Weak Interactions and Presupernova Evolution*

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Abstract

The role of weak interactions, particularly electron capture and β^- decay, in presupernova evolution is discussed. The present uncertainty in these rates is examined and the possibility of improving the situation is addressed.

1. Introduction

Stewart Bloom and I were introduced in the summer of 1988 at the Santa Cruz summer workshop on Supernovae. At that time we began to discuss how his Lanczos codes might facilitate the calculation of electron capture rates in stellar environments. Stewart and George Fuller had started to make this effort¹ in 1985, but at that time it was not yet economical to examine many nuclei. Recent work (see Resler in this volume) will now allow such a study. I am delighted to have the opportunity to take part in this study at Stewart's side.

In this paper, I will discuss the weak interactions which are important in a star just before it becomes a type II supernova. I will begin with an overview of where in stellar evolution we are concerned. I will mention how these reactions affect core collapse calculations and what the present uncertainties are in these rates. I will conclude with a discussion of how, with Stewart's help, we hope to ameliorate this situation.

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2. Overview

In this discussion we shall focus on the evolution of "moderately massive" stars (masses between roughly $12M_{\odot}$ and roughly $30M_{\odot}$). These stars are the progenitors of type II supernova and are believed to be the source of many of the elements which are essential for life (see Woosley in this volume). In this paper we further narrow our focus by concentrating on the very last stages of this evolution. We shall concern ourselves with the events just after silicon burning.

At this point the star has the famous 'onion' or 'Russian do'' structure: an iron core at the center, surrounded by layers of silicon, oxygen, neon, carbon, helium, and hydrogen. Each layer is burning, converting the material in that shell into the material beneath it. The iron core is where we must concentrate our effort, because this is where the type II explosion begins.

This core has densities of 10^7g/cc to 10^9g/cc and higher and temperatures of $2 \times 10^9\text{K}$ to $5 \times 10^9\text{K}$ and higher. It is composed of the ashes of silicon burning: iron group elements in nuclear statistical equilibrium (NSE). This material is in NSE because the relatively high temperatures prevalent in the core make strong and electromagnetic interactions essentially instantaneous. Because of this equilibrium, the abundance of various isotopes will depend only upon their binding energy, partition function, the material's temperature, density, and average charge to mass ratio.

This iron core is similar to a white dwarf because it is mainly supported by the degeneracy pressure of electrons. It is standard to keep track of the number of electrons with the variable Y_e , which is defined as the number of electrons per nucleon in the gas. Since the star has zero net charge, Y_e is also the average charge to mass ratio of the gas. For such a core, the mass which the core can support will be roughly proportional to Y_e^2 , as is true for a white dwarf.² Because the electrons are so important, weak interactions can have great effects on the structure of the iron core.

Weak interactions affect presupernova evolution at three points. During hydrogen burning, the weak interaction turns half of the protons in the stellar core into neutrons. This transformation is accompanied by the emission of neutrinos, which carry off the excess lepton number and a small fraction of the energy generated by the burning. From carbon burning onward, weak interactions provide the dominant energy emission mechanism for the star. Neutrino-antineutrino pairs are emitted from the burning regions, drastically speeding up the rate of burning. Because pairs are emitted, the lepton number (here the number of electrons) is not changed.

The last point at which the presupernova evolution is affected by weak interactions is in the newly formed iron core. Electron capture and beta decay are the important weak interactions here. Both of these processes alter Y_e and carry energy from the core by the emission of neutrinos. Because the core is in NSE, alterations in Y_e also affect the abundance of isotopes. Because these reactions affect Y_e and the energy balance in the core, they have an impor-

tant effect on core collapse calculations. They help form the initial conditions for these calculations. Thus, it is important to accurately calculate them and include them in stellar evolution simulations.

3. Stellar Electron Capture and Beta Decay Rates

There are four weak interactions which could directly affect the Y_e profile. In order of importance they are: electron capture, β^- decay, β^+ decay, and positron capture. They are listed in the same order below:

where (Z, A) is the nucleus ${}^A_Z\text{Z}$. The last reaction is completely unimportant because the density of positrons is very small at these temperatures. The third reaction is important after oxygen burning and during silicon burning, but is relatively slow because of the phase space and energetics of the reaction. Such reactions reduce Y_e from 0.5 to roughly 0.48 and have been discussed recently by Nomoto *et. al.*³ and Takahara *et. al.*⁴ We shall not discuss them further here because the first two reactions, electron capture and β^- decay, have much more dramatic effects.

In Figure 1 (see next page), we show Y_e at the center of an $18M_\odot$ star as a function of time and the central density. This star was evolved using the KEPLER stellar evolution code⁵ and the weak interaction rates of Fuller, Fowler and Newman.⁶⁻¹⁰ The Y_e versus time plot is not very instructive; it merely shows the rapid change in time scales which occurs at the end of the core's life. The Y_e vs. ρ plot is much more interesting.

The 'zigzag' in Y_e as it passes through 0.48 is caused by the complicated reactions during silicon burning. Once this burning is completed, Y_e drops very quickly. This is because some of the ashes of silicon burning, such as ${}^{53}\text{Mn}$ and ${}^{55}\text{Fe}$, are excellent electron capture parents. Once these ashes become abundant, many electron captures occur, reducing the electron fraction to roughly 0.44. At this lower value of Y_e , NSE has shifted the abundances such that these two parents are no longer abundant. Now ${}^{60}\text{Co}$ is the dominant electron capture parent. It is interesting that this nucleus does not exhibit as severe a density dependence as did ${}^{53}\text{Mn}$ and ${}^{55}\text{Fe}$, as seen in the changes in Y_e with ρ . The second 'zigzag' in Y_e versus ρ at Y_e of 0.443 is caused by the strong contribution of ${}^{57}\text{Mn}$. This nucleus is strongly β^- reactive and actually stops the drop in the electron fraction, until it is swamped by increasing density and ${}^{60}\text{Co}$.

Now that the effects of these rates has been seen, it is necessary to discuss how one calculates them. We are interested in the rates of change in Y_e and the internal energy of the material due to such weak interactions. In a star each

Figure 1: Y_e vs Time and Central Density

Y_e is plotted from before core silicon burning to the final collapse. The solid line is Y_e versus central density and the dotted line is Y_e versus time. The zero of time is defined to be the beginning of the contraction to silicon ignition. Both curves begin in the upper left corner of the plot.

nucleus is in equilibrium with a thermal bath of photons and so the excited states of each nucleus are thermally populated. These nuclei also reside in a sea of degenerate electrons. Both of these effects must be included in the rate equations. The rates due to a nucleus (Z, A) are given by:

$$\left. \frac{dY_e}{dt} \right|_{(Z,A)} = \pm \ln 2 \frac{X}{A} \sum_{ij} \frac{(2J_i + 1) \exp(-E_i/k_B T)}{\Pi} \frac{1}{ft_{ij}} \varphi_2(T, \mu_e, Q_{ij}) \quad (1)$$

$$\left. \frac{dE_e}{dt} \right|_{(Z,A)} = m_e \ln 2 \frac{X}{A} \sum_{ij} \frac{(2J_i + 1) \exp(-E_i/k_B T)}{\Pi} \frac{1}{ft_{ij}} \varphi_3(T, \mu_e, Q_{ij}) \quad (2)$$

where X is the mass fraction of nucleus (Z, A) , i is the sum over all mother states, j is the sum over all daughter states, J_i and E_i are the spin and energy of the i^{th} state, Π is the nuclear partition function, T is the temperature, μ_e is the electron chemical potential, ft_{ij} is the ft value which connects the mother and daughter states, and Q_{ij} is the nuclear energy difference between the mother and daughter states, respectively. The sign of equation 1 is positive for β^- decay and negative for electron capture. The phase space integrals are represented by φ_n :

$$\varphi_n^{\text{decay}} = \int_0^{\sqrt{q^2-1}} dp \, p^2 (q - \epsilon)^n \frac{F(Z, m_e \epsilon)}{1 + \exp \frac{\epsilon - \mu}{\tau}} \quad (3)$$

$$\varphi_n^{\text{capture}} = \int_{\epsilon}^{\infty} dp \, p^2 (q + \epsilon)^n \frac{F(Z, m_e \epsilon)}{1 + \exp \frac{\epsilon - \mu}{\tau}} \quad (4)$$

where q , μ , and τ are Q_{ij} , μ_e , and T , respectively, in units of electron rest mass, $F(Z, E_e)$ is the Fermi function and $\mathcal{L}$ has the value 0 for $q > -1$ and $\sqrt{q^2 - 1}$ otherwise.

The presence of the Fermi-Dirac distribution functions has drastic effects on the rates. For β^- decay, increasing the density will completely damp out the rate, since the electrons will become more degenerate and it will be more difficult to place an electron in the Fermi sea. This effect was seen in the case of ^{57}Mn . For electron capture, increasing electron degeneracy allows electron capture transitions which sample very high daughter states. This is where the collective Gamow-Teller strength lies.

Every term in the four equations listed above can be easily calculated, except for the ft values. We have subsumed all of our ignorance in this factor. The ft value can be written as

$$\frac{1}{ft_{ij}} = \frac{1}{6250} \left(\left| \langle j | \tau_{\mp} | i \rangle \right|^2 + \left(\frac{g_A}{g_V} \right)^2 \left| \langle j | \vec{\sigma} \tau_{\mp} | i \rangle \right|^2 \right) \quad (5)$$

where the first term is the allowed Fermi transition term, the second term is the allowed Gamow-Teller term, and $g_{V(A)}$ are the vector(axial vector) coupling constants. The value $\frac{1}{6250}$ in front of equation 5 is approximate, but adequate for these applications. Only allowed transitions are considered because the forbidden terms are much smaller and thus unimportant here. For the iron peak nuclei which we are considering, only the Gamow-Teller term is relevant because, either the Fermi transitions are blocked (in the electron capture direction), or they are energetically inaccessible (in the beta decay direction).

In principle, we need these nuclear matrix elements for all allowed, accessible transitions between mother and daughter states. The small fraction which have been measured have $\log ft$ values which range between roughly 4.5 and 6.5. But in the case of Gamow-Teller resonances, collective effects can reduce ft by factors of 100, down to $ft \sim 10^3$. In the case of β^- decay, these resonances are inaccessible because of energetics, but electron captures directly into this resonance are facilitated by degenerate electrons in the iron core. It is therefore necessary to compute where these resonances lie in the daughter spectrum and how the strength is distributed. The estimate of the location must be fairly accurate. Equation 4 shows that, for $Q_{ij} \ll 0$, φ goes as $\exp \frac{\mu_e + Q_{ij}}{k_B T}$, which shows that mistakes in energy affect the rate exponentially.

Such estimates were made for the first time in the stellar rates of Fuller, Fowler, and Newman (hereafter FFN),⁶⁻⁹ but they had very few experimental results to use for guidance. Only in the last five years have charge-exchange experiments been done in the electron capture direction. Such an experiment requires a monochromatic beam of neutrons above roughly 200 MeV, so that the Gamow-Teller piece of the neutron-proton coupling potential dominates. It is very difficult make such beams. Recently, a group at TRIUMF¹¹ has reported the results of an (n,p) experiment on ^{54}Fe . This is our first chance to see how the Gamow-Teller electron capture strength is actually distributed in an iron

Figure 2: ^{54}Fe and its Gamow-Teller Strength Function
The $\Delta L = 0$ portion of the experimentally measured Gamow-Teller strength function of the ground state of ^{54}Fe is shown as a function of the daughter excitation energy.

peak nucleus. Figure 2, on the next page, shows the result of this experiment.

In a recent paper,¹² I have used the information which this experiment gives about ^{54}Fe to guide my calculations for the nuclei which are nearby and are important in stellar evolution. I used the experiment to tell me where and how much Gamow-Teller strength was located in ^{54}Mn , and then used the FFN approach⁷ to extrapolate to other nuclei. I then compared our rates for ^{54}Fe , ^{55}Fe , ^{53}Mn , ^{59}Co , and ^{60}Co . What I found was that the rates agreed to within a factor of 2 (what FFN estimated as their uncertainty) when the electrons did not have enough degeneracy energy to capture directly to the Gamow-Teller resonance. This was the case for ^{54}Fe and ^{55}Fe . But when the Fermi sea was high enough to reach the Gamow-Teller peak, our rates could differ by as much as a factor of 7. Figure 3 (see next page) demonstrates this difference for ^{53}Mn .

The rate I have calculated is a factor of 3 to 5 larger than the FFN result. The FFN estimate is smaller because they placed the Gamow-Teller strength higher above the daughter ground state. The strength is so high that the captures to it do not make much of a contribution to the rate. In my rate, the resonance is responsible for the increase of the rate. If my estimate is correct, ^{53}Mn would be the most important electron capture parent during the large drop in Y_e shown in Figure 1. This nucleus would make the drop in Y_e even faster.

As was said above, after Y_e has dropped to roughly 0.44, ^{60}Co becomes the dominant electron capture parent. It is therefore interesting to use the

Figure 3: Stellar Electron Capture Rates for ^{53}Mn

The log of the rates are plotted versus temperature for $\rho = 10^8$ g/cc and Y_e of 0.5. The temperatures relevant for presupernova evolution are within the vertical lines. The solid curve is my calculation and the dashed curve is the FFN rate.

experimental results to estimate the electron capture rate for this nucleus. This estimate is compared with the FFN result in Figure 4. The comparison is made at higher density because ^{60}Co only becomes important later in the evolution, as was seen in figure 1. At this density, the electrons have degeneracy energies of roughly 4 MeV, and the rate is dominated by captures to the Gamow-Teller resonance. The rates differ by a factor of 7 because of the uncertainties in where to place the resonance and how much strength to put there. FFN are larger for two reasons. Firstly, they placed all of the strength in one place above the daughter, rather than spreading it over several states as implied in the experiment. The lower dash-dotted curve shows what happens if we concentrate all of our strength in one place. This accounts for roughly a factor of three difference. Secondly, FFN had to estimate the total strength without any help from experiments. They estimated almost twice the amount of strength which we would have. This accounts for the rest of the difference between our rates. The upper dash-dotted curve shows what happens if we use their estimate of the total strength and place it in one place above the daughter. This rate and the FFN rate agree very well within the relevant temperature range. The differences which occur at high and low temperatures come from differences in the physics which do not have much affect on the stellar rates.

These results indicate that stellar electron capture rates are extremely sensitive to how the Gamow-Teller strength is treated, once the electrons are degenerate enough to reach the resonance. Unfortunately, all of my rates were

Figure 4: Stellar Electron Capture Rates for ^{60}Co

The log of the rates are plotted versus temperature for $\rho = 10^9$ g/cc and Y_e of 0.5. The temperatures relevant for presupernova evolution are within the vertical lines. The solid curve is my calculation and the dashed curve is the FFN rate. The two dash-dotted curves are discussed in the text.

extrapolated from the results of one experiment. Clearly some work must be done to improve our estimates. This will be discussed in next section.

As was stated above, the β^- decays do not have access to the Gamow-Teller strength because of energetics. These rates will thus be the sum of several transitions of roughly the same strength. Preliminary work on ^{63}Co , a possible decay parent when Y_e approaches 0.44, indicates that these rates are relatively insensitive to the uncertainties in nuclear matrix elements. This is particularly true in this environment because the high degeneracy of the electron Fermi sea blocks most decays, thus damping these rates.

4. Conclusions and Future Directions

The nuclei discussed above have indicated that electron capture rates become uncertain when most of the contribution is from the Gamow-Teller resonances. This uncertainty is the result of our poor ability to estimate the location and strength of such resonances. Unfortunately, the nuclei which cause most of the reduction in Y_e have strong rates because their Gamow-Teller resonances are relatively low in the daughter spectrum. We are the most uncertain when the nuclei are the most important.

Some trends are apparent. The larger rate for ^{53}Mn will make the drop from $Y_e = 0.48$ to 0.44 even faster than in the current KEPLER runs. But once

the electron fraction reaches roughly 0.44, further decreases in Y_e will be much slower, as was seen in the case of ^{60}Co . This may lead to iron cores with a larger value of Y_e in the center than in present models, but with a flatter profile farther out in the core, since it will be easier to get down to $Y_e = 0.44$ at lower densities. What this would do to the iron core mass and collapse calculations is not clear. The system is too nonlinear to make easy predictions. The rates will have to be put into KEPLER in order to see their effect.

It would be useful to know how sensitive the iron core evolution really is to these electron capture rates. At present, the KEPLER code has only the FFN rates in it. Running the code with different rate tabulations would allow one to determine how Y_e and internal energy depend upon individual nuclei. Our knowledge of what nuclei to study would be much more certain. Five years ago, such a program would not be feasible. But with the rise of the superminicomputers, such parameter studies will be possible. I hope to do such work very soon.

The electron capture rates which I have calculated are by no means definitive. With only one experiment for extrapolation, the location and total strengths used in my rates are still suspect. What is needed is a way to gain a better feel for how these strength functions act in several nuclei. There is also the question of what happens to the strength function of excited parent states. Until more information can be obtained, the rates will remain uncertain.

Happily, there is hope for the near future. Charge-exchange experiments are proceeding for many iron group elements and we soon should have more data points for checking the normalization of total strengths and how their distribution varies from nucleus to nucleus. Also, the development of codes such as SUPERCRUNCHER (see Resler in this volume) will allow us to do more detailed studies of how these strength functions behave for excited parent states and for nuclei which don't exist on Earth. The near future will allow us to answer many of these questions. I am looking forward to exploring these issues with Stewart at Livermore.

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