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Simulations of Second-Order Fermi Acceleration of Electrons: Solving the Injection Problem

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ABSTRACT

The boosting of electrons from a Maxwellian distribution into a suprathermal power-law tail has long been recognized as an important bottleneck governing the subsequent acceleration of some of these electrons to relativistic energies. This is the seed or injection problem. I study this boosting process using a test-particle simulation code, following the full equations of motion of tens of thousands of electrons chosen from a thermal population as they move through general time-dependent magnetic fields. Inhomogeneities in the magnetic field are provided by finite swarms of moving current loops with Maxwellian velocity distributions and power-law distributions of loop size and dipole moment strength.

Whether bulk heating or boosting occurs is found to depend on the size of the swarm thermal speed compared to the electron thermal speed. When the swarm thermal speed is comparable to the electron thermal speed the entire electron population is heated by encounters with the rapidly moving current loops, approximately preserving the Maxwellian character of the electron distribution. On the other hand, at very low swarm thermal speeds there is no bulk heating; instead one percent or fewer of the electrons are boosted into a power-law suprathermal tail with a differential energy spectral index between -1 and -2 . Individual boosts of 2000 and more have been observed in samples of 50,000 electrons. Most of the strongly boosted electrons have initial energies that are well below the peak of the initial Maxwellian.

INTRODUCTION

Stochastic acceleration, or second-order Fermi acceleration, is usually treated using a diffusion or Fokker-Planck approach^{1,2}. The production of a power-law output energy spectrum from a monoenergetic input is readily accomplished, and there are well-known expressions for the acceleration efficiency and for the diffusion coefficients.

In this paper I am interested in the more general problem of what stochastic acceleration does to a Maxwellian input energy spectrum. This is related to the famous "seed" problem: How does the process of particle energization get started? Arguably, most of the diffuse matter in the universe has an energy spectrum that is roughly Maxwellian, and the extraction of energetic particles from that thermal matter is a core part of the problem of the production of cosmic rays. A diffusion approximation is not well suited to this extraction problem. The second law of thermodynamics demands that the production of a suprathermal tail be an inefficient process, selecting a very few particles for special treatment. Any process that acts on the majority of particles can only transform one Maxwellian into another, that is, by bulk heating or cooling.

Related to this consideration is the fact that a thermal population of particles in contact with a population of scatterers will tend toward thermodynamic equilibrium with the scatterers. Only if the configuration is limited in space or time, allowing accelerated particles to escape, can the production of a suprathermal tail occur. In this

case the conditions favoring equilibrium are violated: the very few energized particles depart from the system carrying their excess energy with them. This turns out to be a very effective way for a plasma to get rid of excess free energy.

The application to thermal electrons is complicated by the fact that these particles typically have fast timescales and small gyroradii. The electromagnetic field in the interstellar and interplanetary medium generally varies over much larger time and spatial scales, and thermal electrons are generally thought to respond "only" adiabatically to the relatively small changes on the electron scales.

It is worth noting, however, that adiabatic changes can be sufficient for acceleration, *if* there is a way to "lock in" the gains made during the favorable phases of the adiabatic cycles. An example of this is acceleration in a converging magnetic trap¹. This is essentially a first-order Fermi process in which an electron is trapped on a flux tube between two approaching magnetic mirrors. The electron gains energy on each encounter with one of the mirrors until it has gained enough parallel momentum to enter the loss cone of one of the mirrors. As it escapes through the loss cone the electron loses some energy, roughly the energy gained in its last approach to the mirror, but previous adiabatic gains are preserved. During each encounter with an approaching mirror, the electron is accelerated by the inductive electric field produced by the motion of the mirror.

THE SIMULATIONS

I choose to address this problem using the techniques of numerical simulation. A thermal population of particles is dropped into a distribution of randomly moving magnetic inhomogeneities, and its evolution is observed. Because the extraction process is posited to be inefficient, I have done *test-particle* simulations rather than fully self-consistent ones. There are two reasons for this choice, one physical, the other numerical. Physically, since the process is inefficient, the few particles that are extracted into the suprathermal tail do not greatly influence the evolution of the field inhomogeneities. Numerically, for every extracted particle one needs to compute thousands of particles that remain in the core of the distribution, and this makes self-consistent calculations expensive and impractical.

An example of this approach is given in Gisler and Lemons¹, where we presented simulations of electron acceleration in a closed flux tube with converging magnetic mirrors (*i.e.* first order Fermi acceleration), with application to bow shocks. In the present work I use a substantially upgraded version of that earlier code.

The magnetic scattering centers are represented as moving magnetic dipoles. The particles (which may be either electrons or ions) are followed in three dimensions along their orbits in the magnetic fields of these dipoles, using an adaptive Runge-Kutta scheme which amply resolves the particle gyrofrequency. The code is fully relativistic. There is no grid; the field sources all contribute to the field at any particle position. A particle-Hamiltonian approach is used for the equations of motion, so the acceleration of a particle is a result of the local inductive electric field at the particle position from the motion of nearby dipoles.

The code is implemented in THINK Pascal on the Apple Macintosh computer. Menus allow run-time adjustment of all important physics parameters in the code, and provide control of the real-time plotting and other outputs from the code.

Particles are chosen at random from a 3-dimensional thermal population, and dropped into a swarm of moving magnetic dipoles. In this paper, the dipole swarm also has a 3 dimensional Maxwellian velocity distribution, and flat distributions in position, dipole moment, and "hardness". Because of the finite size of the swarm and its thermal character, the configuration is always expanding, and particles almost

always manage to escape. There are twenty dipoles in the swarm, and to avoid any statistical peculiarities caused by this small number, a new swarm from the same population is chosen after every hundred particles have been run. The dipoles are unaccelerated, that is, they are infinitely massive compared to the particles. Some individual swarms have been found to be much more effective than the average at producing suprathermal particles, and we have used these to try to understand how particles are selected for acceleration.

Since the full equation of motion is used, particles will scatter off the magnetic dipoles or not, according to the appropriate physics. Particle trapping can also occur, as follows. Two moving dipoles with opposing fields approach each other so that the field between them is temporarily reduced. A particle enters this low-field region on an open field line and becomes trapped in the field of one of the dipoles as the dipoles move apart. Subsequently the particle executes bounce orbits in the field of one of the dipoles, and is freed only by an encounter with another moving dipole. Three-body "recombination" events of this form are frequently observed in the simulations, but they result in no net energy gain for the particle. Somewhat less common are events in which a particle is trapped between two or more dipoles, but these events are exceedingly important for particle acceleration because when these traps are converging, very large energy gains can be realized.

We accumulate histograms of the initial and final particle energies, measure the amount of bulk heating, the index of the power-law tail (if any), the efficiency of boosting (defined as the number of particles with final energies greater than ten times the initial thermal energy divided by the total number of particles). We also examine initial and final energies of the boosted particles, to characterize the population from which energized particles are drawn. These statistics are summarized in Table I.

TABLE I. TABLE OF RUNS

Geometry	Swarm Speed	Number of Particles	Efficiency of Boosting	Index of power law tail	Bulk Heating
spherical	0.002	44729	0.29%	-1.76	1.02
spherical	0.006	62474	0.39%	-1.22	1.04
spherical	0.02	74503	0.44%	-1.51	1.14
spherical	0.04	45953	0.97%	-1.27	1.33
spherical	0.1	58744	1.18%	-1.15	1.80
spherical	0.3	45025	8.70%	-2.68	3.85
spherical	0.38	58235	13.32%	-2.27	4.66
disk, Bz	0.02	41230	0.19%	-1.27	1.07
disk, Bx	0.02	40950	0.21%	-1.56	1.06
disk	0.02	41079	0.23%	-1.09	1.09
optimal swarm*	0.02	10564	11.05%	-2.64	5.01

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* optimal swarm: one of the ~745 mirror swarms used in the spherical 0.02 case was found to be spectacularly successful at accelerating particles (13/100) compared to a Poisson distribution with the expected value of 0.44/100). This swarm was isolated and run, without reshuffling the swarm.

RESULTS

When the swarm thermal speed is only a little less than the thermal speed of the particle population, as in Figure 1, the output spectrum looks very much like the initial one, just shifted to higher energy. The particle population has simply been heated by its contact with the dipole swarm.

Figure 1. Initial and final energy histograms from a simulation in which the mirror swarm thermal velocity is comparable to the particle population thermal velocity. A total of 45,025 particles were run in this simulation. The population is mostly bulk heated, with very little production of a power-law tail to high energy.

On the other hand, when the dipole swarm thermal speed is small compared to the particle population thermal speed, as in Figures 2 and 3, there is no bulk heating, but instead the production of a modest suprathermal tail. About as many particles are dropped into a low-energy tail also. As expected, the acceleration process is relatively inefficient, and both the boosting efficiency and the slope of the power-law tail depend on the relative size of the swarm thermal speed.

Figure 2. Initial and final energy histograms from a simulation in which the mirror swarm thermal velocity is $1/20$ times the particle population thermal velocity. A total of 74,503 particles were run, and 327 of them were boosted into the suprathermal tail with final energies > 1000 , for an acceleration efficiency of 0.44%. The high energy tail has a power-law index of -1.34 . There is no bulk heating of the population.

Figure 3. As Figure 2, but now the mirror swarm thermal velocity is $1/200$ times the particle population thermal velocity. A total of 44,729 particles were run, and 130 of them were boosted into the suprathermal tail with final energies > 1000 , for an acceleration efficiency of 0.29%. The high energy tail has a power-law index of -1.55 . Again there is no bulk heating of the population.

An analysis of those particles in the suprathermal tail in all the runs of Table I yields the somewhat surprising result that most of them originate at energies well below the most probable energy in the original distribution (see Figure 4). In fact, because of the finite extent of the dipole swarm, particles that start out at higher energies tend to have fewer encounters with the mirrors before escaping, so have fewer opportunities to gain energy. The particles starting out at low energies suffer more encounters and hence are more likely to be involved in a trapping event. Single

dipole traps, as mentioned earlier, do not result in energy gains, but converging multi-dipole traps can produce enormous energy gains with the result that a particle is scattered far from its original energy.

The trapping of a particle in a converging multi-dipole trap is a relatively improbable event, so those particles that end up in the suprathermal tail are a very small fraction of the total number of particles studied.

Figure 5. Comparison of initial and final energies for boosted particles from all runs. Boosted particles are defined as those particles with final energies greater than ten times the initial thermal energy. Also shown here is the shape of the initial Maxwellian, scaled to the total number of boosted particles. The boosted particles are found to originate in a distribution that is substantially cooler than the bulk of the population.

As mentioned before, the swarm of dipoles is undergoing, as a whole, a thermal expansion. One might expect that a typical particle in such a swarm would *lose* energy, since receding mirror velocities predominate. Indeed many particles lose a substantial amount of energy, as evidenced by the low-energy tails in Figures 3 and 4. But also occurring is the production of a power-law high-energy tail.

Thermal expansions are extremely common in astrophysics. Stellar atmospheres and winds, HII regions heated by hot young stars, supernova remnants, and galactic jets are but a few of the examples of such expansions. While the obvious result of a thermal expansion is an adiabatic cooling of the expanding gas, the acceleration of a few particles to high energies may also occur.

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REFERENCES

1. M. A. Forman, R. Ramaty, and E. G. Zweibel, in *Physics of the Sun*, ed. P. A. Sturrock, T. E. Holzer, D. M. Mihalas, and R. K. Ulrich; D. Reidel, Volume 2, pp 249-289 (1986).
2. M. A. Miller, N. Guessoum, and R. Ramaty, *Astrophys. J.* **361**, 701 (1990).
3. G. Gisler and D. Lemons, *J. Geophys. Res.* **95**, 14925 (1990).