MASTER

ADDED MASS AND DAMPING COEFFICIENTS
FOR HEXAGONAL TUBE ARRAYS^{*}

by

Dennis E. Wilson[†]

BASE TECHNOLOGY

MASTER

August 1979

^{*} Work performed under the Faculty Research Participation Program sponsored by the Center for Educational Affairs

[†] Faculty Research Participant

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TECHNICAL MEMORANDUM

MASTER

ANL-CT-79-51
LMFBR Reactor Core
Systems (UC-79e)

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ABSTRACT

An analytical investigation of the fluid coupling effects from an array of hexagonal cylindrical ducts undergoing harmonic oscillations is presented. A closed form solution for the velocity and pressure is obtained under a "thin" gap approximation for the case of moderate frequencies. From this solution, the usual viscous and inertial fluid coupling coefficients are easily obtained. These analytically derived coefficients indicate a strong dependence upon gap spacing and oscillating Reynolds number.

I. INTRODUCTION

Nuclear reactor components are typically immersed in a liquid coolant environment. In addition, they are often closely spaced. Hexagonal ducts in the core of a fast breeder reactor is an example where both of these conditions exist. To permit the removal and insertion of ducts under reactor shutdown conditions, while limiting the free deflection of ducts during operation, small gaps exist between adjacent ducts at the load pad elevations. These gaps contain an essentially stagnant fluid under normal conditions. However, during a seismic event, the structural components vibrate and due to the small gaps, large velocity gradients may exist. It is well known that the fluid in the gaps has important effects on the resulting structural response due to the inertial and viscous coupling of the fluid. This viscous coupling effect is even more pronounced when the gap is small. Consequently, there is a need to have a good mathematical model which describes this effect. In particular, it is desirable to have closed form analytical solutions which indicate precisely the effect various dynamic and geometric structural parameters plus the associated fluid properties have upon these coefficients.

At present, the so-called added mass effect of various structural components in inviscid fluids can be readily determined [1]. However, the situation is quite different for the case of viscous fluids, where the flow field is more complex due to the temporal and spatial phasing introduced by viscosity. As a result, most of the early work [2], [3] took an experimental approach. Only recently have several analytical [4], [5] and numerical [6], [7] approaches been reported. Unfortunately, these studies have considered small amplitude oscillations. This assumption greatly simplifies the differential equations but limits the applicability to large gaps between adjacent structural components.

In the present analysis, the "thin" gap fluid equations are derived. They are then solved by an asymptotic expansion in terms of a non-dimensional frequency parameter. The resulting equations are of sufficiently simple form so as to allow the application of the boundary condition at the instantaneous boundary position. This eliminates "linearizing" the boundary conditions by the usual Taylor series technique, and thus allows for a more accurate input of the fluid physics into the solution.

II. MATHEMATICAL FORMULATION

A. Two Dimensional Model

In order to derive analytical expressions for the viscous and inertial fluid coupling between adjacent hexagonal cylindrical ducts of a fast breeder reactor, the following model is considered: From the geometry shown in Figure 1, it is reasonable to assume planar oscillations between the two adjacent faces of the hexagonal ducts. This implies that the gap, h , is constant in the z -coordinate direction, see Figure 2. In addition, we initially assume fluid motion to occur only in the x - y plane, i.e. no longitudinal influx is allowed.

The gap is filled with a viscous incompressible fluid of density ρ and viscosity μ . The oscillations are assumed to be simple harmonic with amplitude a and frequency ω . Under the assumptions, the fluid motion is described by

$$\frac{\partial}{\partial t} \nabla^2 \psi + \frac{\partial \psi}{\partial y} \nabla^2 \frac{\partial \psi}{\partial x} - \frac{\partial \psi}{\partial x} \nabla^2 \frac{\partial \psi}{\partial y} = \nu \nabla^4 \psi \quad (1)$$

where

$$u = \frac{\partial \psi}{\partial y} \quad \text{and} \quad v = \frac{\partial \psi}{\partial x} \quad (2a,b)$$

Adopting the coordinate system shown in Figure 3 the relative motion between any two adjacent faces can then be represented by η_H and η_V given below:

$$\eta_H = h + (a \cos \theta) \sin \omega t \quad (3)$$

$$\eta_V = (a \sin \theta) \sin \omega t \quad (4)$$

When $\theta = 0$ the oscillation is normal to the x - z plane and $\eta_V = 0$

Using this model, the boundary conditions on the velocities become

$$\frac{\partial \psi}{\partial y} = 0 \quad \text{At} \quad y = 0 \quad (5a)$$

$$\frac{\partial \psi}{\partial y} = \frac{d\eta_V}{dt} \quad \text{At} \quad y = \eta_H \quad (5b)$$

$$-\frac{\partial \psi}{\partial x} = 0 \quad \text{At } y = 0 \quad (6a)$$

$$-\frac{\partial \psi}{\partial x} = \frac{d\eta_H}{dt} \quad \text{At } y = \eta_H \quad (6b)$$

In addition, some prescribed pressure at $x = \pm L/2$ should be specified for completeness. This pressure will depend upon the instantaneous configuration of the remaining hexagonal cylinders and hence will be some function of time only. For the case of infinite parallel planes, it could be taken as constant and hence would have no effect upon the fluid coupling. At this point in the analysis, for the finite planes it will simply be taken as $f_0(t)$.

B. "Thin" Gap Equations

Before developing the simplified equations describing the fluid motion in the region bounded by $0 < y < \eta_H$ and $-L/2 < x < L/2$ it is important that the proper non-dimensionalization be employed. The characteristic length in the x-direction is L and the y-direction is taken to be h . The characteristic velocity in the y-direction will be ωa and from continuity the x-direction velocity will be $\omega a \frac{L}{h}$. Using these, the following non-dimensional variables and parameters are defined:

$$\bar{x} \equiv \frac{x}{L}, \quad \bar{y} \equiv \frac{y}{h}, \quad \bar{\eta}_{V,H} \equiv \frac{\eta_{V,H}}{h}, \quad \tau \equiv \omega t$$

$$\bar{u} \equiv \epsilon \frac{u}{\omega a}, \quad \bar{v} \equiv \frac{v}{\omega a}, \quad \bar{\psi} \equiv \frac{\psi}{\omega a L} \quad (7)$$

$$\epsilon \equiv \frac{h}{L}, \quad \beta \equiv \frac{a}{h}, \quad M^2 \equiv \frac{\omega h^2}{\nu}$$

The parameter, M , has an important physical interpretation, which can be seen by writing it as

$$M = \frac{h}{(\nu/\omega)^{1/2}}$$

From an analogy with Stokes second problem, it can be shown that the denominator is proportional to the Stokes shear layer, i.e. $\delta_s \sim (\nu/\omega)^{1/2}$. This represents the penetration depth of the vorticity which is generated at the oscillating wall. Thus, when the frequency is "high", i.e. $M \gg 1$, the unsteady nonlinear effects will be confined to a thin region near the oscillating wall and the remainder of the flow will be quasi-steady. Conversely, when the frequency is "low", i.e. $M \ll 1$, the entire flow will be quasi-steady. And finally, when $M \sim O[1]$, the nonlinear effects diffuse throughout the domain creating a complicated fluid flow phenomena, and corresponding complex mathematical analysis.

Substituting (7) into (1) yields,

$$\frac{\partial}{\partial \tau} \nabla^2 \bar{\psi} + \beta \frac{\partial \bar{\psi}}{\partial \bar{y}} \nabla^2 \frac{\partial \bar{\psi}}{\partial \bar{x}} - \beta \frac{\partial \bar{\psi}}{\partial \bar{x}} \nabla^2 \frac{\partial \bar{\psi}}{\partial \bar{y}} = \frac{1}{M^2} \nabla^4 \bar{\psi} \quad (8)$$

where

$$\nabla^2 \equiv \epsilon^2 \frac{\partial^2}{\partial \bar{x}^2} + \frac{\partial^2}{\partial \bar{y}^2} \quad (9)$$

Similarly, the boundary conditions become

$$\frac{\partial \bar{\psi}}{\partial \bar{y}} = \begin{cases} 0 & \text{At } \bar{y} = 0 \\ \frac{\epsilon}{\beta} \frac{d\bar{\eta}_V}{d\tau} & \text{At } \bar{y} = \bar{\eta}_H \end{cases} \quad (10a)$$

$$(10b)$$

$$-\frac{\partial \bar{\psi}}{\partial \bar{x}} = \begin{cases} 0 & \text{At } \bar{y} = 0 \\ \frac{1}{\beta} \frac{d\bar{\eta}_H}{d\tau} & \text{At } \bar{y} = \bar{\eta}_H \end{cases} \quad (11a)$$

$$(11b)$$

Notice that in equation (10b) the x-component boundary velocity is multiplied by ϵ , which is consistent with the "thin" gap scaling.

Expanding the stream function as,

$$\bar{\psi}(\bar{x}, \bar{y}, \tau; \epsilon, \beta, M) = \bar{\psi}_0(\bar{x}, \bar{y}, \tau; \beta, M) + \epsilon \bar{\psi}_1(\bar{x}, \bar{y}, \tau; \beta, M) + O[\epsilon^2] \quad (12)$$

then substituting into (8), yields for ϵ° ,

$$\frac{\partial^3 \bar{\psi}_0}{\partial \tau \partial \bar{y}^2} + \beta \frac{\partial \bar{\psi}_0}{\partial \bar{y}} \frac{\partial^3 \bar{\psi}_0}{\partial \bar{x} \partial \bar{y}^2} - \beta \frac{\partial \bar{\psi}_0}{\partial \bar{x}} \frac{\partial^3 \bar{\psi}_0}{\partial \bar{y}^3} = \frac{1}{M^2} \frac{\partial^4 \bar{\psi}_0}{\partial \bar{y}^4} \quad (13)$$

In a similar manner the boundary conditions for ϵ° become

$$\frac{\partial \bar{\psi}_0}{\partial \bar{y}} = 0 \quad \text{At } \bar{y} = 0 \quad \text{and} \quad \bar{y} = \bar{\eta}_H \quad (14a, b)$$

$$-\frac{\partial \bar{\psi}}{\partial \bar{x}} = \begin{cases} 0 & \text{At } \bar{y} = 0 \\ \cos \theta \cos \tau & \text{At } \bar{y} = \bar{\eta}_H \end{cases} \quad (15a)$$

$$(15b)$$

where

$$\bar{\eta}_V = \beta \sin \theta \sin \tau \quad \text{and} \quad \bar{\eta}_H = 1 + \beta \cos \theta \sin \tau \quad (16a, b)$$

Equations (13) through (16) then represent the first approximation for the flow in the thin gap, i.e. $h \ll L$.

It is interesting to note that for $\theta = 0$ the $\bar{\psi}_1$ equation is identical to (13) with homogenous boundary conditions and thus has only the trivial solution. Therefore, for $\theta = 0$ only, the solution of equations (13) through (16) are accurate to $O[(\frac{h}{L})^2]$, whereas for arbitrary θ they are accurate to $O[h/L]$.

In order to determine the viscous and inertial fluid coupling effects between two adjacent hexagonal cylindrical ducts, it will be necessary to determine the "thin" gap pressure gradient relations for both the x and y coordinate directions. To obtain the correct equations, the proper scaling for the pressure gradient is crucial. From an analogy with slider block lubrication theory [8] it follows that $dp/dx \sim \mu U/h^2$. For this problem $U \sim \omega a L/h$, therefore

$$\bar{p} = \frac{p}{\mu \omega a L^2 / h^3} \quad (17)$$

Using this, the non-dimensional momentum equation becomes

$$\frac{1}{M^2} \frac{\partial \bar{p}}{\partial \bar{x}} = \frac{1}{M^2} \left(\frac{\partial^3 \bar{\psi}}{\partial \bar{y}^3} + \epsilon^2 \frac{\partial^3 \bar{\psi}}{\partial \bar{x}^2 \partial \bar{y}} \right) - \frac{\partial^2 \bar{\psi}}{\partial \tau \partial \bar{y}} - \beta \left(\frac{\partial \bar{\psi}}{\partial \bar{y}} \frac{\partial^2 \bar{\psi}}{\partial \bar{x} \partial \bar{y}} - \frac{\partial \bar{\psi}}{\partial \bar{x}} \frac{\partial^2 \bar{\psi}}{\partial \bar{y}^2} \right) \quad (18)$$

Using (12) and writing

$$\frac{\partial \bar{p}}{\partial \bar{x}} = \frac{\partial \bar{p}_0}{\partial \bar{x}} + \epsilon \frac{\partial \bar{p}_1}{\partial \bar{x}} + O[\epsilon^2] \quad (19)$$

yields

$$\frac{1}{M^2} \frac{\partial \bar{p}_0}{\partial \bar{x}} = \frac{1}{M^2} \frac{\partial^3 \bar{\psi}_0}{\partial \bar{y}^3} - \frac{\partial^2 \bar{\psi}_0}{\partial \bar{x} \partial \bar{y}} - \beta \left(\frac{\partial \bar{\psi}_0}{\partial \bar{y}} \frac{\partial^2 \bar{\psi}_0}{\partial \bar{x} \partial \bar{y}} - \frac{\partial \bar{\psi}_0}{\partial \bar{x}} \frac{\partial^2 \bar{\psi}_0}{\partial \bar{y}^2} \right) \quad (20)$$

Similarly, for the y-momentum equation, we get

$$\frac{1}{\epsilon^2 M^2} \frac{\partial \bar{p}}{\partial \bar{y}} = \frac{1}{M^2} \left(\epsilon^2 \frac{\partial^3 \bar{\psi}}{\partial \bar{x}^3} + \frac{\partial^3 \bar{\psi}}{\partial \bar{x} \partial \bar{y}^2} \right) - \beta \left(\frac{\partial \bar{\psi}}{\partial \bar{y}} \frac{\partial^2 \bar{\psi}}{\partial \bar{x}^2} - \frac{\partial \bar{\psi}}{\partial \bar{x}} \frac{\partial^2 \bar{\psi}}{\partial \bar{x} \partial \bar{y}} \right) \quad (21)$$

The transverse pressure gradient is expanded in ϵ , yielding for the first two equations

$$\frac{\partial \bar{p}_0}{\partial \bar{y}} = 0 \quad \text{and} \quad \frac{\partial \bar{p}_1}{\partial \bar{y}} = 0 \quad (22a,b)$$

This is to be expected due to the coordinate scaling, which is similar to results involving thin viscous (boundary) layers. Having formulated the "thin" gap equations, we turn now to obtaining simple analytical solutions relevant to seismic conditions in fast breeder reactors.

III. LOW FREQUENCY SOLUTION

For the moderate ($\omega \leq 10$ Hz) frequencies and small ($h \approx .001$ ft) gaps involved in liquid sodium fast breeder reactors the value of $M \leq 1$. Consequently, by taking $M \ll 1$ and expanding ψ_0 in an asymptotic series in terms of M , a solution valid for small M , i.e. low frequencies, can easily be obtained. By including several terms, sufficient accuracy can be obtained as $M \rightarrow 1$. We begin by expanding $\bar{\psi}_0$ as

$$\bar{\psi}_0 = x_0 + Mx_1 + M^2x_2 + O[M^3] \quad (23)$$

Substituting (23) in (13) yields

$$\frac{\partial^4 x_0}{\partial \bar{y}^4} = 0 \quad (24)$$

$$\frac{\partial^4 x_1}{\partial \bar{y}^4} = 0 \quad (25)$$

$$\frac{\partial^4 x_2}{\partial \bar{y}^4} = \frac{\partial^3 x_0}{\partial \tau \partial \bar{y}^2} + \beta \frac{\partial x_0}{\partial \bar{y}} \frac{\partial^3 x_0}{\partial \bar{x} \partial \bar{y}} - \beta \frac{\partial x_0}{\partial \bar{x}} \frac{\partial^3 x_0}{\partial \bar{y}^3} \quad (26)$$

The associated boundary conditions are

$$\frac{\partial x_0}{\partial \bar{y}} = 0 \quad \text{At } \bar{y} = 0 \quad \text{and} \quad \bar{y} = \bar{\eta}_H \quad (27a,b)$$

$$-\frac{\partial x_0}{\partial \bar{x}} = \begin{cases} 0 & \text{At } \bar{y} = 0 \\ \cos \theta \cos \tau & \text{At } \bar{y} = \bar{\eta}_H \end{cases} \quad (28b)$$

and for $n \geq 1$

$$\frac{\partial x_n}{\partial \bar{y}} = 0 \quad \text{At } \bar{y} = 0 \quad \text{and} \quad \bar{y} = \bar{\eta}_H \quad (29a,b)$$

$$\frac{\partial x_n}{\partial \bar{x}} = 0 \quad \text{At } \bar{y} = 0 \quad \text{and} \quad \bar{y} = \bar{\eta}_H \quad (30a,b)$$

The solution to equation (24) is given by

$$\chi_0 = f_0(\bar{x}, \tau) + f_1(\bar{x}, \tau) \bar{y} + f_2(\bar{x}, \tau) \bar{y}^2/2! + f_3(\bar{x}, \tau) \bar{y}^3/3! \quad (31)$$

Applying the boundary conditions yields

$$\chi_0 = F_0(\tau) + \bar{x} \cos \theta \cos \tau \left[2 \left(\frac{\bar{y}}{\bar{\eta}_H} \right)^3 - 3 \left(\frac{\bar{y}}{\bar{\eta}_H} \right)^2 \right] \quad (32)$$

The solution to equation (25) with the homogenous boundary conditions is

$$\chi_1 = F_1(\tau) + F_3(\tau) \left[\frac{\bar{y}^3}{6} - \frac{\bar{y} \bar{\eta}_H}{4} \right] \quad (33)$$

In both equations (32) and (33) the functions F_0 , F_1 , and F_3 are related to the imposed pressure gradient which is zero for the infinite parallel plane problem, hence the solution to χ_1 is the trivial solution for that situation.

Using the solution given by equation (32), χ_2 can be determined. The result is somewhat lengthy and is given by

$$\begin{aligned} \chi_2 = & \left\{ F_2(\tau) + f_2 \bar{y}^2/2! + f_3 \bar{y}^3/3! \right. \\ & + \left(\frac{\bar{x} 5 \sin \tau}{4 \bar{\eta}_H^2} \right) \bar{y}^4 + \left(\frac{3 \bar{x} \beta \cos^2 \tau}{10 \bar{\eta}_H^4} - \frac{\bar{x} \sin \tau}{10 \bar{\eta}_H^3} \right) \bar{y}^5 \\ & \left. - \left(\frac{\bar{x} \beta \cos^2 \tau}{5 \bar{\eta}_H^5} \right) \bar{y}^6 + \left(\frac{4 \bar{x} \beta \cos^2 \tau}{70 \bar{\eta}_H^6} \right) \bar{y}^7 \right\} \cos \theta \end{aligned} \quad (34)$$

where f_2 and f_3 are given below

$$\begin{aligned} f_2 &= \frac{\sin \tau}{10} \bar{x} + \frac{16 \beta \cos^2 \tau}{35 \bar{\eta}_H} \bar{x} \\ f_3 &= -\frac{6 \sin \tau}{5 \bar{\eta}_H} \bar{x} - \frac{81 \beta \cos^2 \tau}{35 \bar{\eta}_H^2} \bar{x} \end{aligned}$$

IV. FLUID COUPLING COEFFICIENTS

To determine the fluid coupling effect on the inner cylinder due to its simple harmonic oscillation, we need only compute the pressure and shear forces. To find the pressure, we return to equations (20) and (21) and expand $\partial \bar{p}_0 / \partial \bar{x}$ in term of M as

$$\frac{\partial \bar{p}_0}{\partial \bar{x}} = \left(\frac{\partial \bar{p}}{\partial \bar{x}} \right)_0 + M \left(\frac{\partial \bar{p}}{\partial \bar{x}} \right)_1 + M^2 \left(\frac{\partial \bar{p}}{\partial \bar{x}} \right)_2 + O[M^3] \quad (35)$$

Substituting (35) into (20) yields

$$\left(\frac{\partial \bar{p}}{\partial \bar{x}} \right)_0 = \frac{\partial^3 \chi_0}{\partial \bar{y}^3} \quad (36)$$

$$\left(\frac{\partial \bar{p}}{\partial \bar{x}} \right)_1 = \frac{\partial^3 \chi_1}{\partial \bar{y}^3} \quad (37)$$

$$\left(\frac{\partial \bar{p}}{\partial \bar{x}} \right)_2 = \frac{\partial^3 \chi_2}{\partial \bar{y}^3} - \frac{\partial^2 \chi_0}{\partial \tau \partial \bar{y}} - \beta \left(\frac{\partial \chi_0}{\partial \bar{y}} \frac{\partial^2 \chi_0}{\partial \bar{x} \partial \bar{y}} - \frac{\partial \chi_0}{\partial \bar{x}} \frac{\partial^2 \chi_0}{\partial \bar{y}^2} \right) \quad (38)$$

From equation (36) we find that

$$\left(\frac{\partial \bar{p}}{\partial \bar{x}} \right)_0 = 12 \frac{\bar{x}}{\bar{\eta}_H^3} \cos \theta \cos \tau \quad (39)$$

Similarly, (37) yields

$$\left(\frac{\partial \bar{p}}{\partial \bar{x}} \right)_1 = 0 \quad (40)$$

Using the solution for both χ_0 and χ_2 in the expression for $(\partial \bar{p} / \partial \bar{x})_2$ we find the interesting result that

$$\left(\frac{\partial \bar{p}}{\partial \bar{x}} \right)_2 = f_3(\bar{x}, \tau) \quad (41)$$

The fact that the y -dependence completely cancels out of the right hand side of (38) when expanded is to be expected since from equation (22a) we have $\partial \bar{p}_0 / \partial \bar{y} = 0$ which implies $(\partial \bar{p} / \partial \bar{y})_2 = 0$. Evaluating $f_3(\bar{x}, \tau)$ from (29b), (30b), and (34) yields

$$\left(\frac{\partial \bar{p}}{\partial \bar{x}}\right)_2 = -\frac{6\text{Sin}\tau}{5\bar{\eta}_H} \bar{x} - \frac{818\text{Cos}^2\tau}{35\bar{\eta}_H^2} \bar{x} \quad (42)$$

Notice that the pressure gradient does not contain a term in phase with the velocity but does possess a lower and higher harmonic given by the $\text{Cos}^2\tau$ term. In addition, the first term is in phase with the acceleration and is related to the so-called added mass effect. The two groups of terms arise from inertia effects and convective acceleration effects respectively.

It is now a simple procedure to compute the fluid coupling coefficients from the known pressure distribution. We start by integrating (39) from some $\bar{x}$ to $\bar{x} = 1/2$. The result is

$$\bar{p}(\bar{x}, \tau) = \bar{p}(1/2, \tau) + \frac{6}{\bar{\eta}_H^3} \text{Cos}\theta \text{Cos}\tau \left[\bar{x}^2 - \frac{1}{4} \right] + O[M^2] \quad (43)$$

where $\bar{p}(1/2, \tau)$ is the pressure at the exit which will be denoted $\bar{p}_0(\tau)$. Notice that the pressure is increasing at $\bar{x} = 0$ when the planes are contracting and decreasing when the planes are expanding. The relation is simply

$$\bar{p}(0, \tau) = \bar{p}_0(\tau) - \frac{6}{\bar{\eta}_H^3} \text{Cos}\theta \text{Cos}\tau + O[M^2] \quad (44)$$

Writing (43) in dimensional form yields,

$$p(x, t) = p_0(t) + \frac{6\mu\omega aL^2 \text{Cos}\theta \text{Cos}\omega t}{(h + a \text{Cos}\theta \text{Sin}\omega t)^3} \left[\left(\frac{x}{L}\right)^2 - \frac{1}{4} \right] \quad (45)$$

Integrating this from $-L/2$ to $L/2$, we find the force/unit width to be

$$F/W = \frac{6\mu\omega a L^2 \cos\theta \cos \omega t}{(h + a \cos\theta \sin \omega t)^3} \left(\frac{L}{2}\right) \quad (46)$$

where $p_o(\tau)$ has been taken as zero.

Defining a fluid damping coefficient as

$$C_v \equiv \frac{F/W}{\rho L^2 \omega U} \quad (47)$$

where $U = \omega a \cos \omega t$, we find from (46) that C_v is given by

$$C_v = \frac{5 \cos\theta}{2(g/L)^3} \frac{1}{R_\omega} \quad (48)$$

In this expression g is given by

$$g(t) = h + a \cos\theta \sin \omega t \quad (49)$$

and

$$R_\omega = \frac{\omega L^2}{\nu} \quad (50)$$

Repeating the identical process for the second order pressure gradient given by (42) we find that there is no second order correction to C_v . In addition, the third order solution is trivial. Thus, through fourth order we have

$$C_v = \frac{5 \cos\theta}{2(g/L)^3} \frac{1}{R_\omega} \quad (51)$$

However, a second order damping term containing a higher and lower harmonic due to the interaction of the convective acceleration terms does exist.

Finally, the viscous damping associated with the shear stress can easily be computed from the solution to the x-component of velocity. Looking only at the

$O[M^0]$ term, we find from equation (32) that

$$\bar{u}_0 = 6\bar{x} \cos\theta \cos \tau \left[\frac{\bar{y}^2}{\bar{\eta}_H^3} - \frac{\bar{y}}{\bar{\eta}_H^2} \right] \quad (52)$$

The force/unit width can then be written as

$$F/W = 2 \int_0^{L/2} \mu \left. \frac{\partial u}{\partial y} \right|_{y = \bar{\eta}_H} dx = 2\mu \int_0^{1/2} \omega a \left(\frac{L}{h} \right)^2 \left. \frac{\partial \bar{u}}{\partial y} \right|_{\bar{y} = \bar{\eta}_H} dx \quad (53)$$

Integrating we get

$$F/W = \frac{3}{2} \rho L^2 \left(\frac{v}{\omega L^2} \right) \omega U \frac{L^2}{\bar{\eta}_H^2} \cos \theta \quad (54)$$

From this we find the fluid damping coefficient to be

$$C_v = \frac{3}{2} \frac{\cos \theta}{(g/L)^2} \frac{1}{R_\omega} \quad (55)$$

where $g(t)$ and R_ω are defined by (49) and (50) respectively.

The added mass coefficient can also be calculated in a similar manner from both the pressure and shear stress integrated over the cylinder. The force is in phase with the acceleration, $\ddot{U}$, where $U = -\omega a \sin \omega t$. Considering only the $\sin \omega t$ term in (42) and writing

$$\left(\frac{\partial \bar{p}_0}{\partial \bar{x}} \right)_{\sin \tau} = M^2 \left(\frac{\partial \bar{p}}{\partial \bar{x}} \right)_2 = -M^2 \frac{6 \sin \tau}{5 \bar{\eta}_H} \bar{x}, \quad (56)$$

we find

$$F/W = \frac{(\rho L^2 \omega U) L \cos \theta}{2(h + a \cos \theta \sin \omega t)} \quad (57)$$

Then, defining an added mass coefficient, C_M , as

$$C_M \equiv \frac{F/W}{\rho L^2 \omega \bar{U}} \quad (58)$$

yields

$$C_M = \frac{\cos \theta}{2(g/L)} \quad (59)$$

In (59), g is given by (49) and we find through fourth order that C_M is independent of R_ω .

Using (34), the added mass coefficient due to the oscillating shear stress can also be computed. The result, again through fourth order is simply

$$C_M = \frac{\cos \theta}{40} \quad (60)$$

In this case, not only is C_M independent of the oscillating Reynolds number, but it is also independent of the nondimensional gap.

V. RESULTS AND DISCUSSION

In this investigation, a closed form analytical solution which approximates the fluid coupling coefficients for an array of hexagonal cylindrical ducts was obtained. The mathematical model employed in this analysis contains the following assumptions:

- (1) The hexagonal cylinder is assumed to execute simple harmonic motion in the x-y plane.
- (2) From the geometry and planar motion of the cylinder, the fluid motion is assumed to be two-dimensional. In addition, the fluid is taken as viscous and incompressible.
- (3) The resulting two-dimensional Navier-Stokes equations are simplified from dimensional scaling to yield the "thin" gap equation given by (13).
- (4) The effect of gap geometry from the neighboring hexagonal cylinders upon the pressure distribution between adjacent cylinders is neglected. This is equivalent to setting $p_0(t) = \text{const.}$

Under the four sets of assumptions, an analytical solution for the velocity and pressure distribution is obtained for the case of low frequencies, i.e. $M \ll 1$. From these solutions, the fluid damping coefficients, C_v , is determined. The result due to both pressure and shear stress is given by equations (51) and (55) respectively. The results can easily be compared with a finite element solution [7] in which the equations of motion and the boundary conditions have been linearized. In Yang and Moran's work, they define an oscillating Reynolds number, Re , as $Re = \omega d^2 / \nu$. In addition, their nondimensional gap is given by g/d , where $g \approx \text{constant}$ for their small amplitude analysis. Using their notation, the two fluid damping coefficients can be written as

$$(C_v)_{\text{pressure/side}} = \frac{5}{2\sqrt{3}} \frac{1}{(g/d)^3} \frac{1}{Re} \quad (56)$$

and

$$(C_v)_{\text{shear/side}} = \frac{3}{4} \frac{1}{(g/d)^2} \frac{1}{Re} \quad (57)$$

In equation (56) and (57) the oscillation was taken normal to one face. A total fluid damping coefficient, $(C_v)_{\text{total}}$, can now be written as

$$(C_v)_{\text{total}} = \frac{5}{\sqrt{3}} \frac{1}{(g/d)^3} \frac{1}{Re} + \frac{\sqrt{3}}{2} \frac{1}{(g/d)^2 Re} \quad (58)$$

In a similar manner, the added mass due the pressure and shear distribution can be written as

$$(C_M)_{\text{pressure/side}} = \frac{\cos \theta}{2\sqrt{3} (g/d)} \quad (59)$$

and

$$(C_M)_{\text{shear/side}} = \frac{\cos \theta}{40} \quad (60)$$

From these two coefficients a total C_M can be written for oscillations normal to one side as

$$(C_M)_{\text{total}} = \frac{1}{\sqrt{3} (g/d)} + \frac{\sqrt{3}}{60} \quad (61)$$

Having developed a "thin" gap mathematical model which approximates the fluid dynamic behavior between closely spaced hexagonal ducts, a few remarks regarding the assumptions and possible refinements in the model are in order.

Although the resulting model is consistent with the physical problem, some approximations naturally introduce more error than others. From simple laminar flow considerations, it is apparent that the last assumption represents the most severe restriction. Depending on thickness of the neighboring gaps, it is obvious that $p_0(t)$ can become comparable to the pressure distribution generated by the relative motion of the two adjacent cylinders. Under this

condition, the fluid coupling coefficient is strongly affected by the neighboring hexagonal ducts. It is also obvious that the influence drops off as $1/2^n$, due to the continual branching of the paths, where n represents the number of paths (branches) that the fluid may follow.

By determining an impedance from each successive branch, the effect can be related to the imposed pressure, $p_0(t)$, which in turn is related to the local fluid damping coefficient. In this way, the effect from the configuration of the neighboring hexagonal cylinders can be included, at least conceptually, in an overall fluid damping coefficient.

An additional refinement to the existing model can be made with considerably less effort. This involves relaxing the two-dimensional assumption in (2). In reality some efflux/influx occurs in the z -direction, producing a three-dimensional flow. This additional degree of freedom will effect the pressure distribution between adjacent cylinders. From the fluid physics it is reasonable to expect that it will lower the induced pressure. This lower pressure distribution will produce less resistance and hence a lower value for the fluid damping coefficient.

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FIGURE 1. TYPICAL FUEL ASSEMBLY

Figure 2. Two Dimensional Model

Figure 3. Thin Gap Model