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Vacuum Ultra Violet Monochromators*

by

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Introduction

The general field of spectrometers for the ultra violet has been covered at great length in the past and much invaluable material is on record (1-5). Here we limit our attention to monochromators rather than spectrographs and consider only those which are of use in the vacuum ultra violet region (say 10-1800 Å for Synchrotron Radiation work. We further restrict our consideration to those which use reflection gratings although it is clear that the traditional domination of the field by these devices is being increasingly challenged by the development of transmission gratings of various kinds. (6,7)

Even with these limitations the number of different optical arrangements that have been used is still very large so we will make some attempt to see how the general classes of Rowland circle, plane grating and the new Toroidal grating monochromators (TGMs) have succeeded in the past. We then make some fairly subjective evaluations of which approaches hold the best promise for the future and discuss

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in more detail the basis of these approaches and how they might be implemented.

Present Technologies

Some areas of our wavelength range of interest are covered better than others by existing methods. Table I⁽²⁾ summarizes the position. We see that for the range 300-1800 Å where normal incidence optics can be used and optical systems with low aberrations can be designed that the requirements can be met. It is the grazing incidence region (10-300 Å) where most problems arise. There are a bewildering variety of desirable features that every user would like to have plus more mundane pressures like shortage of money and time and questions about whether to commit valuable scientific manpower to the task of acquiring expertise in UV optics. The waters are further muddled by the influence of external factors like source geometry (especially size), beamline length and so on and by an understandable desire that an expensive grazing monochromator should provide some coverage of the normal incidence range as well. In the next section we try to catalogue this complex set of requirements to form a basis for evaluating the available design approaches.

User Requirements

Table III⁽⁴⁾ attempts to collect together a plausible set of requirements and to give very rough one word assessments of how well the three classes of grazing monochromator considered can handle the requirements.

Before studying the scorecard, we may note that the light gathering power of a monochromator (line 2) is expressed as the collection (solid angle) $\times$ (area) or étendue. This quantity is really the traditional two

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dimensional 'Helmholtz invariant'⁽⁸⁾ which is invariant through any loss-free optical system. It is a function of resolving power for all cases and is the only way to compare the light gathered by a Rowland instrument (large solid angle, small aperture (slit)) and that by a plane grating instrument (small solid angle, large aperture).

Turning to Table II we can best absorb a message from this if we view it as a trial of the Rowland circle approach. We can identify two primary difficulties which the special situation of SR raises for this case. These are #5: fixed in and out directions and (normally) #6 rejection of higher diffracted orders. These are caused by the immovability and spectral continuity of the synchrotron source. Various secondary difficulties are caused in turn by these. In particular, the only way to accomodate #5 is by extra reflections and special mechanism's. This gives problems of transmission efficiency and complications of manufacture and use. The only way #6 has been solved in Rowland designs has been by the use of predis persers.⁽¹⁰⁾ This adds further to the difficulties with large numbers of reflections, increased sensitivity to contamination and inconvenient and expensive mechanisms.

These disadvantages of the Rowland circle approach make us turn back to its major advantage i.e. its capability for good resolution. This is based on the fact that certain aberrations vanish on the Rowland circle. We expecially need a small wavelength bandpass at low wavelength where a small ΔE is required. Consequently we look at the resolution achieved by use of the Rowland Circle in the difficult 10-100 Å range. If we continue to confine our attention to instruments with constant deviation then around 0.04Å (or a resolving

of 250-2500) seems to be the lower limit. This is not an aberration limit but seems to arise indirectly from the transmission difficulties mentioned above and the punishing loss of flux that occurs as both slits are closed down to widths of several microns. There is also a rather limited demand for resolution better than $.04\lambda$.

In the light of these considerations we must ask ourselves whether it is necessary to continue to wrestle with the formidable problems of the constant deviation Rowland mount or whether we cannot achieve as good or better resolution using plane grating instruments. These offer the possibility of fewer reflections, simpler mechanisms, cheaper gratings but a resolving power which depends on the source size (see later). Thus the suitability of the plane grating principle is dependent in part on the source size. This has been discussed recently by Pouett⁽²⁰⁾. A recent design study of a plane grating instrument⁽²¹⁾ for use at the Brookhaven storage ring has indicated a source size limited resolution (with a 1200 λ/mm grating) of less than $.025\lambda$ for the range 25-100 λ . This is excellent resolution and shows that with a present generation, dedicated storage ring plane grating monochromators (PGM's) look very good.

TGM's and PGM'S

Now that we have argued that TGM's and PGM's are a promising way to meet a large number of the requirements of table II let us look at some of the features they have in common and discuss some design concepts.

The general requirement of fixed deviation plus our choice of the simple rotation scanning principle for both TGM's & PGM's implies that we must consider the inward and outward direction as fixed. This is

shown in figure 1. Using the notation and sign convention shown in that figure we may write down a description of our system:

(i) We have the grating equation

$$Nk\lambda = \sin\alpha + \sin\beta \quad (1)$$

and for fixed in and out directions we also have

$$\alpha - \beta = 2\theta \quad (2)$$

Hence if we know N and θ we have

$$\beta = \sin^{-1} \left[\frac{Nk\lambda}{2\cos\theta} \right] + \theta$$

$$\alpha = \sin^{-1} \left[\frac{Nk\lambda}{2\cos\theta} \right] + \theta$$

and the configuration is well defined.

(ii) Another result of working with constant deviation is that we

see the so called 'horizon' effect i.e. there is a limit λ_H to the wavelength scan which occurs when α or $\beta = 90^\circ$. In fact

$$\lambda_H = \frac{2\cos^2\theta}{Nk} \quad (3)$$

(iii) If we use the light source as an entrance slit then its

finite size (s) and distance (r) provide a limit on the

wavelength resolution $\Delta\lambda_s$ given by

$$\Delta\lambda_s = \frac{s \cos\alpha}{Nkr} \quad (4)$$

(iv) From (1) and (2) we see that

$$Nk\lambda = 2\cos\theta\sin(\theta + \beta)$$

Now since $(\theta + \beta) = \phi$ the angle the grating has turned from

zero order, we have λ proportional to $\sin \phi$. Hence we can have an exactly linear wavelength scale if we use a sine bar drive with $\phi = 0$ at zero order. This fits well to the simple rotation scanning principle we are talking about.

(v) The magnification of a TGM is easily seen to be given by:

$$\text{Magn.} = \frac{r' \cos \alpha}{r \cos \beta}$$

For PGM's we have nearly parallel light so we must talk of an angular magnification given by

$$\text{Ang. Magn.} = \frac{\cos \alpha}{\cos \beta}$$

These general statements apply equally well to both PGM's and TGM's but they give no indication of how, from a design point of view, one might choose N and θ . It turns out that we have very little freedom to manipulate θ which is determined entirely by the short wavelength performance that is required. The margin of possible error is rather wide in practice since reflectance^(22,23) varies considerably with deposition method and contamination state even for nominally identical surfaces. Consequently there is considerable spread in the available data. However, figure 2 gives enough of an overview of actual measurements to enable a reasonable choice to be made for any particular spectrometer.

The choice of N is best made on the basis of the horizon wavelength (equation 3). The best choice is the maximum groove density (best resolution) that gives a λ_H which is larger than the longest wavelength of interest by a reasonable margin. We can choose to cover a desired range with more than one grating. For example if we had a design with

$N = 1200 \text{ l/mm}$ $\theta = 87.5^\circ$ then $\lambda_H \sim 32\text{\AA}$ so we might hope to cover 10-25 $\text{\AA}$.

If we had a design that worked from an aberration and focus point of view over this range then we could also have the full scheme:

$N = 1200 \text{ l/mm}$	$\lambda = 10-25 \text{\AA}$	$\lambda_H = 32 \text{\AA}$
$N = 600 \text{ l/mm}$	$\lambda = 20-50 \text{\AA}$	$\lambda_H = 63 \text{\AA}$
$N = 300 \text{ l/mm}$	$\lambda = 40-100 \text{\AA}$	$\lambda_H = 127 \text{\AA}$

This is because $N\lambda$ is the same and hence also α and β . (eqn 1)

The outstanding parameter left to choose is the order k . For PQM's the issues are limited but for TGM's there are a number of questions. These concern the collection aperture, r and r' values, the source size resolution limit and magnification behaviour.

Toroidal Grating Analysis

Following normal practice we can define a suitable coordinate system (Figs. 1 and 2) and write down an expression for the optical path F of the ray APB. (For simplicity we confine ourselves to the case of A in the x, y plane.) This can be expressed as a power series in the aperture co-ordinates y, z .

$$F = \sum_{ij} F_{ij} = r+r' + (\sin\alpha + \sin\beta) y + C_{20} \frac{y^2}{R} + C_{30} \frac{y^3}{R^2} \\ + C_{02} \frac{z^2}{R} + C_{12} \frac{yz^2}{R^2} + C_{40} \frac{y^4}{R^3} + C_{04} \frac{z^4}{R^3} + \frac{y^2 z^2}{R^3} \\ + \text{higher order terms}^{(5)}$$

Each $F_{ij} \left(= C_{ij} \frac{y^i z^j}{R^{i+j-1}} \right)$ corresponds to some particular type of

aberration. Following Haber we can derive expressions for the C_{ij}

(see Appendix 1) but here we have factored the R dependence out and expressed the C_{ij} 's as functions only of r^* , $r^{*'}$, ρ^* , α and β where

the * indicates that the length is expressed in units of R. This means the C_{ij} 's are functions only of the shape, not the size of the configuration. Fermat's principle tells us that the conditions for a kth order diffracted image at B are

$$\frac{\partial F}{\partial y} = Nk\lambda \quad \text{and} \quad \frac{\partial F}{\partial z} = 0$$

Combining these with (5) we obtain the grating equation (1) and the condition that all the C_{ij} 's are zero i.e. no aberrations.

When $\frac{\partial F}{\partial z} \neq 0$ then we have aberrations and the rays arriving in the image plane are displaced by some amount from the ideal Gaussian image point B. The significance of the non-zero differentials^(17,25) is that of a change in the appropriate direction cosine (γ) of the emergent ray from the 'correct' value i.e.

$$\frac{\partial F}{\partial y} = \delta(\cos \gamma_y) = d(\sin \beta) = \cos \beta \, d\beta = \cos \beta \, \frac{\Delta Y}{r'}$$

the significance of Y is shown in fig. 2. It is the displacement of the ray AP when it arrives in the image plane. Thus

$$\Delta Y = \frac{r' \frac{\partial F}{\partial y}}{\cos \beta} \text{ - - - - - (6)}$$

$$\text{Similarly } \Delta Z = r' \frac{\partial F}{\partial z} \text{ - - - - - (7)}$$

We also have from (1) an expression for the reciprocal linear dispersion

$$\frac{\Delta \lambda}{\Delta Y} = \frac{\cos \beta}{r' Nk} \text{ - - - - - (8)}$$

thus we find

$$\Delta \lambda = \frac{1}{Nk} \frac{\partial F}{\partial y}$$

and if we imagine an overall $\Delta \lambda$ comprising a lot of $\Delta \lambda_{ij}$'s arising from the various aberration terms then we have

$$\Delta \lambda_{ij} = \frac{1}{Nk} \frac{\partial F_{ij}}{\partial y} = \frac{1}{Nk} C_{ij} \, i y^{i-1} \, z^{*ij}$$

giving the important conclusion that the aberration limited resolution is independent of R ! Since the resolution of practical TGM's is normally aberration limited we may ask why we should not have a very small R and hence a small monochromator. The answer is that although increasing R does not change the resolution, it does (via (8)) increase the corresponding slit widths. If we assume that the slit length is about 1-2 mm for all cases then we conclude that the étendue is proportional to R . Consequently other things being equal the light collected is proportional to R . One can imagine that it might sometimes be attractive to reduce the size and hence the cost of a monochromator substantially if the intended light source were bright enough.

Astigmatism:

The primary purpose of using a toroidal grating is to reduce astigmatism. This is achieved at a wavelength corresponding to angles α and β if we set C_{02} (see equation A3) equal to zero

$$\text{i.e.} \quad \rho^* = \frac{\cos\alpha + \cos\beta}{\frac{1}{r^*} + \frac{1}{r^{*'}}} \quad \text{--- (9)}$$

One can readily show that if $\alpha - \beta = 2\theta > 150^\circ$ then $(\cos\alpha + \cos\beta)$ is constant within about 1-2%. Thus if astigmatism is corrected at one wavelength then the correction is good to sufficient accuracy over the whole range.

Focus:

Assuming we have a value for ρ^* we must now proceed to design a configuration which will optimise the resolution determining aberrations over some chosen wavelength range. The most important ones in practice

turn out to be focus $\left(C_{20}\frac{y^2}{R}\right)$ and astigmatic coma $\left(C_{12}\frac{yz^2}{R^2}\right)$ or

$$\Delta\lambda_{20} = \frac{1}{Nk} \frac{2y}{R} C_{20} \quad \text{and} \quad \Delta\lambda_{12} = \frac{1}{Nk} \frac{z^2}{R^2} C_{12}$$

From the form of the terms it is obvious that, as always, one can get a better image by reducing the aperture. In particular reducing the ruled width (2y) improves focus and reducing the groove length (2z) improves astigmatic coma.

We have two degrees of freedom to work with, namely choice of r^* and $r^{*'}.$ In figure 3 we show schematically how the term C_{20} varies with wavelength. Following Lapere²⁶ (who treated the aberration corrected case) we can see that we can fix on two wavelengths λ_1 and λ_2 and arrange to have a good focus at these wavelengths. Suppose that N and θ are given and α_1 and β_1 correspond to λ_1 and α_2 and β_2 to λ_2 . Then putting $C_{20} = 0$ at λ_1 & λ_2 gives

$$\frac{1}{2} \left[\left(\frac{\cos^2 \alpha_1}{r^*} - \cos \alpha_1 \right) + \left(\frac{\cos^2 \beta_1}{r^{*'}} - \cos \beta_1 \right) \right] = 0$$

$$\frac{1}{2} \left[\left(\frac{\cos^2 \alpha_2}{r^*} - \cos \alpha_2 \right) + \left(\frac{\cos^2 \beta_2}{r^{*'}} - \cos \beta_2 \right) \right] = 0$$

Clearly one can solve these for r^* and $r^{*'}.$ By this process one can easily arrange to have an approximate focus over the range between λ_L and λ_R . For spectroscopy where a given $\Delta\lambda$ is required one would choose λ_1 and λ_2 symmetrically between λ_L and λ_R . For experiments where a given ΔE is required λ_1 & λ_2 would be pushed toward the important short wave-length end of the range.

Astigmatic coma:

If imperfect focus were the only important aberration then the optimum solution would be close at hand. However, we have already used our only two degrees of freedom and we have not yet addressed the

contribution from $\Delta\lambda_{12}$. To get a feeling for the form of this we proceed

as follows: set $\frac{\partial C_{20}}{\partial \lambda} = 0$ with the assumption (see earlier) that $\cos\alpha + \cos\beta = \text{const.}$ This gives us a condition that is satisfied at the wavelength $(\lambda_{\min})$ for which C_{20} has a minimum.. i.e.

$$\frac{\sin\alpha \cos\alpha}{r^*} = - \frac{\sin\beta \cos\beta}{r^{*'}} \quad \text{--- (10)}$$

Now consider the form of C_{12}

$$C_{12} = \frac{1}{2} \left[\frac{\sin\alpha}{r^*} \left(\frac{1}{r^*} - \frac{\cos\alpha}{\rho^*} \right) + \frac{\sin\beta}{r^{*'}} \left(\frac{1}{r^{*'}} - \frac{\cos\beta}{\rho^*} \right) \right]$$

If we consider the terms containing ρ^* we can see that C_{12} is independent of ρ^* if (10) is satisfied. In fact we see that the C_{12} curves corresponding to various ρ^* values pass through a point P at $\lambda_{\min}$ as shown in fig. 3.

Clearly we would like P to lie on the λ axis and we might hope that a ρ^* value corresponding to a fairly horizontal C_{12} curve might also correspond to an acceptable amount of astigmatism correction.

Before we address this question let us examine the nature of astigmatic coma⁽²⁷⁾ in a little more detail. Its effect is similar to coma for the case of near zero astigmatism (see fig. 4) and more like spectrum line curvature if significant astigmatism is present. Fig 5 shows a realistic case where some defocus is present as well. In both cases the lines joining the dots of the ray trace corresponds to lines of constant y on the grating and the very heavy lines to the top and bottom edge of the grating. By using (6) and (7) we can compute the amount of Y and Z deflection and understand the origin of the parabolic curves. We can also see the effect of increasing $y_{\max}$ or $z_{\max}$ on the size and shape of the image in the Y, Z system.

Returning to the design question let us try to move the point P to

reduce C_{12} . We know that at P, equation (10) holds. If in addition we insist that $C_{12} = 0$ i.e. P lies on the λ axis of fig 3 then we find from (A4)

$$\frac{\sin \alpha}{r^*2} = \frac{\sin \beta}{r^*12}$$

Solving this with (10) leads to the conditions

$$\alpha = -\beta \quad r^* = r^{*1} \quad \text{and} \quad \lambda_{\min} = 0$$

So that P in fact lies at the origin of fig 3. Here we have adopted a second extreme approach. Previously we saw how to optimise C_{20} whilst ignoring C_{12} . Now we have found a scheme that turns out to optimise C_{12} rather well but largely ignores C_{20} . We can still use one degree of freedom to determine one wavelength of exact focus, λ_F say (see fig 6). This involves choosing a value for $r^* = r^{*1}$. To do this we make use of the parameter $\Lambda = \lambda / \lambda_H$.

From (3) we can rewrite (2a) and (2b) as

$$\beta = \sin^{-1} (\Lambda \cos \theta) - \phi$$

$$\alpha = \sin^{-1} (\Lambda \cos \theta) + \phi$$

Substituting these into (A1) and setting $C_{20} = 0$ for the wavelength

λ_F we find

$$r^* = \frac{\cos \theta \left(1 - \cos 2\theta \Lambda_F^2 \right)}{\sqrt{1 - \Lambda_F^2 \cos^2 \theta}} \quad \text{----- (11)}$$

This gives us our monochromator configuration. The ray trace (fig 7) shows as expected that the curvature from astigmatic coma is much reduced and defocus dominates. (11) is an interesting equation containing a number of fruitful cases which are discussed elsewhere.^(28,29)

Overall optimisation:

Whilst the two extreme approaches we have discussed here are physically revealing, neither of them leads to the best TQM designs. To get the

best design one must search parameter space with a computer program to find the best performance at a sequence of chosen wavelengths within the range of interest. This will represent the best compromise between control of C_{20} and C_{12} which are the main resolution determining aberrations. Once the configuration is established the actual resolution/flux trade-off can be established by choice of the grating width and height. The method used at Brookhaven for doing this is discussed in another contribution to this meeting⁽³⁰⁾.

One might legitimately ask how the adequacy of these theoretical and computational design procedures is checked. It is essential to have a good ray trace program for this purpose and fortunately modern microprocessor technology makes this reasonably easy to achieve on a self contained basis. Our approach has actually been to have more than one program using completely different methods of calculation. Since we are dealing with low resolving power, poor imaging, systems it is safe to assume that the geometrical optics approximation is valid.

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Figure Captions

1. Notation and sign convention for the grating equation
2. Measured data from various sources giving approximate cut-off wavelength as a function of grazing angle.

X	J. B. West et al actual mon'r threshold
□	θ (5%) measured from Pt V. Rehn private comm.
⊙	$E = 4.8 \times 10^3 / \theta$ (eV and deg) Pt: X-ray range
— · — · —	Calculated threshold from n and k values (ref. 31)
————	Calculated threshold from n and k values (ref. 32)
△	θ (5%) measured for Pt Romer (ref 4, P. 73)
⊗	Henke θ (5%) measured for Au
⊞	θ (5%) measured for Au (ref. 1, P. 36)
⊕	θ (5%) measured for Au (ref. 34)
+	θ (5%) measured for Au (ref. 33)

Since the scatter is so great there seems little point in separating the Au and Pt values

3. Co-ordinate systems used to discuss the 'perfect ray' APB to the Gaussian focus B and actual ray APE'.
4. Schematic variation of C_{20} and C_{12} with wavelength.
5. Toroidal grating image with C_{12} the dominant aberration. The r' value is chosen for exact focus and the astigmatism is small. Equations (6), (7) and (A) are used to compute ΔZ and ΔY and show how a parabolic curve arises for each y value. Y and Z are in 100's of microns and the parameters are

$$R = 100 \quad \rho = 6.1 \quad r = 38.5 \quad r' = 17.14 \text{ (all cm)}$$

$$\theta = 150^\circ \quad n = 1200 \text{ Å/mm.} \quad \lambda = 150 \text{ Å} \quad k = -1$$
6. More realistic image for the same grating as fig. 5. This time $r' = 17.5$ cm the other parameters are the same. Now both defocussing and astigmatism are significant. ΔZ and ΔY are computed as before with the assumption that ΔZ is determined by C_{02} and ΔY by C_{20} and C_{12} . Again the observed parabolic shape is explained and the role of the two aberrations becomes clear.
7. Schematic of the variation with wavelength of C_{20} and C_{12} for the case $r = r'$. Note the point P has moved to the origin and $\lambda_{\min} = 0$. The instrument can now be operated in either positive or negative order.
8. Image produced by the same toroidal grating as shown in figs. 5 and 6. This time configured with conjugates $r = r' = 26.41$ cm. These correspond to choosing $\lambda_F = 0.15\lambda_H$ giving $\lambda_F = 167\text{Å}$. The image is ray traced for $\lambda = 250 \text{ Å}$ which is considerably out of focus. The dominant influence of C_{20} (defocus) and the small effect of C_{12} is clearly seen.

Appendix

Here we list a few of the terms of the expansion (5).

Focussing term $C_{20} \frac{v^2}{R}$

$$C_{20} = \frac{1}{2} \left[\left(\frac{\cos^2 \alpha}{r^*} - \cos \alpha \right) + \left(\frac{\cos^2 \beta}{r^{*'}} - \cos \beta \right) \right] - - - - - (A1)$$

Coma term $C_{30} \frac{v^3}{R^2}$

$$C_{30} = \frac{1}{2} \left[\frac{\sin \alpha}{r^*} \left(\frac{\cos^2 \alpha}{r^*} - \cos \alpha \right) + \frac{\sin \beta}{r^{*'}} \left(\frac{\cos^2 \beta}{r^{*'}} - \cos \beta \right) \right] - - - (A2)$$

Astigmatism term $C_{02} \frac{z^2}{R}$

$$C_{02} = \frac{1}{2} \left[\left(\frac{1}{r^*} - \frac{\cos \alpha}{\rho^*} \right) + \left(\frac{1}{r^{*'}} - \frac{\cos \beta}{\rho^{*'}} \right) \right] - - - - - (A3)$$

Astigmatic coma term $C_{12} \frac{v z^2}{R^2}$

$$C_{12} = \frac{1}{2} \left[\frac{\sin \alpha}{r^*} \left(\frac{1}{r^*} - \frac{\cos \alpha}{\rho^*} \right) + \frac{\sin \beta}{r^{*'}} \left(\frac{1}{r^{*'}} - \frac{\cos \beta}{\rho^{*'}} \right) \right] - - - - - (A4)$$

The remaining terms are of little importance for approximate focussing situations. However, they can be found in ref. 24.

TABLE I

<u>WAVELENGTH RANGE ($\text{\AA}$)</u>	<u>TECHNOLOGY</u>	<u>REMARKS</u>
OVER 1200	WINDOWS POSSIBLE	FEW PROBLEMS
300 - 1200	NORMAL INCIDENCE OPTICS O. K. REFL. AU $\gtrsim$ FEW %	SMALL ABERRATIONS STANDARD DESIGNS O. K.
45-300	GRAZING INCIDENCE OPTICS NEEDED	BAD ABERRATIONS, STANDARD DESIGNS MEET SOME OF THE REQUIREMENTS, NO DESIGN SATISFIES ALL
10-45	CRYSTALS & TRANSMISSION DEVICES COMPETE WITH REFLECTION GRATINGS	EXTRA DIFFICULT REGION

TABLE II

	<u>ROWLAND</u> <u>CIRCLE</u>	<u>PLANE</u> <u>GRATING</u>	<u>TOROIDAL</u> <u>GRATING</u>
1. High transmission--few reflections, good vacuum	POOR	medium	good
2. Large collection solid angle x area (étendue)	good	good	very good
3. Capability for good resolution	good	source limited	poor
4. Wide wavelength coverage	possible	easy	possible
5. Fixed in and out directions	<u>HARD</u>	easy	easy
6. Good rejection of second and higher diffracted orders	<u>HARD</u>	fairly easy	poor
7. Highly polarized output	various	various	various
8. Convenience factors			
simple mechanism	POOR	good	very good
easy to operate	POOR	good	good
low cost	POOR	medium	good
horizontal output	various	various	various
Examples:	Grasshopper ⁹	Kunz/ ¹² Dietrich	Madden/ Ederer ¹⁷
	Jaegle ¹⁰	Mijake ¹³⁻¹⁵	J. Y. LHT 30 ¹⁸
	Codling/ ¹¹ Mitchell	Flipper ¹⁶	Wisconsin ¹⁹ 'TGM'

Fig. 1

ORDER: k IS + IF $|\beta| < |\alpha|$

Fig. 2

Fig. 3

Fig. 4

Fig. 5

Fig. 6

Fig. 7

Fig. 8