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A REVIEW OF ELECTRON-BEAM HEATING OF MAGNETIC-MIRROR CONFINED PLASMAS, WITH APPLICATION TO THE TANDEM MIRROR EXPERIMENT

Milos Seidl

April 10, 1979

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ABSTRACT

This report reviews results from early electron-beam heating experiments and more recent basic beam-plasma physics experiments as well as present theoretical understanding. We find that rather simple electron-beams could be employed on the Tandem Mirror Experiment (TMX) at Lawrence Livermore Laboratory to carry out electron heating experiments.

1. INTRODUCTION

Recently there has been renewed interest in using the rf fields generated by injected electron beams to produce hot-electron plasmas in magnetic mirror confinement experiments. Electron beams offer several possibilities, including stabilization of the drift-cyclotron loss-cone (DCLC) instability mode, formation of target plasmas, and enhancement of the performance of tandem mirror machines. One of the advantages of electron beams for these purposes is that they are relatively cheap and easy to produce.

Stabilization of the DCLC mode was first successfully demonstrated by Klinkowstein and Smullin.¹ Similar experiments were carried out at the Lawrence Livermore Laboratory (LLL) on the 2XIIB. The electron beam produced a target plasma in 2XIIB that was then used for buildup of a hot-ion plasma by neutral-beam injection. Although the electron beam did indeed suppress the ion-cyclotron fluctuations, there was no improvement in plasma confinement time.

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With the success in 2XIIB, the decision was made to use electron beams to produce the target plasmas in the end plugs of the Tandem Mirror Experiment (TMX) at LLL. Several advantages (in addition to DCLC stabilization) should accrue:

- Target plasma produced from electron beams should contain fewer impurities than that produced with a plasma gun.
- The higher electron temperatures obtained with electron beams would reduce the rate of ion cooling, increase the ambipolar potential well, and contribute to the heating of solenoid ions (thereby leading to more collision-free conditions).
- Use of electron-beam heating in conjunction with thermal particle pumping would improve the overall performance of the tandem mirror configuration.

The feasibility of electron heating in mirror machines by electron beams was demonstrated in numerous experiments performed in the 1960's.³⁻¹⁷ As often happens, most of the work was rather abruptly stopped, leaving many questions unanswered. Parallel to the heating experiments, and surviving them, there has been ongoing research into the basic processes in beam-plasma interactions.²³⁻⁴⁷ The overlap between these two efforts is rather small for the obvious reason that basic processes are studied in simple experiments whereas the inhomogeneities and high power levels characteristic of mirror plasmas make detailed measurements and analysis very difficult. The revival of electron-beam heating experiments may lead to a beneficial interaction between practical and basic aspects.

In this report, we attempt to bring these two aspects together. First the results of the main electron beam heating experiments will be presented. Then the physics of beam-plasma interaction will be briefly reviewed, plasma heating mechanisms will be discussed, and problems that need more work will be identified. (Note: Except in Sec. 3, nonrelativistic electron beams are considered throughout.) Finally, we consider applications of electron-beam heating to the Tandem Mirror Experiment (TMX).

2. PLASMA PRODUCTION BY ELECTRON BEAMS IN SIMPLE MAGNETIC MIRRORS

The first experiments on production and heating of mirror confined plasmas were done at the Kharkov Institute,³ the Massachusetts Institute of Technology (MIT),⁴ and Oak Ridge National Laboratory (ORNL)⁵ in the early sixties. More experiments quickly followed.⁶⁻¹⁶ It was soon recognized that the plasma produced by an electron beam is actually an rf discharge in which the rf field is produced by interaction between the electron beam and the background plasma.

Descriptions and results of the published mirror experiments are summarized in Table 1. Although a simple mirror configuration (no minimum-B geometry) was used in these experiments, there were no indications of strong magnetohydrodynamic (MHD) instabilities in the plasmas—a fact that awaits full explanation.

With the exception of the experiment by Bredikhin et al.,^{17,18} the electron gun was always placed in the mirror peak. In that one experiment, the electron gun was located in the weak field 25 cm from the mirror peak, making it possible to vary the ratio $E_{\perp}/E_{\parallel}$ of the beam electron energy in the mirror region from zero to 0.6 by changing the pitch angle of the injected beam. More efficient plasma heating was observed for large $E_{\perp}/E_{\parallel}$.¹⁷

Simple planar diodes with disc-shaped cathodes were used in most experiments. The MIT experiments used magnetron diodes. In all experiments, a drift tube placed in front of the gun was used to reduce the flow of plasma into the diode.

The electron density in the beam as listed in Table 1 refers to the beam leaving the gun. The Burnout experiments operated in a Penning mode: a floating electrode (anti-cathode) was placed in the mirror not occupied by the gun. Because this electrode got charged up to the cathode potential (Ref. 7, p. 793), beam electrons reflexed several times between the mirrors, thereby probably increasing the effective beam density by a factor close to five.

In most experiments, sufficient hydrogen gas was admitted into the central region of the chamber to produce an ambient pressure of 10^{-4} to 10^{-5} Torr. The Burnout experiments used deuterium gas. The experiment of Bredikhin et al.¹⁷ used argon admitted into the center plane in the form of a molecular beam. In the initial stage of Burnout IV, gas was admitted into the drift tube of the injector. In the MIT(D) experiment, hydrogen gas was

pulsed into the electron-beam collector. Blinov et al.¹⁰ and Plakhov et al.¹⁴ injected plasma from titanium-washer guns.

The plasma parameters listed in Table 1 correspond to optimum performance as judged from x-ray emissions and diamagnetic signals. In each experiment, critical adjustments of gas pressure, electron-beam current and voltage, and magnetic field were necessary. Each plasma consisted of a cold and hot component. The cold components had densities of 10^{12} to 10^{13} cm⁻³; electron temperatures were probably from 50 to 100 eV. The hot components had densities of 10^{10} to 10^{11} cm⁻³ (a few percent of the cold component) and temperatures were 10 to 200 keV. Temperatures were estimated from the measured x-ray spectrum and densities of the hot components were then obtained from the diamagnetic signals. The latter gave typically an energy density of 10^{15} eV/cm³. In the high-field experiment of Bredikhin et al., a high energy density of 10^{18} eV/cm³ was measured.¹⁸ The average energy per electron was 3.5 keV (the electron distribution was not resolved into a hot and cold component).

The ratio of beam density to plasma density was typically $n_{\text{beam}}/n_{\text{plasma}} \approx 1\%$ when n_{beam} was measured in front of the gun and n_{plasma} in the midplane. With the exception of the Burnout experiment, maximum electron heating always occurred for $\omega_{pe}/\omega_{ce} \approx 1.5$ to 3 in the midplane.

Ion heating was not observed in most experiments. However, in the Burnout experiments, which operated at low densities, $\omega_{pe} < \omega_{ce}$, ion heating did occur. The average ion energy in Burnout V was 100 to 500 eV and 10% of the ions had energies in excess of 1 keV. Later work²⁰ showed that the ion heating resulted from an $E \times B$ instability that was driven by a radial dc electric field. The plasma potential was close to the cathode potential, as in a low-pressure Penning discharge.

TABLE 1
Summary of electron-beam production and heating experiments

Experiment	MIT (A)	MIT (D)	Burnout IV	Burnout V	Demir- khanov	Blinov	Tarumov	Seidl	Plakhov	Bredikhin
Reference Number	8,19	8,19	7	13	9	10	11	12,15	14	17,18
<u>Magnetic Field</u>										
Center field (kG)	1	1	9	20	1.5	2	1	0.7	2	40
Mirror ratio	3	6	1.5	2	1.5	5.5	5	2	5	1.4
Distance between mirrors (cm)	60	100	20	50	-	80	-	60	-	25
<u>Electron Beam</u>										
Voltage (kV)	10	10	10	10	4	25	6	8	30	60
Current (A)	10	10	0.7	6	0.3	18	3	0.2	20	70
Pulse length (ms)	0.7	0.7	DC	DC	-	0.1	10	2	0.25	0.17
Diameter (cm)	2/1.6	2/1.6	1	2.5	-	1.2	1	0.5	1	0.5
<u>Electron</u>										
Density (cm^{-3})	2×10^{10}	2×10^{10}	2×10^9	3×10^9	-	3×10^{10}	10^{10}	10^9	4×10^{10}	3×10^{11}
<u>Plasma</u>										
n_{cold} (cm^{-3})	10^{12}	10^{13}	10^{12}	10^{12}	10^{12}	10^{12}	10^{12}	10^{11}	10^{12}	4×10^{11a}
n_{hot} (cm^{-3})	10^{10}	10^{10}	10^{11}	-	10^{11}	4×10^{10}	10^{11}	10^{10}	10^{10}	
T_e hot (keV)	50	50	50	50	20	50	100	10	200	3.5 ^a
$\langle nT \rangle$ (eV/cm^3)	5×10^{14}	-	-	-	-	2×10^{15}	6×10^{15}	10^{14}	-	10^{18}
Diameter of cold plasma (cm)	7.5	20	10	-	-	18	15	6	24	6
Diameter of hot plasma (cm)	-	-	-	-	-	10	3	-	12	5
$n_{\text{beam}}/n_{\text{plasma}}$ (%)	2	0.2	0.2	0.3	-	3	1	1	4	0.1

^a Bulk density and temperatures

3. HEATING MIRROR-CONFINED PLASMAS WITH RELATIVISTIC ELECTRON BEAMS

I am aware of only two published experiments in which a relativistic electron beam was injected into a target plasma in a mirror geometry. (Another experiment is underway at the University of Tsukuba in Japan.) The main parameters of the two experiments are summarized in Table 2.

TABLE 2

Summary of relativistic electron-beam heating experiments.

	Abrashitov ^a	Ekdahl ^b
Magnetic field		
B _{max} (kG)	25	5.2
B _{min} (kG)	15	2.6
Length between mirrors (cm)	230	-
Electron beam		
kV	1000	400
kA	10	20
Diameter (cm)	2	5
Pulse length (ns)	70	60
Plasma		
n _{cold} (cm ⁻³)	3x10 ¹⁴	10 ¹³
n _{hot} (cm ⁻³)	5x10 ¹³	-
T _{hot} (keV)	1	
< nT > (eV/cm ³)	10 ¹⁶	
Diameter (cm)	8	
Decay time of hot component(μs)	0.5	1
n _{beam} /n _{plasma} (%)	0.6	0.4

^aRef. 21

^bRef. 22

Note that the ratio of the beam density to the plasma density is again around 1% as in the nonrelativistic experiments. The temperature of the hot electrons is only 1 keV, and the decay time of the hot component is only 1 μs. No explanation of these results is given.

4. BEAM-PLASMA INTERACTION

The collective interaction of an electron beam with a plasma has been studied in numerous papers dating back as far as 1961³; a monograph appeared in 1964.²³ Rather than attempt a comprehensive list of references, I will review those processes that are both well understood and capable of being briefly summarized. The reader is referred to these references as well as to Ref. 24 for a more complete discussion of this topic.

The linear theory of the interaction of a weak electron beam with homogeneous plasma in uniform magnetic field is well understood and useful for the discussion of more complex cases. Any wave whose frequency ω and longitudinal wave number k_z simultaneously satisfy the plasma dispersion equation and the resonance condition

$$\omega = k_z V_b + m \omega_{ce} \quad (1)$$

may be unstable. Here V_b is the longitudinal velocity of the electrons in the beam and m is an integer. The plasma dispersion equation has two branches: the lower branch ($\omega < \omega_{ce}$, L wave) and the upper branch ($\omega > \omega_{ce}$, H wave). The potentially unstable waves have ω and k_z values that are close to the intersections of the two plasma dispersion curves ($1 + \epsilon_p = 0$) with the resonance lines, Eq. (1), as shown in Fig. 1. For $\omega_{pe} \gg \omega_{ce}$, the upper branch has a flat dispersion curve so that $\omega \approx \omega_{pe}$.

The integer m defines the interaction mode. For the space-charge wave mode, $m = 0$, the growing wave has a longitudinal phase velocity $\omega/k_z \approx V_b$. The slow cyclotron modes, $m < 0$, have a phase velocity smaller than the beam velocity:

$$\frac{\omega}{k_z} \approx \frac{V_b}{1 + |m| (\omega_{ce}/\omega_{pe})} \quad (2)$$

The fast cyclotron modes, $m > 0$, are faster than the beam electrons:

$$\frac{\omega}{k_z} \approx \frac{V_b}{1 - |m| (\omega_{ce}/\omega_{pe})} \quad (3)$$

Fig. 1. Beam resonance lines, $\omega - k_z V_b - m\omega_c = 0$, and the plasma dispersion curve for the H and L waves.

The growth rate ω_i of these modes depends on the distribution function of the beam electrons and on the perpendicular wave number $k_\perp$. In particular, if the beam has no transverse velocity, only the $m = 0$ and $m = -1$ modes can be unstable. The growth rate of the $m = 0$ mode reaches the maximum value

$$\omega_i/\omega \approx (\bar{n}_b/n_p)^{1/3} \quad (4)$$

for small $k_\perp$ ($k_\perp v_b/\omega_{ce} \ll 1$) and for small spread in the beam ($\Delta v_b/v_b \leq \omega_i/\omega$).

The growth rate of the $m = -1$ mode has a maximum

$$\omega_i/\omega \approx 0.5 (\bar{n}_b/n_p)^{1/2} \quad (5)$$

for large $k_\perp$ ($k_\perp v_b/\omega_{ce} \geq 5$), and for small beam spread. In these formulas, $\bar{n}_b$ is the effective beam density and n_p is the plasma density. More general formulas with numerical values may be found in Ref. 24. The effective beam density is the beam density averaged over a cross-sectional area occupied by the waves. The H waves are localized in the immediate vicinity of the beam because a small drop in plasma density in the transverse direction leads to the hybrid resonance (see Sec. 6). Consequently, the effective beam density $\bar{n}_b$ is only a few times smaller than the actual density in the beam. Usually the L wave is spread out much more than the H wave; hence, the effective density is much smaller than the beam density.

The value of the smallest possible $k_\perp$ is determined by the transverse boundary conditions. The $m = 0$ mode will have the highest growth rate for this smallest $k_\perp$. It is not easy to calculate $k_\perp$ in realistic situations. In cylindrical geometry, the largest growth rate of the $m = 0$ mode occurs for $k_\perp \approx 1/a$, where a is the radius of the beam.²⁵

The growth rate of the $m = -1$ mode increases with $k_\perp$ so that it peaks at a large $k_\perp$ limited by thermal effects. However, the $m = 0$ mode usually will grow faster and suppresses the $m = -1$ mode by nonlinear effects. This suppression may be prevented in very thin systems, which do not allow low values of $k_\perp$. In particular, the $m = -1$ mode will have a higher growth rate than the $m = 0$ mode if the system admits only those $k_\perp$ values satisfying the inequality

$$(k_\perp v_b/\omega_{ce})^2 > \omega_{ce}/\omega_b, \quad (6)$$

where ω_b is the plasma frequency of the beam.²⁴ This inequality can only be satisfied in very thin systems and in very weak magnetic fields if one

assumes reasonable beam densities ($< 10^{10} \text{ cm}^{-3}$). One can thus draw the conclusion that in typical mirror machine experiments an electron beam with no transverse energy will excite only the space-charge wave modes $m = 0$. This is true also for a hot parallel beam (large spread in parallel velocities, $\bar{n}_b/n_p \ll (\Delta v_b/v_b)^2$). The growth rate of the $m = 0$ mode excited by the hot beam is

$$\frac{\omega_i}{\omega_p} \approx \frac{\bar{n}_b}{n_p} \left(\frac{v_b}{\Delta v_b} \right)^2, \quad (7)$$

which is of course much smaller than the growth rate, Eq. (4), of the cold beam. Numerical examples of intermediate cases can be found in Ref. (24).

It should be emphasized that waves with the smallest possible $k_\perp$ will dominate. Therefore, because the electric field of the waves will then have only a small component perpendicular to the magnetic field, it is not a favorable situation for perpendicular heating.

The situation becomes quite different when the beam has a distribution with transverse velocities peaked at some velocity $v_\perp$ comparable to the longitudinal velocity v_b . With such a beam, the fast cyclotron modes, $m > 0$, may also be unstable. Their growth rates were calculated in Ref. 24 for the following beam distribution:

$$f_b = \bar{n}_b \left(\frac{1}{\pi v_b^2} \right)^{1/2} \frac{1}{\pi v_\perp^2} \left(\frac{v_\perp}{v_\perp} \right)^2 \exp \left(- \frac{(v_\perp - v_\perp)^2}{v_\perp^2} - \frac{(v_z - v_b)^2}{v_b^2} \right). \quad (8)$$

Note that there is a misprint in Eq. (27) and in Table 1 of Ref. 24.

The growth rate of the $m = 2$ mode is plotted in Fig. 2 as a function of the excited frequency ω_r for different relative beam spreads v_b/v_b . The remarkable feature of this figure is the fact that the growth rate rapidly decreases with increasing beam spread at all frequencies except close to $\omega = 2\omega_c$, where it is not affected by the growth rate. In general, the growth rate of the m -th mode ($m > 1$) is not affected by the beam spread in the vicinity of the m -th cyclotron harmonics. This is so because the beam spread is unimportant as long as the inequality

$$\left| \frac{\omega - m\omega_c - k_z v_b}{k_z v_b} \right| \approx \left| \frac{\omega_i}{k_z v_b} \right| \geq 1 \quad (9)$$

Fig. 2. Growth rate of the $m = 2$ fast cyclotron mode as function of the excited frequency, ω_r , for different relative beam spreads, v_b/V_b , assuming $\omega_b/\omega_c = 0.1$, $k_{\perp} v_b/\omega_c = 10$, and $\nu = k_{\perp}^2 v_{\perp}^2 / 2\omega_c^2 = 25$.

holds. This criterion is certainly satisfied for $k_z \approx 0$, which occurs for $\omega \approx m\omega_c$ according to the resonance condition, Eq. (1).

The maximum growth rate

$$\frac{\omega_i}{\omega} \approx \left(\frac{n_b}{n_p} \right)^{1/2} \frac{1}{10} \left(\frac{m^2 - 1}{m^3} \right)^{1/2} \quad (10)$$

occurs for a large $k_\perp$, satisfying the equation

$$v = \frac{1}{2} \left(\frac{k_\perp v_\perp}{\omega_c} \right)^2 \approx \frac{5}{8} (4m^2 - 1). \quad (11)$$

The condition in Eq. (9) determines the bandwidth of the growing waves around the m -th harmonic:

$$\left| \frac{\omega - m\omega_c}{\omega} \right| < \frac{\omega_i}{\omega} \frac{v_b}{v} \quad (12)$$

Outside this bandwidth, the growth rate is very small, as shown in Fig. 2. On the other hand, cyclotron damping caused by the plasma will suppress the growth of waves in the immediate vicinity of $\omega = m\omega_c$. As a result, the instability peak will be split into a doublet, as shown in Fig. 3. The width of the valley may be estimated from the condition (growth rate with no plasma

Fig. 3. Effect of plasma temperature on the growth rate of the fast cyclotron modes.

losses) = (damping rate of the plasma alone). In this way, one gets the inequality

$$\left| \frac{\omega - m\omega_c}{\omega} \right| \geq \frac{\sqrt{\pi}}{2} \frac{m^2 - 1}{2^m m!} v^{m-1} \left(\frac{v_t}{V_L} \right)^{2(m-1)} \frac{v_b}{v_t} \exp \left[- (v_b/v_t)^2 \right] \frac{\omega}{\omega_i}, \quad (13a)$$

which determines the frequency bands for which the plasma temperature can be neglected. The plasma has been assumed Maxwellian with a thermal velocity v_t .

As a numerical example, let us assume $m = 2$, $\bar{n}_b/n_p = 10^{-3}$, $V_b/v_b = 2$, $V_L = V_b$, $v_b = 2v_t$. We obtain $\omega_i/\omega = 2 \times 10^{-3}$ for $v_{opt} = 9.5$. The width of the doublet is

$$4 \times 10^{-3} \geq \left| \frac{\omega - 2\omega_c}{\omega} \right| \geq 5 \times 10^{-5}.$$

The growth rate of the $m = 0$ mode is $\omega_i/\omega = 4 \times 10^{-3}$. If we assume that $V_b = 8 \times 10^9$ cm/s (20 keV) and that plasma density $n_p = 10^{14}$ cm $^{-3}$, the wave length of the $m = 0$ mode is $\lambda_0 = V_b/f_p = 0.09$ cm, and the wave length at the outer edge of the band, Eq. (12), of the $m = 2$ mode is $\lambda_2 = (f_p/V_b) (\omega/\omega_i) (v_b/V_b) = 14$ cm. Note that we have assumed a very large beam spread as well as a high temperature plasma.

The fast cyclotron mode can be unstable only when V_L exceeds a critical value²⁴ given by the following inequality:

$$v = \frac{1}{2} \left(\frac{k_L V_L}{\omega_c} \right)^2 > m^2 + \frac{1}{2}.$$

However, because, k_L is limited by plasma damping to a value $0.5 (k_L v_t/\omega_c)^2 \leq 0.1$, the threshold condition for V_L is

$$\left(\frac{V_L}{v_t} \right)^2 \geq 10(m^2 + \frac{1}{2}). \quad (13b)$$

5. NONLINEAR EFFECTS

The nonlinear evolution of the space-charge wave instability, $m = 0$, has been studied in great detail. References 26 through 34 are some of the papers recently published. In contrast, the nonlinear behavior of the cyclotron instabilities is not well known. Some of the older work is in Refs. 12 and 35 to 38; more work is very much needed. For this reason, we shall restrict our brief review to the nonlinear development of the space-charge wave instability. Again, we assume a long, uniform, plasma column in a homogeneous magnetic field.

Since the instability has the highest growth rate for the smallest possible $k_{\perp}$, the ratio $k_{\perp}/k_{\parallel} \approx v_b/a\omega_p$ will be much smaller than one for reasonably dense ($n_p > 10^{12}$) and fat ($a > 5$ cm) plasmas. That means that the electric field of the waves is directed along the magnetic field and that it affects only the parallel velocity of the electrons. Even if there is a perpendicular electric field present, it has a negligible effect on the beam trajectories because it is nonresonant.

The development of the instability in space can be divided into three distinct stages. In the first stage, immediately in front of the injector, a narrow band of the most unstable waves grows exponentially as $\exp(k_z z)$ with a spatial growth rate

$$\frac{k_z}{k_z} = \frac{\omega_i}{\omega} \left| \frac{v_{ph}}{v_{gr}} \right|. \quad (14)$$

Here ω_i/ω is the temporal growth rate used in the previous section and k_z is the parallel wave number. The ratio v_{ph}/v_{gr} of the phase velocity to the group velocity of the plasma eigenmode (unperturbed by the beam) depends on the slope of the plasma dispersion curve. This ratio is fairly large (10 or more) for the upper branch (H wave), since the dispersion curve is very flat (see Fig. 5 of Ref. 25). The waves grow from an initial level much higher than accounted for by thermal fluctuations because there is usually some feedback in the system. If the feedback is large, the linear growth is completely absent.^{30,32,33} The fastest growing wave saturates by beam trapping at an amplitude given by

$$\frac{\phi}{U} \approx \left(\frac{k_i}{k_z} \right)^2, \quad (15)$$

where ϕ is the potential of the wave and U is the injection voltage. All this happens only if the beam is initially mono-energetic (cold). Typically, for $\bar{n}_b/n_p = 10^{-3}$ one obtains $k_i/k_z \approx 0.1$ to 0.3 and $\phi/U \approx 0.01$ to 0.1 .

In the second stage of the development of the instability, the beam evolves into a quasi-linear plateau as shown in Fig. 4. The right-hand curves show distribution functions measured at various distances z from the gun. The left set of curves shows the corresponding spectra. As the beam gets trapped ($z = 15$ cm), its distribution function changes in such a way that the positive slope occurs at a somewhat lower velocity than the injection velocity. This gives rise to the growth of slower waves that appear as the right-hand sideband at $z = 30$ cm. These waves modify the distribution further, and so on. This cascading process leads to the development of a quasi-linear plateau as predicted a long time ago (Ref. 39). In systems long enough, this plateau merges with the plasma distribution as shown in Fig. 5. At this stage, the beam carries one half of the injected power. The other half of the injected power flows into the waves and from them into the plasma.

At high beam densities, $n_b/n_p > 0.01$, the beam distribution spreads out into a high-energy tail as shown in Fig. 5. This is due to the appearance of new waves whose phase velocity exceeds the initial beam velocity. These waves are generated by mode coupling. They show up in the left-hand sideband of Fig. 4. Plasma heating gets stronger also, so that the hot electron tail of the plasma electrons reaches deep into the beam distribution. The resulting total distribution acquires a quasi-Maxwellian shape.

Other nonlinear effects like mode conversions, Ref. 12, back-scattering Refs. 12 and 40, and plasma trapping, Refs. 41 and 42, were observed with strong beams. Their significance for heating of large mirror plasmas is not clear.

Fig. 4. Wave spectrum and beam distribution function for various distances z from the injector.³¹

6. PLASMA HEATING MECHANISMS

An open-ended plasma system using electron beam heating is shown schematically in Fig. 6. By measuring the input and output beam powers, one can directly measure the power lost by the beam, $W = P_{in} - P_{out}$, in steady operation. Usually $W/P_{in} \leq 0.5$. This lost power goes into the waves produced by beam-plasma interaction. These waves are trapped in the plasma: they do not radiate because they are electrostatic, and they do not propagate out of the plasma because they are absorbed, or reflected, at plasma boundaries. In steady-state operation, the waves must be absorbed by the plasma

Fig. 5. Typical beam distribution for long systems:
 (a) no interaction,
 (b) quasi-linear plateau,
 (c) additional plasma and beam heating.

electrons at the same rate that they are generated by the beam electrons. This is expressed by the power balance

$$W = \int dV \, 2\gamma \xi \quad (16)$$

where

$$\xi = \left\langle \frac{E^2}{8} \left(\frac{\partial \epsilon}{\partial \omega} \right) \omega \right\rangle \quad (17)$$

is the energy density of all the waves, γ is their effective damping rate, and the integration is over the plasma volume. Hence, in steady-state operation the plasma temperature adjusts itself to a value satisfying Eq. (16). This is possible because plasma damping is a very sharp function of plasma temperature, while the energy of the waves and W depend on plasma temperature only weakly. The amplitude of the waves is determined by their growth rates [Eq. (15) gives the largest amplitude], which are only weak functions of plasma temperature (compare e.g. Fig. 2 of Ref. 30). This of course assumes that there are damping processes sufficiently strong to make γ sufficiently large.

While this condition seems to be satisfied in typical experiments, it is not easy to identify the particular damping mechanisms.

Fig. 6. Schematic representation of a beam-heated plasma.

In a uniform plasma, there seem to be only two major damping mechanisms: electron-cyclotron damping and nonlinear Landau damping.

Electron-cyclotron damping is large in the vicinity of cyclotron harmonics,

$$\left| \frac{\omega - n\omega_{ce}}{k_z v_t} \right| < 0.5, \quad (18)$$

and for $k_{\perp}$ sufficiently large that the n -th order Bessel function $I_n(k_{\perp}^2 v_t^2 / 2\omega_{ce}^2)$ is not negligible. The condition (18) corresponds to the horizontal, wedge-shaped shaded regions in Fig. 1. The damping rate can reach large values $\gamma/\omega > 0.01$ for moderate temperatures $(k_{\perp}^2 v_t^2 / 2\omega_{ce}^2) \approx 0.02$. An illustrative plot of γ vs ω is in Fig. 6 of Ref. 25. Since $\omega \approx \omega_{ce}$, electron-cyclotron damping can be effective only for dense plasmas, $\omega_{pe}/\omega_{ce} > 1$. Energy is fed into the perpendicular motion of the electrons. The experimentally observed fact that beam-produced plasmas have a tendency to exist at discrete densities such that $\omega_{pe} \approx n\omega_{ce}$ (Ref. 12) is probably related to absorption of waves by electron-cyclotron damping. Electron-cyclotron heating was recognized a long time ago as a possible mechanism for heating plasmas produced by electron beams.⁴³

When $\omega_{pe}/\omega_{ce} > 5$ or when there is no magnetic field at all, electron-cyclotron damping is ineffective and nonlinear Landau damping becomes dominant. Mixing two adjacent, beam-produced waves generates a quasi-wave whose phase velocity is equal to the group velocity of the unperturbed plasma wave. Since

the group velocity is of the same order as the thermal velocity, the wave is Landau-damped by the plasma electrons. Energy is fed into longitudinal motion of the electrons.

The situation becomes much more complicated in an inhomogeneous plasma and magnetic field, as in mirror-confined plasmas.

First, the inhomogeneities make it more difficult to excite the waves. To obtain wave growth, the local growth rate must satisfy the inequality

$$\frac{k_{\perp}}{k_z} \geq \frac{\partial k_z / \partial z}{k_z^2} \quad (19)$$

This simple rule of thumb has been experimentally verified.⁴⁴ The excited wave has a frequency corresponding to the local dispersion equation of the plasma. As the wave propagates in the inhomogeneous plasma, its wave number k_z changes in accordance with the local dispersion equation while its frequency stays constant. Depending on the circumstances, the wave may be stopped by the following processes:

- It may encounter a resonance $k_z \rightarrow \infty$ that leads to total absorption of the wave by the plasma electrons^{12,15,45,46};
- It may encounter a resonance $k_z = 0$ that leads to reflection of the wave^{12,45}; or
- Its phase velocity may increase so that it is re-absorbed by the beam.^{44,47}

The Clemmow-Mullaly-Allis (CMA) diagram⁴⁸ is useful for figuring out the possible resonances. In the simplified version of the CMA diagram for a cold plasma shown in Fig. 7, areas accessible to the H waves ($\omega > \omega_{ce}$) and the L waves ($\omega < \omega_{ce}$) are shown bounded by the three resonance lines. The horizontal line $k_{\perp}/k_z = 0$ corresponds to cyclotron resonance, $\omega = \omega_{ce}$, the vertical line $k_{\perp}/k_z = 0$ corresponds to plasma resonance, $\omega = \omega_{pe}$, and the line segment $k_{\perp}/k_z = \infty$ is the upper-hybrid resonance.

Let us first consider longitudinal inhomogeneities. Let an H wave be excited at a point designated by H in Fig. 7. If we assume that its frequency ω and perpendicular wave number $k_{\perp}$ will not change as it propagates in the inhomogeneous plasma, the path of the wave in the CMA diagram is uniquely determined by the local values of ω_{pe} and ω_{ce} in WKB approximation. Four particularly simple paths for an H wave are shown in Fig. 7. It may be seen that the wave hits the plasma resonance $k_{\perp}/k_z = 0$ when the plasma

density increases in the direction of propagation (direction of group velocity). Since k_1 is assumed constant, $k_z \rightarrow \infty$ at the plasma resonance and the wave is totally absorbed. However, all the energy is fed into

Fig. 7. CMA diagram for a cold plasma.

longitudinal motion of the plasma electrons. When the plasma density decreases in the direction of energy propagation, the wave hits the hybrid resonance $k_1/k_z = \infty$, implying $k_z = 0$ and total reflection. Reflection also occurs when the H wave propagates into a decreasing magnetic field. On the other hand, propagation of the H wave into an increasing magnetic field leads to cyclotron resonance $k_1/k_z = 0$ implying $k_z \rightarrow \infty$ and absorption. In this case, energy is fed into the perpendicular motion of the plasma electrons.^{45,46} One can see that H waves excited in an inhomogeneous plasma tend to be confined in small regions in space bounded by the resonances.

A glance at Fig. 7 shows that the L waves behave differently. Starting again from an excitation point designated by L, the wave can either reach the plasma resonance, and be absorbed, when it propagates in the direction of decreasing plasma density or it can hit the cyclotron resonance, and be absorbed, when it propagates in the direction of decreasing magnetic field. There is no resonance when propagation is in the direction of increasing magnetic field or increasing density. That does not guarantee that the wave will make it too far. Most likely, it will be re-absorbed by the beam electrons and produce a high-energy tail on the beam. This is so because the phase velocity of the wave increases as the wave propagates in the direction of rising density or magnetic field, as can be seen from the curves of constant k_1/k_z indicated in Fig. 7.

This effect has been both theoretically analyzed⁴⁷ and experimentally observed.⁴⁴ Also, efficient heating of plasma electrons was observed in two experiments in which electron beams were injected in the direction of decreasing plasma density.^{49,50} In this situation, the phase velocity of the L wave decreases, making it possible to extract more power from the beam than the 50% limit corresponding to the beam plateau in a uniform plasma.

The inhomogeneity of the plasma density in the direction perpendicular to the magnetic field (present even in longitudinally uniform plasma) may lead to enhanced absorption of H waves at the outer boundary of the plasma.^{12,45} This may be seen from the CMA diagram (Fig. 7) if one assumes propagation of the H wave perpendicularly to the magnetic field in the direction of falling density. The wave hits the hybrid resonance $k_1/k_z = \infty$ which implies $k_1 \rightarrow \infty$ because k_z is fixed. Some experimental evidence of this effect has been reported.^{51,17}

7. HEATING MIRROR-CONFINED PLASMAS WITH ELECTRON BEAMS

For effectively heating mirror-confined plasmas to high temperatures, the electrons must stochastically gain transverse energy either on the slope or in the bottom of the magnetic trap. Typically, in the center of the trap, $\omega_{pe}/\omega_{ce} \approx 2$ to 3, $n_b/n_p \approx 10^{-3}$. It was shown in Sec. 4 that only two instability modes can be excited under these conditions: the space-charge

mode and the fast-cyclotron mode. Let us discuss the possible heating mechanisms associated with these modes.

When the electron gun is placed in a high enough magnetic field where the electrons are adiabatic, the electron beam is nearly parallel to the magnetic field and only the space-charge waves will be initially excited. This applies also to the magnetron gun. All the experiments in Table 1 had parallel beams with the exceptions of Seidl^{12,15} and Bredikhin.^{17,18} Such space-charge waves have small $k_{\perp}$ and a short longitudinal wave length ($n_z \approx 3$ mm for $n_p \approx 10^{13} \text{ cm}^{-3}$ and a 20-keV beam). Both L and H waves can be excited on the slopes of the confinement region and also in the center. An enhancement of $k_{\perp}$ could occur because of cyclotron resonances at the slopes of the mirrors, as can be seen from the CMA diagram. At the expected beam strength $\bar{n}_b/\bar{n}_p \approx 10^{-3}$, all interactions should be completed (the beam should be left with no positive slope in the distribution function) in a distance of 100 wavelengths (about 30 cm). Table 1 provides enough experimental evidence to show that effective transverse heating is possible with a parallel beam, if the gun is placed in the mirror throat. However, to prevent the electron gun structure from perturbing the hot plasma, it is highly desirable to place the gun as far away from the mirror as possible. Then, one must consider the beam transport problem discussed in the next section.

To excite the fast-cyclotron mode in the region between the mirrors, the beam must have an $f(v_{\perp}^2)$ distribution peaked at a large $V_{\perp}$ so that it satisfies the condition (13b) as it enters the mirror region. Such a helical beam can be produced by a gun located far away from the mirror in a weak magnetic field. The initial perpendicular velocity can be produced either by tilting the gun with respect to the magnetic field lines¹⁷ or by passing the beam through a short, nonadiabatic section.⁵² The perpendicular energy builds up adiabatically to the final value, close to total energy, in the mirror peak.

Such a helical beam could launch the $m = 2$ fast-cyclotron wave at the mirror slope in the confinement region. This wave should be very effective in perpendicular heating of the plasma electrons since it has a frequency close to the cyclotron harmonic and a high $k_{\perp}$. Furthermore, since the beam electrons are close to the loss-cone boundary when they enter the mirror region, a small scattering by the fluctuating fields would trap some of the beam electrons. This may help stabilize the DCLC mode in the mirror-confined plasma. Since the fast-cyclotron instability is not sensitive to the longitudinal spread in

the beam distribution, beam transport problems may be less severe than for the parallel beams.

There is experimental evidence that heating mirror-confined plasmas with helical beams is possible and effective.^{12,15,17,18} Although the excited modes have not been identified in the mirror-confined plasma, it is very likely that the helical beams have preferentially excited the fast-cyclotron waves.

8. BEAM TRANSPORT

The electron gun should be far from the hot plasma to keep the density of high-Z atoms at a reasonably low level. Although the acceptable distance is not well defined, let us assume that the gun must be placed at the outer slope of the mirror field. In a machine the size of Beta II or TMX, the distance from the gun to the mirror throat would be 1 m or more. One then faces the problem of transporting the electron beam from the gun to the mirror without causing significant deterioration of the positive slope of the parallel or perpendicular velocity distribution. This is a problem that has not yet been analyzed in its entirety.

The beam distribution will not be affected if all instabilities are either absent or sufficiently weak. This can be achieved, at least with respect to beam-plasma instabilities, if the beam-to-plasma density ratio along the injection path is kept either small ($n_b/n_p < 10^{-4}$) or large ($n_b/n_p > 1$). The first solution is possible, as shown in the experiments on the transport of relativistic electron beam.⁵³ The second solution, $n_b/n_p > 1$, fits more naturally into the setup of mirror experiments since the plasma along the injection path is much weaker than in the confinement region. However, various low-frequency instabilities can be excited by an electron beam propagating either in a vacuum or in a dilute plasma in a magnetic field. There is extensive literature on this subject produced by microwave tube workers^{54,55,57-59} as well as by plasma physicists.^{56,60-62} Most of the reported work was done for simple cylindrical geometry: a cylindrical beam of radius a moving along the axis of a metal cylinder of radius R in a uniform magnetic field. Although this is quite different from the actual injection geometry, the results may still be useful, especially if the geometrical correction factors were worked out.

When the beam moves in a vacuum, magnetic forces caused by beam rotation (Brillouin flow) compensate for space-charge forces.⁵⁴ At a certain limiting beam current, the space charge depresses the potential in the beam to the value of the cathode potential. A virtual cathode forms and electrons are reflected. When the beam completely fills the tube ($a = R$), this limiting current is given by the formula (Ref. 54, p. 167)

$$I_{\text{spc}} = 32 \times 10^{-6} U^{3/2}, \quad (20)$$

where U is the injection voltage in volts and I_{spc} is the beam current in amperes. When the beam does not completely fill the tube ($a < R$), the limiting current is⁵⁵

$$I_{\text{spc}} = 13 \times 10^{-6} U^{3/2} / \ln(R/a). \quad (21)$$

Hollow electron beams in a vacuum are subject to the diocotron instability⁵⁶ that is caused by the $E \times B$ rotation of the beam electrons in their own space-charge field. This instability leads to the formation of electron filaments that are regularly distributed around the circumference of the beam.^{57,59} Similar filamentation occurs in thin, strip beams.⁵⁸ The diocotron instability does not have a threshold current for thin beams. However, it has been shown⁵⁹ that the beam breaks up into clearly distinguishable filaments when the current exceeds the value

$$I = 9.8 B d^2 U / (nL), \quad (22)$$

where I is in amperes, B is in teslas, d is the diameter of the hollow beam in metres, U is in volts, L is the length of the beam in metres, and n is the number of filaments around the circumference of the hollow beam ($n \approx 8$ was typical in the experiments). The current, Eq. (22), was obtained from the condition $k_i L \approx 1$, where k_i is the spatial growth rate of the instability.

The diocotron instability may not totally destroy the beam quality. The beam may be stabilized either by a radial diffusion of the electrons that changes the beam width or by partial neutralization of the space charge. There are, however, no solid data on the nonlinear saturation of this instability.

When the beam space charge is neutralized by positive ions or when the beam propagates in a dilute plasma, several low-frequency instabilities are possible when the beam current exceeds a certain critical value. These instabilities have been reviewed in Refs. 60 and 61.

The critical current has the highest value when very heavy ions compensate for the space charge of the beam. In this situation, the beam electrons interact with the walls of the conducting tube of radius R . The stability condition for this "Pierce instability" is⁶⁰

$$\omega_b^2 < k^2 V^2, \quad (23)$$

where ω_b is the plasma frequency of the beam electrons, V is the velocity of the electrons, and $k^2 = k_z^2 + k_r^2$, where k_z and k_r are the longitudinal and radial wave numbers. The latter has the value

$$k_r = 2.4/a \quad \text{for } a = R, \quad (24)$$

$$k_r^2 = 2 [a^2 \ln(R/a)]^{-1} \quad \text{for } a \ll R. \quad (25)$$

If we assume that $k_z \ll k_r$ and use Eq. (25), the stability condition (23) may be written as $I < I_p$, where I is the beam current and I_p is the Pierce current given by $I_p = (ma^2/4e)k^2 V^3$. Numerically,

$$I_p = 66 \times 10^{-6} U^{3/2} / \ln(R/a). \quad (26)$$

The beam voltage U is in volts and I_p is in amperes.

When the ions have a finite mass M , an electron-ion, two-stream instability can be excited inside the electron beam. This instability results from the relative motion of the electrons with respect to the ions (Buneman instability). The stability condition is⁶⁰

$$(\omega_i k/k_z)^{2/3} + \omega_b^{2/3} < (kV)^{2/3}, \quad (27)$$

where $\omega_i = (4\pi n_i e^2/M)^{1/2}$ is the ion plasma frequency. Using the Pierce current and the beam current I , this condition can be written as

$$I < I_p \left[1 + (mk^2/Mk_z^2)^{1/3} \right]^{-3}. \quad (28)$$

Not far from the stability limit (28), the excited frequency is

$$\omega = k_z V \left[1 + \left(Mk_z^2/mk^2 \right)^{1/3} \right]^{-1}. \quad (29)$$

When the beam moves through a plasma, electron-electron instabilities inside the beam become dominant. The stability condition for this electron-electron, two-stream instability is⁶⁰

$$\omega_b^{2/3} + \omega_p^{2/3} < (kV)^{2/3}, \quad (30)$$

where $\omega_p = 4\pi n_p e^2/m)^{1/2}$ is the plasma frequency of the plasma electrons. The instability is absent when

$$I < I_p \left[1 + (n_p/n_b)^{1/3} \right]^{-3}, \quad \omega_p < kV \quad (31)$$

and the excited frequency is

$$\omega = k_z V \left[1 + (n_b/n_p)^{1/3} \right]^{-1}, \quad (32)$$

close to the instability threshold. On the other hand, for a dense plasma, $\omega_p > kV$, the instability sets in at any beam density (see Sec. 4).

A drift-type, nonaxisymmetric instability is associated with the radial gradient of the beam density.⁶² This beam-drift instability is most dangerous when the beam density is equal to the ion density and plasma electrons are absent. The stability condition for this case is⁶⁰

$$\omega_b^2 < k^2 V^2 \left[1 + (2sV/a^2 \omega_{ce} k_z) \right]^{-1}, \quad (33)$$

or

$$I < I_p \left[1 + (2sV/a^2 \omega_{ce} k_z) \right]^{-1}. \quad (34)$$

The positive integer s is the mode number. The presence of plasma electrons increases the threshold current. In particular, the instability disappears when the radial density gradient of the plasma electrons is sufficiently large:

$$n_p/n_b > R_p^2/a^2, \quad (35)$$

where R_p is the plasma radius.

When the beam current is smaller than any of the three limiting currents given by the formulas (28), (31), and (34), no low-frequency instabilities will be present. Of course, these formulas should be improved by taking into account the actual beam geometry and longitudinal inhomogeneities.

A particularly important geometry is that of a hollow electron beam of mean radius a and thickness w ($w \ll a$). Such a beam is produced, for example, by the magnetron gun. The limiting currents have been calculated for a hollow beam tightly fitting into a conducting tube.⁶¹ Since in our example the beam is far from conducting walls, we shall estimate the limiting currents using simple approximations.

To obtain the Pierce current, we use the stability condition (23) with $k \approx 1/w$ and obtain $I_p \approx (a/w)(m/2e)V^3$; numerically,

$$I_p \approx 66 \times 10^{-6} U^{3/2} (a/w), \quad (36)$$

where I_p is in amperes and U is in volts. Similarly, a hollow beam of arbitrary shape and circumference L and width w ($w \ll L$) will have a Pierce current

$$I_p \approx (L/2\pi w)(m/2e)V^3, \text{ or}$$

$$I_p \approx U^{3/2} (L/w) \times 10^{-5}, \quad (37)$$

where I_p is in amperes and U is in volts.

The same formula would apply for a thin, strip beam of length L and width w . Both formulas show that thin hollow or strip beams have much higher Pierce currents than full cylindrical beams, Eq. (26).

While the formulas (28) and (31) for the stability limits of two-stream instabilities remain valid also for hollow and strip beams, the formula (34) for the beam-drift instability must be modified. Keeping in mind that the term $2s/a^2$ in Eq. (34) has its origin in the product $k_\theta (\partial n / \partial r)$, we recognize that for a hollow beam it must be replaced by s/aw ($k_\theta \approx s/a$, $\partial n / \partial r \approx 1/w$). The stability limit for the beam-drift instability of a thin, hollow beam thus becomes

$$I < I_p \left(1 + sV/awk_z \omega_{ce} \right)^{-1}. \quad (38)$$

The stability limit for a hollow beam of circumference L or a strip beam of length L follows from (38) when a is replaced by $L/2\pi$:

$$I < I_p (1 + 2\pi sV/L\omega k_z \omega_{ce})^{-1} . \quad (39)$$

Even when some of the instabilities are present, the beam may not be necessarily destroyed. More work on the nonlinear regime is needed for quantitative estimates.

The effect of the electron-electron, two-stream instability on the beam distribution function can be estimated from the phase velocity ω/k_z of the most unstable wave, given by Eq. (32). Three typical situations are shown in Fig. 8. When $n_b \ll n_p$, the phase velocity is very close to the beam velocity, $\omega/k_z \approx V$, the wave strongly interacts with the beam electrons, modifying their distribution by trapping, and subsequent cascading leads to a quasi-linear plateau discussed in Sec. 5. On the other hand, the phase velocity is very small when $n_b \gg n_p$. The wave traps the plasma electrons and does not destroy the positive slope of the beam distribution. In the particular case $n_b = n_p$, the phase velocity is $\omega/k_z = V/2$ and the wave interacts with both plasma and beam electrons equally strongly until a plateau extending from $v_z \approx 0$ to $v_z \approx V$ is formed.⁶³ The positive slope of the beam distribution has been completely eliminated. The final distributions shown in Fig. 8 will, of course, occur only in sufficiently long systems.

9. BEAM TRANSPORT IN THE TANDEM MIRROR EXPERIMENT

We shall now discuss beam transport in the TMX,⁶⁴ using the limiting currents of Sec. 8. Figure 9 shows the magnitude of the magnetic field along the injection path, assuming that the electron gun is placed at the end of the TMX vacuum tank ($z = 0$), approximately 3.5 m from the first mirror of the plug. While operationally most convenient, this long distance makes beam transport more difficult. Two modes of operation are considered: plasma production and plasma heating. In the plasma production mode, the beam must travel 6 to 9 m into the central cell, where it interacts with the gas admitted into the gas box. It takes about 50 to 100 μ s for the plasma to stream out from the central cell. During this time, the beam travels in a near vacuum. In the heating mode, the beam interacts with the plasma confined in the plug. It must travel about 4 m through a plasma whose density varies from 10^9 to $5 \times 10^{13} \text{ cm}^{-3}$, as indicated in Fig. 9.

Fig. 8. Effect of the electron-electron, two-stream instability on the beam distribution function.

Fig. 9. Plot of magnetic field B and plasma density n_p along the beam trajectory in the TMX. The dashed curve near the end wall is the effect of the magnetron gun coil.

The beam is produced by a magnetron gun placed in a 0.2-T magnetic field. The dotted curve in Fig. 9 indicates the stray field of the magnetron gun coil. The gun produces a hollow cylindrical beam of mean radius $a = 1.5$ cm and width $w = 0.3$ cm at $z = 0$. At 20 kV, the expected beam current is 50 A and the electron density in the beam at $z = 0$ is $n_b \approx 8 \times 10^9 \text{ cm}^{-3}$. Comparing this with the plasma density of Fig. 9 gives $n_b/n_p \approx 1$ in front of the gun. This ratio stays constant along the whole injection path up to the mirror throat since the motions of both beam and plasma electrons are adiabatic. The transverse energy of the beam electrons at the exit from the gun is relatively small ($W_\perp \approx 0.4$ keV for $W = 20$ keV). The following terms are also adiabatic invariants along the injection path: a/w , I_p , $a^2\omega_{ce}$, $aw\omega_{ce}$, L/w , $Lw\omega_{ce}$. The symbols were defined in Sec. 8.

Let us calculate stability limits for the hollow beam using the following numerical values: $U = 20$ kV, $V = 8.5 \times 10^9$ cm/s, $a/w = 5$, $n_b/n_p = 1$, $aw\omega_{ce} = 1.6 \times 10^{10} \text{ cm}^2/\text{s}$, and $M/m = 3672$. According to Eq. (36), the Pierce current is

$$I_p = 933 \text{ A}, \quad (40)$$

and is invariant along the injection path. The stability limit for the electron-electron, two-stream instability, Eq. (31), is

$$I_{e-e} < I_p/8 = 116 \text{ A}, \quad (41)$$

also invariant along the injection path

The electron-ion, two-stream instability is most likely to occur in the region $z = 0.5$ to $z = 1$ m in Fig. 9, where the plasma has almost constant density. Accordingly, we use $k_z = \pi/50 \text{ cm}^{-1}$ in formula (28). We also use $M/m = 3672$, $k \approx 1/w = 1/0.3 \text{ cm}^{-1}$. The stability limit is

$$I_{e-i} < I_p/2.44 = 383 \text{ A}. \quad (42)$$

The beam-drift instability is strongest at long wavelengths, according to Eq. (38), and it is also most likely to occur in the region $z = 0.5$ to $z = 1$ m. Assuming $k_z = \pi/50 \text{ cm}^{-1}$ and $s = 2$, we obtain from Eq. (38)

$$I_{B-D} < I_p/17.9 = 52 \text{ A}. \quad (43)$$

A higher mode number s would make the limiting current smaller; however, plasma electrons may have a stabilizing effect.

During plasma buildup, the beam-drift instability may be very easily excited in the central cell because the long section allows a long wavelength $\lambda/2 \approx 5$ m. Inserting $k_z = \pi/500$ into Eq. (38) gives us a limiting current

$$I_{B-D} < I_p/172 = 5.5 \text{ A} . \quad (44)$$

Two different situations may thus be encountered during plasma production depending on the location of the gas box. When the latter is located in the far side of the central cell, beam-drift instability will occur, leading to beam disruption and ion heating as observed by Nezlin.⁶⁰ On the other hand, if the gas box is located in the near side of the central cell (closer to the injector, $z \approx 5$ m in Fig. 9), the beam will interact with a dense plasma at the very beginning of the central cell and will produce a relatively quiet, cold-ion plasma by means of electron-electron, two-stream instability.

At the very beginning of the plasma buildup, the beam moves in a vacuum and the only instability is the diocotron instability. Let us use Eq. (22) to estimate the current at which the beam starts breaking up into filaments. Assuming that the most dangerous region is the uniform magnetic field section extending from $z = 0.5$ to $z = 1$ m in Fig. 9, we put in Eq. (22) $L = 0.5$ m, $d = 0.03$ m, $B = 0.01$ T, $U = 20$ kV, $n = 8$, and obtain $I_{dioc} = 0.44$ A. As mentioned earlier, it is not known what damage is caused to the beam distribution by the diocotron instability in the nonlinear regime.

In the presence of the dilute plasma, the most dangerous instability along the injection path is the beam-drift instability, as can be seen from the limiting currents, Eqs. (41) to (43). The limiting current for the beam-drift instability can be written in a more transparent form if one makes the plausible assumption that the highest unstable mode has the mode number $s \approx 2\pi a/2w$. Assuming further that the second term in the parentheses of Eq. (38) dominates, and combining Eq. (38) and (36), we obtain

$$I_{B-D} < 3.5 \times 10^{-13} U w \omega_{ce} k_z a, \quad (45)$$

where the current is in amperes, the voltage U is in volts, and all lengths

are in centimetres. This condition clearly shows two simple means for increasing the limiting current:

- Avoid long, uniform sections of the plasma (make k_z large) and
- Use a high injection voltage (the limiting current is proportional to the voltage).

Finally, Eqs. (37) and (39) indicate that it may be advantageous to use elongated hollow beams or thin, strip beams. Both would naturally fit into the fan-like geometry of the magnetic field. They could be produced by a planar magnetron gun. As an example, a beam of circumference $L = 100$ cm and a width $w = 0.3$ cm at $U = 40$ kV has a Pierce current

$$I_p = 26.7 \text{ kA}, \quad (46)$$

according to Eq. (37). Inserting into Eq. (39) $s = L/2w$, $V = 1.2 \times 10^{10}$ cm/s, $k_z = \pi/20$, $\omega_{ce} = 3.5 \times 10^{10} \text{ s}^{-1}$ ($B = 2$ kG), we obtain a beam drift limit

$$I_{B-D} < I_p/76 = 345 \text{ A}. \quad (47)$$

Notice that this high current has been obtained with a mode number $s = 166$. Although more work is needed to improve the formulas, present results indicate that elongated hollow beams might be an attractive solution to the beam transport problem in the TMX.

10. CONCLUSIONS

An electron beam injected into a plasma can generate rf fields that may heat the plasma electrons. In a uniform plasma, up to 50% of the beam power can be fed into the plasma. In mirror-confined plasmas, 5 to 10% of the beam power has been observed to go into heating a hot-electron component. Even at this efficiency, electron-beam heating of mirror-confined plasmas is an attractive technique since it is relatively easy and cheap to produce electron beams carrying megawatts of power in millisecond pulses.

Any electron-beam heating scheme consists of two parts: beam transport and plasma heating. In the beam transport section, the beam must be trans-

ported from the electron gun to the mirror throat without significant deterioration of its distribution function. To achieve this, all beam instabilities must be either absent or sufficiently weak. Plasma heating occurs within the confinement region of the magnetic mirror. Here, the beam-plasma interaction must be strong enough to ensure efficient heating of the electrons.

To control beam-plasma instabilities, we require detailed knowledge of their properties. Although all major instability modes have been identified and are qualitatively understood, more work is needed for making reliable quantitative predictions. Let us briefly summarize the present state of knowledge and areas needing additional research.

BEAM TRANSPORT

Beam transport occurs in a region filled with dilute, inhomogeneous plasma ($n_b \geq n_p$). Three instability modes are possible: electron-electron, two-stream instability; electron-ion, two-stream instability; and beam-drift instability. The first instability will not occur as long as the plasma and beam densities are sufficiently low to satisfy the condition (30). The stability threshold depends on the total wave number k , which in turn is determined by the transverse dimension of the beam. For this reason, thin, hollow beams have high thresholds to the electron-electron, two-stream instability mode. The other two instabilities prefer long wavelengths. Consequently, their threshold currents can be kept high when long, uniform regions are avoided along the beam transport path. Under normal conditions, the beam-drift instability has the lowest threshold, given by Eq. (34) for a full cylindrical beam and by Eq. (38) or (45) for a hollow beam. Although the equations for full cylindrical beams have been experimentally verified, the equations for hollow beams are tentative.

More work is needed in the following areas:

- Theory and experimental verification of the stability of hollow and strip beams;
- Effect of longitudinal inhomogeneities on stability of beams;
- Nonlinear effects of low-frequency beam instabilities on beam distribution; and
- Nonlinear development of the diocotron instability and its effect on the beam distribution.

PLASMA HEATING

Plasma heating occurs in the confinement region where the plasma density is high ($n_b/n_p \approx 10^{-3}$). Only two instability modes can be excited: the space-charge mode and the fast-cyclotron mode. The space-charge mode will be excited when the beam has a positive slope in the longitudinal velocity distribution only; the fast-cyclotron mode needs a beam with a positive slope in the perpendicular distribution as well. There is experimental evidence that both modes can heat the plasma electrons. However, it is not clear which mode is more effective or whether plasma inhomogeneities play a decisive role in plasma heating. Electron heating is a diffusion in phase space: electrons stochastically gain transverse energy on the slope or in the bottom of the magnetic trap. Formation of the high-energy tail in the electron distribution results from a better confinement of electrons with large transverse energy.

More work is needed in the following areas:

- Nonlinear evolution of the fast-cyclotron instability;
- Effect of plasma inhomogeneities on space-charge and fast-cyclotron instabilities; and
- Plasma heating by both instability modes.

ELECTRON BEAM HEATING OF THE TANDEM MIRROR EXPERIMENT

In considering beam transport and plasma heating for parameters characteristic of the LLL Tandem Mirror Experiment (TMX), we find that certain steps can be taken to minimize instability modes in TMX beam transport from the end regions to the plugs. For example, optimum transport is achieved for high-voltage, low-current, thin electron beams being transported in short, nonuniform transport paths. We estimate that with such attention to beams, it would be possible to transport several hundred amperes of a 40-keV beam in an elongated hollow beam through a 4-m-long injection path in the TMX. This corresponds to power levels in the 10-MW range, a figure comparable to the neutral-beam power available on each TMX end plug. Assuming 10% heating efficiency, about 1 MW of power would go from the electron beam into heating the plasma electrons in the plug. This is considerably more than the 0.4-MW heating power estimated to go into electron heating from neutral beams.

The 10-MW electron gun has yet to be designed and built. However, the four 1-MW (20 kV, 50 A) guns that were built for the 2XIIB machine can be used for preliminary heating experiments on TMX. With two guns per plug, about 0.2 MW of heating power could be delivered to the plasma electrons in each plug. This is about 50% of the electron heating power from neutral beams, a fraction sufficiently large to cause a measurable change in electron temperature.

We suggest starting the heating experiments with simple parallel beams and trying the more complex helical beams at some later stage. It should be emphasized that these experiments will need special diagnostics and considerable experimental operating time devoted to optimizing heating efficiency.

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