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**DETERMINATION OF CROSS PLASMA EQUILIBRIUM
FROM MAGNETIC MULTIPOLES**

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Abstract

A new approximate technique to determine the gross plasma equilibrium parameters, major radius, minor radius, elongation and triangularity for an up-down symmetric plasma is developed. It is based on a multipole representation of the externally applied poloidal magnetic field, relating specific terms to the equilibrium parameters. The technique shows reasonable agreement with free boundary MHD equilibrium results. The method is useful in dynamic simulation and control studies.

1. Introduction

In the modelling of the dynamic behavior of tokamak plasmas, a determination of the successive equilibria is necessary. Changes in the electromagnetic environment and plasma internal conditions will alter the plasma's equilibrium characteristics, such as major radius, minor radius, and shape. To determine these plasma characteristics, a free boundary MHD calculation would have to be made at each time step in the discharge simulation. However, these calculations are complex and time-consuming, which excludes their use in a simulation modelling a full tokamak discharge. It is thus required that a new approximate technique be developed that can yield gross plasma equilibrium parameters (those characterizing only the outer flux surface) in an acceptable computational time.

A method has been developed to perform this task, and it is based on the premise that the externally applied poloidal magnetic field uniquely determines the plasma equilibrium characteristics, given a toroidal plasma current distribution, a plasma poloidal beta, a toroidal field, and a total plasma current. This is best illustrated by the equilibrium vertical field formula of Shafranov.⁽¹⁾

$$R = - \frac{\mu_0 I_p}{4\pi B_v} \left[\ln \frac{8R}{a} + \beta_p + \frac{l_1}{2} - \frac{3}{2} \right]$$

R is the plasma major radius, I_p the total plasma current, B_v the externally applied vertical field, a the plasma minor radius, β_p the plasma poloidal beta, and l_1 the plasma internal self-inductance per unit length (representing the plasma current density). The plasma major radius (center of the outermost flux surface) is determined by the applied vertical field and is dependent on the above plasma properties.

This premise arises also from the fact that if a strictly vertical field is applied to a tokamak plasma, the plasma would assume a circular cross-sectional shape. This is assuming the aspect ratio is sufficiently large, since it has been shown that very small aspect ratio tokamaks, spherical torii, (2) have a natural elongation in a vertical field. The application of a radial component to the external field can cause elongation and triangularity depending on the magnitude of the field and its direction (positive or negative) relative to the radial field produced by the plasma current. Thus, if one can correlate the external field characteristics to the plasma characteristics, a method can be developed to quickly determine the outer flux surface geometry from knowledge of the fields produced by coils and eddy currents.

II. Description of Tokamak Fields

In a conventional tokamak there are three primary magnetic fields, the strong toroidal field, the plasma poloidal field, and the external poloidal field. The toroidal field is produced by poloidal currents in coils normally located outside of the vacuum vessel and poloidal field coils. The plasma poloidal field is produced by a toroidal current in the plasma itself. This current in the plasma is either driven by an OH transformer or some form of RF current drive. The external poloidal field is produced by several poloidal field coils with current in the toroidal direction, located in the outer radial region of the tokamak. A schematic of the various features of a conventional tokamak are shown in Fig. II-1.

From MHD equilibrium theory, an external vertical field (B_z) is required to restrain the plasma outward motion due to toroidal plasma current/poloidal plasma field, external toroidal field/poloidal plasma current, and plasma

pressure forces. The first two forces are due to the fact that the plasma poloidal field and external toroidal field are stronger on the inside of the torus than the outside. The interaction of the vertical field and plasma toroidal current produces an inward force countering the expansion.

Equilibrium is now satisfied, however stability to plasma motion in the radial and vertical directions must be guaranteed. A circular plasma in a strictly vertical field is neutrally stable to vertical motion. A radial field component is added to the vertical field to produce a curved external field, often called a "barrel" field. The radial field is in the same direction as the radial field produced by the plasma current and is such that its interaction with the toroidal plasma current creates a downward force on the plasma if it moves upward and an upward force if the plasma moves downward. An illustration of the fields and currents in a tokamak are shown in Fig. II-2.

For radial stability one would intuitively think that if the vertical field strength increases with R then any outward motion would be forced back inward to the equilibrium point and visa versa. However, the need to produce the radial field for vertical stability has confined us to vertical fields which decrease with R . This can be illustrated by one of Maxwell's equations in the plasma region.

$$\nabla \times B = 0$$

The right hand side is zero because the external coil fields are produced by currents outside the plasma region. These fields are produced by strictly toroidal currents so we examine the component of the above equation in the ϕ -direction only.

$$\frac{\partial B_Z}{\partial R} - \frac{\partial B_R}{\partial Z} = 0$$

$$\frac{\partial B_Z}{\partial R} = \frac{\partial B_R}{\partial Z}$$

Now we want B_R to increase in the Z-direction to restrain the plasma vertical motion. Thus the derivative $\partial B_Z / \partial R$ must also be positive. However, the vertical field is in the negative Z-direction for the convention of a counter-clockwise toroidal plasma current. So for a negative quantity to increase, its magnitude must decrease. So we must have a vertical field decreasing in magnitude in the radial direction to achieve vertical stability.

This is not disastrous because the plasma is stable radially even with a decreasing vertical field. This is so because the rate of increase of the plasma outward force as it expands is slower than the decrease in the external vertical field so long as the decrease scale length or field decay index is less than one. The field decay index is defined, for up-down symmetric circular plasmas, as

$$n = - \frac{R}{B_Z} \frac{\partial B_Z}{\partial R}$$

In addition, if one includes the fact that the plasma would probably move on a time scale faster than the time for magnetic flux diffusion (toroidal and poloidal magnetic flux conservation) the field can decay at a rate as fast as 3/2. The well-known stability limits for both vertical and radial stability of an up-down symmetric circular plasma are given by^(3,4)

$$0 < n < \frac{3}{2}$$

In the case of an elongated plasma the situation is different. In order to elongate the plasma a radial field is set up opposite to the radial field produced by the plasma current (this is the most efficient means of elongation⁽⁵⁾). However, this radial field is opposite to that needed for stability. So elongated plasmas are inherently vertically unstable. Returning to Maxwell's equation

$$\frac{\partial B_z}{\partial R} = \frac{\partial B_R}{\partial z}$$

In this case the radial field is negative, although it still increases with increasing z (this is physically due to the fact that one is getting closer to the coils that produce it). Thus, the vertical field which still points in the negative z -direction must increase in magnitude as R increases. So the elongated plasma is clearly radially stable, but always vertically unstable. The circular and elongated cases are shown in Fig. II-3.

The decay index for vertical stability, $n > 0$ will determine a maximum elongation allowed before the plasma becomes unstable. This is generally less than 1.1 for sufficiently large aspect ratio. Again, in the case of spherical torii the natural elongation allows much greater elongation before instability.⁽²⁾ The stability limits have been derived quite rigorously including the effects of poloidal beta, plasma current distribution, and elongation.⁽⁶⁾ In virtually all experiments the fields are designed to generate an overall stable plasma (excluding vertical motion in an elongated plasma), however both radial and vertical plasma position are feedback controlled.

If the plasma is not up-down symmetric, as is the case in most divertor configurations, it has been proposed to examine another field scale length⁽⁷⁾

derived from another of Maxwell's equations

$$\nabla \cdot \mathbf{B} = 0$$

$$\frac{\partial B_z}{\partial z} + \frac{1}{R} \frac{\partial}{\partial R} (R B_R) = 0$$

so that a new decay index to reflect the asymmetry is introduced

$$K = - \frac{R}{B_z} \frac{\partial B_z}{\partial z}$$

With this brief discussion of the magnetic field configuration in a tokamak the multipole representation of the external field will follow.

III. Representation of External Magnetic Field by Multipoles

The up-down symmetric case will be treated in this study, so the plasma shape parameters will be elongation and triangularity. The technique involves representing the externally applied poloidal field by a magnetic multipole expansion; that is a dipole, quadrupole, and hexapole. The octapole field would be related to rectangularity, however, this and higher multipoles do not contribute substantially to a dee-shaped plasma, and so are neglected.

The dipole field is represented as a strictly vertical field. The quadrupole field is represented as a curving of the field lines, that is, a radial component has been added. This curvature can be characterized by the radial derivative of the vertical field. The hexapole field is represented as a change in the curvature of the field lines and is characterized as the second radial derivative of the vertical field. These fields are represented simplistically for a straight cylindrical plasma in Fig. III-1.

In a tokamak configuration these simple pictures are distorted due to toroidal geometry and the discreteness of the coils, however the general trends remain. Realistic field contours are shown in Fig. III-2 for various elongations and triangularities for INTOR⁽⁸⁾ plasmas at nearly zero poloidal beta (actually $\beta_p = 10^{-4}$). The circular plasma has the "barrel" external field providing the vertical field for equilibrium and the radial component for vertical stability. In Figs. III-2b to 2d the direction of the radial component changes in order to elongate the plasma, and higher elongations require sharper curvature to the field lines. In Figs. III-2e and 2f triangularity is imposed on a circular plasma with $\delta = .1$ and $\delta = .2$, respectively. The changing curvature can be seen quite clearly, the field lines bulging to the right on the right side and to the left on the left side. This bulging is more pronounced at higher triangularity. It can be seen that there are quite distinctive characteristics to the various magnetic multipoles which correspond to specific plasma shapes.

With this curvature characterization of the external field a natural choice is to expand the external poloidal magnetic field along the midplane in a Taylor's series about the plasma center,

$$B_p(R) = B_z(R_0) + (R - R_0) \left. \frac{\partial B_z}{\partial R} \right|_{R_0} + \frac{(R - R_0)^2}{2} \left. \frac{\partial^2 B_z}{\partial R^2} \right|_{R_0} + \dots$$

where R_0 is the plasma major radius. This expansion is then cast into a multipole expansion as the following

$$B_p(R) = B_D = \left(\frac{R - R_0}{a} \right) B_Q + \left(\frac{R - R_0}{a} \right)^2 B_H + \dots$$

with the following multipole definitions

$$B_D = B_Z(R_0)$$

$$B_Q = a \left. \frac{\partial B_Z}{\partial R} \right|_{R_0}$$

$$B_H = \frac{a^2}{2} \left. \frac{\partial^2 B_Z}{\partial R^2} \right|_{R_0}$$

It should be mentioned that since we are considering the up-down symmetric case only the Z-component of the external poloidal field appears. If the plasma was not up-down symmetric one would have B_R components and derivatives with respect to Z. This is given below

$$B_p(R, Z=0) = B_p(R_0, 0) + \sum_{m=1}^{\infty} \frac{1}{m!} \left(\Delta R \frac{\partial}{\partial R} + \Delta Z \frac{\partial}{\partial Z} \right)^m B_p \Big|_{(R_0, 0)}$$

where m is both a power and derivative order. In addition to the radial field component higher multipoles are also likely to be necessary.

The dipole field is the usual vertical field required to restrain the plasma from expanding radially. The quadrupole field is related to plasma elongation and the field decay index for vertical and radial stability. The hexapole field is related to both triangularity and elongation.

One expects the multipoles to be related to total plasma current, aspect ratio, poloidal beta, plasma internal self-inductance per unit length, and plasma shape parameters (κ -elongation and δ -triangularity).

$$B_m = B_m(I_p, \frac{R}{a}, \beta_p, l_i, \kappa, \delta)$$

The Shafranov expression mentioned earlier illustrated this, however it is valid for low poloidal beta and circular plasmas. A vertical field equilibrium formula modified for non-circular cross-sections and high poloidal beta is given by

$$B_v = - \frac{\mu_0 I_p}{4\pi R (\frac{1+\kappa}{2})^{1/4}} \left[\ln \frac{8R}{a} + \beta_p + \frac{l_i}{2} - \frac{3}{2} - \frac{1}{2} \frac{a^2}{R^2} \left(\beta_p + \frac{l_i}{2} \right)^2 \right]$$

which is similar to that used for ISX-B.⁽⁹⁾ An illustration of this expression versus the Shafranov expression is given in Fig. III-3. Included are MHD equilibrium calculations verifying the nonlinearity in the variable $(\beta_p + \frac{l_i}{2})$. The modified expression also displays the fact that less vertical field is required to restrain an elongated plasma than a circular one of the same minor radius.

In order to relate the other multipoles to plasma shape parameters several fixed/free boundary MHD calculations are done using NEQ⁽¹⁰⁾ developed at ORNL/FEDC. The plasma position and shape are specified, along with total plasma current, toroidal magnetic field, poloidal beta, and various profile parameters, and the required coil currents are determined. The coil currents are calculated interactively with the plasma internal conditions, such as plasma current distribution and pressure profile. With the coil currents known the poloidal magnetic field distribution along the midplane can be calculated and from it the multipoles. These multipoles are then related to the plasma elongation and triangularity specified.

The results of this procedure are displayed in Fig. III-4 and 5. The quadrupole field shows a cubic dependence on the plasma elongation and linear

shifting in poloidal beta. The hexapole field is found to have a linear dependence on the ratio of triangularity to the square of elongation. These are also shifted linearly in poloidal beta. Fitting these data points yield the following multipole to shape correlations, for INTOR⁽⁸⁾ parameters

$$B_Q = \frac{\mu_0 I_p}{2\pi a} [-1.488 + 2.543\kappa - 1.338\kappa^2 + .235\kappa^3 + .006g_p]$$

$$B_H = \frac{\mu_0 I_p}{2\pi a} [-\delta/\kappa^2 + .0185g_p - .01378]$$

where κ is the plasma elongation and δ is the plasma triangularity. With these empirical correlations one can determine the shape parameters of the plasma by knowing the currents in the external coils and eddy paths. No dependence on plasma internal self-inductance per unit length or aspect ratio has been developed. It is expected that the actual analytical correlations may be quite complicated.

IV. Use of Magnetic Multipoles to Determine Gross Equilibrium

With the multipole field characterization it is possible to determine the plasma major radius, elongation, and triangularity from the external coil and eddy currents directly. The minor radius can be determined by toroidal flux conservation, knowledge of the plasma edge safety factor, or by setting $a + R = \text{constant}$ to simulate the initial growing phase. Toroidal flux conservation and the plasma edge safety factor are given by

$$\psi_c = \int B_c Z dR \approx \pi B_c a^2 \kappa (1 - 3/8 \delta^2)$$

$$q_I = \frac{5a^2 B_c}{R I_p} \left(\frac{1 + \kappa^2}{2} \right)$$

To determine the four equilibrium parameters the poloidal beta, plasma internal self-inductance per unit length, total plasma current, toroidal magnetic field, and the total currents in each coil and eddy path must be known.

The expressions describing each of the parameters are strongly coupled to each other. The major radius formula depends on elongation and minor radius. The elongation and triangularity formulas depend on minor radius and the major radius where the field derivatives are evaluated. The safety factor equation to determine the minor radius depends on the major radius and elongation (and also triangularity if one replaces $1 + \kappa^2$ by $1 + \kappa^2(1 + \delta^2)$ to account for triangularity). The coupling is too severe for a closed solution so that an iteration procedure is required.

The iteration procedure involves starting with previous time step equilibrium parameters, evaluating the dipole, quadrupole and hexapole fields at the old major radius and minor radius with the new fields. A new elongation and triangularity are generated. Then a new minor radius and major radius are calculated, and the procedure is repeated. This is continued until a tolerance is reached in both minor and major radii. This procedure works well for elongated plasmas, however for circular plasmas the iteration diverges. This is due to the fact that the vertical field decreases with major radius.

This can be circumvented by determining the major radius by the radial plasma velocity and then entering the iteration scheme for minor radius, elongation, and triangularity. And, in fact, this is more convenient in a dynamic simulation where the plasma velocity is required for the circuit equations. The radial plasma velocity is determined by a balance between the actual vertical field applied and that required by equilibrium. The force

balance for a filamentary plasma is given by

$$m\ddot{R} = 2\pi R I_p (B_v - B_z)$$

where m is the plasma mass, R the plasma major radius, I_p the plasma current. The vertical field required to maintain the plasma at R and the actual vertical field imposed are given by

$$B_v = - \frac{\mu_0 I_p}{4\pi R (\frac{1+\kappa^2}{2})^{1/4}} \left[\ln \frac{8R}{a} + \beta_p + \frac{1}{2} - \frac{3}{2} - \frac{1}{2} \frac{a^2}{R^2} \left(\beta_p + \frac{1}{2} \right)^2 \right]$$

$$B_z = \frac{\mu_0}{4\pi} \sum_{i=1}^N f_i(R_i, Z_i; R, Z) I_i$$

where $f_i(R_i, Z_i; R, Z)$ is the function determining the vertical magnetic field at (R, Z) created by a circular current loop at (R_i, Z_i) . N is the total number of coils and eddy currents.

The plasma mass will be neglected, eliminating Alfvén time scale motion, retaining only the slower "circuit" time scale motion. This neglect of Alfvén time scale motions requires that the eddy currents produced in the structure surrounding the plasma are sufficient to restrain the plasma for at least as long as the Alfvén time scale. Since Alfvén times are generally on the order of microseconds and the eddy current decay times are on the order of milliseconds, this assumption should be valid.

The remaining equation is differentiated with respect to time

$$\frac{dB_v}{dt} = \frac{dB_z}{dt}$$

$$\frac{dR}{dt} = \frac{\frac{\partial B_v}{\partial I} \frac{\partial I_p}{\partial t} + \frac{\partial B_v}{\partial \kappa} \frac{\partial \kappa}{\partial t} + \frac{\partial B_v}{\partial a} \frac{\partial a}{\partial t} + \frac{\partial B_v}{\partial \beta_p} \frac{\partial \beta_p}{\partial t} + \frac{\partial B_v}{\partial I_i} \frac{\partial I_i}{\partial t} - \frac{\mu_0 N}{4\pi} \sum_{i=1}^N f_i \frac{\partial I_i}{\partial t} - \frac{\mu_0 N}{4\pi} \sum_{i=1}^N \frac{\partial f_i}{\partial Z} \frac{\partial Z}{\partial t} I_i}{\frac{\mu_0 N}{4\pi} \sum_{i=1}^N \frac{\partial f_i}{\partial R} I_i - \frac{\partial B_v}{\partial R}}$$

The time rates of change of the plasma parameters are determined from previous time step information.

The vertical plasma position can also be determined from the vertical plasma velocity in the same way, neglecting the plasma mass. The radial magnetic field is given by

$$B_R = \frac{\mu_0 N}{4\pi} \sum_{i=1}^N g_i(R_i, Z_i; R, Z) I_i$$

where g_i is similarly defined as f_i except that it is the radial field at (R, Z) produced by a circular current loop at (R_i, Z_i) . Taking the time derivative yields

$$\frac{dZ}{dt} = \frac{-\frac{\mu_0 N}{4\pi} \sum_{i=1}^N g_i \frac{\partial I_i}{\partial t} - \frac{\mu_0 N}{4\pi} \sum_{i=1}^N \frac{\partial g_i}{\partial R} \frac{\partial R}{\partial t} I_i}{\frac{\mu_0 N}{4\pi} \sum_{i=1}^N \frac{\partial g_i}{\partial Z} I_i}$$

The radial and vertical positions are determined by the plasma velocities, and the plasma minor radius, elongation and triangularity are determined iteratively. If the plasma vertical position gets too far off the $Z=0$ midplane the shape determinations will not be valid, however, this can be corrected by a feedback scheme on both radial and vertical plasma position.

Comparisons of the completely iterative method with free boundary MHD results are shown in Figs. IV-1 and 2. The iterative method provides a quite good approximation to the actual solution. The plasma velocity and iteration

scheme showing the dynamic behavior of the equilibrium as time progresses during a discharge simulation is not available at present.

V. Conclusions

A successful technique for estimating the gross equilibrium parameters of an up-down symmetric tokamak plasma has been developed. The method relies on characterizing the externally applied magnetic field by multipoles that correspond as directly as possible to given plasma shape characteristics. With such a correspondence the shape, size and position of the plasma outer flux surface can be determined from coil and eddy currents directly. The technique shows good agreement with actual free-boundary MHD calculations and is ideally suited for studying the dynamic behavior of tokamak discharges.

Acknowledgments

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VII. Figure Captions

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- Fig. II-2. Illustration of the fields and currents in a conventional circular plasma tokamak.
- Fig. II-3. Illustration of the fields and currents in a tokamak for a circular and elongated plasma.
- Fig. III-1. Schematic of the various multipole contributions to a strictly vertical field showing the resulting field and plasma shapes for a straight cylindrical plasma. The plasma current is into the page. (A) A weak quadrupole field is superimposed on a vertical field to produce a circular plasma with a slight radial field. (B) A strong quadrupole field is superimposed on a vertical field to produce a horizontally elongated plasma with a strong radial field. (C) A strong quadrupole field of the opposite polarity is superimposed on a vertical field to produce a vertically elongated plasma. (D) A hexapole field is superimposed on a vertical field to produce a triangular plasma with a radial field varying along the midplane. (E) An octupole field is superimposed on a vertical field to produce a rectangular plasma with a varying radial field in the vertical direction.
- Fig. III-2. Actual magnetic field contours for the INTOR tokamak: (A) - circular plasma, (B) - 1.3 elongation plasma, (C) - 1.5 elongation plasma, (D) - 1.8 elongation plasma, all with zero triangularity; (E) - 0.1 triangularity plasma, (F) - 0.2 triangularity plasma, both with zero elongation.
- Fig. III-3. Comparison of (S) - the linear Shafranov equilibrium vertical field expression, (triangle) - the modified quadratic equilibrium expression, and (diamond) - actual MHD results averaged for elongations between 1.0 and 1.8 and triangularities between 0.0 and 0.2.
- Fig. III-4. Normalized quadrupole field as a function of plasma elongation for poloidal beta at 0.0, 0.5, 1.0, 1.5, and 2.0. The lowest curve is for zero poloidal beta and increases upward.
- Fig. III-5. Normalized hexapole field as a function of triangularity divided by elongation squared for poloidal beta at 0.0, 0.5, 1.0, 1.5, and 2.0. The lowest curve is for zero poloidal beta and increases upward.
- Fig. IV-1. Comparison of the multipole method for gross plasma equilibrium and actual free boundary MHD results for a 1.3 elongation and 0.2 triangularity plasma. This is for the INTOR tokamak.
- Fig. IV-2. Comparison of the multipole method for gross plasma equilibrium and actual free boundary MHD results for a 1.5 elongation and 0.2 triangularity plasma. This is for the INTOR tokamak.

Fig. II-1.

Fig. II-1. The various features of the conventional tokamak.

Fig. II-2. Illustration of the fields and currents in a conventional circular plasma tokamak.

Fig. 11-3. Illustration of the fields and currents in a tokamak for a circular and elongated plasma.

(A) A weak quadrupole field is superimposed on a vertical field to produce a circular plasma with a slight radial field.

(B) A strong quadrupole field is superimposed on a vertical field to produce a horizontally elongated plasma with a strong radial field.

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Fig. III-1. Schematic of the various multipole contributions to a strictly vertical field showing the resulting field and plasma shapes for a straight cylindrical plasma. The plasma current is into the page.

(D) A hexapole field is superimposed on a vertical field to produce a triangular plasma with a radial field varying along the midplane.

(E) An octapole field is superimposed on a vertical field to produce a rectangular plasma with a varying radial field in the vertical direction.

Fig. III-1. Schematic of the various multipole contributions to a strictly vertical field showing the resulting field and plasma shapes for a straight cylindrical plasma. The plasma current is into the page.

(A)

(B)

Fig. III-2. Actual magnetic field contours for the INTOR tokamak. (A) - circular plasma, (B) - 1.3 elongation plasma, both with zero triangularity.

(C)

(D)

Fig. III-2. Actual magnetic field contours for the INTOR tokamak. (C) - 1.5 elongation plasma, (D) - 1.8 elongation plasma, both with zero triangularity.

(E)

(F)

Fig. III-2. Actual magnetic field contours for the INTOR tokamak. (E) - 0.1 triangularity plasma, (F) - 0.2 triangularity plasma, both with zero elongation.

EQUILIBRIUM VERTICAL FIELD VERSUS (POL. BETA + INT. SELF IND.)
 TRIANGULARITY=0.0-0.2 ELONGATION=1.0-1.8

Fig. III-3. Comparison of (S) - the linear Shafranov equilibrium vertical field expression, (triangle) - the modified quadratic equilibrium expression, and (diamond) actual MHD results averaged for elongations between 1.0 and 1.8 and triangularities between 0.0 and 0.2.

QUADRUPOLE FIELD VERSUS PLASMA ELONGATION

TRIANGULARITY=0.0-0.2 ELONGATION=1.0-1.8

Fig. III-4. Normalized quadrupole field as a function of plasma elongation for poloidal beta at 0.0, 0.5, 1.0, 1.5, and 2.0. The lowest curve is for zero poloidal beta and increases upward.

HEXAPOLE FIELD VERSUS TRIANGULARITY/ELONGATION**2 TRIANGULARITY=0.0-0.2 ELONGATION=1.0-1.8

Fig. III-5. Normalized hexapole field as a function of triangularity divided by elongation squared for poloidal beta at 0.0, 0.5, 1.0, 1.5, and 2.0. The lowest curve is for zero poloidal beta and increases upward.

MHD EQUILIBRIUM PLASMA SHAPE

DIAMONDS=EXACT MHD

STARS=MULTIPOLE MHD

Fig. 1V-1. Comparison of the multipole method for gross plasma equilibrium and actual free boundary MHD results for a 1.3 elongation and 0.2 triangularity plasma. This is for the INTOR tokamak.

MHD EQUILIBRIUM PLASMA SHAPE

DIAMONDS=EXACT MHD

STARS=MULTIPOLE MHD

Fig. IV-2. Comparison of the multipole method for gross plasma equilibrium and actual free boundary MHD results for a 1.5 elongation and 0.2 triangularity plasma. This is for the INTOR tokamak.