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SPHEROMAK REACTOR-DESIGN STUDY

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ABSTRACT

A general overview of spheromak reactor characteristics, such as MHD stability, start up, and plasma geometry is presented. In addition, comparisons are made between spheromaks, tokamaks and field reversed mirrors.

The computer code Sphero is also discussed. Sphero is a zero dimensional time independent transport code that uses particle confinement times and profile parameters as input since they are not known with certainty at the present time. More specifically, Sphero numerically solves a given set of transport equations whose solutions include such variables as fuel ion (deuterium and tritium) density, electron density, alpha particle density and ion, electron temperatures.

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1.0 INTRODUCTION

Since toroidal devices such as tokamaks have complicated magnetic coil geometries that may limit their future commercial applications, scientists have been investigating a new class of configurations called compact tori. This class of machines includes the field-reversed mirror and more recently the spheromak. The classical spheromak is shown in Figure 1-1; notice that a mirror like external magnetic field surrounds the plasma.

Because the spheromak is a relative newcomer to the fusion community, its characteristics are not well known at this time, however, the spheromak does possess some good reactor qualities such as MHD stability and potentially simple reactor geometry which makes it a good candidate for further study. Therefore the purpose of this report is to: define the spheromak more clearly, show the spheromak's reactor potential, and describe a method for modeling a beam driven spheromak.

Figure 1-1. The Classical Spheromak.

2.0 SPHEROMAK REACTOR OVERVIEW

2.1 SPHEROMAK DEFINITION

Historically the spheromak is not a new concept. In fact, the spheromak plasma configuration was postulated in the mid-1950's in an astrophysical context by such noted astrophysicists as Chandrasekhar¹, but only recently has the spheromak been looked at for fusion related applications.

The main feature of the spheromak is that the plasma is a force-free configuration. This means that the plasma currents are parallel to the internal magnetic fields. More specifically, referring again to Figure 1-1, the poloidal and toroidal plasma currents produce the toroidal and poloidal magnetic fields respectively. This implies that the spheromak has a closed magnetic field geometry.

From a reactor view point we consider Figure 2-1, which is one spheromak design envisioned by the people at the Princeton Plasma Physics Laboratory. Notice that this reactor geometry has three main geometrical advantages over other toroidal devices such as tokamaks. The three advantages are 1) no toroidal field coils are required, 2) the spheromak has a natural divertor action and 3) no ohmic heating coil is needed in the central column. Thus the spheromak can be compact in size. Note, however, that external field coils are needed for the positioning of the plasma. The magnetic field strength inside the plasma for Figure 2-1 can be obtained from Figure 2-2. In Figure 2-2, part (a) is just a cross section of the plasma found in Figure 2-1, while part (b) is a plot of the magnetic field strengths of the toroidal (B_T) and poloidal (B_p) components vs. the radial distance from the axis of symmetry. It is interesting at this point to notice that for tokamaks $B_T > B_p$, while for spheromaks $B_T \sim B_p$.

Finally, comparing the tokamak plasma geometry with those in Figures 1-1 and 2-1, one realizes that a spheromak is just a low aspect ratio tokamak with no toroidal linking magnetic coils².

Figure 2-1. The PPPL Spheromak Design

(PPPL 793333)

Figure 2-2. Oblate Spheromak Configuration

2.2 THEORETICAL AND EXPERIMENTAL WORK

The work being done on the spheromak is still in its infancy and hence the amount of information available is limited. So far though, in the experimental research area, there are two recent verifications of spheromak plasma formation.

The first well known experiment was done by Professor Goldenbaum at the University of Maryland. Goldenbaum used a Z-theta pinch apparatus that created a spheromak plasma that lasted about thirty microseconds³. The second verification came from the Princeton Plasma Physics Laboratory using a newly developed quasistatic set-up. This method produced a spheromak plasma that lasted approximately twenty microseconds⁴.

In the theoretical area there has been a lot of study dealing with the MHD stability of spheromaks. Papers by Rosenbluth and Bussac⁵, Okabayashi and Todd⁶ are good examples of the type of stability analysis now being done. Reactor studies on the other hand are currently being published and these show some promising results^{7,8}.

2.3 QUALITATIVE MHD ANALYSIS

At this time the tilting mode seems to be the most damaging instability for spheromak plasmas. The tilting mode is characterized by a change in the orientation of the plasma's axis of symmetry. Since the stability properties of the three various geometrical shapes of a spheromak plasma (spherical, prolate, oblate) are different, we must consider each separately, as will be done below.

The oblate spheromak plasma is a configuration that is elongated perpendicular to the axis of symmetry (see Figure 2-1). This shape has the most favorable MHD properties because given sufficient oblateness and/or a close fitting conducting shell, the plasma is ideal MHD stable⁹ and hence is stable against the tilting mode. This is in contrast to the spherically shaped

spheromak plasma which is only marginally stable against the tilting mode because of complete spherical symmetry.

The prolate spheromak plasma is a body that is elongated along the axis of symmetry. This plasma geometry is unstable against the tilting mode even with a close fitting conducting shell. In fact, a resistive MHD analysis, where the plasma is treated as a resistive medium, shows that magnetic field line reconnection is possible after a given initial perturbation. The scenario mentioned above can be clarified with the help of Figure 2-3. In (a) we initially have a prolate spheromak with a vertical axis of symmetry. A perturbation sets the plasma in rotation (b) until its axis of symmetry is horizontal (c). Finally, the internal plasma magnetic field lines connect with external magnetic field lines which lead to the outside, thus creating large plasma particle losses.

2.4 START-UP AND PLASMA FORMATION

The start-up of a spheromak plasma configuration seems to be the biggest problem from an experimental standpoint because of the complicated current geometries that must be generated. Even though this is true there are, so far, three experimentally proven methods of forming a spheromak plasma. They are the coaxial gun method, the reverse Z-theta pinch, and the quasistatic method.

The coaxial gun technique is an old (1950's) method that uses the plasma's inertia for formation. Referring to Figure 2-4, notice that in (a) a current is passed from electrode A to B creating a toroidal plasma with a toroidal magnetic field component H . The resulting magnetic pressure then forces the plasma down the tube, thus reaching a radial magnetic field created by an end magnet. In (b) the plasma is ejected and pulls some of the magnetic field lines with it. The magnetic field lines reconnect in (c), creating a poloidal magnetic field for the toroidal plasma. Finally, with external magnetic field coils, the plasma can be positioned as desired.

The reverse Z-theta pinch is shown in Figure 2-5. This was the technique used by Dr. Goldenbaum at the University of Maryland as mentioned previously in

Figure 2-3. A Prolate Compact Torus

Figure 2-4. Generation of a Spheromak Plasma by the Coaxial Gun Technique.
(PPPL 806446)

PREIONIZATION

IMPLOSION

RECONNECTION

EQUILIBRIUM

Figure 2-5. Reverse Z-Theta Pinch

Section 2.2. Here a cylindrical plasma is created in a mirror-like magnetic field. Then a sudden implosion of the plasma is initiated by reversing the current in the bias magnetic field coils. Next, magnetic field line reconnection takes place, thus forming a plasma with toroidal and poloidal magnetic field components. Finally, a spheromak equilibrium is established.

The two methods described above not only require high currents for their operation, but they are dynamic, i.e., the time scale of plasma formation is on the order of a few Alfvén wave transit times. To reduce the power requirements of the system, the plasma start-up should proceed over a larger time scale. This means that a plasma formation scheme is needed that doesn't rely on the plasma's inertial properties.

The quasi-static method developed by the Princeton Plasma Physics Laboratory is shown in Figure 2-6 and uses a slower technique when compared to the coaxial gun and reverse Z-theta pinch. Part (a) of Figure 2-6 shows a toroidal iron core, in an external vertical field, with an internal coil that produces the poloidal magnetic field, while a coil wrapped around the core produces a toroidal magnetic field. Pulsing the current in the toroidal field coil creates an electric field E_p which breaks down the surrounding neutral gas and hence creates a plasma. The poloidal field coil current is then reversed in (b) creating line reconnection and finally creating a spheromak plasma (c) held in position by the vertical magnetic field.

2.5 REACTOR GEOMETRY

From a reactor engineering standpoint any spheromak reactor design must minimize radiation damage to equipment, and at the same time the reactor system must maximize energy efficiencies. Fortunately, because the spheromak plasmoid can be positioned using external coils, the two design criteria mentioned above can be realized.

For example, consider the reactor geometry given by Figure 2-7 (see reference (7)). At step (1) we create a low density toroidal shaped spheromak plasma

Figure 2-6. Quasi-static Technique for Spheromak Formation. (PPL 806488)

using the quasi-static technique mentioned previously in Section 2.4. In step (2) the plasmoid is compressed and then positioned in a cylindrical chamber so that the plasma can be fueled, possibly with deuterium and/or tritium, and heated by external means. Notice that a shutter is used to isolate the delicate start-up apparatus from the radiation environment of the reactor chamber. Then in step (3), by changing the axial magnetic field in the chamber the plasmoid is moved to the expansion chamber as fuel is burned. This relocation of the plasma more evenly distributes the neutron radiation energy along the chamber walls and thus reduces thermal stresses in components. In the last step, (4), the external magnetic field pressure is reduced and the plasma expands, thereby giving up stored internal magnetic energy. This stored energy can then be collected by surrounding coils at high efficiency. Also notice that charged particle collection in the expansion chamber is possible, thus leading to an even higher conversion efficiency.

Figure 2-7. A Spheromak "Chimney" Reactor

3.0 REACTOR COMPARISONS

3.1 SPHEROMAK AND TOKAMAK

The essential differences between a spheromak and a tokamak are: the spheromak needs no toroidal field coils, the spheromak has a natural divertor action, the spheromak in an oblate shape is ideal MHD stable, and the spheromak has a higher engineering beta as will be discussed later on in this report.

Comparing magnetic field strength as a function of distance from the axis of symmetry, consider Figure 3-1. Notice that for the most part $B_{Tor} \gg B_{Pol}$ for a tokamak while $B_{Tor} \sim B_{Pol}$ for spheromaks.

The main similarity between spheromaks and tokamaks is that both plasmas have closed magnetic field geometries. Nested magnetic surfaces are formed. This is a great advantage because a tokamak analysis can be applied to spheromaks even though the amount of information on spheromaks is small.

3.2 SPHEROMAKS AND FIELD REVERSED MIRRORS

A field reversed mirror is shown in Figure 3-2. Compare this to Figures 1-1 and 2-1 and notice that an azimuthal plasma current usually created by neutral beams closes the field lines. Next, using Figure 3-1, compare the magnetic field strengths of the spheromak and field-reversed mirror. Notice that $B_{Tor} \sim 0$ for a field reversed mirror. Thus the main difference between the field reversed mirror and the spheromak is that the spheromak contains a toroidal magnetic field component and the field-reversed mirror does not. A more pictorial view of this difference is given by Figure 3-3. Note that if (A) had a toroidal component such that $B_{Tor} \sim B_{Pol}$ we would have a prolate spheromak. Tables 3.1 and 3.2 sum up the various compact toruses in a simple way. In Table 3.1 notice that a description of a compact torus relies on the poloidal Larmor radius (ρ_{Pol})

TOKAMAK

REVERSE
FIELD
MIRROR

SPHEROMAK
(OBLATE)

Figure 3-1. Magnetic Field Strengths in Various Reactors

Figure 3-2. Field Reversed Mirror

- (a) FIELD-REVERSED MIRROR
(b) OBLATE SPHEROMAK

Figure 3-3. Comparison of the Field Reversed Mirror and Spheromak Configurations

TABLE 3.1
COMPARISON OF COMPACT TORUSES

	$\rho_{POL} < a_{PLASMA}$	$\rho_{POL} > a_{PLASMA}$
$B_{POL} \gg B_{TOR}$	Field-Reversed Mirror or Θ -Pinch	E-Layers P-Layers
$B_{POL} \sim B_{TOR}$	Spheromak or "Null-Field Z-Pinch"	REB-Injected Toruses

(See Reference (9))

relative to the plasma radius (a_{plasma}) and that $\rho_{\text{pol}} < a_{\text{plasma}}$ for a field reversed mirror and spheromak.

3.3 REACTOR PARAMETERS

Table 3.2 given on the next few pages gives a rough idea on how a spheromak (beam driven), field reversed mirror, and compact tokamak compare in various parameters. A compact tokamak was chosen for this table because the machine more closely resembles the other two devices in terms of size. This can be seen by referring to the aspect ratio in Table 3.2.

A word of caution must be made about Table 3.2. Since the information listed in the table are essentially from "paper" studies, the data is subject to large errors and revisions as refinements are made.

TABLE 3.2
REACTOR PARAMETER COMPARISONS

REACTOR TYPE	MAGNETIC FIELD (kG)	TEMPERATURE (keV)	DENSITY (cm ⁻³)	OPERATION	$n\tau$ sec/cm ³
SPHEROMAK (Ref. 2)	$B_{TOR} = 120$ $B_{POL} \sim 120$	20	2×10^{14}	Steady State (Beam Driven)	$\sim 10^{14}$
FIELD REVERSED MIRROR (Ref. 10)	$B_0 \sim 41$ (Initial Vacuum Field)	~ 200 (Injection Temperature)	$\sim 1.3 \times 10^{15}$ (Peak)	Steady State (Beam Driven)	$\sim 10^{14} - 10^{15}$ (Estimated)
COMPACT TOKAMAK (Ref. 11)	$B_{TOR} \sim 60$ $B_{POL} \ll B_{TOR}$	20 (Average)	10^{14}	Pulsed	10^{14}
REACTOR TYPE	BETA (β)(%)	CURRENT (MA)	FUSION POWER OUTPUT (MW)	ASPECT RATIO	Q VALUE
SPHEROMAK (Ref. 2)	2 (Engineering Beta)	6	40	~ 1 (Low)	2.5 (or Larger)
FIELD REVERSED MIRROR (Ref. 10)	~ 2.8 (Peak)	22×10^{-6} (Injection Current)	22	~ 2	~ 5
COMPACT TOKAMAK (Ref. 11)	7.5	7.7	1233	3.5	∞ (Ignited)

4.0 SPHEROMAK REACTOR MODELING

4.1 BASIC ANALYSIS

A detailed particle and energy balance of a spheromak plasma is given in Figures 4-1 and 4-2 respectively. In Figure 4-1, the left side represents particle source rates while the right hand side denotes particle loss rates, thus S implies a source rate while L means a loss rate. A similar logic holds for Figure 4-2¹², but this time the left hand side denotes input power (Q_S) and the right hand side means power loss (Q_L). Figure 4-2 also contains the power exchange between species, such as Q_{fe} , $Q_{f\alpha}$, $Q_{\alpha e}$. Also note that in both figures the neutrons created by deuterium, tritium fusion are ignored. The reason for this is since the neutrons are neutral and relatively high in energy (~ 14 MeV), they leave the plasma quickly with practically no interaction.

To describe mathematically the balance of energy and particles we must use the transport equations given below:

PARTICLE BALANCE:

$$\frac{\partial n_K}{\partial t} + \underline{v} \cdot (n_K \underline{v}_K) = S_K \quad (\text{Units of } (\text{Volume})^{-1}(\text{Time})^{-1}) \quad (4.1.1)$$

ENERGY BALANCE:

$$\frac{3}{2} \frac{\partial}{\partial t} (n_K T_K) + \underline{v} \cdot \left(\frac{3}{2} n_K T_K \underline{v}_K \right) + n_K T_K \underline{v} \cdot \underline{v}_K + \underline{v} \cdot \underline{q}_K = Q_K \quad (4.1.2)$$

(Units of (Energy)(Volume)⁻¹(Time)⁻¹)

Where: $\underline{r}$ = position vector

t = time

K = Kth species

PARTICLE
INJECTION

PARTICLE
LOSS

Figure 4-1. Plasma Particle Balance

Figure 4-2. Plasma Power Balance

$n_K = n_K(\underline{r}, t)$ = density of Kth species

$T_K = T_K(\underline{r}, t)$ = temperature of Kth species

$V_K = V_K(\underline{r}, t)$ = fluid velocity of Kth species

$q_K = q_K(\underline{r}, t)$ = heat conduction vector of Kth species

$S_K = S_K(\underline{r}, t)$ = particle source term of Kth species

$Q_K = Q_K(\underline{r}, t)$ = power source term to Kth species

and the assumption of pressure isotropy has been made. Here S_K , and Q_K are the algebraic sums of the terms appearing in Figure 4-1 and 4-2 respectively.

Next consider the beam driven spheromak given back in Figure 2-1. Assuming that the prolate torus can be modeled by a cylindrical torus, the procedure given by Borra β^{13} (see also Section 3.1 of this report) is applicable. There Borra β applies the transport equations to a tokamak to obtain a zero dimensional model.

A zero dimensional, time independent, mathematical analysis was performed for a beam driven spheromak using equations (4.1.1), (4.1.2) and reference (13) as a guide. Since the resulting equations are cumbersome the detailed analysis is left for Appendix A. However, it will be sufficient to say that in the analysis it was assumed that the plasma contained the following species, deuterium, tritium, alpha particles, electrons, and an impurity. Also the following profiles for the density n , and temperature T were assumed for each specie:

$$n(r) = n_0 [1 - (r/a)^2]^x \quad (4.1.3)$$

$$T(r) = n_0 [1 - (r/a)^2]^y \quad (4.1.4)$$

where

a = mean minor radius of the plasma

x,y are numbers

Such an assumption on profiles is needed since they are unknown, and notice that the parabolic nature of the profiles resembles those of tokamaks. The resulting analysis produced six simultaneous nonlinear algebraic equations: two particle balance equations, two energy balance equations, and two conservation equations. These six simultaneous equations must then be solved numerically.

4.2 INTRODUCTION TO SPHERO

Sphero is a zero-dimensional, time independent transport code that is designed to numerically solve the six equations mentioned in Section 4.1. Sphero uses the subroutine ZSYSTEM from the International Mathematical and Statistical Library (I.M.S.L.) to do the iterative computations for the set of simultaneous equations.

Sphero also has particle confinement times and the profile shaping exponents (see Equations (4.1.3) and (4.1.4)) as inputs. This is an advantage since these values are unknown experimentally for spheromaks.

The main disadvantage with Sphero is that the code is very sensitive to the initial guess values that deal with the temperature and densities of the species. This was confirmed by using a set of simultaneous equations whose solutions were known, and the subroutine Z system. The sensitivity problem is obviously due to the non-linear nature of the transport equations.

The output structure of sphero lists information that is relevant to a reactor study, such as cyclotron, bremsstrahlung power losses and reactor power output. The code Sphero is compiled in fortran on the Magnetic Fusion Energy's (M.F.E.) C.D.C. 7600 computer at Lawrence Livermore Laboratory. More specific information, on input and output formatting is left for Appendix B of this report.

4.3 TYPICAL PARAMETRIC STUDY

Because of time limitations, a detailed study of a spheromak reactor using sphero was not performed. However, a representative study of a spheromak is presented in Table 4.1 from reference (8). Here Miley et al., assume a geometry similar to that previously shown in Figure 2-7. The most important thing to notice in this table is that the reactor engineering betas are extremely high (41%), this is in contrast to tokamaks whose engineering betas are on the order of a few percent. This is shown more explicitly in Figure 4-3, taken from reference (9), notice that a complete spheromak is shown while only half of a tokamak is presented. In this figure $\langle \beta^* \rangle_{\text{coil}}$ is the engineering beta mentioned above and is defined as:

$$\langle \beta^* \rangle_{\text{coil}} = 8\pi \langle p^2 \rangle^{\frac{1}{2}} / B_{\text{coil}}^2,$$

where

$\langle p^2 \rangle$ = average of the squared plasma kinetic pressure

B_{coil} = magnetic field at inner edge of external coil.

Thus, as can be seen by the definition, $\langle \beta^* \rangle_{\text{coil}}$ determines how efficiently the magnetic field is utilized in containing the plasma. This implies that the spheromak requires less external magnetic energy than a tokamak for the same plasma kinetic pressure, (because the spheromak produces its own poloidal and toroidal fields) hence power consumption can be reduced in a spheromak.

SPHEROMAK

$$\langle \beta^* \rangle_0 = 5 - 10 \%$$

$$\langle \beta^* \rangle_{Coil} = 30 - 60 \%$$

TOKAMAK

$$\langle \beta^* \rangle_0 = 5 - 10 \%$$

$$\langle \beta^* \rangle_{Coil} = 1.5 - 3 \%$$

Figure 4-3. Comparison of Beta Values in Spheromaks and Tokamaks. (PPL 806483)

TABLE 4.1
 REPRESENTATIVE PARAMETERS FOR A LINEAR
 SPHEROMAK FUSION REACTOR
 (The Spheromak Fusion Reactor, Miley, et al.)

<u>REACTOR PARAMETERS</u>	<u>100 MWe</u>	<u>1000 MWe</u>
Plasma Minor Radius (m)	0.45	0.75
Plasma Major Radius (m)	0.90	1.5
Engineering Beta (%)	41	41
On-Axis Beta (%)	5.8	5.8
On-Axis Temperature (keV)	15	15
Initial Solenoidal Field (T)	6.0	7.0
Magnetic Field Energy		
Investment/Plasmoid (MJ)	125	980
Energy Storage Time (s)	10	10
Plasmoid Burn Time (s)	25	30
First Wall Radius (m)	2.5	4.5
Maximum Thermal First Wall		
Loading (MW/m ²)	1.5	2.6
Fusion Yield/Plasmoid (MJ)	2.7×10^3	2.9×10^4
Q _{Plasma}	21	29

5.0 CONCLUSION

5.1 REACTOR POTENTIAL

The future reactor potential of the spheromak was presented by looking at the various aspects of reactor engineering, such as MHD stability and reactor geometry. It was found that the spheromak being a force-free configuration, is a good candidate for further detailed study because the spheromak requires no toroidal field coils, no ohmic heating coils, has a natural divertor action, is ideal MHD stable with an oblate shape, and can obtain high engineering betas if initial studies are correct. In terms of reactor geometry, the best choice is a "chimney" type reactor. In this system the spheromak plasmoid is formed in one chamber, then fueled and burned while moving along a linear chamber. Stored internal energy is collected in an expansion chamber.

5.2 TRANSPORT CODE SPHERO

The zero-dimensional, time independent transport code Sphero which modeled a beam driven spheromak reactor was discussed. The main advantage of Sphero is that confinement times and profile shaping exponents are used as input since they are unknown at the present time. The main disadvantage of Sphero is that the code is very sensitive to the initial guess used because of the non-linear nature of the problem. Thus it is planned to include an algorithm that will produce desirable initial conditions.

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APPENDIX A
DERIVATION OF THE TRANSPORT EQUATIONS

In this appendix the zero-dimensional, time independent transport equations are derived. The proceeding derivation will closely follow that of Borra β (Reference 13). Begin with the equations of transport: (equations (4.1.1) and (4.1.2))

$$\frac{\partial n_K(\underline{r}, t)}{\partial t} + \underline{\nabla} \cdot (n_K(\underline{r}, t) \underline{v}_K(\underline{r}, t)) = S_K(\underline{r}, t) \quad (A.1)$$

$$\begin{aligned} \frac{\partial}{\partial t} \left[\frac{3}{2} n_K(\underline{r}, t) T_K(\underline{r}, t) \right] + \underline{\nabla} \cdot \left[\frac{3}{2} n_K(\underline{r}, t) T_K(\underline{r}, t) \underline{v}_K(\underline{r}, t) \right] \\ + n_K(\underline{r}, t) T_K(\underline{r}, t) \underline{\nabla} \cdot \underline{v}_K(\underline{r}, t) + \underline{\nabla} \cdot \underline{q}_K(\underline{r}, t) = Q_K(\underline{r}, t) \end{aligned} \quad (A.2)$$

where the above variables have already been defined in Section 4.1. Next, if we assume that the variables depend only on the magnitude of the minor radius, i.e., $n_K(\underline{r}, t) = n_K(r, t)$, $T_K(\underline{r}, t) = T_K(r, t)$, $\underline{v}_K(\underline{r}, t) = \underline{v}_K(r, t)$, $S_K(\underline{r}, t) = S_K(r, t)$, $\underline{q}_K(\underline{r}, t) = \underline{q}_K(r, t)$, $Q_K(\underline{r}, t) = Q_K(r, t)$, then diffusion and convection occur only in the radial direction, i.e., $\underline{q}_K(\underline{r}, t) = q_r^K(r, t)$, $\underline{v}_K = v_r^K(r, t)$ where the subscripts denote the radial component. Thus in cylindrical geometry equations (A.1) and (A.2) become:

$$\frac{\partial n_K}{\partial t} + \frac{1}{r} \frac{\partial}{\partial r} (r n_K v_r^K) = S_K \quad (A.3)$$

$$\frac{\partial}{\partial t} \left(\frac{3}{2} n_K T_K \right) + \frac{1}{r} \frac{\partial}{\partial r} \left(\frac{3}{2} r n_K T_K v_r^K \right) + n T \frac{1}{r} \frac{\partial}{\partial r} (r v_r^K) + \frac{1}{r} \frac{\partial}{\partial r} (r q_r^K) = Q_K \quad (A.4)$$

Notice we can rewrite equation (A.4) by differentiating as:

$$\frac{\partial}{\partial t} \left(\frac{3}{2} n_K T_K \right) + \frac{3}{2} \frac{1}{r} \left[n_K T_K \frac{\partial}{\partial r} (r v_r^K) + r v_r^K \frac{\partial}{\partial r} (n_K T_K) \right] \\ + n_K T_K \frac{1}{r} \frac{\partial}{\partial r} (r v_r^K) + \frac{1}{r} \frac{\partial}{\partial r} (r q_r^K) = Q_K$$

summing and subtracting terms:

$$\frac{\partial}{\partial t} \left(\frac{3}{2} n_K T_K \right) + \frac{5}{2} \frac{n_K T_K}{r} \frac{\partial}{\partial r} (r v_r^K) + \frac{5}{2} v_r^K \frac{\partial}{\partial r} (n_K T_K) \\ - v_r^K \frac{\partial}{\partial r} (n_K T_K) + \frac{1}{r} \frac{\partial}{\partial r} (r q_r^K) = Q_K$$

thus (A.4) becomes:

$$\frac{\partial}{\partial t} \left(\frac{3}{2} n_K T_K \right) + \frac{1}{r} \frac{\partial}{\partial r} \left(\frac{5}{2} n_K T_K r v_r^K \right) - v_r^K \frac{\partial}{\partial r} (n_K T_K) + \frac{1}{r} \frac{\partial}{\partial r} (r q_r^K) = Q_K \quad (\text{A.4'})$$

Next we assume that we have deuterium, tritium, electrons, alpha particles, and an impurity and that the ions (deuterium, tritium, alpha particles, impurity) have the same temperature. Thus adding the transport equations together produces (using A.3 and A.4'):

PARTICLE BALANCE:

Fuel Ions (D + T):

$$\frac{\partial n_f}{\partial t} + \frac{1}{r} \frac{\partial}{\partial r} (r n_f v_r^f) = S_f \quad (\text{A.5})$$

$n_f = n_d + n_t$ = density deuterium + density tritium.
= fuel density

Alpha Particles:

$$\frac{\partial n_\alpha}{\partial t} + \frac{1}{r} \frac{\partial}{\partial r} (r n_\alpha v_r^\alpha) = S_\alpha \quad (\text{A.6})$$

where

α denotes alpha particle.

Impurity Ion:

$$\frac{\partial n_{\sigma}}{\partial t} + \frac{1}{r} \frac{\partial}{\partial r} (r n_{\sigma} v_r^{\sigma}) = S_{\sigma} \quad (\text{A.7})$$

σ denotes the impurity. It will be assumed that the impurity has a charge Z_{σ} .

Electrons:

$$n_e = n_f + 2n_{\alpha} + Z_{\sigma} n_{\sigma} \quad (\text{A.8})$$

subscript e denotes electrons. Notice that a neutrality condition is assumed.

It is also found that the total ion density must be conserved hence:

Ions:

$$n_i = n_f + n_{\alpha} + n_{\sigma} \quad (\text{A.9})$$

where the subscript i denotes total ions ($D + T + \alpha + \sigma$)

ENERGY BALANCE:

Ions:

$$\begin{aligned} & \frac{\partial}{\partial t} \left(\frac{3}{2} n_i T_i \right) + \frac{1}{r} \frac{\partial}{\partial r} \left(r \left[\frac{5}{2} T_i (n_f v_r^f + n_{\alpha} v_r^{\alpha} + n_{\sigma} v_r^{\sigma}) \right. \right. \\ & \left. \left. + (q_r^f + q_r^{\alpha} + q_r^{\sigma}) \right] \right) - v_r^f \frac{\partial}{\partial r} (n_f T_i) - v_r^{\alpha} \frac{\partial}{\partial r} (n_{\alpha} T_i) \\ & - v_r^{\sigma} \frac{\partial}{\partial r} (n_{\sigma} T_i) = Q_i, \quad n_i = n_f + n_{\alpha} + n_{\sigma} \end{aligned} \quad (\text{A.10})$$

Electrons:

$$\begin{aligned} \frac{\partial}{\partial t} \left(\frac{3}{2} n_e T_e \right) + \frac{1}{r} \frac{\partial}{\partial r} (r \left[\frac{5}{2} T_e n_e V_r^e + q_r^e \right]) \\ - V_r^e \frac{\partial}{\partial r} (n_e T_e) = Q_e \end{aligned} \quad (\text{A.11})$$

Now defining the particle flux as $n_x V_r^x = \Gamma_x$, where x is a given species and assuming ambipolar diffusion, i.e., $\Gamma_f + \Gamma_\alpha + \Gamma_\sigma \sim \Gamma_e$ we rewrite our balance equations as:

PARTICLE BALANCE:

fuel:

$$\frac{\partial n_f}{\partial t} + \frac{1}{r} \frac{\partial}{\partial r} (r \Gamma_f) = S_f \quad (\text{A.5'})$$

Alpha Particles:

$$\frac{\partial n_\alpha}{\partial t} + \frac{1}{r} \frac{\partial}{\partial r} (r \Gamma_\alpha) = S_\alpha \quad (\text{A.6'})$$

Impurity Ion:

$$\frac{\partial n_\sigma}{\partial t} + \frac{1}{r} \frac{\partial}{\partial r} (r \Gamma_\sigma) = S_\sigma \quad (\text{A.7'})$$

Electrons:

$$n_e = n_f + 2n_\alpha + Z_\sigma n_\sigma$$

Ions:

$$n_i = n_f + n_\alpha + n_\sigma$$

ENERGY BALANCE:

Ions:

$$\begin{aligned} & \frac{\partial}{\partial t} \left(\frac{3}{2} n_i T_i \right) + \frac{1}{r} \frac{\partial}{\partial r} \left(r \left[\frac{5}{2} T_i \Gamma_e + (q_r^f + q_r^\alpha + q_r^\sigma) \right] \right) \\ & - \frac{\Gamma_f}{n_f} \frac{\partial}{\partial r} (n_f T_i) - \frac{\Gamma_\alpha}{n_\alpha} \frac{\partial}{\partial r} (n_\alpha T_i) - \frac{\Gamma_\sigma}{n_\sigma} \frac{\partial}{\partial r} (n_\sigma T_i) = Q_i \end{aligned} \quad (\text{A.10'})$$

Electrons:

$$\begin{aligned} & \frac{\partial}{\partial t} \left(\frac{3}{2} n_e T_e \right) + \frac{1}{r} \frac{\partial}{\partial r} \left(r \left[\frac{5}{2} T_e \Gamma_e + q_r^e \right] \right) \\ & - \frac{\Gamma_e}{n_e} \frac{\partial}{\partial r} (n_e T_e) = Q_e \end{aligned} \quad (\text{A.11'})$$

Next we define our volume average as:

$$\bar{f} = \bar{f}(r, t) = \frac{1}{V} \int f(r, t) dV$$

since $V = 2\pi a^2 R$ where a = plasma minor radius, R = plasma major radius we find:

$$\bar{f} = \frac{2}{a^2} \int_0^a f(r, t) r dr$$

Using the definition of volume average and assuming the following profiles:

$$n_f = n_{f0}(t) \left(1 - \left(\frac{r}{a} \right)^2 \right)^{x_f} = \text{fuel density profile}$$

$$n_e = n_{e0}(t) \left(1 - \left(\frac{r}{a} \right)^2 \right)^{x_e} = \text{electron density profile}$$

$$n_{\alpha} = n_{\alpha_0}(t) \left(1 - \left(\frac{r}{a}\right)^2\right)^{x_{\alpha}} = \text{alpha particle density profile}$$

$$n_{\sigma} = n_{\sigma_0}(t) \left(1 - \left(\frac{r}{a}\right)^2\right)^{x_{\sigma}} = \text{impurity density profile}$$

$$n_i = n_{i_0}(t) \left(1 - \left(\frac{r}{a}\right)^2\right)^{x_i} = \text{ion density profile}$$

$$T_i = T_{i_0}(t) \left(1 - \left(\frac{r}{a}\right)^2\right)^{y_i} = \text{ion temperature profile}$$

$$T_e = T_{e_0}(t) \left(1 - \left(\frac{r}{a}\right)^2\right)^{y_e} = \text{electron temperature profile}$$

and using these relationships:

$$\begin{aligned} \overline{\frac{1}{r} \frac{\partial}{\partial r} [f(r,t)]} &= \frac{2}{a^2} \int_0^a \left(\frac{1}{r} \frac{\partial}{\partial r} [f(r,t)]\right) r dr \\ &= \frac{2}{a^2} [f(r,t)]_{r=a} \text{ for } f(0,t) = 0 \end{aligned}$$

$$\bar{n} = \overline{n_0(t) \left[1 - \left(\frac{r}{a}\right)^2\right]^x} = \frac{n_0(t)}{(1+x)}$$

$$\bar{T} = \overline{T_0(t) \left[1 - \left(\frac{r}{a}\right)^2\right]^y} = \frac{T_0(t)}{(1+y)}$$

$$\begin{aligned} \overline{nT} &= \overline{n_0(t) T_0(t) \left[1 - \left(\frac{r}{a}\right)^2\right]^x \left[1 - \left(\frac{r}{a}\right)^2\right]^y} \\ &= \frac{n_0(t) T_0(t)}{(x+y+1)} \end{aligned}$$

$$\overline{\frac{\Gamma}{n} \frac{\partial}{\partial r} (nT)} = \frac{2}{a^2} \int_0^a dr \, r \left[\frac{\Gamma}{n} \frac{\partial}{\partial r} (nT) \right]$$

assuming $r\Gamma \sim \text{constant}$

$$\therefore \overline{\frac{\Gamma}{n} \frac{\partial}{\partial r} (nT)} \sim \frac{2}{a^2} [r\Gamma]_{r=a} \int_0^a dr \left(\frac{\partial T}{\partial r} + \frac{T}{n} \frac{\partial n}{\partial r} \right)$$

then using the following facts:

$$\frac{\partial T}{\partial r} = \frac{-2T_0(t)y}{a^2} r \left(a - \left(\frac{r}{a} \right)^2 \right) (y-1)$$

$$\frac{\partial n}{\partial r} = \frac{-2n_0(t)x}{a^2} r \left(1 - \left(\frac{r}{a} \right)^2 \right) (x-1)$$

thus we find:

$$\overline{\frac{\Gamma}{n} \frac{\partial}{\partial r} (nT)} \sim \frac{-2}{a^2} [r\Gamma]_{r=a} T_0(t) \left[1 + \frac{x}{y} \right]$$

we finally obtain the volume averaged equations of (A.5')-(A.7'), (A.8), (A.9), (A.10'), (A.11'):

PARTICLE BALANCE:

fuel:

$$\frac{\partial \bar{n}_f}{\partial t} + \frac{2}{a^2} [r\Gamma_f]_{r=a} = \bar{S}_f \quad (\text{A.12})$$

alpha particle:

$$\frac{\partial \bar{n}_\alpha}{\partial t} + \frac{2}{a^2} [r\Gamma_\alpha]_{r=a} = \bar{S}_\alpha \quad (\text{A.13})$$

Impurity ion:

$$\frac{\partial \bar{n}_\sigma}{\partial t} + \frac{2}{a^2} [r\Gamma_\sigma]_{r=a} = \bar{S}_\sigma \quad (\text{A.14})$$

Electrons:

$$\bar{n}_e = \bar{n}_f + 2\bar{n}_\alpha + z_\sigma \bar{n}_\sigma \quad (\text{A.15})$$

Ions:

$$\bar{n}_i = \bar{n}_f + \bar{n}_\alpha + \bar{n}_\sigma \quad (\text{A.16})$$

ENERGY BALANCE:

Ions:

$$\begin{aligned} & \frac{(x_i + 1)(y_i + 1)}{(x_i + y_i + 1)} \frac{\partial}{\partial t} \left(\frac{3}{2} \bar{n}_i \bar{T}_i \right) + \frac{2}{a^2} \left[r \left(\frac{5}{2} \bar{T}_i \Gamma_e + (q_r^f + q_r^\alpha + q_r^\sigma) \right) \right]_{r=a} \quad (\text{A.17}) \\ & + \bar{T}_i (1 + y_i) \left[1 + \frac{x_f}{y_i} \right] \left(\frac{2}{a^2} [r\Gamma_f]_{r=a} \right) \\ & + \bar{T}_i (1 + y_i) \left[1 + \frac{x_\alpha}{y_i} \right] \left(\frac{2}{a^2} [r\Gamma_\alpha]_{r=a} \right) \\ & + \bar{T}_i (1 + y_i) \left[1 + \frac{x_\sigma}{y_i} \right] \left(\frac{2}{a^2} [r\Gamma_\sigma]_{r=a} \right) = \bar{Q}_i \end{aligned}$$

electron:

$$\frac{(x_e + 1)(y_e + 1)}{(x_e + y_e + 1)} \frac{\partial}{\partial t} \left(\frac{3}{2} \bar{n}_e \bar{T}_e \right) + \frac{2}{a^2} \left[r \left(\frac{5}{2} T_e \Gamma_e + q_r^e \right) \right]_{r=a} \quad (A.18)$$

$$+ \bar{T}_e (1 + y_e) \left[1 + \frac{x_e}{y_e} \right] \left(\frac{2}{a^2} [r \Gamma_e]_{r=a} \right) = \bar{Q}_e$$

We will next assume that Γ_x and q_r^x of a species x follows the laws:

$$\Gamma_x = -D_x(r) \frac{\partial n_x}{\partial r}, \quad q_r^x = -n_x \chi_x(r) \frac{\partial T_x}{\partial r}$$

where D_x , χ_x are the particle diffusion and heat conduction coefficients respectively. Also because of the way our profiles have been chosen they are zero at the boundary, $r = a$. Hence we will chose our variables to be evaluated at $r = \lambda a$ to eliminate any mathematical difficulty. Note that λ is a constant, usually chosen to be around 0.70. After much algebra and setting $a \rightarrow \lambda a$ produces the following equations:

PARTICLE BALANCE:

Fuel:

$$\frac{\partial \bar{n}_f}{\partial t} + \bar{n}_f \left\{ \frac{4\lambda^2}{a^2} x_f (x_f + 1) D_f(\lambda a) (1 - \lambda^2)^{(x_f - 1)} \right\} = \bar{S}_f \quad (A.19)$$

Alpha Particle:

$$\frac{\partial \bar{n}_\alpha}{\partial t} + \bar{n}_\alpha \left\{ \frac{4\lambda^2}{a^2} x_\alpha (x_\alpha + 1) D_\alpha(\lambda a) (1 - \lambda^2)^{(x_\alpha - 1)} \right\} = \bar{S}_\alpha \quad (A.20)$$

Impurity

$$\frac{\partial \bar{n}_\sigma}{\partial t} + \bar{n}_\sigma \left\{ \frac{4\lambda^2}{a^2} x_\sigma (x_\sigma + 1) D_\sigma(\lambda a) (1 - \lambda^2)^{(x_\sigma - 1)} \right\} = \xi_\sigma \quad (\text{A.21})$$

Electron:

$$\bar{n}_e = \bar{n}_f + 2\bar{n}_\alpha + z_\sigma \bar{n}_\sigma \quad (\text{A.22})$$

Ions:

$$\bar{n}_i = \bar{n}_f + n_\alpha + n_\sigma \quad (\text{A.23})$$

ENERGY BALANCE:

Ions:

$$\frac{(x_i + 1)(y_i + 1)}{(x_i + y_i + 1)} \frac{\partial}{\partial t} \left(\frac{3}{2} \bar{n}_i T_i \right) + \bar{n}_i T_i F + G = \bar{Q}_i \quad (\text{A.24})$$

where:

$$F = \frac{4\lambda^2}{a^2} (1 - \lambda^2)^{(y_i - 1)} \left[\frac{5}{2} \left(\frac{\bar{n}_e}{\bar{n}_i} \right) \frac{(x_e + 1)}{(x_i + 1)} x_e D_e(\lambda a) (1 - \lambda^2)^{x_e} \right. \\ \left. + \frac{y_i}{(x_i + 1)} \left(\begin{aligned} & \left(\frac{\bar{n}_f}{\bar{n}_i} \right) (x_f + 1) x_f (\lambda a) (1 - \lambda^2)^{x_f} \\ & + \left(\frac{\bar{n}_\alpha}{\bar{n}_i} \right) (x_\alpha + 1) x_\alpha (\lambda a) (1 - \lambda^2)^{x_\alpha} \\ & + \left(\frac{\bar{n}_\sigma}{\bar{n}_i} \right) (x_\sigma + 1) x_\sigma (\lambda a) (1 - \lambda^2)^{x_\sigma} \end{aligned} \right) \right]$$

$$G = \frac{4\lambda^2}{a^2(x_i + 1)} \left(\begin{aligned} & \left[\left(\frac{\bar{n}_f}{\bar{n}_i} \right) x_f (x_f + 1) D_f(\lambda a) (1 - \lambda^2)^{(x_f - 1)} \right] \left[1 + \frac{x_f}{y_i} \right] \\ & + \left[\left(\frac{\bar{n}_\alpha}{\bar{n}_i} \right) x_\alpha (x_\alpha + 1) D_\alpha(\lambda a) (1 - \lambda^2)^{(x_\alpha - 1)} \right] \left[1 + \frac{x_f}{y_i} \right] \\ & + \left[\left(\frac{\bar{n}_\sigma}{\bar{n}_i} \right) x_\sigma (x_\sigma + 1) D_\sigma(\lambda a) (1 - \lambda^2)^{(x_\sigma - 1)} \right] \left[1 + \frac{x_\sigma}{y_i} \right] \end{aligned} \right)$$

Electrons:

$$\frac{(x_e + 1)(y_e + 1)}{(x_e + y_e + 1)} \frac{\partial}{\partial t} \left(\frac{3}{2} \bar{n}_{ee} \right) + \bar{n}_{ee} \bar{T}_{ee} (H) (x_e + 1)(y_e + 1) = \bar{Q}_e \quad (A.25)$$

where:

$$H_e = \frac{4\lambda^2}{a^2} \left(\begin{aligned} & (1 - \lambda^2)^{(x_e + y_e + 1)} \left[\frac{5}{2} D_e(\lambda a) x_e + y_e x_e(\lambda a) \right] \\ & + [x_e D_e(\lambda a) (1 - \lambda^2)^{(x_e - 1)}] \left(1 + \frac{x_e}{y_e} \right) \end{aligned} \right)$$

Now the source terms represented by $\bar{S}_f$, $\bar{S}_\alpha$, $\bar{S}_\sigma$, $\bar{Q}_i$, $\bar{Q}_e$ will be discussed.

Since the derivations are rather involved the results will be presented only. Here most of the results will rely on reference (13), otherwise a new reference will be cited.

AVERAGE FUEL PARTICLE SOURCE RATE:

$$\bar{S}_f (\text{cm}^{-3} \text{sec}^{-1}) = \bar{S}_f^I + \bar{S}_f^B + \bar{S}_f^F \quad (A.26)$$

$$\bar{S}_f^I = \begin{array}{l} \text{Average} \\ \text{Beam} \\ \text{Injection} \\ \text{of Fuel} \end{array} = 3.16 \times 10^{20} \frac{P_{inj}}{E_o a^2 R} \theta(t_H - t) \quad \begin{array}{l} \text{(Deuterons} \\ \text{make up} \\ \text{Beam)} \end{array}$$

$$\bar{S}_f^B = \begin{array}{l} \text{Average} \\ \text{Burn up of} \\ \text{Fuel Due to} \\ \text{Fusion} \end{array} = -2.45 \times 10^{-13} H(1 - H) \frac{\bar{n}_f^2}{\bar{T}_i^{2/3}} \frac{(x_f + 1)^2}{(y_e + 1)^{2/3}} S(\bar{T}_i)$$

$$\bar{S}_f^F = \begin{array}{l} \text{Average} \\ \text{Refueling} \\ \text{Rate} \end{array} = \bar{N}_f$$

where:

$$P_{inj} = \text{Total Beam Power In MW}$$

$$E_o = \text{Injection Energy in KeV}$$

$$a = \text{Minor Radius in cm}$$

$$R = \text{Major Radius in cm}$$

$$H = \frac{n_d}{n_f} = \frac{\text{deuterium density}}{\text{fuel density}}$$

H is assumed to be approximately constant.

N_F = refueling rate assumed to be uniform and constant

$$\theta(t - t) = \begin{array}{l} \text{step} \\ \text{function} \end{array} = \begin{cases} 1 & t \leq t_H \\ 0 & t > t_H \end{cases}, \quad t_H = \text{Heating Time}$$

$$S(\bar{T}_i) = 4 \sum_{n=0}^4 \left(\frac{2n+1}{10}\right)^\alpha e^{\frac{-19.94}{P} \left(\frac{2n+1}{10}\right)^Y} + 2 \sum_{n=1}^4 \left(\frac{n}{5}\right)^\alpha e^{\frac{-19.94}{P} \left(\frac{n}{5}\right)^Y} + e^{\frac{-19.94}{P}}$$

where:

$$\alpha = 2x_f - \frac{2}{3} y_i$$

$$\gamma = \frac{y_i}{3}$$

$$P = T_i^{\frac{1}{3}} (1 + y_i)^{\frac{1}{3}}$$

AVERAGE ALPHA PARTICLE SOURCE RATE:

$$\bar{S}_\alpha (\text{cm}^{-3} \text{sec}^{-1}) = \bar{S}_\alpha^B + \bar{S}_\alpha^{ZH} \quad (\text{A.27})$$

$$\bar{S}_\alpha^B = \begin{array}{l} \text{Average Alpha} \\ \text{Production} \\ \text{From Thermalized} \\ \text{Fuel Ions} \end{array} = -\bar{S}_f/2$$

$$\bar{S}_\alpha^{ZH} = \begin{array}{l} \text{Average Alpha Production from Beam Deuterons} \\ \text{Slowing Down on thermal Fuel Ions} \end{array}$$

$$\bar{S}_\alpha^{ZH} = \frac{6.84 \times 10^{20}}{Ra^2 E_o^{5/3}} P_{inj} (1 - H) \frac{\bar{n}_f \bar{T}_e}{\bar{n}_e} \frac{(x_f + 1)(y_e + 1)}{(x_e + 1)} I_\alpha^{ZH}$$

$$I_\alpha^{ZH} = \int_0^1 u^{(x_f + y_e + x_e)} du \int_0^1 dx \frac{e^{\frac{-19.94}{E_o^{1/3} x^{1/3}}} \theta(t_H - t)}{x^{1/6} (x^{3/2} + \frac{4.64(H+2)}{E_o^{3/2}}) (y_e + 1)^{3/2} T_u^{3/2} y_e^{3/2}}$$

AVERAGE IMPURITY PARTICLE SOURCE RATE:

$$\bar{S}_\sigma (\text{cm}^{-3} \text{sec}^{-1}) = \text{assumed constant referring to equation (A.21), } \bar{S}_\sigma \text{ a constant implies that } \bar{n}_\sigma \text{ is constant when } \partial \bar{n}_\sigma / \partial t = 0. \quad (\text{A.28})$$

AVERAGE ION POWER SOURCE:

$$\bar{Q}_i (\text{KeV cm}^{-3} \text{sec}^{-1}) = \bar{Q}_i^\Delta + \bar{Q}_i^N + \bar{Q}_i^{\text{ZHO}} + \bar{Q}_i^{\text{ZHF}} \quad (\text{A.29})$$

$\bar{Q}_i^\Delta$ = Average Power Exchanged Between Ions and Electrons in Thermal Background

$\bar{Q}_i^N$ = Average Heating of Ions from Alphas Produced By Thermal Background Fusions

$\bar{Q}_i^{\text{ZHO}}$ = Average Heating of Ions by Deuteron Beam Slowing Down on Background

$\bar{Q}_i^{\text{ZHF}}$ = Average Heating of Ions From Alphas Produced by Deuteron Beam Slowing Down on Background

$$\bar{Q}_i^\Delta = 3.01 \times 10^{-12} \left\{ \begin{aligned} & \frac{2+H}{6} \left[\frac{\bar{T}_e \bar{n}_e \bar{n}_f}{\bar{T}_i^{3/2}} B_1 - \frac{\bar{n}_e \bar{n}_f}{\bar{T}_i^{1/2}} B_2 \right] \\ & + \left[\frac{\bar{T}_e \bar{n}_e \bar{n}_\alpha}{\bar{T}_i^{3/2}} B_3 - \frac{n_e n_\alpha}{\bar{T}_i^{1/2}} B_4 \right] \\ & + \frac{Z_\sigma}{2} \left[\frac{\bar{T}_e \bar{n}_e \bar{n}_\sigma}{\bar{T}_i^{3/2}} B_5 - \frac{n_e n_\sigma}{\bar{T}_i^{1/2}} B_6 \right] \end{aligned} \right\}$$

where:

$$B_1 = \frac{(y_e + 1)(x_e + 1)(x_f + 1)}{(y_i + 1)^{3/2}(y_e + x_e + x_f - \frac{3}{2}y_i + 1)}$$

$$B_2 = \frac{(x_e + 1)(x_f + 1)}{(y_i + 1)^{1/2}(x_e + x_f - \frac{y_i}{2} + 1)}$$

$$B_3 = \frac{(y_e + 1)(x_e + 1)(x_\alpha + 1)}{(y_i + 1)^{3/2}(y_e + x_e + x_\alpha - \frac{3}{2}y_i + 1)}$$

$$B_4 = \frac{(x_e + 1)(x_\alpha + 1)}{(y_i + 1)^{1/2}(x_e + x_\alpha - \frac{y_i}{2} + 1)}$$

$$B_5 = \frac{(y_e + 1)(x_e + 1)(x_\sigma + 1)}{(y_i + 1)^{3/2}(y_e + x_e + x_\sigma - \frac{3}{2}y_i + 1)}$$

$$B_6 = \frac{(x_e + 1)(x_\sigma + 1)}{(y_i + 1)^{1/2}(x_e + x_\sigma - \frac{y_i}{2} + 1)}$$

$$\bar{Q}_i^N = 1.29 \times 10^{-8} H(1 - H) \left\{ \begin{aligned} & (0.000457(H + 2)^{2/3}) \frac{\bar{n}_f^2 \bar{T}_e}{\bar{T}_i^{2/3}} \frac{(x_f + 1)^2 (y_e + 1)}{(y_i + 1)^{2/3}} (\bar{\tau}(T_i)) \\ & - (0.0000283(H + 2)) \frac{\bar{n}_f^2 \bar{T}_e^{3/2}}{\bar{T}_i^{2/3}} \frac{(x_f + 1)^2 (y_e + 1)^{3/2}}{(y_i + 1)^{2/3}} (\varphi(\bar{T}_i)) \end{aligned} \right\}$$

where:

$$\tau(\bar{T}_i) = 4 \sum_{n=0}^4 \left(\frac{2n+1}{10}\right)^a e^{\frac{-19.94}{P} \left(\frac{2n+1}{10}\right)^\gamma} + 2 \sum_{n=1}^4 \left(\frac{n}{5}\right)^a e^{\frac{-19.94}{P} \left(\frac{n}{5}\right)^\gamma} + e^{\frac{-19.94}{P}}$$

$$\varphi(\bar{T}_i) = 4 \sum_{n=0}^4 \left(\frac{2n+1}{10}\right)^b e^{\frac{-19.94}{P} \left(\frac{2n+1}{10}\right)^\gamma} + 2 \sum_{n=1}^4 \left(\frac{n}{5}\right)^b e^{\frac{-19.94}{P} \left(\frac{n}{5}\right)^\gamma} + e^{\frac{-19.94}{P}}$$

$$a = 2x_f - \frac{2}{3} y_i + y_e$$

$$b = 2x_f - \frac{2}{3} y_i + \frac{3}{2} y_e$$

$$\gamma = \frac{y_i}{3}$$

$$P = \bar{T}_i^{1/3} (1 + y_i)^{1/3}$$

$$\bar{Q}_i^{ZHO} = 1.06 \times 10^{20} \frac{P_{inj}}{Ra^2} \int_0^1 du (\tilde{K} \bar{T}_e u^{y_e}) \left\{ \ln \left(\frac{1 - (\tilde{K} \bar{T}_e u^{y_e})^{1/2} + (\tilde{K} \bar{T}_e u^{y_e})}{1 + 2(\tilde{K} \bar{T}_e u^{y_e})^{1/2} + (\tilde{K} \bar{T}_e u^{y_e})} \right) \right\} \Theta(t_H - t)$$

$$+ 3.66 \times 10^{20} \frac{P_{inj}}{Ra^2} \int_0^1 du (\tilde{K} \bar{T}_e u^{y_e}) \left\{ \arctan \left(\frac{2 - (\tilde{K} \bar{T}_e u^{y_e})}{\sqrt{3} (\tilde{K} \bar{T}_e u^{y_e})} \right) \right\} \Theta(t_H - t)$$

$$+ 1.92 \times 10^{20} \frac{P_{inj} \tilde{K} \bar{T}_e}{(y_e + 1) Ra^2} \Theta(t_H - t)$$

where:

$$\tilde{K} = \frac{10(H+2)^{2/3}}{E_0} (y_e + 1)$$

$$\bar{Q}_i^{ZHF} = 9.13 \times 10^{21} \frac{P_{inj}}{Ra^2} \frac{(1-H)(H+2)^{2/3}}{E_0^{5/3}} \bar{T}_e^{5/2} \frac{\bar{n}_f}{\bar{n}_e} \frac{(y_e + 1)^{5/2} (x_f + 1)}{(x_e + 1)}.$$

$$\int_0^1 du u^{(5/2 y_e + x_f + x_e)} (*) \cdot (**) \theta(t_H - t)$$

$$(*) = \left\{ \ln \left(\frac{1 - (\tilde{S} \bar{T}_e u^{y_e})^{1/2} + (\tilde{S} \bar{T}_e u^{y_e})}{1 + 2(\tilde{S} \bar{T}_e u^{y_e})^{1/2} + (\tilde{S} \bar{T}_e u^{y_e})} \right) \right.$$

$$\left. + 2\sqrt{3} \arctan \left(\frac{2 - (\tilde{S} \bar{T}_e u^{y_e})}{\sqrt{3} (\tilde{S} \bar{T}_e u^{y_e})^{1/2}} \right) + \frac{\pi}{\sqrt{3}} \right\}$$

$$(**) = \int_0^1 dx \frac{\frac{-19.94}{E_0^{1/3} x^{1/3}}}{x^{1/6} (x^{3/2} + (\frac{10(H+2)^{2/3} (y_e + 1) \bar{T}_e u^{y_e}}{E_0})^{3/2})}$$

$$\bar{S} = 5.65 \times 10^{-3} (H+2)^{2/3} (y_e + 1)$$

AVERAGE ELECTRON POWER SOURCE

$$\bar{Q}_e (\text{KeV cm}^{-3} \text{sec}^{-1}) = \bar{Q}_e^{\Delta} + \bar{Q}_e^J + \bar{Q}_e^N + \bar{Q}_e^{\text{Br}} + (\bar{Q}_e^{\text{Li}} + \bar{Q}_e^{\text{Re}}) + \bar{Q}_e^{\text{Zy}} + \bar{Q}_e^{\text{ZHO}} + \bar{Q}_e^{\text{ZHF}} \quad (\text{A.30})$$

$$\bar{Q}_e^A = -\bar{Q}_i^A$$

$$\bar{Q}_e^J = \text{Average Ohmic Heating Power to Electrons}$$

$$\bar{Q}_e^N = \text{Average Heating of Electrons From Alphas Produced by Thermal Background Fusions}$$

$$\bar{Q}_e^{Br} = \text{Average Bremsstrahlung Power From Electrons}$$

$$(\bar{Q}_e^{Li} + \bar{Q}_e^{Re}) = \text{Average Line and Recombination Power From Electrons on Impurities}$$

$$\bar{Q}_e^{ZY} = \text{Average Cyclotron Radiation Power From Electrons}$$

$$\bar{Q}_e^{ZHO} = \text{Average Heating of Electrons From Deuteron Beam Slowing Down on Background}$$

$$\bar{Q}_e^{ZHF} = \text{Average Heating of Electrons From Alphas Produced By Deuteron Beam Slowing Down on Background}$$

$$\bar{Q}_e^N = 4.31 \times 10^{-10} H(1-H) \frac{\bar{n}_f^2}{\bar{T}_i^{2/3}} \frac{(x_f + 1)^2}{(y_i + 1)^{2/3}} S(\bar{T}_i) - \bar{Q}_i^N$$

$$\bar{Q}_e^{Br} = -3.95 \times 10^{-15} \bar{T}_e^{1/2} \bar{n}_e (y_e + 1)^{1/2} (x_e + 1) [\bar{n}_f K_1 + \bar{n}_\alpha K_2 + \bar{n}_\sigma K_3]$$

where:

$$K_1 = \frac{(x_f + 1)}{(\frac{y_e}{2} + x_e + x_f + 1)}, \quad K_2 = \frac{4(x_\alpha + 1)}{(\frac{y_e}{2} + x_e + x_\alpha + 1)}$$

$$K_3 = \frac{(x_\sigma + 1) Z_\sigma^2}{(\frac{y_e}{2} + x_e + x_\sigma + 1)}$$

$$(\bar{Q}_e^{Li} + \bar{Q}_e^{Re}) = (\text{from reference (15)})$$

$$= - 1.14 \times 10^{-16} \frac{(x_\sigma + 1)(x_e + 1)Z_\sigma^4}{(x_\sigma + x_e - \frac{y_e}{2} + 1)} \frac{\bar{n}_\sigma \bar{n}_e \bar{T}_e^{-1/2}}{(y_e + 1)^{1/2}}$$

$$- 2.58 \times 10^{-18} \frac{(x_\sigma + 1)(x_e + 1)Z_\sigma^6}{(x_\sigma + x_e - \frac{3}{2}y_e + 1)} \frac{\bar{n}_\sigma \bar{n}_e \bar{T}_e^{-3/2}}{(y_e + 1)^{3/2}}$$

$$\bar{Q}_e^{ZHO} = 3.17 \times 10^{20} \frac{P_{inj}}{a_R^2} \theta(t_H - t) - \bar{Q}_i^{ZHO}$$

$$\bar{Q}_e^{ZHF} = 4.84 \times 10^{24} \frac{P_{inj}(1-H)}{Ra^2 E_0^{5/3}} \frac{\bar{n}_f \bar{T}_e^{3/2}}{\bar{n}_e} \frac{(x_f + 1)(y_e + 1)^{3/2}}{(x_e + 1)} I_e^{ZMF} - \bar{Q}_i$$

$$I_e^{ZHF} = \int_0^1 du u^{(x_f + \frac{3}{2}y_e - x_e)} \int_0^1 dx \frac{e^{\frac{-19.94}{E_0^{1/3} x^{1/3}} \theta(t_H - t)}}{x^{1/6} (x^{3/2} + \frac{4.64(H+2)}{E_0^{3/2}}) (y_e + 1)^{3/2} \bar{T}_e^{3/2} u^{3/2 y_e}}$$

$$\bar{Q}_e^{ZY} = (\text{equation (2.20) from reference (12)})$$

$$\bar{Q}_e^{ZY} = 3.13 \times 10^{-16} \frac{(x_e + 1)^2 (y_e + 1)^2}{[2(x_e + y_e) + 1]} \bar{n}_e^2 \bar{T}_e^2$$

$$\bar{Q}_e^J = (\text{equation (63) from reference (14)})$$

$$= \left(\frac{1.42 \times 10^{21} I_T^2}{(1 - \frac{3}{2} y_e)(y_e + 1)^{3/2} a^4} \right) \frac{1}{T_e^{3/2}}$$

I_T = Toroidal Current in MA

Finally, combine equations (A.19) to (A.30) together. Now assume that the variables are time independent, this means time derivatives are set to zero. Then from reference (13) and (16) we find:

$$x_i \sim D_i$$

$$x_e \sim 3D_e$$

$$\tau_p = \frac{\text{Particle}}{\text{Confinement Time}} = \frac{\bar{n}a}{-2D_{\perp} \frac{dN}{dr} \big|_{r=\lambda a}}$$

here $D_{\perp} = D_i$ or D_e

notice for $n = n_0 \left[1 - \left(\frac{r}{a} \right)^2 \right]^x$

$$D_{\perp} = \frac{1}{\frac{4\lambda^2}{a^2} x (x+1) (1 - \lambda^2)^{(x-1)} \tau_p}$$

Using this information plus the equations (A.19) to (A.30) provide the time independent equations shown on the last few pages. Note here that these equations are used in the computer code sphero.

TIME INDEPENDENT TRANSPORT EQUATIONS

1. FUEL PARTICLE BALANCE:

$$\frac{\bar{n}_f}{\tau_{pf}} + \xi_e \frac{\bar{n}_f^2}{\bar{T}_i^{2/3}} S(\bar{T}_i) - 3.16 \times 10^{20} \frac{P_D}{E_0} - \bar{N}_F = 0$$

2. ALPHA PARTICLE BALANCE:

$$\frac{\bar{n}_\alpha}{\tau_{p\alpha}} - \xi_i \frac{\bar{n}_f^2}{\bar{T}_i^{2/3}} S(\bar{T}_i) - \frac{\bar{n}_f \bar{T}_e}{\bar{n}_e} I_\alpha^{ZH}(\bar{T}_e) = 0$$

3. ELECTRON PARTICLE BALANCE:

$$\bar{n}_e - \bar{n}_f - 2\bar{n}_\alpha - Z_\sigma \bar{n}_\sigma = 0$$

4. ION PARTICLE BALANCE:

$$\bar{n}_i - \bar{n}_f - \bar{n}_\alpha - \bar{n}_\sigma = 0$$

5. ION POWER BALANCE:

$$\bar{n}_i \bar{T}_i \{CC\} + C_{i1}^F \bar{n}_e \bar{T}_e + C_{i1}^{HC} \bar{n}_f \bar{T}_i + C_{i2}^{HC} \bar{n}_\alpha \bar{T}_i + C_{i3}^{HC} \bar{n}_\sigma \bar{T}_i$$

$$- \frac{\bar{T}_e \bar{n}_e}{\bar{T}_i^{3/2}} \{ \bar{n}_f C_1^\Delta + \bar{n}_\alpha C_3^\Delta + \bar{n}_\sigma C_3^\Delta \} + \frac{\bar{n}_e}{\bar{T}_i^{1/2}} \{ \bar{n}_f C_2^\Delta + \bar{n}_\alpha C_4^\Delta + \bar{n}_\sigma C_6^\Delta \}$$

TIME INDEPENDENT TRANSPORT EQUATIONS (Continued)

$$- C_{i1}^N \frac{\bar{n}_f^2 \bar{T}_e}{\bar{T}_i^{2/3}} \varphi(\bar{T}_i) + C_{i2}^N \frac{\bar{n}_f^2 \bar{T}_e^{3/2}}{\bar{T}_i^{2/3}} \varphi(\bar{T}_i) - I_{i1n}^{ZHO}(\bar{T}_e) - I_{iARCTAN}^{ZHO}(\bar{T}_e)$$

$$- C_i^{ZHO} \bar{T}_e - \frac{\bar{n}_f}{\bar{n}_e} \bar{T}_e^{5/2} I_i^{ZHF}(\bar{T}_e) = 0$$

6. ELECTRON POWER BALANCE:

$$\bar{n}_e \bar{T}_e [H_e] (x_e + 1)(y_e + 1) + \frac{\bar{T}_e \bar{n}_e}{\bar{T}_i^{3/2}} \{ \bar{n}_f C_1^\Delta + \bar{n}_\alpha C_3^\Delta + \bar{n}_\sigma C_3^\Delta \}$$

$$- \frac{\bar{n}_e}{\bar{T}_i^{1/2}} \{ \bar{n}_f C_2^\Delta + \bar{n}_\alpha C_4^\Delta + \bar{n}_\sigma C_6^\Delta \} - C_e^J \frac{1}{\bar{T}_e^{3/2}} - C_e^N \frac{\bar{n}_f^2}{\bar{T}_i^{2/3}} S(\bar{T}_i) + C_{i1}^N \frac{\bar{n}_f^2 \bar{T}_e}{\bar{T}_i^{2/3}} \varphi(\bar{T}_i)$$

$$- C_{i2}^N \frac{\bar{n}_f^2 \bar{T}_e^{3/2}}{\bar{T}_i^{2/3}} \varphi(\bar{T}_i) + C_{e1}^{Br} \bar{n}_f \bar{n}_e \bar{T}_e + \bar{n}_e \bar{T}_e (C_{e1}^{Br} \bar{n}_f + C_{e2}^{Br} \bar{n}_\alpha + C_{e3}^{Br} \bar{n}_\sigma)$$

$$+ C_e^{ZY} \frac{\bar{n}_e^2 \bar{T}_e^2}{\bar{n}_e \bar{n}_\sigma} (C_{i1}^{Li} + Re \frac{1}{\bar{T}_e^{1/2}} + C_{e2}^{Li} + Re \frac{1}{\bar{T}_e^{3/2}}) - 3.17 \times 10^{20} P_D$$

$$+ \{ I_{i1n}^{ZHO}(\bar{T}_e) + I_{iARCTAN}^{ZHO}(\bar{T}_e) + C_i^{ZHO} \bar{T}_e \} - \frac{\bar{n}_f \bar{T}_e^{3/2}}{\bar{n}} I_e^{ZHF}(\bar{T}_e)$$

$$+ \frac{\bar{n}_f}{\bar{n}_e} \bar{T}_e^{5/2} I_i^{ZHF}(\bar{T}_e) = 0$$

DEFINITIONS:

$$P_D = \frac{P_{inj}}{a^2 R}, \quad \xi_e = 2.45 \times 10^{-13} H(1-H) \frac{(x_f + 1)^2}{(y_i + 1)^{2/3}} = \frac{\xi_i}{2}$$

$$CC = \frac{1}{\tau_{pf}} \frac{(1 + \frac{x_f}{y_i})}{(x_i + 1)} + \frac{1}{\tau_{p\alpha}} \frac{(1 + \frac{x_\alpha}{y_i})}{(x_i + 1)} + \frac{1}{\tau_{p\sigma}} \frac{(1 + \frac{x_\sigma}{y_i})}{(x_i + 1)}$$

τ_{pf} = FUEL PARTICLE CONFINEMENT TIME

$\tau_{p\alpha}$ = ALPHA PARTICLES CONFINEMENT TIME

$\tau_{p\sigma}$ = IMPURITY PARTICLE CONFINEMENT TIME

τ_{pe} = ELECTRON PARTICLE CONFINEMENT TIME

$$H_e = \frac{1}{(x_e + 1) \tau_{pe}} \left\{ (1 - \lambda^2)(y_e + 2) (2.5 + 3 \frac{y_e}{x_e}) + (1 + \frac{x_e}{y_e}) \right\}$$

$$C_1^\Delta = 3.01 \times 10^{-12} \frac{(2+H)}{6} \frac{(y_e + 1)(x_e + 1)(x_f + 1)}{(y_i + 1)^{3/2} (y_e + x_e + x_f - \frac{3}{2} y_i + 1)}$$

$$C_2^\Delta = 3.01 \times 10^{-12} \frac{(2+H)}{6} \frac{(x_e + 1)(x_f + 1)}{(y_i + 1)^{1/2} (y_e + x_e + x_f - \frac{3}{2} y_i + 1)}$$

$$C_3^\Delta = 3.01 \times 10^{-12} \frac{(y_e + 1)(x_e + 1)(x_\alpha + 1)}{(y_i + 1)^{3/2} (y_e + x_e + x_\alpha - \frac{3}{2} y_i + 1)}$$

$$C_4^\Delta = 3.01 \times 10^{-12} \frac{(x_e + 1)(x_\alpha + 1)}{(y_i + 1)^{1/2} (x_e + x_\alpha - \frac{y_i}{2} + 1)}$$

DEFINITIONS (Continued):

$$C_5^{\Delta} = 3.01 \times 10^{-12} \left(\frac{Z}{2}\right) \frac{(y_e + 1)(x_e - 1)(x_{\sigma} + 1)}{(y_i + 1)^{3/2} (y_e + x_e + x_{\sigma} - \frac{3}{2} y_i + 1)}$$

$$C_6^{\Delta} = 3.01 \times 10^{-12} \left(\frac{Z}{2}\right) \frac{(x_e + 1)(x_{\sigma} + 1)}{(y_i + 1)^{1/2} (x_e + x_{\sigma} - \frac{y_i}{2} + 1)}$$

$$C_{i1}^N = 1.29 \times 10^{-8} H (1 - H)(0.000457 (H + 2)^{2/3}) \frac{(x_f + 1)^2 (y_e + 1)}{(y_i + 1)^{2/3}}$$

$$C_{i2}^N = 1.29 \times 10^{-8} H (1 - H)(0.0000283 (H + 2)) \frac{(x_f + 1)^2 (y_e + 1)^{3/2}}{(y_i + 1)^{2/3}}$$

$$I_{i1n}^{ZHO}(\bar{T}_e) = 1.06 \times 10^{20} P_n \int_0^1 du \left(\frac{\tilde{K} \bar{T}_e u^{y_e}}{(K \bar{T}_e u^{y_e})} \right) \left\{ \ln \left(\frac{1 - (\tilde{K} \bar{T}_e u^{y_e})^{1/2} + (\tilde{K} \bar{T}_e u^{y_e})}{1 + 2 (\tilde{K} \bar{T}_e u^{y_e})^{1/2} + (\tilde{K} \bar{T}_e u^{y_e})} \right) \right\}$$

$$\tilde{K} = \frac{10 (H + 2)^{2/3}}{E_0} (y_e + 1)$$

$$I_{i \text{ ARCTAN}}^{ZHO}(\bar{T}_e) = 3.66 \times 10^{20} P_D \int_0^1 du \left(\frac{\tilde{K} \bar{T}_e u^{y_e}}{(K \bar{T}_e u^{y_e})} \right) \left\{ \arctan \left(\frac{2 - (\tilde{K} \bar{T}_e u^{y_e})}{\sqrt{3} (\tilde{K} \bar{T}_e u^{y_e})} \right) \right\}$$

DEFINITIONS (Continued):

$$C_i^{ZHO} = 1.92 \times 10^{20} \frac{P_D \tilde{K}}{(y_e + 1)}$$

$$C_{e1}^{Li + Re} = 1.14 \times 10^{-16} \frac{(x_\sigma + 1)(x_e + 1) z_\sigma^4}{(x_\sigma + x_e - \frac{y_e}{2} + 1)(y_e + 1)^{1/2}}$$

$$C_{e2}^{Li + Re} = 2.58 \times 10^{-18} \frac{(x_\sigma + 1)(x_e + 1) z_\sigma^6}{(x_\sigma + x_e - \frac{3}{2} y_e + 1)(y_e + 1)^{3/2}}$$

$$C_{e1}^{Br} = \frac{3.95 \times 10^{-15} (y_e + 1)^{1/2} (x_e + 1)(x_f + 1)}{(\frac{y_e}{2} + x_e + x_f + 1)},$$

$$C_{e2}^{Br} = \frac{1.58 \times 10^{-14} (y_e + 1)^{1/2} (x_e + 1)(x_\alpha + 1)}{(\frac{y_e}{2} + x_e + x_\alpha + 1)},$$

$$C_{e3}^{Br} = \frac{3.95 \times 10^{-15} (z_\sigma^2)(y_e + 1)^{1/2} (x_e + 1)(x_\sigma + 1)}{(\frac{y_e}{2} + x_e + x_\sigma + 1)}$$

$$C_e^N = 4.31 \times 10^{-10} H(1 - H) \frac{(x_f + 1)^2}{(y_i + 1)^{2/3}}, \quad C_e^{Zy} = 3.13 \times 10^{-16} \frac{(x_e + 1)^2 (y_e + 1)^2}{[2(x_e + y_e) + 1]}$$

DEFINITIONS (Continued):

$$C_e^J = \left(\frac{1.42 \times 10^{21} I_T^2}{(1 - \frac{3}{2} y_e)(y_e + 1)^{3/2} a^4} \right), \quad C_i^F = \frac{2.5 (1 - \lambda^2)^{y_i} y_i}{(x_i + 1) \tau_{pe}}$$

$$C_{i1}^{HC} = \frac{(1 - \lambda^2)^{y_i} y_i}{(x_i + 1) \tau_{pf} x_f}, \quad C_{i2}^{HC} = \frac{(1 - \lambda^2)^{y_i} y_i}{(x_i + 1) \tau_{p\alpha} x_\alpha}$$

$$C_{i2}^{HC} = \frac{(1 - \lambda^2)^{y_i} y_i}{(x_i + 1) \tau_{p\sigma} x_\sigma}$$

$$I_i^{ZHF} = (\bar{T}_e) = 9.13 \times 10^{21} \frac{P_D}{E_0^{5/3}} (1 - H)(H + 2)^{2/3} \frac{(y_e + 1)^{5/2} (x_f + 1)}{(x_e + 1)}.$$

$$\int_0^1 du u^{\left(\frac{5}{2} y_e + x_f + x_e\right)} \bar{f}(\bar{T}_e) f(\bar{T}_e)$$

$$f(\bar{T}_e) = \left\{ \ln \left(\frac{1 - (\tilde{S} \bar{T}_e u^{y_e})^{1/2} + (\tilde{S} \bar{T}_e u^{y_e})}{1 + 2 (\tilde{S} \bar{T}_e u^{y_e})^{1/2} (\tilde{S} \bar{T}_e u^{y_e})} \right) + 2 \sqrt{3} \arctan \left(\frac{2 - (\tilde{S} \bar{T}_e u^{y_e})^{1/2}}{\sqrt{3} (\tilde{S} \bar{T}_e u^{y_e})^{1/2}} + \frac{\pi}{\sqrt{3}} \right) \right\}$$

$$\tilde{S} = 5.65 \times 10^{-3} (H + 2)^{2/3} (y_e + 1)$$

DEFINITIONS (Continued):

$$\bar{f}(\bar{T}_e) = \int_0^1 dx \frac{e^{\frac{-19.94}{E_0^{1/3} x^{1/3}}}}{x^{1/6} \left(x^{3/2} + \frac{10 (H+2)^{2/3} (y_e + 1) \bar{T}_e u^{y_e}}{E_0} \right)}$$

$$I_{\alpha}^{ZH}(\bar{T}_e) = \frac{6.84 \times 10^{20} P_D}{E_0^{5/3}} (1 - H) \frac{(x_f + 1)(y_e + 1)}{(x_e + 1)} \int_0^1 du u^{(x_f + y_e + x_e)}$$

$$\int_0^1 dx \frac{e^{\frac{-19.94}{E_0^{1/3} x^{1/3}}}}{x^{1/6} \left(x^{3/2} + \frac{4.64 (H+2)}{E_0^{3/2}} (y_e + 1)^{3/2} \bar{T}_e^{3/2} u^{3/2} y_e \right)}$$

$$I_e^{ZHF}(\bar{T}_e) = \frac{4.84 \times 10^{24} P_D}{E_0^{5/3}} (1 - H) \frac{(x_f + 1)(y_e + 1)^{3/2}}{(x_e + 1)} \int_0^1 du u^{(x_f + \frac{3}{2} y_e - x_e)}$$

$$\int_0^1 dx \frac{e^{\frac{-19.94}{E_0^{1/3} x^{1/3}}}}{x^{1/6} \left(x^{3/2} + \frac{4.64 (H+2)}{E_0^{3/2}} (y_e + 1)^{3/2} \bar{T}_e^{3/2} u^{3/2} y_e \right)}$$

Definitions of other variables can be found elsewhere in this appendix. Note:

$\bar{T}_e$ in keV, E_0 in keV, P_{inj} in MW

APPENDIX B

SPHERO DOCUMENTATION

The flow diagram for Sphero is shown in Figure B.1. Notice that all of the computations done on the six transport equations given in Appendix A are done by subroutine trans, and as mentioned previously in the text the I.M.S.L. subroutine ZSYSTEM is used.

The input to Sphero is in a namelist format. A list of the input variables, their definitions and the initial values are listed in Table B.1. Notice that the letters and numbers in parentheses after the variable names in Table B.1 denote which namelist block the variable belongs to.

Sphero can be run on the CDC-7600 computer at Lawrence Livermore Laboratory, using the CHATR compiler. Specific questions on syntax should be referred to the M.F.C.C. (Magnetic Fusion Computer Center) reference manuals.

The output of Sphero consists of the inputted data as well as calculated values. So far the output consists of the calculated values:

AVERAGE FUEL DENSITY IN CM-3

AVERAGE ALPHA DENSITY IN CM-3

AVERAGE TOTAL ION DENSITY IN CM-3

AVERAGE ELECTRON DENSITY IN CM-3

AVERAGE ION TEMPERATURE IN keV

AVERAGE ELECTRON TEMPERATURE IN keV

AVERAGE BREMSTRAHLUNG POWER LOSS IN keV CM-3 SEC-1

AVERAGE CYCLOTRON POWER LOSS IN CM-3 SEC-1

AVERAGE LINE, RECOMBINATION POWER LOSS IN keV CM-3 SEC-1

AVERAGE FUSION POWER OUTPUT IN keV CM-3 SEC-1

AVERAGE TOTAL FUSION POWER OUTPUT IN keV/SEC

AVERAGE $n\tau$ OF FUEL IN CM-3 SEC.

Of course more data can be extracted from Sphero but these seem like important variables to consider.

Finally the source program of Sphero is given on the last few pages of this Appendix for further reference.

Figure B-1. Flow Diagram of Sphero

TABLE B.1
INPUT VARIABLES

<u>VARIABLE NAME</u>	<u>VARIABLE DESCRIPTION</u>	<u>DEFAULT VALUE</u>
n (IN1)	INTERVAL MESH NUMBER FOR SOME OF THE FUNCTIONS CALLED IN SPHERO	1
EPS (IN1)	FIRST STOPPING CRITERION FOR ZSYSTEM (CONVERGENCE INTERVAL)	1.0×10^{-6}
NSIG (IN1)	SECOND STOPPING CRITERION FOR ZSYSTEM (SIGNIFICANT DIGITS)	5
ITMAX (IN1)	MAXIMUM NUMBER OF ITERATIONS ALLOWED TO ZSYSTEM	100
XE (IN3)	ELECTRON DENSITY PROFILE EXPONENT	1.0
XF (IN3)	FUEL DENSITY PROFILE EXPONENT	1.0
XA (IN3)	ALPHA DENSITY PROFILE EXPONENT	1.0
XS (IN3)	IMPURITY DENSITY PROFILE EXPONENT	1.0
XI (IN3)	ION DENSITY PROFILE EXPONENT	1.0
YE (IN3)	ELECTRON TEMPERATURE PROFILE EXPONENT	1.0
YI (IN3)	ION TEMPERATURE PROFILE EXPONENT	1.0
PINJ (IN4)	TOTAL INJECTED BEAM POWER IN MW	0.0
EO (IN4)	DEUTERIUM BEAM INJECTION ENERGY IN keV	120.0
CT (IN4)	TOROIDAL CURRENT IN MA	1.0
HR (IN4)	RATIO OF DEUTERIUM TO FUEL (D + T)	0.5
R (IN5)	PLASMA MAJOR RADIUS IN CM	12.0
A (IN5)	PLASMA MINOR RADIUS IN CM	6.0
LAMDA (IN1)	BOUNDARY SIZING CONDITION USUALLY SET TO 0.70	0.70
ZS (IN5)	EFFECTIVE CHARGE OF IMPURITY	8.0
XNF (IN5)	AVERAGE REFUELING RATE IN $\text{CM}^{-3} \text{ SEC}^{-1}$	1.0×10^{14}
XNS (IN5)	AVERAGE IMPURITY DENSITY IN CM^{-3}	1.0×10^{10}

TABLE B.1 (Continued)

INPUT VARIABLES

<u>VARIABLE NAME</u>	<u>VARIABLE DESCRIPTION</u>	<u>DEFAULT VALUE</u>
TAUPF (IN6)	FUEL PARTICLE CONFINEMENT TIME IN SEC	1.0
TAUPA (IN6)	ALPHA PARTICLE CONFINEMENT TIME IN SEC	1.5
TAUPE (IN6)	ELECTRON PARTICLE CONFINEMENT TIME IN SEC	0.10
TAUPS (IN6)	IMPURITY PARTICLE CONFINEMENT TIME IN SEC	2.0
X1 (IN2)	INITIAL AVERAGE FUEL DENSITY GUESS IN CM ⁻³	1.0×10^{14}
X2 (IN2)	INITIAL AVERAGE ALPHA DENSITY GUESS IN CM ⁻³	4×10^{12}
X3 (IN2)	INITIAL AVERAGE ION DENSITY GUESS IN CM ⁻³	1.0×10^{14}
X4 (IN2)	INITIAL AVERAGE ELECTRON DENSITY GUESS IN CM ⁻³	2×10^{14}
X5 (IN2)	INITIAL AVERAGE ION TEMPERATURE GUESS IN keV	15.0
X6 (IN2)	INITIAL AVERAGE ELECTRON TEMPERATURE GUESS IN keV	15.0


```

1 c*
2 c** program sphero is a very simple steady state , zero dimensional **
3 c** model of a spheromak reactor in beam driven mode **
4 c*
5 c* units are in cgs when possible *
6 c*
7 c** written by john m. les **
8 c*
9 c** input variables are: **
10 c
11 c   r = major radius in cm
12 c   a = minor radius in cm
13 c   pinj = total injected beam power in mw
14 c   hr = ratio of deuterium to total fuel (d+t)
15 c   xe = electron density profile exponent
16 c   xf = fuel density profile exponent
17 c   xa = alpha density profile exponent
18 c   xs = impurity density profile exponent
19 c   xi = total ion density profile exponent
20 c   ye = electron temperature profile exponent
21 c   yi = total ion temperature profile exponent
22 c   taupf = fuel particle confinement time (sec)
23 c   taupe = electron particle confinement time (sec)
24 c   taupa = alpha particle confinement time (sec)
25 c   taups = impurity particle confinement time (sec)
26 c   lamda = boundary sizing condition (usually set to 0.70)
27 c   zs = effective charge of impurity
28 c   xnf = average refueling rate (cm-3 sec-1)
29 c   xns = average impurity density (cm-3)
30 c   e0 = beam energy in kev (deuterium beam only)

```

```

31 c   ct = torodial current in ma
32 c   eps = first stopping criterion for zsystem
33 c   nsig = second stopping criterion for zsystem
34 c   x(6) = solution vector which contains average densities and
35 c           temperatures (initial guess at begining of program)
36 c   itmax = maximum number of iterations allowed to zsystem
37 c   n = interval mesh number needed for some functions called in
38 c       this program
39 c*****
40 c
41     real lamda
42     common x(6)
43     common/cp/ n,eps,nsig,itmax
44     common/exc/ xe,xf,xa,xs,xi,ye,yi
45     common/pow/ pinj,e0,ct,pd,hr
46     common/dim/ r,a,lamda,zs,xnf,xns
47     common/conft/ taupf,taupa,taupe,taups
48 c
49     call link("unit5=(input,open),unit6=(output,create,hc)//")
50 c
51     data n/1/,itmax/100/,nsig/5/,eps/1.0e-6/,lamda/0.70/
52     data x1/1.0e14/,x2/4.0e12/,x3/1.0e14/,x4/2.0e14/,x5/15.0/,x6/15.0/
53     data xe/1.0/,xf/1.0/,xa/1.0/,xs/1.0/,xi/1.0/,ye/1.0/,yi/1.0/
54     data pinj/0.0/,e0/120.0/,ct/1.0/,hr/0.5/
55     data r/12.0/,a/6.0/,zs/8.0/,xn timer/1.0e14/,xns/1.0e10/
56     data taupf/1.0/,taupa/1.5/,taupe/0.10/,taups/2.0/
57     namelist/in1/ n,itmax,nsig,eps,lamda
58     namelist/in2/ x1,x2,x3,x4,x5,x6
59     namelist/in3/ xe,xf,xa,xs,xi,ye,yi
60     namelist/in4/ pinj,e0,ct,hr
61     namelist/in5/ r,a,zs,xnf,xns
62     namelist/in6/ taupf,taupa,taupe,taups
63 c
64     read(5,in1)
65     read(5,in2)
66     read(5,in3)
67     read(5,in4)
68     read(5,in5)
69     read(5,in6)
70     write(6,10) n,itmax,nsig,eps,lamda
71     10 format("n=",i4/"itmax=",i4/"nsig=",i4/"eps=",e13.5/"lamda=",e13.5)
72     write(6,11) x1,x2,x3,x4,x5,x6
73     11 format("x1=",e13.5/"x2=",e13.5/"x3=",e13.5/"x4=",e13.5/"x5=",e13.5
74     1/"x6=",e13.5)
75     write(6,12) xe,xf,xa,xs,xi,ye,yi
76     12 format("xe=",f10.5/"xf=",f10.5/"xa=",f10.5/"xs=",f10.5/"xi=",f10.5
77     1/"ye=",f10.5/"yi=",f10.5)
78     write(6,13) pinj,e0,ct,hr
79     13 format("pinj=",e13.5/"e0=",e13.5/"ct=",e13.5/"hr=",e13.5)
80     write(6,14) r,a,zs,xnf,xns
81     14 format("r=",e13.5/"a=",e13.5/"zs=",e13.5/"xn timer=",e13.5/"xns=",e13.5

```

```

82      1)
83      write(6,15) taupf,taupa,taupe,taups
84      15 format("taupf=",e13.5/"taupa=",e13.5/"taupe=",e13.5/"taups=",e13.5
85      1//)
86 c
87      x(1)=x1
88      x(2)=x2
89      x(3)=x3
90      x(4)=x4
91      x(5)=x5
92      x(6)=x6
93 c
94 c
95 c**
96 c      note that :
97 c
98 c      x(1) = average fuel density in cm-3
99 c      x(2) = average alpha density in cm-3
100 c      x(3) = average total ion density in cm-3
101 c      x(4) = average electron density in cm-3
102 c      x(5) = average total ion temperature in kev
103 c      x(6) = average electron temperature in kev
104 c
105 c
106 c* call subroutine trans which will try and solve the simultaneous
107 c transport equations for the given input **
108 c
109      call trans(nn)
110 c
111 c ** next if nn=1 then this means that the solution vector x has a
112 c ** zero or negative element. since each element of x represents a
113 c ** real positive quantity we must guess another initial vector.
114 c
115      if(nn .eq. 0) go to 500
116      write(6,3500)
117      3500 format("** warning ** the solution vector x has at least one zero"
118      1/5x,"or negative element, the solution is given by:"//)
119      300 format("x(",i2,")=",e13.5)
120      do 50 i=1,6
121      write(6,300) i,x(i)
122      50 continue
123      go to 400
124 c
125 c* if trans solves the transport equations so that x contains no zero
126 c* or negative elements, then call subroutine wod which will write out
127 c* the output.
128 c
129      500 call wod
130      400 call exit
131      end
132 c

```

```

133 c
134 function zhoiln(te,n)
135 common/pow/ pinj,e0,ct,pd,hr
136 common/exc/ xe,xf,xa,xs,xi,ye,yi
137 if(pinj .eq. 0.) zhoiln=0.
138 if(pinj .eq. 0.) write(6,25) zhoiln
139 if(pinj .eq. 0.) return
140 h=1./n/2.
141 xk=10.*((hr+2.)*0.6667)*(ye+1.)/e0
142 sumo=0.
143 sune=0.
144 sumel=0.
145 do 10 i=1,n
146 uo=h+((i-1)*2.*h)
147 ue=(2.*h)+((i-1)*2.*h)
148 argo=(1.-(sqrt((xk*te*(uo**ye))))+(xk*te*(uo**ye)))/(1.+2.*(sqrt((
149 1xk*te*(uo**ye))))+(xk*te*(uo**ye)))
150 arge=(1.-(sqrt((xk*te*(ue**ye))))+(xk*te*(ue**ye)))/(1.+2.*(sqrt((
151 1xk*te*(ue**ye))))+(xk*te*(ue**ye)))
152 sumo=xk*te*(uo**ye)*alog(argo)+sumo
153 if(i .lt. n) sune=xk*te*(ue**ye)*alog(arge)+sune
154 if(i .eq. n) sumel=xk*te*(ue**ye)*alog(arge)
155 10 continue
156 zhoiln=(h/3.)*((4.*sumo)+(2.*sune)+sumel)*1.06e20*pd
157 write(6,25) zhoiln
158 25 format("zhoiln=",e13.5)
159 return
160 end

```

```

161 c
162 c
163 function zhoiat(te,n)
164 common/pow/ pinj,e0,ct,pd,hr
165 common/exc/ xe,xf,xa,xs,xi,ye,yi
166 if(pinj .eq. 0.) zhoiat=0.
167 if(pinj .eq. 0.) write(6,15) zhoiat
168 if(pinj .eq. 0.) return
169 h=1./n/2.
170 xk=10.*((hr+2.)*0.6667)*(ye+1.)/e0
171 sumo=0.
172 sune=0.
173 sumel=0.
174 do 10 i=1,n
175 uo=h+((i-1)*2.*h)
176 ue=(2.*h)+((i-1)*2.*h)
177 argo=(2.-(xk*te*(uo**ye)))/(1.732*xk*te*(uo**ye))
178 arge=(2.-(xk*te*(ue**ye)))/(1.732*xk*te*(ue**ye))
179 sumo=xk*te*(uo**ye)*atan(argo)+sumo
180 if(i .lt. n) sune=xk*te*(ue**ye)*atan(arge)+sune
181 if(i .eq. n) sumel=xk*te*(ue**ye)*atan(arge)
182 10 continue
183 zhoiat=(h/3.)*(1.571+(4.*sumo)+(2.*sune)+sumel)*3.66e20*pd

```

```

184      write(6,15) zhoiat
185      15 format("zhoiat=",e13.5)
186      return
187      end
188 c
189 c
190      function sx(te,itype)
191      common/exc/ xe,xf,xa,xs,xi,ye,yi
192      go to (1,2,3), itype
193      1 xx=(2.*xf)-(0.66667*yi)
194      go to 10
195      2 xx=(2.*xf)-(0.66667*yi)+ye
196      go to 10
197      3 xx=(2.*xf)-(0.66667*yi)+(1.5*ye)
198      10 gamma=yi/3.
199      p=(te**0.3333)*((yi+1.)**0.3333)
200      sumo=0.
201      sune=0.
202      sumel=0.
203      do 20 i=1,5
204      uo=0.1+((i-1)*0.2)
205      ue=0.2+((i-1)*0.2)
206      argo=-19.94/p/(uo**gamma)
207      arge=-19.94/p/(ue**gamma)
208      sumo=(uo**xx)*exp(argo)+sumo
209      if(i .lt. 5) sune=(ue**xx)*exp(arge)+sune
210      if(i .eq. 5) sumel=(ue**xx)*exp(arge)
211      20 continue
212      sx=(4.*sumo)+(2.*sune)+sumel
213      write(6,26) sx
214      26 format("sx=",e13.5)
215      return
216      end
217 c
218 c
219      function zhx(te,n,itype)
220      common/pow/ pinj,e0,ct,pd,hr
221      common/exc/ xe,xf,xa,xs,xi,ye,yi
222      common/zhxzh/ aa,au,zz,dy,bb,st
223      data w1/0.027778/,w2/0.11111/,w3/0.027778/,w4/0.11111/,w5/0.44444/
224      1,w6/0.11111/,w7/0.027778/,w8/0.11111/,w9/0.027778/
225      if(pinj .eq. 0.) zhx=0.
226      if(pinj .eq. 0.) write(6,200) zhx
227      if(pinj .eq. 0.) return
228      h=1./n/2.
229      zz=1.5*ye
230      bb=-19.94/(e0**0.33333)
231      dy=4.64*(hr+2.)*(ye+1.)**1.5*(te**1.5)/(e0**1.5)
232      st=5.65e-3*((hr+2.)**0.66667)*(ye+1.)*te
233 c
234 c ** 4=electron, 2=alpha, 3=ion for itype **

```



```

235 c ** determine either electron, alpha, or ion double integral **
236 c
237     if(itype .eq. 3) go to 70
238     if(itype .eq. 2) go to 10
239 c
240 c** electron double integral constants **
241 c
242     aa=4.84e24*pd*(1.-hr)*(xf+1.)*((ye+1.)**1.5)/(e0**1.6667)/(xe+1.)
243     eu=xf+(1.5*ye)-xe
244     go to 80
245 c
246 c** alpha double integral constants **
247 c
248     10 aa=6.84e20*pd*(1.-hr)*(xf+1.)*(ye+1.)/(e0**1.6667)/(xe+1.)
249     eu=xf+ye+xe
250     go to 80
251 c
252 c** ion double integral constants **
253 c
254     70 aa=9.13e21*pd*(1.-hr)*((hr+2.)**0.66667)*((ye+1.)**2.5)*(xf+1.)/(e
255     10**1.6667)/(xe+1.)
256     eu=(2.5*ye)+xf+xe
257 c
258 c     ** solve the double integrals **
259 c
260     80 sumz=0.
261     do 100 j=1,n
262         u0=h+((j-1)*2.*h)
263         do 100 i=1,n
264             x0=h+((i-1)*2.*h)
265 c
266             x=x0-h
267             u=u0-h
268             f1=0.
269             if(x .eq. 0. .and. u .eq. 0.) go to 50
270             f1=zh(x,u,itype)
271 c
272     50 x=x0-h
273         u=u0
274         f2=zh(x,u,itype)
275 c
276         x=x0-h
277         u=u0+h
278         f3=zh(x,u,itype)
279 c
280         x=x0
281         u=u0-h
282         f4=zh(x,u,itype)
283 c
284         x=x0
285         u=u0

```

```

286      f5=zh(x,u,itype)
287 c
288      x=x0
289      u=u0+h
290      f6=zh(x,u,itype)
291 c
292      x=x0+h
293      u=u0-h
294      f7=zh(x,u,itype)
295 c
296      x=x0+h
297      u=u0
298      f8=zh(x,u,itype)
299 c
300      x=x0+h
301      u=u0+h
302      f9=zh(x,u,itype)
303 c
304      szhx=f1*w1+f2*w2+f3*w3+f4*w4+f5*w5+f6*w6+f7*w7+f8*w8+f9*w9
305 c
306      sumz=sumz+szhx
307 100 continue
308      zhx=sumz/(n**2)
309      write(6,200) zhx
310 200 format("zhx=",e13.5)
311      return
312      end
313 c
314 c
315      function zh(w,y,m)
316      common/exc/ xe,xf,xa,xs,xi,ye,yi
317      common/zhxzh/ aa,eu,zz,dy,bb,st
318      nml=m-1
319      go to (1,2,1),nml
320 c
321 c ** electron and alpha part **
322 c
323      1 zh=aa*(y**eu)*exp((bb/(w**0.33333)))/(w**0.16667)/((w**1.5)+(dy*(y
324      1**zz)))
325      return
326 c
327 c ** ion part **
328 c
329      2 zh=aa*(y**ye)*((alog(((1.-(sqrt((st*(y**ye))))+(st*(y**ye)))/(1.+(
330      12.*(sqrt((st*(y**ye))))+(st*(y**ye)))))))+(3.464*(atan(((2.-(st*(y
331      2**ye)))/1.732/(sqrt((st*(y**ye)))))))+1.3138)*exp((bb/(w**0.33333
332      3)))/(w**0.16667)/((w**1.5)+(dy*(y**zz)))
333      return
334      end
335 c
336 c

```

```

337     real function f(x,k,par)
338     integer k
339     real x(1),par(1),lamda
340     common/exc/ xe,xf,xa,xs,xi,ye,yi
341     common/pow/ pinj,e0,ct,pd,hr
342     common/conft/ taupf,taupa,taupe,taups
343     common/con/ ee,ei,cd1,cd2,cd3,cd4,cd5,cd6,cn1,cn2,czho,cne,cbri,cb
344     r2,cbri3,clri1,clri2,czhfe,czye,cje,cfi,chi1,chi2,chi3,cc,hh
345     common/dim/ r,a,lamda,zs,xnf,xns
346     common/cp/ n,eps,nsig,itmax
347     write(6,120) k,x(1),x(2),x(3),x(4),x(5),x(6)
348 120 format("k=",i2,6(e10.3,2x))
349     go to (1,2,3,4,5,6), k
350 c
351 c ** fuel particle balance equation **
352 c
353     1 f=(x(1)*(1./taupf))+(ee*((x(1)**2)/(x(5)**0.6667))*sx(x(5),1))-(3.
354     116e20*(pd/e0))-xnf
355     write(6,100) f
356 100 format("fpb=",e13.5)
357     return
358 c
359 c ** alpha particle balance equation **
360 c
361     2 f=(x(2)*(1./taupa))-(ei*((x(1)**2)/(x(5)**0.6667))*sx(x(5),1))-((x
362     1(1)*x(6)/x(4))*zhx(x(6),n,2))
363     write(6,101) f
364 101 format("apb=",e13.5)
365     return
366 c
367 c ** total ion energy balance equation **
368 c
369     3 f=(x(3)*x(5)*cc)+(x(5)*((cfi*x(4))+(chi1*x(1))+(chi2*x(2))+(chi3*x
370     1ns)))-((x(6)*x(4)/(x(5)**1.5))*((cd1*x(1))+(cd3*x(2))+(cd5*xns)))+
371     2((x(4)/(sqrt(x(5))))*((cd2*x(1))+(cd4*x(2))+(cd6*xns)))+((x(1)**2
372     3)*x(6)/(x(5)**0.6667))*((cn2*sqrt(x(6))*sx(x(5),3))-(cn1*sx(x(5),2
373     4))))-zhoiln(x(6),n)-zhoiat(x(6),n)-(czho*x(6))-(((x(1)/x(4))*x(6)
374     5**2.5))*zhx(x(6),n,3))
375     write(6,102) f
376 102 format("tieb=",e13.5)
377     return
378 c
379 c ** electron energy balance equation **
380 c
381     4 f=(x(4)*x(6)*(xe+1.)*(ye+1.)*hh)+((x(6)*x(4)/(x(5)**1.5))*((cd1*x(
382     11))+(cd3*x(2))+(cd5*xns)))-((x(4)/(sqrt(x(5))))*((cd2*x(1))+(cd4*x
383     2(2))+(cd6*xns)))-(cje/(x(6)**1.5))+(((x(1)**2)/(x(5)**0.6667))*((x
384     3(6))*((cn1*sx(x(5),2))-(cn2*sqrt(x(6))*sx(x(5),3))))-(cne*sx(x(5),1
385     4))))+((x(4)*x(6))*((cbri1*x(1))+(cbri2*x(2))+(cbri3*xns)))+(czye*(x(4
386     5)**2)*(x(6)**2))+((x(4)*xns/(sqrt(x(6))))*(clri1+(clri2/x(6))))-3.17
387     6e20*pd+zhoiln(x(6),n)+zhoiat(x(6),n)+(czho*x(6))-((x(1)*x(6)**1.5

```

```

388      7)/x(4))*zhx(x(6),n,4))+((x(1)*(x(6)**2.5)/x(4))*zhx(x(6),n,3))
389      write(6,103) f
390      103 format("eeb=",e13.5)
391      return
392 c
393 c ** electron particle balance **
394 c
395      5 f=x(4)-x(1)-(2.*x(2))- (zs*xns)
396      write(6,104) f
397      104 format("epb=",e13.5)
398      return
399 c
400 c ** total ion particle balance **
401 c
402      6 f=x(3)-x(1)-x(2)-xns
403      write(6,105) f
404      105 format("tipb=",e13.5)
405      return
406      end
407 c
408 c
409      subroutine trans(n)
410      real lamda
411      dimension wa(38),par(1)
412      common x(6)
413      common/cp/ n,eps,nsig,itmax
414      common/exc/ xe,xf,xa,xs,xi,ye,yi
415      common/pow/ pinj,e0,ct,pd,hr
416      common/din/ r,a,lamca,zs,xnf,xns
417      common/conft/ taupf,taupa,taupe,taups
418      common/con/ ee,ei,cd1,cd2,cd3,cd4,cd5,cd6,cn1,cn2,chzo,cne,cbn1,cb
419      1r2,cbn3,cln1,cln2,czhfe,czve,cje,cfi,chn1,chn2,chn3,cc,hh
420      external f
421 c
422 c** the function f contains the transport equations **
423 c
424 c** here subroutine ccon calculates the needed constant coefficients **
425 c
426      call ccon
427 c
428 c** zsystem solves our simultaneous equations of transport **
429 c
430      call zsystem(f,eps,nsig,6,x,itmax,wa,par,ier)
431 c
432 c**check for problems given by ier **
433      if(ier.eq.129) write(6,100) itmax
434      100 format("failed to converge in ",i4," iterations")
435      if(ier.eq.139) write(6,200)
436      200 format("singularity in jacobian matrix has been encountered twice")
437      1)
438 c

```

```

437 c** check to see if solution x contains any zero or negative elements **
440 c
441     nn=0
442     do 10 i=1,6
443         if(x(i) .lt. 0. .or. x(i) .eq. 0.) nn=1
444     10 continue
445     return
446     end
447 c
448 c
449     subroutine ccon
450     real lamda
451     common/con/ ee,ei,cd1,cd2,cd3,cd4,cd5,cd6,cn1,cn2,czho,cne,cbr1,cb
452     1r2,cbr3,clr1,clr2,czhfe,czye,cfi,chl1,chl2,chl3,cc,hh
453     common/pow/ pinj,e0,ct,pd,hr
454     common/din/ r,a,lamda,zs,xnf,xns
455     common/conft/ taupf,taupa,taupe,taups
456     common/exc/ xe,xf,xa,xs,xi,ye,yi
457 c
458 c
459 c ** calculate the constants **
460 c
461     pd=pinj/(a**2)/r
462     ee=2.45e-13*hr*(1.-hr)*(((xf+1.))**2)/((yi+1.))**0.6667))
463     ei=ee/2.
464     cd1=5.02e-13*(2.+hr)*(ye+1.)*(xe+1.)*(xf+1.)/((yi+1.))**1.5)/(ye+xe
465     1+xf-(1.5*yi)+1.)
466     cd2=5.02e-13*(2.+hr)*(xe+1.)*(xf+1.)/(sqrt((yi+1.)))/(xe+xf-(0.5*y
467     1i)+1.)
468     cd3=3.01e-12*(ye+1.)*(xe+1.)*(xa+1.)/((yi+1.))**1.5)/(ye+xe+xa-(1.5
469     1*yi)+1.)
470     cd4=3.01e-12*(xe+1.)*(xa+1.)/(sqrt((yi+1.)))/(xe+xa-(0.5*yi)+1.)
471     cd5=3.01e-12*(zs/2.)*(ye+1.)*(xe+1.)*(xs+1.)/((yi+1.))**1.5)/(ye+xe
472     1+xs-(1.5*yi)+1.)
473     cd6=3.01e-12*(zs/2.)*(xe+1.)*(xs+1.)/(sqrt((yi+1.)))/(xe+xs-(0.5*y
474     1i)+1.)
475     cn1=1.29e-8*hr*(1.-hr)*(0.000457*((hr+2.))**0.6667))*((xf+1.))**2)*
476     1ye+1.)/((yi+1.))**0.6667)
477     cn2=1.29e-8*hr*(1.-hr)*(0.000283*((hr+2.)))*((xf+1.))**2)*((ye+1.))**
478     11.5)/((yi+1.))**0.6667)
479     czho=1.92e21*pd*((hr+2.))**0.6667)/e0
480     cne=4.31e-10*hr*(1.-hr)*((xf+1.))**2)/((yi+1.))**0.6667)
481     cbr1=3.95e-15*(sqrt((ye+1.)))*(xe+1.)*(xf+1.)/((ye*0.5)+xe+xf+1.)
482     cbr2=1.58e-14*(sqrt((ye+1.)))*(xe+1.)*(xa+1.)/((0.5*ye)+xe+xa+1.)
483     cbr3=3.95e-15*(sqrt((ye+1.)))*(xe+1.)*(xs+1.)/((ye*0.5)+xe+xs+1.)
484     clr1=1.14e-16*(xs+1.)*(xe+1.)*(zs**4)/(sqrt((ye+1.)))/(xs+xe-(0.5*
485     1ye)+1.)
486     clr2=2.58e-18*(xs+1.)*(xe+1.)*(zs**6)/(xs+xe-(1.5*ye)+1.)/((ye+1.))
487     1**1.5)
488     czhfe=4.84e24*pd*(1.-hr)*(xf+1.)*((ye+1.))**1.5)/(e0**1.6667)/(xe+1
489     1.)

```



```

490      cze=3.13e-16*((xe+1.)**2)*((ye+1.)**2)/((2.*(xe+ye))+1.)
491      cje=1.42e21*(ct**2)/((1.-(1.5*ye))/((ye+1.)**1.5)/(a**4)
492      cfi=2.5*((1.-(landa**2))*yi)/(xi+1.)/taupe
493      chc1=((1.-(landa**2))*yi*yi/(xi+1.))/xf/taupf
494      chc2=((1.-(landa**2))*yi*yi/(xi+1.))/xa/taupa
495      chc3=((1.-(landa**2))*yi*yi/(xi+1.))/xs/taups
496      cc=((1./taupf)*(1.+(xf/yi))/(xi+1.))+((1./taupa)*(1.+(xa/yi))/(xi+
497      1.))+((1./taups)*(1.+(xs/yi))/(xi+1.))
498      hh=(1./(xe+1.)/taupe)*(((1.-(landa**2))*((ye+2.)*(2.5+(3.*ye/xe)
499      1))-(1.+(xe/ye)))
500      return
501      end
502 c
503 c
504      subroutine wod
505      real landa
506      common x(6)
507      common/cp/ n,eps,nsig,itmax
508      common/exc/ xe,xf,xa,xs,xi,ye,yi
509      common/pow/ pinj,e0,ct,pd,hr
510      common/din/ r,a,landa,zs,xnf,xns
511      common/conft/ taupf,taupa,taupe,taups
512      common/con/ ee,ei,cd1,cd2,cd3,cd4,cd5,cd6,cn1,cn2,czho,cne,cbr1,cb
513      r2,cbr3,clr1,clr2,czhfa,czfe,cje,cfi,chi,chi2,chi3,cc,hh
514 c
515 c ** subroutine wod prints out the output **
516 c
517      write(6,100)
518      100 format("*** here are the input values *****")
519      write(6,110) xe,xf,xa,xs,xi,ye,yi
520      110 format("xe = ",f9.5/"x" = ",f9.5/"xa = ",f9.5/"xs = ",f9.5/"xi = "
521      1,f9.5/"ye = ",f9.5/"yi = ",f9.5//)
522 c
523      write(6,120) pinj,e0,ct,pd,hr
524      120 format("pinj = ",e13.5," eV"/"e0 = ",e13.5," keV"/"ct = ",e13.5,
525      1" na"/"pd = ",e13.5," nm/cn3"/"hr = ",e13.5//)
526 c
527      write(6,130) r,a,landa,zs,xnf,xns
528      130 format("r = ",e13.5," cm"/"a = ",e13.5," cm"/"landa = ",e13.5/"
529      1zs = ",e13.5/"xnf = ",e13.5," cm-3 sec-1"/"xns = ",e13.5," cm3"/
530      2/)
531 c
532      write(6,140) taupf,taupa,taupe,taups
533      140 format("taupf = ",e13.5," sec"/"taupa = ",e13.5," sec"/"taupe =
534      1",e13.5," sec"/"taups = ",e13.5," sec//)
535 c
536      write(6,150)
537      150 format("*****")
538 c
539      write(6,200)
540      200 format(" - - - here are some calculated values _ _ _ _ _")

```

```

541 c
542 write(6,210) x(1),x(2),x(3),x(4),x(5),x(6)
543 210 format("average fuel density = ",e13.5," cm-3"/"average alpha den
544 sity = ",e13.5," cm-3"/"average total ion density = ",e13.5," cm
545 2-3"/"average electron density = ",e13.5," cm-3"/"average ion temp
546 erature = ",e13.5," kev"/"average electron temperature = ",e13.5,
547 4" kev"/)
548 c
549 c ** calculate some output such as line radiation, etc. from the
550 c values given by the vector x **
551 c
552 brl=(x(4)*x(6))*((cbr1*x(1))+(cbr2*x(2))+(cbr3*xns))
553 crl=czye*(x(4)**2)*(x(6)**2)
554 rlr1=(xns*x(4)/sqrt(x(6)))*(clr1+(clr2/x(6)))
555 fpo=(ei*(x(1)**2)/(x(5)**0.6667))*(sx(x(5),1))*17600.
556 xntau=x(1)*taupf
557 tfpo=fpo*19.74*(a**2)*r
558 c
559 c
560 write(6,220) brl
561 220 format("average bremsstrahlung power loss = ",e13.5," kev/cm3 sec"
562 1//)
563 write(6,230) crl
564 230 format("average cyclotron power loss = ",e13.5," kev/cm3 sec"/)
565 write(6,240) rlr1
566 240 format("average line, recombination power loss = ",e13.5," kev/cm
567 13 sec"/)
568 write(6,250) fpo,tfpo
569 250 format("average fusion power output = ",e13.5," kev/cm3 sec"/"ave
570 rage total fusion power output = ",e13.5," kev/sec"/)
571 write(6,260) xntau
572 260 format("average n*tau of fuel = ",e13.5," sec/cm3"/)
573 write(6,270)
574 270 format("- - - - -")
575 return
576 end

```

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