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BNL--42493

QUANTIZATION OF ANOMALOUS GAUGE THEORIES ON A LATTICE *

DE89 010247

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APR 24 1989

We discuss advantages and disadvantages of three equivalent lattice actions proposed before for anomalous chiral gauge theories. Applications to the chiral Schwinger model and the 4 dimensional chiral QED are presented.

1. INTRODUCTION

Lattice regularization¹ provides us with a powerful tool to investigate Quantum Fields Theories, such as QCD or Gauge-Higgs system. However successes of lattice regularization have been limited to purely bosonic systems or to systems with fermion-gauge field vector-like interactions. The reason for this is that we can not define the lattice fermion coupled to gauge field in a chirally invariant way.² This fact is closely related to the local gauge anomaly.³ Because the lattice regularization is well-defined, an anomalous symmetry cannot be maintained on a lattice. In the continuum case, the anomalous symmetry is broken by the regularization, for example, Pauli-Villars regularization or dimensional regularization.

From this observation, it seems possible to construct a chirally gauge invariant action for the anomaly free case. However it is not so easy. There are two ways of breaking chiral gauge symmetry: one is by the anomaly, the other is the breaking which can be written as gauge variant local terms. Anomaly free means that a pure anomaly is canceled out among all fermions contributions; however it does not mean the cancellation of the gauge variant local terms. We should add by hand the gauge variant local counterterms order by order to recover the gauge invariance of the final results. Furthermore the cancellation of a pure anomaly is only true for the infinite cut-off limit. Because of the above two reasons, we cannot generally have chiral gauge invariant action with finite cut-off even for the anomaly free case.

Recently three methods have been proposed to define chiral gauge theories on a lattice.^{4,5} In this talk we will dis-

cuss advantages and disadvantages of these three methods. One important physical motivation for this work is to put the full Weinberg-Salam model (with fermions) on a lattice in order to investigate it by non-perturbative methods such as Strong Coupling Expansion or Monte-Carlo simulation. The other theoretical motivation is to investigate the consistency; for example, renormalizability or unitarity, of anomalous gauge theories. We will show that anomalous gauge theories are renormalizable in the sense of lattice power counting.⁴ We will also apply these methods to the chiral Schwinger model in 2 dimensions as well as the chiral QED in 4 dimensions.

2. LATTICE ACTIONS

We take the Wilson fermion action⁶ for the fermionic part of the action to define chiral gauge theory on a lattice. We call this action *Type I*, which is given by

$$S_F^I = S_{inv} + S_W$$

$$S_F = a^d \sum_{n,\mu} \frac{1}{2a} \bar{\psi}_n [(U_{n,\mu} P_L + P_R) \psi_{n+\hat{\mu}} - (U_{n-\hat{\mu},\mu}^\dagger P_L + P_R) \psi_{n-\hat{\mu}}] \quad (2.1)$$

$$S_W = -a^d \sum_{n,\mu} \frac{r}{2a} \bar{\psi}_n (\psi_{n+\hat{\mu}} + \psi_{n-\hat{\mu}} - 2\psi_n) \quad (2.2)$$

Here S_{inv} is a gauge invariant part of S_F^I and S_W is a gauge variant part of S_F^I , that is, the Wilson term.

Properties of this action are as follows.

1. Unphysical fermion doublings are suppressed by S_W .

We can think ψ_n , $\bar{\psi}_n$ and $U_{n,\mu}$ as the *physical fields*.

*Talk presented at the International Symposium *Lattice '88*, Batavia, IL, USA, September 22-25, 1988.

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2. The gauge symmetry is broken by S_W . Though S_W is irrelevant in the naive continuum limit, we can get the local chiral anomaly from the effect of fermion integration in the presence of S_W .⁷ Therefore, at least in the usual perturbative expansion this lattice action can work as well as other continuum regularizations. Because of the lack of the gauge invariance, renormalizability is not manifest.

3. This action has the reflection positivity in the range that $0 < \tau^2 \leq 1$.⁴

By a change of integration variables, we get *Type II* action form *TYPE I* action without changing the partition function;^{4,5}

$$S_F^{II} = S_{inv} + S_W(g)$$

$$S_W(g) = -a^d \sum_{n,\mu} \frac{\tau}{2a} [\bar{\psi}_n g_n^\dagger P_R (\psi_{n+\hat{\mu}} + \psi_{n-\hat{\mu}} - 2\psi_n) + (\bar{\psi}_{n+\hat{\mu}} + \bar{\psi}_{n-\hat{\mu}} - 2\bar{\psi}_n) g_n P_L \psi_n]. \quad (2.3)$$

$S_W(g)$ represents the violation of gauge invariance and includes all information concerning anomaly.

Properties of this action is as follows.

1. Unphysical fermion doublings are suppressed in the range that $g_n \approx 1$, but the suppression is not clear when g_n is random from site to site.
2. S_F^{II} is invariant under the extended gauge transformation :

$$\begin{aligned} \psi'_n &= (h_n P_L + P_R) \psi_n \\ \bar{\psi}'_n &= \bar{\psi}_n (h_n^\dagger P_R + P_L) \\ U'_{n,\mu} &= h_n U_{n,\mu} h_{n+\hat{\mu}}^\dagger \\ g'_n &= g_n h_n^\dagger. \end{aligned} \quad (2.4)$$

3. We can define the regularized lattice Wess-Zumino term⁸ by

$$S^{WZ} \equiv S_W(g) - S_W(1),$$

and g_n can be called the Wess-Zumino scalar. Then we get

$$S_F^{II} = S_F^I + S^{WZ}(g).$$

4. The power counting can not be applied directly to S_F^{II} , since g field has no kinetic terms.

5. If we add a kinetic term of g by hand, g may be regarded as an angular part of the Higgs field⁹ (Higgs generated Wilson term). In 2 dimension this Higgs generated Wilson term is renormalizable. However it is un-renormalizable in 4 dimensions.

Since there are no coupling among g_n in the action we can integrate the g_n field explicitly.⁴ Now we get *Type III* action form *Type II*;

$$S_F^{III} = S_{inv} + S_R$$

where

$$\exp[S_R] = \prod_n \int dg_n \exp[S_W(g)].$$

Properties of this action are as follows.

1. S_F^{III} has the original gauge invariance.
2. S_F^{III} is renormalizable by the lattice power counting even in 4 dimensions.⁴ This is not strange since the original action does not contain any dimensionful coupling which is un-renormalizable.
3. Because of the Grassmann property of fermions, S_R is a finite polynomial of $\bar{\psi}$ and ψ .
4. Unphysical doubling fermions may exist since the fermion propagator at the lowest order is equal to the one of the naive fermion. To conclude whether the doubling modes exist or not, we have to sum up the effect of S_R . But it is very difficult to calculate higher order terms by this action. For example, if we apply our method to the free Wilson fermion by using the global chiral transformation as a change of the integration variables instead of the local chiral gauge transformation, we get the fermion propagator given by

$$S_F(p) = \frac{-ia\gamma_\mu \cdot \sin p_\mu}{\sin^2 p + A(p)^2} \quad (2.5)$$

where

$$A(p) = \tau \sum_\mu (1 - \cos p_\mu).$$

From the above result it is seen that the fermion propagator has chiral symmetry and contains no doubling modes. However the action which gives the above fermion propagator should be non-local. Therefore the above calculation cannot be easily extended to the case of chiral gauge theories. It is

also noted that the above propagator is similar to the Rebbi's fermion.¹⁰

5. If the doubling mode survive in the continuum limit, we cannot regard ψ , $\bar{\psi}$ and U as the physical fields. We call them the *renormalizable* fields. However it is stressed that we do not attempt to propose the chiral gauge invariant lattice action for the physical fields though Nielsen-Ninomiya's theorem² does not apply to the non-linear action. We introduce these renormalizable fields to show the renormalizability of the theory and the physical interpretation should be made for the physical fields. This relation between the physical fields and the renormalizable fields reminds me of the relation between the unitary gauge and the renormalizable gauge in the gauge-Higgs system.

6. Since the *Type I* action is gauge variant, there exist a non-trivial relation between the physical fields and the renormalizable fields.¹¹ For example, to see the relation between fermion fields with $g_n \in U(1)$, we introduce a generating function for fermion field such as

$$\begin{aligned} Z(\eta, \bar{\eta}) &= \int \mathcal{D}\bar{\psi} \mathcal{D}\psi \mathcal{D}U \\ &\times \exp[S_{inv} + S_W + \bar{\eta}\psi^p + \bar{\psi}^p\eta] \\ &= \int \mathcal{D}\bar{\psi} \mathcal{D}\psi \mathcal{D}U \\ &\times \exp[S_{inv} + \bar{\eta}P_R\psi^r + \bar{\psi}^r P_L\eta] \\ &\times \prod_n I_0(2\sqrt{\bar{D}_n D_n}) \end{aligned} \quad (2.6)$$

where

$$\begin{aligned} \bar{D}_n &= \bar{A}_n + \bar{\psi}_n^r P_R \eta_n, \\ D_n &= A_n + \bar{\eta}_n P_L \psi_n^r. \end{aligned}$$

and suffix p (r) means the physical (renormalizable) field. From the above identity it is easy to get equations such that

$$\begin{aligned} \langle (P_R \psi_n^p)^\alpha (\bar{\psi}_m^p P_L)^\beta \rangle &= \langle (P_R \psi_n^r)^\alpha (\bar{\psi}_m^r P_L)^\beta \rangle > \\ \langle (P_L \psi_n^p)^\alpha (\bar{\psi}_m^p P_L)^\beta \rangle &= \langle (P_L \psi_n^r)^\alpha (\bar{\psi}_m^r P_L)^\beta A_n \rangle \\ \langle (P_L \psi_n^p)^\alpha (\bar{\psi}_m^p P_R)^\beta \rangle &= \delta_{nm} \times \\ &\langle (P_L \psi_n^r)^\alpha (\bar{\psi}_m^r P_R)^\beta \rangle \\ &+ \langle (P_L \psi_n^r)^\alpha (\bar{\psi}_m^r P_R)^\beta A_n \bar{A}_m \rangle \end{aligned}$$

in 2-dimensions. Similar but more complicated equations can be also obtained in 4-dimensions. By using these equation we can calculate all vacuum expectation values of the physical fields if we know all vacuum expectation values of the renormalized fields.

7. If we interpret $\bar{\psi}^p$, ψ^p and A_μ^p as real physical fields we can observe interesting properties as follows (1) The physical fermion field is not confined because gauge invariance does not hold for the physical fields. (2) Mass generation for gauge field by quantum effect is allowed because of the same reason. These two properties may correspond to the fact that the weak gauge bosons have mass and the leptons are not confined.

3. APPLICATIONS

We calculate the effective action of the chiral Schwinger model¹² by the perturbative expansion. After integrating out the fermion field we get

$$\begin{aligned} S_{eff}(A') &= -\sum_n \left[\frac{K+1}{8\pi} A'_{n,\mu} A'_{n,\mu} \right. \\ &\quad + \frac{1}{4\pi} \partial^\mu A'_{n,\mu} \frac{1}{\Delta} \partial^\nu A'_{n,\nu} \\ &\quad \left. + \frac{1}{4\pi} \partial^\mu A'_{n,\mu} \frac{1}{\Delta} \epsilon^{\alpha\beta} \partial_\alpha A'_{n,\beta} + S_G \right] \end{aligned} \quad (3.1)$$

where

$$A'_{n,\mu} = A_{n,\mu}$$

for *Type I*¹³ and

$$A'_{n,\mu} = A_{n,\mu} - \partial_\mu \theta_n$$

for *Type II*,^{4,5}

$$\Delta = \partial^\mu \cdot \partial_\mu, \quad U_{n,\mu} = \exp[iaA_{n,\mu}], \quad g_n = \exp[i\theta_n],$$

$$\begin{aligned} \frac{K}{4\pi} &= \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} \frac{d^2 p}{(2\pi)^2} \left[\frac{\sin^2 p_\mu}{F(p)} \right. \\ &\quad \left. + \frac{(2\sin^2 p_\mu - \sin^2 p) \cdot \cos^2 p_\mu}{F(p)^2} \right] \end{aligned} \quad (3.2)$$

$$\sin^2 p = \sum_\mu \sin^2 p_\mu, \quad F(p) = \sin^2 p + [r \sum_\mu (1 - \cos p_\mu)]^2.$$

From the above effective action we get the Wess-Zumino term;

$$S^{WZ} = \frac{K-1}{4\pi} \theta_n \cdot \partial^\mu A_{n,\mu} + \frac{K-1}{8\pi} \partial^\mu \theta_n \partial_\mu \theta_n - \frac{1}{4\pi} \theta_n \cdot \epsilon^{\alpha\beta} \partial_\alpha A_{n,\beta} \quad (3.3)$$

In the case of the *Type II*, we integrate θ_n field in the range $-\infty < \theta_n < \infty$ and get^{4,5}

$$S_{eff}(A) = -\frac{1}{4} \sum_{n,\mu \neq \nu} F^{\mu\nu}(n) \frac{1}{\Delta} (\Delta - m^2) F_{\mu\nu}(n)$$

where

$$m^2 = \frac{e^2}{4\pi} \cdot \frac{K^2}{(K-1)}.$$

It is noted that the above result is gauge invariant.

The renormalization ambiguity K of the continuum case¹² (denoted as a there) now can be determined. By the numerical integration of K , we find that

$$m^2 > 0, \quad \text{for } 0 < r^2 \leq r_c^2$$

where $r_c \simeq 1.5$. This shows that the theory has unitarity in this range of r as well as gauge invariance: the chiral Schwinger model is renormalizable, gauge invariant and unitary in this range of Wilson parameter r . The condition that $0 < r^2 \leq r_c^2$ is consistent with the condition that $0 < r^2 \leq 1$, which is implied by the reflection positivity,⁴ and thus it shows that the reflection positivity is a *sufficient* condition for unitarity of theories.

If we use the naive fermion instead of the Wilson fermions, we get

$$S_{eff}(A) = -\frac{1}{4} \sum_{n,\mu \neq \nu} F^{\mu\nu}(n) \frac{1}{\Delta} (\Delta - m^2) F_{\mu\nu}(n)$$

where

$$m^2 = \frac{e^2}{\pi} \cdot \frac{1}{2} \cdot 4.$$

Here e^2/π is the dynamically generated mass of the vector Schwinger model, the factor $1/2$ comes from the chiral projection and the factor 4 comes from the number of fermion doubling modes. This shows that the anomaly does not play an essential role to generate a mass of gauge boson. The infra-red singularity in 2 dimensions generate the mass.

If we add a gauge invariant kinetic term for g_n :

$$S_H = \frac{H}{a^2} \sum_{n,\mu} [g_n U_{n,\mu} g_{n+\hat{\mu}}^\dagger + g_{n+\hat{\mu}} U_{n,\mu}^\dagger g_n^\dagger],$$

K is shifted as $K + 2H$.⁴

$$m^2 = \frac{e^2}{4\pi} \cdot \frac{(K + 2H)^2}{(K + 2H - 1)}.$$

Next we calculate the two-point function of gauge boson in the 4 dimensional chiral QED.¹⁴ The result is

$$\Pi_{\mu\nu}(p) = (Z_0 \log(p^2 a^2) + Z_1)(p^2 \delta_{\mu\nu} - p_\mu p_\nu) - \frac{M^2}{a^2} \delta_{\mu\nu} + B p^2 \delta_{\mu\nu} + C p_\mu^2 \delta_{\nu\nu}. \quad (3.4)$$

Here M , Z_0 , Z_1 , B and C are constant. The above result is obtained both for *Type I* and *Type II*. The above result show that the Lorentz non-covariant term as well as gauge non-invariant terms are generated by the fermion integral.¹⁵ In the case of the lattice QCD these unwanted terms are prohibited by Ward identity.¹⁶ In the case of the chiral QED these terms cannot be prohibited since *Type I* and *Type II* do not have non-trivial Ward identity (*Type II* has *trivial* Ward identity.) If we use the naive fermion we get a gauge invariant result:

$$\Pi_{\mu\nu}(p) = 2^4 (Z_0 \log(p^2 a^2) + Z_1)(p^2 \delta_{\mu\nu} - p_\mu p_\nu).$$

Since *Type III* has non-trivial Ward identity, $\Pi_{\mu\nu}$ should be gauge invariant. Of course we have to check it by the explicit calculation in the near future.

4. DISCUSSION

We propose three equivalent action and investigate their properties. *Type I* is physical but gauge-variant. *Type II* is gauge invariant with Wess-Zumino scalar and *Type III* is renormalizable, gauge invariant but non-linear.

An application of *Type II* to the chiral Schwinger model shows that the theory is unitary and gauge invariant. On the other hand for the chiral QED in 4 dimension, the Lorentz non-covariant term as well as gauge non-invariant terms are generated by fermion integral. Before attempting g integration, we should investigate this point more carefully. This situation for the non-Abelian chiral gauge theory is more difficult; we have to deal with the non-linear σ model coupled to gauge fields. Recently it is claimed that the non-linear σ model without gauge fields is renormalizable though the unitarity is not clear.¹⁷ It may be interesting to extend the analysis to the non-linear σ model with gauge fields.

Type III action may be free from the above problems. Of course that should be checked explicitly.

ACKNOWLEDGEMENTS

I would like to thank Prof. M. Creutz, Prof. R. Shrock and Dr. I-Hsiu Lee for useful discussion. This work was supported by the U.S. Department of Energy under contract DE-AC02-76CH00016.

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