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THE STABILITY OF THE HIGH-DENSITY Z-PINCH

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ABSTRACT

Fiber-initiated High Density Z-Pinches at Los Alamos,¹ NRL,² and Karlsruhe³ have shown anomalously good stability. Kink modes are never seen, and sausage modes are at best delayed until late in the discharge. The success of these devices in reaching fusion conditions may depend on maintaining and understanding this anomalous stability. We have developed two numerical methods to study the stability in the regime where fluid theory is valid. While our methods are applicable to all modes, we will describe them only for the $m = 0$ sausage mode. The appearance of sausage modes late in the discharge and the total absence of kink modes suggest that an understanding of sausage modes is more urgent, and it is also simpler.

INTRODUCTION

Figure 1 shows a shadowgraph movie of the Los Alamos experiment HDZP-I. Frames are taken once every 10 ns for 100 ns. The fiber is the vertical line toward the left in each frame. Until about 70 ns, the discharge remains free of instabilities. Then it breaks out into $m = 0$ modes and expands rapidly. This may not be catastrophic because the wall is far away and the current continues to flow. The purpose of our work is to understand this behavior.

The rest of this paper is organized as follows. First we discuss our equations and physical concepts and assumptions about pressure and current profiles. Then we describe our two numerical methods, a nonlinear initial value code and a linear shooting code. Finally we present our results and discuss our conclusions.

DYNAMICAL MODEL

The discharge passes through three regimes of dynamics. Starting from a frozen fiber and going up to about 50 eV, there is a startup regime in which it is too cold and dense to be regarded as a plasma, with the Coulomb logarithm small or negative. Then there is an intermediate regime in which Braginskii's magnetized fluid equations are applicable. Above a few keV, the mean free path exceeds the radius, the plasma becomes collisionless, and a kinetic or CGL description is required. Our treatment is intended to apply only to the intermediate fluid regime, where the pinch spends much of its time.

We work with a one-fluid version of Braginskii's equations⁴. The equations for mass, momentum, energy, and fields,

$$\begin{aligned}\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) &= 0, & \rho \left(\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{v} \right) &= \frac{1}{c} \mathbf{J} \times \mathbf{B} - \nabla p - \nabla \cdot \pi, \\ 3n \left(\frac{\partial T}{\partial t} + \mathbf{v} \cdot \nabla T \right) + \nabla \cdot \mathbf{q} + \nabla \cdot \pi : \nabla \mathbf{v} &= \frac{1}{ne} \mathbf{J} \cdot \mathbf{R}, \\ ne \left(\mathbf{E} + \frac{1}{c} \mathbf{v} \times \mathbf{B} \right) &= \mathbf{R} + \frac{1}{c} \mathbf{J} \times \mathbf{B} - \nabla p_e, & \nabla \times \mathbf{E} &= -\frac{1}{c} \frac{\partial \mathbf{B}}{\partial t}, & \mathbf{J} &= \frac{c}{4\pi} \nabla \times \mathbf{B},\end{aligned}$$

contain dissipative terms for viscosity π , the heat flux $\mathbf{q}$, and the friction force $\mathbf{R}$. These are given by Braginskii as complicated, anisotropic functions of the fluid variables and their derivatives. We include all of Braginskii's terms except electron inertia and viscosity and separate species temperatures.

What physical effects might be important in determining stability? Profiles of pressure and current vs. radius are known to be important in ideal MHD. Dynamical effects such as rate of current rise and radial velocity are suggested by the NRL results. Dissipative effects such

as resistivity, viscosity, and thermal conductivity could be large enough to overcome ideal MHD instability. To be sure of a correct treatment, we use numerical methods capable of accurately including all of these effects at once.

One important effect is a form of viscosity with some surprising properties. This is the term which we normally call unmagnetized or parallel viscosity, but it plays an important role in determining transverse motions as well. It enters the transverse momentum equations and is proportional to transverse gradients of transverse velocities.

$$\rho \left(\frac{\partial v_r}{\partial t} + \mathbf{v} \cdot \nabla v_r \right) = -\frac{1}{r} \frac{\partial}{\partial r} (r \pi_{rr}^0) + \dots, \quad \rho \left(\frac{\partial v_z}{\partial t} + \mathbf{v} \cdot \nabla v_z \right) = -\frac{\partial \pi_{zz}^0}{\partial z} + \dots,$$

$$\pi_{rr}^0 = \pi_{zz}^0 = -\frac{1}{2} \pi_{\theta\theta}^0 = -\frac{1}{3} \eta_0 \left[\left(\frac{\partial v_r}{\partial r} + \frac{\partial v_z}{\partial z} \right) - 2 \frac{v_r}{r} \right], \quad \eta_0 = n T \tau_{ii} \sim T^{5/2}.$$

Braginskii interprets the effect as a departure from isotropic pressure due to transverse compression and the imperfectness of collisional isotropisation. It increases rapidly with temperature. CGL theory⁵ may be thought of as the collisionless limit of this effect.

The importance of resistivity and viscosity for stability can be understood by examining the scaling of the viscous Reynolds number R and the magnetic Reynolds or Lundquist number S with current. The Bennett pressure balance relation⁶ and properties of self-similar equilibrium profiles⁷ allow us to express the density, temperature, and radius of the pinch in terms of the current.

$$NT \sim I^2, \quad I = I_0 \left(\frac{t}{t_0} \right)^\alpha, \quad \alpha = 1, \quad n \sim T \sim I^2 \sim \frac{1}{a^2}.$$

When these relations are introduced into R and S , they show that S scales as I^3 while R scales as $1/I^3$.

$$S \sim \frac{\tau_R}{\tau_A} \sim \sigma a v \sim a T^2 \sim T^{5/2} \sim I^3, \quad R \sim \frac{\tau_v}{\tau_A} \sim \frac{\rho a v}{\eta_0} \sim \frac{N}{a T^2} \sim \frac{N}{T^{5/2}} \sim \frac{N}{I^3}.$$

Since RS is proportional to N , thinner fibers should be more stable, because the gap between resistive and viscous stabilization is smaller.

These points are illustrated in Fig. 2, a graph of the logarithm of the Lundquist and Reynolds numbers as a function of the current. The Lundquist number is small at low current and gets large as the current rises. The Reynolds number does the opposite. When either approaches one, its effect becomes large. The observed stability at low current may be a result of large resistivity at low current. There is a gap in the middle where both effects are too weak to stabilise the mode. The pinch must get through this gap before it reaches a second region of stability at high temperature. The size of the gap is proportional to the line density.

Does this gap in the middle mean that all is lost? No, because profile shape also influences stability. There are two special profiles to consider. When the pinch is stable, one-dimensional dissipative fluid theory predicts that the profile relaxes to self-similarity. The self-similar pressure profile is the flatter one shown in Fig. 3. As the temperature rises and resistive stabilisation becomes ineffective, the sausage mode breaks out. The more peaked profile shown here is the Kadomtsev profile,⁸ which is marginally stable to ideal sausage modes. The nonlinear effect of the sausage mode should be to drive the pinch toward the Kadomtsev profile. This process may be similar to the turbulent relaxation of the Reversed Field Pinch to a Taylor state.⁹ In relaxing from self-similar to marginally stable, the pinch will expand. Because the High Density Z-Pinch has room to expand without hitting the wall, it may survive this transition without catastrophe.

NONLINEAR INITIAL VALUE CODE

We have developed two numerical methods to study sausage stability quantitatively. One is a 3D initial value code which can follow the nonlinear evolution of an instability. The other is

a linear shooting code which is very fast and accurate. The two can mutually verify their results in their region of overlap. The first method, called the DEBS code, was developed originally by Dalton Schnack and then carried much further by Rick Nebel.¹⁰ It treats problems in periodic cylindrical geometry by finite differences in the radial direction, a pseudospectral representation in s and θ , and a semi-implicit time step. The idea of the pseudo-spectral method is to move back and forth between Fourier space and real space on each time step, using Fast Fourier Transforms. Everything is done in the space where it is simplest. For example, products are evaluated in real space while derivatives are evaluated in Fourier space.

The semi-implicit time step involves a compromise between explicit and implicit methods.¹¹ An explicit time step is severely limited by the Courant stability condition, especially in the presence of the Hall term. A fully implicit time step requires the solution of large matrices. The semi-implicit method involves adding and subtracting an extra term, treating one implicitly and the other explicitly, nearly canceling each other. The idea can be illustrated by considering equations for the evolution of the magnetic field in a slab,

$$\begin{aligned}\frac{\partial B_{1y}}{\partial t} &= -HB_0 \frac{\partial^2 B_{1x}}{\partial z^2} + \eta \frac{\partial^2 B_{1y}}{\partial z^2} + F\Delta t \frac{\partial^3 B_{1y}}{\partial z^2 \partial t}, \\ \frac{\partial B_{1x}}{\partial t} &= HB_0 \frac{\partial^2 B_{1y}}{\partial z^2} + \eta \frac{\partial^2 B_{1x}}{\partial z^2} + F\Delta t \frac{\partial^3 B_{1x}}{\partial z^2 \partial t}.\end{aligned}$$

Expressing derivatives in terms of wavelengths, we obtain a dispersion relation,

$$\omega^2 = \frac{H^2 S^2 B_0^2 k^4 + \eta k^2 [2i\omega(1 + k^2 F\Delta t) + \eta k^2]}{(1 + k^2 F\Delta t)^2}.$$

The implicit term relaxes the stability constraint by introducing the factor in the denominator without requiring the solution of large matrices. Excessive distortion the resistive term is avoided by splitting the time step and applying the semi-implicit procedure only to the Hall term, not to the resistive term.

Figure 4 shows an example of a DEBS solution. It shows graphs of the logarithm of the radial velocity vs. radius at equally spaced time steps. The region to the left of the vertical line is the plasma, while the region to the right is a near vacuum with a very high resistivity and an 1% density floor. Steady increase without change of shape indicates convergence to an exponentially growing eigenfunction and allows computation of a complex growth rate. While there is some roughness in the vacuum region, the solution in the plasma region is smooth and well-converged. The eigenfunction is strongly peaked toward the edge.

LINEAR EIGENVALUE CODE

The ability of the DEBS code to follow the instability into the nonlinear regime could be important, and this code is also being developed to study RFP's. Disadvantages of this approach include limited accuracy and long run time. A complementary approach has therefore been developed, based on a linear shooting method, which is faster, more accurate, useful for rapid surveys of parameter space, and capable of providing a check on the other code. Of the four independent variables in Braginskii's equations, θ and s are ignorable and may be represented by a single Fourier component. The time t is also ignorable if we consider only modes which grow much faster than the time scale of equilibrium variation, and may be treated by a Laplace transform.

$$f^{(1)}(r, \theta, s, t) = \hat{f}(r) \exp[i(m\theta + kz - \omega t)].$$

This leaves one independent variable, the radius r , and reduces the original nonlinear partial differential equations to a coupled linear system of ordinary differential equations. These are expressed in vector form,

$$\mathbf{A} \cdot \frac{dy}{dr} = \mathbf{B} \cdot y,$$

with y a complex n -vector of dependent variables and $\mathbf{A}$ and $\mathbf{B}$ complex $n \times n$ matrices.

The perturbed dependent variables include perturbed density ρ , 3 velocity components u_r , u_θ , and u_z , temperature T , and two magnetic field components B_θ and B_z . The radial magnetic field B_r can be expressed algebraically in terms of the other variables. The equations for all these dependent variables except density are dissipative, with viscosity acting on velocity, thermal conduction acting on temperature, and resistivity acting on the magnetic field, making these differential equations second order. In order to express these as a system of coupled 1st-order ODE's, we introduce conjugate variables viscosity π , heat flux q_r , and electric field E . The resulting system is 13th order. For $m = 0$, the last four of these decouple as a stable twisting motion which may be omitted, reducing the order to 9.

Our equations have two singular points, one at the axis and one at the edge. To determine boundary conditions, we use a form of Frobenius analysis at each singular point. In the neighborhood of the axis, $r \rightarrow 0$, the known matrices $\mathbf{A}$ and $\mathbf{B}$ are expanded in power series in r . $\mathbf{A}$ starts with a constant matrix while $\mathbf{B}$ starts with a constant over r .

$$\mathbf{A} = \mathbf{A}_0 + r\mathbf{A}_1 + \dots, \quad \mathbf{B} = \frac{1}{r} [\mathbf{B}_0 + r\mathbf{B}_1 + \dots].$$

We seek a power series solution for y , beginning with a power p which is to be determined.

$$y = r^p [y_0 + ry_1 + \dots].$$

When these expansions are introduced into the equations, the leading order terms yield a generalized 9th-order complex matrix eigenvalue problem for determining y_0 and p ,

$$[\mathbf{B}_0 - p\mathbf{A}_0] \cdot y_0 = 0.$$

With the help of Mathematica,¹² we find that the characteristic polynomial can be factored and this part of the problem can be solved analytically. There are 5 regular solutions and 4 singular ones. The boundary conditions require that the coefficients of the 4 singular solutions vanish. We also find that one of the eigenvalues is large and positive, making the ODE's stiff.

The analysis at the edge is similar but more complicated. The behavior of the equilibrium profiles as the density $\rho \rightarrow 0$ determines the form of the singularity. The density may vanish at a finite or infinite radius, the temperature may go to zero, infinity, or a finite value. The detailed manner in which the density vanishes determines the rank of the singularity. Each case is different, and they must be treated separately. We consider only the case in which the density vanishes linearly at a finite radius and the temperature and magnetic field approach finite values. The variation of the Coulomb logarithm is neglected. Even with these restrictions, the characteristic determinant does not factor and the problem must be solved numerically. The numerical solution spuriously resolves degeneracy and returns several eigenvectors which are nearly linearly dependent. Four of these are selected as regular and linearly independent. The remaining 5 solutions at the edge are singular.

A final complication is caused by a well-known problem for shooting methods. We must determine four independent solutions accurately. Because of a resistive skin effect, some solutions grow rapidly with varying r , some grow slowly, and some decay. Numerical errors in the small solutions cause them to pick up some of the large solutions, which can then prevent accurate determination of the small ones. To avoid this, a procedure known as decoupling is used. Each successive solution is expressed as a linear combination of already known solutions plus a new term.

$$y_j = u_j + \sum_{i=1}^{j-1} u_i \alpha_{ij}.$$

The new degrees of freedom are used to enforce linear independence, and the resulting equations are solved iteratively,

$$Ly_j \equiv A \cdot \frac{dy_j}{dt} - B \cdot y_j = Lu_j + \sum_{i=1}^{j-1} \left[A \cdot u_i \frac{da_{ij}}{dt} + Lu_i a_{ij} \right] = 0.$$

We can now summarise the numerical procedure. The equations are initialized at the edge to each of the four regular solutions. They are integrated to the origin simultaneously and recursively with decoupling, using an implicit form of the stiffly stable Gear package, LSODI.¹⁸ In the neighborhood of the origin, a 4×4 matrix D of singular coefficients is evaluated. The vanishing of the determinant D of this matrix is the condition for the existence of an eigenfunction. This is used as the eigenvalue condition for determining the complex growth rate ω . Nyquist and iterative techniques are used to find roots. The eigenfunction itself may be constructed as a homogeneous solution of D .

Nyquist analysis is used to determine the number of unstable roots and estimate their location for root finding.¹⁴ The graph in the upper left of Fig. 5 shows a contour encircling a portion of the complex normalised growth rate plane. The graph in the upper right shows the image of this contour in the complex D plane. The number of times this contour encircles the origin is equal to the number of roots inside the contour in the omega plane. The graph in the lower left shows that the magnitude of D ranges over 15 powers of 10. The visible portion of the Nyquist plot is only the tip of the iceberg and cannot be used as an accurate visual guide. A better view is seen in the lower right, where we have plotted the phase of D as it winds around the origin. The horizontal lines denote complete cycles. The contour winds around the origin many times but then unwinds, leaving exactly one unstable root.

Once a root has been found by iterative methods, an eigenfunction may be computed and plotted. Figure 6 shows an example of an unstable eigenfunction, the real and imaginary parts of the perturbed radial velocity, with a scaled growth rate of about $1/2$, for the self-similar profile. It is finite but sharply peaked at the edge. The steep behavior there, together with a strong temperature gradient and a strong dependence of viscosity on temperature make the behavior very sensitive to viscosity. The eigenfunction approaches the axis linearly, but diverges near the axis for numerical reasons that effect the eigenfunction but not the growth rate. The eigenfunction and growth rate are similar to those found by the DEBS code.

Figure 7 shows a scan of normalised complex growth rate vs. normalised axial wave number $k_z a$, with a plasma current of 450 kA. The stabilisation at short wave length is due to viscosity. The maximum growth rate is at $k_z a = 7$. Figure 8 shows a scan of normalised growth rate vs. plasma current for $k_z a = 1$. Partial stabilisation at high current is due to viscosity. The behavior at low current is not shown because of a minor numerical difficulty which prevents the code from functioning below about 300 kA. This cause of this difficulty has been understood but it has not yet been corrected.

CONCLUSIONS

We can now summarise our conclusions and make some conjectures about their meaning. The observed stability at low current is probably due to resistive stabilisation, but this cannot be verified until the numerical problem mentioned has been fixed. Our eigenfunctions and eigenvalues are very sensitive to edge behavior because of a high-temperature skin and $T^{5/2}$ scaling of viscosity. Full viscous stabilisation at high temperature does not seem to occur, but only reduction of the growth rate. Above a few hundred eV, there is no fluid effect except profile modification which can stabilise the mode. The existence of a nearby stable Kadomtsev profile should prevent the nonlinear effects from becoming catastrophic and should allow relaxation to this profile in a manner similar to the relaxation of an RFP to a Taylor state. The robustness of the High Density Z-Pinch may be attributable to the fact that it has room to expand and find its stable profile.

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FIGURE CAPTIONS

1. Shadowgraph movie diagnostic of Los Alamos HDZP-I experiment.
2. Common logarithm of Lundquist and Reynolds numbers vs. plasma current I .
3. Plasma pressure vs. radius. Peaked curve is Kadomtsev ideal marginally stable profile, flat curve is self-similar.
4. DEBS code solution for log of perturbed radial velocity vs. radius at equally spaced time steps.
5. Nyquist analysis. a. Contour in complex scaled growth rate plane. b. Image contour in complex determinant plane. c. Log of absolute value of determinant vs. position around contour. d. Number of cycles encircling origin vs. position around contour.
6. Linear eigenfunction for real and imaginary parts of perturbed radial velocity vs. scaled radius.
7. Scaled growth rate vs. scaled wave number.
8. Scaled growth rate vs. plasma current.

**CONSTANT-RADIUS DIFFUSE PINCH, SHOWING
LONG-TIME $m = 0$ AND $m = 1$ STABILITY**

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LUNDQUIST AND REYNOLDS NUMBERS

VS. PLASMA CURRENT

LOG₁₀ OF LUNDQUIST & REYNOLDS NUMBERS

□ REYNOLDS NUMBER

+ LUNDQUIST NUMBER

08:59:40
24-APR-89

$\times 10^4$

12:42:09
31-MAR-89

15:28:37
20-APP-89

16:46:25
1-APP-89

NORMALIZED GROWTH RATE AND FREQUENCY

NORMALIZED GROWTH RATE AND FREQUENCY