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FLUID MODELING OF MAGNETIZED PLASMAS

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ABSTRACT. The simplified description given by fluid models of magnetized plasmas makes it possible to simulate large scale problems such as the interaction of the solar wind with the earth's magnetic field. However, the accurate numerical solution of the fluid equations is made more difficult by the singular nature of the flow when magnetic reconnection occurs. During substorms, for example, details of small features of the flow in the magnetotail appear to cause changes in the global solutions. New methods for treating singular problems have been developed. Some of these are reviewed, including adaptive meshes and other tricks taken from fluid calculations. Several illustrative problems are described.

1. INTRODUCTION

Fluid equations should only be applied to the dynamics of collisional, fully ionized plasmas in a magnetic field. The solar corona, solar wind, and magnetosphere are not collisional, and sometimes not fully ionized. Nevertheless, observations suggest their applicability [Levy, 1964] and fluid equations are widely used, especially in global modeling of the magnetosphere. [Walker, 1983].

Fluid models can be used to describe the global dynamics of the magnetosphere because something very like collisions occur on much shorter length scales. For example, in the earth's bow shock, current driven instabilities drive turbulence that provides coupling on a much shorter length scale than a mean free path. [Forslund et al., 1984; Quest et al., 1983]. Thus, in applying fluid models, one must include these collision-like processes in a phenomenological way through transport coefficients or other, non-ideal effects.

Exactly how these effects are to be included in modeling space plasmas is still an incompletely answered question. While more detailed measurements of plasma properties exist for space than for laboratory plasmas, the very detail and locality of the measurements makes it difficult to reconstruct what is happening on a global scale. [Gosling, et al., 1984]

The principal difficulty in making comparisons among various phenomenological models, aside from the effort and cost, is the difficulty in obtaining accurate enough solutions to the global magnetosphere problem to distinguish their effects. The range of time and space scales is so wide that a single numerical computation cannot encompass them all, and one must choose between leaving some of them out or artificially compressing the range. For example, to make the dimensions of the reconnection region more nearly comparable to those of the whole domain, the Lundquist number in most numerical computations is $O(10^2 - 10^3)$ at best compared with $O(10^5 - 10^6)$ in a space plasma. Luckily, in the case of reconnection, the results are insensitive to the magnitude of the resistivity. [Petschek, 1964]

Here, we identify two aspects of typical numerical computations which offer the greatest opportunities for improvement, namely the calculation of convective transport and the resolution of the flow. After first describing the magnetohydrodynamic (MHD) model and its numerical representation, we show, by example and analysis, the impact of numerical error in calculating convective transport on the results obtained in modeling a substorm. We then describe a variational method for adaptive gridding and illustrate its use in global magnetospheric modeling.

2. THE MODEL MAGNETOSPHERE

As in many early studies of the global magnetosphere [Lyon et al., 1980; C.C. Wu et al., 1981, Lyon et al., 1981], we consider the flow in two dimensions of a magnetized plasma over a line dipole. The model has inherent limitations. There are no steady solutions when the interplanetary magnetic field (IMF) is in the plane of the computation, other than the Dungey model (which requires ten times the observed reconnection rate [Levy et al., 1964]) and a symmetry is imposed that does not obtain in the real world. [Gosling et al., 1984]. However, more and better computations can be done at less cost in two dimensions than in three, and the results in two and three dimensions are similar in some respects. [C.C. Wu, 1981]

The earth's magnetosphere has the structure shown in Figure (1). Prominent features of the magnetosphere are the reconnection of the IMF with the earth's field on the day side, and the stretching of the magnetotail on the night side. The transport of flux from day side to night side causes energy storage in the magnetotail; reconnection leads to its sudden release during substorms. [Hones, 1982].

We also remark the dimensions of the magnetosphere. The reconnection region on the day side and in the magnetotail are small features compared with the standoff distance of the bow shock and the length of the magnetotail. The variation in scales is obviously a difficulty in numerical calculations.

We model the flow by magnetohydrodynamic equations which describe the evolution of the dependent variables: mass, momentum, magnetic field intensity and energy. The momentum equation includes the Lorentz force and viscosity,

$$\rho \frac{d\mathbf{u}}{dt} = -\nabla(p + \frac{B^2}{2}) + \mathbf{B} \cdot \nabla \mathbf{B} + \nabla \mu (\nabla \cdot \mathbf{u}) + \nabla \times (\mu + \lambda) (\nabla \times \mathbf{u}), \quad (1)$$

where ρ is the mass density, $\mathbf{u}$ the fluid velocity, p the pressure, $\mathbf{B}$ the magnetic field intensity, and μ and λ the bulk and shear viscosity coefficients. Faraday's law also includes transport,

$$\frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \mathbf{E}, \quad (2)$$

where $\mathbf{E}$ is the electric field given by,

$$\mathbf{E} = \mathbf{u} \times \mathbf{B} + \eta \mathbf{J} \quad (3)$$

η is the resistivity, and $\mathbf{J}$ is the current, which is calculated from Ampere's law, $\mathbf{J} = \nabla \times \mathbf{B}$. The model does not include thermal conduction or thermoelectric effects.

Among the transport coefficients, μ and λ determine the thickness of shock transitions and boundary layers, and the resistivity determines the rate of magnetic field diffusion and reconnection.

To model reconnection phenomenologically, we introduce a nonlinear resistivity [Sato, 1979],

$$\eta = \eta_0 [1 + |\mathbf{J}|^2 / J_0^2] / L, \quad (4)$$

where J_0 is a reference current, $J_0 = B_s / R_e$, (where B_s is the IMF and R_e the radius of the earth), η_0 is a reference value of the resistivity, $\eta_0 = u_s R_e$, (where u_s is the solar wind speed), and L is the specified Lundquist number for the flow.

As in singular problems generally, the larger L , the smaller the reconnection region will be compared with R_e . However, the nonlinearity of η tends to concentrate the effect of resistivity even when L is $O(10^2 - 10^3)$. The nonlinearity increases the diffusion at the neutral point, and generates a perpendicular field component nearby,

$$\nabla \times \eta \mathbf{J} = -\eta \nabla^2 \mathbf{B} + \nabla \eta \times \mathbf{J}. \quad (5)$$

When $\nabla \eta$ is parallel to the neutral field line, as shown in an idealized case in Figure (2) where two, oppositely directed, curved field lines are in contact at a neutral point, the first term in Equation (5) will cut the field lines at the neutral point, and the second, by generating a perpendicular component, will rejoin them. The variation in η tends to decrease the size of the reconnection region along magnetic field lines, but still preserve the usual model. [Petschek et al., 1964]

The nonlinear dependence of the resistivity on the current has some basis in the results of plasma simulation of the lower hybrid drift instability [Brackbill *et al.*, 1984] and of collisionless shocks, [Forslund *et al.*, 1984]. However, the value of the numerical resistivity is typically much higher than the physical value. This is unavoidable, given the limitations of numerical approximations. Just as in fluid dynamics, where one makes similar arguments about artificial viscosity, the results of numerical calculations are interesting and useful if the dimensions of the reconnection region are small compared with other lengths of interest.

3. CONVECTIVE TRANSPORT

As in other numerical studies [Walker, 1983], we use finite difference methods in our examples. Our grid is shown in Figure (3), and consists of nested ellipses with one foci at the earth. The zones are very small near the earth, and large near the boundary to remove the specified solar wind flow conditions on the boundary as far as possible from the region of interest at an acceptable cost.

Because flow through the grid contributes the dominant terms to the dynamical equations, numerical diffusion in approximating convective transport is also the dominant source of numerical diffusion. In our calculations, in which we solve the equations of motion in Lagrangean form and then project the solutions back on to the grid, we must solve equations of the form of the equation for conservation of mass,

$$\frac{\partial}{\partial t} \int_V \rho dv = - \oint_S (\hat{n} \cdot \underline{u}) \rho ds, \quad (6)$$

for each of the conserved quantities, mass, momentum, energy and magnetic flux.

Differencing Equation (6) requires computing a mass flux ψ , through the surface of each of the cells of the grid shown in Figure (3). In one dimension, an approximation to Equation (6) can be written,

$$\rho_{j+1/2}(t + \Delta t) = \rho_{j+1/2}(t) - \{\psi_{j+1} - \psi_j\} \Delta t / \Delta x, \quad (7)$$

where vertices of the mesh are labeled j , and cell centers $j+1/2$.

The various methods for solving Equation (7) differ in their evaluation of ψ . Two standard methods, donor cell and Lax-Wendroff (LW) differencing, are written, [Richtmyer and Morton, 1964],

$$\psi_j = \begin{cases} \rho_{j-1/2} u_j & , \quad u_j > 0 \\ \rho_{j+1/2} u_j & , \quad u_j < 0 \end{cases}, \quad (8)$$

for donor cell, and

$$\psi_j = \bar{\rho}_j u_j \quad (9)$$

for LW differencing. Equation (9) is time-advanced explicitly by substituting spatial for time derivatives in evaluating $\bar{\rho}$,

$$\frac{\partial \rho}{\partial t} = -u \frac{\partial \rho}{\partial x},$$

yielding,

$$\bar{\rho}_j \equiv 1/2 \{ \rho_{j-1/2} (1 + u_j \theta \Delta t / \Delta x) + \rho_{j+1/2} (1 - u_j \theta \Delta t / \Delta x) \}. \quad (10)$$

For $1/2 < \theta \leq 1$, Equation (9) is linearly stable.

Using donor cell and LW differencing can give very different results. In Figure (4) are shown flux contours for a calculation using LW differencing with $L = 10^{-3}$, no anomalous resistivity ($J_0 = \infty$), and no IMF. The solar wind is supersonic ($A = 5$). Where time is measured in units of (R_E/u_8) , the sequence of contours show a progressive stretching and thinning of the magnetotail, followed by a rapid reconnection and release of a plasmoid to drift in the flow. Also in Figure (4) are shown the corresponding contours with donor cell differencing. No stretching of the magnetotail is evident, nor is there visible reconnection. We can understand the reason for the difference by comparing the velocity plots for the two calculations, shown in Figure (5). With centered differencing, the bow shock is clearly distinguished. With donor cell differencing, the bow shock is spread over so many zones that it is difficult to locate. Evidently, the reason the results are so different is that donor cell differencing is very diffusive.

The diffusivity can be estimated by truncation error analysis [Hirt, 1968] Briefly, one assumes a smooth solution so that each term can be expanded in a Taylor series about (x,t) . When coefficients of each derivative are collected, a differential equation results that contains the equation one is solving plus extra terms of $O(\Delta x, \Delta t)$ due to truncation errors. The differential equation corresponding to Equation (7) contains a diffusion term,

$$\frac{\partial \rho}{\partial t} = -u \frac{\partial \rho}{\partial x} + v \frac{\partial^2 \rho}{\partial x^2} + O(\Delta x^2, \Delta t^2). \quad (11)$$

The diffusivity for donor cell differencing is,

$$v = \frac{\Delta x}{2} |u| \left(1 - \frac{|u| \Delta t}{\Delta x} - \frac{\Delta x}{2} \frac{\partial u}{\partial x} \right) \quad (12)$$

and for LW differencing is,

$$\nu = \frac{\Delta x |u|}{2} (2\theta - 1) \frac{|u| \Delta t}{\Delta x} - \frac{\Delta x}{2} \frac{\partial u}{\partial x} \quad (13)$$

The two diffusivities are similar. The principal differences are the $O(\Delta x)$ term in donor cell differencing (which accounts for the extra diffusion) and the positive sign of the $O(\Delta t)$ diffusivity in Equation (13), which arises because we have calculated ψ at an advanced time.

For stability, both methods require $|u| \Delta t / \Delta x < 1$, and both give negative diffusivity when the gradient scale length of the velocity approaches Δx in rarefactions. Because donor cell differencing is more diffusive, it is more robust where there are strong gradients in the dependent variables. (It is also less likely to cause negative densities.) To reduce overall diffusion, yet employ donor cell differencing where it is better, hybrid schemes are used. [Zalesak, 1980]. In the "centered" calculation, Figure (4), we have used a simple hybrid scheme with ψ given by,

$$\psi_j = 1/2[(1 + \alpha_j)\rho_{j-1/2} + (1 - \alpha_j)(\rho_{j+1/2})] \quad (14)$$

The interpolation coefficient, α depends on u and its gradients. As the length scale of variation of the velocity approaches the mesh spacing, α changes in value from $\theta u \Delta t / \Delta x$ to 1 or -1. α is given by,

$$\alpha_j = (1 - \Delta_j)\theta_j u_j \Delta t / \Delta x + \Delta_j \text{sign}(1, u_j), \quad (15)$$

where $\Delta_j = \min(1, |(u_{j+1} - u_{j-1}) / 2u_j|)$. This method is easily extended to two-dimensional, non rectilinear meshes.

(Methods that represent shock transitions in one zone are compared in a recent review [Woodward and Collela, 1984]. It has been remarked these methods are especially suited to high speed flow. They have not been applied to MHD flow.)

4. ADAPTIVE GRIDDING

With a given finite difference method, one can improve the accuracy only by increasing the number of zones to resolve the data better. If the zones are uniform in size, increasing the resolution of boundary layers or reconnection regions requires increasing the density of zones everywhere. This is obviously more expensive than using variable zoning, with mesh points clustered in regions of strong gradients and dispersed elsewhere. When boundary layers form or move during the course of calculation, it is difficult to anticipate, a priori, where one must cluster points. Instead, one must devise adaptive grid generators to cluster points in response to changes in the data. A calculation using an adaptive grid is illustrated in Fig (6), where the computation mesh and flux plots are shown at a sequence of times. In this example, the IMF is southward and the nonlinear resistivity (Equation (4)) is used with $L = 10^{-3}$. Magnetic flux is introduced on the left by the solar wind flow. It's interaction with

the earth's magnetic field causes reconnection on the day side and transport of magnetic flux to the magnetotail. Eventually, reconnection in the magnetotail forms a plasmoid, which then moves tailward. In response to changes in the current density, the mesh first clusters at the day side reconnection point, then in the magnetotail in response to changes in the current density.

The adaptive mesh generator we have used is based on a variational formulation described in Brackbill and Saltzman (1982). In deriving the variational principle, we observe that a grid of quadrilaterals is the image of a logical grid of square cells of unit dimension in the space of natural coordinates, (ξ, η) . (At vertices, the natural coordinates assume integer values. (i, j)). The connectivity of the mesh is described by the logical grid; the deformation by the metric of the mapping from logical to physical space.

The size of the zones is measured by the Jacobean,

$$J = \det\{\partial(x, y)/\partial(\xi, \eta)\}.$$

The size is controlled by minimizing the integral,

$$I = \int_V dV wJ, \quad (16)$$

where $w(x, y)$ is a weight or control function determined by the data. It can be shown that this integral is minimized when $wJ^2 = \text{const}$. Thus, where w is large, the zones will be small. When w is defined by,

$$w = J^2/B^2, \quad (17)$$

the cells will be smallest where reconnection is most likely to occur (because $\eta \propto J^2$).

In global magnetospheric modeling, we observe that reconnection in the magnetotail is much weaker than on the dayside. The additional resolution in the magnetotail shown in Fig. (6), while modest, is sufficient to allow us to use the same small value of resistivity on day and night sides.

5. PHYSICAL RESULTS

A comparison of the magnetosphere calculation without IMF in Fig. (4), and with IMF in Fig. (1) suggests that a southward pointing field is not essential to produce substorms. In detail, however, the two cases are quite different.

Without an IMF or with a weak one, the magnetotail is stretched and compressed by the solar wind until reconnection releases a plasmoid. The stress is exerted on this magnetotail by a momentum transfer due to viscosity. The dependence of the reconnection on the viscosity can be learned from this case.

With a strong IMF, the structure of the earth's field is altered by reconnection. If we locate the magnetopause by following the junction between a terrestrial and IMF field line from the subsolar x-point to the magnetotail as marked by the kink or shear discontinuity we find the conjugate x-point in the magnetotail is inside the magnetopause. The location appears to coincide with the junction of lobes which have been draped over and under the earth by reconnection. In contrast to the case without IMF, the stress on the magnetotail appears to be exerted by the magnetic field.

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Figure 1 The earth's magnetosphere is depicted in a sequence of magnetic flux plots from a two dimensional calculation. In (a), at $t=50$, an X-point has formed on the day-side (left), allowing transport of flux to the night-side (right). This stresses the magnetotail, causing it to thin (b), at $t=70$, and stretch (c), at $t=75$. Finally, reconnection releases a plasmoid (d), at $t=80$. The center of the plasmoid is at the magnetopause; X-points lie inside and outside. (Time is measured in solar wind transit times across the earth.)

(a)

(b)

(c)

Figure 2 Two, oppositely directed, curved magnetic field lines are in contact at a neutral point, producing reconnection (a). A nonlinear resistivity enhances the normal tendency for merging at the neutral point (b), and reconnection nearby (c).

Figure 3 The grid for the numerical computations is formed by nested ellipses with the earth at one focus. The grid extends 30 R_e sunward (left) and 60 R_e anti-sunward. The 30 radial zones increase geometrically from 0.01 R_e at the center. The 60 angular zones are equally spaced in angle.

in the absence of an IMF, reconnection occurs. However, the results depend on the convection. With donor cell convection (a), which is diffusive, rapid reconnection causes the magnetotail to shrink as the solar wind exerts stress. With Lax-Wendroff convection (b), reconnection is delayed, and when it occurs the stretched magnetotail rapidly relieves accumulated stress by shedding a plasmoid. Times are in earth transit times.

(a)

(b)

(c)

(d)

Figure 5 The larger dissipation in donor cell convection is illustrated by a comparison of results with donor cell (c and d) and Lax-Wendroff convection (a and b). Donor cell convection heats the interior of the magnetosphere (a and c where ion temperature contours are shown), and spreads the shear discontinuity at the bow shock over several zones (b and d where velocity vectors are plotted).

Figure 6 The results of an adaptively zoned calculation are represented by a plot of the grid above, and the magnetic flux contours at the corresponding time below. The mesh points are clustered by adaptive zoning in the dayside X-point, the expansion fan, and the magnetotail where the currents are large.

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