09	0.09	0.10	0.10	0.10	0.10	0.10	0.10	0.11	0.11	0.11	0.11	0.11	0.11	0.11	0.12	0.12	0.12	0.12	0.12	0.12	0.12	0.12
0.09	0.09	0.10	0.10	0.10	0.10	0.10	0.11	0.11	0.11	0.11	0.11	0.12	0.12	0.12	0.12	0.12	0.12	0.13	0.13	0.13	0.13	0.13	0.13	0.13
0.09	0.10	0.10	0.10	0.10	0.11	0.11	0.11	0.11	0.12	0.12	0.12	0.12	0.13	0.13	0.13	0.13	0.13	0.14	0.14	0.14	0.14	0.14	0.14	0.14
0.09	0.10	0.10	0.10	0.11	0.11	0.11	0.11	0.12	0.12	0.12	0.13	0.13	0.13	0.13	0.14	0.14	0.14	0.14	0.14	0.14	0.14	0.15	0.15	0.15
0.10	0.10	0.10	0.11	0.11	0.11	0.12	0.12	0.12	0.12	0.13	0.13	0.13	0.14	0.14	0.14	0.14	0.15	0.15	0.15	0.15	0.15	0.15	0.15	0.16
0.10	0.10	0.11	0.11	0.11	0.12	0.12	0.12	0.12	0.13	0.13	0.13	0.14	0.14	0.14	0.15	0.15	0.15	0.15	0.16	0.16	0.16	0.16	0.16	0.16
0.10	0.10	0.11	0.11	0.12	0.12	0.12	0.12	0.13	0.13	0.14	0.14	0.14	0.15	0.15	0.15	0.16	0.16	0.16	0.16	0.17	0.17	0.17	0.17	0.17
0.10	0.11	0.11	0.11	0.12	0.12	0.12	0.13	0.13	0.14	0.14	0.14	0.15	0.15	0.16	0.16	0.16	0.17	0.17	0.17	0.18	0.18	0.18	0.18	0.18
0.10	0.11	0.11	0.12	0.12	0.12	0.13	0.13	0.14	0.14	0.14	0.15	0.15	0.16	0.16	0.17	0.17	0.18	0.18	0.18	0.19	0.19	0.19	0.19	0.19
0.10	0.11	0.12	0.12	0.12	0.13	0.13	0.14	0.14	0.15	0.15	0.15	0.16	0.16	0.17	0.18	0.18	0.18	0.19	0.19	0.20	0.20	0.20	0.21	0.21
0.11	0.11	0.12	0.12	0.13	0.13	0.14	0.14	0.14	0.15	0.16	0.16	0.17	0.17	0.18	0.18	0.19	0.19	0.20	0.21	0.21	0.21	0.22	0.22	0.22
0.11	0.11	0.12	0.13	0.13	0.13	0.14	0.14	0.15	0.15	0.16	0.17	0.17	0.18	0.19	0.19	0.20	0.21	0.21	0.22	0.22	0.23	0.23	0.23	0.24
0.11	0.12	0.12	0.13	0.13	0.14	0.14	0.15	0.15	0.16	0.17	0.17	0.18	0.19	0.19	0.20	0.21	0.22	0.23	0.23	0.24	0.25	0.25	0.25	0.25
0.11	0.12	0.13	0.13	0.14	0.14	0.15	0.15	0.16	0.16	0.17	0.18	0.19	0.20	0.20	0.21	0.22	0.23	0.24	0.25	0.26	0.26	0.27	0.27	0.28
0.11	0.12	0.13	0.13	0.14	0.14	0.15	0.16	0.16	0.17	0.18	0.19	0.19	0.20	0.21	0.22	0.24	0.25	0.26	0.27	0.28	0.29	0.29	0.30	0.30
0.11	0.12	0.13	0.14	0.14	0.15	0.15	0.16	0.17	0.18	0.19	0.19	0.20	0.21	0.22	0.24	0.25	0.26	0.28	0.29	0.30	0.31			

0.12 0.13 0.14 0.15 0.15 0.16 0.17 0.18 0.19 0.20 0.21 0.23 0.25 0.26 0.29 0.31 0.35 0.38 0.43 0.49 0.56 0.64 0.74 0.84 0.90 0.98
 0.12 0.13 0.14 0.15 0.15 0.16 0.17 0.18 0.19 0.20 0.22 0.23 0.25 0.27 0.29 0.32 0.36 0.40 0.46 0.53 0.62 0.74 0.90 1.09 1.25 1.25
 0.12 0.13 0.14 0.15 0.15 0.16 0.17 0.18 0.19 0.20 0.22 0.23 0.25 0.27 0.30 0.33 0.37 0.42 0.48 0.56 0.67 0.84 1.09 1.50 2.01 2.01
 0.12 0.13 0.14 0.15 0.16 0.16 0.17 0.18 0.19 0.21 0.22 0.24 0.25 0.28 0.30 0.33 0.37 0.42 0.49 0.58 0.70 0.90 1.25 2.01 4.50 4.50
 0.12 0.13 0.14 0.15 0.16 0.16 0.17 0.18 0.19 0.21 0.22 0.24 0.25 0.28 0.30 0.33 0.37 0.42 0.49 0.58 0.70 0.90 1.25 2.01 4.50 4.50
 0.12 0.13 0.14 0.15 0.15 0.16 0.17 0.18 0.19 0.20 0.22 0.23 0.25 0.27 0.30 0.33 0.37 0.42 0.48 0.56 0.67 0.84 1.09 1.50 2.01 2.01
 0.12 0.13 0.14 0.15 0.15 0.16 0.17 0.18 0.19 0.20 0.22 0.23 0.25 0.27 0.29 0.32 0.36 0.40 0.46 0.53 0.62 0.74 0.90 1.09 1.25 1.25
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RIGHT SIDE

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 ITERATION= 1 RESIDUAL= 0.6403 MODE= 27 27 1 ACCELERATION PARAMETER= 1.50
 ITERATION= 2 RESIDUAL= 0.6100 MODE= 28 28 1 ACCELERATION PARAMETER= 1.50

ITERATION= 3 RESIDUAL= 0.1396 NOME= 24 24 2 ACCELERATION PARAMETER= 1.45
 ITERATION= 4 RESIDUAL= 0.1040 NOME= 28 24 1 ACCELERATION PARAMETER= 1.46
 ITERATION= 5 RESIDUAL= 0.0668 NOME= 27 24 1 ACCELERATION PARAMETER= 1.46

FINAL POTENTIAL ARRAY (VOLTS)

LEFT SIDE

0.07	0.08	0.08	0.08	0.08	0.08	0.08	0.08	0.08	0.08	0.08	0.09	0.09	0.09	0.09	0.09	0.09	0.09	0.09	0.09	0.09	0.09	0.09	0.09	0.09	0.09	0.09	0.09
0.08	0.08	0.08	0.09	0.09	0.09	0.09	0.09	0.09	0.09	0.09	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10
0.08	0.09	0.09	0.09	0.10	0.10	0.10	0.10	0.10	0.10	0.11	0.11	0.11	0.11	0.11	0.11	0.11	0.12	0.12	0.12	0.12	0.12	0.12	0.12	0.12	0.12	0.12	0.12
0.09	0.09	0.10	0.10	0.10	0.10	0.10	0.11	0.11	0.11	0.11	0.11	0.12	0.12	0.12	0.12	0.12	0.12	0.13	0.13	0.13	0.13	0.13	0.13	0.13	0.13	0.13	0.13
0.09	0.10	0.10	0.10	0.10	0.11	0.11	0.11	0.11	0.12	0.12	0.12	0.12	0.13	0.13	0.13	0.13	0.13	0.14	0.14	0.14	0.14	0.14	0.14	0.14	0.14	0.14	0.14
0.09	0.10	0.10	0.10	0.11	0.11	0.11	0.11	0.12	0.12	0.12	0.13	0.13	0.13	0.13	0.14	0.14	0.14	0.14	0.14	0.14	0.14	0.15	0.15	0.15	0.15	0.15	0.15
0.10	0.10	0.10	0.11	0.11	0.11	0.12	0.12	0.12	0.12	0.13	0.13	0.13	0.14	0.14	0.14	0.14	0.15	0.15	0.15	0.15	0.15	0.16	0.16	0.16	0.16	0.16	0.16
0.10	0.10	0.11	0.11	0.11	0.12	0.12	0.12	0.12	0.13	0.13	0.13	0.14	0.14	0.14	0.15	0.15	0.15	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16
0.10	0.10	0.11	0.11	0.12	0.12	0.12	0.13	0.13	0.14	0.14	0.14	0.15	0.15	0.15	0.16	0.16	0.16	0.16	0.17	0.17	0.17	0.17	0.17	0.17	0.17	0.17	0.17
0.10	0.11	0.11	0.11	0.12	0.12	0.12	0.13	0.13	0.14	0.14	0.14	0.15	0.15	0.16	0.16	0.16	0.17	0.17	0.17	0.18	0.18	0.18	0.18	0.18	0.18	0.18	0.18
0.10	0.11	0.11	0.12	0.12	0.12	0.13	0.13	0.14	0.14	0.14	0.15	0.15	0.16	0.16	0.17	0.17	0.18	0.18	0.18	0.19	0.19	0.19	0.19	0.19	0.19	0.19	0.19
0.10	0.11	0.12	0.12	0.12	0.13	0.13	0.14	0.14	0.15	0.15	0.16	0.16	0.17	0.18	0.18	0.18	0.19	0.19	0.20	0.20	0.20	0.20	0.21	0.21	0.21	0.21	0.21
0.11	0.11	0.12	0.12	0.13	0.13	0.14	0.14	0.14	0.15	0.16	0.16	0.17	0.17	0.18	0.18	0.19	0.19	0.20	0.21	0.21	0.22	0.22	0.22	0.23	0.23	0.23	0.24
0.11	0.11	0.12	0.13	0.13	0.13	0.14	0.14	0.15	0.15	0.16	0.17	0.17	0.18	0.19	0.19	0.20	0.21	0.22	0.22	0.23	0.23	0.24	0.25	0.25	0.25	0.25	0.25
0.11	0.12	0.13	0.13	0.14	0.14	0.15	0.15	0.16	0.16	0.17	0.18	0.19	0.20	0.20	0.21	0.22	0.23	0.24	0.25	0.26	0.26	0.27	0.27	0.28	0.28	0.28	0.28
0.11	0.12	0.13	0.13	0.14	0.14	0.15	0.16	0.16	0.17	0.18	0.19	0.19	0.20	0.21	0.22	0.24	0.25	0.26	0.27	0.28	0.29	0.29	0.30	0.30	0.30	0.30	0.30
0.11	0.12	0.13	0.14	0.14	0.15	0.15	0.16	0.17	0.18	0.19	0.19	0.20	0.21	0.22	0.24	0.25	0.26	0.28	0.29	0.30	0.31	0.32	0.33	0.33	0.33	0.33	0.33
0.11	0.12	0.13	0.14	0.14	0.15	0.16	0.16	0.17	0.18	0.19	0.20	0.21	0.22	0.24	0.25	0.26	0.28	0.30	0.31	0.33	0.35	0.36	0.37	0.37	0.37	0.37	0.37
0.12	0.12	0.13	0.14	0.15	0.15	0.16	0.17	0.18	0.19	0.21	0.22	0.23	0.25	0.27	0.29	0.31	0.34	0.37	0.41	0.45	0.49	0.53	0.56	0.57	0.57	0.57	0.57
0.12	0.13	0.14	0.14	0.15	0.16	0.17	0.18	0.19	0.20	0.21	0.22	0.24	0.26	0.28	0.30	0.33	0.36	0.40	0.45	0.50	0.56	0.62	0.67	0.70	0.70	0.70	0.70
0.12	0.13	0.14	0.15	0.15	0.16	0.17	0.18	0.19	0.20	0.21	0.23	0.25	0.26	0.29	0.31	0.35	0.38	0.43	0.49	0.56	0.65	0.74	0.84	0.89	0.89	0.89	0.89

0.12 0.13 0.14 0.15 0.15 0.16 0.17 0.18 0.19 0.20 0.22 0.23 0.25 0.27 0.29 0.32 0.36 0.40 0.46 0.53 0.62 0.74 0.93 1.11 1.26 1.25
 0.12 0.13 0.14 0.15 0.15 0.16 0.17 0.18 0.19 0.20 0.22 0.23 0.25 0.27 0.30 0.33 0.37 0.42 0.48 0.56 0.67 0.84 1.11 1.64 1.80 1.82
 0.12 0.13 0.14 0.15 0.16 0.16 0.17 0.18 0.19 0.21 0.22 0.24 0.25 0.28 0.30 0.33 0.37 0.42 0.49 0.57 0.70 0.89 1.26 1.81 5.57 5.62
 0.12 0.13 0.14 0.15 0.16 0.16 0.17 0.18 0.19 0.21 0.22 0.24 0.25 0.28 0.30 0.33 0.37 0.42 0.49 0.57 0.70 0.88 1.26 1.84 5.62 5.63
 0.12 0.13 0.14 0.15 0.15 0.16 0.17 0.18 0.19 0.20 0.22 0.23 0.25 0.27 0.30 0.33 0.37 0.42 0.48 0.56 0.67 0.84 1.10 1.60 1.80 1.83
 0.12 0.13 0.14 0.15 0.15 0.16 0.17 0.18 0.19 0.20 0.22 0.23 0.25 0.27 0.29 0.32 0.36 0.40 0.46 0.53 0.62 0.74 0.92 1.11 1.28 1.25
 0.12 0.13 0.14 0.15 0.15 0.16 0.17 0.18 0.19 0.20 0.21 0.23 0.25 0.26 0.29 0.31 0.35 0.38 0.43 0.49 0.56 0.64 0.74 0.84 0.88 0.89
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147

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TOTAL FIELD APPARENT RESISTIVITY

LEFT SIDE.

[illegible]

RIGHT SIDE

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LISTING OF INPUT DATA AND OUTPUT FOR TEST OF DIPOLE-DIPOLE APPARENT
RESISTIVITY CALCULATION FOR EARTH MODEL WITH TERRAIN.

FENTON HILL DIPOLE-DIPOLE TERRAIN EFFECTS

1	1	0	0	5
55	35	16	1	11
3000.	100.			
23	33	26	30	29
16	41	18	39	20
35	21	38	18	41
0.	25.	125.	250.	375.
875.	1000.	1125.	1250.	1500.
0.	500.	1000.	1250.	1500.
2000.	2125.	2250.	2375.	2500.
3000.	3125.	3250.	3375.	3500.
4000.	4125.	4250.	4375.	4500.
5000.	5125.	5250.	5375.	5500.
6000.	6125.	6250.	6375.	6500.
7000.	7125.	7250.	7375.	7500.
0.	500.	1000.	1250.	1500.
2000.	2125.	2250.	2375.	2500.
3000.	3125.	3250.	3375.	3500.
4000.	4125.	4250.	4375.	4500.
5000.	5125.	5250.	5375.	5500.
6000.	6125.	6250.	6375.	6500.
7000.	7125.	7250.	7375.	7500.
2505.	2509.	2515.	2536.	2500.
2557.	2569.	2575.	2581.	2575.
2565.	2557.	2548.	2548.	2545.
2565.	2581.	2609.	2606.	2521.
2545.	2551.	2557.	2557.	2563.
2557.	2569.	2575.	2581.	2581.
2581.	2569.	2548.	2521.	2521.
2545.	2484.	2487.	2315.	2195.
2557.	2557.	2569.	2566.	2566.
2591.	2581.	2581.	2393.	2590.
2524.	2542.	2587.	2606.	2612.
2484.	2506.	2496.	2530.	2496.
2575.	2581.	2581.	2534.	2534.
2595.	2581.	2599.	2603.	2603.
2543.	2539.	2509.	2321.	2563.
2327.	2533.	2533.	2545.	2539.
2581.	2584.	2584.	2584.	2584.
2609.	2612.	2612.	2506.	2581.
2515.	2509.	2509.	2337.	2296.
2521.	2545.	2557.	2563.	2575.
2569.	2569.	2587.	2536.	2536.
2569.	2569.	2587.	2536.	2536.
2605.	2621.	2618.	2606.	2575.
2569.	2412.	2399.	2434.	2515.
2551.	2542.	2548.	2548.	2551.
2595.	2599.	2606.	2612.	2612.
2581.	2521.	2490.	2484.	2424.
2449.	2436.	2496.	2533.	2551.
2539.	2533.	2551.	2553.	2533.
2612.	2613.	2621.	2624.	2624.
2412.	2424.	2445.	2339.	2451.
2563.	2551.	2551.	2539.	2539.
2494.	2527.	2545.	2533.	2527.
2621.	2596.	2624.	2619.	2605.

2483.2424.2393.2399.2406.2405.2424.2443.2435.2533.2539.2527.2515.2509.2496.
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 2569.2569.2569.2581.2569.2521.2521.2521.2521.2527.2527.2533.2545.2545.2531.
 2593.2599.2545.2509.2484.2424.2393.2381.2369.2357.2345.2339.2321.2381.2397.
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 2435.2387.2375.2353.2353.2351.2327.2339.2363.2375.2563.2563.2557.2557.2553.
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1 54 1 54 1 15100.

FENTON HILL DIPOLE-DIPOLE TERRAIN EFFECTS

(ELEVATION ARRAY (METERS))

LEFT SIDE

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RIGHT SIDE

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SOURCE POSITION AT ELEMENT COORDINATES X= 33 Y= 23

[illegible]

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 0.15 0.16 0.17 0.18 0.19 0.20 0.22 0.23 0.25 0.26 0.28 0.30 0.32 0.36 0.38 0.42 0.46 0.50 0.57 0.64 0.62 0.59 0.57 0.56 0.53 0.49
 0.15 0.16 0.17 0.18 0.19 0.20 0.21 0.23 0.25 0.27 0.28 0.31 0.34 0.37 0.42 0.46 0.50 0.57 0.67 0.76 0.77 0.72 0.68 0.64 0.58 0.54
 0.15 0.17 0.18 0.19 0.20 0.21 0.22 0.23 0.25 0.28 0.29 0.32 0.35 0.39 0.44 0.50 0.56 0.65 0.75 0.92 0.96 0.97 0.84 0.74 0.65 0.59
 0.15 0.17 0.18 0.19 0.20 0.21 0.23 0.24 0.26 0.26 0.31 0.33 0.37 0.41 0.46 0.54 0.63 0.75 0.91 1.16 1.25 1.36 1.18 0.88 0.73 0.64
 0.16 0.17 0.19 0.20 0.20 0.21 0.23 0.25 0.27 0.30 0.33 0.36 0.39 0.43 0.49 0.58 0.71 0.87 1.11 1.47 2.07 1.95 1.60 1.18 0.84 0.67
 0.15 0.17 0.19 0.20 0.21 0.22 0.24 0.25 0.26 0.31 0.34 0.38 0.42 0.47 0.55 0.62 0.75 0.97 1.34 2.07 4.19 4.19 1.95 1.27 0.95 0.72
 0.15 0.17 0.19 0.20 0.21 0.23 0.24 0.26 0.29 0.31 0.34 0.37 0.41 0.45 0.51 0.59 0.71 0.91 1.27 2.19 5.29 4.19 1.82 1.22 0.91 0.70
 0.15 0.16 0.19 0.20 0.21 0.23 0.24 0.26 0.29 0.31 0.34 0.37 0.41 0.45 0.50 0.57 0.68 0.85 1.13 1.53 1.95 1.95 1.43 1.06 0.85 0.66
 0.15 0.16 0.18 0.19 0.21 0.22 0.24 0.26 0.28 0.29 0.33 0.37 0.41 0.46 0.50 0.59 0.70 0.77 0.96 1.20 1.34 1.22 1.07 0.89 0.74 0.61
 0.15 0.16 0.18 0.19 0.20 0.21 0.23 0.24 0.26 0.29 0.31 0.35 0.40 0.44 0.50 0.56 0.61 0.71 0.85 0.96 0.92 0.90 0.81 0.71 0.62 0.55
 0.15 0.16 0.18 0.18 0.20 0.21 0.22 0.23 0.24 0.27 0.28 0.32 0.37 0.42 0.47 0.51 0.58 0.65 0.72 0.73 0.72 0.72 0.68 0.64 0.56 0.49
 0.15 0.16 0.18 0.19 0.20 0.21 0.22 0.23 0.24 0.25 0.27 0.30 0.33 0.37 0.43 0.47 0.52 0.57 0.61 0.63 0.63 0.63 0.68 0.56 0.50 0.45
 0.15 0.16 0.17 0.19 0.19 0.20 0.22 0.22 0.23 0.24 0.26 0.28 0.30 0.35 0.39 0.42 0.43 0.46 0.53 0.55 0.56 0.55 0.54 0.50 0.47 0.43
 0.15 0.16 0.17 0.18 0.19 0.19 0.20 0.22 0.22 0.23 0.25 0.26 0.29 0.31 0.35 0.37 0.38 0.40 0.44 0.46 0.47 0.46 0.46 0.44 0.42 0.39
 0.14 0.16 0.17 0.18 0.19 0.19 0.20 0.21 0.22 0.23 0.24 0.25 0.27 0.29 0.32 0.34 0.34 0.35 0.37 0.40 0.40 0.38 0.38 0.37 0.35 0.33
 0.14 0.15 0.17 0.17 0.18 0.19 0.20 0.20 0.21 0.22 0.23 0.24 0.25 0.27 0.29 0.31 0.32 0.32 0.33 0.35 0.35 0.34 0.33 0.32 0.31 0.30
 0.14 0.15 0.16 0.17 0.17 0.18 0.19 0.19 0.20 0.20 0.22 0.23 0.24 0.25 0.26 0.28 0.29 0.29 0.30 0.31 0.32 0.31 0.30 0.29 0.29 0.27
 0.13 0.15 0.16 0.17 0.17 0.17 0.18 0.18 0.19 0.20 0.21 0.21 0.22 0.23 0.24 0.25 0.26 0.27 0.27 0.28 0.28 0.28 0.28 0.27 0.26 0.25
 0.13 0.14 0.16 0.16 0.16 0.17 0.17 0.18 0.19 0.19 0.20 0.21 0.21 0.22 0.23 0.23 0.24 0.25 0.25 0.25 0.25 0.25 0.26 0.25 0.25 0.24
 0.13 0.14 0.15 0.15 0.16 0.16 0.17 0.18 0.18 0.19 0.20 0.20 0.21 0.21 0.21 0.22 0.22 0.23 0.23 0.23 0.23 0.23 0.23 0.23 0.23 0.23
 0.12 0.14 0.15 0.15 0.16 0.16 0.17 0.17 0.18 0.19 0.19 0.20 0.20 0.20 0.21 0.21 0.22 0.22 0.22 0.22 0.22 0.22 0.22 0.21 0.21 0.21
 0.12 0.13 0.14 0.15 0.15 0.16 0.16 0.17 0.17 0.17 0.18 0.18 0.18 0.19 0.19 0.20 0.20 0.21 0.21 0.21 0.21 0.21 0.21 0.20 0.20 0.19
 0.11 0.13 0.13 0.14 0.14 0.15 0.15 0.16 0.16 0.17 0.17 0.17 0.17 0.18 0.18 0.18 0.18 0.19 0.19 0.19 0.19 0.19 0.19 0.19 0.19 0.19
 0.11 0.12 0.13 0.14 0.14 0.14 0.14 0.15 0.16 0.16 0.16 0.17 0.17 0.17 0.17 0.17 0.17 0.18 0.18 0.18 0.18 0.18 0.18 0.18 0.18 0.18
 0.11 0.12 0.13 0.13 0.13 0.13 0.14 0.14 0.14 0.14 0.15 0.15 0.15 0.15 0.15 0.16 0.16 0.16 0.16 0.16 0.16 0.16 0.16 0.16 0.16 0.16
 0.10 0.11 0.12 0.12 0.12 0.13 0.13 0.13 0.13 0.13 0.13 0.14 0.14 0.14 0.14 0.14 0.14 0.15 0.15 0.15 0.15 0.15 0.15 0.15 0.14 0.14
 0.09 0.10 0.11 0.11 0.11 0.11 0.11 0.12 0.12 0.12 0.12 0.12 0.12 0.12 0.13 0.13 0.13 0.13 0.13 0.13 0.13 0.13 0.13 0.13 0.13 0.13
 0.09 0.09 0.09 0.10 0.10 0.10 0.10 0.10 0.10 0.10 0.11 0.11 0.11 0.11 0.11 0.11 0.11 0.11 0.11 0.11 0.11 0.11 0.11 0.11 0.11

RIGHT SIDE

[illegible]

0.54	0.46	0.43	0.38	0.33	0.31	0.28	0.26	0.24	0.22	0.21	0.20	0.19	0.18	0.18	0.17	0.16	0.15	0.15	0.14	0.14	0.13	0.13	0.12	0.11	0.10
0.59	0.48	0.44	0.40	0.35	0.31	0.29	0.26	0.24	0.23	0.22	0.20	0.19	0.18	0.18	0.17	0.16	0.15	0.15	0.14	0.14	0.13	0.13	0.12	0.11	0.11
0.64	0.52	0.46	0.42	0.36	0.32	0.29	0.27	0.25	0.23	0.22	0.21	0.19	0.18	0.17	0.17	0.16	0.16	0.15	0.14	0.14	0.13	0.13	0.12	0.11	0.11
0.67	0.55	0.48	0.43	0.37	0.33	0.30	0.27	0.25	0.23	0.22	0.21	0.19	0.18	0.17	0.17	0.16	0.16	0.15	0.14	0.14	0.13	0.13	0.12	0.11	0.11
0.72	0.57	0.50	0.44	0.37	0.33	0.30	0.27	0.25	0.23	0.22	0.20	0.19	0.18	0.17	0.17	0.16	0.16	0.15	0.15	0.14	0.13	0.13	0.12	0.11	0.11
0.70	0.58	0.48	0.43	0.38	0.34	0.31	0.28	0.25	0.23	0.22	0.20	0.19	0.18	0.17	0.17	0.16	0.16	0.15	0.15	0.14	0.13	0.13	0.12	0.11	0.11
0.66	0.55	0.47	0.42	0.37	0.34	0.30	0.28	0.25	0.23	0.22	0.20	0.19	0.18	0.17	0.17	0.16	0.16	0.15	0.15	0.14	0.13	0.13	0.12	0.11	0.11
0.61	0.51	0.45	0.40	0.35	0.32	0.29	0.27	0.25	0.23	0.21	0.20	0.19	0.18	0.17	0.17	0.16	0.16	0.15	0.15	0.14	0.13	0.13	0.12	0.11	0.10
0.55	0.48	0.42	0.38	0.34	0.31	0.28	0.26	0.25	0.23	0.21	0.20	0.19	0.18	0.17	0.17	0.16	0.16	0.15	0.14	0.14	0.13	0.13	0.12	0.11	0.10
0.49	0.45	0.41	0.37	0.33	0.30	0.28	0.26	0.24	0.22	0.21	0.19	0.18	0.17	0.17	0.16	0.16	0.15	0.15	0.14	0.14	0.13	0.12	0.12	0.11	0.10
0.45	0.43	0.39	0.35	0.32	0.29	0.27	0.25	0.23	0.22	0.20	0.19	0.18	0.17	0.16	0.16	0.16	0.15	0.15	0.14	0.14	0.13	0.12	0.12	0.11	0.10
0.43	0.40	0.36	0.32	0.29	0.28	0.26	0.24	0.22	0.21	0.20	0.19	0.18	0.17	0.16	0.16	0.15	0.15	0.15	0.14	0.14	0.13	0.12	0.12	0.11	0.10
0.39	0.37	0.34	0.31	0.28	0.26	0.24	0.23	0.22	0.20	0.19	0.18	0.17	0.17	0.16	0.16	0.15	0.15	0.14	0.14	0.13	0.13	0.12	0.12	0.11	0.10
0.33	0.33	0.31	0.29	0.27	0.25	0.23	0.22	0.21	0.20	0.19	0.18	0.17	0.16	0.16	0.16	0.15	0.15	0.14	0.14	0.13	0.13	0.12	0.11	0.11	0.10
0.30	0.30	0.28	0.27	0.26	0.24	0.23	0.21	0.20	0.19	0.18	0.17	0.17	0.16	0.16	0.15	0.15	0.14	0.14	0.13	0.13	0.12	0.12	0.11	0.10	0.10
0.27	0.27	0.26	0.25	0.24	0.23	0.22	0.21	0.19	0.19	0.18	0.17	0.16	0.16	0.15	0.15	0.15	0.14	0.14	0.13	0.12	0.12	0.11	0.11	0.10	0.10
0.25	0.24	0.24	0.23	0.22	0.22	0.21	0.20	0.19	0.18	0.17	0.16	0.16	0.16	0.15	0.15	0.14	0.14	0.14	0.13	0.12	0.12	0.11	0.11	0.10	0.10
0.24	0.23	0.22	0.22	0.21	0.20	0.20	0.19	0.18	0.17	0.16	0.16	0.16	0.15	0.15	0.14	0.14	0.14	0.13	0.13	0.12	0.11	0.11	0.11	0.10	0.09
0.23	0.22	0.21	0.20	0.20	0.19	0.19	0.18	0.18	0.17	0.16	0.15	0.15	0.15	0.14	0.14	0.14	0.13	0.13	0.12	0.12	0.11	0.11	0.10	0.10	0.09
0.21	0.21	0.20	0.19	0.19	0.18	0.18	0.17	0.17	0.16	0.16	0.15	0.14	0.14	0.14	0.14	0.13	0.13	0.13	0.12	0.12	0.11	0.11	0.10	0.10	0.09
0.19	0.19	0.19	0.18	0.18	0.17	0.17	0.16	0.16	0.16	0.15	0.14	0.14	0.14	0.14	0.13	0.13	0.13	0.12	0.12	0.11	0.11	0.10	0.10	0.10	0.09
0.19	0.18	0.18	0.17	0.17	0.16	0.16	0.16	0.15	0.15	0.14	0.14	0.14	0.14	0.13	0.13	0.13	0.12	0.12	0.12	0.11	0.11	0.10	0.10	0.10	0.09
0.18	0.17	0.17	0.17	0.16	0.16	0.16	0.15	0.15	0.14	0.14	0.14	0.13	0.13	0.13	0.13	0.13	0.12	0.12	0.11	0.11	0.10	0.10	0.10	0.10	0.09
0.16	0.15	0.15	0.15	0.15	0.15	0.14	0.14	0.14	0.13	0.13	0.13	0.13	0.12	0.12	0.12	0.12	0.11	0.11	0.11	0.10	0.10	0.10	0.10	0.10	0.09
0.14	0.14	0.14	0.14	0.14	0.14	0.13	0.13	0.13	0.12	0.12	0.12	0.12	0.12	0.12	0.12	0.11	0.11	0.11	0.10	0.10	0.10	0.10	0.10	0.10	0.09
0.13	0.12	0.12	0.12	0.12	0.12	0.11	0.11	0.11	0.11	0.11	0.11	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.09
0.11	0.11	0.11	0.11	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.10	0.09
ITERATION=	1	RESIDUAL=	0.7296	NOFF=	23	33	1	ACCELERATION	PARAMETER=	1.50															
ITERATION=	2	RESIDUAL=	0.6906	NOFF=	24	34	1	ACCELERATION	PARAMETER=	1.50															
ITERATION=	3	RESIDUAL=	0.1428	NOFF=	24	32	1	ACCELERATION	PARAMETER=	1.51															
ITERATION=	4	RESIDUAL=	0.1061	NOFF=	24	34	1	ACCELERATION	PARAMETER=	1.51															
ITERATION=	5	RESIDUAL=	0.0975	NOFF=	24	33	1	ACCELERATION	PARAMETER=	1.52															

SOURCE POSITION AT ELEMENT COORDINATES X= 73 Y= 73

LEFT SIDE

[illegible]

0.15 0.16 0.17 0.18 0.19 0.20 0.21 0.22 0.23 0.24 0.25 0.27 0.29 0.31 0.34 0.37 0.40 0.43 0.46 0.49 0.51 0.52 0.52 0.51 0.48 0.45 0.42
 0.15 0.16 0.17 0.18 0.19 0.20 0.22 0.23 0.24 0.26 0.28 0.30 0.33 0.36 0.39 0.43 0.48 0.52 0.56 0.59 0.61 0.61 0.60 0.56 0.52 0.47
 0.15 0.16 0.18 0.19 0.20 0.21 0.22 0.24 0.25 0.27 0.29 0.32 0.35 0.38 0.42 0.47 0.53 0.59 0.65 0.70 0.74 0.74 0.71 0.66 0.59 0.52
 0.15 0.16 0.18 0.19 0.20 0.21 0.23 0.24 0.26 0.28 0.30 0.33 0.36 0.40 0.45 0.51 0.59 0.68 0.78 0.89 0.94 0.92 0.88 0.79 0.68 0.59
 0.15 0.17 0.18 0.19 0.20 0.22 0.23 0.25 0.26 0.29 0.31 0.34 0.38 0.42 0.48 0.55 0.65 0.78 0.97 1.16 1.32 1.31 1.14 0.95 0.77 0.64
 0.15 0.17 0.18 0.19 0.21 0.22 0.23 0.25 0.27 0.29 0.32 0.35 0.39 0.44 0.50 0.59 0.70 0.88 1.16 1.70 1.85 1.88 1.74 1.17 0.86 0.68
 0.15 0.17 0.18 0.19 0.21 0.22 0.23 0.25 0.27 0.29 0.32 0.36 0.40 0.45 0.52 0.60 0.73 0.93 1.31 1.86 5.62 5.66 1.95 1.35 0.92 0.71
 0.15 0.17 0.18 0.19 0.21 0.22 0.24 0.25 0.27 0.30 0.33 0.36 0.40 0.46 0.52 0.62 0.74 0.93 1.31 1.89 5.71 5.66 1.89 1.32 0.92 0.72
 0.15 0.17 0.18 0.19 0.21 0.22 0.23 0.25 0.27 0.30 0.32 0.36 0.40 0.45 0.52 0.60 0.72 0.90 1.16 1.74 1.94 1.90 1.67 1.15 0.88 0.68
 0.15 0.17 0.18 0.19 0.20 0.22 0.23 0.25 0.27 0.29 0.32 0.35 0.39 0.44 0.50 0.57 0.67 0.80 0.97 1.16 1.34 1.31 1.15 0.98 0.77 0.63
 0.15 0.16 0.18 0.19 0.20 0.21 0.23 0.24 0.26 0.28 0.31 0.34 0.37 0.41 0.47 0.53 0.61 0.70 0.79 0.88 0.93 0.94 0.89 0.78 0.67 0.58
 0.15 0.16 0.18 0.19 0.20 0.21 0.22 0.24 0.26 0.27 0.30 0.32 0.35 0.39 0.43 0.48 0.54 0.60 0.66 0.71 0.75 0.75 0.71 0.65 0.58 0.52
 0.15 0.16 0.18 0.18 0.19 0.21 0.22 0.23 0.25 0.26 0.28 0.31 0.33 0.36 0.40 0.44 0.48 0.52 0.56 0.60 0.61 0.60 0.58 0.55 0.51 0.46
 0.15 0.16 0.17 0.18 0.19 0.20 0.21 0.22 0.24 0.25 0.27 0.29 0.31 0.34 0.37 0.40 0.43 0.46 0.49 0.50 0.51 0.51 0.50 0.47 0.45 0.41
 0.14 0.16 0.17 0.18 0.19 0.20 0.20 0.22 0.23 0.24 0.26 0.27 0.29 0.31 0.34 0.37 0.39 0.41 0.43 0.44 0.44 0.44 0.43 0.41 0.39 0.37
 0.14 0.15 0.17 0.17 0.18 0.19 0.20 0.21 0.22 0.23 0.24 0.26 0.27 0.29 0.31 0.33 0.35 0.37 0.38 0.39 0.39 0.39 0.38 0.37 0.36 0.34
 0.14 0.15 0.16 0.17 0.18 0.18 0.19 0.20 0.21 0.22 0.23 0.24 0.26 0.27 0.28 0.30 0.32 0.33 0.34 0.34 0.35 0.35 0.34 0.34 0.32 0.31
 0.14 0.15 0.16 0.17 0.17 0.18 0.19 0.19 0.20 0.21 0.22 0.23 0.24 0.25 0.26 0.28 0.29 0.30 0.30 0.31 0.31 0.31 0.31 0.30 0.29 0.28
 0.13 0.14 0.16 0.16 0.17 0.17 0.18 0.19 0.20 0.20 0.21 0.22 0.23 0.24 0.25 0.26 0.26 0.27 0.28 0.28 0.28 0.28 0.28 0.27 0.27 0.26
 0.13 0.14 0.15 0.16 0.16 0.17 0.18 0.18 0.19 0.19 0.20 0.21 0.22 0.22 0.23 0.24 0.24 0.25 0.25 0.26 0.26 0.26 0.25 0.25 0.25 0.24
 0.13 0.14 0.15 0.15 0.16 0.16 0.17 0.17 0.18 0.19 0.19 0.20 0.20 0.21 0.22 0.22 0.23 0.23 0.23 0.24 0.24 0.24 0.24 0.23 0.23 0.22
 0.12 0.13 0.14 0.15 0.15 0.16 0.16 0.17 0.17 0.18 0.18 0.19 0.19 0.20 0.20 0.21 0.21 0.21 0.22 0.22 0.22 0.22 0.22 0.22 0.21 0.21
 0.12 0.13 0.14 0.14 0.15 0.15 0.16 0.16 0.17 0.17 0.18 0.18 0.18 0.19 0.19 0.20 0.20 0.20 0.20 0.20 0.21 0.21 0.20 0.20 0.20 0.20
 0.12 0.13 0.14 0.14 0.14 0.15 0.15 0.16 0.16 0.16 0.17 0.17 0.18 0.18 0.18 0.18 0.19 0.19 0.19 0.19 0.19 0.19 0.19 0.19 0.19
 0.11 0.12 0.13 0.14 0.14 0.14 0.15 0.15 0.15 0.16 0.16 0.16 0.17 0.17 0.17 0.17 0.18 0.18 0.18 0.18 0.18 0.18 0.18 0.18 0.18
 0.11 0.12 0.12 0.13 0.13 0.13 0.14 0.14 0.14 0.14 0.15 0.15 0.15 0.15 0.16 0.16 0.16 0.16 0.16 0.16 0.16 0.16 0.16 0.16 0.16
 0.10 0.11 0.12 0.12 0.12 0.13 0.13 0.13 0.13 0.13 0.14 0.14 0.14 0.14 0.14 0.15 0.15 0.15 0.15 0.15 0.15 0.15 0.15 0.15 0.14
 0.09 0.10 0.10 0.11 0.11 0.11 0.11 0.12 0.12 0.12 0.12 0.12 0.12 0.12 0.12 0.13 0.13 0.13 0.13 0.13 0.13 0.13 0.13 0.13 0.12
 0.09 0.09 0.09 0.10 0.10 0.10 0.10 0.10 0.10 0.10 0.11 0.11 0.11 0.11 0.11 0.11 0.11 0.11 0.11 0.11 0.11 0.11 0.11 0.11 0.11

RIGHT SIDE

[illegible]

0.52 0.47 0.42 0.37 0.34 0.31 0.28 0.26 0.24 0.23 0.21 0.20 0.19 0.18 0.17 0.17 0.16 0.15 0.15 0.14 0.14 0.13 0.13 0.12 0.11 0.11
0.59 0.51 0.45 0.39 0.35 0.32 0.29 0.27 0.25 0.23 0.22 0.20 0.19 0.18 0.18 0.17 0.16 0.15 0.15 0.14 0.14 0.13 0.13 0.12 0.11 0.11
0.64 0.55 0.47 0.41 0.37 0.33 0.30 0.27 0.25 0.23 0.22 0.21 0.19 0.18 0.18 0.17 0.16 0.16 0.15 0.14 0.14 0.13 0.13 0.12 0.11 0.11
0.68 0.57 0.49 0.42 0.37 0.33 0.30 0.28 0.26 0.25 0.24 0.22 0.21 0.19 0.19 0.18 0.17 0.16 0.16 0.15 0.14 0.14 0.13 0.13 0.12 0.11 0.11
0.71 0.58 0.50 0.43 0.38 0.34 0.30 0.28 0.26 0.24 0.22 0.21 0.20 0.19 0.18 0.17 0.16 0.16 0.15 0.14 0.14 0.13 0.13 0.12 0.11 0.11
0.72 0.58 0.49 0.43 0.38 0.34 0.30 0.28 0.25 0.24 0.22 0.21 0.19 0.18 0.18 0.17 0.16 0.16 0.15 0.14 0.14 0.13 0.13 0.12 0.11 0.11
0.68 0.56 0.48 0.42 0.37 0.33 0.30 0.27 0.25 0.23 0.22 0.21 0.19 0.18 0.18 0.17 0.16 0.16 0.15 0.14 0.14 0.13 0.13 0.12 0.11 0.11
0.63 0.54 0.46 0.41 0.36 0.33 0.30 0.27 0.25 0.23 0.22 0.20 0.19 0.18 0.17 0.17 0.16 0.15 0.15 0.14 0.14 0.13 0.13 0.12 0.11 0.10
0.58 0.50 0.44 0.39 0.35 0.32 0.29 0.27 0.25 0.23 0.21 0.20 0.19 0.18 0.17 0.17 0.16 0.15 0.15 0.14 0.14 0.13 0.13 0.12 0.11 0.10
0.52 0.46 0.41 0.37 0.33 0.30 0.28 0.26 0.24 0.22 0.21 0.20 0.19 0.18 0.17 0.17 0.16 0.15 0.15 0.14 0.14 0.13 0.13 0.12 0.11 0.10
0.46 0.42 0.38 0.35 0.32 0.29 0.27 0.25 0.23 0.22 0.20 0.19 0.18 0.18 0.17 0.16 0.16 0.15 0.15 0.14 0.14 0.13 0.13 0.12 0.11 0.10
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0.11 0.11 0.11 0.11 0.10 0.10 0.10 0.10 0.10 0.10 0.10 0.10 0.10 0.09 0.09 0.09 0.09 0.09 0.09 0.09 0.09 0.09 0.08 0.08 0.07

DIPOL-DIPOLF

INPUT CURRENT IN PEAK TO PEAK AMPS = 25.0

FENTON HILL DIPOL-DIPOLF TERRAIN EFFECTS

N=1	109	114	115	99	119	112	117	121
N=2	120	110	77	111	80	120	113	104 121
N=3	127	78	104	103	85	119	104	106
N=4	92	102	99	107	94	121	106	
N=5	121	100	86	106	85	119		
N=6	114	89		147	100			

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