FUSION REACTIONS AT LOW ENERGY

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CONF-850376--1

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CONF-850376--1

1. INTRODUCTION

DE85 010561

One of the most dramatic and fundamental reactions in nature is that of the fusion of atomic nuclei. In this process two nuclei come together fusing to form a Bohr¹ compound nucleus which then decays either through particle and gamma-ray emission or by fissioning. Formation of the compound nucleus is inhibited primarily by the electrostatic repulsion between the collision partners. The full radially-dependent ion-ion or interaction potential consists of the sum of Coulombic, nuclear and centrifugal terms. As shown in Figure 1 this potential reaches a maximum at the separation distance where long-range Coulombic and short-range nuclear forces just balance one another. At smaller radial separations there is an attractive pocket in the potential which traps the ions. Fusion occurs once the ions either pass over or penetrate through the outer maximum in this potential.

To calculate the cross sections for fusion we solve the Schrödinger equation subject to the appropriate boundary conditions. We may use the ingoing wave boundary condition method²⁻⁴ or we may use the WKB method⁵. In the latter case, we may express the fusion cross section $\sigma^{CF}(E)$ as a sum over partial waves

$$\sigma^{CF}(E) = \pi \lambda^2 \sum_{\ell} (2\ell+1) T_{\ell}(E) \quad (1)$$

with transmission functions

$$T_{\ell}(E) = [1 + \exp 2K_{\ell}(E)]^{-1} \quad (2)$$

and action integral

$$K_{\ell}(E) = \mp \int_{r_a}^{r_b} \left[\frac{2\mu}{\hbar^2} |V_{\ell}(r) - E| \right]^{1/2} dr \quad (3)$$

In the above, λ is the reduced deBroglie wavelength of the incident ion, μ is the reduced mass and $V_{\ell}(r)$ is the ion-ion potential. We will see shortly that this simple, intuitive picture fails in a rather spectacular way for massive nuclear systems.

2. FUSION MEASUREMENTS AT LOW ENERGY

The evaporation residues, i.e., the particle and gamma-ray emission decay products, differ only slightly in mass and charge from the compound nucleus.

FIGURE 1

Plots of two interaction potentials (solid and dashed lines) as a function of radial separation between the centers of the nuclei for representative partial waves for $^{58}\text{Ni}+^{58}\text{Ni}$.

Their direct detection provides us with an unambiguous measure that fusion rather than, say, quasielastic or deep-inelastic scattering has occurred. The major difficulty in detecting evaporation residues is that their angular distributions are sharply peaked at 0° . As a consequence they are frequently embedded in a background of beam-like ions 10^{11} or 10^{12} times larger. To detect the rare events of physical interest we make use of the difference in velocity, mass and charge between the residues and the beam-like ions employing various combinations of velocity selectors and energy loss and/or time-of-flight telescopes.

The evaporation residues are formed as highly excited, highly ionized atoms. Typical ion recoil velocities are in the range 2 to 5% of the speed of light. While moving through the target medium an equilibrium ionic charge distribution is established characteristic of the newly synthesized atomic species. The ionic charge distribution, however, may be affected by the deexcitation process. Although the particle emission state is over before the ions exit the target material, the gamma-ray emission stage may last long enough for the residues to

travel some distance beyond the target as illustrated in Figure 2. When this happens internal conversion and Auger processes can perturb the ionic charge distribution. Several studies of ionic charge distributions have been carried out ^{6,7} which show shifts in mean ionic charge away from empirical values⁸ or values predicted by the theory of Bohr and Lindhard⁹.

FIGURE 2

Schematic representative of the products of a typical fusion reaction.

The velocity selectors used in the fusion and charge-state measurements contain electrostatic deflectors sensitive to the ionic electric rigidity E_{kin}/q . In the measurements^{6,10-12} carried out at BNL using the MIT-BNL velocity selector the electrostatic deflector was followed by a Wein (ExB or velocity) filter plus a ΔE -E telescope. Other system of this general type have been used for fusion measurements at low energy at the GSI¹³, Munich¹⁴ and ANL¹⁵ and, most recently, at ORNL¹⁶ and Legnaro¹⁷.

3. FUSION CROSS SECTIONS

We now examine our experimental results beginning with the $^{58}\text{Ni}+^{58}\text{Ni}$, $^{58}\text{Ni}+^{64}\text{Ni}$ and $^{64}\text{Ni}+^{64}\text{Ni}$ triad of systems ^{10,18}. Displayed in Figure 3 are both the measured and the predicted excitation functions for complete fusion. The data span energy ranges from some 7 MeV below the barrier to about 14 MeV above the barrier, and exhibit exponential-like increases in magnitude with increasing energy typical of a barrier penetration process. The calculations were carried out using the WKB method. At above barrier energies these calculations, performed with no parameter adjustments, are in good agreement with the data. At energies not too far above the barrier we find that the cross sections for these as well as other massive systems behave classically. First obtained by Weisskopf¹⁹ the classical expression for capture of a charged-particle by a nucleus is

$$\sigma_{CF}(E) = \pi R_0^2 (1 - V_0/E) \quad (4)$$

FIGURE 3

Excitation functions for complete fusion of $^{58}\text{Ni}+^{58}\text{Ni}$, $^{58}\text{Ni}+^{64}\text{Ni}$ and $^{64}\text{Ni}+^{64}\text{Ni}$ as a function of average center-of-mass energy. Smooth curves denote the results of WKB calculations.

FIGURE 4

Plot of cross sections for complete fusion of $^{74}\text{Ge}+^{74}\text{Ge}$ versus $1/E_{c.m.}$. The barrier height is given by the intercept of the linear fit and the abscissa. The insert provides an expanded view of the near and subbarrier regimes.

We see an example of classical behavior in Figure 4. When plotted as a function of $1/E$ the cross sections for the $^{74}\text{Ge}+^{74}\text{Ge}$ system⁶ increase linearly over a broad range of above-barrier energies.

The departure of the data displayed in Figure 4 from linearity at low energies signals the onset of quantal tunnelling. Returning to Figure 3 we observe that although theory and experiment are not in obvious disagreement with one another at above-barrier energies the quantal tunnelling calculations do not describe the near-and subbarrier behaviour of the fusion data. Instead, the WKB predications fall progressively further below the experimental results as the energy is lowered. At far subbarrier energies the measured cross sections exceed the WKB predications by more than three orders of magnitude.

These findings are not restricted to the Ni+Ni systems. In Figure 5 we show results of measurements carried out by us¹¹ of complete fusion of $^{40}\text{Ca}+^{40}\text{Ca}$, ^{44}Ca , ^{48}Ca . These data span energy ranges from 4 MeV below the barrier to about 20 MeV above the barrier. Again, while the quantum tunnelling predictions are in good agreement with the data at above barrier energies they fail to describe the extent of the subbarrier fusion. The underestimates in cross section magnitude may be expressed in terms of an effective shift in energy scale at the lowest energies. The shifts in energy scale for the Ni+Ni systems are on the order of 3 to 5 MeV. The shifts in energy scale for the Ca+Ca systems range from 2 to 3 MeV.

FIGURE 5

Excitation functions for complete fusion of $^{40}\text{Ca}+^{40}\text{Ca}$, $^{40}\text{Ca}+^{44}\text{Ca}$ and $^{40}\text{Ca}+^{48}\text{Ca}$ as a function of average center-of-mass energy. Smooth curves denote the results of WKB calculations.

The nuclear potentials used in the tunnelling calculations were of the liquid-drop or proximity type²⁰. Representative potential barriers generated using the Krappe-Nix-Sierk (KNS)²¹ potential as well as the more strongly attractive Akyuz-Winther (AW)²² potential were displayed in Figure 1. The KNS potential was employed in the calculations presented in Figure 3 and the AW potential was used for the predictions shown in Figure 5. The cross sections obtained using these potentials are similar to one another for these systems.

A possible explanation for the discrepancies between the tunnelling calculations and the data is that the nuclear potentials used to generate the potential barriers are incorrect. It has long been recognized that subbarrier fusion data can be used to probe the nuclear potential as far into the nuclear interior as the inner turning-point of the ion-ion potential. Making use of this observation, a procedure has been developed²³ for inverting subbarrier fusion data to obtain the corresponding potential barrier. Assumed is the existence of a real, local energy-independent potential barrier possessing a simple quadratic maximum. Also assumed is the WKB relationship between the potential barrier and the transmission function, and Eq. (1). The barrier thickness is then related to the action $K_0(E)$ by

$$t(V) = \frac{2}{\pi} \sqrt{\frac{\hbar^2}{2\mu}} \int_V^{V_0} dE \frac{\partial K_0(E)/\partial E}{\sqrt{E-V}} \quad (5)$$

and, in turn, the action $K_0(E)$ is determined from the experimental $\sigma^{CF}(E)$

$$K_0(E) = \frac{1}{2} \ln \left[\frac{1}{\frac{\partial}{\partial E} \left(\frac{E \sigma^{CF}(E)}{\pi R_E^2} \right)} - 1 \right] \quad (6)$$

An auxiliary condition is used to specify R_E . Potential barriers obtained from subbarrier fusion data by inversion are displayed in Figure 6. Included in this figure are results for fusion of lp-shell nuclei with one another. For these light mass systems discrepancies in energy dependence between data and predictions are largely absent (see, for example Ref. 24). As can be seen in Figure 6 the potential barriers extracted from these data by inversion are similar in shape to those obtained from the liquid-drop potentials. The situation changes for massive systems such as Ni+Ni and Ni+Ge where we now observe that there is no physically reasonable nuclear potential consistent with the data. From this we infer that the failure of the model to describe the low energy data is not primarily due to the choice of potential, but rather is due to inadequacies in the underlying model.

FIGURE 6

Barrier shapes determined by inversion of subbarrier fusion data for $^{13}\text{C}+^{13}\text{C}$, $^{14}\text{N}+^{14}\text{N}$, $^{13}\text{C}+^{16}\text{O}$, $^{40}\text{Ca}+^{40}\text{Ca}$, $^{64}\text{Ni}+^{64}\text{Ni}$ and $^{64}\text{Ni}+^{74}\text{Ge}$. The outer turning points were determined from a liquid drop potential. The distance between outer and inner turning points is the thickness function $t(V)$. Shaded areas denote the error envelop.

4. STRUCTURAL VARIATIONS IN SUBBARRIER FUSION

Perhaps the most striking and hitherto unexpected feature of the data for subbarrier fusion is the strong dependence of the fusion probability upon the nuclear valence structure. This aspect emerges when we directly compare excitation functions for neighboring systems after removing systematic $A^{1/3}$ -like differences in barriers. Shown in Figure 7 are reduced excitation functions for fusion of ^{58}Ni , ^{64}Ni and ^{74}Ge with one another. We observe that, while the excitation functions approach one another at above-barrier energies, they vary considerably from system to system at subbarrier energies. We see that there is a substantial increase in fusion when one of the ^{58}Ni collision partners is replaced by ^{64}Ni . However a comparable increase in fusion does not occur when the second ^{58}Ni collision partner is replaced by ^{64}Ni . Further increases in fusion do occur whenever ^{58}Ni or ^{64}Ni is replaced by ^{74}Ge .

FIGURE 7

Plot of reduced excitation functions for $^{58}\text{Ni}+^{58}\text{Ni}(x)$, $^{58}\text{Ni}+^{64}\text{Ni}$ (open circles), $^{64}\text{Ni}+^{64}\text{Ni}(+)$, $^{58}\text{Ni}+^{74}\text{Ge}$ (filled circles), $^{64}\text{Ni}+^{74}\text{Ge}$ (open squares) and $^{74}\text{Ge}+^{74}\text{Ge}$ (open triangles).

Excitation functions for other systems have been found which exhibit complex variations in magnitude and shapes. In systems which are not so massive that the barriers deduced by inversion are reentrant the variations are reflected in the effective barrier shapes. Displayed in Figure 8 are the barriers deduced by inversion²⁵ of the data¹¹ for subbarrier fusion of $^{40}\text{Ca}+^{40}\text{Ca}$ and $^{40}\text{Ca}+^{44}\text{Ca}$. We see that the addition of four valence neutrons to the ^{40}Ca core has a pronounced effect upon the deduced barrier shape. Structure influences of this type are present in data for lp-shell and sd-shell systems. Among lp-shell systems the differences in shape of the excitation functions for the $^{12}\text{C}+^{12}\text{C}$, $^{12}\text{C}+^{13}\text{C}$ and $^{13}\text{C}+^{13}\text{C}$ triad are noteworthy.

The influence of the valence structure of the collision partners upon the fusion process is pronounced in the vicinity of closed shells. Particularly large isotopic effects have been found in the $^{32,36}\text{S}+^{92-100}\text{Mo}$ systems¹⁴ and in the $^{40}\text{Ar}+^{144-154}\text{Sm}$ systems^{13,26}.

The presence of a few valence nucleons has an appreciable influence upon fusion even in systems involving nearly 200 nucleons. Displayed in Figure 9 are

FIGURE 8

Barrier shapes determined by inversion of subbarrier fusion data for $^{40}\text{Ca}+^{40}\text{Ca}$ and $^{40}\text{Ca}+^{44}\text{Ca}$. Cross-hatched and solid curves denote the inner and outer turning points. Shaded and dashed-dotted curves were generated using global, liquid-drop potentials.

excitation functions for evaporation residue formation following complete fusion of $^{81}\text{Br}+^{90}\text{Zr}$, ^{94}Zr , ^{96}Mo , ^{104}Ru and $^{90}\text{Zr} + ^{90}\text{Zr}$, ^{94}Zr . We observe that the excitation functions for these systems do not approach one another at above-barrier energies as did those for neighboring, less massive systems. Instead, evaporation residue formation declines at above barrier energies as the systems become more massive. Part, but not all, of this decline is due to an onset of fission competition at progressively lower energies relative to the barrier. Shown in Figure 10 are excitation functions for $^{81}\text{Br} + ^{90}\text{Zr}$, ^{94}Zr , ^{96}Mo and ^{104}Ru , plotted as a function of $E_{c.m.}-V_0$, where V_0 is the fusion barrier height. In this figure we see that as valence nucleons are added to the ^{90}Zr core the subbarrier fusion extends progressively further below the barrier. The shifts in energy scale between $^{81}\text{Br}+^{90}\text{Zr}$ and $^{81}\text{Br}+^{104}\text{Ru}$ of some 10 MeV is considerable.

The fusion of $^{90}\text{Zr}+^{90}\text{Zr}$ has been studied¹² as well. In systems such as $^{40}\text{Ca}+^{40}\text{Ca}$ and $^{90}\text{Zr}+^{90}\text{Zr}$ we find that the excitation functions rise more steeply than do those for neighboring systems at both near and subbarrier energies.

FIGURE 9

Excitation functions for evaporation residue formation following complete fusion of $^{81}\text{Br}+^{90}\text{Zr}$, ^{94}Zr , ^{96}Mo , ^{104}Ru and $^{90}\text{Zr}+^{90}\text{Zr}$, ^{94}Zr versus average center-of-mass energy. Smooth curves are visual guides and are not fits to the data.

FIGURE 10

Comparison of excitation functions for evaporation residue formation for the $^{81}\text{Br}+^{90}\text{Zr}$, ^{94}Zr , ^{96}Mo and ^{104}Ru systems.

5. CHANNEL COUPLING AND QUANTAL TUNNELLING

We have seen that structural variations are an essential feature of the quantal tunnelling process extending as they do from light mass $1p$ -shell systems to some of the heaviest systems yet formed. We have also seen that fusion is enhanced over quantal tunnelling predictions even in closed-shell systems such as $^{40}\text{Ca}+^{40}\text{Ca}$, $^{58}\text{Ni}+^{58}\text{Ni}$ and $^{90}\text{Zr}+^{90}\text{Zr}$. The largest shifts in energy scale occur in fairly massive systems. These shifts are less pronounced in the $\text{Ca}+\text{Ca}^{11}$ and $\text{S}+\text{Mg}^{27}$ systems, and are largely absent (with some puzzling exceptions) in $1p$ shell systems.

We now examine how these phenomena may come about as a consequence of the couplings of intrinsic degrees of freedom of the dinuclear system to that of

the relative motion. The consequences of these couplings upon the tunnelling have been demonstrated in an elegant manner using analytic models^{4,28-30} which maintain the clarity of the WKB and IWBC approaches. In the simplest of these models, a channel (2) representing, say, the excitation of a low-lying collective level in one of the collision partners, or the transfer of a neutron from one ion to the other, is coupled to the entrance channel (1). The coupled Schrödinger equations for the wave function $U(x)$ in one dimension

$$\begin{aligned} \left[-\frac{d^2}{dx^2} + K_1^2(x) \right] U_1(x) &= \frac{2\mu_F(x)}{\hbar^2} U_2(x) \\ \left[-\frac{d^2}{dx^2} + K_2^2(x) \right] U_2(x) &= \frac{2\mu_F(x)}{\hbar^2} U_1(x) \end{aligned} \quad (7)$$

with $K_\alpha^2(x) = 2\mu[E_\alpha - V(x)]/\hbar^2$ are solved subject to the appropriate boundary conditions. It is assumed that there is a wave incident from the right in the entrance channel (1), and reflected waves to the right of the barrier and transmitted waves to the left in both channels. The decoupling of the Schrödinger equations is accomplished by introducing the wave functions $U_\pm(x) = U_1(x) \pm U_2(x)$ and assuming that $E_1 = E_2 = E$ to give

$$\left[-\frac{\hbar^2}{2\mu} \frac{d^2}{dx^2} + V(x) \pm F(x) - E \right] U_\pm(x) = 0 \quad (8)$$

The total transmission is found to be the sum of half the transmission for a barrier $V(x)+F(x)$ and half the transmission for a barrier $V(x)-F(x)$. The resulting penetration of a barrier in the presence of a coupling interaction given by this model is depicted in Figure 11.

The upper portion of this figure shows the bifurcation of the original barrier on the left-hand-side by the coupling interaction. The total transmission is presented in the lower portion of the figure as a function of center-of-mass energy. The essential point is that the transmission is increased by the coupling interaction for energies below the top of the no-coupling barrier and the transmission is decreased for energies above the barrier top. Thus, the couplings enhance fusion below the barrier and diminish fusion above the barrier.

The two-channel model has been generalized to any number of channels and non-zero Q-values. In the case where the coupling interaction can be factored into relative motion and intrinsic parts, the total transmission is found to be

FIGURE 11

Upper figure: Bifurcation of the potential barrier in the two-channel, spatially constant coupling model. Lower figure: Influence of bifurcation upon the transmission coefficient. The dashed line is the result of penetration through the unmodified barrier of height V_b . The solid line gives the penetrability in the presence of the coupling interaction.

$$T = \sum_{\beta} | \langle 1 | \beta \rangle |^2 T[E, V(x) + \lambda_{\beta}] \quad (9)$$

The quantities λ_{β} are the eigenvalues of the hamiltonian for the interacting intrinsic system and play the rôle of the coupling strengths F in the two-channel model. From Eq. 9 we see that the incident flux now confronts a spectrum of barriers $V(x) + \lambda_{\beta}$ and the total transmission is obtained by summing the transmission coefficients for the barriers in the spectrum after weighting by the overlap factors $| \langle 1 | \beta \rangle |^2$, where 1 denotes the entrance channel, as before.

The eigenvalues λ_{β} depend upon the coupling strenghts F and Q -values. These models yield total transmission factors which are sensitive functions of the valence structure of the collision partners, in accordance with the experimental observations. For the two-channel model generalized to include non-zero Q -values

and

$$\lambda_{\pm} = (-Q \pm \sqrt{4F^2 + Q^2})/2 \quad (10)$$

$$P_{\pm} = |\langle 1 | \pm \rangle|^2 = \frac{F^2}{F^2 + \lambda_{\pm}^2}$$

Results³¹ of calculations of the influences upon fusion of inelastic couplings are exhibited in Figure 12. These calculations were carried out by numerically integrating the coupled Schrödinger equations using ingoing wave boundary conditions. Included in the figure are the results of couplings to a progressively larger number of low-lying collective levels. We first observe that the coupling of the entrance channel to the first 2^+ level at 1.454 MeV increases the cross sections for fusion at subbarrier energies over the static no-coupling case and decreases the cross sections for fusion at above-barrier energies. Inclusion of the 3^- level at 4.47 MeV increases the cross sections at subbarrier energies slightly and increases the cross sections at above-barrier

FIGURE 12

Comparison of measured and calculated excitation functions for complete fusion of $^{58}\text{Ni}+^{58}\text{Ni}$. The nuclear potential was of a Woods-Saxon form with $V = -82.3$ MeV, $r_0 = 1.149$ fm and $a = 0.63$ fm. The calculated barrier height is 101.5 MeV.

energies, as well. This somewhat different behavior is due to the higher excitation energy and not to the multipolarity. Inclusion of higher-lying 2^+ levels and the first 4^+ level has only a small effect, especially as one goes further below the barrier. The net consequence of the couplings is a modification of the slope of the excitation function and a substantial increase in fusion at

subbarrier energies.

Results similar to those just described have been obtained³² in an optical model approach in which the imaginary potential is restricted to the nuclear interior. The influence upon fusion of transfer couplings has been studied³³ for the $^{58}\text{Ni}+^{58}\text{Ni}$ and $^{58}\text{Ni}+^{64}\text{Ni}$ systems. As was the case for the inelastic couplings, the transfer couplings were shown to influence the fusion cross sections.

A simplified approach to the exact channel coupling and analytic model methods is given by the zero-point-motion picture³⁴. This geometric approach produces a spectrum of barriers as a result of zero-point-fluctuations in the surface-to-surface distance. It is most useful in situations where the coupled levels are low in energy and dynamic polarization influences are small^{35,36}.

6. QUASIELASTIC SCATTERING

In order to further our understanding of the low energy reaction processes involving massive nuclei measurements were carried out^{37,38} of quasielastic scattering in the Ni+Ni and Ca+Ca systems. By quasielastic scattering we refer to inelastic scattering and few nucleon transfer reactions which take place during weak contact of the collision partners. Shown in Figure 13 are the results³⁷ of our measurements of one-neutron transfer in the $^{58}\text{Ni}+^{64}\text{Ni}$ and $^{64}\text{Ni}+^{64}\text{Ni}$ systems. The Q-values for ground-state transfer in these systems are -0.66 and -3.56 MeV, respectively. There are two observations to make in Fig. 13. First, the energy dependence of the transfer cross sections differs from that of the fusion cross sections, falling-off less rapidly as the bombarding energy is lowered. For $^{58}\text{Ni}+^{64}\text{Ni}$, the excitation function for one-neutron transfer crosses over that for fusion at an energy near the barrier and is still 1 mb at energies nearly 20 MeV below the barrier. Second, the isotopic variations in the one-neutron transfer channel parallel those seen in the fusion channel.

A similar result is found for quasielastic scattering of $^{40}\text{Ca}+^{40,44}\text{Ca}$. For those systems inelastic scattering and transfer cross sections were measured³⁸ over energy ranges from just above the barrier to the highest energy of the fusion data. The isotopic variations in both inelastic and transfer channels were shown to parallel those of the fusion channel.

CONCLUSION

We have established that atomic nuclei fuse far more readily at low energies than would be expected from quantal tunnelling considerations alone. We have found that the behavior of the cross sections for fusion depends strongly upon the valence structure of the collision partners. This structural dependence

FIGURE 13

Excitation functions for complete fusion and for one-neutron transfer versus center-of-mass energy relative to the s-wave fusion barrier height. Circles denote data for $^{58}\text{Ni}+^{64}\text{Ni}$; squares represent data for $^{64}\text{Ni}+^{58}\text{Ni}$; and triangles signify data for $^{64}\text{Ni}+^{64}\text{Ni}$. Open symbols give the results for population of the 155.5 keV level in ^{63}Ni , filled symbols represent the complete fusion results.

extends from light lp-shell systems to systems involving nearly 200 nucleons.

These new phenomena may be viewed as characterizing the tunnelling of a quantal system with many degrees of freedom. The failure of standard tunnelling models may be understood as resulting from the ability of the dinuclear system to tunnel into the classically forbidden region by means of couplings to intrinsic degrees of freedom.

ACKNOWLEDGEMENT

The author wishes to acknowledge the valuable assistance of his colleagues at MIT with whom the measurements reported herein were carried out. The author particularly wishes to thank H.A. Enge, L. Grodzins, R.J. Ledoux, M.K. Salomaa, A. Sperduto and J. Wiggins.

The Joint Institute for Heavy Ion Research has as member institutions the University of Tennessee, Vanderbilt University, and the Oak Ridge National Laboratory; it is supported by the members and by the Department of Energy through Contract No. DE-AS05-76ER0-4936 with the University of Tennessee.

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