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by

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SURVEY OF STRUCTURES REVEALED IN NUCLEON-NUCLEON SCATTERING EXPERIMENTS AND DIBARYON RESONANCES†

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ABSTRACT

We review structures appearing in various experimental data (particularly those with polarized beams) in nucleon-nucleon systems. We present evidence for the existence of dibaryon resonances with an emphasis on a diproton resonance in ${}^3F_2(J^P = 3^-)$ state.

I. INTRODUCTION

Since a striking structure in the difference between proton-proton total cross sections for pure longitudinal initial spin states, $\Delta\sigma_L = \sigma^{\text{Tot}}_{(\uparrow\uparrow)} - \sigma^{\text{Tot}}_{(\uparrow\downarrow)}$, was discovered around $p_{\text{lab}} = 1.5$ GeV/c at Argonne ZGS,¹ many other authors have suggested the possibilities of the existence of diproton resonances.²⁻⁸

Hidaka et al.² investigated such a possibility by studying the structure in $\Delta\sigma_L$ and the energy dependence of the Legendre expansion coefficients of differential cross-section and polarization data in proton-proton elastic scattering around the incident proton momentum of 1.5 GeV/c. They found

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a partial wave whose behavior is consistent with the Breit-Wigner resonance formula, namely the spin-triplet 3F_3 wave. From the dispersion analysis of a forward pp scattering amplitude using the data on $\Delta\sigma_L$, Grein and Kroll³ showed that the Argand plot of the amplitude has a clear resonancelike behavior around 1.5 GeV/c that strongly suggests the existence of a spin-triplet diproton resonance. Hoshizaki⁴ performed a phase-shift analysis of p-p elastic scattering including $\Delta\sigma_L$ data and suggested the possibility of 3F_3 resonance around 1.5 GeV/c.

Since then, others⁵ have further investigated the existence of such a resonance by studying the recent data on the spin-spin correlation parameter $C_{LL} = (L,L;0,0)$ in p-p elastic scattering. In this article we review such structures revealed in nucleon-nucleon scattering experiments.

Here we remark briefly on some theoretical background of dibaryon resonances. Several models predict the existence of dibaryon resonances: For example, (i) the bag model⁹ predicts a dibaryon resonance, which is made of 6 quarks inside a cavity, depicted in Fig. 1; e.g. Aerts et al. predicted many dinucleon resonances with mass of 2.1 to 2.5 GeV/c.

FIGURE 1 Dibaryon resonance in bag model. Solid line and circles represent cavity and quarks respectively.

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(ii) The string model¹⁰ gives a dibaryon resonance, which is made of 6 quarks combined together by strings, such as shown in Fig. 2. (iii) The nucleon-nucleon potential model also

FIGURE 2 Dibaryon Resonance in String Model.

predicts many nucleon-nucleon weak bound states and resonances.¹¹ On the other hand, if dibaryon resonances of type (i) and (ii) that are not baryon-baryon bound states (nuclei) but are 6-quark bound states (hadrons) really exist, then the dual-diagram rules¹² will be broken. For example, the imaginary part of a nonplanar diagram shown in Fig. 3 will not vanish! Then we need a new dual-diagram rule. Dibaryon resonance search is important for the theories of hadron structure and duality. In Section II experimental observables in nucleon-nucleon scattering are explained. In Section III experimental evidence for a diproton resonance 3F_3 (2260) is shown. In Section IV other possible dibaryon resonances are described. Section V is devoted to discussions and conclusions.

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FIGURE 3 Nonplanar Dual Diagram of Baryon-Baryon Scattering

II. EXPERIMENTAL OBSERVABLES IN NUCLEON-NUCLEON SCATTERING

When spin is considered, one can measure three independent total cross sections in the proton-proton or proton-neutron system. They are the spin-averaged total cross section σ^{Tot} and the differences between total cross sections ($\Delta\sigma_T$ and $\Delta\sigma_L$) for parallel and antiparallel spin states (transverse and longitudinal, respectively).

These cross sections are defined as

$$\begin{aligned}\sigma^{\text{Tot}} &= 1/2[\sigma^{\text{Tot}}(\uparrow) + \sigma^{\text{Tot}}(\downarrow)] \\ &= 1/2[\sigma^{\text{Tot}}(++) + \sigma^{\text{Tot}}(+-)] , \\ \Delta\sigma_T &= \sigma^{\text{Tot}}(++) - \sigma^{\text{Tot}}(+-), \text{ and} \\ \Delta\sigma_L &= \sigma^{\text{Tot}}(\uparrow) - \sigma^{\text{Tot}}(\downarrow) ,\end{aligned}$$

where the arrows denote the spin directions in the laboratory frame. These cross sections can be expressed in terms of the

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forward s-channel helicity amplitudes $\phi_1 = \langle ++ | \phi | ++ \rangle$,
 $\phi_2 = \langle -- | \phi | ++ \rangle$, and $\phi_3 = \langle +- | \phi | +- \rangle$ as follows:¹³

$$\sigma^{\text{Tot}} = (2\pi/k) \text{Im}[\phi_1(0) + \phi_3(0)] ,$$

$$\Delta\sigma_T = -(4\pi/k) \text{Im}[\phi_2(0)] , \text{ and}$$

$$\Delta\sigma_L = (4\pi/k) \text{Im}[\phi_1(0) - \phi_3(0)] ,$$

where k is the center-of-mass momentum.

In terms of partial-wave amplitudes,

$$\sigma^{\text{Tot}}_{(+)} = 4\pi/k \text{Im}\phi_1(0) = 4\pi/k^2 \sum_J \text{Im}\{ (2J+1)R_J + (J+1)R_{J+1,J} + JR_{J-1,J} + 2[J(J+1)]^{1/2}R^J \} , \quad (1)$$

$$\sigma^{\text{Tot}}_{(\pm)} = 4\pi/k \text{Im}\phi_3(0) = 4\pi/k^2 \sum_J \text{Im}\{ (2J+1)R_{JJ} + JR_{J+1,J} + (J+1)R_{J-1,J} + 2[J(J+1)]^{1/2}R^J \} , \quad (2)$$

$$\Delta\sigma_L = 4\pi/k \text{Im}[\phi_1(0) - \phi_3(0)] = 4\pi/k^2 \sum_J \text{Im}\{ (2J+1)(R_J - R_{JJ}) + R_{J+1,J} - R_{J-1,J} + 4[J(J+1)]^{1/2}R^J \} , \text{ and } (3)$$

$$\begin{aligned} \Delta\sigma_T &= -(4\pi/k) \text{Im}\phi_2(0) \\ &= -(4\pi/k^2) \sum_J \text{Im}\{ -(2J+1)R_J + (J+1)R_{J+1,J} \\ &\quad + JR_{J-1,J} + 2[J(J+1)]^{1/2}R^J \} , \end{aligned} \quad (4)$$

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where R_{JJ} and $R_{J\pm 1, J}$ are spin-triplet partial-wave amplitudes with $J = L = \text{odd}$ and $J = L \mp 1 = \text{even}$, respectively, R^J is a mixing term of $L = J \pm 1$ states, and R_J is a spin-singlet wave with $J = L = \text{even}$ for proton-proton system. These partial-wave amplitudes can be expressed in terms of phase shifts δ , elasticities η , and so on¹⁴; e.g.,

$$R_{JJ} = (\eta_{JJ} e^{2i\delta_{JJ}} - 1)/(2i) . \quad (5)$$

The experimental observables in elastic scattering can be written as $(B, T; S, R)$ where B , T , S , and R describe the spin state of the beam, target, forward-scattered protons, and slow recoil protons, respectively; the number of elastic events, I , is expressed as¹⁵

$$I = I_0 [1 + P_B \cdot (B, 0; 0, 0) + P_T \cdot (0, T; 0, 0) + P_B \cdot P_T \cdot (B, T; 0, 0) + \dots],$$

where I_0 is the number of spin-averaged events. We define $\vec{N}$ as the normal to the scattering plane, $\vec{L}$ as being along the direction of motion of the proton being considered, and $\vec{S} = \vec{N} \times \vec{L}$, as illustrated below:

N: NORMAL TO THE SCATTERING PLANE
 L: LONGITUDINAL DIRECTION
 $S = N \times L$ IN THE SCATTERING PLANE

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We use short-hand notations for these observables:

$$\begin{aligned}
 P &= (N,0;0,0) = (0,N;0,0); \text{ polarization parameter,} \\
 \left. \begin{aligned}
 C_{NN} &= (N,N;0,0) \\
 C_{SS} &= (S,S;0,0) \\
 C_{LL} &= (L,L;0,0) \\
 C_{SL} &= (S,L;0,0)
 \end{aligned} \right\} &\text{ spin-spin correlation parameter,} \\
 D_{ij} &= (0,i;0,j); \text{ depolarization parameter, etc.}
 \end{aligned}$$

These observables are expressed in terms of s-channel helicity amplitudes as shown in Table I.

TABLE I Observables in Terms of Helicity Amplitudes

Observables (B,T;S,R)	Helicity Amplitude
σ^{Tot}	$\frac{1}{2} \text{Im}\{\phi_1(0) + \phi_3(0)\}$
$\Delta\sigma_L^{\text{Tot}}$	$\text{Im}\{\phi_1(0) - \phi_3(0)\}$
$\Delta\sigma_T^{\text{Tot}}$	$-\text{Im}\phi_2(0)$
$d\sigma/dt \equiv \sigma = (0,0;0,0)$	$\frac{1}{2}\{ \phi_1 ^2 + \phi_2 ^2 + \phi_3 ^2 + \phi_4 ^2 + 4 \phi_5 ^2\}$
$P = (0,N;0,0)$	$\text{Im}\{(\phi_1 + \phi_2 + \phi_3 - \phi_4)^* \phi_5\}/\sigma$
$C_{NN} = (N,N;0,0)$	$\text{Re}\{\phi_1^* \phi_2 - \phi_3^* \phi_4\} + 2 \phi_5 ^2/\sigma$
$C_{SS} = (S,S;0,0)$	$\text{Re}(\phi_1^* \phi_2 + \phi_3^* \phi_4)/\sigma$
$C_{SL} = (S,L;0,0)$	$\text{Re}\{(\phi_1 + \phi_2 - \phi_3 + \phi_4)^* \phi_5\}/\sigma$
$C_{LL} = (L,L;0,0)$	$\frac{1}{2}\{- \phi_1 ^2 - \phi_2 ^2 + \phi_3 ^2 + \phi_4 ^2\}/\sigma$

Note - Kinematical factor $4\pi/k$ (for total cross section) and π/k^2 (for differential cross section) are assumed.

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III. EXPERIMENTAL EVIDENCE FOR A DIPROTON RESONANCE 3F_3 (2260)

Experimentally there is evidence for a diproton 3F_3 (2260). This comes from the observed structures in $\Delta\sigma_L$, $P(d\sigma/d\Omega)$, σ_{el}^{Tot} , and $C_{LL}(d\sigma/d\Omega)$ data, and from a dispersion analysis and a phase-shift analysis in pp scattering.

A. Evidence in $\Delta\sigma_L$ Data (Refs. 1 and 2)

A remarkable structure around $p_{lab} = 1.5$ GeV/c in the difference between pp total cross sections, $\Delta\sigma_L = \sigma_L^{Tot}(+) - \sigma_L^{Tot}(-)$, was discovered at Argonne ZGS.¹ The data are shown in Fig. 4. As is shown in Fig. 5 the structure appears in the parallel cross section $\sigma_L^{Tot}(+)$ rather than in the antiparallel cross section $\sigma_L^{Tot}(-)$. As is seen in Eqs. 1 and 2, the partial wave characteristic to the parallel cross section and

FIGURE 4 Total Cross-Section Difference $\Delta\sigma_L = \sigma_L^{Tot}(+) - \sigma_L^{Tot}(-)$.

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FIGURE 5 Total Cross Sections for Pure Initial Spin States.
The dotted curves are only to guide the eye.

not to the antiparallel cross section is R_{JJ} . The structure is therefore due to R_{JJ} wave. If this structure of R_{JJ} is considered to be due to a resonance, its mass and width are about 2260 and 200 MeV, respectively. Note that the total cross-section difference $\Delta\sigma_L$ has a very small background because of a cancellation of the large backgrounds of $\text{Im}\phi_1(0)$ and $\text{Im}\phi_3(0)$ (by unitarity, $\text{Im } R \geq 0$, except for R^J), while the unpolarized total cross-section

$$\sigma^{\text{Tot}} = \frac{1}{2} \left\{ \sigma^{\text{Tot}}(\uparrow) + \sigma^{\text{Tot}}(\downarrow) \right\} = 2\pi/k \text{ Im} \left\{ \phi_1(0) + \phi_3(0) \right\} \quad (6)$$

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contains a huge background. The structure therefore shows up much more clearly in $\Delta\sigma_L$ than in σ^{Tot} . For this reason, it is difficult to recognize the structure in σ^{Tot} data around 1.5 GeV/c (see Fig. 6).

FIGURE 6 pp Total Cross Section.

B. Evidence in $P \, d\sigma/d\Omega$ Data

Using polarization data, we investigate whether an R_{J_1} partial wave has the behavior of the Breit-Wigner formula.² The effect of resonance can be studied through the energy dependence of the Legendre expansion coefficients obtained from the product of differential cross-section and polarization data. We expand differential cross section and polarization as follows:

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$$d\sigma/d\Omega = 1/k^2 \sum_{n=0}^{\infty} a_n P_n(\cos \theta) \text{ and} \quad (7)$$

$$P d\sigma/d\Omega = 1/k^2 \sum_{n=2}^{\infty} b_n P_n^{(1)}(\cos \theta) . \quad (8)$$

Note that, because of symmetries, n must be even. All the observed coefficients a_n and b_n with $n \geq 8$ almost vanish in $p_{lab} = 1.0 - 2.0$ GeV/c. This means we can ignore partial waves with $J > 4$ and $L > 4$ in this energy range. Therefore the possible R_{JJ} resonance is $R_{11} = {}^3P_1$ or $R_{33} = {}^3F_3$. As shown in Fig. 7, all the coefficients b_n have a strong energy dependence around 1.5 GeV/c, where $\Delta\sigma_L$ has the prominent structure. We examine whether 3P_1 or 3F_3 resonance can explain all these structures by studying each coefficient. The coefficients a_n and b_n are expressed in terms of partial-wave amplitudes as shown in Appendix.

FIGURE 7 Results of Legendre Coefficient Analysis of Polarization Data; the Coefficient b_n of Eq. 8 vs. Laboratory Momentum.

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1. Coefficient b_6

The coefficient b_6 is related with partial-wave amplitudes as follows:

$$b_6 \approx 1.8 (\text{Im}^3 F_3 \text{Re}^3 F_4 - \text{Re}^3 F_3 \text{Im}^3 F_4) + \dots, \quad (9)$$

where the residual terms (...) include neither 3F_3 nor 3P_1 . A rapid increase with energy in b_6 can be explained with the 3F_3 wave following the Breit-Wigner formula and the other amplitudes varying slowly with energy. By unitarity, $\text{Im}^3 F_4$ is positive. Therefore, if we assume that the 3F_3 wave has a Breit-Wigner behavior such as

$$^3F_3 = \frac{\epsilon + i}{\epsilon^2 + 1} \frac{r_{e1}}{r} \quad (10)$$

with $\epsilon = 2/r (M - E)$, $M = 2260$ MeV, and $r = 200$ MeV, then each term of Eq. 9 behaves as shown in Fig. 8. In any case, the sum of the first two terms increases rapidly with energy as a whole. Here note that 3P_1 resonance can not explain this increase with energy because b_6 does not include the 3P_1 wave.

FIGURE 8 Plots of Each Term of Eq. 9 Against Laboratory
(Figure 8 Caption Continued on Next Page)

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Momentum (Vertical Scale is Arbitrary). Solid curve represents the second term $-1.8 (\text{Re}^3 F_3) (\text{Im}^3 F_4)$. Dashed curve and dotted curve show the first term $1.8(\text{Im}^3 F_3)(\text{Re}^3 F_4)$ corresponding to positive and negative $\text{Re}^3 F_4$ respectively.

2. Coefficients b_2 and b_4

Here we examine whether the structure in b_2 and b_4 can be interpreted with 3F_3 following the Breit-Wigner formula of Eq. 10. In terms of partial-wave amplitudes, b_2 and b_4 are given as:

$$b_2 = (\text{Im}^3 F_3)(\text{Re} A) - (\text{Re}^3 F_3)(\text{Im} A) + (\text{terms without } ^3F_3) \quad (11)$$

and

$$b_4 = (\text{Im}^3 F_3)(\text{Re} A') - (\text{Re}^3 F_3)(\text{Im} A') + (\text{terms without } ^3F_3) \quad (12)$$

where

$$A = 2.0 \ ^3P_2 - 2.0 \ ^3F_2 + 0.8 \ R^2 + 2.8 \ ^3F_4 \quad (13)$$

$$A' = 1.2 \ ^3P_2 - 1.5 \ ^3F_3 + 0.6 \ R^2 + 2.5 \ ^3F_4 \quad (14)$$

In the region around 1.5 GeV/c, both phase-shift analyses of MacGregor et al.¹⁶ and Hoshizaki⁴ give

$$\text{Re } A \gg \text{Im } A > 0 \quad (15)$$

$$\text{Re } A' \gg \text{Im } A' > 0. \quad (16)$$

The second term in Eqs. 11 and 12 can be neglected, and the first term in the same equations has a peak at 1.5 GeV/c. We therefore expect b_2 and b_4 to have a bump around 1.5 GeV/c. Indeed, b_2 and b_4 have such a behavior, as shown in Fig. 7.

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Thus we can say that strong energy dependences of b_2 , b_4 and b_6 can be explained with 3F_3 consistent with the Breit-Wigner formula.¹⁷ From $\Delta\sigma_L$ data we can estimate the elasticity Γ_{el}/Γ of the possible 3F_3 resonances as 0.15-0.25.¹⁸

C. Evidence in σ_{el}^{Tot} Data

The elastic pp total cross section is expressed in terms of partial waves as

$$\sigma_{el}^{Tot} = 2\pi/k^2 \sum_J (2J+1) \{ |R_{JJ}|^2 + |R_{J+1,J}|^2 + |R_{J-1,J}|^2 + |R_J|^2 + |R^J|^2 \} \quad (17)$$

Data¹⁹ on σ_{el}^{Tot} show a bump around 1.5 GeV/c as shown in Fig. 9. The bump size is consistent with the possible 3F_3 resonance with the elasticity of 0.15-0.25 estimated from the $\Delta\sigma_L$ data. This experimental fact seems to support our 3F_3 resonance picture.

FIGURE 9 Elastic Total Cross Section.

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D. Evidence in C_{LL} $d\sigma/d\Omega$ Data

Recently, structures around 1.5 GeV/c in C_{LL} have been observed at Argonne ZGS⁵ for $70^\circ \geq \theta_{c.m.} \geq 110^\circ$. The data at $\theta_{c.m.} = 90^\circ$ are shown in Fig. 10. These strong energy dependences can be explained with the possible 3F_3 resonance, and the other partial-wave amplitudes varying slowly with energy.^{2,5} Let us study first the energy dependence of C_{LL} $d\sigma/d\Omega$ at $\theta_{c.m.} = 90^\circ$. The 3F_3 contribution to $C_{LL}(90^\circ)$ is given as:

$$k^2 C_{LL} d\sigma/d\Omega (90^\circ) = 0.77 |{}^3F_3|^2 + \text{Im} A \text{Im} {}^3F_3 \\ + \text{Re} A \text{Re} {}^3F_3 + (\text{terms without } {}^3F_3), \quad (18)$$

where

$$A = -2.63 {}^3P_1 + 1.75 {}^3F_2 + 2.63 {}^3P_2 - 4.29 R^2 \\ - 3.281 {}^3F_4 + 5.87 R^4, \quad (19)$$

FIGURE 10 C_{LL} at $\theta_{c.m.} \approx 90^\circ$ vs. p_{lab} . (Weighted averaged of data at angles adjacent to 90° .) The dashed curve is only to guide the eye.

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the value of which can be estimated from the results of phase-shift analyses.^{4,16} In the region around 1.5 GeV/c, both phase-shift analyses of Refs. 4 and 16 give approximately $\text{Re}A \approx 0.9$ and $\text{Im}A \approx -1.2$. By substituting these values and the 3F_3 resonance at 2260-MeV mass with a 200-MeV width, and with an elasticity of 0.25 into Eq. 18, we find that the 3F_3 resonance reproduces well the rapid decrease of the data, as shown in Fig. 11a.

So far we have shown the agreement at $\theta_{\text{c.m.}} = 90^\circ$. Similar conclusions were reached at other angles (See Figs. 11 a-c).

FIGURE 11a

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FIGURE 11b

FIGURES 11a-c

$k^2 C_{LL} d\sigma/d\Omega$ at $\theta_{c.m.}$
 $\approx 90^\circ$ (a), 80° (b) and
 70° (c) vs. p_{lab} . The
solid curve is the con-
tribution of the 3F_3
resonance over a smooth
(constant) background.

FIGURE 11c

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E. Evidence in Dispersion Analysis

With the dispersion analysis of a forward pp scattering amplitude $\phi_1(0) - \phi_3(0)$ using the data on $\Delta\sigma_L = (4\pi/k) \cdot \text{Im}[\phi_1(0) - \phi_3(0)]$, Grein and Kroll³ have calculated $\text{Re}[\phi_1(0) - \phi_3(0)]$ and shown that the Argand plot of the amplitude $-A_3 \equiv -p_{lab}/k [\phi_1(0) - \phi_3(0)]$ has a clear resonancelike behavior around 1.5 GeV/c. The Argand plot is shown in Fig. 12.

In terms of partial-wave amplitudes, $-A_3$ is given as

$$-A_3 = p_{lab}/k^2 \sum_J \{ (2J+1)(R_{JJ} - R_J) - R_{J+1,J} + R_{J-1,J} - 4[J(J+1)]^{1/2} R^J \} . \quad (20)$$

FIGURE 12 Argand Plot of $-A_3 = -p_{lab}/k[\phi_1(0) - \phi_3(0)]$
Obtained from the Dispersion Analysis of Ref. 3

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For the reason explained in III-A above, this circle seems to suggest the existence of an R_{JJ} resonance, probably a 3F_3 resonance. If this circle is taken to be due to the 3F_3 resonance, its mass, width, and elasticity are about 2260 MeV, and 0.15-0.25, respectively.²⁰

F. Evidence in Phase-Shift Analysis

Next is an evidence in the phase-shift analysis done by Hoshizaki.⁴ His input data are as follows: σ^{Tot} , $\sigma_{\text{inel}}^{\text{Tot}}$, $\text{Re}[\phi_1(0) + \phi_3(0)]/\text{Im}[\phi_1(0) + \phi_3(0)]$, $\text{Re}[\phi_1(0) - \phi_3(0)]$, $\Delta\sigma_T$, $\Delta\sigma_L$, $d\sigma/d\Omega$, P , C_{KP} , C_{NN} , C_{LL} , C_{SS} , C_{qkn} , D , R , and A where $\text{Re}[\phi_1(0) - \phi_3(0)]$ is from the dispersion calculation of Grein and Kroll,³ and C_{KP} , C_{NN} , C_{LL} , C_{SS} , C_{qkn} , D , R , and A are spin-spin correlation parameters. He performs a χ^2 fit at each energy. The number of data points at each energy is about 130, and the number of free parameters is about 20. At each energy χ^2 is about 100. He uses one-pion exchange (OPE) for higher partial-wave contributions, that is, waves with $J > 6$ and $L > 6$. Outputs are a possible 3F_3 resonance around 1.5 GeV/c and a possible 1D_2 resonance around $p_{\text{lab}} = 1.2$ GeV/c. Note here that changing OPE to OBE (one-boson exchange) does not change the results very much. The resonance parameters of the 1D_2 and 3F_3 diprotons were obtained by estimating a background: The energy dependence of 1D_2 and 3F_3 partial waves at lower energies was extrapolated to the resonance region. The resonant parts thus extracted are shown in Figs. 13 and 14. The 3F_3 resonance parameters are estimated to be $M = 2.22$ GeV, $\Gamma = 100$ -150 MeV, and $\Gamma_{\text{el}}/\Gamma = 0.2$, consistent with the estimations from $\Delta\sigma_L$ data in III-A and III-B.²¹

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FIGURE 13 Argand Diagram of the 3F_3 Resonant Amplitude.
Numbers are p_{lab} in GeV/c.

FIGURE 14 Argand Diagram of the 1D_2 Resonant Amplitude.
Numbers are p_{lab} in GeV/c.

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IV. OTHER POSSIBLE DIBARYON RESONANCES

A. Diproton Resonance at Mass ≈ 2430 MeV

Results of $\Delta\sigma_T$ measurements are shown in Fig. 15. The 2.0-GeV/c peak is clearly visible in $\Delta\sigma_T$. We observe a hint of structure in $\sigma^{\text{Tot}}(\pm)$ near 2.0 GeV/c as shown in Fig. 5, although it is not as remarkable as the one appearing in $\Delta\sigma_T$. From Eq. 4 one can say that if the peak is due to a resonance it is one of the singlets 1S_0 , 1D_2 , 1G_4 , The structure around 2 GeV/c is absent in the data of Fig. 11c, where the values of $k^2 C_{LL}$ ($d\sigma/d\Omega$) at $\theta_{\text{c.m.}} = 70^\circ$ are plotted as a function of beam momentum. Only the 1G_4 contribution among the singlets vanishes at $\theta_{\text{c.m.}} = 70^\circ$. Thus we may conclude that the 1G_4 wave is responsible for the structure.

FIGURE 15 Total Cross-Section Difference $\Delta\sigma_T = \sigma^{\text{Tot}}(\pm) - \sigma^{\text{Tot}}(\pm)$.²²

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On the other hand, Hoshizaki's phase-shift analysis⁴ shows hints of a possible dibaryon resonance in the singlet 1S_0 state with a mass center around $p_{lab} = 2.0$ GeV/c.

B. 1D_2 Diproton Resonance

We observe a structure in both $\Delta\sigma_L$ and $\Delta\sigma_T$ around 1.2 GeV/c (Figs. 4 and 15), which suggests a possible singlet resonance.

As explained in III-F, Hoshizaki's phase-shift analysis shows a hint of the possible 1D_2 diproton resonance around $p_{lab} = 1.2$ GeV/c. His estimates of the resonance parameters are $M \approx 2.17$ GeV, $\Gamma \approx 50$ -100 MeV, and $r_{el}/r \approx 0.1$.

The observed sharp peak in produced π^- laboratory momentum spectrum in a reaction $pd \rightarrow \pi^- + X$ (which can be interpreted as $pn \rightarrow \pi^- + X^{++}$) suggests a diproton resonance with $M = 2.16$ GeV and $\Gamma = 20$ MeV. This may be identical with the possible 1D_2 diproton resonance.²³ The observed sharp peak at 1.2 GeV/c in the forward $pp \rightarrow \pi^+d$ differential cross section also suggests a possible resonance (presumably 1D_2 resonance),²⁴ though there is a qualitative explanation for the peak with an exchange mechanism.²⁵

There is already a history for the possible 1D_2 resonance. For example, in SU(6) theory, Dyson and Xuong²⁶ predicted the 1D_2 resonance with $M = 2.18$ MeV which would predominately decay into s-wave $N\Delta$. Developing coupled channel N/D equations (pp, $N\Delta$), Arndt²⁷ predicted a bound state in the $N\Delta$ channel. In a model similar to that of Arndt, Suzuki²⁸ found a resonance in the spin-singlet 1D_2 p-p state with a mass of 2.16 GeV. In the analysis of pp total cross sections in transverse spin states, Kane and Thomas²⁹ discussed the possibility of a resonance in the 1D_2 state with a mass of 2.16 GeV.

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C. Possible Dinucleon Resonance

Kamae et al.^{6,30} suggested the possible existence of a dinucleon resonance by measuring the proton polarization in the reaction $\gamma d \rightarrow p^+ n$. They observed a prominent energy dependence in the polarization. They interpret this as an indication of an existence of a deeply bound Δ - Δ state. Their estimates of mass and width of the possible resonance are 2380 and 200 MeV. From the angular dependence of their polarization data, they estimated spin parity and isospin as $(J^P, I) = (3^+, 0)$. They obtained reasonable fits to the polarization data by introducing their dibaryon resonance with mass around 2380 MeV in addition to the 3F_3 dinucleon resonance with mass around 2260 MeV described in Section III. Without the 3F_3 resonance, they could not obtain good fits by using only one resonance of 2380 MeV. This fact also supports the 3F_3 diproton resonance picture.

D. Possible $p\Lambda$, $\Lambda\Lambda$, and $p\Lambda\Lambda$ Resonances

CERN/Hidelberg/Munich collaboration³¹ suggested the possible existence of a dibaryon resonance from an invariant mass distribution of the p - Λ system in the reaction $k^- d \rightarrow (p\Lambda)\pi^-$ which has a narrow sharp peak. Mass and width are estimated as 2129 and 6 MeV. Their estimates of spin and parity are $J^P = 1^+$. At the Tokyo conference, Shahbazian et al.³² reported three strange multibaryon resonances suggested by their propane bubble-chamber experiments with π^- and neutron beams at Dubna. They observed many peaks in the invariant-mass distributions of $p\Lambda$, $\Lambda\Lambda$, and $p\Lambda\Lambda$ systems. For $p\Lambda$, $(M, \Gamma) = (2128 \text{ MeV}, 1 \sim 3 \text{ MeV})$ and $(2256, \leq 15)$, for $\Lambda\Lambda$, $(M, \Gamma) = (2350, \leq 50)$, and for $p\Lambda\Lambda$, $(M, \Gamma) = (3568, \leq 60)$.

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E. $I=0$ Dinucleon Resonance

An Argonne group³³ has recently measured the difference between isoscalar nucleon-nucleon total cross sections for pure longitudinal initial spin states, $\Delta\sigma_L(I=0)$, using a polarized proton beam and a polarized deuteron target. In the $\Delta\sigma_L(I=0)$ preliminary data, a significant structure is observed around 1.5 GeV/c. This seems to suggest the existence of a new isoscalar spin-singlet dinucleon resonance. More detailed investigations will take place in the future.

V. DISCUSSION AND CONCLUSION

Here we discuss threshold effects and exchange models concerned with the possible 3F_3 (2260) resonances described in Section III.

A. Threshold Effects

The s-wave N- Δ system can couple only to 1D_2 wave of p-p system by parity and angular-momentum conservations. Therefore, the s-wave N- Δ threshold effect, which is supposed to be the most important one in the energy region in question, is allowed to appear only in the single 1D_2 partial-wave amplitude, if any. [Note p_{lab} (N- Δ threshold) $\approx$ 1.3 GeV/c.] So at least we can say the following:

1. The remarkable structure in $\Delta\sigma_L$ (III-A) and A_3 (III-E) is due to the triplet R_{JJ} wave. Thus the structure has nothing to do with the s-wave N- Δ threshold effect.

2. Moreover, polarization P $d\sigma/d\Omega$ (III-B) does not include singlet terms, so that the observed structure in polarization also has nothing to do with this threshold effect.

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3. The structure of the triplet 3F_3 solution obtained in the phase-shift analysis of Hoshizaki (III-F) also has nothing to do with this threshold effect.

There are many standard issues concerning the resonance interpretations and the threshold effects.

B. Exchange Models (Inelastic Threshold Models)

Next we remark about exchange models that try to explain the structure of $\Delta\sigma_L$ data at 1.5 GeV/c. There are three models.

1. "Deck" Model³⁴

The "Deck" contribution to $\Delta\sigma_L$ data is given by the diagram shown in Fig. 16. The results are shown in Fig. 17. The "Deck" contribution cannot explain the structure of $\Delta\sigma_L$ data.

2. OPE Three-Body Theory

Kloet et al.³⁵ assume that $NN \rightarrow NN$ and $NN \rightarrow N\Delta$ amplitudes satisfy the relations which yield the integral equations for the two amplitudes. They translate these equations into those for partial-wave amplitudes, solve them, and obtain the results shown in Fig. 18. Their model cannot explain the $\Delta\sigma_L$ structure.

FIGURE 16 The "Deck" Diagram Contributing to $\Delta\sigma_L$; Small Arrows Mean the Longitudinal Spin States.

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FIGURE 17 The Plot of "Deck" Contribution to $\Delta\sigma_L$ Compared With the Data (Ref. 34).

FIGURE 18 The OPE-Model Prediction Compared With the $\Delta\sigma_L$ Data (Ref. 35).

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3. OBE Inelastic-Threshold Model³⁶

In this model, elastic and inelastic parts of $\Delta\sigma_L$ are given graphically by the diagrams shown in Fig. 19. The results are shown in Fig. 20. The OBE model cannot explain the structure of the data. So far, none of the exchange models explain the structure of $\Delta\sigma_L$ at 1.5 GeV/c.

FIGURE 19 The OBE Diagrams Contributing to (a) $\Delta\sigma_L^{el}$ and (b) $\Delta\sigma_L^{inel}$. Small arrows represent the longitudinal spin states.

FIGURE 20 The OBE Model Prediction for $\Delta\sigma_L^{el}$ Compared With the "Data": $\Delta\sigma_L^{el}$ ("data") $\equiv \Delta\sigma_L^{inel}$ (OBE Model).

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C. Conclusion

The pp-data analyses indicate the existence of a diproton resonance with a mass near 2260 MeV, a width of 100-200 MeV, and an elasticity between 0.15 and 0.25, namely a ${}^3F_3(J^P = 3^-)$ resonance named $B^2(2.22)$. The pp data also indicate the possible existence of several diproton resonances, that is, 1D_2 resonance at 1.2 GeV/c, 1S_0 or 1G_4 resonance at 2.0 GeV/c, and higher resonances. These possible diproton resonances are summarized in Table II.³⁷

The 3F_3 (2260) resonance corresponds to $b = 0.9$ fm of the impact parameter between the incident protons, which is small as compared with the deuteron size $b = 1.7$ fm and is consistent with the size of ordinary baryon resonances. Experimentally, resonances in meson-baryon scattering are mainly formed by peripheral ($b = 1$ fm) interaction. This suggests strongly that the 3F_3 resonance is not a nucleus but a six-quark bound state (single hadron).

TABLE II Diproton Resonances

Name	$B^2(2.14)$	$B^2(2.22)$	$B^2(2.43)$
I	1	1	1
Mass, GeV	2.14 - 2.17	2.20 - 2.26	2.43 - 2.50
Width, MeV	50-100	100-200	~150
Elasticity	~0.1	~0.4	
J^P	2^+	3^-	0^+ or 4^+
pp State	1D_2	3F_3	1S_0 or 1G_4
Remarks	$\Delta\sigma_L, \Delta\sigma_T, C_{LL}$ Arndt et al. Hoshizaki	$\Delta\sigma_L, P, C_{LL}, \sigma_{el}^{Tot}$ Hidaka et al. Hoshizaki Grein-Kroll	$\Delta\sigma_T, C_{LL}$ Hoshizaki

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tation presented in this section. Hidaka et al.² take
into account partial waves up to $J=4$ or $L=4$, which cor-
responds to the impact parameter $b=1.3$ fm in 1-2-GeV/c
region. From the interaction-range argument, Minami
insisted that it was necessary to take the waves up to
at least $L=6$. However, all the experimental data on
 a_n and b_n with $n \geq 8$ almost vanish in $p_{lab} = 1.0 \sim 2.0$
GeV/c. This clearly indicates we can safely neglect
the waves with $J \geq 5$ and $L \geq 5$. Moreover, according
to Hoshizaki's phase-shift analysis⁴ with the waves up
to $L=7$, $\delta(^3H_4)$, $\delta(^3H_6)$, and $\delta(^3J_6)$ are all much smaller
than $\delta(^3F_4)$ in 1-2 GeV/c. This also suggests that includ-
ing higher waves does not change our argument and con-
clusion on b_n . Note here that polarization P $d\sigma/d\Omega$
includes only spin-triplet waves.
 18. The name $B^2(2.22)$, of the possible 3F_3 resonance was
adopted during the Tokyo conference (August, 1978).

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Kamo and Watari⁷ showed that all the existing data on total and differential cross sections and polarization of $pp \rightarrow \pi^+ d$ scattering can be well explained with the formation of the diproton resonances of 1D_2 and 3F_3 in addition to the neutron exchange Born term and $N\Delta$ intermediate states contribution. Without these resonances, they could not reproduce all the data well. For these resonances they obtained values of masses and widths consistent with those estimated in the analyses

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of pp elastic scattering in the section III and IVB. These two analyses of $\pi d \rightarrow \pi d$ and $pp \rightarrow \pi^+ d$ scatterings also seem to support our 1D_2 and 3F_3 diproton resonance picture.

MacGregor³⁸ pointed out that these resonances 1D_2 , 3F_3 and 1G_4 accurately correspond to rotational levels of a virtual $pp\pi$ (2020) bound state.

With a bootstrap calculation Balasz and Nicolescu have obtained many dinucleon Regge trajectories which give resonances with the masses, spins and parities just corresponding to these resonances 1D_2 , 3F_3 and 1G_4 (L.A.P. Balasz and B. Nicolescu, in preparation).

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APPENDIX

TABLE 1 $An(J,i,J',i')$ and $Bn(J,i,J',i')$

$a_0 = 1/2 \sum (\text{coefficient}) \operatorname{Re}(A_{J,i} \cdot A_{J',i'}^*)$

	$1S_0^*$	$3P_0^*$	$3P_1^*$	$1D_2^*$	$3F_3^*$	$3P_2^*$	R^2^*	$3F_3^*$	$1G_4^*$	$3H_4^*$	$3F_4^*$	R^4^*
$1S_0$	2.00	0	0	0	0	0	0	0	0	0	0	0
$3P_0$	0	2.00	0	0	0	0	0	0	0	0	0	0
$3P_1$	0	0	6.00	0	0	0	0	0	0	0	0	0
$1D_2$	0	0	0	10.00	0	0	0	0	0	0	0	0
$3F_2$	0	0	0	0	10.00	0	0	0	0	0	0	0
$3P_2$	0	0	0	0	0	10.00	0	0	0	0	0	0
R^2	0	0	0	0	0	0	20.00	0	0	0	0	0
$3F_3$	0	0	0	0	0	0	0	14.00	0	0	0	0
$1G_4$	0	0	0	0	0	0	0	0	18.00	0	0	0
$3H_4$	0	0	0	0	0	0	0	0	0	18.00	0	0
$3F_4$	0	0	0	0	0	0	0	0	0	0	18.00	0
R^4	0	0	0	0	0	0	0	0	0	0	0	36.00

$$b_2 = \sum (\text{coefficient}) \operatorname{Im}(A_{J,i} \cdot A_{J,i}^*)$$

	${}^3F_2^*$	${}^3P_2^*$	R^2^*	${}^3F_3^*$	${}^1G_4^*$	${}^3H_4^*$	${}^3F_4^*$	R^4^*
3P_0	-2.00	2.00	0.82	0	0	0	0	0
3P_1	-3.00	3.00	1.22	0	0	0	0	0
1D_2	0	0	0	0	0	0	0	0
3F_2	0	0.71	2.33	2.00	0	-7.94	0.22	1.77
3P_2	0	0	-2.04	-2.00	0	0	7.71	0
R^2	0	0	0	0.82	0	0	3.15	0
3F_3	0	0	0	0	0	-2.78	2.78	0.62
1G_4	0	0	0	0	0	0	0	0
3H_4	0	0	0	0	0	0	0.19	2.44
3F_4	0	0	0	0	0	0	0	-2.40

b_4	b_4						
	$3P_2^*$	R^2^*	$3F_3^*$	$1G_4^*$	$3H_4^*$	$3F_4^*$	R^4^*
$3P_0$	0	0	0	0	-2.00	2.00	0.45
$3P_1$	0	0	0	0	-3.00	3.00	0.67
$1D_2$	0	0	0	0	0	0	0
$3F_2$	4.29	1.75	1.50	0	-1.12	1.23	1.57
$3P_2$	0	0	-1.50	0	-0.89	0.79	-1.12
R^2	0	0	-0.61	0	-2.40	2.36	0
$3F_3$	0	0	0	0	-2.45	2.45	0.55
$1G_4$	0	0	0	0	0	0	0
$3H_4$	0	0	0	0	0	0.73	2.28
$3F_4$	0	0	0	0	0	0	-2.12
R^4	0	0	0	0	0	0	0

a_6	b_6									
	$3F_3^*$	$1G_4^*$	$3H_4^*$	$3F_4^*$	R^4^*	$3F_3^*$	$3H_4^*$	$3F_4^*$	R^4^*	
$1D_2$	0	81.82	0	0	0					
$3F_2$	0	0	5.94	60.61	37.95	$3F_2$	-0.14	5.05	1.13	
$3P_2$	0	0	58.91	0	0	$3P_2$	-4.91	0	0	
R^2	0	0	37.41	0	119.50	R^2	02.00	0	0	
$3F_3$	23.86	0	29.70	37.12	-66.40	$3F_3$	-1.77	1.77	0.40	
$1G_4$	0	32.73	0	0	0	$1G_4$	0	0	0	
$3H_4$	0	0	9.31	17.82	36.43	$3H_4$	0	1.91	1.95	
$3F_4$	0	0	0	5.68	28.46	$3F_4$	0	0	-1.52	
R^4	0	0	0	0	32.36	R^4	0	0	0	

a_8	b_8			
	$1G_4^*$	$3H_4^*$	$3F_4^*$	R^4^*
$1G_4$	61.68	0	0	0
$3H_4$	0	2.47	98.69	44.13
$3F_4$	0	0	0	0
R^4	0	0	0	98.69
		$3H_4$	$3F_4$	R^4
			6.17	1.38
			0	0
			0	0

$$\begin{aligned}
 \text{Notes - } a_0 &= 1/2\{2.00 \operatorname{Re}(^1S_0 \cdot ^1S_0^*) + 2.00 \operatorname{Re}(^3P_0 \cdot ^3P_0^*) + \dots\} \\
 a_2 &= 1/2\{20.00 \operatorname{Re}(^1S_0 \cdot ^1D_2^*) + 12.00 \operatorname{Re}(^3P_0 \cdot ^3F_2^*) + \dots\} \\
 b_2 &= -2.00 \operatorname{Im}(^3P_0 \cdot ^3F_2^*) + 2.00 \operatorname{Im}(^3P_0 \cdot ^3P_2^*) + \dots\} \\
 a_4 &= 1/2\{36.00 \operatorname{Re}(^1S_0 \cdot ^1G_4^*) + 20.00 \operatorname{Re}(^3P_0 \cdot ^3H_4^*) + \dots\} \\
 b_4 &= -2.00 \operatorname{Im}(^3P_0 \cdot ^3H_4^*) - 3.00 \operatorname{Im}(^3P_1 \cdot ^3H_4^*) + \dots\} \\
 a_6 &= 1/2\{81.82 \operatorname{Re}(^1D_2 \cdot ^1G_4^*) + \dots\} \\
 b_6 &= -0.14 \operatorname{Im}(^3F_2 \cdot ^3H_4^*) + \dots\} \\
 a_8 &= 1/2\{61.68 \operatorname{Re}(^1G_4 \cdot ^1G_4^*) + \dots\} \\
 b_8 &= 6.17 \operatorname{Im}(^3H_4 \cdot ^3F_4^*) + \dots\}
 \end{aligned}$$