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ACTION IN A CONFINED PLASMA COLUMN

BY

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PLASMA PHYSICS
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Conditions for Soft X-Ray Lasing Action in a Confined Plasma Column

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ABSTRACT

The idea of using a multi-Z (e.g., carbon, oxygen) thin plasma column as a medium for soft x-ray lasing action is presented. A plasma confined by a strong magnetic field is first heated by a CO_2 -laser, and then cools rapidly by radiation losses. This leads to a level population inversion of hydrogen-like carbon or oxygen ions.

Two computational models are presented. One uses given electron temperature, $T_e(t)$, evolutions. The other uses $T_e(t)$ calculated from an energy balance equation with CO_2 -laser beam power as a parameter. According to calculations, a total gain of $G > 100$ is expected for $3+2$ and $G > 10$ for $4+2$ transitions ($\lambda = 182 \text{ \AA}$ and $\lambda = 135 \text{ \AA}$, respectively) for CVI ions using a CO_2 -laser beam with power $\sim 5 \times 10^{10} \text{ W}$ for plasma column heating.

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1. EXISTENCE OF LASERS

A brief survey of the work carried out in this field from 1965 to 1970 has been given in [1]. In the last few years the interest in this problem has increased. This has been reflected in a steadily increasing number of papers published in the Soviet Union and abroad. The number of papers published in the Soviet Union alone has increased from 10 in 1969 to 20 in 1970. The number of papers published abroad has increased from 10 in 1969 to 15 in 1970. The following is a brief survey of the work carried out in this field.

At the present time, the most promising approach to the solution of the problem of the existence of lasers is the approach proposed in [2]. This approach is based on the use of a recombining plasma as a medium for the generation of soft x-ray radiation. The idea of using a recombining plasma as a medium for the generation of soft x-ray radiation was first proposed in [3]. The authors of [3] proposed that a recombining plasma could be used for soft x-ray lasers, one of the most difficult problems in the field of soft x-ray lasers.

There are a number of approaches to obtaining the population inversion, proposed in [4, 5]. The authors of [4] proposed that a recombining plasma could be used for soft x-ray lasers.

- (1) photoionization collision ionization of a recombining plasma, $\omega \approx 10^{18}$ cm⁻¹ [6];
- (2) photoionization of a recombining plasma, $\omega \approx 10^{18}$ cm⁻¹ [7];
- (3) photoionization of a recombining plasma, $\omega \approx 10^{18}$ cm⁻¹ [8].

There have also been some controversial experiments [9, 10] which have been analyzed and strongly criticized in several papers [11-13]. The results of these experiments are given here.

It seems to us that approach (3) is the most promising for obtaining soft x-ray lasers working in the regime of amplified spontaneous emission (ASE). The idea of using a recombining plasma to create a population inversion was first proposed by Gudzenko and Shelepov [14] and developed in a series of theoretical papers by Gudzenko [15]. Recent data by Jaggle et al. [7a] supported their previous claim about negative absorption of A2 IV (112 Å line).

et al.²⁷⁻²⁹ and others.^{30-38,41} The principle of this idea is that a plasma by cooling rapidly becomes strongly non-equilibrium and is dominated by electron-ion collisional-radiative recombination. The equilibrium electron density $N_e(\text{eq.})$ corresponding to this T_e drops very quickly. If N_e decreases much more slowly than $N_e(\text{eq.})$ the relaxation processes (collisions) cannot keep up with the changes at some intermediate level n' and thus create the inversion near such n' . To cool the plasma, Gudzenko et al.^{26,27} proposed to use adiabatic decompression. However, in such a plasma N_e is also decreasing quite rapidly. But the main problem was with achieving a short cooling time in comparison to the relaxation time of radiative processes.

Strong support for approach (3) arose through experiments on the interaction of powerful lasers with solid targets. The plasma produced in this manner expands quickly into a low density region (e.g., vacuum) and its temperature (and also its electron density) decreases rapidly by adiabatic decompression. In this way, Irons and Peacock³⁹ obtained one of the first population inversions for soft x-ray transitions. A plasma was formed by irradiating a polyethylene target with a pulsed neodymium-glass laser beam. They observed inversion in hydrogen-like carbon ions CVI between levels $n = 3, 4, 5$ and $m = 2$. However, total gain was very low, $G \approx 10^{-6} - 10^{-5}$, because ℓ was of the order of $1 \mu\text{m}$. Dewhurst et al.⁴⁰ later reported higher gain in a similar type of experiment, but total gain still did not exceed $G \approx 10^{-2}$.

Also, calculations predict the possibility of obtaining an inversion in that kind of plasma with respect to the ground level ($m = 1$) for hydrogen-like ions.^{32,33,37,38} Jones and Ali³² predicted high inversion of $2 \rightarrow 1$ transitions of CVI ions by solving a set of rate equations for populations and assuming an instantaneous decrease in electron temperature. Green and Solfast³³ performed similar work for HeII. In more extensive calculations, Pert³⁷ and Tallents,³⁸ used the time dependent change of temperature in an expanding plasma (Pert³⁷ also included heat return to electrons by recombination). However, in all these calculations it is assumed that the plasma in its initial condition (steady state) consists only of totally stripped ions and electrons,

and when the temperature starts to decrease at $t = 0$, ions and electrons begin to recombine. This type of approach leads to seemingly optimistic results, since in the early stage of expansion there would be a tendency toward populating the higher levels more than the ground level. However, if one includes fast ionization of the plasma during heating, then the population of ground level hydrogen-like ions should be quite high when strong recombination starts (see Sec. V). This effect makes inversion with respect to the ground level much more difficult to achieve experimentally.

Vinogradov and Sobelman⁴⁴ proposed using a charge-exchange process for generating a sufficiently high inversion in an expanding plasma. They analyzed the plasma produced by laser evaporation of a solid target (pellet) placed in a helium atmosphere. According to them, the charge-exchange cross section has a resonance character for a certain level n^* . This cross section can be as high as

$$\sigma_{\text{ex}} \approx \pi a_0^2 z^2 \approx 10^{-16} z^2 \text{ (cm}^2\text{)} \quad (2)$$

Thus, n^* will be preferentially populated and, in turn, will lead to its overpopulation in comparison to lower levels. For example, for carbon ($z = 6$) the charge-exchange collisions

$$\text{C}^{6+} + \text{C}^0 \rightarrow (\text{C}^{5+})^* + \text{C}^+ + \Delta E \quad (3)$$

are resonant for the excited level $n^* = 4$ of the product ion $(\text{C}^{5+})^*$ according to Refs. 45,46, although Vinogradov and Sobelman⁴⁴ suggested that $n^* = 6$ should be preferentially populated. The energy ΔE is the difference between the binding energies of the initial and final state.⁴⁵

Dixon and Flton⁴⁶ have performed experiments to show that the resonance charge-exchange process can be efficient for selective enhancement of populations leading to inversion. According to Dixon et al.⁴⁷ this process can even be a dominant cause of inversion in a plasma expanding into a buffer gas. The conclusion was made

on the basis of numerical modeling of observed line intensity inversion in Lyman and Balmer spectra of C^{5+} ions. Calculations of level populations were done using steady state approximations. This assumption may lead to wrong conclusions in the case of a rapidly expanding plasma. Time dependent calculations for modeling the results of Dixon et al.^{46,47} as well as the influence of charge-exchange on population inversion in a plasma with $N_e \sim 5 \times 10^{18} - 5 \times 10^{19} \text{ cm}^{-3}$ will be discussed in Sec. VI of this paper.

The main purpose of this paper is to point out another scheme for obtaining inversion in a recombining plasma when particle confinement time τ_p is much longer than electron temperature decay time τ_E . It is based on calculations⁴² and observations of population inversion for H I and He II in the Spherator FM-1.⁴³ The concept and proposed experiment are the subject of the next section. In Sec. III we will discuss the set of equations and the computer program. The results will be presented in Secs. IV-VII. A discussion of the results and the possibility of stimulating lasing action will appear in Sec. VIII.

II. INVERSION IN A CONFINED PLASMA COLUMN

We suggest using a multi-Z plasma column with a large ratio of length to radius r_0 ($l/r_0 \geq 10^2$) as a medium for lasing action. The plasma would be confined by a magnetic field (or the chamber wall) and surrounded by a relatively high pressure neutral gas. In this case, the particle confinement time τ_p would be much longer than the electron temperature decay time τ_E . This difference is caused primarily by large radiation and charge-exchange losses. It is possible to achieve these conditions in a plasma column produced by CO_2 laser-induced breakdown of neutral gas with high initial pressure (10-50 Torr) of CH_4 , CO_2 or $CH_4 + Xe$ in a strong solenoidal magnetic field.⁵³⁻⁵⁵ In such a plasma, density N_e is highest midway ($r \approx r_0/2$)

[†] The idea and the experimental arrangement discussed here were presented by one of us (S.S.) at the Plasma Physics Laboratory in January 1976.

between the axis and the side of the column^{53,54} ($N_e = 1-2 \times 10^{18} \text{ cm}^{-3}$ for relatively low CO_2 -laser power, $P_L \approx 3 \times 10^8 \text{ W}$) where electron temperature $T_e = 10-20 \text{ eV}$. The electron temperature has a maximum ($T_e = 100-200 \text{ eV}$) at the center of the column ($r = 0$, $N_e(r) = 3 \times 10^{17} \text{ cm}^{-3}$), and rapidly decreases toward the outside. In this case, totally stripped ions from the center of the column can recombine in the high density and low temperature region near the side, thereby providing better conditions for inversion. The particle confinement time τ_p is much longer than temperature decay time τ_E as can be deduced from Refs. 54,55. Decay time τ_E can be decreased by increasing radiative losses. This situation can be achieved easily by adding a few percent of a high-Z substance to the plasma (e.g., Xe). After the CO_2 laser pulse ends and while T_e is dropping, the electron density N_e is much higher than the equilibrium density $N_{e(\text{eq})}$. Hence, after the breakdown, along the plasma column the conditions for high inversion for soft x-ray transitions are suitable, as will be shown in Secs. IV and V.

The principle differences between the confined plasma column (CPC) and expanding plasma (EP) are:

- Particle confinement time in CPC is much longer than for EP. Since CPC has a much more slowly decreasing N_e , there is the possibility of obtaining a higher gain G .
- In CPC the long column is beneficial for total laser gain G while the small radius is useful from the point of view of the pumping power requirement and inversion (the plasma can be optically thin in a radial direction).
- In CPC the maximum inversion can be obtained in a region close to maximum plasma density.

There is some disadvantage of CPC compared to EP due to a lower probability of charge-exchange in CPC caused by the lower interaction energy of A^{Z+} ions with neutral atoms A^0 along the side of the column. However, if in some way we

can increase the velocity of neutrals so that they penetrate the plasma column fast enough, it would increase neutral density as well as the charge-exchange cross section at $r = r_0/2$.

A suggested scheme of experiment based on a CO_2 -laser gas breakdown plasma⁵³⁻⁵⁵ is presented in Fig. 1. It corresponds to a "short plasma column" ($l \approx 5-10$ cm) with $r_0 \approx 0.1$ cm and laser energy of order $W_L \approx 50-100$ joules. A "long plasma column" ($l \approx 100$ cm) can be obtained with higher laser energy ($W_L \approx 500-1000$ joules) by focusing it along the column using cylindrical lenses. In Fig. 1 the laser beam (a CO_2 or Nd-glass laser could be used) is focused through an orifice onto a gas target in a strong magnetic field B_z ($B_z \approx 100-150$ kGauss). The B_z is created by a low inductance solenoid supplied by a condenser bank. Without B_z the plasma would expand rapidly in a radial direction leading to a rapidly decreasing electron density.⁵⁷ (There is also the possibility of using a cylindrical chamber for plasma radial confinement⁵⁸). In Fig. 1 the radial distribution of N_e is also shown and in Fig. 2 we see the temporal evolution of N_e and T_e in a CO_2 -laser heated plasma⁵³⁻⁵⁵ ($P_L \approx 10^9$ W cm⁻³). Initially, we propose to use CH_4 and $\text{CH}_4 + \text{Xe}$ as a gas target (working gas) in order to observe an inversion in H-like carbon C VI ions (Fig. 3) between levels $n = 3, 4, 5$ and $m = 2$ ($\lambda_{3,2} = 182 \text{ \AA}$, $\lambda_{4,2} = 135 \text{ \AA}$ and $\lambda_{5,2} = 120 \text{ \AA}$). By comparing experiments with different initial concentrations of Xe it is possible to predict the optimum efficiency of radiation cooling for given laser energy. The same scheme can be used for oxygen or atoms with higher Z in order to obtain inversion for shorter wavelength (in such a case the energy of the CO_2 laser should be higher for the same size plasma column). Using $\text{He} + \text{Ar}$ or $\text{He} + \text{Xe}$ mixtures it will be possible to obtain an inversion in He II for transitions with $\lambda = 1215 \text{ \AA}$ and $\lambda = 1025 \text{ \AA}$.

III. COMPUTATIONAL MODEL

For about the first 30-70 ns after breakdown the plasma is heated by a laser beam leading to an increase in electron temperature T_e and density N_e . When the laser pulse is terminated, T_e decreases rapidly, primarily by radiation losses, while N_e changes only slightly. If the plasma consists almost exclusively of ions both fully stripped and hydrogen-like, the ratio of plasma energy to radiation losses by resonance lines gives an order of magnitude estimate of decay time as follows:

$$\tau_E \approx \frac{N_e T_e(t)}{N_2 A_{21} h\nu_{21}} \quad (4)$$

For example, in a carbon plasma with $N_e = 5 \times 10^{19} \text{ cm}^{-3}$, $T_e = 500 \text{ eV}$ ($T_i \ll T_e$), $N_2 \approx 5 \times 10^{14} \text{ cm}^{-3}$, $A_{21} = 7 \times 10^{11} \text{ sec}^{-1}$ and $h\nu_{21} \approx 370 \text{ eV}$, we get $\tau_E \approx 4 \times 10^{-8} \text{ sec}$. Taking into account radiation losses by ions in the lower ionization stages we may obtain an even shorter τ_E . A more accurate estimate of τ_E was obtained using self-consistent calculations for $T_e(t)$, $N_e(t)$, radiation W_{rad} and ionization W_{ioniz} losses all as a function of beam power P_L and neutral gas density N_0 . The calculations presented in the last part of this section give a value for τ_E not far from the one above. In most of our calculations we use τ_E in the range $10^{-8} - 10^{-7} \text{ sec}$ for the time evolution of $T_e(t)$ if the energy balance equation is not used. $N_e(t)$, the level population densities $N_n^{Z-1}(t)$ of hydrogen-like ions and the ion densities for different stages of ionization $N_l^j(t)$ as a function of time were calculated from a set of rate equations for ionization:

$$\begin{aligned} \frac{dN_l^j}{dt} = & \left(N_l^{j-1} S_l^{j-1} - N_l^j S_l^j \right) N_e + \left(\alpha_l^{j+1} + N_e \beta_l^{j+1} \right) N_l^{j+1} N_e \\ & - \left(\alpha_l^j + N_e \beta_l^j \right) N_l^j N_e + \delta_{j0} \psi^0 - \psi_l^j, \end{aligned} \quad (5)$$

$$j = 0, 1, 2, \dots, Z-1, Z,$$

$$N_l^{j-1}, S_l^{j-1} \equiv 0 \quad \text{for } j = 0; \quad N_l^{j+1}, S_l^j, \alpha_l^{j+1}, \beta_l^{j+1} \equiv 0 \quad \text{for } j = Z$$

and for populations:

$$\begin{aligned} \frac{dN_n^{Z-1}}{dt} = & N_e \sum_{\substack{k=1 \\ k \neq n}}^{k_{\max}+m} \left(N_k^{Z-1} S_{kn}^{Z-1} - N_n^{Z-1} S_{nk}^{Z-1} \right) \\ & - \sum_{k=1}^{n-1} N_n^{Z-1} A_{nk}^{Z-1} \gamma_{nk}^{Z-1} + \sum_{k=n+1}^{k_{\max}+m} N_k^{Z-1} \left(A_{kn}^{Z-1} + N_e S_{kn}^{Z-1} \right) \\ & - N_e N_n^{Z-1} S_n^{Z-1} + N_l^Z N_e \left(\alpha_n^Z + N_e \beta_n^Z \right) - \psi_n^{Z-1} \\ & + \delta_n^* n \cdot \delta_{1s} N_l^0 N_l^Z R \quad ; \end{aligned} \quad (6)$$

$$n = 2, \dots, k_{\max} \quad ; \quad s = \frac{(t-t_0)}{|t-t_0|} \quad .$$

and the plasma neutrality equation:

$$N_e = \sum_{j=1}^Z j N_l^j \quad , \quad (7)$$

where Z is the atomic number, δ_{jk} is Kronecker's delta. $\delta_{jk} = 0$ for $j \neq k$ and $\delta_{jk} = 1$ for $j = k$. ψ_n^0 is the influx of neutrals, ψ_n^j is the outflux (or influx) in the state with quantum number n . The S_n^j , α_n^j and β_n^j are the rate coefficients for ionization, photo-recombination and three body recombination, respectively (to or from level n). S_{kn}^{Z-1} is the rate coefficient for collisional excitation ($k < n$) or de-excitation ($k > n$) of hydrogen-like ions ($j = Z-1$). A_{nk}^{Z-1} is the spontaneous transition probability. γ_{nk}^{Z-1} is the Biberman-Holstein radiation opacity coefficient.^{60,61}

which can be used in analytical form, presented in Ref. 62. For $i \rightarrow k$ transitions $\gamma_{ik} = 1$ for an optically thin plasma and $\gamma_{ik} = 1/2$ for an optically thick plasma. In most of our present calculations, we had $\gamma_{tr} = 1$, since the plasma was optically thin in a radial direction. For some calculations, where the density N_1^{Z+1} was high ($N_1^{Z+1} = 10^{18} \text{ cm}^{-3}$), we took also account the fact that $\gamma_{ul} < 1$, where γ_{ul} was calculated from analytical approximation⁶².

A resonance charge-exchange process with rate coefficient $R_{ex} = \sigma_{ex} v_0$ is included in Eq. (6) for $n = n^*$ and $t \geq t_0$. Time t_0 corresponds to a delay in the start of interaction between atoms and ions in the charge-exchange process. In R we used charge-exchange cross sections σ_{ex} given by Vmogradov and Solodov⁶⁴ (Eq. 2) or by Salop and Olson⁶³ and Salop⁶⁵ with relative particle velocity in the range $v = 10^6 - 10^7 \text{ cm/sec}$. Most of the calculations were performed with $v = 5 \times 10^6 \text{ cm/sec}$. Spontaneous transition probabilities A_{nk}^1 were taken from tables by Wiese et al.⁶⁵ Electron impact ionization and excitation rate coefficients were used in the form given by Drawin.⁶⁶ Rate coefficients for reverse processes (de-excitation and three-body recombination) were obtained from detailed balance relations. There are several other forms of cross-section giving better agreement with experiment. However, Drawin's form scales with quantum number n quite well, which is important in finding optimum conditions for a maximum population inversion. Also, Drawin's cross sections are simple and can be used for atoms and ions with different Z (e.g., H, He, C, O). Seaton's approximation^{66,67} was used for photo-recombination. If the gas consisted of two kinds of atoms, e.g., carbon and hydrogen (CH_4), an additional equation was used for the ionization of hydrogen

$$\begin{aligned} \frac{dN_H}{dt} &= -N_H N_e S_1^H + N_p \left(\alpha_1^P + N_e \beta_1^P \right) ; \\ \frac{dN_H}{dt} &= -\frac{dN_p}{dt} , \end{aligned} \quad (8)$$

and the proton density N_p was included in the right hand side of Eq. (7).

The set of Eq. (5) gives the time evolution of ion density in ground state N_1^j for $j \neq Z-1$ whereas N_1^{Z-1} is obtained from Eq. (6). Except for hydrogen-like ions ($j = Z-1$), we neglected ionization from excited levels and recombination into these levels as being less important in determining N_1^j . This exclusion simplified the calculation.

Summation over k in Eq. (6) was ended at $k = k_{\max}$ on the assumption that for levels with $k > k_{\max}$ Saha equilibrium with the ground state of the next stage of ionization is satisfied,

$$\frac{N_{k_{\max}}^{j-1} \cdot m}{N_1^j \cdot N_e} = \frac{g_{k_{\max}}^{j-1} \cdot m}{2g_1^j} \left(\frac{2\pi m_e T_e}{h^2} \right)^{-3/2} \exp \left(\frac{E_1^j - E_{k_{\max}}^{j-1}}{T_e} \right) \quad (9)$$

E_n^j is the energy of level n of j -ions in relation to the ground level, and its statistical weight is g_n^j . In most of the calculations we had $k_{\max} = 10$ and $m = 3$. As a check we used $k_{\max} = 20$ and $m = 5$. We found that the difference is of the order of a few percent between the two calculations for levels $n = 1, 2, \dots, 8$.

As initial conditions we used a steady state approximation for Eqs. (5) and (6) assuming that the plasma is ionized prior ($t < 0$) to laser beam heating

$$\frac{dN_n^j}{dt} = 0 \text{ for all } j \text{ and } n, \quad (10)$$

$T_e = T_{e0}$, where $T_{e0} = 1-10$ eV

with:

- (a) Constant fluxes: neutral influx $\psi_0^0 = N_0/\tau_0$, neutral outflux $\psi_1^0 = N_1^0/\tau_0^0$ and ion outflux $\psi_1^j = N_1^j/\tau^j$ where $\tau^j = \ell/v^j$, ℓ is the characteristic length, and v^j is the velocity of the particles.

or

- (b) Given atomic density at $t = 0$, $N_1^0 = C$ and $\psi_n^j = 0$ for all j and n . [In this case, the equation for N_1^0 in the set of Eq. (5) should be omitted at $t = 0$.]

The initial values for N_0^I and N_e were found by iteration since N_e appears nonlinearly in Eqs. (5) and (6).

Integration of the set of differential Eqs. (5) and (6) was performed for two cases:

$$\frac{dN_1^I}{dt} = F/G, \quad \frac{dN_n^{Z+1}}{dt} = F/G_{\text{max}} \quad (Z+2 \text{ differential equations}) \quad (11a)$$

$$\frac{dN_1^I}{dt} = F/G, \quad \frac{dN_n^{Z+1}}{dt} = 0 \quad \text{for } n \geq 2 \quad (Z+1 \text{ differential equations}) \quad (11b)$$

Approximation (11b) is valid if relaxation times at the excited levels are much shorter than those of ground level. Indeed, as can be seen in Fig. 4, CVI level populations calculated with approximation (11b) are very close to those calculated with all $dN_n/dt \neq 0$. Also the results were close for high N_e and rapidly changing T_e (beginning of evolution) unless $dT_e/dt < 10\text{-}20$ eV/nsec and $N_e < 10^{20} \text{ cm}^{-3}$. In all such cases we applied (11b) which shortened the computation time very much.

IV. POPULATION INVERSION (PRESCRIBED $T_e(t)$ PROFILES)

In Fig. 5 relative populations N_n/g_n of four levels ($n = 2, 3, 4, 5$) and the population N_1^{Z+1} of the ground level of CVI are shown. The calculations were performed for the temperature evolution close to one expected experimentally⁵³⁻⁵⁵ with maximum $T_{e(\text{max})} = 140$ eV and cooling decay time $\tau_E = 4 \times 10^{-8}$ sec. The sets of Eqs. (5) and (6) are nonlinear in electron density, N_e . Therefore, N_e was calculated by iterating Eq. (7) together with the sets of Eqs. (5) and (6). The initial conditions (10b) were chosen. The neutral density, $N_1^0 = N_0 = 3.3 \times 10^{10} \text{ cm}^{-3}$, for $t = 0$ was found from the steady state solution of Eqs. (5) and (6) with $T_e(0) = 10$ eV. With electron temperature dropping rapidly and electron density changing much more slowly, we can see a buildup of population inversion for levels $n = 3$ and $n = 4$ with relation to level $m = 2$ when T_e drops below 60 eV. The inversion $N_{\text{inv}} = N_n - N_m g_n/g_m$ reaches

a maximum $N_{inv} = 1.1 \times 10^{16} \text{ cm}^{-3}$ for $n = 3$ and $m = 2$ at $t = 1.0 \times 10^{-7} \text{ sec}$ and $N_{inv} = 5 \times 10^{15} \text{ cm}^{-3}$ for $n = 4$ and $m = 2$ at $t = 1.5 \times 10^{-7} \text{ sec}$.

For oxygen, the maximum temperature should be higher (Fig. 6) than for carbon in order to get the densities of totally stripped ions, hydrogen-like ions and electrons high enough. However, as one may see from Fig. 6, increasing the temperature is not sufficient to obtain an inversion as high as that for carbon. To increase the inversion for oxygen, faster cooling is required.

The same results as in Figs. 5 and 6 were obtained with initial condition (10a) with $N_O^0 = N_O = 5 \times 10^{17} \text{ cm}^{-3}$ and $\Psi_O^0 = \Psi_O = 5 \times 10^{23} \text{ cm}^{-3} \text{ sec}^{-1}$.

From Figs. 5 and 6 it can be seen that an inversion appears at a time when the plasma parameters are changing much more slowly than at the beginning of laser heating. This can imply that by using a steady state approximation it is possible to obtain similar results. However, the steady state approximation gives results that are quite different although all the parameters are the same as in the time dependent case (this will be discussed in Sec. VI).

V. CALCULATIONS OF POPULATION INVERSION WITH $T_e(t)$ FROM ENERGY BALANCE EQUATION

Before we describe the results of the calculations with $T_e(t)$ obtained from the energy balance equation, we will present an estimation for the necessary laser beam energy W_L needed to maintain the balance with ionization W_{ioniz} , radiation W_{rad} and thermal W_{th} energies of CPC.

The energy for gas ionization was estimated from the Thomas-Fermi model.

$$W_{ioniz} = 16Z^{7/3} N_I^Z V \times 1.6 \times 10^{-19} \text{ (joules)} \quad , \quad (12)$$

where V is the plasma volume. For totally ionized carbon with density $N_I^Z = 10^{18} \text{ cm}^{-3}$ and "short" plasma column ($l = 10 \text{ cm}$, $r_0 = 1 \text{ mm}$) we have

$W_{\text{ioniz}} \approx 50$ joules. In such a column with average $\bar{T}_e \approx 100$ eV and $\bar{N}_e \approx 10^{19} \text{ cm}^{-3}$, the thermal energy is

$$W_{\text{th}} \approx \frac{3}{2} N_e T_e V \times 1.6 \times 10^{-19} \approx 70 \text{ (joules)} \quad (13)$$

where it is assumed that $\bar{T}_i \ll \bar{T}_e$ and $N_i^Z \ll N_e$. The main radiation losses W_{rad}^* of this plasma can be expected for resonance transitions $2 \rightarrow 1$ of CIV and CV ($\lambda \approx 50 \text{ \AA}$).

$$W_{\text{rad}}^* \approx N_2^{Z-1} A_{21} h\nu_{21} V \Delta t \times 1.6 \times 10^{-19} \text{ (joules)} \quad (14)$$

For CIV the spontaneous transition probability is $A_{21} \approx 7 \times 10^{11} \text{ sec}^{-1}$ for photons with energy $h\nu_{21} \approx 400$ eV. If average density $N_2^{Z-1} \approx 5 \times 10^{19} \text{ cm}^{-3}$ exists during time $\Delta t \approx 10^{-8} \text{ sec}$, the radiation losses W_{rad}^* will exceed ≈ 65 joules. Taking into account the fact that resonance radiation losses for CV are similar to those for CIV and including non-resonance radiation losses (factor of 1.5) we get $W_{\text{rad}} \approx 3 W_{\text{rad}}^* \approx 200$ joules. Hence, the laser beam should balance the total energy $W_{\text{rad}} + W_{\text{ioniz}} + W_{\text{th}} \approx 320$ joules. Assuming that the efficiency η of plasma heating by a CO_2 -laser is about 50% we come to the conclusion that laser beam energy should be of the order $W_L \approx 600$ joules, and its power should be $P_L \approx 10\text{-}20 \text{ GW}$ for heating during 30-50 nsec.

The laser beam energy W_L or power P_L plays the role of an independent parameter in the energy balance equation which is solved consistently with the set of Eqs. (5) and (6). This additional equation for energy rate is

$$\frac{3}{2} \frac{d}{dt} (N_e T_e) = \eta \frac{P_L}{V} - P_{\text{rad}} - P_{\text{ioniz}} + P_{\text{rec}} - P_{\Delta T} \quad (15)$$

where laser beam power P_L is a constant parameter during time Δt . Radiation losses P_{rad} are included in the form

$$P_{\text{rad}} = F \sum_{n=2}^{k_{\text{max}}} N_n^{Z-1} A_{n1}^{Z-1} h\nu_{n1}^{Z-1} \quad (16)$$

where the factor $F \approx 3$ takes into account the radiation losses of the lower stages of ionization. Terms for ionization P_{ioniz} and recombination P_{rec} have the forms

$$P_{\text{ioniz}} = \sum_{j=0}^{Z-1} N_e N_l^j S_l^j E_l^j, \quad (17)$$

$$P_{\text{rec}} = \sum_{j=1}^Z N_e N_l^{j-1} E_l^{j-1} \left(\alpha_l^j + N_e \beta_l^j \right), \quad (18)$$

and the rate of heat transfer $P_{\Delta T}$ from electrons (mass m_e) to ions (mass M) is

$$P_{\Delta T} = \frac{2m_e}{M} v_{e\ell} N_e (T_e - T_i), \quad (19)$$

where $v_{e\ell}$ is the frequency of elastic collisions of electrons with ions. In Eq. (19) we assumed that $T_i \ll T_e$, which is the usual condition in laser heating of a plasma column. Here, we used the same initial conditions as in the previous section.

In Fig. 7a the results of a calculation of T_e , N_e and level populations N_n/g_n ($n = 2, 3, 4, 5$) of CIV are shown for effective laser beam power $P_L = 1.5 \times 10^{10}$ W introduced into the plasma. We simulated heating of the plasma column ($\ell = 3$ cm, $r = 0.1$ cm) during the time interval from $t = 0$ to $t = 3 \times 10^{-8}$ sec. The populations of the ground states of C^{4+} , C^{5+} and C^{6+} ions are shown in Fig. 7b. In the calculations, the initial conditions (10a) were used with influx $\psi_0^0 = \psi_0 = 3 \times 10^{23} \text{ cm}^{-3} \text{ sec}^{-1}$ and density of neutrals $N_0^0 = N_0 = 3 \times 10^{17} \text{ cm}^{-3}$. A temperature of $T_e \approx 120$ eV is reached in the first few nsec of heating and stays at this level until the end of CO_2 -laser heating. Under these plasma conditions ionization and radiation losses do not allow a further increase in temperature for chosen CO_2 -laser beam power. When heating is terminated, the temperature starts to drop due to radiation cooling in this relatively high density ($N_e \approx 1-1.5 \times 10^{19} \text{ cm}^{-3}$) plasma column. While the temperature decreases radiation losses increase as a result of population levels increasingly excited by collisional

(three-body) recombination of C^{6+} . This avalanche process produces a very rapidly decreasing T_e and a very rapidly increasing population inversion. Relative populations N_n/g_n for levels $n = 5, 4, 3, 2$ reach values 1×10^{15} , 1.4×10^{15} , 1.7×10^{15} and $3 \times 10^{14} \text{ cm}^{-3}$, respectively, when T_e drops to 1 eV. If the maximum plasma temperature is higher (e.g., when plasma density is lower for the same laser energy as in Fig. 7) then ionization results in a decrease in density of C VI, which in turn causes lower radiation loss. This leads to a more slowly decreasing T_e after laser beam termination (Fig. 8). Of course, when the temperature finally drops significantly, the processes of increasing inversion (radiation losses) and decreasing temperature again becomes fast. Unfortunately, in Figs. 7a and 8, the peak of the population density, its duration and its subsequent decrease are not shown because of an unresolved problem with our computer integration for a very rapidly dropping temperature near 1 eV and very large $\exp(\Delta E_{nl}/T_e)$. Therefore, the results presented correspond only to the rising part of the buildup of the level population inversion but not to its maximum.

In order to obtain a high $N_e (10^{19} - 10^{20} \text{ cm}^{-3})$, with a correspondingly high inversion, the neutral gas density or influx should be high ($N_0 = 10^{17} - 10^{18} \text{ cm}^{-3}$, $\phi_0 = 10^{23} - 10^{24} \text{ cm}^{-3} \text{ sec}^{-1}$) and of course, laser beam power should be relatively high ($P_L = 10^{10} - 10^{12} \text{ W}$) for a plasma column with $z = 1-10 \text{ cm}$ and $r_0 = 0.1-0.2 \text{ cm}$.

VI. EFFECT OF CHARGE-EXCHANGE ON LEVEL POPULATIONS

The papers of Vinogradov and Sobell'man⁴⁴ and Dixon et al.^{46,47} stimulated our interest in the influence of charge-exchange on level populations. We included the charge-exchange process (3) for carbon in Eq. (6) in the form $N^{Z+1}_0 \langle \sigma_{c,ex} v \rangle$ for level $n = n^*$. Following Salop and Olson,⁶³ $n^* = 4$ was chosen for C VI. The same term was subtracted in Eq. (5) for the ground state of populations of C^{6+} and C^0 and added for the C^{1+} equation. In most of our calculations we used the Salop and Olson

charge-exchange cross section. They suggest that for process (3) it should be similar to the one for the process

with asymptotic value $\sigma_{\text{C.ex}} \approx 1.4 \times 10^{-6} v^{-3/2} (\text{cm}^2)$ for particles with relative velocity $v \geq 5 \times 10^7 \text{ cm/sec}$. For lower velocities, e.g., $v = 0.5, 1.0$ and $3 \times 10^7 \text{ cm/sec}$, the corresponding cross sections^{63,64} are $\sigma_{\text{C.ex}} \approx 1, 5,$ and $20 \times 10^{-16} \text{ cm}^2$, respectively. We used two values of the rate coefficient, $\langle \sigma_{\text{C.ex}} v \rangle = 5 \times 10^{-10}$ and $5 \times 10^{-9} \text{ cm}^3 \text{ sec}^{-1}$.

The influence of the charge-exchange process on the formation of an inversion can be seen in Fig. 9 where results of calculations with ($\langle \sigma_{\text{C.ex}} v \rangle = 5 \times 10^{-10} \text{ cm}^3 \text{ sec}^{-1}$) and without it are shown. The calculations for CVI level populations ($n = 2-10$) correspond to two different times of population evolution, $t_1 = 140 \text{ nsec}$ and $t_2 = 148 \text{ nsec}$ from the beginning of the simulation of plasma heating (they correspond to 20 nsec and 98 nsec, respectively, from the end of plasma heating). These times were chosen to lie on the rapidly rising part of the curves. Electron temperature and density reached their maximum values, $T_e = 490 \text{ eV}$ and $N_e = 1.8 \times 10^{19} \text{ cm}^{-3}$, respectively, at the end of plasma heating. A plasma column ($L = 10 \text{ cm}, r_0 = 0.1 \text{ cm}$) was assumed to be heated by a CO_2 -laser beam of power $P_L = 4.5 \times 10^{10} \text{ W}$ during time $\Delta t = 50 \text{ nsec}$. For this temperature evolution with a relatively high peak, radiation losses start to become large enough at $t = 135 \text{ nsec}$ to cool the plasma rapidly. From Fig. 9 it can be seen that for relatively high electron density, resonance charge-exchange increases the population inversion. However, this process is enhanced more by the increased rate of plasma cooling as opposed to an increase in population of a certain level. In Fig. 9 neutral density N_1^0 increases rapidly between t_1 and t_2 changing the value of the charge-exchange term $N_1^0 N^{O6+} \langle \sigma_{\text{C.ex}} v \rangle$ from $10^3 \text{ cm}^{-3} \text{ sec}^{-1}$ at t_1 to $10^4 \text{ cm}^{-3} \text{ sec}^{-1}$ at t_2 . (Please note that N_1^0 is not the same as N_{O_1} . The latter is related to the initial conditions.)

The density of the totally stripped ions, $N^{6+} = 2 \times 10^{18} \text{ cm}^{-3}$, does not change significantly. However, for a much larger charge-exchange term, the depopulation of C^{6+} by charge-exchange can be strong enough to have a negative effect on the population inversion in this high density plasma. Charge-exchange increases the population of level n^* . Also, rapid relaxation processes (collisional and radiative) increase the populations of levels $n < n^*$ when T_e is low. This is particularly true for level $n = 1$. In such a case, the decrease of the density of C^{6+} is faster than the increase of the population of level n^* of C^{5+} , which leads to a decreased time interval when a high degree of population inversion exists.

For a better understanding of the effect of charge-exchange on population inversions as well as the results of Dixon et al.,^{46,47} we provide calculations which model plasma heating in the first 30 nsec followed by radiation cooling and adiabatic decompression. At $t_0 = 45 \text{ nsec}$ the charge-exchange term was "turned on" in the equations (a wait of time t_0 was used to simulate a delay in interaction between neutrals and ions). The charge-exchange term $N_0 \langle \sigma v \rangle_{ex} = 1.5 \times 10^5 \text{ sec}^{-1}$ was similar to the one reported by Dixon et al.⁴⁷ Figure 10a shows the time evolutions of T_e , N_e , N_1^{Z-1} and N_n/g_n ($n = 2, 3, 4, 5$). When the temperature drops below 10 eV inversion increases rapidly. However, the rapid decrease of N_e causes the peak of the inversion to be short in time. At $t = 53 \text{ nsec}$ the temperature drops to 1.5 eV (similar to the temperature in Dixon et al.⁴⁷). From this moment on we kept temperature constant, $T_e = 1.5 \text{ eV}$, in order to observe the relaxation of populations to steady state conditions. The level population plots in Fig. 10b are for certain times of interest chosen from the time evolutions in Fig. 10a. At $t_1 = 20 \text{ nsec}$ ($N_1^Z = 1.3 \times 10^{18} \text{ cm}^{-3}$, $N_1^{Z-1} = 1.7 \times 10^{18} \text{ cm}^{-3}$), when collisional ionization and excitation processes dominate, inversion is not created. When the temperature has dropped to 13 eV at $t_2 = 50 \text{ nsec}$ ($N_1^Z = 8.6 \times 10^{16} \text{ cm}^{-3}$, $N_1^{Z-1} = 3.5 \times 10^{16} \text{ cm}^{-3}$), the highest population has level $n = 4$. However, only a small effect of charge-exchange is noticeable (increasing the charge-exchange term 10^3 times higher

produces only an increase of 10 times in populations of levels $n = 2, 3$, and 4). At the peak of the inversion, $t_3 = 53$ nsec, where the population is approximately an order of magnitude higher than that at t_2 , the influence of charge-exchange is negligible. Even for a charge-exchange term 10^3 times higher there was practically no change in population distribution (at t_3 we have $N_1^{\sim} = 6.2 \times 10^{15} \text{ cm}^{-3}$ and $N_1^{\sim-1} = 2.5 \times 10^{15} \text{ cm}^{-3}$). In the steady state, the maximum population shifted toward $n = 7$ under the condition that the population density was 200 times lower than that for 53 nsec. A somewhat higher population of level $n = 4$ may be seen with charge-exchange taken into account.

The charge-exchange process significantly modified the time evolution of the level populations in the plasma with $N_e < 10^{17} \text{ cm}^{-3}$. This can be seen by comparing Fig. 11a (with charge-exchange) and Fig. 11b (without charge-exchange). Calculations were performed in both cases with the other conditions the same. Temperature and density rise was calculated for lower laser beam power. Then, N_e was kept constant ($N_e \approx 6 \times 10^{16} \text{ cm}^{-3}$) while T_e was made to decrease rapidly, somewhat arbitrarily. Charge-exchange had a strong effect on the buildup of population inversion, but only for electron temperature $T_e > 20$ eV when three-body recombination was low. For $T_e < 5$ eV, the effect of charge-exchange diminished and later reached a point where it had a negative effect on population inversions. Therefore, in order to make a judgement about the effect of charge-exchange on population inversion in an experiment, the absolute densities of level populations as a function of time are required.

VII. GAINS

The crucial point of this paper is the value of the expected total gain $G = \int \kappa dx$ in a confined plasma column, where (x, t, λ_0) is the gain per unit length at the center of line λ_0 . In gain calculation, for (x, t, λ_0) the following expression (see e.g., Refs. 70,71) was used:

$$\kappa(x, t, \lambda_0) = \sigma_{nm}(\lambda_0) N_{inv}(x, t), \quad (22)$$

where $\sigma_{nm}(\lambda)$ is the cross section for stimulated emission. For the Gaussian profile of the line, $\sigma_{nm}(\lambda)$ has the form:

$$\sigma_{nm}(\lambda) = \frac{\lambda^4}{8\pi c} \left(\frac{4 \ln 2}{\pi} \right)^{1/2} \frac{A_{nm}}{\Delta\lambda_D} \exp \left[- \left(\frac{\lambda - \lambda_0}{\Delta\lambda_D} \right)^2 \ln 2 \right], \quad (23)$$

By inserting the expressions for N_{inv} and σ_{nm} , the gain $\kappa(x, t, \lambda_0)$ can be written as

$$\kappa(x, t, \lambda_0) = 1.24 \times 10^{-12} \left(\frac{\lambda^4 A_{nm}}{\Delta\lambda_D} \right) \left[N_n(x, t) - \frac{N_{inv}(x, t) \lambda_n}{v_{th}} \right], \quad (24)$$

For lines broadened by the Stark effect the coefficients σ_{nm} and κ have different forms:

$$\sigma_{nm}(\lambda) = \frac{\lambda^4}{8\pi c} \frac{A_{nm}}{\Delta\lambda_s} \frac{1}{1 + 4 \left(\frac{\lambda - \lambda_0}{\Delta\lambda_s} \right)^2}, \quad (25)$$

and

$$\kappa(x, t, \lambda_0) = 8.4 \times 10^{-13} \left(\frac{\lambda^4 A_{nm}}{\Delta\lambda_s} \right) N_{inv}(x, t), \quad (26)$$

where $\Delta\lambda_s$ is the half intensity width of the line broadened by the Stark effect. In Eqs. (23) and (25) λ_0 and $\Delta\lambda$ are in cm, N_{inv} is in cm^{-3} , and κ is in cm^{-1} . If $\Delta\lambda_D$ and $\Delta\lambda_s$ have comparable values in formulas for $\sigma_{nm}(\lambda)$, the Voigt profile should be used. The Doppler width $\Delta\lambda_D$ has been calculated from the formula

$$\Delta\lambda_D = 7.6 \times 10^{-5} \lambda_0 \sqrt{\frac{T_i}{M}}, \quad (27)$$

where ion temperature T_i is in eV, and M is the atomic weight of the ion. The widths of $\Delta\lambda_s$ for different λ_0 of hydrogen-like ions CVI (or OVIII) were estimated from

formulas for the quasi-static linear Stark effect $\Delta\lambda_{s1}$ and the electron impact broadening $\Delta\lambda_{s2}$. $\Delta\lambda_s$ was chosen as the lesser of $\Delta\lambda_{s1}$ and $\Delta\lambda_{s2}$.⁷² The $\Delta\lambda_{s1}$ and $\Delta\lambda_{s2}$ were calculated from the following formulas^{72,73}

$$\Delta\lambda_{s1} \approx (n^2 - m^2) \lambda_o^2 \frac{h}{\pi^2 c m} \frac{\bar{p}}{Z} N_p^{2/3}, \quad (27)$$

and

$$\Delta\lambda_{s2} \approx \frac{3n^4 \lambda_o^2}{2\pi c} \frac{h}{m^2} \frac{N_e}{v_e}, \quad (28)$$

where Z is the ion charge and $\bar{p}$ is the average perturbed charge of density N_p . n and m are the principal quantum numbers of the upper and lower levels of line λ_o . v_e is the electron velocity, which can be approximated by the electron thermal velocity. By comparing $\Delta\lambda_D$ and $\Delta\lambda_s$ from Eqs. (26) - (28) it can be seen that for $N_e \geq 10^{19} \text{ cm}^{-3}$, $\lambda \geq 100 \text{ \AA}$ and $T_i \leq 5 \text{ eV}$ the broadening of the lines is due mainly to the Stark effect.

In Table I the calculated total gain G in a plasma column, $l = 10 \text{ cm}$, is given for maximum inversions from Figs. 5, 7a, and 9. This very high gain for the $3 \rightarrow 2$ transition ($\lambda_o = 132 \text{ \AA}$) indicates that even a shorter plasma column with an electron density one order of magnitude lower should be sufficient for obtaining strong lasing action. If the rate coefficients used were too favorable for calculating inversion (an uncertainty factor of 2-3 in rate coefficients may be expected) then the real gain K would be lower. Experimentally, one can compensate for this uncertainty by simply modifying the length of the plasma column.

VIII. CONCLUSION

A new proposal for soft x-ray lasing in a confined plasma column was presented. Inversion was considered in carbon and oxygen columns without mirrors and with a small ratio of column radius to length. It was assumed that the plasma was heated by a CO_2 -laser and terminated in the radial direction by a strong

solenoidal magnetic field. This approach allowed for a slow decrease of $N_e(t)$ and a faster decrease of $T_e(t)$ by radiation losses which, in turn, provided good conditions for population inversion by recombination processes in the plasma column. Resonance charge-exchange can lead to the increase of populations of selected levels in a plasma with electron density $N_e \approx 10^{17} \text{ cm}^{-3}$. For a higher electron density ($N_e \approx 10^{19} \text{ cm}^{-3}$) the charge-exchange process stimulated an increased inversion through rapid decrease of $T_e(t)$ rather than by selective level population.

By extensive calculations of the time evolution of level populations it was shown that for a CO_2 -laser beam with power $P_L \approx 10^{10} \text{ W}$ an inversion for CIV ions of order $N_{\text{inv}} \approx 10^{14} - 10^{15} \text{ cm}^{-3}$ can be obtained in the spectral range 100-200 Å. For a shorter wavelength ($\lambda_0 \approx 27 \text{ Å}$) the laser beam power should be about three orders of magnitude higher in order to reach $T_e \approx 2000 \text{ eV}$ in a dense plasma column ($N_e \approx 10^{20} \text{ cm}^{-3}$). This power would be necessary for strong depopulation by ionization of the ground state of CIV during plasma heating.

Calculated gain k for different heating and plasma conditions indicates the possibility of obtaining strong lasing action ($G > 100$) in the amplified spontaneous emission regime (without mirrors) in the spectral range 100-200 Å. It is possible to obtain this G for a relatively short plasma column ($l \approx 3-15 \text{ cm}$) and moderate laser power.

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TABLE I. Total gain G in $L = 10$ cm plasma column for CVI maximum level population inversions presented in Figs. 5, 7a, and 9.

Wavelength, $n \rightarrow m$	Figure 5		Figure 7a		Figure 9	
	$N_{inv}(cm^{-3})$	G	$N_{inv}(cm^{-3})$	G	$N_{inv}(cm^{-3})$	G
$\lambda = 182 \text{ \AA}$ $3 \rightarrow 2$	1.1×10^{16}	1.9×10^2	1.6×10^{16}	2.8×10^2	7.6×10^{15}	1.4×10^2
$\lambda = 135 \text{ \AA}$ $4 \rightarrow 2$	5×10^{15}	3.6	1.9×10^{16}	1.3×10^1	3.8×10^{15}	2.8

FIGURE CAPTIONS

Fig. 1. Scheme of experiment for creating a level population inversion in a magnetically confined plasma column heated by CO_2 -laser. Distributions of electron density N_e along the radius and column axis of the plasma heated by relatively low power CO_2 -laser ($P_L \approx 3 \times 10^8 \text{ W}$) are taken from T. K. Chu and L. C. Johnson.⁵³

Fig. 2. Time evolution of N_e and T_e in a plasma column heated by a lower power CO_2 -laser ($P_L \approx 3 \times 10^8 \text{ W}$) (from Refs. 54,55).

Fig. 3. Energy levels diagram for CV and CVI ions.

Fig. 4. Comparison of calculations for level population ratios N_n/N_2 using rate equations with $dN_n/dt \neq 0$ for all n (solid line) and $dN_n/dt = 0$ for $n > 2$ (dashed line).

Fig. 5. Time evolution of relative level populations N_n/g_n for four levels ($n = 2,3,4,5$), population of ground level N_1^{Z-1} of CVI and electron density N_e calculated for given $T_e(t)$ profile.

Fig. 6. Time evolution of relative level populations N_n/g_n of hydrogen-like oxygen (OVIII) and electron density calculated for given $T_e(t)$ profile.

Fig. 7. (a) Time evolution of relative level populations of CVI with $T_e(t)$ calculated from energy balance equation. (b) Time evolution of ground state densities of C^{4+} , C^{5+} and C^{6+} with $T_e(t)$ and $N_e(t)$ as in (a).

Fig. 8. Relative level populations of CVI with maximum $T_e = 300 \text{ eV}$ [$T_e(t)$ from energy balance equation].

Fig. 9. Relative level populations of CVI calculated with and without charge-exchange for times $t_1 = 140 \text{ nsec}$ and $t_2 = 148 \text{ nsec}$ from beginning of population evolution. Times t_1 and t_2 are chosen on the fast raising part of level populations.

Fig. 10. (a) Time evolution of CVI level populations for laser heated, radiation cooled and adiabatically decompressed plasma. For $t > 53$ nsec relaxation of population to steady state conditions are shown. Charge-exchange term is included for $t \geq 45$ nsec. (b) Relative populations for levels with $n = 2, 3, \dots, 9$ at $t_1 = 20$ nsec, $t_2 = 50$ nsec, $t_3 = 53$ nsec and for steady state conditions with and without charge-exchange taken into account.

Fig. 11. (a) Time evolution of CVI level populations calculated with charge-exchange term taken into account. Relatively low, constant electron density and rapidly decreasing electron temperature was assumed during cooling simulation. (b) Results of the same calculation as for (a), but without charge-exchange.

Fig. 1. 783419

Fig. 2. 783918

Fig. 3. 793167

Fig. 4. 786487

Fig. 5. 793760

Fig. 6. 793758

Fig. 7a. 793166

Fig. 7b. 793779

Fig. 8. 793754

Fig. 9. 793647

Fig. 10a. 793757

Fig. 10b. 793762

Fig. 11a. 793633

Fig. 11b. 793648