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Analytic Model of the Radiation-Dominated Decay of a Compact Toroid*

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The coaxial-gun, compact-torus experiments at LLNL and LASNL are believed to be radiation-dominated,^{1,2} in the sense that most or all of the input energy is lost by impurity radiation. This paper presents a simple analytic model of the radiation-dominated decay of a compact torus, and demonstrates that several striking features of the experiment (finite lifetime, linear current decay, insensitivity of the lifetime to density or stored magnetic energy) may also be explained by the hypothesis that impurity radiation dominates the energy loss. The model incorporates the essential features of the more elaborate 1 1/2-D simulations of Shumaker et al.,³ yet is simple enough to be solved exactly. Based on the analytic results, a simple criterion is given for the maximum tolerable impurity density.

The model assumes that power flow in the compact torus decay is dominated by the following processes: magnetic energy is transferred to the electrons by ohmic heating; the electrons then lose their energy by radiation. Other loss channels, such as electron-ion equilibration, electron cross-field heat conduction and ion cross-field heat conduction are assumed to be negligible. Given the observed temperatures and densities, neglecting these other loss channels is readily shown to be justified. The power flow equations are then

$$\frac{dW_B}{dt} = -P_{OH} \quad (1)$$

$$\frac{dW_{T,e}}{dt} = P_{OH} - P_{RAD} \quad (2)$$

where W_B is the magnetic energy, $W_{T,e}$ the electron thermal energy, P_{OH} the ohmic heating power, and P_{RAD} is the radiated power. Model expressions for W_B , P_{OH} and P_{RAD} will be discussed next.

The plasma is in MHD equilibrium inside the flux conserver. The equilibrium will be modeled by the spherical force-free Spheromak equilibrium of Morikawa,⁴ and Rosenbluth and Bussac.⁵ Essentially identical results would be obtained if the force-free cylindrical equilibria of Bondeson et al.⁶ were used. For the Spheromak equilibrium, the magnetic energy is $W_B = bI^2R$ where $b = \text{constant}$, $I = \text{total toroidal current}$, $R = \text{separatrix radius}$. For this equilibrium, the current is parallel to the magnetic field. Thus, using the Spitzer parallel resistivity,

$$P_{OH} = \int n_{||} J^2 d^3x = \frac{1}{\eta_{||}} \int J^2 d^3x \quad (3)$$

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where $\bar{\eta}_{||}$ is an average parallel resistance. For the Spheromak equilibrium, P_{OH} is then $P_{OH} = aI^2/(RT^{3/2})$ where $a = \text{constant}$, $T = \text{electron temperature}$. If I is measured in amperes, R in cm, T in eV, then the constants are $a = .6$, $b = 1.8 \times 10^{-9}$.

The radiated power will be modeled by a power law fit to the coronal equilibrium radiation calculations of D. E. Post et al.⁷ Thus, the radiated power is taken to be $P_{RAD} = cn_e n_i V T^k$ where c and k are constants, n_e and n_i are the electron and impurity densities, respectively, and V is the plasma volume. As an example, for oxygen radiation in the range $5 < T < 20$ eV, a decent fit to the coronal equilibrium calculations is $c = 4.2 \times 10^{-28}$, $k = 1.92$ (if P_{RAD} is measured in watts, V in cm^3 , and T in eV). For carbon, the appropriate values are $c = 7 \times 10^{-28}$, $k = 2.55$ for $3 < T < 10$ eV. Similar parameterizations are also reasonable above the radiation barriers. For example, for oxygen in the temperature range $30 < T < 100$ eV, $c = 4.5 \times 10^{-21}$, $k = -3.8$ provides a reasonable fit. However, for these higher temperatures the neglect of cross-field ion heat transport may not be correct, so these temperature ranges will not be discussed further.

To derive soluble equations, three further assumptions will be made: (1) the separatrix radius R is constant. This is indeed approximately true for the decay of plasma in a flux conserver, for which $R \sim \text{flux conserver radius}$, (2) n_e , n_i are constant during the discharge. This is certainly true classically, since the particle loss rate is of order β times the field decay rate ($\beta = 8\pi P/B^2$), and (3) $P_{OH} = P_{RAD}$. This is reasonable because, in a low beta plasma, the electron heat capacity is so small that the electron energy sources and sinks must very closely balance. All of these assumptions agree well with the results of numerical simulations.

As a result of these assumptions, Eqs. (1) and (2) become, respectively

$$bR \frac{dI^2}{dt} = - \frac{aI^2}{RT^{3/2}} \quad (4)$$

$$\frac{aI^2}{RT^{3/2}} = cn_e n_i V T^k \quad (5)$$

Equation (5), the electron power balance equation, determines the electron temperature:

$$T^{3/2} = \left(\frac{aI^2}{cR V n_e n_i} \right)^\alpha \quad (6)$$

where $\alpha = 3/(3 + 2k)$. Combining Eqs. (4) and (6) and solving the resulting equation, the result is

$$I(t) = I(0) \left(1 - \frac{t}{t_E} \right)^{1/2\alpha} \quad (7)$$

where the "extinction time" t_E is

$$t_E = \frac{(I(0))^{2\alpha}}{\gamma} \quad (8)$$

$$\gamma = \frac{a\alpha}{bR^2} \left(\frac{cn_e n_I R V}{a} \right)^\alpha$$

The magnetic field components will also obey an equation like Eq. (7) because, for fixed K , the fields are proportional to the current I . Equations (7) and (8) have the following consequences:

- A.) Finite lifetime--Obviously, the currents and fields are extinguished in a finite time t_E .
- B.) Lifetime scaling--Since $W_B = bI^2R$, the extinction time scales as

$$t_E \sim \frac{(W_B)^\alpha (R)^{2-5\alpha}}{(n_e n_I)^\alpha} \quad (9)$$

For oxygen, $\alpha = .44$, and so t_E is only weakly sensitive to the magnetic energy, the impurity level or the radius (2 - 5 $\alpha = -.19$).

- C.) Essentially linear current decay--For oxygen, $1/2\alpha = 1.14$, so

$$I(t) = I(0) \left(1 - \frac{t}{t_E} \right)^{1.14}$$

which is almost indistinguishable from linear decay.

These results are in good qualitative agreement with the experiments which indeed exhibit finite lifetimes, apparently linear current decay, and insensitivity to W_B . The impurity levels in the experiment are not known; thus detailed comparisons with the theory are difficult.

The analytic results are in good quantitative agreement with a 1/2-D simulation performed by Shumaker et al.³ The parameters of this simulation are $K \sim 38$ cm, the average electron density was $\sim .6 \times 10^{15} \text{ cm}^{-3}$, the oxygen impurity density was $n_I = 10^{13} \text{ cm}^{-3}$, and the poloidal field at the geometric center of the plasma ($r = z = 0$) was $B_C = 3.1$ kG. For the spheromak equilibrium, $B_C = 1.398 IK$ (if B_C is measured in Gauss, I in amperes). Using this relation Eqs. (12) and (14) become, for oxygen impurity radiation

$$\gamma = .70 \left(\frac{B_C^2}{4.28 \times 10^{-28} n_e n_I R^2} \right)^{.293} \text{ eV} \quad (10)$$

$$t_E = 3.9 \times 10^{-9} \frac{(B_C)^{.88} (R)^{1.12}}{(4.28 \times 10^{-28} n_e n_I)^{.44}} \text{ sec.} \quad (11)$$

(B_C measured in Gauss)

For the parameters of the simulation, this gives $T = 6.9$ eV, $t_f = 180$ μ sec. The simulation finds $T = 6.1$ eV at the 0 point, after the electron temperature equilibrates, i.e., after T adjusts so that $P_{RAD} = P_{OH}$. The analytical and simulation results for T are in reasonable agreement. In the simulation, the current does decay roughly linearly, but it does not decay to zero at 180 μ sec. This is probably because the power law fit overestimates P_{RAD} at these low temperatures ($T \sim 3.5$ eV at 180 μ sec.). However, the current decay time at $t = 20$ μ sec, when the electrons are first equilibrated is

$$\frac{1}{I} = \begin{cases} 200 \text{ } \mu\text{sec, simulation} \\ 2\alpha t_E = 160 \text{ } \mu\text{sec, theory} \end{cases}$$

The overall agreement between theory and simulation appears to be good.

The results of this analysis demonstrate that several features of the coaxial-gun CT experiments at LLNL and LASNL, viz., finite lifetime, linear decay, and insensitivity to magnetic stored energy and to density, may be simply explained by impurity radiation. This analysis thus gives support to the hope that the performance of these devices will improve if the impurity level is decreased sufficiently. With this in mind, it is useful to derive a criterion for the tolerable impurity density. Clearly, the plasma will burn through the peak of the oxygen radiation curve at ~ 20 eV if the equilibrated electron temperature is greater than 20 eV, i.e., if the temperature derived from Eq. (10) exceeds 20 eV. This requires

$$n_e n_{\text{oxygen}} < 2.6 \times 10^{22} \frac{B_C^2}{R^2} \quad (12)$$

(where B_C is the field at the geometric center $r = z = 0$, measured in Gauss, R is the separatrix radius in cm, and n is measured in cm^{-3}). As an example for $n_e = 2 \times 10^{14} \text{ cm}^{-3}$, $B_C = 5$ kG, $R = 40$ cm, which is typical of the low density higher current Beta-II experiments, burn through requires $n_i < 2 \times 10^{12} \text{ cm}^{-3}$; i.e., less than 1% oxygen is required. Burning through the carbon peak at ~ 10 eV requires $n_e n_{\text{carbon}} < 3.6 \times 10^{22} B_C^2 / R^2$. Caramana and Perkins have derived a similar criterion.⁸

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