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**FAMILY NUMBER NON-CONSERVATION
INDUCED BY THE SUPERSYMMETRIC
MIXING OF SCALAR LEPTONS***

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1. Introduction

1.1 WHY SUPERSYMMETRY?

Many of the original motivations for supersymmetry have long since vanished. It was recognized immediately that such a symmetry must be severely broken in the low energy world. In its early days supersymmetry was looked on principally as a hopeful solution to the gauge hierarchy problem (the question of why there should exist such a discrepancy between the two energy scales M_{weak} at 100 GeV, and M_{GUT} at 10^{16} GeV or more). In order to be useful in this respect the scale at which supersymmetry breaks cannot exceed the weak scale by more than an order of magnitude if the breaking mechanism is to be relatively simple. Thus we have replaced one symmetry-breaking hierarchy problem by another. Admittedly the radiative hierarchy problem is resolved but this was but a chimeral problem to begin with (presumably if we possessed the mathematical techniques, and the stamina, to solve the theory to all orders, including non-perturbative effects, we would only have to adjust the overall scales once). Early aspirations of uniting known particles into supermultiplets failed utterly with the result of an instantaneous doubling of the number of states conjectured to exist. None of these new states has yet been seen.

Despite this the supersymmetric principle has become increasingly popular. The true reason for this is the most compelling of motivations, that same aesthetic belief in unification which spawned the Grand Unified Theories, and therefore the gauge hierarchy problem. Supersymmetric theories are believed to be the only theories capable of unifying fundamental bosonic and fermionic fields. This is undoubtedly due to our imperfect understanding of what spin is at a fundamental, topological, level. Surely uniting these two disparate sectors of the material world is at least as profound as considering ever-greater gauge groups to fragment.

When supersymmetry is raised to a local symmetry the Lorentz group manifests itself as a subgroup. Thus gravitation becomes incorporated into the theory

in a natural way. In addition, the currently popular string models find the inclusion of supersymmetry highly desirable, if not irrecusable.

1.2 THE FAMILY PUZZLE

One of the outstanding puzzles in elementary particle physics is that of particle generations. The material constituents of the world, as opposed to the mediators of the fundamental forces, appear to be replicated in families or generations. Each seems to be a simulacrum of the previous family, but of greater mass. At this time three such generations are known. Why this should occur is a complete mystery.

This mystery is particularly profound in the leptonic, or weakly interacting, sector where each family would seem to possess an absolutely conserved quantum number, generally referred to as electron, muon and tau number or, collectively, as family number. In all reactions which have been thus far observed the amount of "electron-ness" has neither increased nor decreased. This is in stark contrast to the quark (strongly-interacting) sector where mixing between the states is known to occur at a fundamental level. In the light of the "standard model" it is understood why this should be the case. In extensions beyond this it is frequently difficult to retain this symmetry in an appealing fashion.

A symmetry may be realized locally or globally. If local such a "horizontal" symmetry would have profound implications, requiring a new family-changing gauge sector. A global symmetry, if not the consequent of some local symmetry, but if imposed or maintained artificially (for instance by constraining some parameter to be small) is to be viewed as, at best, unnatural and probably untenable. Considered in this light it is interesting to ponder whether lepton family number might not be broken to some small degree. We will discuss a mechanism which might accomplish this.

1.3 SCALAR LEPTON MIXING

The most egregious aspect of ($N=1$) supersymmetric theories is that each particle state is accompanied by a 'super-partner', a state with identical quantum numbers save that it differs in spin by one half unit. For the leptons these are scalars and are called "sleptons", or scalar leptons. These consist of the charged sleptons (selectron, smuon, stau) and the scalar neutrinos ('sneutrinos').

We examine a model of supersymmetry with soft breaking terms in the electroweak sector. Explicit mixing among the scalar leptons results in a number of effects, principally non-conservation of lepton family number. Comparison with experiment permits us to place constraints upon the model.

Each supersymmetric family has charged sleptons associated with both the left and right-handed helicity states of the corresponding lepton. These, too, may combine (within a single generation). The effect of this will be discussed in chapter two in the context of the anomalous magnetic moment of the muon and the electron. These were selected due to the extremely precise knowledge of their experimental values. In chapter three we will examine the result of intergenerational slepton-mixing for the process $e^+e^- \rightarrow Z^0 \rightarrow \tau^\pm \mu^\mp$. Since several Z^0 -factories, in particular the SLC, are expected to produce these particles copiously, this process, should it occur at a reasonable rate, could prove experimentally interesting. These matters will be discussed. In chapter four we consider the classic family number violating process, radiative muon decay: $\mu \rightarrow e\gamma$. These three chapters are followed by a string of appendices, most of which are associated with a particular chapter. These are referred to within the chapters and contain details and derivations not necessary to the cursory understanding of a given chapter.

2. Contribution to Anomalous Magnetic Moment from Slepton Mixing

2.1 INTRODUCTION

In Chapter 1 we saw that the sleptons might mix among themselves in a supersymmetric theory. In this chapter we illustrate the manner in which the anomalous magnetic moments of the electron (a_e) and muon (a_μ) are altered when the theory is made supersymmetric in a minimal way. We then permit the sleptons to mix together which will lead to new effects. We will discover, despite the fact that a_e has been determined experimentally to much greater accuracy than a_μ , that the results of the a_μ calculation tend to place the more stringent restrictions on supersymmetry parameters when compared with experiment.

We will confine ourselves to pure QED and SQED (supersymmetric QED) in this chapter. The extension to the full electroweak theory is straightforward but tedious. Exactly how this should be done will be made obvious in the later chapters and their corresponding appendices.

The calculation will be performed in a somewhat pedagogical fashion. As two-component notation is natural for supersymmetric calculations some basic notation has been introduced in Appendix A. In Appendix B we examine QED in its two and four component forms and consider the interpretation of two-component propagators. Thus armed, in Appendix C we calculate a_e in four-component form and in Appendix D we repeat the calculation in two-component form so that they may be carefully compared. In Appendix E we next compute Δa_e , the additional contribution from supersymmetrizing the theory (standard SQED), before the sleptons have been allowed to mix. The most important contribution from the mixing of the sleptons comes from $\tilde{e}_L - \tilde{e}_R$ mixing as detailed in Appendix F. If we permit intergenerational mixing then the contribution to a_e will be spread over a greater number of terms. For this reason we restrict ourselves to a single generation here. This has the added benefit of reducing 25

angles and phases to three angles (one per generation assuming three generations). The reverse of this argument will be used in the next chapter when the left-right mixing will be ignored in favor of the intergenerational mixing. The penultimate appendix (Appendix G) related to chapter two will show plots of the various results. In Appendix H we derive a few useful formulae. In the first section certain Feynman integrals, which appear frequently in these pages, are presented. In the second section we discuss generalized versions of the Gordon Decomposition, a generally useful result.

As can be readily seen most of the actual work will be presented in the appendices. This chapter is primarily a summary of the results, comparison with experiment, consequences and commentary. In Section 2.2 we review the concept of anomalous magnetic moments. In 2.3 the results of the appendix calculations are summarized and compared with experiment in 2.4.

2.2 THE MAGNETIC MOMENT

From a Hamiltonian viewpoint the only interaction between the ψ (electron) and A (electromagnetic) fields is via the term $\mathcal{H}_{int} = j_\mu^\dagger A_\mu$ where $j_\mu^\dagger = c\bar{\psi}\gamma_\mu\psi$. The Gordon Decomposition (Appendix H) can be used in conjunction with the Dirac equation in the presence of an A -field $(\not{D} + m)\psi = 0$ where $D_\mu = \partial_\mu + ieA_\mu$, to divide the current into two pieces.^[1] In the non-relativistic limit the first reduces to the electric charge current density of non-relativistic quantum mechanics. It is the other term which interests us. This yields an interaction piece

$$\mathcal{H}_{int}^{mag} = j_\mu A^\mu \sim e/4m \bar{\psi} \not{F}^{\mu\nu} \sigma_{\mu\nu} \psi + \text{total derivative.}$$

Non-relativistically

$$\bar{\psi} \not{F}^{\mu\nu} \sigma_{\mu\nu} \psi \approx 2\psi^\dagger \underline{\sigma} \cdot \underline{B} \psi$$

and therefore $\mathcal{H}_{int}^{mag} \rightarrow e/2m \psi^\dagger \underline{\sigma} \cdot \underline{B} \psi$. But we know that classically $\mathcal{H}_{int}^{mag} = \underline{\mu} \cdot \underline{B}$ where $\underline{\mu}$ is the magnetic moment of the electron. Thus we can identify $\underline{\mu} = e/m \underline{S}$

or, in greater generality, $\underline{\mu} = g e/2m \underline{S}$ where g is the gyromagnetic ratio which here is two.^{*} Classically g is determined by the charge current distribution of an infinitesimal current loop (magnetic dipole). $g = 2$ corresponds to a point dipole. Deviations from $g = 2$ are given by $g = 2(1 + a)$ where a is called the anomalous magnetic moment.

From the two-component composition of the $\bar{\psi}^* \gamma^\mu \psi^*$ and $\bar{\psi} \sigma^{\mu\nu} \psi$ vertices (see Appendix B) we may observe their helicity structure. Ignoring the mass of the electron we see that the $A_\mu \bar{\psi}^* \gamma^\mu \psi^*$ term connects e_L to e_L (equivalently e_L^- to e_L^+) while $F_{\mu\nu} \bar{\psi}^* \sigma^{\mu\nu} \psi^*$ connects e_L to e_R (or e_L^- to e_L^+). Thus photonic emission via the electric charge current leaves the helicity of the state unchanged while the magnetic dipole interaction flips the helicity.

Such observations have been made in conjunction with the lowest-order vertex (as illustrated in Fig. 2.1a). The one-loop QED vertex correction is as in the classic infundibular (triangle) diagram of Fig. 2.1b. This will contribute the interactions

$$\bar{u}(p) \Gamma_\mu(p', p) u(p) = \bar{u}(p) \left[\gamma_\mu F_1(q^2) + \frac{i}{2m} \sigma_{\mu\nu} q^\nu F_2(q^2) \right] u(p').$$

$F_1(q^2)$ contributes to the (infinite) wavefunction renormalization while, on shell ($q^2 = 0$), $F_2(q^2)$ contributes to the magnetic moment and thus to $a_e = (g_e - 2)/2$. Indeed, comparing the definition of a_e we see that $a_e = F_2(q^2 \rightarrow 0)$. Since QED is a renormalizable theory, and no F_2 -type term appears in the QED Lagrangian, we know that the F_2 term resulting from first order contributions must be finite (since there is no bare parameter to absorb an infinity). The anomalous moment can be thought of as measuring an effective radius of the charge current distribution engendered by radiative corrections to the simple bare "point" vertex.

* Here $\hbar = c = 1$ so that g is in units of Bohr Magnetons. Otherwise $\underline{\mu} = g\hbar e/2mc$.

It is often the case than when we consider interactions at large energies, or have particles in internal loops whose masses are large with respect to the external lepton masses, we will find that the magnetic ($\sigma^{\mu\nu}$) contribution to the cross section will be reduced relative to that of the electric (γ^μ) term (by $O(m_{\text{lepton}}^2/q^2, m_{\text{heavy}}^2)$) and can thus often be ignored. If for some reason, such as gauge invariance, the γ^μ term cannot appear then the interaction must go through the convective ($\sigma^{\mu\nu}$) term. We will see this happen when we examine $\mu \rightarrow e\gamma$ on shell ($q^2 = 0$) but not for $Z^0 \rightarrow e^+\mu^-$ ($q^2 \neq 0$).

2.3 RESULTS

In this section the results of computing the various contributions to $a_\mu = (g_\mu - 2)/2$ and $a_e = (g_e - 2)/2$, as algebraically detailed in the first few appendices, are ingeminated. We work to one-loop (α) order. The one-loop vertex correction from pure QED (Appendices 2 and 3) yields

$$a_i = a_\mu = \frac{\alpha}{2\pi}. \quad (2.1)$$

When supersymmetric QED (SQED) contributions (assuming no mixing amongst the sleptonic states) are appended, as in Appendix D, we discover the additional contribution

$$\Delta a_i = \frac{\alpha}{2\pi} \{I_L + I_R\} \quad (2.2)$$

where

$$I_{L,R} = \int_0^1 dx \int_0^1 dz \frac{x(1-x)}{-x^2 + \frac{m_e^2 - m_{\tilde{e}_{L,R}}^2 + m_\lambda^2}{m_e^2} x - \frac{m_\lambda^2}{m_e^2}} \quad (2.3)$$

m_e is the mass of the electron.

$m_\lambda = m_{\tilde{\gamma}}$ is the mass of the photino.

$m_{\tilde{e}_{L,R}}$ are the masses of the left and right selectron.

$$\text{For } \Delta a_\mu, m_e \text{ is replaced by } m_\mu \text{ and } m_{\tilde{e}_{L,R}} \text{ by } m_{\tilde{\mu}_{L,R}}. \quad (2.4)$$

We remark on some of the limiting cases which are presented in the appendix.

A likely limit is $m_{\tilde{e}_{L,R}} \gg m_\lambda \gg m$ (equation E.24). Then

$$\Delta a_e \doteq \Delta a_e^{(m_\lambda=0)} = -\frac{\alpha}{12\pi} \left\{ \frac{m_e^2}{m_{\tilde{e}_L}^2} + \frac{m_e^2}{m_{\tilde{e}_R}^2} \right\}. \quad (2.5)$$

This is precisely Fayet's 1974 result.^[11] If we relax the condition that $m_\lambda \ll m_{\tilde{e}_{L,R}}$ we find (eqn E.32) that, to the next order in $m_\lambda/m_{\tilde{e}_{L,R}}$,

$$\Delta a_e = \Delta a_e^{(m_\lambda=0)} \left[1 - 2 \frac{m_\lambda^2}{m_{\tilde{e}_L}^2 + m_{\tilde{e}_R}^2} \left(\frac{m_{\tilde{e}_L}^2}{m_{\tilde{e}_L}^2} + \frac{m_{\tilde{e}_R}^2}{m_{\tilde{e}_L}^2} \right) \right]. \quad (2.6)$$

In limits where supersymmetry remains unbroken $m_{\tilde{e}_L} = m_{\tilde{e}_R} = m_e$ and $m_\lambda = 0$. In this limit we find that $\Delta a_e = -\alpha/2\pi$ so that, for (2.1),

$$a_e = a_e^{QED} + \Delta a_e = 0. \quad (2.7)$$

This is gratifying as we might have anticipated that there would be no anomalous magnetic moment to any order in an exactly supersymmetric theory (for the same reason that loop divergences cancel). This was first demonstrated by Ferrara and Remidi.^[14]

We now consider the effect of permitting the left and right sleptons to mix. Indeed, in realistic models with Higgs fields, it is often difficult to prevent them from doing so.^[11] Since these states differ in hypercharge and weak isospin we see that $SU(2) \times U(1)_Y$ is broken, although $U(1)_{em}$ is not. (This may be induced both radiatively^[11] and at the tree level.^[11]) Most treatments only consider the case where the photino is massless or nearly so. We shall leave m_λ as a free

parameter.* This introduces a number of important new effects which would appear in a complete electroweak treatment^[7] and thus represents a suitable decoction but with fewer complications.

Let us first consider muons. The mass eigenstates may be taken to be

$$\begin{aligned}\tilde{\mu}_1 &= \cos \phi \tilde{\mu}_L + \sin \phi \tilde{\mu}_R \\ \tilde{\mu}_2 &= -\sin \phi \tilde{\mu}_L + \cos \phi \tilde{\mu}_R.\end{aligned}\quad (2.8)$$

It is found (in Appendix F) that, to lowest order in $m_\mu^2/(m_{\tilde{\mu}_1}^2 - m_\lambda^2)$,

$$\Delta a_\mu = \frac{\alpha}{2\pi} \frac{m_\lambda m_\mu \sin \phi \cos \phi}{(m_{\tilde{\mu}_2}^2 - m_\lambda^2)^2} \left\{ \frac{1}{2} (m_{\tilde{\mu}_2}^2 - m_\lambda^2) - m_\lambda^2 m_{\tilde{\mu}_2}^2 \ell n \frac{m_{\tilde{\mu}_2}^2}{m_\lambda^2} \right\} - (m_{\tilde{\mu}_2}^2 \rightarrow m_{\tilde{\mu}_1}^2). \quad (2.9)$$

We can write (2.9) as

$$\Delta a_\mu = C(f(X_2) - f(X_1)) = \Delta a_\mu^2 - \Delta a_\mu^1 \quad (2.10)$$

where we have let

$$\Delta a_\mu^i = C f(X_i) \quad (2.11)$$

and

$$X_i = \frac{m_{\tilde{\mu}_i}^2}{m_\lambda^2} \quad (2.12)$$

$$f(x) = \frac{\frac{1}{2}(x^2 - 1) - x \ell n x}{(x - 1)^2} \quad f(1) = \frac{1}{6} \quad (2.13)$$

$$C = \frac{\alpha \sin \phi \cos \phi}{2\pi} \frac{m_\mu}{m_\lambda} \quad f'(1) = -\frac{1}{12}. \quad (2.14)$$

* There are astrophysical bounds on m_λ but these are model dependent.

Note that Δa_μ^i is perfectly regular at $m_{\tilde{\mu}_i} = m_\lambda$ and is given by

$$|\Delta a_\mu^i| = \frac{1}{6} G = \frac{\alpha}{12\pi} \frac{m_\mu}{m_\lambda} \sin \phi \cos \phi \quad [m_{\tilde{\mu}_i} = m_\lambda \gg m_\mu]. \quad (2.15)$$

The important point is to note that $\Delta a_\mu \sim m_\mu/m_{SUSY}$ here whereas, prior to slepton mixing, we find that $\Delta a_\mu \sim (m_\mu/m_{SUSY})^2$. The salience of this becomes apparent when we make contact with experimental limits in the next section. When the anomalous moment is linearly dependent upon the *lepton* mass we have substantially greater sensitivity to new effects. Concomitant with this statement is the assertion that we can place more stringent limits on the supersymmetry parameters of the theory. It may be noted, however, that the Feynman diagrams which are responsible for the m_μ^2 terms are still present (Diagrams I, II, III, IV in Fig. F.2 of Appendix F) and there are terms of $O(m_\mu^2)$ which have been dropped in (2.9) (see equation F.15). The $O(m_\mu^2)$ term is (for $m_\mu \ll m_{\tilde{\mu}_{1,2}}, m_\lambda$)

$$\Delta a_\mu^{(m_\mu^2)} = -\frac{\alpha}{12\pi} \left\{ \frac{m_\mu^2}{m_{\tilde{\mu}_1}^2} [1 + F(\mathcal{R}_1)] + \frac{m_\mu^2}{m_{\tilde{\mu}_2}^2} [1 + F(\mathcal{R}_2)] \right\}$$

$$F(z) = z(6z^2 - 9z + 2) + z^2(1 - z)^2 \ln \left(\frac{z-1}{z} \right) \quad (2.16)$$

$$\mathcal{R}_{1,2} = \frac{m_\lambda^2}{m_\lambda^2 - m_{\tilde{\mu}_{1,2}}^2}$$

which can become dominant if $|\theta|$ or $|\pi - \theta| \lesssim O(m_\mu/m_{SUSY})$. In the limit that $m_{\tilde{\mu}_{1,2}} \gg m_\lambda$ (2.16) becomes

$$\Delta a_\mu = -\frac{\alpha}{12\pi} \left\{ \frac{m_\mu^2}{m_{\tilde{\mu}_1}^2} + \frac{m_\mu^2}{m_{\tilde{\mu}_2}^2} \right\} \quad (2.17)$$

which is essentially (2.5) and would give similar limits for slepton masses. For the remainder of this section we will restrict our attention to the terms which are linear in m_μ .

Appendix G contains various plots related to $\Delta\alpha_\mu$. In Fig. G.1 $\frac{1}{C} \Delta\alpha_\mu^1$ is plotted against X_i (as define in (2.12)). We see that $\frac{1}{C} \Delta\alpha_\mu^1$ decreases monotonically from $\frac{1}{2}$ at $X_i = 0$ to about 0.1 at $X_i = 2$ and thence asymptotically to zero. Since $\Delta\alpha_\mu = \Delta\alpha_\mu^1 - \Delta\alpha_\mu^2$, we have plotted $\frac{1}{C} \Delta\alpha_\mu$ versus X_1 for $m_{\tilde{\mu}_2} = 1.01 m_{\tilde{\mu}_1}$ and $m_{\tilde{\mu}_2} = 2m_{\tilde{\mu}_1}$ in figures G.2 and G.3 respectively. We note the general feature of a maximum for $|\Delta\alpha_\mu|$, call it $|\Delta\alpha_\mu|_{\max}$, which occurs at some $X_1 = X_{1 \max}$. As $y \equiv m_{\tilde{\mu}_2}^2/m_{\tilde{\mu}_1}^2$ increases $X_{1 \max}$ and $|\Delta\alpha_\mu|_{\max}$ change. Thus $|\Delta\alpha_\mu|_{\max} = |\Delta\alpha_\mu|_{\max}(y)$ and $X_{1 \max} = X_{1 \max}(y)$. In particular for $y = 0$ ($m_{\tilde{\mu}_2} = \infty$) we find that $\Delta\alpha_\mu^1|_{\max} = \frac{C}{2}$ and $X_{1 \max} = \infty$. Since $\Delta\alpha_\mu = \Delta\alpha_\mu^1 - \Delta\alpha_\mu^2$ we see that $X_{1 \max}(y) = (1/y) X_{1 \max}(1/y)$ and $\Delta\alpha_\mu|_{\max}(y) = -\Delta\alpha_\mu|_{\max}(1/y)$. Thus $X_{1 \max} = 0$ for $y = \infty$.

The value of $|\Delta\alpha_\mu|_{\max}$ is important in that it will place the most stringent limits on the supersymmetric parameters. The maxima occur at solutions to the dimensionless transcendental equation (here $x_i = X_i$)

$$\ln x_1 = \frac{y[(x_1 y + 5)(x_1 y - 1) - 2(2x_1 y + 1) \ln y](x_1 - 1)^4 - (x_1 + 5)(x_1 - 1)(x_1 y - 1)^4}{2[y(2x_1 y + 1)(x_1 - 1)^4 - (2x_1 + 1)(x_1 y - 1)^4]} \quad (2.18)$$

(where $y \neq 1$ when $x_1 = 1$)

$$x_1 = \frac{m_{\tilde{\mu}_1}^2}{m_{\tilde{\mu}_2}^2} \quad x_2 = \frac{m_{\tilde{\mu}_2}^2}{m_{\tilde{\mu}_1}^2} \quad y = \frac{x_2}{x_1} = \frac{m_{\tilde{\mu}_2}^2}{m_{\tilde{\mu}_1}^2}. \quad (2.19)$$

In Fig. G.4 we have plotted $X_{1 \max}$ as a function of y whereas $\frac{1}{C} \Delta\alpha_\mu|_{\max}$ versus y appears in Fig. G.5-6. It must always be remembered that $C \sim 1/m_\lambda$. Unsurprisingly the maximal value achievable by $|\Delta\alpha_\mu|$ is $\frac{1}{2} C$ at $y = 0$ and $y = \infty$ (at which points $X_{1 \max} = \infty, 0$). Thus $\frac{1}{C} |\Delta\alpha_\mu|$ achieves its greatest value when $m_{\tilde{\mu}_1} \rightarrow 0$ (or $m_{\tilde{\mu}_2} \rightarrow 0$ while $m_{\tilde{\mu}_2}$ (or $m_{\tilde{\mu}_1}$) $\rightarrow \infty$). Since $C \sim m_\mu/m_\lambda$ we expect that $|\Delta\alpha_\mu|$ is maximized when the supersymmetric masses are as small as possible and the mass-splitting in the sleptonic sector is maximal ($m_{\tilde{\mu}_2} \gg m_{\tilde{\mu}_1}$). Indeed this is

a generic feature which we will find common to all processes examined in this paper. (Since we have assumed that $m_\lambda, m_{\tilde{\mu}_2} \gg m_\mu$ in the above expressions, it would be improper to take both $m_{\tilde{\mu}_2}$ and m_λ to zero). This is, however, partially an artifact from dividing (2.9) through by m_λ^2 to get (2.13). This new contribution depends fundamentally upon the photino flipping helicity, contributing a factor of m_λ which replaces one factor of m_μ , and so vanishes as $m_\lambda \rightarrow 0$. As long as the photino possesses any finite non-zero value ($m_\lambda \neq 0$) we maximize $|\Delta\alpha_\mu|$ by taking either $m_{\tilde{\mu}_1}^-$ or $m_{\tilde{\mu}_2}^-$ to be as light as is consistently possible with experiment and the other to be much heavier. Since experimentally $m_{\tilde{\mu}_1}^- \gtrsim 25$ GeV we might consider the case $m_\lambda \ll m_{\tilde{\mu}_1}^- \ll m_{\tilde{\mu}_2}^-$. In this limit (2.9) becomes

$$\Delta\alpha_\mu = -\frac{\alpha}{4\pi} \frac{m_\lambda m_\mu}{m_{\tilde{\mu}_1}^2} \sin \phi \cos \phi. \quad (2.20)$$

Although such "large mass-splitting" cases are not ruled out one generally finds, if explicit supersymmetry-breaking terms have their ultimate origin via spontaneous supersymmetry-breaking at some larger mass scale, that states tend to have meager mass-splitting (in this case $\delta m_{\tilde{\mu}} = m_{\tilde{\mu}_2} - m_{\tilde{\mu}_1} \ll m_{\tilde{\mu}_{1,2}}$). Indeed one is led in some models⁽¹¹⁾ to the conclusion that, should the spontaneous breaking arise in the gauge ("D") terms of the superpotential, then the mass-splitting is expected to be very small. In such models $\tilde{\mu}_L$ and $\tilde{\mu}_R$ possess equal masses and are unmixed at the tree level but develop a small mixing term at one loop via diagrams such as that in Fig.2.2.

The result is a smuon mass matrix of the form

$$(\tilde{\mu}_L \ \tilde{\mu}_R)^* \begin{pmatrix} m^2 & \delta m^2 \\ \delta m^2 & m^2 \end{pmatrix} \begin{pmatrix} \tilde{\mu}_L \\ \tilde{\mu}_R \end{pmatrix}$$

where

$$\delta m^2 \sim O\left(\frac{\alpha}{\pi}\right) m_\mu v$$

$$m^2 \sim O(v^2)$$

$$v \sim O(100 \text{ GeV})$$

and $|m_{\tilde{\mu}_1}^2 - m_{\tilde{\mu}_2}^2| = 2\delta m^2 \ll m_{\tilde{\mu}_{1,2}}^2$. The mixing angle is near maximal.*

Should the supersymmetry-breaking terms arise in the Yukawa ("F") sector certain scenarios¹⁰⁾ suggest that the $\tilde{\mu}_L - \tilde{\mu}_R$ mixing angle, ϕ , would be small although the mass splitting could be larger.

When the mass-splitting is small we find (for $m_\lambda < m_{\tilde{\mu}}^2$)

$$\Delta a_\mu = -\frac{\delta m^2}{m_{\tilde{\mu}}^2} \frac{\alpha}{4\pi} \sin \phi \cos \phi \frac{m_\lambda m_\mu m_{\tilde{\mu}}^2}{(m_{\tilde{\mu}}^2 - m_\lambda^2)^4} \times \left\{ 1 + 4 \frac{m_\lambda^2}{m_{\tilde{\mu}}^2} \left[1 - \ln \frac{m_\lambda^2}{m_{\tilde{\mu}}^2} \right] - \frac{m_\lambda^2}{m_{\tilde{\mu}}^2} \left[5 + 2 \ln \frac{m_\lambda^2}{m_{\tilde{\mu}}^2} \right] \right\} \quad (2.21)$$

where

$$\delta m^2 = m_{\tilde{\mu}_1}^2 - m_{\tilde{\mu}_2}^2 \quad m_{\tilde{\mu}} = m_{\tilde{\mu}_{1,2}}$$

or

$$\Delta a_\mu = -\frac{\delta m^2}{m_{\tilde{\mu}}^2} \frac{\alpha}{4\pi} \sin \phi \cos \phi \frac{m_\mu}{m_\lambda} p(x)$$

where

$$x = m_{\tilde{\mu}}^2/m_\lambda^2$$

$$p(x) = \frac{x}{(x-1)^4} \{ (x+5)(x-1) - 2(2x+1)\ln x \}$$

$$p(x \rightarrow 0) \rightarrow -4x \ln x \quad (2.22)$$

$$p(x \rightarrow 1) \rightarrow \frac{1}{6}$$

$$p(x \rightarrow \infty) \rightarrow \frac{1}{x}$$

(Note that this formulation is equivalent to that of reference 8 provided that one allows for the difference in the definition of α used there.)

* Since this is actually a two-loop process there are a number of other diagrammatic configurations of comparable magnitude which were not included in the original treatment. This must be done if a more precise model is required.

2.4 COMPARISON WITH EXPERIMENT

Sooner or later it behooves us to make contact with reality. Here we use the experimental limits on measurements of a_e and a_μ to place restrictions on the contributions considered in the previous subsection. It should be noted that g_e has been so well measured that a significant fraction of the uncertainty which is engendered when the empirical and theoretical values are compared may be ascribed to the error inherent in high-order calculations *within the standard model*, particularly from the hadronic contributions.

Current limits on the deviation of $\Delta a_{\mu,e}$ from predicted standard model values are:^{10,11)}

$$|\Delta a_\mu| \lesssim 2 \times 10^{-8} \quad (2.23)$$

$$|\Delta a_e| \lesssim 3 \times 10^{-10} \quad (2.24)$$

First let us consider the implications in the case of unmixed sleptons. When $m_{\tilde{\mu}_{1,2}} \gg m_\lambda \gg m_\mu$ we obtained the result of (2.5) which, using (2.23), translates to

$$\frac{1}{m_{\tilde{\mu}_L}^2} + \frac{1}{m_{\tilde{\mu}_R}^2} \lesssim \frac{1}{(10 \text{ GeV})^2}$$

This would exclude a circle of the radius $(10 \text{ GeV})^{-1}$ centered at the origin in the $m_{\tilde{\mu}_L}^{-1} - m_{\tilde{\mu}_R}^{-1}$ plane or, equivalently, the region indicated in Fig. 2.3.

This implies an absolute limit on the smuon masses of

$$m_{\tilde{\mu}} \gtrsim 10 \text{ GeV} \quad (2.25)$$

From (2.5) and (2.24) in the $m_{\tilde{\mu}} \gg m_\lambda \gg m_e$ limit we would find a corresponding

constraint on the selectron mass from Δa_e of only

$$m_{\tilde{e}} \gtrsim 0.4 \text{ GeV} . \quad (2.26)$$

In passing, we note that should the photino be substantially more massive than the stauon ($m_{\lambda} \gg m_{\tilde{\nu}} \gg m_{\mu}$) then (2.25) would become $m_{\lambda} \gtrsim 5 \text{ GeV}$.

Turning now to the $\tilde{\mu}_L - \tilde{\mu}_R$ mixed case we will first consider the implications of substantial mass-splitting of the eigenstates $\tilde{\mu}_1$ and $\tilde{\mu}_2$ and then will return to the slightly split scenario. From (2.20) and (2.25) we see that when

$$m_{\mu} < m_{\lambda} < m_{\tilde{\mu}_1} < m_{\tilde{\mu}_2}$$

that

$$|\Delta a_{\mu}| = \frac{\alpha}{4\pi} \frac{m_{\lambda} m_{\mu}}{m_{\tilde{\mu}_1}^2} \sin \phi \cos \phi \lesssim 2 \times 10^{-8}$$

or, assuming maximal mixing ($\phi = \pi/4$),

$$m_{\tilde{\mu}_1} \gtrsim 39 \text{ GeV} \sqrt{m_{\lambda}(\text{in GeV})} . \quad (2.27)$$

We obtain a similar limit on $m_{\tilde{e}}$ under such conditions. In light of the assumption $m_{\tilde{\mu}_1} \gg m_{\lambda}$ this provides a constraint on $m_{\tilde{\mu}_{1,2}}$ for light photinos ($m_{\lambda} < 1 \text{ TeV}$). Note that if we let $m_{\lambda} = m_{\mu}$ in (2.27) then the inequality reduces roughly to that of (2.25) in accordance with our observation in the previous section that one power of photino mass has replaced a factor of lepton mass in Δa_{μ} . This has its most profound effect, of course, when we consider selectrons, as in (2.26), and when the photino mass is comparable or greater than the mass of the slepton (if $m_{\tilde{\nu}} \approx m_{\lambda}$ then both must exceed 0.5 TeV).

If the mass-splitting is small we might consider the case corresponding to (2.27), i.e.

$$\frac{\delta m^2}{m_{\tilde{\mu}}^2} < 1 \quad m_{\mu} < m_{\lambda} < m_{\tilde{\mu}_{1,2}}$$

Then from (2.22),

$$\Delta a_{\mu} \sim \frac{\delta m^2}{m_{\tilde{\mu}}^2} \frac{\alpha}{4\pi} \sin \phi \cos \phi \frac{m_{\mu} m_{\lambda}}{m_{\tilde{\mu}}^2}$$

and from (2.23) and (2.24) we obtain

$$\frac{\delta m^2}{m_{\tilde{\mu}}^2} \frac{m_{\mu} m_{\lambda}}{m_{\tilde{\mu}}^2} \lesssim 7 \times 10^{-6} \quad (2.28)$$

or $\lesssim 10^{-6}$ in the $\tilde{e}$ case.

This is not a terribly useful limit. If, for instance, we were to take $m_{\lambda} \sim 10 \text{ GeV}$, $m_{\tilde{\nu}} \sim 100 \text{ GeV}$ we would find

$$\frac{\delta m^2}{m_{\tilde{\mu}}^2} \lesssim O(1) .$$

We note that the above treatment differs only in detail from a full supersymmetric electroweak treatment and the results are similar. The principal difference in the latter is that there are a plethora of contributions from additional gauginos and higgsinos (which may mix amongst themselves). While we could have $m_{\tilde{G}} < m_{\tilde{\mu}}$ for a gaugino $\tilde{G}$, we might equally well have $m_{\tilde{G}_2} > m_{\tilde{\mu}}$ for a second gaugino. The complication in placing bounds is evident when we point out that if $m_{\tilde{\nu}} \approx m_{\lambda}$ then (2.28) will become (using (2.15))

$$\frac{\delta m^2}{m_{\tilde{\mu}}^2} \lesssim 4 \times 10^{-3} m_{\lambda}(\text{in GeV}) \quad (2.29)$$

which is, potentially, significantly more restrictive. We include such complications when we consider mixing among sleptons of different generations.

FIGURE CAPTIONS

1. (a) Tree-level vertex for a massless electron.
(b) One Loop QED correction.
2. Mass-Mixing in the Model of Ref. 8.
3. Excluded region of the $m_{\tilde{\mu}_L} - m_{\tilde{\mu}_R}$ plane if $m_{\tilde{\mu}_{L,R}} \gg m_\lambda \gg m_\mu$ for case of no left-right mixing.

Fig. 2.1

Fig. 2.2

Fig. 2.3

3. $Z^0 \rightarrow \tau\mu$

3.1 INTRODUCTION

In this chapter we examine the possibility of real Z^0 decay in a way which violates lepton family number. We permit the admixture of the scalar partners of the leptons (sleptons) from different generations. This chapter duplicates, to a large extent, the work contained in a previous paper.^[14] We will work in the more familiar four-component Dirac notation.^[15] Our notation is based upon that of Refs. 12 and 13. Useful sample calculations using this notation may be found in several places.^[13,14]

Our principal consideration will be the process $e^+e^- \rightarrow Z^0 \rightarrow \tau^\pm\mu^\mp$. Should such an interaction occur with a moderate cross-section it should be observable at the emerging generation of colliders. The choice of the final states is somewhat arbitrary. It is partially based on the possibility that the sleptonic masses might progress in the same order as their leptonic counterparts and that more massive states might have a greater tendency to mix. Neither of these assumptions is, of course, necessary, however it is highly desirable to consider that most of the mixing occurs between two generations in order to reduce the number of angular parameters present. There is also the observation that there are additional box diagrams present if one of the final states is an electron. When dealing with on-shell Z^0 's such contributions are negligible and the results would be equally applicable to mixing in the $e - \mu$ or $e - \tau$ sectors. It must be noted that such computations may be, as will be seen in Chapter 4, readily extended off-shell.

As in the previous chapter the bulwark of detail will be presented in related appendices (Appendices I-N). Appendix I provides a brief overview of supersymmetry, motivates the construction of supersymmetric Lagrangia in general, establishes the notation and presents the "standard supersymmetric Lagrangian". Appendix J considers the diagonalization of the various mass matrices in different bases. In Appendix K the Feynman rules are derived and detailed examples of

their use given. The computation of the matrix element is done in Appendix L, notation established and various integral functions defined. These are analyzed in detail, both analytically and numerically, in Appendix M. Finally in Appendix N the various background calculations are presented.

3.2 THE MODEL

We consider a minimal supersymmetric extension of the standard model and restrict our attention to the leptonic sector. We admit the inclusion of explicit soft supersymmetry-breaking terms, that is those which do not introduce new quadratic divergences. Thus this "supersymmetric standard model" will develop arbitrary scalar lepton masses. Presumably all such terms arise at some higher energy scale in a more fundamental way (such as supersymmetry-breaking in a manner which is not family-independent). We shall not concern ourselves here with the ultimate origin of these terms but consider the effective low-energy Lagrangian as given. In this way we may proceed in a reasonably model-independent fashion (subject to the not insubstantial restrictions of using minimal supersymmetry).

Given scalar leptons of arbitrary mass it is only lepton family number, an empirically observed global (presumably) symmetry, which would prevent them from mixing. There is some feeling that exact global symmetries are very unnatural^[16] (perhaps representing the approximate remnants of some more fundamental local symmetry at larger energy scales). Thus we allow the sleptons to have arbitrary mass and mix with arbitrary angle. Family-number violation might also come about through any number of other effects in conjunction with this or independently.^[14] We shall not consider this point further other than to note that specific experimental observations will generally be able to distinguish one model from another (given sufficient statistics). The same sort of analysis may be applied to the hadronic decay of the Z^0 leading to possible enhancements of flavour symmetry violation in that sector.^[10,11]

We have seen that the left and right sleptons may readily mix in each of the $\tilde{e}$, $\tilde{\mu}$ and $\tilde{\tau}$ sectors. We therefore anticipate mixing among six charged sleptons. In general, mixing among N states may be described by $\frac{1}{2}N(N-1)$ real angles and $\frac{1}{2}(N-1)(N-2)$ complex phases. The latter will typically result in CP violation. In the above example (three generations) we would need 15 real angles and 10 phases. In addition there are three neutral sneutrino states (six if we permit right-handed sneutrinos in the spectrum) resulting in three more angles and one additional phase. In order to reduce the burgeoning number of parameters somewhat we will consider the case where only the two 'heaviest' generations mix significantly with little left-right mixing. Thus the only mixings are between:

$$\begin{aligned}\tilde{\mu}_L \text{ and } \tilde{\tau}_L \text{ with angle } \theta_L \\ \tilde{\mu}_R \text{ and } \tilde{\tau}_R \text{ with angle } \theta_R \\ \tilde{\nu}_\mu \text{ and } \tilde{\nu}_\tau \text{ with angle } \theta_\nu\end{aligned}\quad (3.1)$$

So that

$$\begin{aligned}\tilde{e}_{L1} &= \tilde{\mu}_L \cos \theta_L + \tilde{\tau}_L \sin \theta_L \\ \tilde{e}_{L2} &= -\tilde{\mu}_L \sin \theta_L + \tilde{\tau}_L \cos \theta_L \\ \tilde{e}_{R1} &= \tilde{\mu}_R \cos \theta_R + \tilde{\tau}_R \sin \theta_R \\ \tilde{e}_{R2} &= -\tilde{\mu}_R \sin \theta_R + \tilde{\tau}_R \cos \theta_R \\ \tilde{\nu}_1 &= \tilde{\nu}_\mu \cos \theta_\nu + \tilde{\nu}_\tau \sin \theta_\nu \\ \tilde{\nu}_2 &= -\tilde{\nu}_\mu \sin \theta_\nu + \tilde{\nu}_\tau \cos \theta_\nu\end{aligned}$$

with $\tilde{e}_{L,R1}$, $\tilde{e}_{L,R2}$ and $\tilde{\nu}_{1,2}$ the physical mass eigenstates.

The particle spectrum of the supersymmetric "standard" model is illustrated in Table I-1 in Appendix I. The superpotential of the chiral superfields must be homogeneous (no terms involving both chiral and antichiral fields as one finds in the field strength)^(11,12) with the result that a minimum of two Higgs doublets are required to give the u and d quarks (and thus charged leptons) masses.⁽¹³⁾ In this minimal model we will ignore the possible inclusion of a Higgs singlet, $\tilde{H}$.

These points are discussed in Appendix I. The fields which will be needed in this chapter are presented in Table 3-1.

A major complication arises in the gaugino sector of the theory. Let us suppose that supersymmetry breaks at an energy scale above that at which $SU(2) \times U(1)$ does. At large energies we will have exact supersymmetric $SU(2) \times U(1)$. Thus we have massless $\tilde{W}^0, \tilde{W}^\pm$ $SU(2)$ gauge bosons and a massless $\tilde{B}^0$ $U(1)$ gauge boson with corresponding supersymmetric partners, all massless fermions, $\tilde{\tilde{W}}^0, \tilde{\tilde{W}}^\pm$ and $\tilde{\tilde{B}}^0$. Similarly there are two Higgs doublets H_1 and H_2 with their superpartners $\tilde{\tilde{H}}_1$ and $\tilde{\tilde{H}}_2$. Note that the above are Weyl spinors ($\tilde{\tilde{\cdot}}$ is notation differs slightly from that of Ref. 12). It is frequently convenient to write these as four-component objects. For the gauginos we shall use the same symbol.

$$\tilde{W}^+ = \begin{pmatrix} -i\tilde{W}^+ \\ i\tilde{W}^- \end{pmatrix} \quad \tilde{W}^0 = \begin{pmatrix} -i\tilde{W}^0 \\ i\tilde{W}^0 \end{pmatrix} \quad \tilde{B}^0 = \begin{pmatrix} -i\tilde{B}^0 \\ i\tilde{B}^0 \end{pmatrix} \quad (3.2)$$

whereas we will distinguish the Higgs four component object as $\tilde{\tilde{H}}$,

$$\tilde{\tilde{H}}^+ = \begin{pmatrix} \tilde{\tilde{H}}_1^+ \\ \tilde{\tilde{H}}_2^+ \end{pmatrix} \quad \tilde{\tilde{H}}_1^0 = \begin{pmatrix} \tilde{\tilde{H}}_1^0 \\ \tilde{\tilde{H}}_2^0 \end{pmatrix} \quad \tilde{\tilde{H}}_2^0 = \begin{pmatrix} \tilde{\tilde{H}}_1^0 \\ \tilde{\tilde{H}}_2^0 \end{pmatrix} \quad (3.3)$$

The neutral states are Majorana. Note that $\tilde{\tilde{H}}^+$ is built from both higgsinos. When supersymmetry breaks, effective low energy terms of the form

$$\begin{aligned}\mathcal{L}_B = & -\frac{1}{2} M \tilde{W}^0 \tilde{W}^0 - \frac{1}{2} M' \tilde{B}^0 \tilde{B}^0 - \mu \tilde{\tilde{H}}_1^0 \tilde{\tilde{H}}_2^0 - M \tilde{W}^+ \tilde{W}^- + \mu \tilde{\tilde{H}}_1^+ \tilde{\tilde{H}}_2^+ \\ & + m_{ij} \tilde{e}_i \tilde{e}_j + \tilde{m}_{ij} \tilde{\nu}_i \tilde{\nu}_j + h.c.\end{aligned}\quad (3.4)$$

develop. These are the most general bilinear soft supersymmetry-breaking terms which may develop.⁽¹⁴⁾ Note that explicit diagonal Higgsino mass terms are *not* soft.

At the weak scale H_1^0 and H_2^0 develop vacuum expectation values $v_1 = \langle H_1^0 \rangle$ and $v_2 = \langle H_2^0 \rangle$. The standard electroweak VEV is replaced by

$$v = (v_1^2 + v_2^2)^{1/2}.$$

The terms $f^i H_1^0(\ell_L)_i(\ell_R)_i^c$ ($i = e, \mu, \tau$), $W^0 H_1^0 H_2^0$ and $W^\pm H_1^\mp H_2^0$ of the standard theory (with two Higgses) have their supersymmetric counterparts $f^i H_1^0(\tilde{\ell}_L)_i(\tilde{\ell}_R)_i^c$, $\tilde{W}^0 \tilde{H}_1^0 H_2^0$ and $\tilde{W}^\pm \tilde{H}_1^\mp H_2^0$ (the general rule is to replace every pair of fields by their supersymmetric partners if R parity is conserved as is here assumed). Now when the neutral Higgs develop VEVs we develop more quadratic terms in $\mathcal{L}_{\text{effective}}$. The first terms gives mass to the slepton (this would be equal to the lepton mass if it weren't for the additional term in (3.4)). The others yield

$$\begin{aligned} \mathcal{L}_{\text{Mix}} = & \frac{i}{2} (g' \tilde{B}^0 - g \tilde{W}^0) \left(v_1 \tilde{\psi}_{H_1}^0 - v_2 \tilde{\psi}_{H_2}^0 \right) \\ & - \frac{ig}{\sqrt{2}} [v_1 \tilde{W}^+ \tilde{\psi}_{H_1}^- + v_2 \tilde{W}^- \tilde{\psi}_{H_2}^+] + \text{h.c.} \end{aligned} \quad (3.5)$$

It is convenient to define

$$\tan \theta_v \equiv \frac{v_1}{v_2} \quad (3.6)$$

(this is called $\cot \beta$ in Ref. 13).

The final term in (3.5) will induce the charged gaugino and higgsino states of (3.4) to mix. The first term in (3.5) will cause $\tilde{B}$ and $\tilde{W}$ to mix with both $\tilde{H}_1^0$ and $\tilde{H}_2^0$. Since $\tilde{H}_1^0$ and $\tilde{H}_2^0$ are mixed by (3.5) (by virtue of possessing opposite hypercharges) all four neutral states mix. We refer to the two charged mass eigenstates as "charginos" and represent them by $\tilde{\chi}_i^\pm$. The four neutral mass eigenstates are "neutralinos" represented by $\tilde{\chi}_i^0$. If there are Higgs singlets then the number of neutralinos will be increased accordingly. It is also possible to assign a VEV to the sneutrinos: $\langle \tilde{\nu}_i \rangle \neq 0$. This interesting possibility leads to further mixing due to terms in $\mathcal{L}$ such as $\tilde{H}^0 \tilde{\nu} \nu$ and have been examined elsewhere.^[10] Such theories explicitly violate R -parity, lepton and family number and are therefore of a somewhat more radical nature than will be considered here.

Since higgsinos and gauginos couple quite differently, the rate of the interaction $e^+e^- \rightarrow \tau^+\tau^-$ and similar processes will depend upon the details of the mass spectrum of the physical eigenstates. These effects may be more profound away from the Z^0 resonance since there will be contributions from charginos and neutralinos in the t -channel.^[11]

3.3 MASS EIGENSTATES

The Lagrangian may be concisely written in terms of chargino and neutralino mass matrices. The unitary matrices which diagonalize the Lagrangian are called N (4×4) for the neutralino sector and U and V (2×2) for the charginos. The details are given in Appendix J. These matrices will perforce appear in the Feynman rules as derived in appendix K. The most involved combinations will occur in the non-Abelian vertices which contain more than one gaugino or higgsino. Following Haber and Kane^[12] we represent the chargino combination by matrices O' and neutralinos by O'' . Since the parameters in the gaugino mass matrices may vary over a wide range of parameters one might be wary of finding that one or more mass eigenvalues have become negative. This does indeed appear to occur but this does not imply the existence of a tachyon. It asserts an inappropriate set of basis states were selected for parameters in this regime. These may be redefined or, equivalently, an altered diagonalization procedure used (which accomplishes the same end). Indeed in the simple 2×2 chargino system the latter approach has been adopted leading to slight modifications in the Feynman rules as this threshold is broached. It is also possible, and often desirable, to retain the negative states for computational purposes.

The chargino masses are given by the explicit expressions

$$\begin{aligned}
m_{\tilde{\chi}_1^0} &= \frac{1}{2} (M_+ + M_-) \\
m_{\tilde{\chi}_2^0} &= \frac{1}{2} (M_+ - M_-) \text{sign}[D] \\
M_{\pm} &= \sqrt{(M + \mu)^2 + 2m_W^2(1 \mp \sin 2\theta_e)} \\
D &= M\mu - m_W^2 \sin 2\theta_e
\end{aligned} \tag{3.7}$$

where M, μ, θ_e are the breaking parameters described in (3.4) and (3.6).

The neutralino masses are given by Ref. 12 as the eigenvalues of the matrix

$$Y = \begin{pmatrix} M' & 0 & -m_Z \sin \theta_e \sin \theta_w & m_Z \cos \theta_e \sin \theta_w \\ 0 & M & m_Z \sin \theta_e \cos \theta_w & -m_Z \cos \theta_e \cos \theta_w \\ -m_Z \sin \theta_e \sin \theta_w & m_Z \sin \theta_e \cos \theta_w & 0 & -\mu \\ m_Z \cos \theta_e \sin \theta_w & -m_Z \cos \theta_e \cos \theta_w & -\mu & 0 \end{pmatrix} \tag{3.8}$$

The general solutions are the roots of a quartic equation. This can be solved exactly but proves to be unilluminating.^[11] Furthermore, if a Higgs singlet is added Y becomes a 5×5 matrix and the corresponding quintic root equation will have no closed form solution in general.^[12] Note that in specific cases, such as those discussed in appendix J, simplified forms are frequently possible.

Mass matrices for the sleptons may be similarly constructed. There are on-diagonal contributions from the lepton mass and the explicit breaking terms. The off-diagonal entries derive solely from the soft terms. Since lepton masses are expected to be much smaller than the supersymmetry-breaking terms they may effectively be ignored except in special circumstances (such as lifting a degeneracy or preventing the matrix from becoming singular). Since all of the entries are then "arbitrary", it is sufficient to merely consider the physical eigenstates and characterize them by given mixing angles and masses without reference to the unmixed precursor states.

3.4 MATRIX ELEMENTS

The diagrams contributing to $Z \rightarrow \mu\bar{\mu}$ are found in Fig. 1. Details may be found in Appendix N using the Feynman rules derived in the previous appendix. The explicit expression for the leading order terms in the matrix element, M^μ , is given by

$$M^W = M_L \xi^W_L + M_R \xi^W_R \quad (3.9)$$

$$\xi^W_{L,R} = \bar{u}^{\nu}(p_1) \gamma_\mu \gamma^\nu \gamma_\pm v^{\nu}(p_2) \quad \gamma_\pm = \frac{1}{2} (1 \pm \gamma_5) \quad (3.9a)$$

$$\begin{aligned} M_L = & \frac{K_L}{\sin^2 \theta_w \sin 2\theta_w} \sum_{i=1}^2 \left\{ |V_{ii}|^2 [\Delta G(T_i^+, R_i^+) + \cos 2\theta_w \Delta G(T_i^+, 0)] \right. \\ & + 2 \sum_{j=1}^2 V_{ii}^* V_{ji} \left[\sqrt{S_{ij}^+} O_{ij}^{R+} \Delta I_{[1]}(T_i^+, S_{ij}^+, R_i^+) \right. \\ & \quad \left. - R_i^+ O_{ij}^{L+} \Delta I_{[\bar{2}, -2]}(T_i^+, S_{ij}^+, R_i^+) \right. \\ & \quad \left. + O_{ij}^{L+} \Delta \hat{G}(T_i^+, S_{ij}^+, R_i^+) \right] \Big\} \\ & - \frac{K_L}{2 \sin^2 \theta_w \sin 2\theta_w} \sum_{i=1}^4 \left\{ (\tan \theta_w N_{i1} + N_{i2})^2 \cos 2\theta_w \Delta \hat{G}(T_L^0, R_i^0) \right. \\ & \quad \left. - 2 \sum_{j=1}^4 (\tan \theta_w N_{i1}^* + N_{i2}^*) (\tan \theta_w N_{j1} + N_{j2}) \right. \\ & \quad \left[\sqrt{S_{ij}^0} O_{ij}^{R''} \Delta I_{[1]}(T_L^0, S_{ij}^0, R_i^0) \right. \\ & \quad \left. - R_i^0 O_{ij}^{L''} \Delta I_{[\bar{2}, -2]}(T_L^0, S_{ij}^0, R_i^0) \right. \\ & \quad \left. + O_{ij}^{L''} \Delta \hat{G}(T_L^0, S_{ij}^0, R_i^0) \right] \Big\} \quad (3.9b) \end{aligned}$$

$$\begin{aligned} M_R = & \frac{4K_R}{\sin^2 \theta_w \sin 2\theta_w} \sum_{i=1}^4 \left\{ (\tan \theta_w N_{i1})^2 \sin^2 \theta_w \Delta \hat{G}(T_R^0, R_i^0) \right. \\ & + \sum_{j=1}^4 (\tan \theta_w N_{i1}) (\tan \theta_w N_{j1}) \\ & \left[\sqrt{S_{ij}^0} O_{ij}^{L''} \Delta I_{[1]}(T_R^0, S_{ij}^0, R_i^0) \right. \\ & \quad \left. - R_i^0 O_{ij}^{R''} \Delta I_{[\bar{2}, -2]}(T_R^0, S_{ij}^0, R_i^0) \right. \\ & \quad \left. + O_{ij}^{R''} \Delta \hat{G}(T_R^0, S_{ij}^0, R_i^0) \right] \Big\} \quad (3.9c) \end{aligned}$$

$$K_\nu = \frac{i}{16\pi^2} e^3 \sin \theta_\nu \cos \theta_\nu$$

$$K_L = \frac{i}{16\pi^2} e^3 \sin \theta_L \cos \theta_L \quad (3.9d)$$

$$K_R = \frac{i}{16\pi^2} e^3 \sin \theta_R \cos \theta_R$$

$R, S,$ and T are ratios of masses:

$$\begin{aligned} T_i^+ &= \frac{m_{\tilde{L}_i^+}^2}{m_{\tilde{X}_i^+}^2} & T_{L,R}^0 &= \frac{m_{\tilde{L}_{L,R}^0}^2}{m_{\tilde{X}_i^0}^2} \\ S_{ij}^+ &= \frac{m_{\tilde{X}_i^+}^2}{m_{\tilde{X}_j^+}^2} & S_{ij}^0 &= \frac{m_{\tilde{X}_i^0}^2}{m_{\tilde{X}_j^0}^2} \\ R_i^+ &= \frac{q^2}{4m_{\tilde{X}_i^+}^2} & R_i^0 &= \frac{q^2}{4m_{\tilde{X}_i^0}^2} \end{aligned} \quad (3.10)$$

where $m_{\tilde{L}_{1,2}}$ and $m_{\tilde{L}_{3,4}}$ are the masses of the charged slepton of the first generation mixture (see (3.1)) and $m_{\tilde{\nu}_1}$ is the mass of the first sneutrino ($\tilde{\nu}_1 = \tilde{\nu}_\mu \cos \theta_\nu + \tilde{\nu}_\tau \sin \theta_\nu$). The neutralino masses are $m_{\tilde{\chi}_i^0}$ ($i = 1, 2, 3, 4$) and chargino masses are $m_{\tilde{X}_i^\pm}$ ($i = 1, 2$). The energy of the Z^0 is $q^2 = m_Z^2$ since we are working on-shell. The integral functions ($\Delta I_{[N]}, \Delta G, \Delta \hat{G}, \Delta \tilde{G}$) are defined in Appendix N and analyzed, analytically and numerically, in the subsequent appendix. The matrix element M^W cannot be examined in all of its complexity (there are, after all, 120 terms in Eq. (3.9)) but there are a number of special cases which prove instructive. These principally involve taking the breaking parameters to extreme values (zero or infinity) and a number of examples have been examined. We find, in accordance with the decoupling theorem,^[14] that as the physical mass of a matrix becomes large its contributions to the cross-section become progressively

less important, finally vanishing. The matrix element remains finite for all values of the parameters involved.⁶

3.5 PRODUCTION THRESHOLD

If any of the supersymmetric particles is sufficiently light ($m < \frac{1}{2} m_Z$) then real production occurs. If such states are unstable (generally only the least massive supersymmetric state will be stable, and only if R is conserved) then final state processes with four or more particle will dominate. If the lightest supersymmetric particle is a neutral then all such decays are characterized by "missing mass" in the final state. Such signals are notoriously amenable to alternate interpretations yet remain the hallmark of experimental supersymmetric searches. Below threshold there are still contributions to closed loop processes, such as the one under consideration, however the matrix elements become complex.

Let us assume that a nearly massless sneutrino were the lightest supersymmetric particle and, furthermore, was stable. The reaction $e^+e^- \rightarrow Z^0 \rightarrow \tilde{\nu}\tilde{\nu}$ would appear as $e^+e^- \rightarrow$ nothing. The real particle production analogues of the processes in Fig. 3.1 would be

$$e^+e^- \rightarrow Z^0 \rightarrow \tilde{\chi}^- \tilde{\chi}^+ \begin{cases} \downarrow \\ \tilde{\nu}\nu_r^+ \rightarrow \tilde{\nu}\nu_\mu\nu_r\mu^+ \\ \downarrow \\ \tilde{\nu}\mu^- \end{cases}$$

and

$$e^+e^- \rightarrow Z^0 \rightarrow \tilde{\ell}^+ \tilde{\ell}^- \begin{cases} \downarrow \\ \tilde{\nu}\nu_r^- \rightarrow \tilde{\nu}\nu_\mu\nu_r\mu^- \\ \downarrow \\ \tilde{\nu}\mu^+ \end{cases}$$

Since $\nu_r, \tilde{\nu}, \tilde{\nu}$ and $\tilde{\nu}$ escape the detector their reactions appear as $e^+e^- \rightarrow \mu^+\nu_r\tilde{\ell}^+$

⁶ In the infrared limit if we take the superparticle mass to be small we must be careful to reinstate lepton mass terms which have been ignored. This is demonstrated for the off-shell $\mu\nu\gamma$ vertex in the appendices associated with Chapter 4.

missing energy. This six-particle final state would be difficult to distinguish from far more quotidian occurrences such as $e^+e^- \rightarrow \nu^+\nu^- \rightarrow \mu^+\mu^-\nu_\mu\nu_r\nu_r\nu_\mu$.

3.6 SMALL MASS SPLITTINGS

The largest cross-sections occur when the mass-splitting between $\tilde{\ell}_{L,R_1}(\tilde{\nu}_1)$ and $\tilde{\ell}_{L,R_2}(\tilde{\nu}_2)$ is large. In many models we may restrict ourselves to the case where the relative mass-splitting is small, i.e.

$$\delta_{\tilde{\ell}_L} \ll 1, \quad \delta_{\tilde{\ell}_R} \ll 1, \quad \delta_{\tilde{\nu}} \ll 1$$

where

$$\delta_{\tilde{\ell}_{L,R}} \equiv \frac{m_{\tilde{\ell}_{L,R_1}}^2 - m_{\tilde{\ell}_{L,R_2}}^2}{m_{\tilde{\ell}_{L,R_1}}^2}, \quad \delta_{\tilde{\nu}} \equiv \frac{m_{\tilde{\nu}_1}^2 - m_{\tilde{\nu}_2}^2}{m_{\tilde{\nu}_1}^2} \quad (3.11)$$

These models¹⁰⁾ assume (quite reasonably) that supersymmetry breaks spontaneously in a flavour-independent manner. If the breaking scale is not too large then small mass splitting follows naturally. This has not been assumed in this paper. In such scenarios the matrix elements are generally at least an order of magnitude smaller. In this limit we can write

$$\Delta F(T_i^+, S_{ij}^+, R_i^+) = \delta_{\tilde{\nu}} F'(T_i^+, S_{ij}^+, R_i^+) \quad (3.12)$$

$$\Delta F(T_{L,R}^0, S_{ij}^0, R_i^0) = \delta_{\tilde{\ell}_{L,R}} F'(T_{L,R}^0, S_{ij}^0, R_i^0) \quad (3.13)$$

$$\Delta F \equiv F(\tilde{\ell}_1) - F(\tilde{\ell}_2) \doteq \delta_{\tilde{\ell}} F'(\tilde{\ell}_1) \quad (3.14)$$

$$F'(T, S, R) \equiv T \frac{\partial F(T, S, R)}{\partial T} = \frac{\partial F(T, S, R)}{\partial \ln T} \quad (3.15)$$

In this case we may replace each ΔJ and ΔG term in (3.9) by $\delta_{\tilde{\ell}_\nu} J'$ and $\delta_{\tilde{\ell}_\nu} G'$. (See Appendix N for these functional forms).

3.7 CROSS-SECTION AND SPECIAL CASES

Given the matrix element, M^w , in (3.9) we may derive

$$\frac{\sigma_{e^+e^- \rightarrow \tau^+\mu^-}}{\sigma_{e^+e^- \rightarrow \mu^+\mu^-}} = \frac{(|M_L|^2 + |M_R|^2)}{8\pi\alpha(\cot^2 2\theta_w + \tan^2 \theta_w)} \quad (3.16)$$

Note that if any of the supersymmetric particles has a mass of less than $\frac{1}{2}M_X$ that real production can occur and that M_L and M_R are complex. It is the modulus which is important in (3.16).

In order to get a feeling for the range of $\frac{\sigma_{e^+e^- \rightarrow \tau^+\mu^-}}{\sigma_{e^+e^- \rightarrow \mu^+\mu^-}}$ we examine (3.9) in two extreme gaugino limits. The first is the "supersymmetry" limit in which we eliminate the gaugino supersymmetry-breaking terms by letting $M', M, \mu \rightarrow 0$ and let $v_1 = v_2$. Thus the only supersymmetry-breaking terms are the explicit slepton mass terms. Then (3.9) reduces to

$$\begin{aligned} M_{SUSY}^w = & \frac{K_L \xi_L}{\sin^2 \theta_w \sin 2\theta_w} \left\{ \Delta G(T_1^+, R_1^+) + \cos 2\theta_w \Delta G(T_1^+, 0) \right. \\ & \left. - \cos 2\theta_w \Delta I_{[1]}(T_1^+, R_1^+) \right. \\ & \left. + 2\cos^2 \theta_w \left[R_1^+ \Delta I_{[\bar{2}, -2]}(T_1^+, R_1^+) - \Delta \tilde{G}(T_1^+, R_1^+) \right] \right\} \\ & - 2K_L \xi_L \cot 2\theta_w \left\{ \Delta \tilde{G}(T_L^0, R_1^0) + \cot^2 2\theta_w \Delta \tilde{G}(T_L^0, R_2^0) \right\} \\ & + 2K_R \xi_R \tan \theta_w \left\{ \Delta \tilde{G}(T_R^0, R_1^0) + \tan^2 \theta_w \Delta \tilde{G}(T_R^0, R_2^0) \right\} \end{aligned}$$

with

$$\Delta I_{[N(z, \bar{z})]}(T, R) = \Delta I_{[N(z, \bar{z})]}(T, S \equiv 1, R) \quad \tilde{G}(T, R) = \Delta \tilde{G}(T, S \equiv 1, R) \quad (3.17)$$

where the physical masses are now

$$m_{\tilde{\chi}_1^+} = (M_W, M_W) \quad m_{\tilde{\chi}_1^0} = (0, M_X, M_X, 0) \quad (3.18)$$

and the relevant mass ratios are

$$\begin{aligned} T_1^+ &= \frac{m_{\tilde{\chi}_1^+}^2}{M_W^2} & T_{L,R}^0 &= 0 & T_{L,R}^0 &= \frac{m_{\tilde{L},R}^2}{M_X^2} \\ R_1^+ &= \frac{1}{4\cos^2 \theta_w} & R_1^0 &= 0 & R_2^0 &= 1/4 \end{aligned} \quad (3.19)$$

$\frac{\sigma_{e^+e^- \rightarrow \tau^+\mu^-}}{\sigma_{e^+e^- \rightarrow \mu^+\mu^-}}$ is plotted in this limit in Fig. 2. Since experiments rule out charged supersymmetry partners of mass less than about 24 GeV,¹⁰⁴ and in some instances much stronger limits have been placed. ASP¹⁰⁵ has placed limits on the selectron mass and the wino mass of approximately 60 GeV. When combined with the results of other groups they find that $m_{\tilde{e}} \gtrsim 84$ GeV. While such strong limits do not, as yet, exist for $m_{\tilde{\mu}}$ or $m_{\tilde{\tau}}$, and hence for $m_{\tilde{L},R}$, it would be somewhat surprising to find $m_{\tilde{\mu}} < m_{\tilde{\tau}}$. We see that for light slepton masses with large mass-splitting that $\frac{\sigma_{e^+e^- \rightarrow \tau^+\mu^-}}{\sigma_{e^+e^- \rightarrow \mu^+\mu^-}}$ could be as large as 3×10^{-6} . The significance of this will be discussed shortly. In order to achieve such a value parameters conducive to a large cross-section have been selected. We have allowed the sleptons to mix maximally so that $\theta_L = \theta_R = \theta_R = \frac{\pi}{4}$. We have further assumed maximal mass-splitting between the two slepton sectors, i.e. $m_{\tilde{e}}, m_{\tilde{L}_1}$ and $m_{\tilde{\tau}}, m_{\tilde{L}_2}$ are relatively light but $m_{\tilde{\mu}}, m_{\tilde{L}_1}$ and $m_{\tilde{\tau}}, m_{\tilde{L}_2}$ are large and decouple.

Another extreme limit is the "unmixed" limit in which the Higgsino and Gaugino sectors have been disentangled from one another. There are several ways of achieving this. Here we consider the limiting case $M \rightarrow \infty, M' \rightarrow 0$ and $\mu \rightarrow 0$ (θ_w arbitrary). The physical mass states become

$$M_{\tilde{\chi}_1^+} = (M \rightarrow \infty, 0) \quad M_{\tilde{\chi}_1^0} = (M \rightarrow \infty, 0, 0, 0). \quad (3.20)$$

Note that $\tilde{\chi}_1^0 = \tilde{W}_3^0$, $\tilde{\chi}_2^0 = \tilde{B}^0$ and thus are purely gaugino whereas $\tilde{\chi}_{3,4}^0$ are purely Higgsino. Thus $T_{L,R}^0 \equiv \frac{m_{\tilde{L},R}^2}{M_{\tilde{\chi}_1^0}^2} \rightarrow \infty$ and $R_2^0 \equiv \frac{q^2}{4M_{\tilde{\chi}_2^0}^2} \rightarrow \infty$. The

matrix element is given by

$$M_{Unmixed}^{\nu} = \frac{2K_R \xi^{\nu} R}{\cos^2 \theta_w} \tan \theta_w \Delta \tilde{G}(T_R^0, R_2^0) - \frac{K_L \xi^{\nu} L}{2 \cos^2 \theta_w} \cot 2\theta_w \Delta \tilde{G}(T_L^0, R_2^0) \quad (3.21)$$

Note that this result is independent of θ_w . The corresponding cross-section is plotted in Fig. 3. Again the sleptons are assumed to have maximal mixing and mass-splitting. Note, however, that in this limiting case we have a massless chargino (a charged Higgsino). Since experimentally the mass limit for new charged particles is ~ 24 GeV, we conclude that this case is merely illustrative.

3.8 EXPERIMENTAL CONSIDERATIONS

The question arises as to whether such a process would be observable at SLC or LEP. These machines will operate as Z -factories and so are ideally suited for such a search. We see from Fig. 2 that, at best, the cross-section for $e^+e^- \rightarrow Z^0 \rightarrow \tau^+\mu^-$ will be $\sim 3 \times 10^{-6}$ that of $e^+e^- \rightarrow Z^0 \rightarrow \mu^+\mu^-$. Thus we expect

$$BR(Z^0 \rightarrow \tau^+\mu^-) \lesssim O(10^{-7}) \quad (3.22)$$

where we have used the relevant standard branching ratios^[10]

$$BR(Z^0 \rightarrow \tau\tau) \simeq BR(Z^0 \rightarrow \mu\mu) \simeq BR(Z^0 \rightarrow ee) \simeq 3\% \quad (3.23)$$

We also note that

$$BR(\tau \rightarrow \mu\nu) \simeq BR(\tau \rightarrow e\nu) \simeq 17.5\% \quad (3.24)$$

The experimental signature for such a decay would be μ and τ back-to-back with the μ having $E = \frac{M_Z}{2}$ and no missing energy. The principal background will be from $e^+e^- \rightarrow Z^0 \rightarrow \tau^+\tau^-$ followed by $\tau^\pm \rightarrow \mu^\pm\nu$, with the μ having nearly all of the τ momentum. The τ will travel approximately 2.5 mm before decaying in

this case and, since the τ and μ are nearly collinear, the kink in the track will be unobservable. Clearly the number of μ , $N(\Delta\epsilon)$, produced in the energy range from $\frac{M_Z}{2} - \Delta\epsilon$ to $\frac{M_Z}{2}$ is of paramount importance. We find that (see Appendix N)

$$N(\Delta\epsilon) = \frac{3-\alpha}{9+\alpha} \cdot 24 \frac{\Delta\epsilon^2}{M_Z^2} \quad (3.25)$$

where α measures the degree of polarization ($\alpha = 0$ for the unpolarized case; $\alpha = 1$ for complete polarization). At SLAC we expect $\alpha \approx 1$ and thus $N_{pol}(\Delta\epsilon) \simeq \frac{24}{5} \frac{\Delta\epsilon^2}{M_Z^2}$. From this and (19b) we see that we need $\Delta\epsilon \lesssim 0.1\%(\frac{M_Z}{2})$ to achieve a background of 10^{-7} or less which would compare with (3.22). Thus for muons with $E \approx \frac{M_Z}{2}$ we need $\frac{\Delta p}{p} \approx 0.1\%$.

At the SLC the MARK II detector will have an energy resolution for e 's and μ 's of $\sim 0.3\%/GeV$ without a vertex detector and, perhaps, as low as $0.1\%/GeV$ with the planned vertex detector.^[11] For $\frac{M_Z}{2} \approx 50$ GeV this means that $\frac{\Delta p}{p} \sim 5\%$ and from (3.25) the background will swamp the signal by at least a factor of 200. It appears unlikely that detector momentum resolutions will be improved by the required two orders of magnitude in the near future (there remains the formidable problem of increased multiple scatterings as detector mass is added). Thus the process $e^+e^- \rightarrow Z^0 \rightarrow \tau^+\mu^-$ will not be experimentally observable at the emerging generation of machines. Furthermore, because of (3.22), when SLC achieves its eventual target luminosity of 10^6 Z^0 /year, the $Z^0 \rightarrow \tau^\pm\mu^\mp$ production rate will be at best ~ 0.2 event/year.

Should the mixing be primarily between $\tilde{e}$ and $\tilde{\mu}$ there will be an important additional restriction to consider. Under the same conditions described above (maximal mixing; large mass-splitting; Z^0 on shell) the production rate would appear to be the same. It is found, however, that the experimental limits on the process $\mu \rightarrow e\gamma$ place severe constraints on $\tilde{e} - \tilde{\mu}$ mixing in the neutralino sector. If we assume maximal mixing and large mass splitting then (chapter 4) we find that the lightest slepton has $m \gtrsim 1$ TeV and so decouples. Thus the

charginos provide the sole contribution to $Z \rightarrow \mu^+ e^-$ (other than in exceptional circumstances). The matrix element is then proportional to the sneutrino mass splitting. Assuming that this is also large we find, for cases of interest, that the chargino sector contribution is generally much larger than that of the neutralino sector *even before the above constraint has been imposed*. Thus the removal of the neutralino contribution, while considerably simplifying calculations, would not greatly affect the final cross-section.

For the situation illustrated in Fig. 3(b), when applied to $Z \rightarrow \mu^+ e^-$, we find that imposing the above constraints on $m_{\tilde{L}}$ affects the results by, at most, 15%. Furthermore, in the region of interest, this change results in an *increase* in the cross-section (because the real parts of the neutralino and chargino contributions enter with opposite signs). Note that the preceding argument (except the quoted percentage) also follows in the cases of small mass-splitting and non-maximal mixing.

When considering the process $Z^0 \rightarrow \tau^+ \mu^-$ current limits from $\tau \rightarrow \mu \gamma^{(*)}$ do not impose serious constraints on the neutralino sector. In the case of large mass-splitting and maximal mixing we find only that $m_{\tilde{L}} \gtrsim 13$ GeV. In particular substantial effects persist in this case (arising from the neutralino sector) even when $m_{\tilde{\nu}_\tau} = m_{\tilde{\nu}_\mu}$.

We can do somewhat better on background if we assume that the principal slepton mixing occurs in the $\tilde{e} - \tilde{\mu}$ sector. The principal background will still be due to $Z^0 \rightarrow \tau^+ \tau^-$ with $\tau^\pm \rightarrow \mu^\pm \bar{\nu} \nu$ and $\tau^\pm \rightarrow e^\pm \bar{\nu} \nu$. The decay $Z^0 \rightarrow \mu \mu$ will not be a problem since, at these energies, muons will not decay in the detector. Now misidentification of a $\tau^+ \tau^-$ pair as a back-to-back $\mu^+ e^-$ requires that both the μ and e emerge with energies near $\frac{M_Z}{2}$. Assuming complete polarization we find that $N(\Delta p)$, the number of back-to-back $e - \mu$ pairs emerging with $E = \frac{M_Z}{2}$, to within the experimental momentum resolution Δp , is given by $N(\Delta p) \doteq 1.4 \left(\frac{\Delta p}{p} \right)^4$. Thus with $\frac{\Delta p}{p} \sim 5\%$ we find $N(\Delta p) \approx 9 \times 10^{-6}$, which corresponds to roughly 0.02 misidentified events per year, assuming a sample of

$10^6 Z^0$. It would appear that, under the most opportune of conditions, the signal might stand well above the background. It must be cautioned, however, that such propitious circumstances are singularly unlikely. The actual $e^+ e^- \rightarrow \mu^+ e^-$ production rate is almost assuredly much less than the near-maximal value which we have been considering and this idealized background calculation has omitted a number of important experimental effects. This result is, none the less, encouraging particularly in light of the observation that statistics and detector sensitivities will only improve with time.

3.9 CONCLUSION

In conclusion, even in the most favourable scenarios, it appears unlikely that $e^+ e^- \rightarrow \tau^+ \mu^-$ will be observed at SLC within the first few years of operation if the sole contribution is from slepton family mixing. The production rate is simply insufficient and the background rate overly severe. The decay $e^+ e^- \rightarrow \mu^+ e^-$ is equally rare but the background problems may prove more tractable. The full parameter space of the gaugino-higgsino sector has yet to be explored. Since the cross-sections predicted in the limiting cases presented come within an order of magnitude of being experimentally interesting at the SLC, realistic symmetry-breaking parameters might exist which would increase the cross-section to observable levels. In any instance the projected luminosity available at LEP might render such searches feasible.

Table 3.1 Fields in This Chapter

Superfield	Ordinary Matter	Superpartners	Weak Isospin	y
<i>Vector (Gauge) Multiplets</i>				
$\hat{V}$	$W^{\pm,0}$	$\tilde{W}^{\pm,0}$	Triplet	0
$\hat{V}'$	B^0	$\tilde{B}^0$	Singlet	0
<i>Scalar Multiplets</i>				
$\hat{L}_i$	ν_i, ℓ_{Li}^-	$\tilde{\nu}_i, \tilde{\ell}_{Li}$	Doublet	-1
$\hat{R}_i$	$(\ell_i^+)_{L_1} = (\ell_{R_i}^-)^*$	$(\tilde{\ell}_{R_i}^-)^*$	Singlet	2
$\hat{H}_1$	H_1^0, H_1^-	$\tilde{\psi}_{H_1}^0, \tilde{\psi}_{H_1}^-$	Doublet	-1
$\hat{H}_2$	H_2^+, H_2^0	$\tilde{\psi}_{H_2}^+, \tilde{\psi}_{H_2}^0$	Doublet	1

FIGURE CAPTIONS

- Diagrams contributing to $Z^0 \rightarrow \tau^+ \mu^-$.
  - Chargino Diagrams.
  - Neutralino Diagrams.
- Ratio of cross-sections, $\frac{\sigma_{\tau^+ \tau^- \rightarrow \tau^+ \mu^-}}{\sigma_{\tau^+ \tau^- \rightarrow \mu^+ \mu^-}}$, in "supersymmetry" limit.
 $M = M' = \mu = 0$. $\theta_a = \pi/4$.
  - Varying all slepton masses equally. [Ignoring $m_{\tilde{L}_1}$ and $m_{\tilde{L}_2}$].
  - Varying $m_{\tilde{L}_1}$ while holding all other mass parameters equal to the indicated values.
- Ratio of cross-sections, $\frac{\sigma_{\tau^+ \tau^- \rightarrow \tau^+ \mu^-}}{\sigma_{\tau^+ \tau^- \rightarrow \mu^+ \mu^-}}$, in the "unmixed" limit.
 $M = M' = \mu = 0$. $\theta_a = \pi/4$.
 Vary $m_{\tilde{L}_1} = m_{\tilde{L}_2}$ assuming $m_{\tilde{L}_1} = m_{\tilde{L}_2} = \infty$.

4. $\mu \rightarrow e\gamma$

4.1 INTRODUCTION

This is the classic lepton family number violating process. Of such processes it is the best-studied, both theoretically and experimentally, and that which has provided the keenest limits on model-building. In terms of the present model such a decay would occur due to mixing in the selectronic and smuonic sectors. We could motivate the form of the matrix elements by first considering the related decay $\tau \rightarrow \mu\gamma$ in which the mixing is principally between $\tilde{\mu}$ and $\tilde{\tau}$ (and the corresponding scalar neutrinos). We would then utilize the wisdom gleaned in the previous chapter by developing crossing relations to go from $Z^0 \rightarrow \tau^+\mu^-$ to $\tau \rightarrow \mu\gamma$ directly. This would give us the leading order "vectorial" (γ^μ) interaction term which can be shown to vanish due to the reasons discussed in section 4.2.

The lowest-order non-trivial matrix elements will be presented (Appendix O) and discussed (in section 4.3). We will consider off-shell behaviour and observe that the "vectorial" terms may indeed prove important in this case. We will not ignore the masses of the fermions in this case since, unlike the previous chapter, the lepton masses are comparable to the decay energy. Finally, comparison with experiment will be examined in section 4.4 and special cases of interest presented.

4.2 THE FORM OF THE MATRIX ELEMENT

The first observation which we make is that, in the approximation of massless leptons (particularly muons and electrons), $\mu \rightarrow e\gamma$ must be principally mediated by a convective $\sigma^{\mu\nu}$ interaction. The reason is that the photon is massless and therefore transverse. Thus it has spin quantum numbers (on shell)

$$(s, s_3) = (1, \pm 1)$$

but no $s_3 = 0$ longitudinal part. Hence, denoting the 'z-component' of the spin of the (anti)muon by $s_3(\mu)$, let us consider the related process of $\gamma \rightarrow \mu e$ depicted

Fig. 3.3

in Fig. 4.1. Since $s_3(\gamma) = s_3(\mu) + s_3(e)$ and the leptons emerge back-to-back they must have spins aligned in the same direction and so have opposite chiralities. In the weak basis $\tilde{W}^{\pm,0}$ only couple to left-handed particles (and likewise for their supersymmetric partners) and so only $\tilde{B}^0$ would contribute in the indicated fashion. This conclusion is true if Fig. 4.1 were rotated to illustrate $\mu \rightarrow e\gamma$. Of course, since both $\tilde{X}^0$ and $\tilde{\gamma}$ contain a piece of $\tilde{B}^0$ they would both contribute to such a process. Higgsinos couple with a strength which is proportional to the mass of the *lepton* and so they also do not contribute. In an arbitrary neutralino basis, of the forms described in the previous chapter, the $\tilde{B}^0$ portion of each $\tilde{X}_i^0$ would contribute.

When electron and muon masses are included then helicity flipping becomes possible along the lepton lines and $\tilde{W}^{\pm,0}$, $\tilde{H}^{\pm,0}$ diagrams will contribute (suppressed by m_μ/M_{SUSY} , of course). Both terms^[10] of the form $(s + d\gamma_5)\sigma^{\mu\nu}$, which flip helicity, and $(e + d\gamma_5)\gamma^\mu$, which do not, are possible. As the helicity-preserving terms are down by $m_\ell/M_{\tilde{L}W}$ most papers simply ignore them and assume an interaction of the form $\sigma^{\mu\nu}q_\nu$. Since

$$\begin{aligned}\bar{\psi}\psi &= \psi_R^\dagger\psi_L + \psi_L^\dagger\psi_R \\ \bar{\psi}\gamma^\mu\psi &= \psi_R^\dagger\sigma^\mu\psi_R + \psi_L^\dagger\bar{\sigma}^\mu\psi_L \\ \bar{\psi}\sigma^{\mu\nu}q_\nu\psi &= \psi_R^\dagger\bar{\sigma}^\mu\sigma^\nu\psi_L + \psi_L^\dagger\sigma^\mu\bar{\sigma}^\nu\psi_R\end{aligned}\quad (4.1)$$

we can rotate $\psi_L^{(i)}$ (i is a generation index) and $\psi_R^{(i)}$ independently when there are no mass terms present. Then $\psi_L^{(i)\dagger}\sigma^\mu\psi_L^{(i)}$ may always be rotated to $\psi_L^{(i)\dagger}\sigma^\mu\psi_L^{(i)}$. This informs us that the coefficients of the σ^μ terms must always be proportional to some power of the lepton masses.

We will use the somewhat non-standard notation for the pure SQED case:

$$\begin{aligned}M_{\mu e\gamma}^\mu &= K\bar{\psi}^e(p', s')\left\{ [F_1(q^2) + F_1^2(q^2)\gamma_5] \frac{g\sigma^{\mu\nu}q_\nu}{m_\gamma^2} \right. \\ &\quad \left. + [F_2(q^2) + F_2^2(q^2)\gamma_5] \sigma^{\mu\nu}q_\nu \right\} \psi^\mu(p, s)\end{aligned}\quad (4.2)$$

where K is a constant which can be shown to be equal to

$$K = -\frac{\alpha^{3/2}\sin\theta\cos\theta}{144\sqrt{\pi}} \quad (4.3)$$

and

$$q_\nu = p_\nu - p'_\nu$$

in this instance.

The $\tilde{e}$ - $\tilde{\mu}$ mixing angle is θ (here we are tacitly taking $\theta_L = \theta_R \equiv \theta$). Weinberg and Feinberg^[10] refer to these form factors as $f_{B0}(q^2)$, $f_{M0}(q^2)$, $f_{M1}(q^2)$, and $f_{E1}(q^2)$ respectively (up to factors of q^2 etc.). The $g\sigma^{\mu\nu}q_\nu$ term is actually a γ^μ -type term written in this way to more closely resemble the form of $\sigma^{\mu\nu}q_\nu$. This follows from

$$g\sigma^{\mu\nu}q_\nu = [gq^\mu - q^2\gamma^\mu]. \quad (4.4)$$

We see that the F_1 , F_1^2 terms (f_{B0} , f_{M0}) will not contribute for real, free, massless photons since the cross-section, σ , goes like $\sigma \propto |M^\mu \cdot \epsilon_\mu|^2$ and $q \cdot \epsilon = 0$ as well as $q^2 = 0$. For most off-shell processes, such as $\mu \rightarrow e e e$ via an off-shell photon, the q^μ term will vanish in any case. With this in mind it is not surprising that the leading-order coefficients of $F_1(q^2)$ and $F_1^2(q^2)$ are found to vanish.

4.3 DECAY RATE

SQED Result

From Appendix O we find that

$$\begin{aligned}M_{\mu e\gamma} &= K\bar{\psi}^e(p', s')\left\{ [F_1(q^2) + F_1^2(q^2)\gamma_5] \frac{g\sigma^{\mu\nu}q_\nu}{m_\gamma^2} \right. \\ &\quad \left. + [F_2(q^2) + F_2^2(q^2)\gamma_5] \sigma^{\mu\nu}q_\nu \right\} \psi^\mu(p, s)\end{aligned}$$

with

$$K = -\frac{\alpha^{3/2}\sin\theta\cos\theta}{144\sqrt{\pi}} \quad q_\nu = p_\nu - p'_\nu$$

and that, when the muon decays radiatively on-shell to a photon and electron,

that the F_1 and F_1^2 terms are unimportant but that

$$\begin{aligned} F_2(q^2 = 0) &\doteq \frac{0m_\mu}{m_\lambda^2} [\Delta f_2(T_R) + \Delta f_2(T_L)] \\ F_2^2(q^2 = 0) &\doteq \frac{0m_\mu}{m_\lambda^2} [\Delta f_2(T_R) - \Delta f_2(T_L)] \end{aligned} \quad (4.5)$$

where m_λ is the photino mass, which is not necessarily small, and we are considering a purely SQED result. We have abbreviated the mass ratios (Chapter 3 and Appendix O) to:

$$T_L = \frac{m_{\tilde{L}_1}^2}{m_\lambda^2} \quad T_R = \frac{m_{\tilde{R}_1}^2}{m_\lambda^2} \quad (4.6)$$

From the appendix

$$\begin{aligned} \Delta f_2(T_R) &\equiv f_2 \left(\frac{m_{\tilde{R}_1}^2}{m_\lambda^2} \right) - f_2 \left(\frac{m_{\tilde{L}_1}^2}{m_\lambda^2} \right) \\ \Delta f_2(T_L) &\equiv f_2 \left(\frac{m_{\tilde{L}_1}^2}{m_\lambda^2} \right) - f_2 \left(\frac{m_{\tilde{R}_1}^2}{m_\lambda^2} \right) \end{aligned} \quad (4.7)$$

$$f_2(x) = \frac{(1-x)(x^2-5x-2) - e^{-1/x}}{(1-x)^4} \quad f_2(1) = -\frac{1}{6} \quad (4.8)$$

This has been plotted in Fig. 4.1. We find that the decay width is

$$\Gamma_{\mu e \gamma} = \Gamma_0 \sin^2 \theta \cos^2 \theta \left(\frac{m_\mu}{m_\lambda} \right)^4 \left(|\Delta f_2(T_L)|^2 + |\Delta f_2(T_R)|^2 \right) \quad (4.9)$$

where

$$\begin{aligned} \Gamma_0 &\equiv \frac{\alpha^3}{1024\pi^2} \frac{m_\mu c^2}{\hbar} \\ &\simeq 6.17 \times 10^{12} \text{ sec}^{-1} \end{aligned} \quad (4.10)$$

Small Mass-Splitting

In the case of small slepton mass-splitting, when

$$\begin{aligned} \Delta f_2(T_L) &\simeq \delta_{\tilde{L}} f_2'(T_L) & \Delta f_2(T_R) &\simeq \delta_{\tilde{R}} f_2'(T_R) \\ \delta_{\tilde{L}} &\equiv \frac{m_{\tilde{L}_1}^2 - m_{\tilde{L}_2}^2}{m_{\tilde{L}_1}^2} & f_2'(T_L) &\equiv T_L \frac{\partial f_2(T_L)}{\partial T_L} \end{aligned} \quad (4.11)$$

$$\begin{aligned} \delta_{\tilde{R}} &\equiv \frac{m_{\tilde{R}_1}^2 - m_{\tilde{R}_2}^2}{m_{\tilde{R}_1}^2} & f_2'(T_R) &\equiv T_R \frac{\partial f_2(T_R)}{\partial T_R} \\ \delta_{\tilde{L}} &\ll 1 & \delta_{\tilde{R}} &\ll 1 \end{aligned} \quad (4.12)$$

we have

$$\Gamma_{\mu e \gamma} \simeq \Gamma_0 \sin^2 \theta \cos^2 \theta \left(\frac{m_\mu}{m_\lambda} \right)^4 \left[\delta_{\tilde{L}}^2 |f_2'(T_L)|^2 + \delta_{\tilde{R}}^2 |f_2'(T_R)|^2 \right] \quad (4.13)$$

Note that

$$f_2'(x) = \frac{x}{3(1-x)^5} \{ (1-x)(x^2-8x-17) - 6(6x+1) \ln x \} \quad (4.14)$$

$$\begin{aligned} f_2'(0) &= 0 & f_2'(x \rightarrow 0) &\sim -\frac{1}{3}x(17+6 \ln x) \\ f_2'(1) &= \frac{1}{10} \\ f_2'(\infty) &= 0 & f_2'(x \rightarrow \infty) &\sim \frac{1}{3x} \end{aligned} \quad (4.15)$$

The peak of f' is found at

$$f_2'(0.273) \simeq 0.12407 \quad (4.16)$$

This has been plotted in Fig. 4.2b. The physical meaning of this is that such a

contribution will be maximal when

$$m_A \approx 2m_{\tilde{t}_1}.$$

Note that $f_2^i(T_L) = -2J_{|z(1-z)}(T_L, 0)$ which is given in Table L.1.

Electroweak Calculation

As discussed in the first section, in a full supersymmetric electroweak (SGWS) calculation, within this model, the only gaugino which would contribute would be a pure $\tilde{B}^0$, should it be a mass eigenstate.

Thus the calculation would be identical with that of the purely SQED case except that the photino λ ($\tilde{\gamma}$) would be supplanted by a Bino. Higgsino contributions would enter into $M_{\mu \rightarrow \tau}^H$ with a factor of m_μ^4 , down even from the m_μ^2 coefficients which allowed us to ignore the F_1 and F_1^* terms. In using a photino we have been, in effect, using only a piece of the full $\tilde{B}^0$ contribution (i.e. if the $\tilde{\gamma} - \tilde{Z}^0$ were the correct mass eigenbasis we would have been neglecting the Zino terms). The Bino and photino couplings are similar except that we must make the following substitutions:

$$\begin{aligned} ie \sin \theta; ie \cos \theta &\rightarrow -\frac{i}{2} g' \sin \theta; -\frac{i}{2} g' \cos \theta & \text{for } \tilde{t}_L \\ ie \sin \theta; ie \cos \theta &\rightarrow ig' \sin \theta; ig' \cos \theta & \text{for } \tilde{t}_R \end{aligned} \quad (4.17)$$

where $g' = \frac{e}{\cos \theta_w}$ is the $U(1)_Y$ hypercharge coupling constant ($Y_{e_L} = -1$; $Y_{e_R} = 2$). Using this (4.9) becomes

$$\Gamma_{\mu \tau} = \Gamma_0 \frac{\sin^2 \theta \cos^2 \theta}{\cos^4 \theta_w} \left(\frac{m_\mu}{m_{\tilde{B}^0}} \right)^4 \left(\frac{|\Delta f_2(T_L)|^2}{16} + |\Delta f_2(T_R)|^2 \right). \quad (4.18)$$

This is exactly correct if $\tilde{B}^0$ is indeed an eigenstate. If the photino and Zino are closer to the true eigenbases then we would use the SQED result and add in a

similar $\tilde{Z}^0$ contribution obtainable by the following substitutions from the SQED result:

$$\begin{aligned} ie \sin \theta; ie \cos \theta &\rightarrow -ie \cot 2\theta_w \sin \theta; -ie \cot 2\theta_w \cos \theta & \text{for } \tilde{t}_L \\ ie \sin \theta; ie \cos \theta &\rightarrow ie \tan \theta_w \sin \theta; ie \tan \theta_w \cos \theta & \text{for } \tilde{t}_R \end{aligned} \quad (4.19)$$

which follows from the general fermion-fermion- $\tilde{Z}^0$ vertex

$$\frac{g}{\cos \theta_w} [T_3 - Q \sin^2 \theta_w] \quad (4.20)$$

where $g = e/\sin \theta_w$ is the $SU(2)$ coupling constant. Once again we have set the left and right mixing angles equal (to θ).

Pursuing this line of reasoning further the integral function terms may also be readily extended to include both photinos and Zinos:

$$\begin{aligned} \frac{\Delta f_2(T_L)}{m_{\tilde{A}}^2} &\rightarrow \frac{\Delta f_2(T_L^{\tilde{t}_L})}{m_{\tilde{A}}^2} + \cot^2 2\theta_w \frac{\Delta f_2(T_L^{\tilde{t}_L^*})}{m_{\tilde{Z}}^2} \\ \frac{\Delta f_2(T_R)}{m_{\tilde{A}}^2} &\rightarrow \frac{\Delta f_2(T_R^{\tilde{t}_R})}{m_{\tilde{A}}^2} + \tan^2 \theta_w \frac{\Delta f_2(T_R^{\tilde{t}_R^*})}{m_{\tilde{Z}}^2} \end{aligned} \quad (4.21)$$

where

$$\begin{aligned} T_L^{\tilde{t}_L} &= \frac{m_{\tilde{t}_L}^2}{m_{\tilde{A}}^2} & T_R^{\tilde{t}_R} &= \frac{m_{\tilde{t}_R}^2}{m_{\tilde{A}}^2} \\ T_L^{\tilde{t}_L^*} &= \frac{m_{\tilde{t}_L}^2}{m_{\tilde{Z}}^2} & T_R^{\tilde{t}_R^*} &= \frac{m_{\tilde{t}_R}^2}{m_{\tilde{Z}}^2} \end{aligned} \quad (4.22)$$

The width then becomes

$$\Gamma_{\mu e \gamma} = \Gamma_0 \sin^2 \theta \cos^2 \theta m_\mu^4 \left\{ \left| \frac{\Delta f_2(T_L^A)}{m_\lambda^2} + \frac{\cot^2 2\theta_\omega \Delta f_2(T_L^{B*})}{m_{2s}^2} \right|^2 + \left| \frac{\Delta f_2(T_R^A)}{m_\lambda^2} + \frac{\tan^2 \theta_\omega \Delta f_2(T_R^{B*})}{m_{2s}^2} \right|^2 \right\} \quad (4.23)$$

It is clear how we would extend this to the instance of general neutralino mixing in the fashion used in chapter three and its associated appendices.

4.4 COMPARISON WITH EXPERIMENT

The radiative decay of the muon has been intensively searched for. Thus far no positive evidence has surfaced but improved limits are being reported on a regular basis. We will use the limit^[60]

$$\frac{\Gamma(\mu \rightarrow e \gamma)}{\Gamma(\mu \rightarrow e \nu \bar{\nu})} < 4.9 \times 10^{-11} \quad (4.24)$$

and

$$\Gamma_{TOT} \simeq \Gamma_{\mu \rightarrow e \nu \bar{\nu}} = 455137 \text{ s}^{-1} \quad (4.25)$$

therefore

$$\Gamma_{\mu \rightarrow e \gamma}^{exp} \leq 2.2 \times 10^{-8} \text{ s}^{-1} \quad (4.26)$$

and so

$$\Gamma_{\mu \rightarrow e \gamma}^{exp} / \Gamma_0 \lesssim 3.6 \times 10^{-10} . \quad (4.27)$$

It must be commented that (4.24) represents a remarkably accurate limit and is the culmination of much dedicated work by many people.

From (4.9) and (4.18) we may immediately obtain

$$\sin^2 \theta \cos^2 \theta \left(\frac{m_\mu}{m_\lambda} \right)^4 \left(|\Delta f_2(T_L)|^2 + |\Delta f_2(T_R)|^2 \right) \leq 3.6 \times 10^{-10} \quad (4.28)$$

for SQED and

$$\frac{\sin^2 \theta \cos^2 \theta}{\cos^4 \theta_\omega} \left(\frac{m_\mu}{m_{2s}} \right)^4 \left(\frac{|\Delta f_2(T_L)|^2}{16} + |\Delta f_2(T_R)|^2 \right) \leq 3.6 \times 10^{-10} \quad (4.29)$$

for SGWS if the Bino were a mass eigenstate. Note that if

$$m_{\tilde{L}_1} \simeq m_{\tilde{L}_2}, \quad m_{\tilde{L}_3} \simeq m_{\tilde{L}_4}, \quad \theta = \frac{\pi}{4}$$

that (SQED)

$$\frac{|\Delta f_2(T_{L,R})|}{m_\lambda^2} \leq 2.4 \times 10^{-7} \text{ GeV}^{-2} \quad (4.30)$$

Small Mass-Splitting Limit

If the differences in the mass of $m_{\tilde{L}_1}$ and $m_{\tilde{L}_2}$ is small (and the same for the "right" sleptons), i.e.

$$\delta_{\tilde{L}_L}^2 \ll 1 \quad \delta_{\tilde{L}_R}^2 \ll 1$$

as in (4.11) then we have

$$\Gamma_{\mu e \gamma} \simeq \Gamma_0 \sin^2 \theta \cos^2 \theta \left(\frac{m_\mu}{m_\lambda} \right)^4 \left[\delta_{\tilde{L}_L}^2 |f_2'(T_L)|^2 + \delta_{\tilde{L}_R}^2 |f_2'(T_R)|^2 \right] .$$

Since $f_2'(0) = f_2'(\infty) = 0$ we know that

$$\Gamma_{\mu \rightarrow e \gamma} = 0 \text{ as } \begin{array}{ll} (i) & m_\lambda \rightarrow \infty \\ (ii) & m_i \rightarrow \infty \\ (iii) & m_i \rightarrow m_{\tilde{i}} \end{array} \quad (4.31)$$

From (4.16) $f'_2 \lesssim \frac{1}{8}$. Should it be that

$$\delta_{\tilde{L}} \simeq \delta_{\tilde{R}} \simeq \delta_i \ll 1$$

then, for SQED,

$$\Gamma_{\mu\tau} \lesssim \frac{\delta_i^2}{32} \Gamma_0 \sin^2 \theta \cos^2 \theta \left(\frac{m_\mu}{m_\lambda} \right)^4$$

and since $\sin^2 \theta \cos^2 \theta \leq \frac{1}{4}$

$$\Gamma_{\mu\tau} \lesssim \frac{\delta_i^2}{128} \Gamma_0 \left(\frac{m_\mu}{m_\lambda} \right)^4. \quad (4.32)$$

When we compare with experiment using (4.27) we obtain a limit on how large the relative splitting may be.

$$\delta_i \lesssim 1.9 \times 10^{-6} m_\lambda^2 \quad (4.33)$$

where m_λ is given in GeV. This should be compared with (4.30). Thus for $m_\lambda \sim 100 \text{ GeV}$ the slepton splitting would be less than 2% while for $m_\lambda \sim 1 \text{ GeV}$ the slepton splitting would be less than $2 \times 10^{-4}\%$. These would occur near the maximal cross-section, when $m_\lambda \approx 2m_i$.

The condition that $m_\lambda \approx 2m_i$ is a rather unnatural one. A popular scenario among designers of specific models is that the neutralino state which is principally photino-like tends to be on the light side. Indeed some authors tend to assume that $m_\lambda \ll m_i$ almost axiomatically. Although we will not take this view it is, however, interesting to examine the results in this limit. If

$$m_\lambda \ll m_{\tilde{L},R}$$

then

$$T_{L,R} \rightarrow \infty$$

and

$$f'_2(T_{L,R}) \rightarrow \frac{1}{3T_{L,R}}. \quad (4.34)$$

In this case (4.13) becomes

$$\Gamma_{\mu\tau} \doteq \Gamma_0 \sin^2 \theta \cos^2 \theta \left(\frac{m_\mu}{m_\lambda} \right)^4 \left(\frac{\delta_L^2}{9T_L^2} + \frac{\delta_R^2}{9T_R^2} \right) \quad (4.35)$$

and so if, in addition, $\delta_{\tilde{L}} \simeq \delta_{\tilde{R}}$ then (SQED)

$$\Gamma_{\mu\tau} \approx \frac{2}{9} \Gamma_0 \sin^2 \theta \cos^2 \theta \left(\frac{m_\mu}{m_\lambda} \right)^4 \delta_{\tilde{L}}^2$$

and if $\theta = \pi/4$ (maximal mixing)

$$\frac{\delta_{\tilde{L}}}{m_{\tilde{L}}^2} \lesssim 7.2 \times 10^{-7} \text{ GeV}^{-2}. \quad (4.36)$$

If the Bino is an eigenstate and $m_{\tilde{B}_0} \ll m_i$ this would become ($\cos^2 \theta_w = 0.77$)

$$\frac{\delta_{\tilde{L}}}{m_{\tilde{L}}^2} \lesssim 7.6 \times 10^{-7} \text{ GeV}^{-2}. \quad (4.37)$$

In their classic paper^[24] Ellis and Nanopoulos made the first rough estimates for such processes. Working exclusively in the small mass-splitting regime (as discussed in chapter three) they found

$$\delta_{\tilde{L}}/m_{\tilde{L}}^2 < O(10^{-7}) \text{ GeV}^{-2}$$

assuming maximal mixing and that $\tilde{B}^0$ is an eigenstate. At the time which that paper was written the best experimental limit was $\Gamma_{\mu\tau}^{\text{exp}}/\Gamma_{TOT} < 1.9 \times 10^{-10}$.

Using this value (4.37) would alter to

$$\frac{\delta \tilde{L}}{m_{\tilde{L}}^2} \lesssim 3 \times 10^{-6} \text{ GeV}^{-2}.$$

which is a factor of thirty off of their naive estimate. Thus a careful computation reveals that we are only able to place somewhat weaker limits than one might have initially hoped.

Large Mass-Splitting Limit

In this limit $m_{\tilde{L}_2} \gg m_{\tilde{L}_1}$ and $m_{\tilde{R}_2} \gg m_{\tilde{R}_1}$. Therefore

$$\Delta f_3(T_{L,R}) \rightarrow f_3(T_{L,R}).$$

Where $f_3(x)$ was given in (4.8). Note that

$$f_3(x \rightarrow \infty) \rightarrow -\frac{1}{3x}. \quad (4.38)$$

If we further consider the case where $m_{\tilde{L}} \gg m_{\tilde{R}}$, then (for $\tilde{X}^0 = \tilde{B}^0$) from (4.18)

$$\begin{aligned} \Gamma_{\mu\tau\gamma} &\rightarrow \Gamma_0 \frac{\sin^2 \theta \cos^2 \theta}{\cos^4 \theta_w} \cdot \frac{1}{9} \left(\frac{m_\mu}{m_{\tilde{B}^0}} \right)^4 \left(\frac{m_{\tilde{B}^0}^4}{16m_{\tilde{L}_1}^4} + \frac{m_{\tilde{B}^0}^4}{m_{\tilde{L}_1}^4} \right) \\ &= \frac{1}{9} \Gamma_0 \frac{\sin^2 \theta \cos^2 \theta}{\cos^4 \theta_w} \cdot m_\mu^4 \left[\frac{1}{16m_{\tilde{L}_1}^4} + \frac{1}{m_{\tilde{L}_1}^4} \right] \end{aligned} \quad (4.39)$$

For $\theta = \theta_L = \theta_R = \pi/4$ and using $\cos^2 \theta_w = 0.77$ and (4.27) we find

$$\left(\frac{1}{16m_{\tilde{L}_1}^4} + \frac{1}{m_{\tilde{L}_1}^4} \right) \lesssim \frac{7.7 \times 10^{-17}}{m_\mu^4} \quad (4.40)$$

Further specializing to

$$m_{\tilde{L}_1} = m_{\tilde{R}_1} \equiv m_{\tilde{L}}$$

would result in

$$m_{\tilde{L}} \gtrsim 1.1 \text{ TeV} \quad (4.41)$$

since $m_\mu = 0.105659 \text{ GeV}$.

This result is not "viable" since it was obtained by considering the limit $m_{\tilde{L}_2}, m_{\tilde{R}_2} \gg m_{\tilde{L}} \approx 1 \text{ TeV}$ and we would like the supersymmetry-breaking scale to be $\lesssim O(1 \text{ TeV})$ if it is to alleviate the gauge hierarchy problem in a 'natural' way. If the mass eigenstates are photinos and Zinos we should anticipate a similar result in this limit. If $m_{\tilde{g}} \sim m_{\tilde{z}}$ then $m_{\tilde{L}} \gg m_{\lambda, \tilde{z}}$, and since $T_L^1 = T_R^1$ and $T_L^{2*} = T_R^{2*}$ we would have from (4.18)

$$\Gamma_{\mu\tau\gamma} \rightarrow \Gamma_0 \sin^2 \theta \cos^2 \theta \cdot \frac{1}{9} \left(\frac{m_\mu}{m_{\tilde{L}_1}} \right)^4 \left\{ (1 + \cot^2 2\theta_w)^2 + (1 + \tan^2 \theta_w)^2 \right\}.$$

The term in the curly brackets is about 3.7. If only the photino part were included it would have been 2.0 while the corresponding term for the Bino was about 1.8. In the photino-Zino case the limit in (4.41) would become

$$m_{\tilde{L}} \gtrsim 1.4 \text{ TeV}. \quad (4.42)$$

Such a constraint would, of course, be weakened if the angular mixing were not maximal.

Implication of ASP Results

The ASP single photon search at SLAC has placed^[11] limits on $m_{\tilde{L}}$ of about 60 GeV. When combined with the results of MAC and other collaborations they

find^[10] (under a set of parameter assumptions) that

$$m_{\tilde{e}} \gtrsim 84 \text{ GeV} . \quad (4.43)$$

The limit comes from considerations of the process

$$e^+e^- \rightarrow \gamma\tilde{\gamma}\tilde{\gamma}$$

whose leading contribution arises from the diagram in Fig. 4.3. Under the assumptions that $m_{\tilde{e}_L} = m_{\tilde{e}_R}$, the photino is a mass eigenstate and is the only light neutral gaugino, and that $m_{\tilde{\gamma}} \ll m_{\tilde{e}_{L,R}}$, they quote

$$\sigma(e^+e^- \rightarrow \gamma\tilde{\gamma}\tilde{\gamma}) \sim \alpha^3 \frac{s}{m_{\tilde{e}}^2} < 0.06 \text{ pb} . \quad (4.44)$$

The specific search is for $e^+e^- \rightarrow \gamma + \text{energy}$ where the standard model calculation of $e^+e^- \rightarrow \gamma\nu\bar{\nu}$, for three light neutrino generations, has been accounted for. Thus the search for new light species of neutrinos (as well as Higgsinos, sneutrinos and so forth) complements the photino search, since the existence of such states would only improve the limit in (4.43).

The above analysis assumes that there is no intergenerational mixing. Permitting $\tilde{e} - \tilde{\mu}$ mixing, with angle θ , changes (4.44) to $(m_{\tilde{e}_L} = m_{\tilde{e}_R})$

$$\sigma(e^+e^- \rightarrow \gamma\tilde{\gamma}\tilde{\gamma}) \sim \alpha^3 s \left(\frac{\cos^2 \theta}{m_{\tilde{e}_L}^2} - \frac{\sin^2 \theta}{m_{\tilde{e}_R}^2} \right)^2 < 0.06 \text{ pb} \quad (4.45)$$

so instead of

$$\frac{1}{m_{\tilde{e}}^2} < \frac{1}{(84 \text{ GeV})^2}$$

which follows from (4.43) we have

$$\frac{\cos^2 \theta}{m_{\tilde{e}_L}^2} - \frac{\sin^2 \theta}{m_{\tilde{e}_R}^2} < \frac{1}{(84 \text{ GeV})^2} . \quad (4.46)$$

Since $\sigma(\mu \rightarrow e\gamma)$ goes as $\sin^2 \theta \cos^2 \theta$, whereas $\sigma(ee \rightarrow \gamma\tilde{\gamma}\tilde{\gamma})$ has θ dependence as

in (4.46), we see that, when these processes are compared, that the mixing angle ($\theta = \theta_L = \theta_R$) enters non-trivially.

First let us consider the case of large mass-splitting, where $m_{\tilde{e}_L} \gg m_{\tilde{e}_R}$.

Then (4.46) reduces to

$$\frac{\cos^2 \theta}{m_{\tilde{e}_L}^2} < \frac{1}{(84 \text{ GeV})^2}$$

i.e.

$$m_{\tilde{e}_L} > 84 \text{ GeV} \cos \theta \quad (4.47)$$

which is weaker than (4.43). For $\theta = \pi/4$ (maximal mixing) this limit becomes

$$m_{\tilde{e}_L} > 59 \text{ GeV} . \quad (4.48)$$

We have found that $\sigma_{e^+e^- \rightarrow \gamma\tilde{e}^+\tilde{e}^-}$ and $\sigma_{\mu\mu \rightarrow e\gamma}$ go as

$$\sin^2 \theta \cos^2 \theta (\Delta F_1(T^0) + \Delta F_2(T^+))$$

where $T^0 = \frac{m_{\tilde{e}_L}^2}{m_{\tilde{e}_R}^2}$ and $T^+ = \frac{m_{\tilde{\nu}_\tau}^2}{m_{\tilde{e}_R}^2}$. $\Delta F_{1,2}$ are some functions which decrease with large, increasing arguments. For $\mu \rightarrow e\gamma$ the F_2 term is absent. Thus as the limit from (4.47) becomes stronger (increasing $\cos \theta$) then $\max[\Delta F_1]$ decreases. However $\sin^2 \theta \cos^2 \theta$ peaks at $\theta = \pi/4$. Thus as θ increases from $\theta < \pi/4$ to $\theta = \pi/4$, $\max[\Delta F_1]$ decreases while $\sin^2 \theta \cos^2 \theta$ increases. As θ increases beyond $\theta = \pi/4$ $\max[\Delta F_1]$ increases while $\sin^2 \theta \cos^2 \theta$ decreases. Thus there is always competition between these two effects (as far as obtaining experimental limits from $e^+e^- \rightarrow \mu^+e^-$ and $\mu \rightarrow e\gamma$ are concerned). Note that at $\cos \theta = 0, 1$ that both processes vanish.

Let us now consider the small mass-splitting case:

$$m_{\tilde{L}_1}^2 - m_{\tilde{L}_2}^2 \equiv m_{\tilde{L}_1}^2 \cdot \delta_{\tilde{L}} \ll m_{\tilde{L}_1}^2.$$

Then

$$\frac{1}{m_{\tilde{L}_0}^2} \simeq \frac{1 - \delta_{\tilde{L}}}{m_{\tilde{L}_1}^2}$$

and therefore

$$\frac{\cos^2 \theta}{m_{\tilde{L}_1}^2} - \frac{\sin^2 \theta}{m_{\tilde{L}_2}^2} \approx \frac{\cos^2 \theta - \sin^2 \theta}{m_{\tilde{L}_1}^2} + \frac{\sin^2 \theta \delta_{\tilde{L}}}{m_{\tilde{L}_1}^2}. \quad (4.49)$$

If we are near "maximal mixing" then $\theta \simeq \pi/4$ and

$$\frac{\cos^2 \theta}{m_{\tilde{L}_1}^2} - \frac{\sin^2 \theta}{m_{\tilde{L}_2}^2} \approx + \frac{\delta_{\tilde{L}}}{2m_{\tilde{L}_1}^2}$$

and (4.46) becomes

$$\frac{\delta_{\tilde{L}}}{2m_{\tilde{L}_1}^2} \leq \frac{1}{(84 \text{ GeV})^2}$$

i.e.

$$\frac{\delta_{\tilde{L}}}{m_{\tilde{L}_1}^2} \lesssim 3 \times 10^{-4} \text{ GeV}^{-2} \quad (4.50)$$

which is much weaker than the $\mu \rightarrow e\gamma$ limit

$$\frac{\delta_{\tilde{L}}}{m_{\tilde{L}_1}^2} \lesssim 7 \times 10^{-7} \text{ GeV}^{-2}$$

which we found in (4.36).

The weakness of the limit lies in the supposition that $\theta = \pi/4$. This is due to the particular form of (4.49). If we took $\theta = \pi/3$ then the limit in (4.36) would change to

$$\frac{\delta_{\tilde{L}}}{m_{\tilde{L}_1}^2} \lesssim 10^{-6} \text{ GeV}^{-2} \quad (4.51)$$

from $\mu \rightarrow e\gamma$ while $m_{\tilde{L}_1} > 42 \text{ GeV}$ from $e^+e^- \rightarrow \gamma\tilde{\gamma}\tilde{\gamma}$. When θ is not near $\pi/4$ (4.46) and (4.49) imply that

$$m_{\tilde{L}_1} > 84 \text{ GeV} \cos 2\theta \quad (4.52)$$

as opposed to (4.47). Note that limits that ASP places on m_W from the analogous process $e^+e^- \rightarrow \gamma\tilde{\nu}\tilde{\nu}$ are both weaker and subject to a greater number of model-dependent assumptions.

FIGURE CAPTIONS

1. The vertex $\gamma \rightarrow \mu e$.
2. (a) The function $f_2(x)$ vs. x
 (b) The function $f_2'(x)$ vs. x .
3. Principle contribution to the process

$$e^+e^- \rightarrow \gamma \tilde{\tau} \tilde{\tau}$$

used by ASP to place limits on the selectron mass.

Fig. 4.1

Fig. 4.2a

Fig. 4.2b

Summary

We have seen that intergenerational mixing of sleptons may occur at experimentally acceptable rates. When compared with experiment we may begin to eliminate certain regions from an otherwise unconstrained parameter space. It is not anticipated that lepton family number violation, arising from this mechanism, will be seen at the SLC due to insufficient production rates.

The low-energy processes of chapters two and four provide more substantial constraints. If the mass-splitting between the two sectors (left-right or species one-species two) is small then reasonably strong limits on the splitting may be obtained, depending upon the specifics involved. If a large mass-splitting is assumed then some quite respectable limits of supersymmetric masses become possible, but this is again highly dependent upon specifics.

In conclusion we feel that the search for both family number non-conservation and evidence for the existence of supersymmetry is of great importance. Then interplay between these two areas can provide endless entertainment for both experimentalists and theorists alike.

Fig. 4.3

APPENDIX A

Two-Component Weyl Notation

The two-component Weyl spinor formalism is a more fundamental representation of spinorial states and algebra than that of Dirac. In most theories the Dirac notation is a convenient shorthand for summarizing the actions of the different chiral states and this explains its immense utility and popularity. In supersymmetric theories the states of different chirality are dealt with independently and two-component notation is the natural language to use. Greater familiarity with the manipulation of Dirac objects will prompt us to use them in the more involved computations but the irrecusable consequence will be the proliferation of conjugation and helicity projection operators.

There are many excellent references^{1,12,18} whose appendices summarize definitions (which vary from one author to the next) and properties of the Weyl algebra. We will use notation similar to that of reference 12. The most appropriate representation of the gamma matrices to use when we work in four-component notation is not the Dirac representation but the chiral representation. Note that the chiral representation used here differs from that of Itzykson and Zuber.¹⁰

A.1 SPINORS

We write the electron as:

$$\psi = \begin{pmatrix} \varphi_a \\ \bar{\chi}^{\dot{a}} \end{pmatrix} \equiv \begin{pmatrix} \psi_- \\ \bar{\psi}_+ \end{pmatrix} \quad \begin{array}{l} L.H. \ e^- \\ R.H. \ e^- \end{array} \quad (A.1)$$

$$\bar{\psi} = (\chi^a \quad \bar{\varphi}_{\dot{a}}) \equiv (\psi_+^a \quad \bar{\psi}_{-\dot{a}}) \quad \begin{array}{l} L.H. \ e^+ \ R.H. \ e^+ \end{array} \quad (A.2)$$

where the position of the indices in conventional. Barred spinors ($\bar{\psi}$) and dotted

indices denote right-handed $(0, \frac{1}{2})$ representations of the Lorentz Group. Unbarred spinors (ψ) and undotted indices denote left-handed $(\frac{1}{2}, 0)$ representations.

A.2 SIGMA AND GAMMA MATRICES

$$\text{The metric used is}^{11} \quad g^{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

The gamma matrices, γ^μ , have the structure

$$\gamma^\mu = \begin{pmatrix} 0 & (\sigma^\mu)_{a\dot{b}} \\ (\bar{\sigma}^\mu)^{\dot{a}b} & 0 \end{pmatrix} \quad \mu = 0, 1, 2, 3 \quad \gamma^5 = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \quad (A.3)$$

where

$$\begin{aligned} \sigma^0 &= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} & \sigma^1 &= \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \\ \sigma^2 &= \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} & \sigma^3 &= \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \end{aligned} \quad (A.4)$$

Definition:

$$\sigma^{\mu\nu} = \frac{i}{2} [\gamma^\mu, \gamma^\nu] \quad (4\text{-component}) \quad (A.5)$$

¹¹ Note that Wess and Bagger¹ use

$$\eta^{\mu\nu} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \text{ and } \sigma^0 = - \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\partial^{\mu\nu} = \frac{i}{2} [\partial^\mu \partial^\nu - \partial^\nu \partial^\mu] \quad (2\text{-component}) \quad (\text{A.6})$$

$$\sigma^{\mu\nu} = \frac{i}{2} [\sigma^\mu \partial^\nu - \sigma^\nu \partial^\mu] \quad (2\text{-component}).$$

$$\sigma^{\mu\nu} \{4\text{-component}\} = \begin{pmatrix} \sigma^{\mu\nu} & 0 \\ 0 & \partial^{\mu\nu} \end{pmatrix} = \begin{pmatrix} (\sigma^{\mu\nu})_{\alpha\beta} & 0 \\ 0 & (\partial^{\mu\nu})^{\dot{\alpha}\dot{\beta}} \end{pmatrix} \quad (\text{A.7})$$

$$\bar{\psi} \sigma^{\mu\nu} \psi \{\text{Dirac}\} = \psi_+ \sigma^{\mu\nu} \psi_- + \bar{\psi}_- \partial^{\mu\nu} \bar{\psi}_+ \quad (\text{A.8})$$

$$\not{a} \not{b} = a \cdot b - i \sigma_{\mu\nu} a^\mu b^\nu$$

$$\gamma_5 \not{a} \not{b} = \not{a} \not{b} \gamma_5 = \gamma_5 [a \cdot b - i \sigma^{\mu\nu} a_\mu b_\nu] \quad (\text{A.9})$$

$$(\sigma \cdot p)(\sigma \cdot q) = p \cdot q - i \sigma^{\mu\nu} p_\mu q_\nu$$

$$(\partial \cdot p)(\partial \cdot q) = p \cdot q - i \sigma^{\mu\nu} p_\mu q_\nu$$

A special case is

$$(\sigma \cdot p)^2 = (\sigma^\mu)_{\alpha\beta} p_\mu (\sigma^\mu)^{\dot{\beta}\dot{\alpha}} p_\nu = p^2 \delta_{\alpha\dot{\alpha}} \quad (\text{A.10})$$

A.3 INDEX GYMNASTICS

Raise and lower indices with ϵ as follows

$$\psi^\alpha = \epsilon^{\alpha\beta} \psi_\beta \quad \psi_\alpha = \epsilon_{\alpha\beta} \psi^\beta \quad \bar{\psi}^{\dot{\alpha}} = \epsilon^{\dot{\alpha}\dot{\beta}} \bar{\psi}_{\dot{\beta}} \quad \bar{\psi}_{\dot{\alpha}} = \epsilon_{\dot{\alpha}\dot{\beta}} \bar{\psi}^{\dot{\beta}} \quad (\text{A.11})$$

$$\epsilon^{\alpha\beta} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \quad \epsilon_{\alpha\beta} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \quad (\text{in matrix form}) \quad (\text{A.12})$$

$$\epsilon_{\alpha\beta} \epsilon^{\beta\gamma} = \delta_\alpha^\gamma \quad \epsilon^{\dot{\alpha}\dot{\beta}} \epsilon_{\dot{\beta}\dot{\gamma}} = \delta^{\dot{\alpha}}_{\dot{\gamma}} \quad (\text{A.13})$$

In actuality $\partial^\mu = \sigma^\mu$. $(\partial^\mu)^{\dot{\alpha}\alpha}$ is just the raised index version of $(\sigma^\mu)_{\alpha\dot{\alpha}}$ and no more. Once we have chosen the convention that (σ^μ) has index structure $(\sigma^\mu)_{\alpha\dot{\alpha}}$

then (σ^μ) always refers to the operator of complementary index ordering.

$$\begin{aligned} (\sigma^\mu)^{\dot{\alpha}\alpha} &= \epsilon^{\dot{\alpha}\dot{\beta}} \epsilon^{\alpha\beta} (\sigma^\mu)_{\beta\dot{\beta}} \\ &= \epsilon^{\dot{\alpha}\dot{\beta}} [(\sigma^\mu)_{\beta\dot{\beta}}^T]_{\dot{\beta}\beta}^T [\epsilon^T]^{\beta\alpha} \\ &= -(\epsilon [(\sigma^\mu)^T \epsilon])^{\dot{\alpha}\alpha} \\ &= (\sigma^\mu [(\sigma^\mu)^T \sigma^\mu])^{\dot{\alpha}\alpha} \end{aligned} \quad (\text{A.14})$$

since $\epsilon^{\alpha\beta} = i(\sigma^3)^{\alpha\beta}$. However

$$(\sigma^i, (\sigma^i)^T) = -2\delta^{ii} \quad |(\sigma^i, (\sigma^0)^T) = 0 \quad i = 1, 2, 3$$

therefore $\sigma^3(\sigma^{1,2})^T = -\sigma^{1,2}\sigma^3$, $\sigma^3(\sigma^3)^T = -1$ and $\sigma^3(\sigma^0)^T = \sigma^0\sigma^3$ so we find that

$$(\sigma^\mu)^{\dot{\alpha}\alpha} = \begin{cases} [(\sigma^\mu)\sigma^3\sigma^3]^{\dot{\alpha}\alpha} & \mu = 0 \\ -[\sigma^\mu\sigma^3\sigma^3]^{\dot{\alpha}\alpha} & \mu = 1, 2, 3 \end{cases} = \begin{cases} (\sigma^\mu)^{\dot{\alpha}\alpha} & \mu = 0 \\ -(\sigma^\mu)^{\dot{\alpha}\alpha} & \mu = 1, 2, 3 \end{cases} \quad (\text{A.15})$$

$$(\sigma^\mu)_{\alpha\dot{\alpha}} = (I_{\alpha\dot{\alpha}}, \sigma_{\alpha\dot{\alpha}}^i) \quad (\sigma^\mu)^{\dot{\alpha}\alpha} = (I^{\dot{\alpha}\alpha}, -(\sigma^i)^{\dot{\alpha}\alpha})$$

where $I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \sigma^0$ is the unit matrix. Since $(\sigma^\mu)_{\alpha\dot{\alpha}} = (1, \sigma^i)_{\alpha\dot{\alpha}}$ and $(\sigma^\mu)^{\dot{\alpha}\alpha} = (1, -\sigma^i)^{\dot{\alpha}\alpha}$ from (A.15) then $g_{\mu\nu} = \text{diag}(1, -1, -1, -1)$ lowers the Lorentz index to give

$$g_{\mu\nu}(\sigma^\nu)_{\alpha\dot{\alpha}} \equiv (\sigma_\mu)_{\alpha\dot{\alpha}} = (1, -\sigma^i)_{\alpha\dot{\alpha}}; \quad g_{\mu\nu}(\partial^\nu)^{\dot{\alpha}\alpha} \equiv (\partial_\mu)^{\dot{\alpha}\alpha} = (1, \sigma^i)^{\dot{\alpha}\alpha}. \quad (\text{A.16})$$

Note that when we raise one index and lower another

$$\bar{\chi}_{\dot{\alpha}} \bar{\psi}^{\dot{\alpha}} = -\bar{\chi}^{\dot{\alpha}} \bar{\psi}_{\dot{\alpha}} \quad (\text{A.17})$$

Notation:

$$\chi^\alpha \phi_\alpha \equiv \chi \phi \quad (\text{A.18})$$

but

$$\begin{aligned} \psi \chi &= \psi^\alpha \chi_\alpha = (\epsilon^{\alpha\beta} \psi_\beta)(\epsilon_{\alpha\gamma} \chi^\gamma) = \psi_\beta (\epsilon^{\alpha\beta} \epsilon_{\alpha\gamma}) \chi^\gamma \\ &= \psi_\beta (-\epsilon^{\beta\alpha} \epsilon_{\alpha\gamma}) \chi^\gamma = -\psi_\beta \delta^\beta_\gamma \chi^\gamma \\ &= -\psi_\beta \chi^\beta \end{aligned} \quad (\text{A.19})$$

$$\psi^\alpha \chi_\alpha = -\psi_\alpha \chi^\alpha = \chi^\alpha \psi_\alpha \text{ since } \chi, \psi \text{ are fermions} = \chi \psi. \text{ Thus} \quad (\text{A.20})$$

$$\psi \chi = \chi \psi. \quad (\text{A.21})$$

A.4 CONJUGATION

$$(\psi_\alpha)^{\dagger} = (\psi^\dagger)_\alpha = \bar{\psi}_\alpha \quad (2 \text{ component spinors})$$

$$C = \begin{pmatrix} \epsilon_{\alpha\beta} & 0 \\ 0 & \epsilon^{\dot{\alpha}\dot{\beta}} \end{pmatrix} \quad (\text{charge conjugation operator})$$

$$\psi^C = C \bar{\psi}^T \quad \text{in general}$$

$$\chi_\alpha = \epsilon_{\alpha\beta} (\bar{\chi})^{\dagger\beta}$$

also note

$$(\chi \psi)^{\dagger} = \bar{\psi} \bar{\chi} = \bar{\chi} \bar{\psi}.$$

A.5 USEFUL RELATIONSHIPS.

A plethora of useful relationships may be found in Wess and Bagger "Supersymmetry and Supergravity"¹. Some of these and others of interest are (here $m, n=0,1,2,3$):

$$(\sigma^m \sigma^n + \sigma^n \sigma^m)_{\alpha\beta} = (\sigma^m)_{\alpha\gamma} (\sigma^n)^{\gamma\beta} + (\sigma^n)_{\alpha\gamma} (\sigma^m)^{\gamma\beta} = 2g^{mn} \delta_{\alpha\beta} \quad (\text{A.22})$$

$$(\partial^m \sigma^n + \sigma^n \partial^m)_{\alpha\beta} = 2g^{mn} \delta_{\alpha\beta} \quad (\text{A.23})$$

$$\text{tr } \sigma^m \sigma^n = (\sigma^m)_{\alpha\beta} (\sigma^n)^{\beta\alpha} = 2g^{mn} \quad (\text{A.24})$$

$$\sigma_{\alpha\dot{\alpha}}^m \sigma_m^{\beta\dot{\beta}} = -2\delta_{\alpha}^{\beta} \delta_{\dot{\alpha}}^{\dot{\beta}} \quad (\text{A.25})$$

$$(\sigma^m)^{\dot{\alpha}\alpha} (\sigma_m)_{\beta\dot{\beta}} = -2\delta_{\beta}^{\alpha} \delta_{\dot{\beta}}^{\dot{\alpha}} \quad (\text{A.26})$$

$$\sigma_{\alpha\dot{\alpha}}^m \sigma_m^{\dot{\alpha}\alpha} = -2 \quad (\text{A.27})$$

$$\partial_m \sigma^n \partial^m = (\partial^m)^{\dot{\alpha}\alpha} (\sigma^n)_{\alpha\dot{\beta}} (\partial^m)^{\dot{\beta}\beta} = -2(\partial^n)^{\dot{\alpha}\beta} \quad (\text{A.28})$$

$$\sigma_m \sigma^n \sigma^m = (\sigma_m)_{\alpha\dot{\alpha}} (\partial^n)^{\dot{\alpha}\beta} (\sigma^m)^{\beta\dot{\beta}} = -2(\sigma^n)^{\alpha\dot{\beta}} \quad (\text{A.29})$$

$$\sigma^m \partial^n \sigma_m = -2(\partial^n)_{\alpha\dot{\beta}} \quad (\text{A.30})$$

$$\partial^m \sigma^n \partial_m = -2(\partial^n)^{\dot{\alpha}\beta} \quad (\text{A.31})$$

$$(\sigma_\mu \partial^\nu \sigma^\mu \partial^\mu)_{\alpha\beta} = 4g^{\nu\mu} \delta_{\alpha\beta} = (\sigma^\mu \partial^\nu \sigma^\mu \partial_\mu)_{\alpha\beta} \quad (\text{A.32})$$

$$(\partial_\mu \sigma^\nu \partial^\mu \sigma^\mu)^\lambda_\beta = 4g^{\nu\lambda} \delta^\mu_\beta = (\partial^\mu \sigma^\nu \partial^\mu \sigma_\mu)^\lambda_\beta \quad (\text{A.33})$$

$$(\sigma^\mu \partial_\nu \sigma^\nu \partial_\mu)_\alpha^\beta = 4\delta_\nu^\mu \delta_\alpha^\beta = (\sigma_\mu \partial_\nu \sigma^\nu \partial^\mu)_\alpha^\beta \quad (\text{A.34})$$

$$(\sigma^\mu \partial^\nu \sigma_\mu \partial_\nu)_\alpha^\beta = 4\delta_\alpha^\beta = (\sigma_\mu \partial^\nu \sigma_\nu \partial^\mu)_\alpha^\beta \quad (\text{A.35})$$

$$(\sigma^\mu \partial_\nu \sigma_\mu \partial_\nu)_\alpha^\beta = 4\delta_\alpha^\beta = (\sigma_\mu \partial_\nu \sigma_\nu \partial^\mu)_\alpha^\beta \quad (\text{A.36})$$

$$(\partial_\nu \sigma^\mu \partial^\mu \sigma^\mu \partial^\nu)^\lambda_\beta = -2(\partial^\mu \sigma^\mu \partial^\mu)^\lambda_\beta = (\partial^\nu \sigma^\mu \partial^\mu \sigma^\mu \partial_\nu)^\lambda_\beta \quad (\text{A.37})$$

$$(\sigma_\nu \partial^\mu \sigma^\mu \partial^\mu \sigma^\nu)_\alpha^\beta = -2(\sigma^\mu \partial^\mu \sigma^\mu)_\alpha^\beta = (\sigma^\nu \partial^\mu \sigma^\mu \partial^\mu \sigma_\nu)_\alpha^\beta \quad (\text{A.38})$$

If F is anything which has no " μ " dependence and " $\bar{F}$ " means replace all σ^ν 's in F by $\bar{\sigma}^\nu$'s, σ_ν 's by $\bar{\sigma}_\nu$'s, ∂^ν 's by $\bar{\partial}^\nu$'s, etc. then

$$\begin{aligned} \sigma^\mu F \sigma_\mu &= \sigma^\mu \bar{F} \bar{\sigma}_\mu \\ \sigma^\mu F \bar{\sigma}_\mu &= \bar{\sigma}^\mu F \sigma_\mu \\ \sigma_\mu F \sigma^\mu &= \sigma_\mu \bar{F} \bar{\sigma}^\mu \\ \sigma_\mu F \bar{\sigma}^\mu &= \bar{\sigma}_\mu F \sigma^\mu \end{aligned} \quad (\text{A.39})$$

$$\begin{aligned} F_1 \sigma^\mu F_2 &= F_3 \sigma^\mu F_4 \Rightarrow F_1 \bar{\sigma}^\mu F_2 = F_3 \bar{\sigma}^\mu F_4 \\ F_1 \sigma^\mu F_2 &= F_3 \bar{\sigma}^\mu F_2 \Rightarrow F_1 \bar{\sigma}^\mu F_2 = F_3 \sigma^\mu F_4. \end{aligned} \quad (\text{A.40})$$

Using (A.39) and (A.40) we can generate many more identities like (A.28) - (A.38).

In addition there are identities in reference 1 such as

$$\sigma^a \bar{\sigma}^b \sigma^c + \sigma^c \bar{\sigma}^b \sigma^a = 2(g^{ab} \sigma^c + g^{bc} \sigma^a - g^{ac} \sigma^b). \quad (\text{A.41})$$

Fierz Rearrangement Formula:

$$(\psi \phi) \bar{\chi}_\beta = -\frac{1}{2} (\phi \sigma^\mu \bar{\chi}) (\psi \sigma_\mu)_\beta \quad (\text{A.42})$$

Proof:

$$\begin{aligned} -\frac{1}{2} (\phi \sigma^\mu \bar{\chi}) (\psi \sigma_\mu)_\beta &= -\frac{1}{2} (\phi^\alpha (\sigma^\mu)_{\alpha\beta} \bar{\chi}^\beta) (\psi^\rho (\sigma_\mu)_{\rho\beta}) \\ &= -\frac{1}{2} (\psi^\rho \phi^\alpha \bar{\chi}^\beta (\sigma^\mu)_{\alpha\beta} (\sigma_\mu)_{\rho\beta}) \end{aligned}$$

and from (A.26) and (A.14)

$$-\frac{1}{2} (\phi \sigma^\mu \bar{\chi}) (\psi \sigma_\mu)_\beta = -\frac{1}{2} (-2\psi^\rho \phi_\rho \chi_\beta) = (\psi \phi) \bar{\chi}_\beta$$

as required.

APPENDIX B

Two-Component QED

We use the two-component Weyl notation, as presented in Appendix A, to express the QED Lagrangian and Feynman rules.

B.1 DIRAC EQUATION

$$(\gamma^\mu \bar{p}_\mu - m)\psi(p, s) = 0 = \bar{\psi}(p', s')(\gamma^\mu \bar{p}'_\mu + m) \quad \text{4-component form} \quad (\text{B.1})$$

Letting $q_\nu = p'_\nu - p_\nu$

$$[(\sigma^\mu)_{\alpha\dot{\alpha}} p_\mu] \bar{\psi}_\dot{\alpha}(p, s) = m \psi_{-\alpha}(p, s)$$

$$[(\sigma^\mu)_{\alpha\dot{\alpha}} p'_\mu] \bar{\psi}_\dot{\alpha}(p, s) = m \psi_{-\alpha}(p, s) + [(\sigma^\mu)_{\alpha\dot{\alpha}} q_\mu] \bar{\psi}_\dot{\alpha}(p, s)$$

$$[(\bar{\sigma}^\mu)^{\dot{\alpha}\alpha} p_\mu] \psi_{-\alpha}(p, s) = m \bar{\psi}_\dot{\alpha}(p, s)$$

$$[(\bar{\sigma}^\mu)^{\dot{\alpha}\alpha} p'_\mu] \psi_{-\alpha}(p, s) = m \bar{\psi}_\dot{\alpha}(p, s) + [(\bar{\sigma}^\mu)^{\dot{\alpha}\alpha} q_\mu] \psi_{-\alpha}(p, s)$$

(B.2)

$$\bar{\psi}_{-\dot{\alpha}}(p', s') [(\bar{\sigma}^\nu)^{\dot{\alpha}\alpha} p'_\nu] = -m' \psi_\alpha^{\dot{\alpha}}(p', s')$$

$$\bar{\psi}_{-\dot{\alpha}}(p', s') [(\bar{\sigma}^\nu)^{\dot{\alpha}\alpha} p_\nu] = -m' \psi_\alpha^{\dot{\alpha}}(p', s') - \bar{\psi}_{-\dot{\alpha}}(p', s') [(\bar{\sigma}^\nu)^{\dot{\alpha}\alpha} q_\nu]$$

$$\psi_\alpha^{\dot{\alpha}}(p', s') [(\sigma^\nu)_{\alpha\dot{\alpha}} p'_\nu] = -m' \bar{\psi}_{-\dot{\alpha}}(p', s')$$

$$\psi_\alpha^{\dot{\alpha}}(p', s') [(\sigma^\nu)_{\alpha\dot{\alpha}} p_\nu] = -m' \bar{\psi}_{-\dot{\alpha}}(p', s') - \psi_\alpha^{\dot{\alpha}}(p', s') [(\sigma^\nu)_{\alpha\dot{\alpha}} q_\nu]$$

B.2 QED LAGRANGIAN

In four-component form

$$\mathcal{L}_{QED} = i\bar{\psi}\gamma^\mu\partial_\mu\psi - e\bar{\psi}\not{A}\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - m\bar{\psi}\psi. \quad (\text{B.3})$$

Consider the two-component version of $\mathcal{L}_{KIN} = i\bar{\psi}\not{\partial}\psi$ first.

$$\mathcal{L}_{KIN} = i\left\{\psi_\alpha^{\dot{\alpha}}(\sigma^\mu)_{\alpha\dot{\alpha}}\partial_\mu\bar{\psi}_\dot{\alpha} + \bar{\psi}_{-\dot{\alpha}}(\bar{\sigma}^\mu)^{\dot{\alpha}\alpha}\partial_\mu\psi_{-\alpha}\right\}.$$

Now it is $\int d^4x \mathcal{L}$ which will be of interest in a path integral so we may integrate the second term by parts. We also anticommute the spinors in the first term to obtain (ignoring surface terms)

$$\mathcal{L}_{KIN} = i\left\{-\partial_\mu\bar{\psi}_\dot{\alpha}(\sigma^\mu)_{\alpha\dot{\alpha}}\psi_\alpha^{\dot{\alpha}} + \partial_\mu\bar{\psi}_{-\dot{\alpha}}(\bar{\sigma}^\mu)^{\dot{\alpha}\alpha}\psi_{-\alpha}\right\}.$$

Now we lower the spinor indices with $\epsilon_{\alpha\beta}$ and raise them on $(\sigma^\mu)_{\alpha\dot{\alpha}}$ with $\epsilon^{\alpha\beta}$ to obtain

$$\mathcal{L}_{KIN} = -i\left\{\partial_\mu\bar{\psi}_{+\dot{\alpha}}(\sigma^\mu)^{\dot{\alpha}\alpha}\psi_{+\alpha} + \partial_\mu\bar{\psi}_{-\dot{\alpha}}(\bar{\sigma}^\mu)^{\dot{\alpha}\alpha}\psi_{-\alpha}\right\} \quad (\text{B.4})$$

which is the form often found in the literature.¹ Similarly the remainder of the Lagrangian is

$$\begin{aligned} \mathcal{L}_{QED} = \mathcal{L}_{KIN} - e\left\{\psi_\alpha^{\dot{\alpha}}(\sigma^\mu)_{\alpha\dot{\alpha}}A_\mu\bar{\psi}_\dot{\alpha} + \bar{\psi}_{-\dot{\alpha}}(\bar{\sigma}^\mu)^{\dot{\alpha}\alpha}A_\mu\psi_{-\alpha}\right\} \\ - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - m[\psi_{-\alpha}\psi_\alpha^{\dot{\alpha}} + \bar{\psi}_\dot{\alpha}\bar{\psi}_{-\dot{\alpha}}]. \end{aligned} \quad (\text{B.5})$$

The field operators of (B.5) should be thought of as annihilating the corresponding fields. Therefore, since $\psi_{-\alpha}$ is a *left-handed electron* and $\psi_\alpha^{\dot{\alpha}}$ is a *left-handed positron*, $-m\psi_{-\alpha}\psi_\alpha^{\dot{\alpha}}$ destroys an $(e^-)_L$ and an $(e^+)_L$, which is the same as destroying an $(e^-)_L$ and creating an $(e^-)_R$, with strength "m".

B.3 CHIRALITY-CHANGING OPERATORS

The operators which change one chiral state to another are

Particles:

$$\begin{aligned} \frac{(\sigma^\mu)_{\alpha\beta} p_\mu}{m} \bar{\psi}_+^\alpha(p, s) &= \psi_{-\alpha}(p, s) \\ \frac{(\sigma^\mu)^{\dot{\alpha}\dot{\beta}} p_\mu}{m} \psi_{-\dot{\alpha}}(p, s) &= \bar{\psi}_{\dot{\beta}}^\dagger(p, s) \end{aligned} \quad (\text{B.6})$$

Antiparticles:

$$\begin{aligned} \bar{\psi}_{-\alpha}(p, s) \frac{(\bar{\sigma}^\mu)^{\dot{\alpha}\dot{\beta}} p_\mu}{m} &= \psi_{\dot{\beta}}^\dagger(p, s) \\ \psi_{\dot{\alpha}}^\dagger(p, s) \frac{(\sigma^\mu)_{\alpha\beta} p_\mu}{m} &= \bar{\psi}_{\beta}^\dagger(p, s) \end{aligned}$$

These relations follow directly from the Dirac equation.

B.4 PROPAGATORS

We can decompose the electron Dirac propagator into

$$\langle 0 | T \psi_\alpha(x) \bar{\psi}_\beta(y) | 0 \rangle = \left\langle 0 \left| T \begin{pmatrix} \psi_{-\alpha} \psi_{\dot{\beta}}^\dagger & \psi_{-\alpha} \bar{\psi}_{-\dot{\beta}} \\ \bar{\psi}_{\dot{\alpha}}^\dagger \psi_{\dot{\beta}}^\dagger & \bar{\psi}_{\dot{\alpha}}^\dagger \bar{\psi}_{-\dot{\beta}} \end{pmatrix} \right| 0 \right\rangle \quad (\text{B.7})$$

or

$$\frac{i(\not{p} + m)_{\alpha\beta}}{p^2 - m^2} = \Delta(p) \begin{pmatrix} m\delta_{\alpha\beta} & (\sigma^\mu)_{\alpha\beta} p_\mu \\ (\bar{\sigma}^\mu)^{\dot{\alpha}\dot{\beta}} p_\mu & m\delta^{\dot{\alpha}\dot{\beta}} \end{pmatrix} \quad (\text{B.8})$$

with $\Delta(p) \equiv \frac{i}{p^2 - m^2}$

which we can interpret as

$$\epsilon_L^- \rightarrow \epsilon_L^- : \quad \langle 0 | T \psi_{-\alpha} \bar{\psi}_{-\dot{\beta}} | 0 \rangle = \Delta(p) (\sigma^\mu)_{\alpha\dot{\beta}} p_\mu \quad \begin{array}{c} \psi_{-\alpha} \\ \xrightarrow{7-87} \\ 5773A66 \end{array} \rightarrow \bar{\psi}_{-\dot{\beta}} \quad (\text{B.9})$$

To get $\epsilon_L^- \rightarrow \epsilon_R^-$ we apply the helicity-changing operator $(\bar{\sigma}^\mu)^{\dot{\alpha}\beta} p_\mu$ to get

$$\Delta(p) (\sigma^\mu)_{\alpha\dot{\alpha}} p_\mu \frac{(\bar{\sigma}^\nu)^{\dot{\alpha}\beta} p_\nu}{m}$$

But since $(\sigma \cdot p)_{\alpha\dot{\alpha}} (\bar{\sigma} \cdot p)^{\dot{\alpha}\beta} = p^2 \delta_{\alpha}^{\beta} = m^2 \delta_{\alpha}^{\beta}$ (on shell) then

$$\epsilon_L^- \rightarrow \epsilon_R^- : \quad \langle 0 | T \psi_{-\alpha} \psi_{\dot{\beta}}^\dagger | 0 \rangle = m \Delta(p) \delta_{\alpha}^{\dot{\beta}} \quad \begin{array}{c} \psi_{-\alpha} \\ \xrightarrow{7-87} \\ 5773A67 \end{array} \rightarrow \bigotimes \rightarrow \psi_{\dot{\beta}}^\dagger \quad (\text{B.10})$$

which we could have read off of (B.8) directly ($\bigotimes$ indicates helicity flip). Similarly

$$\epsilon_R^- \rightarrow \epsilon_R^- : \quad \langle 0 | T \bar{\psi}_{\dot{\alpha}}^\dagger \psi_{\dot{\beta}}^\dagger | 0 \rangle = \Delta(p) (\bar{\sigma}^\mu)^{\dot{\alpha}\dot{\beta}} p_\mu \quad \begin{array}{c} \bar{\psi}_{\dot{\alpha}}^\dagger \\ \xrightarrow{7-87} \\ 5773A68 \end{array} \rightarrow \psi_{\dot{\beta}}^\dagger \quad (\text{B.11})$$

$$\epsilon_R^- \rightarrow \epsilon_L^- : \quad \langle 0 | T \bar{\psi}_{\dot{\alpha}}^\dagger \bar{\psi}_{-\dot{\beta}} | 0 \rangle = m \Delta(p) \delta_{\dot{\alpha}}^{\dot{\beta}} \quad \begin{array}{c} \bar{\psi}_{\dot{\alpha}}^\dagger \\ \xrightarrow{7-87} \\ 5773A69 \end{array} \rightarrow \bigotimes \rightarrow \bar{\psi}_{-\dot{\beta}} \quad (\text{B.12})$$

We may apply ϵ 's to these which will raise and lower indices as desired using the rules of Appendix A.3. For example, applying $\epsilon_{\alpha\beta}$ to (B.10) results in

$$\langle 0 | \psi_{-\alpha} \psi_{\dot{\beta}}^\dagger | 0 \rangle = -m \Delta(p) \epsilon_{\alpha\dot{\beta}}$$

In general the helicity-preserving operators are proportional to $\sigma^\mu p_\mu$ while the helicity-changing operators are proportional to m . We do not consider multiple

flips since

$$\begin{array}{c} \text{L} \quad \text{R} \quad \text{L} \\ \rightarrow \text{---} \text{---} \text{---} \rightarrow \\ \text{5773A70} \end{array} = m\Delta(p) \frac{(\sigma^\nu)^{\dot{\alpha}\beta}}{m} p_\nu \cdot \frac{(\sigma^\mu)_{\beta\dot{\beta}} p_\mu}{m} = m\Delta(p) \frac{p^2}{m^2} \delta^{\dot{\alpha}}_{\dot{\beta}} = m\Delta(p) \delta^{\dot{\alpha}}_{\dot{\beta}}$$

$$= \begin{array}{c} \text{L} \\ \rightarrow \text{---} \rightarrow \\ \text{5773A71} \end{array}$$

In some sense $\rightarrow$ can be thought of as the appropriately normalized sum of

$$\begin{array}{c} \rightarrow + \text{---} \otimes \text{---} + \text{---} \otimes \otimes \text{---} + \dots \\ \text{7-87} \qquad \qquad \qquad \text{5773A73} \end{array}$$

and $\rightarrow \otimes \rightarrow$ as the sum of

$$\begin{array}{c} \text{---} \otimes \text{---} + \text{---} \otimes \otimes \text{---} + \dots \\ \text{7-87} \qquad \qquad \qquad \text{5773A75} \end{array}$$

(normalized so that "m" is the physical mass).

Note that when external massive fermions are present in a process (i.e. the diagrams or Green's functions have not been truncated) such fields may change helicity in the legs of the diagram.

B.5 PROJECTION OPERATORS

The analogue of the Dirac projection operators for positive and negative

energy states

$$\sum_{\pm s} u_\alpha(p, s) \bar{u}_\beta(p, s) = \left(\frac{\not{p} + m}{2m} \right)_{\alpha\beta}$$

are:

$$\sum_{\pm s} \psi_{-\alpha}(p, s) \bar{\psi}_{-\beta}(p, s) = \frac{(\sigma^\mu)_{\alpha\beta} p_\mu}{2m}$$

$$\sum_{\pm s} \bar{\psi}_{+\alpha}(p, s) \psi_{+\beta}(p, s) = \frac{(\sigma^\mu)^{\dot{\alpha}\dot{\beta}} p_\mu}{2m} \quad (\text{B.13})$$

$$\sum_{\pm s} \psi_{-\alpha}(p, s) \psi_{+\beta}(p, s) = \frac{1}{2} \delta_{\alpha\beta}$$

$$\sum_{\pm s} \bar{\psi}_{+\alpha}(p, s) \bar{\psi}_{-\beta}(p, s) = \frac{1}{2} \delta_{\alpha\beta}$$

In the case where spin is not summed over we project out both helicity and energy states. In Dirac notation we have

$$u_\alpha(p, s) \bar{u}_\beta(p, s) = \left[\frac{\not{p} + m}{2m} \frac{1 + \gamma_5 \not{s}}{2} \right]_{\alpha\beta}$$

where $s^\mu = (0, \vec{s})$ in the rest frame. Let us consider the instance of a particle moving along the $+\hat{z}$ -axis very slowly ($\vec{p} \rightarrow 0$) with spin in the $+\hat{z}$ direction for right-handed particles. Then

$$s_L^\mu = (0, 0, 0, 1) \quad s_R^\mu = (0, 0, 0, -1)$$

For $s_L^\mu = (0, 0, 0, 1)$ we find

$$\begin{aligned}
 \psi_{-a}(p, s) \bar{\psi}_{-\beta}(p, s) &= \frac{1}{4m} [(\sigma^\mu)_{a\beta} p_\mu + m(\sigma^3)_{a\beta}] \\
 \psi^{+a}(p, s) \psi_+^\beta(p, s) &= \frac{1}{2m} [(\bar{\sigma}^\mu)^{a\beta} p_\mu + m(\bar{\sigma}^3)^{a\beta}] \\
 \psi_{-a}(p, s) \psi_+^\beta(p, s) &= \frac{1}{4m} [m\delta_a^\beta - (\sigma^\mu)_{a\dot{a}} p_\mu (\bar{\sigma}^3)^{\dot{a}\beta}] \\
 \bar{\psi}_+^{\dot{a}}(p, s) \bar{\psi}_{-\beta}(p, s) &= \frac{1}{4m} [m\delta_{\dot{a}}^\beta + (\bar{\sigma}^\mu)^{\dot{a}a} p_\mu (\sigma^3)_{a\beta}]
 \end{aligned} \tag{B.14}$$

remembering that $\bar{\sigma}^3 = -\sigma^3$.

For photons

$$\sum_\lambda \epsilon_\mu(k, \lambda) \epsilon_\nu(k, \lambda) = -g_{\mu\nu} . \tag{B.15}$$

B.6 SIMPLE VERTICES

Photon emission does not flip helicity (since the current is transverse). Thus the simple (truncated) vertices appear as

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APPENDIX C

Calculation of Anomalous Magnetic Moment I:

Four-Component Spinors.

The standard calculation of the anomalous magnetic moment of the electron in four-component spinor notation is presented here in detail. The rationale being that this is the prototype for all processes considered in this paper. It provides a direct comparison with the two-component version which follows. We follow the approach of Ref. 2 but use the metric and spinor representations of Appendix A.

We examine the vertex correction

$$-ie\gamma_{\alpha\beta}^{\mu} \rightarrow -ie\Gamma_{\alpha\beta}^{\mu}(p, p') = -ie[\gamma^{\mu} + \Lambda^{\mu}(p, p')]_{\alpha\beta} \quad (\text{C.1})$$

as illustrated in figure 1 and whose lowest order contribution is given by the diagram in figure 2. Since we will only be interested in the Λ^{μ} piece we will not distinguish between Γ^{μ} and Λ^{μ} but simply refer to the first order correction as $\Gamma^{\mu}(p, p')$. To guard against, *a priori*, infrared divergence problems we will let the mass of the photon be μ , which we shall let go to zero at the end. We see that

$$\begin{aligned} -ie\Gamma_{\alpha\beta}^{\mu}(p, p') &= (-ie)^3 \int \frac{d^4k}{(2\pi)^4} \left\{ \left(\frac{-\not{q}\not{p} + \text{gauge terms}}{k^2 - \mu^2 + i\epsilon} \right) (\gamma^{\mu})_{\alpha\gamma} \right. \\ &\quad \times \frac{i}{\not{p}' - \not{k} - m + i\epsilon} (\gamma^{\mu})^{\gamma\eta} \frac{i}{\not{p} - \not{k} - m + i\epsilon} (\gamma^{\mu})_{\eta\beta} \left. \right\}. \end{aligned} \quad (\text{C.2})$$

We will do away with the gauge terms in the photon by recognizing that such

terms vanish in Feynman gauge when $\mu \rightarrow 0$. Hence

$$\begin{aligned} \Gamma^{\mu}(p, p')_{\alpha\beta} &= (-ie)^2 \int \frac{d^4k}{(2\pi)^4} \left\{ \left(\frac{-1}{k^2 - \mu^2 + i\epsilon} \right) \right. \\ &\quad \times \left[\gamma^{\nu} \frac{i(\not{p}' - \not{k} + m)}{(\not{p}' - \not{k})^2 - m^2 + i\epsilon} \gamma^{\mu} \frac{i(\not{p} - \not{k} + m)}{(\not{p} - \not{k})^2 - m^2 + i\epsilon} \gamma^{\nu} \right]_{\alpha\beta} \left. \right\} \end{aligned} \quad (\text{C.3})$$

We will evaluate this on-shell where $\not{p} = \not{p}' = m$ and then let $q^2 \rightarrow 0$, i.e. $p' \rightarrow p$. We ignore the other graphs which occur at this order as they only contribute to the self-energy renormalizations. In order to regulate the integral we employ the traditional Pauli-Villars⁵⁰ subtraction scheme. This introduces a "massive photon" of mass $\Lambda \rightarrow \infty$ which utilizes an incorrect metric. The "incorrect metric" in this case is the negative of the one given in section A.2. The result is that the heavy photon propagator always appears with sign opposite to that of the light ($\mu \rightarrow 0$) propagator. Thus the leading divergences cancel. Although such a propagator is not physically meaningful, as we take $\Lambda \rightarrow \infty$ such contributions vanish. Such a scheme is not overtly gauge invariant during the intermediate steps of the calculations but, of course, gauge invariance must be restored once the limit is taken. Consequently we may represent the combined photon propagators by

$$\frac{1}{k^2 - \mu^2} - \frac{1}{k^2 - \Lambda^2} = - \int_{\mu^2}^{\Lambda^2} \frac{dt}{(k^2 - t)^2} \quad (\text{C.4})$$

resulting in

$$\Gamma^\mu(p, p') = -ic^2 \int \frac{d^4 k}{(2\pi)^4} \int_{\mu^2}^{\Lambda^2} -dt \frac{N^\mu}{D} \quad (C.5)$$

$$N^\mu = \gamma^\nu \{i(\not{p}' - \not{k} + m)\} \gamma^\mu \{i(\not{p} - \not{k} + m)\} \gamma_\nu$$

$$D = (k^2 - t^2)^2 ((p' - k)^2 - m^2) ((p - k)^2 - m^2) .$$

Now using the standard gamma matrix contraction identities of the form $\gamma^\mu [\dots] \gamma_\nu$ which can be found in the appendices of Bjorken and Drell³⁴ we find that

$$\begin{aligned} N^\mu &= i^2 \{ -2(\not{p} - \not{k}) \gamma_\nu (\not{p}' - \not{k}) - 2m^2 \gamma^\mu + 2m \{ (\not{p}' - \not{k}) \gamma^\mu + \gamma^\mu (\not{p}' + \not{k}) \\ &\quad + (\not{p} - \not{k}) \gamma^\mu + \gamma^\mu (\not{p} - \not{k}) \} \} \\ &= 2 \{ (\not{p} - \not{k}) \gamma^\mu (\not{p}' - \not{k}) + m^2 \gamma^\mu - m \{ \gamma^\mu (\not{p}' + \not{p} - 2\not{k}) + (\not{p}' + \not{p} - 2\not{k}) \gamma^\mu \} \} \end{aligned} \quad (C.6)$$

where we have used $\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu}$ and therefore $2p^\mu = \not{p} \gamma^\mu + \gamma^\mu \not{p}$. Since $p^2 = m^2$ on-shell we have $(p + k)^2 - m^2 = k^2 - 2p \cdot k$ and so

$$D = (k^2 - t^2)^2 (k^2 - 2p' \cdot k) (k^2 - 2p \cdot k) . \quad (C.7)$$

Using Feynman's trick

$$\frac{1}{a^2 b c} = 6 \int_0^1 dx \int_0^{1-x} dy \int_0^{\Lambda^2} dt \frac{1-x-y}{[a + (b-a)x + (c-a)y]^4} \quad (C.8)$$

from (H.5) to give

$$\frac{1}{D} = 6 \int_0^1 dx \int_0^{1-x} dy \int_{\mu^2}^{\Lambda^2} dt \frac{(1-x-y)}{[k^2 - t - (2p \cdot k - t)x - (2p' \cdot k - t)y]^4} . \quad (C.9)$$

Now use (H.7) to evaluate this realizing that

$$p \rightarrow px + p'y, \quad s = -t + tx + ty = -t(1-x-y)$$

to yield

$$\int \frac{d^4 k}{(2\pi)^4} \frac{N^\mu}{D} = 6 \int_0^1 dx \int_0^{1-x} dy \int_{\mu^2}^{\Lambda^2} dt \frac{iM^\mu}{96x^2[-t(1-x-y) - px + p'y]^2} \quad (C.10)$$

with

$$\begin{aligned} M^\mu &= 2 \not{p} \gamma^\mu \not{p}' - 2 \not{p} \gamma^\mu (\not{p}x + \not{p}'y) + (\not{p}x + \not{p}'y) \gamma^\mu \not{p}' \\ &\quad - 2m^2 \gamma^\mu + [-t(1-x-y) - (\not{p}x + \not{p}'y)^2] \gamma^\mu \gamma_\nu \\ &\quad - m \{ \gamma^\mu (\not{p}' + \not{p} - 2\not{p}x - 2\not{p}'y) + (\not{p}' + \not{p} - 2\not{p}x - 2\not{p}'y) \gamma^\mu \} \\ &\quad + 2(\not{p}x + \not{p}'y) \gamma^\mu (\not{p}x + \not{p}'y) \end{aligned} \quad (C.11)$$

and using $\gamma^\nu \gamma^\mu \gamma_\nu = -2\gamma^\mu$ we may factor out a "2". Since we are really examining $\bar{u}(p') \Gamma^\mu(p, p') u(p)$ we may use the Dirac equation

$$\bar{u}(p') (\not{p}' - m) = (\not{p} - m) u(p) = 0 \quad \text{with} \quad p' = p + q \quad (C.12)$$

on M^μ . Further, from (H.17 b) $(px + p'y)^2 = m^2(x+y)^2 - q^2xy$ so that

$$\begin{aligned}
 M^\mu &= 2 \left\{ (m - \not{q})\gamma^\mu(m + \not{q}) + (m - \not{q})x + my \right\} \gamma^\mu \{ mx + (m + \not{q})y \} \\
 &\quad - \{ (m - \not{q})x + my \} \gamma^\mu (m + \not{q}) + (m - \not{q})\gamma^\mu \{ mx + (m + \not{q})y \} \\
 &\quad - m \{ \gamma^\mu (m + \not{q}) + m - 2mx - 2(m + \not{q})y \} \\
 &\quad + \{ m + (m - \not{q}) - 2(m - \not{q})x - 2my \} \gamma^\mu \} \\
 &\quad + \{ t(1 - x - y) + (m^2[x + y]^2 - q^2xy) \} \gamma^\mu - m^2 \gamma^\mu \} \\
 &= 2 \left\{ \gamma^\mu \{ t(1 - x - y) + m^2(x + y)^2 - q^2xy + m^2 - 2m^2(x + y) \right. \\
 &\quad \left. - 4m^2(1 - x - y) + m^2(x + y)^2 - m^2 \right\} \\
 &\quad + \gamma^\mu \not{q} \{ m - m(x + y) - my - m + 2my + m(x + y)y \} \\
 &\quad + \{ -m + mx + m(x + y) + m - 2mx - m(x + y)x \} \not{q} \\
 &\quad + \not{q} \gamma^\mu \not{q} \{ -1 - x - y - xy \} \} .
 \end{aligned} \tag{C.13}$$

We have exercised our gauge freedom when we chose to calculate in the Feynman gauge. This is one of the R_ξ (or Lorentz) gauges which are covariant, i.e. satisfy $q^\nu A_\nu(q^2) = 0$ (in this case $\xi = 1$). In a physical process we have terms like $\Gamma^\mu A_\mu$ and so all terms proportional to q^μ in M^μ will vanish. We explicitly impose this gauge condition: $q^\mu A_\mu(q) = 0$. Now

$$\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu}$$

thus

$$\{\gamma^\mu, \not{q}\} = \{\gamma^\mu, \gamma^\nu q_\nu\} = 2g^{\mu\nu} q_\nu = 2q^\mu$$

which vanishes in M^μ . Therefore

$$\gamma^\mu \not{q} = -\not{q} \gamma^\mu \tag{C.14}$$

and the anti-commutation term does not contribute. Also

$$\not{q} \gamma^\mu \not{q} = -\gamma^\mu \not{q} \not{q} = -\gamma^\mu q^2.$$

Letting

$$\begin{aligned}
 A &= 2 \left\{ t(1 - x - y) + 2m^2 \{ (x + y)^2 - 2(1 - x - y) - (x + y) \right. \\
 &\quad \left. - q^2xy - q^2(1 - x)(1 - y) \} \right\}
 \end{aligned}$$

(C.13) becomes

$$\begin{aligned}
 M^\mu &= \gamma^\mu \{ A \} \\
 &\quad + \{ m \not{q} \gamma^\mu \{ (x + y) - x(x + y) - x \} - m \gamma^\mu \not{q} \{ (x + y) - y(x + y) - y \} \} \\
 &= \gamma^\mu \{ A \} + 2m \{ \not{q} \gamma^\mu \{ -x^2 - xy + y \} - \gamma^\mu \not{q} \{ -y^2 - xy + x \} \} \\
 &= \gamma^\mu \{ A \} \\
 &\quad + 2m \left\{ \not{q} \gamma^\mu \left[-\frac{x^2}{2} - xy + \frac{y^2}{2} + \frac{x}{2} - \frac{x^2}{2} + \left(-\frac{x^2}{2} - \frac{x}{2} + \frac{y^2}{2} + \frac{y}{2} \right) \right] \right. \\
 &\quad \left. - \gamma^\mu \not{q} \left[-\frac{y^2}{2} - xy + \frac{x}{2} + \frac{y}{2} - \frac{x^2}{2} + \left(-\frac{y^2}{2} - \frac{y}{2} + \frac{x^2}{2} + \frac{y}{2} \right) \right] \right\}
 \end{aligned}$$

but

$$\not{q} \gamma^\mu \left(-\frac{x^2}{2} - \frac{x}{2} + \frac{y^2}{2} + \frac{y}{2} \right) - \gamma^\mu \not{q} \left(-\frac{y^2}{2} - \frac{y}{2} + \frac{x^2}{2} + \frac{x}{2} \right) = 0$$

by (C.14) thus

$$\begin{aligned} M^\mu &= \gamma^\mu \{ \Lambda \} + \frac{g \gamma^\mu - \gamma^\mu g}{2} [2m(x+y-z^2-y^2-2xy)] \\ &= \gamma^\mu \{ \Lambda \} + \frac{[\gamma^\mu, \gamma^\mu] g}{2} [2m(x+y)(1-x-y)] \end{aligned}$$

with

$$\Gamma^\mu(p, p') = 6 \int_0^1 dx \int_0^{1-x} dy \frac{-1}{96\pi^2} (ie^2) \int_{\mu^2}^{\Lambda^2} (-dt) \frac{i(1-x-y)M^\mu}{[t(1-x-y) + (px+py)^2]^2}.$$

Now

$$\sigma^{\mu\nu} = \frac{i}{2} [\gamma^\mu, \gamma^\nu]$$

and

$$\Gamma^\mu(p, p') = \gamma^\mu F_1(q^2) + \frac{i}{2m} \sigma^{\mu\nu} q_\nu F_2(q^2) \quad (\text{C.15})$$

with the anomalous magnetic moment

$$a_e \equiv F_2(q^2 \rightarrow 0) \quad a \equiv \frac{g-2}{2}. \quad (\text{C.16})$$

Therefore

$$iF_2(q^2 \rightarrow 0) = \frac{2}{i} \cdot \frac{-6(i)^2 e^2}{-96\pi^2} \int_0^1 dx \int_0^{1-x} dy \int_{\mu^2}^{\Lambda^2} dt \frac{(1-x-y)2m^2(x+y)(1-x-y)}{[t(1-x-y) + m^2(x-y)^2]^2}$$

$$ia_e = -i \frac{\alpha m^2}{\pi} \int_0^1 dx \int_0^{1-x} dy \int_{\mu^2}^{\Lambda^2} dt \frac{(x+y)(1-x-y)^2}{[t(1-x-y) + m^2(x-y)^2]^2}$$

where $\alpha \equiv e^2/4\pi$. Performing the t -integration

$$\begin{aligned} ia_e &= -i \frac{\alpha m^2}{\pi} \int_0^1 dx \int_0^{1-x} dy \frac{(x+y)(1-x-y)^2}{(1-x-y)} \\ &\quad \times \left\{ \frac{1}{\Lambda^2(1-x-y) + m^2(x-y)^2} - \frac{1}{\mu^2(1-x-y) + m^2(x-y)^2} \right\} \end{aligned}$$

and taking $\Lambda \rightarrow \infty$

$$ia_e = i \frac{\alpha}{\pi} \int_0^1 dx \int_0^{1-x} dy \frac{(x+y)(1-x-y)}{(\mu^2/m^2)(1-x-y) + (x-y)^2}.$$

Let $x = z+y$

$$a_e = \frac{\alpha}{\pi} \int_0^1 dz \int_z^1 dx \frac{x(1-x)}{(\mu^2/m^2)(1-x) + x^2}$$

and we see that this remains finite where $\mu^2 = \text{mass of photon} \rightarrow 0$. Doing this

$$\begin{aligned} a_e &= \frac{\alpha}{\pi} \int_0^1 dz \int_z^1 dx \frac{1-x}{x} \\ &= \frac{\alpha}{\pi} \int_0^1 dx [\ell n x - x] \Big|_x^1 \\ &= \frac{\alpha}{\pi} \int_0^1 dx [-\ell n x - 1 + x] \\ &= \frac{\alpha}{\pi} \left[-x \ell n x - x + \frac{x^2}{2} \right] \Big|_0^1 \\ &= \frac{\alpha}{2\pi} \end{aligned}$$

Thus we have found

$$\kappa_1 \approx \frac{\alpha}{2\pi} \quad (\text{C.17})$$

which is the Schwinger result.

FIGURE CAPTIONS

1. Vertex correction

$$-ie\gamma_{\alpha\beta}^{\mu} \rightarrow -ie\Gamma_{\alpha\beta}^{\mu}(p,p') \approx -ie\{\gamma^{\mu} + A^{\mu}(p,p')\}_{\alpha\beta}$$

2. Lowest order contribution to the vertex correction

Fig. C.1

Fig. C.2

APPENDIX D

Calculation of Anomalous Magnetic Moment II:

Two-Component Spinors.

We repeat the calculation performed in Appendix C but employ two-component Weyl spinors in order to illustrate their use. We will specifically examine the vertex correction to

$$e_{L(R)}^-(p, s) + \gamma(q = p' - p) \rightarrow e_{L(R)}^-(p', s'). \quad (D.1)$$

Certain comments should be made. The processes $e_L^- + \gamma \rightarrow e_R^-$ and $e_R^- + \gamma \rightarrow e_L^-$ are expected to provide equivalently to the anomalous moment a . The processes $e_L \rightarrow e_L$ and $e_R \rightarrow e_R$ contribute solely to the vertex (wavefunction) renormalization. The process $\gamma \rightarrow e_{L(R)}^+ e_{L(R)}^-$ is equivalent to $\gamma + e_{L(R)}^- \rightarrow e_{R(L)}^-$. Consider $e_L^-(p, s) + \gamma(q) \rightarrow e_R^-(p', s')$ as illustrated in Fig. D.1.

We would naively consider these to be the sole contributions. To see that this does not lead to the correct result we proceed to evaluate them. Remembering from Eqn. (C.1) that the vertex is given by $M^\mu = -ie\psi_+ \Gamma^\mu \psi_-$ we find:

Diagram I:

$$\begin{aligned} & \psi_+^\dagger(p', s') \Gamma_1^\mu(p, p') \gamma_5 \psi_-(p, s) \\ &= ie^2 \int \frac{d^4 k}{(2\pi)^4} m \Delta(p-k) \Delta(p'-k) \frac{1}{k^2} \\ & \quad \times \psi_+^\dagger(\sigma^\mu)_{\gamma\gamma'} (\bar{\sigma}^\lambda)^{\beta\beta'} (p'-k)_\lambda (\bar{\sigma}^\mu)_{\beta\beta'} \delta^{\beta\beta'} (\bar{\sigma}^\nu)^{\alpha\alpha'} \psi_{-\alpha\beta\nu} \\ &= ie^2 \int \frac{d^4 k}{(2\pi)^4} \frac{N_1^\mu}{D} \end{aligned} \quad (D.2)$$

where we have introduced the notation

$$\Delta(p) \equiv i/p^2 - m^2 + i\epsilon$$

we find that

$$\begin{aligned} N_1^\mu &= m\psi_+^\dagger(\sigma_\nu)_{\gamma\gamma}(\sigma^\lambda)^{\dagger\beta}(\sigma^\mu)_{\beta\delta}(\sigma^\nu)^{\delta\alpha}\psi_{-\alpha}(p' - k)_\lambda \\ &= m\psi_+^\dagger[4g^{\mu\lambda}(p' - k)_\lambda\delta_{\gamma\gamma}^{\alpha\alpha}]\psi_{-\alpha} \quad \text{from (A.32)} \\ &= 4m(p' - k)^\mu\psi_+^\dagger\psi_{-\alpha} \end{aligned} \quad (\text{D.3})$$

Diagram II:

$$\begin{aligned} &\psi_+^\dagger(p', s')\Gamma_2^\mu(p, p')\gamma^\alpha\psi_{-\alpha}(p, s) \\ &= ie^3 \int \frac{d^4 k}{(2\pi)^4} m\Delta(p - k)\Delta(p' - k) \frac{1}{k^2} \\ &\quad \times \psi_+^\dagger(\sigma^\nu)_{\gamma\gamma}\delta_{\beta}^{\dagger}(\sigma^\mu)^{\beta\delta}(\sigma^\lambda)_{\delta\alpha}(p - k)_\lambda(\sigma^\nu)^{\delta\alpha}\psi_{-\alpha}g_{\mu\nu} \\ &= ie^3 \int \frac{d^4 k}{(2\pi)^4} \frac{N_2^\mu}{D} \end{aligned} \quad (\text{D.4})$$

$$\begin{aligned} N_2^\mu &= m\psi_+^\dagger(\sigma_\nu)_{\gamma\beta}(\sigma^\mu)^{\beta\delta}(\sigma^\lambda)_{\delta\alpha}(\sigma^\nu)^{\delta\alpha}\psi_{-\alpha}(p - k)_\lambda \\ &= m\psi_+^\dagger[4g^{\mu\lambda}(p - k)_\lambda\delta_{\gamma\gamma}^{\alpha\alpha}]\psi_{-\alpha} \\ &= 4m(p - k)^\mu\psi_+^\dagger\psi_{-\alpha} \end{aligned} \quad (\text{D.5})$$

and, as in Eqn. (C.7),

$$D = k^2(k^2 - 2p \cdot k)(k^2 - 2p' \cdot k) . \quad (\text{D.6})$$

The total is given by

$$\psi_+^\dagger\Gamma^\mu(p, p')\gamma^\alpha\psi_{-\alpha} = ie^2 \int \frac{d^4 k}{(2\pi)^4} \frac{N^\mu}{D} \quad \text{with} \quad (\text{D.7})$$

$$N^\mu = N_1^\mu + N_2^\mu = 4m\psi_+^\dagger(p', s')[p' - k)^\mu + (p - k)^\mu]\psi_{-\alpha}(p, s).$$

As in Appendix C (C.4) we could use a Pauli-Villars regulator ($\Lambda \rightarrow \infty$) and let the photon acquire a small mass ($\mu \rightarrow 0$) so that

$$\frac{1}{k^2} \rightarrow - \int_{\mu^2}^{\Lambda^2} dt \frac{dt}{(k^2 - t^2)^2} \quad (\text{D.8})$$

and

$$\begin{aligned} D &\rightarrow (k^2 - t^2)^2(k^2 - 2p' \cdot k)(k^2 - 2p \cdot k) \\ \int \frac{d^4 k}{(2\pi)^4} &\rightarrow - \int \frac{d^4 k}{(2\pi)^4} \int_{\mu^2}^{\Lambda^2} dt . \end{aligned} \quad (\text{D.9})$$

Then using (C.8)

$$\int \frac{d^4 k}{(2\pi)^4} \frac{1; k_\mu; k_\mu k^\nu}{(k^2 - 2k \cdot p + s - i\epsilon)^4} = \frac{i}{96\pi^2(s - p^2)^2} [1; p_\mu; p_\mu p^\nu + \frac{1}{2}(s - p^2)\delta_\mu^\nu] \quad (\text{D.10})$$

from (H.7) with $s = -t(1 - x - y)$ and $p = px + p'y$. We have seen in Appendix C that this was unnecessary since, as we had argued from general principles, the helicity-changing part is finite. Thus we use instead

$$\frac{1}{abc} = 2 \int_0^1 dx \int_0^{1-x} dy \frac{1}{[a + (b-a)x + (c-a)y]^3} \quad (\text{D.11})$$

and

$$\int \frac{d^4 k}{(2\pi)^4} \frac{1; k_\mu}{(k^2 - 2k \cdot p + s - i\epsilon)^3} = \frac{i[1; p_\mu]}{32\pi^2(s - p^2)} \quad (\text{D.12})$$

from (H.6). Using (D.11) and (D.8) yields

$$\frac{1}{D} = 2 \int_0^1 dx \int_0^{1-x} dy \frac{1}{[k^2 - 2p \cdot kx - 2p' \cdot ky]^2}. \quad (D.13)$$

Note, however, that we can equally well interchange "b" and "c" in (D.11) (as outlined in (H.20)) in which case (D.13) would have become

$$\frac{1}{D} = 2 \int_0^1 dx \int_0^{1-x} dy \frac{1}{[k^2 - 2p' \cdot kx - 2p \cdot ky]^2}. \quad (D.14)$$

We see that the net result is to interchange x and y in the integrand. Using the notation of (D.12) we see, in the case of (D.13), that $p \rightarrow px + p'y$ while for (D.14) $p \rightarrow py + p'x$. In both cases $a = 0$.

Using (D.12) and (D.7) [with (D.13) being used for $1/D$] yields

$$\psi_+ \Gamma^\mu(p, p') \psi_- = 2ie^2 \int_0^1 dx \int_0^{1-x} dy \frac{-i}{32\pi^3} \frac{M^\mu}{(px + p'y)^2} \quad (D.15)$$

and, again, from (H.17 b) $(px + p'y)^2 = m^2(x+y)^2 - q^2xy$ where $q = p' - p$.

$$M^\mu = 4m\psi_+(p') [p^\mu(1-2x) + p'^\mu(1-2y)]\psi_-(p) \quad (D.16)$$

thus

$$\psi_+ \Gamma^\mu(p, p') \psi_- = \frac{e^2}{16\pi^2} \int_0^1 dx \int_0^{1-x} dy \frac{4m\psi_+[p^\mu(1-2x) + p'^\mu(1-2y)]\psi_-}{m^2(x+y)^2 - q^2xy}. \quad (D.17)$$

Now if we had used (D.14) instead of (D.13) (i.e. let "b" $\leftrightarrow$ "c") then we must

interchange x and y in the integrand to get

$$\psi_+ \Gamma^\mu(p, p') \psi_- \rightarrow \frac{e^2}{16\pi^2} \int_0^1 dx \int_0^{1-x} dy \frac{4m\psi_+[p^\mu(1-2y) + p'^\mu(1-2x)]\psi_-}{m^2(x+y)^2 - q^2xy}. \quad (D.18)$$

Note that the denominator is the same. We may now average these two expressions to obtain

$$\psi_+ \Gamma^\mu(p, p') \psi_- = \frac{m e^2}{4\pi^2} \int_0^1 dx \int_0^{1-x} dy \frac{\psi_+(p + p')^\mu(1-x-y)\psi_-}{m^2(x+y)^2 - q^2xy}. \quad (D.19)$$

This procedure of switching "b" and "c" in expressions like (D.11) and averaging the results after integrating out the $\int d^4k$ we shall refer to (see (H.20)) as the "b $\leftrightarrow$ c trick". It is quite general and can be also used in the regulated version presented in Appendix C. The result is generally a simple function of $x+y$ and $x-y$ (subtleties arise off-shell).

The expression in (D.19) represents one half of the anomaly (the $e_L \rightarrow e_R$ part). We may similarly compute the other ($e_R \rightarrow e_L$) part. We find that it is the same as (D.19) except that

$$\psi_+^\mu(p') \psi_- \rightarrow \bar{\psi}_-^\mu(p') \bar{\psi}_+^\mu(p). \quad (D.20)$$

This sum can be represented by

$$\frac{m e^2}{4\pi^2} \int_0^1 dx \int_0^{1-x} dy \frac{M_{sum}^\mu}{m^2(x+y)^2 - q^2xy} \quad (D.21)$$

where

$$M_{sum}^\mu = (1-x-y)[\psi_+(p')\psi_-(p) + \bar{\psi}_-(p')\bar{\psi}_+(p)](p+p')^\mu. \quad (D.22)$$

Now we may employ the generalized Gordon identity of Appendix H (Eqn. H.43)

which states

$$\begin{aligned} & (\psi_+(p')\psi_-(p) + \bar{\psi}_-(p')\bar{\psi}_+(p))\{(p+p')^\mu = \\ & -i[\psi_+(p')\sigma^{\mu\nu}q_\nu\psi_-(p) + \bar{\psi}_-(p')\bar{\sigma}^{\mu\nu}q_\nu\bar{\psi}_+(p)] \quad (D.23) \\ & + 2m[\bar{\psi}_-(p')\bar{\sigma}^{\mu\nu}\psi_+(p) + \psi_+(p')\sigma^{\mu\nu}\psi_-(p)] \end{aligned}$$

Using this in (D.22) we may ignore the final term as it contributes solely to the vertex renormalization. Thus (D.22) may be written as

$$\Gamma^\mu(p, p') = \frac{-ime^2}{4\pi^2} [\psi_+(p')\sigma^{\mu\nu}q_\nu\psi_-(p) + \bar{\psi}_-(p')\bar{\sigma}^{\mu\nu}q_\nu\bar{\psi}_+(p)] I \quad (D.24)$$

where

$$I(q^2) = \int_0^1 dx \int_0^{1-x} dy \frac{(1-x-y)}{m^2(x+y)^2 - q^2xy}.$$

From (A.8) we see that this can be written in four-component Dirac form as

$$\bar{\psi}(p)\Gamma^\mu(p, p')\psi(p) = \frac{-im\alpha I(q^2)}{\pi} \bar{\psi}(p')\sigma^{\mu\nu}q_\nu\psi(p), \quad (D.25)$$

where we have used $\alpha = e^2/4\pi$, or simply

$$\Gamma^\mu(p, p') = \frac{-im\alpha I(q^2)}{\pi} \sigma^{\mu\nu}q_\nu. \quad (D.26)$$

From (C.15) this implies

$$F_2(q^2) = -\frac{2m^2\alpha I(q^2)}{\pi} \quad (D.27)$$

and from (C.16)

$$a_e = -\frac{2m^2\alpha I(0)}{\pi}. \quad (D.28)$$

From (D.24)

$$I(0) = \int_0^1 dx \int_0^{1-x} dy \frac{(1-x-y)}{m^2(x+y)^2}$$

using (H.10 c)

$$\int_0^1 dx \int_0^{1-x} dy = \frac{1}{2} \int_0^1 dx \int_{-x}^x d\tilde{z} \quad (D.29)$$

therefore

$$I(0) = \frac{1}{m^2} \int_0^1 dx \int_0^x d\tilde{z} \frac{1-x}{\tilde{z}^2} = \frac{1}{m^2} \int_0^1 dz \frac{1-z}{z}. \quad (D.30)$$

Hence

$$a_e = -\frac{2\alpha}{\pi} \int_0^1 dz \left(1 - \frac{1}{z}\right) \quad (D.31)$$

which is divergent!

The reason that the naive contributions do not reproduce the Schwinger result is that the F_1 and F_2 parts of Γ^μ are intimately tied together for massive spinors. This is manifestly evident in the Gordon decomposition (Appendix H). In (D.23) we say that we generated a " γ^μ " type term which is indicative of a helicity-preserving vertex. This is in spite of the fact that our diagrammatic representations in Fig. D.1 contained explicit helicity flips (indicated by $\oplus$). Similarly " $\sigma^{\mu\nu}$ " type terms may be generated from the diagrams connecting e_L to e_L and e_R to e_R . Carefully following the derivation in Appendix C we see that this is indeed the case. There we start and end with $\bar{\psi}$ and ψ which contain both left and right-handed parts. To take this over to the two-component case we would have to add the four amplitudes for e_L or $R + \gamma \rightarrow e_L$ or R together.

This is somehow distasteful. We would like to be able to start with a specified state, say e_L , and end with e_R without references to additional processes. One can obscure this disagreeable situation by permitting the external legs to be chirality-changing. This is not an option for truncated diagrams but we really are dealing with $\bar{\psi}\Gamma^\mu\psi$, not Γ^μ , since we use the Dirac equation and Gordon identities.

Adopting this approach we have six additional diagrams which should be added to diagrams I and II in Fig. D.1. These are found in Fig. D.2.

Following the same procedure as before but regulating the integral as in Appendix C (see (C.4)) we find

$$M^\mu = ie\psi_+\Gamma^\mu(p,p')\psi_- = -ie^3 \int \frac{d^4k}{(2\pi)^4} \int_0^1 dt \frac{N^\mu}{D} \quad (D.32)$$

with D as in (D.6) and

$$\begin{aligned} N^\mu = \psi_+(p) \left\{ \frac{\sigma \cdot p'}{m} \bar{\sigma}^\nu m \sigma^\mu \bar{\sigma} \cdot (p-k) \sigma_\nu \frac{\bar{\sigma} \cdot p}{m} \right. \\ + \frac{\sigma \cdot p'}{m} \bar{\sigma}^\nu \sigma \cdot (p'-k) m \bar{\sigma}^\mu \sigma_\nu \frac{\bar{\sigma} \cdot p}{m} + \frac{\sigma \cdot p'}{m} \bar{\sigma}^\nu m^2 \sigma^\mu \bar{\sigma}_\nu \\ + \frac{\sigma \cdot p'}{m} \bar{\sigma}^\nu \sigma \cdot (p'-k) \bar{\sigma}^\mu \sigma \cdot (p-k) \bar{\sigma}_\nu + \sigma^\nu \bar{\sigma} \cdot (p'-k) m \sigma^\mu \bar{\sigma}_\nu \\ + \sigma^\nu m \bar{\sigma}^\mu \sigma \cdot (p-k) \bar{\sigma}_\nu \\ \left. + \sigma^\nu \bar{\sigma} \cdot (p'-k) \sigma^\mu \bar{\sigma} \cdot (p-k) \sigma_\nu \frac{\bar{\sigma} \cdot p}{m} + \sigma^\nu m^2 \bar{\sigma}^\mu \sigma_\nu \frac{\bar{\sigma} \cdot p}{m} \right\} \psi_-(p') \quad (D.33) \end{aligned}$$

We make use of the relations of Section (A.3) to obtain

$$\begin{aligned} N^\mu = \psi_+(p) \left\{ \frac{2}{m} \sigma \cdot p' [(\bar{\sigma}^\mu \sigma \cdot (p-k) + \bar{\sigma} \cdot (p-k) \sigma^\mu) \right. \\ + (\sigma^\nu \sigma \cdot (p'-k) + \bar{\sigma} \cdot (p'+k) \sigma^\mu)] \bar{\sigma} \cdot p - 2m[\sigma \cdot p' \bar{\sigma}^\mu + \sigma^\mu \bar{\sigma} \cdot p] \\ - \frac{2}{m} [\sigma \cdot p' \bar{\sigma} \cdot (p-k) \sigma^\mu \bar{\sigma} \cdot (p'-k) + \sigma \cdot (p-k) \bar{\sigma}^\mu \sigma \cdot (p'-k) \bar{\sigma} \cdot p] \\ + 2m[\sigma \cdot (p'-k) \bar{\sigma}^\mu - \sigma^\mu \bar{\sigma} \cdot (p'-k) \\ \left. + \sigma \cdot (p-k) \bar{\sigma}^\mu + \sigma^\mu \bar{\sigma} \cdot (p'-k)] \right\} \psi_-(p') \quad (D.34) \end{aligned}$$

Performing the integrals as in (C.9) (but ignoring the δ -term as it contributes solely to renormalization) and using (C.9)

$$M^\mu = -ie^3 \cdot 6 \cdot \frac{i}{96\pi^2} \int_0^1 dt \int_0^1 dz \int_0^{1-z} dy \frac{(1-z-y)M^\mu}{[s-p^2]^2} \quad (D.35)$$

where

$$s-p^2 = -[t(1-z-y) + (pz + p'y)]^2 \quad (D.36)$$

and M^μ is an abbreviation for the expression

$$\begin{aligned}
M^\mu = \psi_+(p') \left\{ \frac{2}{m} \sigma \cdot p' [\bar{\sigma}^\mu \sigma \cdot (p + p' - 2px - 2p'y)] \right. \\
+ \bar{\sigma} \cdot (p + p' - 2px - 2p'y) \sigma^\mu \bar{\sigma} \cdot p \\
- 2m[\sigma \cdot p' \bar{\sigma}^\mu + \sigma^\mu \bar{\sigma} \cdot p] - \frac{2}{m} [\sigma \cdot p' \bar{\sigma} \cdot p \sigma^\mu \bar{\sigma} \cdot p' + \sigma \cdot p \bar{\sigma}^\mu \sigma \cdot p' \bar{\sigma} \cdot p] \\
+ 2m[\sigma \cdot p' \bar{\sigma} (px + py) \sigma^\mu \bar{\sigma} \cdot p' + \sigma \cdot p' \bar{\sigma} \cdot p \sigma^\mu \bar{\sigma} \cdot k \\
+ \sigma \cdot k \bar{\sigma}^\mu \sigma \cdot p' \bar{\sigma} \cdot p + \sigma \cdot p \bar{\sigma}^\mu \sigma \cdot k \bar{\sigma} \cdot p] \\
- 2m[\sigma \cdot p' \bar{\sigma} \cdot (px + p'y) \sigma^\mu \bar{\sigma} \cdot (px + p'y) + \sigma^\mu \bar{\sigma} \cdot (px + p'y)] \\
+ \sigma \cdot (px + p'y) \bar{\sigma}^\mu \sigma \cdot (px + p'y) \bar{\sigma} \cdot p] \\
\left. + 2m[\sigma \cdot (p + p' - 2px - 2py) \bar{\sigma}^\mu + \sigma^\mu \bar{\sigma} \cdot (p + p' - 2px - 2p'y)] \right\} \psi_-(p)
\end{aligned} \tag{D.37}$$

Now use Eqn. (B.2):

$$\bar{\psi}_+(p) = (\bar{\sigma} \cdot p/m) \psi_-(p)$$

and

$$\bar{\psi}_-(p') = \psi_+(p') (\sigma \cdot p'/m)$$

along with $q = p' - p$ to find

$$\begin{aligned}
M^\mu = \bar{\psi}_-(p') \left\{ 2m[\bar{\sigma}^\mu \sigma \cdot (p + p' - 2px - 2p'y) \right. \\
+ \bar{\sigma} \cdot (p + p' - 2px - 2p'y) \sigma^\mu] \bar{\psi}_+(p) \\
- \bar{\psi}_-(p') [2m^2 \bar{\sigma}^\mu] \psi_-(p) - \psi_+(p') [\sigma^\mu 2m^2] \bar{\psi}_+(p) \\
- 2\{\bar{\psi}_-(p') \bar{\sigma} \cdot p \sigma^\mu [\bar{\sigma} \cdot q \psi_-(p) + m \bar{\psi}_+(p)] \\
+ [m \bar{\psi}_-(p') - \sigma \cdot q \psi(p')] \bar{\sigma}^\mu \sigma \cdot p' \bar{\psi}_+(p')\} \\
+ 2\{\bar{\psi}_-(p') \bar{\sigma} \cdot (px + p'y) \sigma^\mu [\bar{\sigma} \cdot q \psi_-(p) + m \bar{\psi}_+(p)] \\
+ \bar{\psi}_-(p') \bar{\sigma} \cdot (p - px - p'y) \sigma^\mu [\bar{\sigma} \cdot q \psi(p) + m(x + y) \bar{\psi}_+(p)]\} \\
+ \{[m(x + y) \bar{\psi}_-(p) - x \psi_+(p) \sigma \cdot q] \bar{\sigma}^\mu \sigma \cdot (p' - px - p'y) \bar{\psi}_+(p) \\
+ [m \bar{\psi}_-(p) - \psi_+(p) \sigma \cdot q] \bar{\sigma}^\mu \sigma \cdot (px + p'y) \bar{\psi}_+(p)\} \\
+ 2m\{2(1 - x - y) m \bar{\psi}_-(p') \bar{\sigma}^\mu \psi_-(p) - (1 - 2x) \psi_+(p) \sigma \cdot q \bar{\sigma}^\mu \psi_-(p)\} \\
+ 2m \psi_+(p') \sigma^\mu \{2m[1 - x - y] \bar{\psi}_+(p) + \bar{\psi}_- \cdot q(1 - 2y) \psi_-(p)\}
\end{aligned} \tag{D.38}$$

Collecting terms as using Eqn. (B.2) again, i.e. $\psi_-(p) = (\sigma \cdot p/m) \bar{\psi}_+(p)$ and $\psi_+(p') = \bar{\psi}_-(p') (\bar{\sigma} \cdot p'/m)$, we obtain (after doing some algebra):

$$\begin{aligned}
M^\mu &= 2m\bar{\psi}_-(p') \left\{ [-y^2 - xy + x] \sigma^\mu \sigma \cdot q - [-x^2 + xy + y] \bar{\sigma} \cdot q \sigma^\mu \right\} \bar{\psi}_+(p) \\
&+ 2m^2 \bar{\psi}_-(p') \left\{ \bar{\sigma}^\mu [- (1-x-y)^2 + (1-2x-2y)] \right\} \psi_-(p) \\
&+ 2m^2 \psi_+(p') \left\{ \sigma^\mu [- (1-x-y)^2 + (1-2x-2y)] \right\} \bar{\psi}_+(p) \\
&+ 2m\psi_+(p') \left\{ (-xy - y^2 + x) \sigma^\mu \bar{\sigma} \cdot q + (x^2 + xy - y) \sigma \cdot q \bar{\sigma}^\mu \right\} \psi_-(p)
\end{aligned} \quad (D.39)$$

We now once again use the "b ↔ c" trick of Eqn. (D.19). By switching what we call "b" and "c" we, in essence, switch $x \leftrightarrow y$ in the integrand of (C.8). Doing this we obtain (D.39) but with $x \leftrightarrow y$. Averaging (D.39) with this new equation we find that

$$\begin{aligned}
M^\mu &= 2m\bar{\psi}_-(p') \left\{ \frac{1}{2} [(x+y) - (x+y)^2] [\sigma^\mu \sigma \cdot q - \bar{\sigma} \cdot q \sigma^\mu] \right\} \bar{\psi}_+(p) \\
&+ 2m^2 [(1-x-y) - (1-x-y)^2 - (x+y)] \\
&\times \{ \bar{\psi}_-(p') \psi_-(p) + \psi_+(p) \sigma^\mu \bar{\psi}_+(p) \} \\
&+ 2m\psi_+(p') \left\{ \frac{1}{2} [(x+y)^2 + (x+y)] \right\} \{ -\sigma^\mu \bar{\sigma} \cdot q + \sigma \cdot q \bar{\sigma}^\mu \} \psi_-(p) \\
&= m(x+y)(1-x-y) \left\{ \bar{\psi}_-(p') [\sigma^\mu \sigma \cdot q - \bar{\sigma} \cdot q \sigma^\mu] \bar{\psi}_+(p) \right. \\
&+ \left. \psi_+(p') [\sigma^\mu \bar{\sigma} \cdot q - \sigma \cdot q \bar{\sigma}^\mu] \psi_-(p) \right\} \\
&- 2m^2 (x+y)^2 \left\{ \bar{\psi}_-(p') \bar{\sigma}^\mu \psi_-(p) + \psi_+(p') \sigma^\mu \bar{\psi}_+(p) \right\}
\end{aligned} \quad (D.40)$$

and the final term is proportional to the γ^μ term (i.e. vertex renormalization).

Thus considering only the first term and noting that

$$\bar{\psi} \sigma^{\mu\nu} q_\nu \psi \text{ (Dirac)} = \psi_+ q_\nu \sigma^{\mu\nu} \psi_- + \bar{\psi}_- q_\nu \bar{\sigma}^{\mu\nu} \bar{\psi}_+$$

(from (A.9))

$$\begin{aligned}
\bar{\psi} \sigma^{\mu\nu} q_\nu \psi &= \psi_+ \left\{ \sigma^{\mu\nu} + \frac{\sigma \cdot p'}{m} \bar{\sigma}^{\mu\nu} \frac{\bar{\sigma} \cdot p}{m} \right\} q_\nu \psi_- \\
&= \frac{i}{2m} \psi_+ \{ m [\sigma^\mu \bar{\sigma}^\nu - \sigma^\nu \bar{\sigma}^\mu] \\
&+ \frac{1}{m} [\sigma \cdot p' \bar{\sigma}^\mu \sigma^\nu \bar{\sigma} \cdot p - \sigma \cdot p' \bar{\sigma}^\nu \sigma^\mu \bar{\sigma} \cdot p] \} q_\nu \psi_-
\end{aligned} \quad (D.41)$$

$$= \frac{i}{2} \left\{ \psi_+ [\sigma^\mu \bar{\sigma} \cdot q - \sigma \cdot q \bar{\sigma}^\mu] \psi_- + \bar{\psi}_-(p') [\bar{\sigma}^\mu \sigma \cdot q - \bar{\sigma} \cdot q \sigma^\mu] \bar{\psi}_- \right\} \quad (D.42)$$

From (D.42) the first term in (D.40) is

$$= m(x+y)(1-x-y) \frac{1}{(i/2)} \bar{\psi} \sigma^{\mu\nu} q_\nu \psi$$

(using Dirac spinor notation for short). From (D.32)

$$M^\mu = -\bar{\psi} \sigma^{\mu\nu} q_\nu \psi \frac{-ie^2}{(i/2)} \frac{i}{16\pi^3} \int_{\mu^2}^{\Lambda^2} dt \int_0^1 dx \int_0^{1-x} dy \frac{m(x+y)(1-x-y)^2}{[-t(1-x-y) - m^2(x+y)^2]^2}. \quad (D.43)$$

Since

$$\Gamma^\mu(p, p') = \gamma^\mu F_1(q^2) + \frac{i}{2m} \sigma^{\mu\nu} q_\nu F_2(q^2) \quad (D.44)$$

with

$$\Delta a_e \equiv F_2(q^2 \rightarrow 0) \equiv \frac{g-2}{2} \quad (D.45)$$

we have

$$\Delta a_e = \frac{2m}{i} \cdot \frac{-ie^2}{i/2} \cdot \frac{i}{16\pi^2} \int_{\mu^2}^{\Lambda^2} dt \int_0^1 dx \int_0^1 dz \frac{mz(1-z)^2}{[i(1-z) + m^2z^2]} \quad (D.46)$$

and so

$$\begin{aligned} \Delta a_e &= -\frac{m^2 e^2}{4\pi^2} \int_{\mu^2}^{\Lambda^2} dt \int_0^1 dx \int_0^1 dz \frac{z(1-z)^2}{[i(1-z) + m^2z^2]} \\ &= \frac{m^2 \alpha}{\pi} \int_0^1 dx \int_0^1 dz \frac{(1-z)z}{i(1-z) + m^2z^2} \Big|_{\mu^2 \rightarrow 0}^{\Lambda^2 \rightarrow \infty} \\ &= -\frac{m^2 \alpha}{\pi} \int_0^1 dx \int_0^1 dz \frac{z(1-z)}{m^2z^2} \quad (D.47) \\ &= -\frac{\alpha}{\pi} \int_0^1 dx [\ell n x - x + 1] \\ &= \frac{\alpha}{2\pi} . \end{aligned}$$

Thus we obtain the correct result again using this *more involved* method.

FIGURE CAPTIONS

1. Naive contributions to $e_L + \gamma \rightarrow e_R$.

Here $\Delta(p) \equiv i/p^2 - m^2 + i\epsilon$.

2. Additional Diagrams Contributing to $e_L + \gamma \rightarrow e_R$.

Fig. D.1

Fig. D.2

APPENDIX E

Supersymmetric QED Contribution to Anomalous Moment

We wish to consider Δa_e , the additional contribution to the anomalous magnetic moment of the electron, arising from a minimal supersymmetric extension of QED.

E.1 LAGRANGIAN FOR SUPERSYMMETRIC QED (SQED)

We use the following notation¹¹ for the superfields and their components:

Superfield	Ordinary Matter	Superpartners
$\hat{\Psi}$	γ	λ
$\hat{L}_i$	ℓ_{Li}	$\tilde{\ell}_{Li}$
$\hat{R}_i$	$(\ell_i^*)_L = (\ell_{Ri}^-)^*$	$(\tilde{\ell}_{Ri}^-)^*$

(E.1)

Note that when we consider a full electroweak theory that L_i will be promoted to a weak doublet which will include neutrinos and their supersymmetric counterparts. The generation index "i" will now be dropped as we proceed to specialize to a single generation.

The superfield strengths are

$$W^a = -\frac{1}{2} \bar{D} \bar{D} D^a \hat{\Psi}$$

$$\bar{W}_a = -\frac{1}{2} D D \bar{D}_a \hat{\Psi}$$

(E.2)

and

$$\mathcal{L}_{SQED} = \int d^4\theta d^2\bar{\theta} \left\{ \frac{1}{4} (W W \delta(\bar{\theta}^2) + \bar{W} \bar{W} \delta(\theta^2)) \right. \\ \left. + L e^{-\kappa V} L + \bar{R} e^{\kappa V} R + m_e ((R L \delta(\bar{\theta}^2) + L \bar{R} \delta(\theta^2)) \right\}$$

(E.3)

where $\delta(\theta) = \theta$ and $\delta(\bar{\theta}) = \bar{\theta}$ due to the rules of Grassman integration (see Appendix I for details of this construction). Expanding this out in component fields and eliminating the auxiliary fields $F_{\pm}$ and D via their equations of motion yields (for electrons only)

$$\mathcal{L}_{SQED} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - i \lambda \sigma^\mu \partial_\mu \bar{\lambda} + \bar{\epsilon}_R \square \tilde{\epsilon}_R + \tilde{\epsilon}_L^* \square \tilde{\epsilon}_L \\ + i (\partial_\mu \bar{\psi}_+ \sigma^\mu \psi_+ + \partial_\mu \bar{\psi}_- \sigma^\mu \psi_-) \\ + \frac{1}{2} e A_\mu \left[\bar{\psi}_+ \sigma^\mu \psi_+ - \bar{\psi}_- \sigma^\mu \psi_- \right. \\ \left. + i (\bar{\epsilon}_R \partial^\mu \tilde{\epsilon}_R^* - \partial^\mu \bar{\epsilon}_R \tilde{\epsilon}_R^* - \tilde{\epsilon}_L^* \partial^\mu \tilde{\epsilon}_L + \partial^\mu \tilde{\epsilon}_L^* \tilde{\epsilon}_L) \right] \\ - \frac{ie}{\sqrt{2}} \left[\bar{\epsilon}_R^* \bar{\psi}_+ \bar{\lambda} - \bar{\epsilon}_R \psi_+ \lambda - \tilde{\epsilon}_L \bar{\psi}_- \bar{\lambda} + \tilde{\epsilon}_L^* \psi_- \lambda \right] \\ - \frac{1}{4} e^2 A_\mu A^\mu [\bar{\epsilon}_R \tilde{\epsilon}_R^* + \tilde{\epsilon}_L^* \tilde{\epsilon}_L] - \frac{1}{4} e^2 [\bar{\epsilon}_R^* \tilde{\epsilon}_R - \tilde{\epsilon}_L^* \tilde{\epsilon}_L]^2 \\ - m[\psi_+ \psi_- - \bar{\psi}_+ \bar{\psi}_-] - m_{\tilde{\epsilon}_R}^2 \tilde{\epsilon}_R^* \tilde{\epsilon}_R \\ - m_{\tilde{\epsilon}_L}^2 \tilde{\epsilon}_L^* \tilde{\epsilon}_L - m_\lambda \bar{\lambda} \lambda$$

(E.4)

In unbroken SQED $m_{\tilde{\epsilon}_L} = m_{\tilde{\epsilon}_R} = m_e$ and $m_\lambda \equiv m_\gamma = 0$. Here explicit breaking terms have been added to give $\tilde{\epsilon}_L$, $\tilde{\epsilon}_R$ and $\tilde{\gamma}$ arbitrary masses. Presumably such

terms would arise in a realistic model as effective residues of some unspecified higher-energy theory. Note that $\tilde{e}_L$ and $\tilde{e}_R$ are independent fields.

E.2 FEYNMAN RULES

- i) There are four types of vertices in this theory. Since QED is left-right blind ψ_e , $\tilde{e}_L$ and $\tilde{e}_R$ are subject to standard QED and scalar electrodynamics photonic coupling vertices. There are also Yukawa vertices $e_L \tilde{e}_L \tilde{\gamma}$ and $e_R \tilde{e}_R \tilde{\gamma}$ of coupling strength $\sqrt{2}e$.
- ii) ψ_e and $\tilde{\gamma}$ have standard (Weyl) fermionic propagators ($\tilde{\gamma}$ is a Majorana particle) and $\tilde{e}_{L,R}$ have standard scalar propagators.
- iii) In considering Δ_{ee} , there are no diagrams with a chirality change in the photino propagator since that would require $\tilde{e}_L - \tilde{e}_R$ mixing in $\mathcal{L}$. Thus we find, when looking for $e_L + \gamma \rightarrow e_R$, that only the two diagrams of Fig. 1 contribute.

E.3 COMPUTATION OF DIAGRAM 1

$$\begin{aligned} \mathcal{M}^\mu &\equiv -ie\psi_+^R(p', s')\Gamma_1^\mu(p', s)\beta^\alpha\psi_{-\alpha}(p, s) \\ &= (-ie)^2 2(i^3) \int \frac{d^4 k}{(2\pi)^4} \\ &\quad \times \frac{\psi_+^R(p')[(\sigma^\mu)_{\beta\alpha} p'_\alpha/m](\bar{\sigma}^\nu)_{\alpha\beta} k_\nu \psi_{-\alpha}(p)(p + p' - 2k)^\mu}{(k^2 - m_\lambda^2)[(p - k)^2 - m_{\tilde{e}_L}^2][(p' - k)^2 - m_{\tilde{e}_L}^2]} \end{aligned} \quad (\text{E.5})$$

We shall work on shell so that $p^2 = p'^2 = m_e^2 \equiv m^2$.

Define

$$\begin{aligned} \bar{m}_L^2 &\equiv m^2 - m_{\tilde{e}_L}^2 + m_\lambda^2 \\ \bar{m}_R^2 &\equiv m^2 - m_{\tilde{e}_R}^2 + m_\lambda^2 \end{aligned} \quad (\text{E.6})$$

from Eqn. (B.6)

$$\psi_+^R(p', s') \frac{(\sigma^\mu)_{\beta\alpha} p'_\alpha}{m} = \bar{\psi}_{-\alpha}(p', s')$$

then (E.5) becomes

$$\begin{aligned} \psi_+ \Gamma_1^\mu \psi_{-\alpha} &= 2ie^2 \int \frac{d^4 k}{(2\pi)^4} \frac{N^\mu}{D} \\ N^\mu &= \bar{\psi}_{-\alpha}(p')(\bar{\sigma}^\nu)^{\alpha\beta} k_\nu \psi_{-\alpha}(p)(p + p' - 2k)^\mu \\ D &= (k^2 - m_\lambda^2)(k^2 - 2p \cdot k + m^2 - m_{\tilde{e}_L}^2)(k^2 - 2p' \cdot k + m^2 - m_{\tilde{e}_L}^2) \end{aligned} \quad (\text{E.7})$$

We now use Eqn. (D.11) to find

$$\frac{1}{D} = 2 \int_0^1 dx \int_0^{1-x} dy (k^2 - 2k \cdot (px + p'y) + \bar{m}_L^2(x+y) - m_\lambda^2)^{-3}. \quad (\text{E.8})$$

We next use Eqn. (D.12) or (H.6) to find

$$\begin{aligned} \psi_+ \Gamma_1^\mu \psi_{-\alpha} &= 2ie^2 \cdot 2 \int_0^1 dx \int_0^{1-x} dy \\ &\quad \times \frac{i\bar{\psi}_{-\alpha}(\bar{\sigma}^\nu)^{\alpha\beta} [px + p'y]_\nu \psi_{-\alpha}(p + p' - 2[px + p'y])^\mu}{32\pi^2(\bar{m}_L^2[x+y] - m_\lambda^2 - [px + p'y]^2)} \end{aligned}$$

where we have ignored the $k_\nu k_\mu$ term which contributes solely to the helicity-preserving piece of the amplitude (i.e. $\bar{\sigma}^\mu$). Since $(px + p'y)^2 = m^2(x+y)^2 - q^2xy$

as before

$$\psi_+ \Gamma_1^\mu \psi_- = -\frac{e^2}{8\pi^2} \int_0^1 dx \int_0^{1-x} dy \times \frac{\bar{\psi}_{-\Delta}(\sigma^\nu)^{\dot{\alpha}\alpha} [px + p'y]_\nu \psi_{-\alpha} [p(1-2x) + p'(1-2y)]^\mu}{m_L^2(x+y) - m_\lambda^2(x+y)^2 - q^2 xy} \quad (\text{E.9})$$

We now perform the " $b \leftrightarrow c$ " averaging trick as in (D.19). The result being that $x \leftrightarrow y$ in the integrand of (E.8), and therefore (E.9), and then we average the two expressions. Before we do this it is prudent to apply the Dirac equations of (B.2).

$$\begin{aligned} & \bar{\psi}_{-\Delta}(p')(\sigma^\nu)^{\dot{\alpha}\alpha} [px + p'y]_\nu \psi_{-\alpha}(p) \\ &= \bar{\psi}_{-\Delta} \left\{ mx \bar{\psi}_+^\dot{\alpha} + my \bar{\psi}_+^\dot{\alpha} + (\sigma^\nu)^{\dot{\alpha}\alpha} q_\nu \psi_{-\alpha} y \right\} \\ &= m(x+y) \bar{\psi}_{-\Delta} \bar{\psi}_+^\dot{\alpha} + y \bar{\psi}_{-\Delta} (\sigma^\nu)^{\dot{\alpha}\alpha} q_\nu \psi_{-\alpha} \end{aligned} \quad (\text{E.10})$$

when we now perform the $x \leftrightarrow y$ average we find

$$\psi_+ \Gamma_1^\mu \psi_- = -\frac{e^2}{8\pi^2} \int_0^1 dx \int_0^{1-x} dy \frac{\lambda f^\mu}{-m^2(x+y)^2 + m_L^2(x+y) - m_\lambda^2 - q^2 xy} \quad (\text{E.11})$$

$$\begin{aligned} \lambda f^\mu &= m(x+y)(1-x-y)(p+p')^\mu \bar{\psi}_{-\Delta} \bar{\psi}_+^\dot{\alpha} \\ &+ \left[y \{ p(1-2x) + p'(1-2y) \} \right. \\ &\left. + x \{ p(1-2y) + p'(1-2x) \} \right] \bar{\psi}_{-\Delta} (\sigma^\mu)^{\dot{\alpha}\alpha} q_\alpha \psi_{-\alpha} \end{aligned}$$

We are interested in the helicity-changing part of this (the $\bar{\psi}_{-\Delta} \bar{\psi}_+^\dot{\alpha}$ term) as

$q^2 \rightarrow 0$ (on shell). This is

$$\psi_+ \Gamma_{1LR}^\mu (q^2 \rightarrow 0) \psi_- = \frac{e^2}{8\pi^2} \frac{(p+p')^\mu}{m} \bar{\psi}_{-\Delta} \bar{\psi}_+^\dot{\alpha} I_L \quad (\text{E.12})$$

where

$$\begin{aligned} I_L &\equiv \int_0^1 dx \int_0^{1-x} dy \frac{(x+y)(1-x-y)}{-(x+y)^2 + \frac{m_L^2}{m^2}(x+y) - \frac{m_\lambda^2}{m^2}} \\ &= \int_0^1 dx \int_x^1 dz \frac{z(1-z)}{-z^2 + \frac{m_L^2}{m^2}z - \frac{m_\lambda^2}{m^2}} \end{aligned} \quad (\text{E.13})$$

where we have let $z = x+y$.

Now in (E.10) we applied the Dirac operator to the *right*. We could equally well have had it operator to the *left*. Using Eqn. (B.2) again we would have obtained

$$\begin{aligned} & \bar{\psi}_{-\Delta}(p')(\sigma^\nu)^{\dot{\alpha}\alpha} [px + p'y]_\nu \psi_{-\alpha}(p) \\ &= \{ [m\psi_+^\alpha - (\sigma^\nu)^{\dot{\alpha}\alpha} q_\nu \bar{\psi}_{-\alpha}] x + m\psi_+^\alpha y \} \psi_{-\alpha} \\ &= m(x+y) \psi_+^\alpha \psi_{-\alpha} - x \bar{\psi}_{-\Delta} (\sigma^\nu)^{\dot{\alpha}\alpha} q_\nu \psi_{-\alpha} \end{aligned} \quad (\text{E.14})$$

(We use $+m$ since we are considering electrons and not positrons.) Note that in (E.10) we have related the $\epsilon_L \rightarrow \epsilon_R$ term to the corresponding $\epsilon_R \rightarrow \epsilon_L$ terms whereas (E.14) expresses $\epsilon_L \rightarrow \epsilon_R$ in terms of $\epsilon_L \rightarrow \epsilon_R$. Using (E.4) instead of (E.10) would change (E.12) to

$$\psi_+ \Gamma_{1LR}^\mu (q^2 \rightarrow 0) \psi_- = \frac{e^2}{8\pi^2} \frac{(p+p')^\mu}{m} \psi_+^\alpha \psi_{-\alpha} I_L. \quad (\text{E.15})$$

Averaging (E.12) and (E.15) and using $\alpha = e^2/4\pi$

$$\psi_+ \Gamma_{1LR}^\mu \psi_- = \frac{\alpha}{4\pi} \frac{(p+p')^\mu}{m} \{ \psi_+^\alpha \psi_{-\alpha} + \bar{\psi}_{-\Delta} \bar{\psi}_+^\dot{\alpha} \} I_L. \quad (\text{E.16})$$

From Appendix H or Eqn. (D.23) we employ the Gordon decomposition producing

$$\psi_+ \Gamma_{LR}^\mu \psi_- = \frac{\alpha}{4\pi m} I_L \left\{ -i[\psi_+ \sigma^{\mu\nu} q_\nu \psi_- + \bar{\psi}_- \bar{\sigma}^{\mu\nu} q_\nu \psi_+] + 2m[\bar{\psi}_- \bar{\sigma}^{\mu\nu} \psi_+ + \psi_+ \sigma^{\mu\nu} \psi_-] \right\} \quad (\text{E.17})$$

We are only interested in the $\sigma^{\mu\nu}$ and $\bar{\sigma}^{\mu\nu}$ terms. Using (A.8) these terms can be combined to yield the Dirac spinor formulation:

$$\bar{\psi} \Gamma_{LR}^\mu \psi = \frac{-i\alpha}{4\pi m} I_L \bar{\psi} \sigma^{\mu\nu} q_\nu \psi. \quad (\text{E.18})$$

From (C.15) and (C.16)

$$\Delta a_e^{(L)} = \frac{\alpha}{2\pi} I_L \quad (\text{E.19})$$

which is the $\tilde{z}_L$ contribution to a_e .

E.4 COMPUTATION OF DIAGRAM II AND TOTAL CONTRIBUTION

We ingeminate the steps of each in Section E.3 beginning with

$$\begin{aligned} -ie\psi_+ \Gamma_5^\mu(p, p') \psi_- &= (-ie)^2 2(i^3) \int \frac{d^4 k}{(2\pi)^4} \\ &\times \frac{\psi_+^\alpha (\sigma^\nu)_{\alpha\beta} k_\nu [(\bar{\sigma}^\mu)^{\delta\beta} p_\mu / m] \psi_-^\beta (p + p' - 2k)^\mu}{[k^2 - m_\lambda^2] [(p-k)^2 - m_{e_L}^2] [(p'-k)^2 - m_{e_R}^2]} \end{aligned} \quad (\text{E.20})$$

and conclude with

$$\Delta a_e^{(R)} = \frac{\alpha}{2\pi} I_R \quad (\text{E.21})$$

where

$$I_R = \int_0^1 dx \int_z^1 dz \frac{z(1-z)}{-x^2 + \frac{m_\lambda^2}{m^2} x - \frac{m^2}{m^2}} \quad (\text{E.22})$$

and $\bar{m}_R$ is given by (E.6). The total contribution to Δa_e , the incremental super-

symmetric contribution to the anomalous moment of the electron a_e , is

$$\begin{aligned} \Delta a_e &= \Delta a_e^{(L)} + \Delta a_e^{(R)} \\ &= \frac{\alpha}{2\pi} (I_L + I_R) \end{aligned} \quad (\text{E.23})$$

$$I_{L,R} = \int_0^1 dx \int_z^1 dz \frac{z(1-z)}{-x^2 + \frac{m_\lambda^2 + m_\lambda^2 - m_{e_L}^2}{m^2} x - \frac{m^2}{m^2}}$$

E.5 SPECIFIC CASES

a) $m_\lambda^2, m_\lambda^2 < m^2$. This case is excluded experimentally. In this limit

$$\begin{aligned} I_{L,R} &= -\frac{1}{2} \\ \Delta a_e &= -\frac{\alpha}{2\pi}. \end{aligned}$$

This is of the same magnitude as the QED contribution but of the opposite sign.

b) $m_\lambda^2 \approx m_\lambda^2 > m$. Rather unlikely in most models but not excluded.

$$\begin{aligned} I_{L,R} &\approx \int_0^1 dx \int_z^1 dz \frac{m^2}{m_\lambda^2} z(z-1) = -\frac{1}{12} \frac{m^2}{m_\lambda^2} \\ \Delta a_e &= -\frac{\alpha}{24\pi} \frac{m^2}{m_\lambda^2} \end{aligned}$$

c) $m_\lambda^2 > m_\lambda^2 > m^2$. The standardly presumed scenario.

$$\begin{aligned} I_{L,R} &= \int_0^1 dx \int_z^1 dz \frac{m^2}{m_{e_L}^2} (1-z) = -\frac{1}{6} \frac{m^2}{m_{e_L}^2} \\ \Delta a_e &= -\frac{\alpha}{12} \left\{ \frac{m^2}{m_{e_L}^2} + \frac{m^2}{m_{e_R}^2} \right\} \equiv \Delta a_e^{(m_\lambda=0)} \end{aligned} \quad (\text{E.24})$$

This is precisely the 1974 result of Fayet.³

d) $m_{\tilde{L}_L} = m_{\tilde{L}_R} = m$; $m_\lambda = 0$. Case of exact supersymmetry

$$I_{L,R} = \int_0^1 dx \int_1^{\frac{1}{x}} dz \frac{x(1-x)}{z^2} = -\frac{1}{2} \quad (\text{E.25})$$

$$\Delta a_e^{(SUSY)} = -\frac{\alpha}{2\pi} = -a_e^{(QED)}.$$

In the exact supersymmetric limit we expect that there will be no anomalous magnetic moment to any order. Therefore we should have anticipated that $\Delta a_e^{(SUSY)}$ would precisely cancel $a_e^{(QED)}$.⁴

e) $m_\lambda = 0$

$$I_{L,R} = \int_0^1 dx \int_1^{\frac{1}{x}} dz \frac{1-x}{\rho - (1-x)} \quad \rho \equiv \frac{m_{\tilde{L},R}^2}{m^2} \\ = \rho(1-\rho)[\ln \rho - \ln(\rho-1)] - \rho + \frac{1}{2}$$

f) $m_\lambda, m_{\tilde{L}} \gg m$. This is the most interesting case.

Defining

$$\mathcal{R}_{L,R} = \frac{m_\lambda^2}{m_{\tilde{L},R}^2} = \frac{m_\lambda^2}{m_\lambda^2 - m_{\tilde{L},R}^2 + m^2} \quad (\text{E.26})$$

$$I_{L,R} = \frac{m^2}{m_{\tilde{L},R}^2} \int_0^1 dx \int_1^{\frac{1}{x}} dz \frac{x(1-x)}{z - \mathcal{R}_{L,R}}$$

noting that

$$\int_0^1 dx \int_1^{\frac{1}{x}} dz = \int_0^1 dx \int_0^{\frac{1}{x}} d\tilde{z}$$

where $\tilde{z} = 2x - z$

$$I_{L,R} = \frac{m^2}{m_{\tilde{L},R}^2} \int_0^1 dx \frac{x^2 - x^3}{x - \mathcal{R}} \\ I = \frac{m^2}{m_{\tilde{L},R}^2} \left[\frac{1}{6} + \frac{1}{2} \mathcal{R} - \mathcal{R}^2 + \mathcal{R}^3(1-\mathcal{R}) \ln \left(\frac{\mathcal{R}-1}{\mathcal{R}} \right) \right]. \quad (\text{E.27})$$

Now we note that

$$\frac{m^2}{m_{\tilde{L},R}^2} = -\frac{m^2}{m_{\tilde{L}}^2} \left(1 - \left[1 + \frac{m^2}{m_\lambda^2} \right] \mathcal{R} \right) \approx -\frac{m^2}{m_{\tilde{L}}^2} (1 - \mathcal{R}) \quad (\text{E.28})$$

if $m \ll m_\lambda$ so

$$I = -\frac{m^2}{m_{\tilde{L}}^2} [1 - \mathcal{R}] h(\mathcal{R}) \quad (\text{E.29})$$

where

$$h(\mathcal{R}) = \frac{1}{6} + \frac{1}{2} \mathcal{R} - \mathcal{R}^2 + \mathcal{R}^3(1-\mathcal{R}) \ln \left(\frac{\mathcal{R}-1}{\mathcal{R}} \right).$$

The "1/6" term is what we found in part (c) when $m_\lambda \rightarrow 0$. This is hardly shocking since as $m_\lambda \rightarrow 0$ we have $\mathcal{R} \rightarrow 0$ and $(1-\mathcal{R})h(\mathcal{R}) \rightarrow 1/6$. Factoring this term out we can write

$$I_{L,R} = -\frac{m^2}{6m_{\tilde{L},R}^2} [1 + f(\mathcal{R}_{L,R})] \quad (\text{E.30})$$

$$f(\mathcal{R}) = 2\mathcal{R} - 9\mathcal{R}^2 + 6\mathcal{R}^3 + 6\mathcal{R}^2(1-\mathcal{R})^2 \ln \left(\frac{\mathcal{R}-1}{\mathcal{R}} \right)$$

and from (E.23) and (E.24)

$$\Delta a_e = \Delta a_e^{(m_\lambda=0)} \left[1 + \frac{m_{\tilde{L}}^2}{m_{\tilde{L}}^2 + m_{\tilde{L},R}^2} \left(\frac{f(\mathcal{R}_L)}{m_{\tilde{L}}^2} + \frac{f(\mathcal{R}_R)}{m_{\tilde{L},R}^2} \right) \right] \quad (\text{E.31})$$

where

$$\mathcal{R}_{L,R} \pm \frac{m_\lambda^2}{m_\lambda^2 - m_{\tilde{e}_{L,R}}^2}$$

since $m_\lambda \gg m$.

Notes:

(i) when $m_{\tilde{e}} \gg m_\lambda$ $\mathcal{R} \rightarrow 0$ and $f(\mathcal{R}) \rightarrow 0$. Then $\Delta a_e \rightarrow \Delta a_e^{(m_\lambda=0)}$ to lowest order.

(ii) The first order correction in the above [to $O(m_\lambda^2/m_\lambda^2)$] comes from noting that $f(\mathcal{R}) = 2\mathcal{R}$ as $\mathcal{R} \rightarrow 0$ where $\mathcal{R} \pm m_\lambda^2/m_\lambda^2$. Thus (E.31) becomes

$$\begin{aligned} \Delta a_e &\rightarrow \Delta a_e^{(m_\lambda=0)} \left[1 + \frac{1}{m_{\tilde{e}_L}^2 + m_{\tilde{e}_R}^2} \left\{ \frac{2m_{\tilde{e}_R}^2 m_\lambda^2}{-m_{\tilde{e}_L}^2} + \frac{2m_{\tilde{e}_L}^2 m_\lambda^2}{m_{\tilde{e}_R}^2} \right\} \right] \\ &\pm \Delta a_e^{(m_\lambda=0)} \left[1 - 2 \frac{m_\lambda^2}{m_{\tilde{e}_L}^2 + m_{\tilde{e}_R}^2} \left(\frac{m_{\tilde{e}_L}^2}{m_{\tilde{e}_R}^2} + \frac{m_{\tilde{e}_R}^2}{m_{\tilde{e}_L}^2} \right) \right] \end{aligned} \quad (\text{E.32})$$

(iii) When $m_\lambda \gg m_{\tilde{e}} \gg m$ $\mathcal{R} \rightarrow 1 + m_\lambda^2/m_\lambda^2$ and $f(\mathcal{R}) \rightarrow -1$. Using (E.29) we see that $I \rightarrow [1 - \mathcal{R}] [-1/3] = -\frac{1}{3} \frac{m_\lambda^2}{m_\lambda^2}$ just as in case (g) which follows.

g) $m_\lambda \gg m_{\tilde{e}} \gg m$. This case is also unlikely in most models but is not excluded.

$$\begin{aligned} I_{L,R} &= \int_0^1 dz \int_0^{\pi} d\tilde{x} \frac{m^2 z(1-\tilde{x})}{-m^2 z^2 + (m_\lambda^2 - m_{\tilde{e}}^2 + m^2) - m_\lambda^2} \\ &= \frac{m^2}{m_\lambda^2} \int_0^1 dz \int_0^{\pi} d\tilde{x} \frac{z(1-\tilde{x})}{z-1} = -\frac{1}{3} \frac{m^2}{m_\lambda^2} \\ \Delta a_e &= -\frac{\alpha}{3} \frac{m^2}{m_\lambda^2}. \end{aligned}$$

FIGURE CAPTIONS

1. Contributions to Δa_e when $\tilde{e}_L$ and $\tilde{e}_R$ are unmixed.

Here $\Delta(p) \equiv i/p^2 - m^2 + i\epsilon$.

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Fig. E.1

APPENDIX F

Contribution to Anomalous Moment from Smuon Mixing

F.1 LAGRANGIAN

We permit the two smuons $\tilde{\mu}_L$ and $\tilde{\mu}_R$ to mix resulting in the mass eigenstates

$$\tilde{\mu}_1 = \cos \phi \tilde{\mu}_L + \sin \phi \tilde{\mu}_R \quad (\text{F.1})$$

$$\tilde{\mu}_2 = -\sin \phi \tilde{\mu}_L + \cos \phi \tilde{\mu}_R$$

or equivalently

$$\tilde{\mu}_L = \cos \phi \tilde{\mu}_1 - \sin \phi \tilde{\mu}_2 \quad (\text{F.2})$$

$$\tilde{\mu}_R = \sin \phi \tilde{\mu}_1 + \cos \phi \tilde{\mu}_2 .$$

We will confine ourselves to the muon here while recognizing that the same analysis goes through for the electron and tau. It is the muon which provides the most stringent bounds from experiment. We assume that $\lambda = (\tilde{\gamma})$ is purely Majorana, although it may be massive (R -parity is always to be considered an inviolable symmetry).

The terms in the Lagrangian of Appendix E (E.4) are altered thusly

$$\begin{aligned} \mathcal{L}_{\gamma\lambda\mu\mu} \rightarrow & -\frac{ie}{\sqrt{2}} \left[\sin \phi \tilde{\mu}_1^* \bar{\psi}_+ \bar{\lambda} + \cos \phi \tilde{\mu}_2^* \bar{\psi}_+ \bar{\lambda} \right. \\ & - \sin \phi \tilde{\mu}_1 \psi_+ \lambda - \cos \phi \tilde{\mu}_2 \psi_+ \lambda \\ & - \cos \phi \tilde{\mu}_1 \bar{\psi}_- \bar{\lambda} + \sin \phi \tilde{\mu}_2 \bar{\psi}_- \bar{\lambda} \\ & \left. + \cos \phi \tilde{\mu}_1^* \psi_- \lambda - \sin \phi \tilde{\mu}_2^* \psi_- \lambda \right] . \end{aligned}$$

Where ψ represents the muon field.

$$\mathcal{L}_{\text{MASS}} \rightarrow -m_{\tilde{\mu}_1}^2 \tilde{\mu}_1^* \tilde{\mu}_1 - m_{\tilde{\mu}_2}^2 \tilde{\mu}_2^* \tilde{\mu}_2$$

$$\begin{aligned} \mathcal{L}_{\text{Scalar-Photon}} \rightarrow & \frac{i}{2} e A_\nu \left[\sin^2 \phi (\tilde{\mu}_1 \partial^\nu \tilde{\mu}_1^* - \tilde{\mu}_1^* \partial^\nu \tilde{\mu}_1) + \cos^2 \phi (\tilde{\mu}_2 \partial^\nu \tilde{\mu}_2^* - \tilde{\mu}_2^* \partial^\nu \tilde{\mu}_2) \right. \\ & + \sin \phi \cos \phi (\tilde{\mu}_1 \partial^\nu \tilde{\mu}_2^* + \tilde{\mu}_2 \partial^\nu \tilde{\mu}_1^* - \tilde{\mu}_2^* \partial^\nu \tilde{\mu}_1 - \tilde{\mu}_1^* \partial^\nu \tilde{\mu}_2) \\ & + \sin^2 \phi (\tilde{\mu}_1 \partial^\nu \tilde{\mu}_1^* - \tilde{\mu}_1^* \partial^\nu \tilde{\mu}_1) + \cos^2 \phi (\tilde{\mu}_2 \partial^\nu \tilde{\mu}_2^* - \tilde{\mu}_2^* \partial^\nu \tilde{\mu}_2) \\ & \left. - \sin \phi \cos \phi (\tilde{\mu}_1 \partial^\nu \tilde{\mu}_2^* + \tilde{\mu}_2 \partial^\nu \tilde{\mu}_1^* - \tilde{\mu}_2^* \partial^\nu \tilde{\mu}_1 - \tilde{\mu}_1^* \partial^\nu \tilde{\mu}_2) \right] \\ & = \frac{i}{2} e A_\nu [\tilde{\mu}_1 \partial^\nu \tilde{\mu}_1^* - \tilde{\mu}_1^* \partial^\nu \tilde{\mu}_1 + \tilde{\mu}_2 \partial^\nu \tilde{\mu}_2^* - \tilde{\mu}_2^* \partial^\nu \tilde{\mu}_2] \end{aligned}$$

F.2 FEYNMAN RULES AND DIAGRAMS I-IV

The Feynman Rules are similar to those of Section E.2 except for the changes illustrated in Fig. F.1. Note that there is no $\tilde{\mu}_1 \tilde{\mu}_2 \gamma$ vertex (Fig. F.1a) but the helicity-flipped photino propagator is now utilized.

In Fig. F.2 the lowest level contributions to $\mu_L + \gamma \rightarrow \mu_R$ are illustrated. Diagrams I-IV are reminiscent of the contributions of Fig. E.1. Indeed, if $m_{\tilde{\mu}_1} = m_{\tilde{\mu}_2}$, it is evident that these would sum to the unmixed result (we could rotate the mixing away all together and make diagrams V-VIII vanish recovering the previous configuration). The new features appear in diagrams V to VIII. Here the helicity flip takes place in the photino line instead of the muon.

We may evaluate the first four diagrams by direct comparison with those of the previous appendix. Comparing I and II of Fig. F.2 with I of Fig. E.1

$$\Delta a_\mu^{(I)} = \frac{\alpha}{2\pi} I_1 \cos^2 \phi \quad \Delta a_\mu^{(II)} = \frac{\alpha}{2\pi} I_2 \sin^2 \phi \quad (\text{F.3})$$

$$I_{1,2} = \int_0^1 dx \int_{\frac{1}{2}}^1 dz \frac{z(1-z)}{-z^2 + \frac{m_{\tilde{\mu}_1}^2}{m_\mu^2} z - \frac{m_{\tilde{\mu}_2}^2}{m_\mu^2}} \quad (\text{F.4})$$

$$m_{1,2}^2 = m_\lambda^2 - m_{\tilde{\mu}_{1,2}}^2 + m_\mu^2 \quad (\text{F.5})$$

and, as before, we note that we can write this using

$$\int_0^1 dz \int_{\frac{1}{2}}^1 dx = \int_0^1 dx \int_0^{\tilde{x}} d\tilde{z} = \int_0^1 dx x \quad (\text{F.6})$$

since there is no x dependence and $\tilde{z} = 2x - x$. Similarly diagrams III and IV give

$$\Delta a_\mu^{(III)} = \frac{\alpha}{2\pi} I_1 \sin^2 \phi \quad \Delta a_\mu^{(IV)} = \frac{\alpha}{2\pi} I_2 \cos^2 \phi \quad (\text{F.7})$$

and so the sum of the first four diagrams gives

$$\Delta a_\mu^{(I-IV)} = -\frac{\alpha}{2\pi} (I_1 + I_2). \quad (\text{F.8})$$

This in itself would place no sterner limits than did (E.23):

$$\Delta a_\mu^{(\text{unmixed})} = \frac{\alpha}{2\pi} (I_L + I_R).$$

In the limit $m_{\tilde{\mu}_{1,2}} \gg m_\lambda \gg m_\mu$ (F.8) becomes

$$\Delta a_\mu = -\alpha^2 \pi \left\{ \frac{m_\mu^2}{m_{\tilde{\mu}_1}^2} + \frac{m_\mu^2}{m_{\tilde{\mu}_2}^2} \right\}$$

which is essentially the same as the unmixed version

$$\Delta a_\mu = -\frac{\alpha}{2\pi} \left\{ \frac{m_\mu^2}{m_{\mu_1}^2} + \frac{m_\mu^2}{m_{\mu_2}^2} \right\}$$

presented in (E.24).

F.3 EVALUATION OF DIAGRAMS V-VIII

Diagram V:

$$\begin{aligned} \mathcal{M}^\mu &= -ie \psi_+^\dagger(p', s') \Gamma_V^\mu(p', s') \psi_-(p, s) \\ &= (-ie)^3 (\sqrt{2})^2 (i^3) \int \frac{d^4 k}{(2\pi)^4} \\ &\quad \times \frac{\psi_+^\dagger \delta p^\alpha \psi_-(p, s) m_\lambda (p + p' - 2k)^\alpha \sin \phi \cos \phi}{\{k^2 - m_1^2\} \{p - k\}^2 - m_{\mu_1}^2 \} \{p' - k\}^2 - m_{\mu_2}^2 \} } \end{aligned} \quad (\text{F.9})$$

Following the steps detailed in Section E.3 we find $(m_1^2 = m_\lambda^2 - m_{\mu_1}^2 + m_\mu^2)$

$$\psi_+ \Gamma_V^\mu \psi_- = \frac{ie^2}{8\pi^2} m_\lambda \int_0^1 dx \int_0^{1-x} dy \frac{i \psi_+^\dagger \psi_-(p + p' - 2(px - p'y))^\mu \sin \phi \cos \phi}{m_1^2 (x+y) - m_\lambda^2 - m_\mu^2 (x+y)^2 - q^2 xy} \quad (\text{F.10})$$

Performing the "b ↔ c" trick as when we went from (E.6) to (E.11)

$$\psi_+ \Gamma_V^\mu \psi_- = \frac{\alpha m_\lambda}{2\pi} \sin \phi \cos \phi \psi_+ \psi_- \int_0^1 dx \int_0^{1-x} dy \quad (\text{F.11})$$

$$\times \frac{(p + p')^\mu (1 - x - y)}{[m_\mu^2 (x+y)^2 + m_1^2 (x+y) - m_\lambda^2 - q^2 xy]}.$$

From Appendix H or Eqn. (D.23), or simply following the step from (E.17) to

(E.19), we use the Gordon decomposition to find on shell ($q^2 = 0$)

$$\Delta a_\mu^{(V)} = \frac{\alpha}{2\pi} \frac{m_\mu m_\lambda}{m_\mu^2} J_1 \quad (\text{F.12})$$

with

$$\begin{aligned} J_1 &= \int_0^1 dx \int_0^{1-x} dy \frac{1 - x - y}{-(x+y)^2 + \frac{m_\lambda^2}{m_\mu^2} (x+y) - \frac{m_\lambda^2}{m_\mu^2}} \\ &= \frac{m_\mu^2}{m_1^2} \int_0^1 dx \int_0^x d\bar{z} \frac{1 - x}{-\frac{m_\mu^2}{m_1^2} x^2 + x - R_1} \\ &\quad x = x + y \quad \bar{z} = x - y \quad R_1 = m_\lambda^2 / m_1^2 \end{aligned} \quad (\text{F.13})$$

where we have used notation similar to that of (E.26).

Diagram V only provides the $\psi_+ \sigma^{\mu\nu} q_\nu \psi_-$ part of this term. The $\bar{\psi} - \sigma^{\mu\nu} q_\nu \psi_+$ piece comes from Diagram VII.

In the most interesting limit, when $m_\lambda, m_{\mu_1} \gg m_\mu$ (and assuming $m_1 \gg m_\mu$) we may expand this in powers of m_μ^2 / m_1^2 . The leading order term is easy to find:

$$\begin{aligned} J_1 &= \frac{m_\mu^2}{m_1^2} \int_0^1 dx \frac{x(1-x)}{x - R_1} = \frac{m_\mu^2}{m_1^2} \int_{-R_1}^{1-R_1} d\xi \frac{-\xi^2 + (1-2R_1)\xi + R_1(1-R_1)}{\xi} \\ &= \frac{m_\mu^2}{m_1^2} \left\{ \frac{1}{2} (1-2R_1) + R_1(1-R_1) \ln \left(\frac{R_1-1}{R_1} \right) \right\} \end{aligned} \quad (\text{F.14})$$

which may be written, including the next order term in m_μ^2 ,

$$J_1 = \frac{m_\mu^2 \left\{ -\frac{1}{2} (m_{\mu_1}^2 - m_\lambda^2) + m_{\mu_1}^2 m_\lambda^2 \ln \frac{m_\lambda^2}{m_{\mu_1}^2} \right\}}{(m_{\mu_1}^2 - m_\lambda^2)^2} + \frac{m_\mu^4 G(m_{\mu_1}, m_\lambda)}{(m_{\mu_1}^2 - m_\lambda^2)^3} + O(m_\mu^6) \quad (\text{F.15})$$

where

$$G(m_{\tilde{\mu}_1}, m_\lambda) = \frac{(m_{\tilde{\mu}_1}^2 + 2m_\lambda^2)(5m_{\tilde{\mu}_1}^2 + m_\lambda^2) + 6m_{\tilde{\mu}_1}^2 m_\lambda^2}{6(m_{\tilde{\mu}_1}^2 - m_\lambda^2)^2} - \frac{2m_{\tilde{\mu}_1}^2 m_\lambda^2}{(m_{\tilde{\mu}_1}^2 - m_\lambda^2)^2} \ln \frac{m_{\tilde{\mu}_1}^2}{m_\lambda^2} \quad (\text{F.16})$$

Eqn. (F.14) agrees with Griffols and Mendes³⁵ and, with some algebra, Ellis, Hagelin and Nanopoulos⁸.

The contributions of diagrams VI and VII are the same as the above except that $\tilde{\mu}_1 \rightarrow \tilde{\mu}_2$ and the overall additional factor of -1 coming from $-\sin \theta$ at one of the Yukawa vertices.

Thus the term of Δa_μ which is linear in m_μ is

$$\Delta a_\mu^{\text{linear}} = \frac{\alpha}{2\pi} \sum_{j=1}^2 \frac{m_\mu m_\lambda \sin \phi \cos \phi}{(m_{\tilde{\mu}_j}^2 - m_\lambda^2)^2} \left[\frac{1}{2} (m_{\tilde{\mu}_j}^4 - m_\lambda^4) - m_\lambda^2 m_{\tilde{\mu}_j}^2 \ln \frac{m_{\tilde{\mu}_j}^2}{m_\lambda^2} \right]. \quad (\text{F.17})$$

When $m_{\tilde{\mu}_i} \approx m_\lambda$ the above formulation would appear to be invalid since $|m_{\tilde{\mu}_i} - m_\lambda|$ might be comparable to m_μ terms which were discarded. If $m_{\tilde{\mu}_i} \gg m_\mu$, however, then (F.17) is still valid and yields

$$\Delta a_\mu(m_{\tilde{\mu}_i} = m_\lambda) \rightarrow -\frac{\alpha}{2\pi} \frac{m_\mu}{m_\lambda} \sin \phi \cos \phi. \quad (\text{F.18})$$

Equation (F.17) can also be written as

$$\begin{aligned} \Delta a_\mu &= C(f(X_2) - f(X_1)) \\ C &= \frac{\alpha}{2\pi} \frac{m_\mu}{m_\lambda} \sin \phi \cos \phi \\ f(x) &= \frac{\frac{1}{2}(x^2 - 1) - x \ln x}{(x - 1)^2} \quad f(1) = \frac{1}{6} \\ X_i &= m_{\tilde{\mu}_i}^2 / m_\lambda^2. \end{aligned} \quad (\text{F.19})$$

F.4 A NOTE ON AXIAL COUPLINGS

Griffols and Mendes³⁶ originally considered the possibility of a more general vertex coupling of the form

$$\bar{\psi}(\not{g}_V + \not{g}_A \gamma_5) \lambda \tilde{\mu}_i.$$

This is more in keeping with the full electroweak theory. Since

$$\bar{\psi} \gamma^5 \psi = \psi_+^\dagger \psi_- - \bar{\psi}_- \psi_+^\dagger$$

we see that this really implies that $\bar{\psi}_+ \bar{\lambda}$ and $\psi_- \lambda$ may couple with unequal coefficients. The relative Yukawa strengths of the vertices and a few of the salient diagrams are illustrated in Fig. F.3.

We note that $g_V - g_A$ comes from the $\tilde{\mu}_R$ part of $\tilde{\mu}_1$ while $g_V + g_A$ stems from the $\tilde{\mu}_L$ part of $\tilde{\mu}_1$. Thus diagrammatic contributions where the helicity flips in the photino propagator go as $(\tilde{\mu}_L \text{ part of } \tilde{\mu}_1)(\tilde{\mu}_R \text{ part of } \tilde{\mu}_1)$ or $g_L^2 - g_A^2$, which is like $\sin \phi \cos \phi$ in the notation which we have adopted. The other two contributions are like $(g_V - g_A)^2 + (g_V + g_A)^2 \propto g_L^2 + g_A^2$. Thus in this case we would find

$$\begin{aligned} \Delta a_\mu &= -\frac{(g_L^2 - g_A^2)}{\pi^2} \sum_{j=1}^2 (-1)^j \frac{m_\mu m_\lambda}{(m_{\tilde{\mu}_j}^2 - m_\lambda^2)^2} \\ &\times \left[\frac{1}{2} (m_{\tilde{\mu}_j}^4 - m_\lambda^4) - m_\lambda^2 m_{\tilde{\mu}_j}^2 \ln \frac{m_{\tilde{\mu}_j}^2}{m_\lambda^2} + m_\mu^2 (m_{\tilde{\mu}_j}^2 - m_\lambda^2) G(m_{\tilde{\mu}_j}^2, m_\lambda^2) \right] \\ &- \frac{(g_L^2 + g_A^2)}{6\pi^2} \sum_{j=1}^2 \frac{m_\mu^3}{m_{\tilde{\mu}_j}^2}. \end{aligned}$$

F.5 SMALL MASS-SPLITTING

When $m_{\mu_1} \approx m_{\mu_2}$, we may simplify the expression in (F.19).

Letting

$$\delta m^2 = m_{\mu_1}^2 - m_{\mu_2}^2 \quad m_{\mu}^2 = m_{\mu_{1,2}}^2$$

we find (for $m_\lambda < m_{\mu}^2$)

$$\Delta a_\mu = -\frac{\delta m^2}{m_\mu^2} \frac{\alpha}{4\pi} \sin \phi \cos \phi \frac{m_\lambda m_\mu m_\mu^2}{(m_\mu^2 - m_\lambda^2)^2} \times \left\{ 1 + 4 \frac{m_\lambda^2}{m_\mu^2} \left[1 - \ln \frac{m_\mu^2}{m_\lambda^2} \right] - \frac{m_\lambda^4}{m_\mu^4} \left[5 + 2 \ln \frac{m_\mu^2}{m_\lambda^2} \right] \right\} \quad (\text{F.20})$$

or

$$\Delta a_\mu = -\frac{\delta m^2}{m_\mu^2} \frac{\alpha}{4\pi} \sin \phi \cos \phi \frac{m_\mu}{m_\lambda} p(x) \quad (\text{F.21})$$

with

$$x = m_\mu^2/m_\lambda^2$$

$$p(x) = \frac{x}{(x-1)^4} \{ (x+5)(x-1) - 2(2x+1)\ln x \}$$

$$p(x \rightarrow 0) \rightarrow -4x \ln x$$

$$p(x \rightarrow 1) \rightarrow \frac{1}{6}$$

$$p(x \rightarrow \infty) \rightarrow \frac{1}{x}$$

When we further have $m_\lambda \ll m_\mu$ (i.e. $x \gg 1$) then

$$\Delta a_\mu \doteq -\frac{\alpha}{4\pi} \sin \phi \cos \phi \frac{m_\mu m_\lambda}{m_\mu^2} \cdot \frac{\delta m^2}{m_\mu^2} \quad (\text{F.22})$$

F.6 EXTREMA OF THE LINEAR TERM

In Chapter 2 we saw that it is the term linear in m_μ which places the most rigorous limits when compared with the "real world". We may divide Δa_μ into

$$\Delta a_\mu = \Delta a_\mu^{\text{lin}} - \Delta a_\mu^{\text{nl}} \quad \Delta a_\mu^{\text{lin}} = C f(X_1) \quad (\text{F.23})$$

using the notation of (F.19). The function $f(x)$ has been plotted in Fig. G.1 for $\Delta a_\mu^{\text{linear}}$. All of the plots related to the linear terms have been collected into Appendix G. We immediately see from the form of $f(x)$ that there will be an extremum in $\Delta a_\mu^{\text{lin}} = C(f(X_2) - f(X_1))$ when $X_1 \neq X_2$. Differentiating to find the maximum $\Delta a_\mu^{\text{lin}}$ we find that it occurs when

$$\ln x_1 = \frac{y[(x_1 y + 5)(x_1 y - 1) - 2(2x_1 y + 1) \ln y](x_1 - 1)^4 - (x_1 + 5)(x_1 - 1)(x_1 y - 1)^4}{2[y(2x_1 y + 1)(x_1 - 1)^4 - (2x_1 + 1)(x_1 y - 1)^4]} \quad (\text{F.24})$$

(where $y \neq 1$ when $x_1 \neq 1$. Here $x_1 = X_1$.)

$$X_1 = \frac{m_{\mu_1}^2}{m_1^2} \quad X_2 = \frac{m_{\mu_2}^2}{m_1^2} \quad y = \frac{X_2}{X_1} = \frac{m_{\mu_2}^2}{m_{\mu_1}^2} \quad (\text{F.25})$$

This extremum, a maximum for $|\Delta a_\mu^{\text{lin}}|$, we shall call $\Delta a_{\mu \text{max}}$. For a given value of y it occurs at $X_1 = X_{1 \text{max}}$ and so $\Delta a_{\mu \text{max}} = \Delta a_{\mu \text{max}}(y)$ and $X_{1 \text{max}} = X_{1 \text{max}}(y)$ while $\Delta a_{\mu \text{max}} = \Delta a_\mu(X_{1 \text{max}})$. In the next appendix we have plotted $f(x)$, $\Delta a_\mu(X)$ for $y = 1.01$ and $y = 2$, $\Delta a_{\mu \text{max}}(y)$ and $X_{1 \text{max}}(y)$ for a range of parameters. Discussion takes place in Chapter 2.

FIGURE CAPTIONS

1. Vertices and Propagators

(a) Photon-Scalar Muon (*smuon*) Vertices.

(b) Photino Yukawa Vertices.

(c) Photino Propagator illustrating flipping and non-flipping propagation.

2. Lowest order contributions to Δa_μ .

3. Sample Diagrams from $\bar{\psi}(\not{g}_V + g_A \gamma_5)\lambda\bar{\mu}$ Vertex.

Fig. F.1

Fig. F.2

Fig. F.3

APPENDIX G

Plots Related to Anomalous Moment

In this appendix we plot some of the results derived in the previous appendix. In each instance this is for the case of smuon mixing. The states $\tilde{\mu}_L$ and $\tilde{\mu}_R$ mix with angle ϕ to produce the eigenstates $\tilde{\mu}_1$ and $\tilde{\mu}_2$. Letting

$$X_i = m_{\tilde{\mu}_i}^2 / m_\lambda^2 \quad i = 1, 2 \quad y = m_{\tilde{\mu}_2}^2 / m_{\tilde{\mu}_1}^2$$

$$C = \frac{\alpha}{2\pi} \frac{m_\mu}{m_\lambda} \sin \phi \cos \phi$$

we found that Δa_μ , the *additional* contribution to a_μ from supersymmetric smuon mixing, was given by

$$\Delta a_\mu = C \{f(x_2) - f(x_1)\}$$

where $f(x)$ is given in Eq. (F19).

FIGURE CAPTIONS

Fig. G.1

$f(x)$ versus x .

Fig. G.2

$10^4 \times \frac{1}{C} \Delta a_\mu$ versus X_1 for $y = 1.01$.

Fig. G.3

$\frac{1}{C} \Delta a_\mu$ versus X_1 for $y = 2$.

Fig. G.4

$X_{1\max}(y)$ versus y where $X_{1\max}$ is the value of X_1 where $\frac{1}{C} |\Delta a_\mu^{\text{lin}}|$ achieves its extremal value for a given y .

Fig. G.5

$\frac{1}{C} (\Delta a_\mu^{\text{lin}})_{\max}$ versus y for y ranging from 0 to 10 where $\frac{1}{C} (\Delta a_\mu^{\text{lin}})_{\max}$ is the extremal value of $\frac{1}{C} (\Delta a_\mu^{\text{lin}})$ for a given y .

Fig. G.6

$\frac{1}{C} (\Delta a_\mu^{\text{lin}})_{\max}$ in vicinity of $y = 1$.

Fig. G.1

Fig. G.2

6-87

5773A3

Fig. G.3

6-87

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Fig. G.4

Fig. G.5

Fig. G.6

APPENDIX H

Useful Relationships

In this appendix we gather certain algebraic formulae which are frequently referred to in this paper.

H.1 FEYNMAN INTEGRALS

A number of "tricks", due to Feynman, permit us to re-parameterize the momentum integrals which frequently arise in the computation of Feynman diagrams. A general expression is³⁶

$$\frac{1}{D_1^{p_1} D_2^{p_2} \dots D_N^{p_N}} = \frac{\Gamma(p_1 + p_2 + \dots + p_N)}{\Gamma(p_1) \Gamma(p_2) \dots \Gamma(p_N)} \int_0^1 dx_1 \dots \int_0^1 dx_N \frac{x_1^{p_1-1} x_2^{p_2-1} \dots x_N^{p_N-1} \delta(x_1 + \dots + x_N)}{(D_1 x_1 + \dots + D_N x_N)^{p_1 + \dots + p_N}} \quad (\text{H.1})$$

We will use a slightly different form of this identity.³ The particular cases of interest are

$$\frac{1}{ab} = \int_0^1 dx \frac{1}{[ax + b(1-x)]^2} \quad (\text{H.2})$$

$$\frac{1}{a^2 b} = 2 \int_0^1 dx \frac{x}{[ax + b(1-x)]^3} \quad (\text{H.3})$$

$$\frac{1}{abc} = 2 \int_0^1 dx \int_0^{1-x} dy \frac{1}{[a + (b-a)x + (c-a)y]^3} \quad (\text{H.4})$$

$$\frac{1}{a^2 bc} = 6 \int_0^1 dx \int_0^{1-x} dy \frac{1-x-y}{[a + (b-a)x + (c-a)y]^4} \quad (\text{H.5})$$

These are particularly useful in conjunction with

$$\int \frac{d^4 k}{(2\pi)^4} \frac{1}{(k^2 - 2p \cdot k + s - i\epsilon)^3} = \frac{i}{32\pi^2 (s - p^2)} \quad (\text{H.6 a})$$

$$\int \frac{d^4 k}{(2\pi)^4} \frac{k^\mu}{(k^2 - 2p \cdot k + s - i\epsilon)^3} = \frac{i p^\mu}{32\pi^2 (s - p^2)} \quad (\text{H.6 b})$$

$$\int \frac{d^4 k}{(2\pi)^4} \frac{1}{(k^2 - 2p \cdot k + s - i\epsilon)^4} = \frac{i}{96\pi^2 (s - p^2)^2} \quad (\text{H.7 a})$$

$$\int \frac{d^4 k}{(2\pi)^4} \frac{k^\mu}{(k^2 - 2p \cdot k + s - i\epsilon)^4} = \frac{i p^\mu}{96\pi^2 (s - p^2)^2} \quad (\text{H.7 b})$$

$$\int \frac{d^4 k}{(2\pi)^4} \frac{k_\mu k_\nu}{(k^2 - 2p \cdot k + s - i\epsilon)^4} = \frac{i [p_\mu p_\nu + \frac{1}{2} (s - p^2) \delta_{\mu\nu}]}{96\pi^2 (s - p^2)^2} \quad (\text{H.7 c})$$

In order to improve convergence we frequently change an integral of the form (H.2) to (H.3) or (H.4) to (H.5) by introducing an additional parametric integral

$$\frac{1}{a} \rightarrow - \int_0^\infty \frac{dt}{(a-t)^2} \quad (\text{H.9 a})$$

In particular if $a = k^2 - m^2$ we introduce the³³ Pauli-Villars regulator, $\Lambda \rightarrow \infty$:

$$\frac{1}{k^2 - m^2} \rightarrow \frac{1}{k^2 - m^2} - \frac{1}{k^2 - \Lambda^2} = - \int_{m^2}^{\Lambda^2} \frac{dt}{(k^2 - t)^2} \quad (\text{H.9 b})$$

Instead of x and y in eqn. (H.7) we frequently find it useful to introduce

$$z \equiv x + y \quad \bar{z} \equiv x - y \quad (\text{H.10 a})$$

$$x = \frac{1}{2} (z + \bar{z}) \quad y = \frac{1}{2} (z - \bar{z}) \quad (\text{H.10 b})$$

whence it becomes easy to show that

$$\int_0^1 dx \int_0^{1-x} dy = \frac{1}{2} \int_0^1 dz \int_{-z}^z d\bar{z} \quad (\text{H.10 c})$$

and, if the integrand is even in $\bar{z}$,

$$\int_0^1 dx \int_0^{1-x} dy = \int_0^1 dx \int_0^{\bar{z}} d\bar{z} \quad (\text{H.10 d})$$

Consider a triangle loop diagram whose legs have momenta p_1, p_2 (fermions) and $q = p_1 + p_2$ with masses m_1, m_2, m_3 and internal particles whose masses are M_1, M_2 and M_3 . A typical Feynman integral for such a triangle diagram would have a piece which looks like

$$\mathcal{M}^\mu \sim \int d^4 k \frac{(\not{p}_1 - \not{k})\gamma^\mu(\not{p}_2 + \not{k})}{[k^2 - M_3^2][(p_1 - k)^2 - M_1^2][(p_2 + k)^2 - M_2^2]} \quad (\text{H.11})$$

and from (H.9), with $\Lambda \rightarrow \infty$,

$$\mathcal{M}^\mu \sim - \int_{M_3^2}^{\Lambda^2} dt \int d^4 k \frac{(\not{p}_1 - \not{k})\gamma^\mu(\not{p}_2 + \not{k})}{[k^2 - t]^3[(p_1 - k)^2 - M_1^2][(p_2 + k)^2 - M_2^2]}. \quad (\text{H.12})$$

The denominator is of the form $\frac{1}{a^3 bc}$ so we may use (H.5) to obtain

$$\begin{aligned} \mathcal{M}^\mu \sim & - \int_{M_3^2}^{\Lambda^2} dt \int d^4 k \int_0^1 dx \int_0^{1-x} dy \\ & \times \frac{(1-x-y)(\not{p}_1 - \not{k})\gamma^\mu(\not{p}_2 + \not{k})}{[(k^2 - t) + (t - 2p_1 \cdot k + p_1^2 - M_1^2)x + (t + 2p_2 \cdot k + p_2^2 - M_2^2)y]^4}. \end{aligned} \quad (\text{H.13})$$

Now, if the fermions with momenta p_1 and p_2 are on shell, then

$$p_1^2 = m_1^2 \quad p_2^2 = m_2^2$$

and (H.13) becomes

$$\begin{aligned} \mathcal{M}^\mu \sim & - \int_{M_3^2}^{\Lambda^2} dt \int d^4 k \int_0^1 dx \int_0^{1-x} dy \\ & \times \frac{(1-x-y)(\not{p}_1 - \not{k})\gamma^\mu(\not{p}_2 + \not{k})}{[(k^2 - t) + (t - 2p_1 \cdot k + m_1^2 - M_1^2)x + (t + 2p_2 \cdot k + m_2^2 - M_2^2)y]^4} \\ = & - \int_{M_3^2}^{\Lambda^2} dt \int d^4 k \int_0^1 dx \int_0^{1-x} dy \\ & \times \frac{(1-x-y)(\not{p}_1 - \not{k})\gamma^\mu(\not{p}_2 + \not{k})}{[k^2 - (p_1 x - p_2 y) \cdot k + t(x+y-1) + (m_1^2 - M_1^2)x + (m_2^2 - M_2^2)y]^4} \\ = & - \int_{M_3^2}^{\Lambda^2} dt \int d^4 k \int_0^1 dx \int_0^{1-x} dy \frac{(1-x-y)(\not{p}_1 - \not{k})\gamma^\mu(\not{p}_2 + \not{k})}{[k^2 - P \cdot k + s]^4} \end{aligned}$$

where

$$\begin{aligned} P &= p_1 x - p_2 y \\ s &= m_1^2 x + m_2^2 y - (M_1^2 x + M_2^2 y) - t(1-x-y) \end{aligned} \quad (\text{H.14})$$

We could equally we have writted (H.11) with the final two terms of the denominator in reverse order. Then, when we applied (H.5) with $b \leftrightarrow c$, instead of getting (H.12) we would have obtain the same expression but with $x \leftrightarrow y$. Thus we may always interchange x and y in the integrand. Keeping this in mind we apply (H.7) to obtain

$$\begin{aligned} \mathcal{M}^\mu \sim & - \int_{M_3^2}^{\Lambda^2} dt \int d^4 k \int_0^1 dx \int_0^{1-x} dy \frac{i}{96\pi^2} \\ & \frac{(1-x-y)[\not{p}_1 - (\not{p}_1 x - \not{p}_2 y)]\gamma^\mu[\not{p}_2 + (\not{p}_1 x - \not{p}_2 y)]}{(s - P^2)^2} \\ & + a \delta_\mu^\nu \text{ term.} \end{aligned} \quad (\text{H.15})$$

Forgetting the δ -term for now we see that

$$\begin{aligned}
 P^2 &= p_1^2 x^2 + p_2^2 y^2 - 2p_1 \cdot p_2 xy \\
 q^2 &= (p_1 + p_2)^2 = p_1^2 + p_2^2 + 2p_1 \cdot p_2 \\
 \text{so } 2p_1 \cdot p_2 &= q^2 - p_1^2 - p_2^2 \\
 &= q^2 - m_1^2 - m_2^2 \quad (\text{H.16}) \\
 \text{and } P^2 &= m_1^2 x^2 + m_2^2 y^2 - (q^2 - m_1^2 - m_2^2)xy \quad (\text{H.17 a}) \\
 &= m^2(x+y)^2 - q^2 xy \quad \text{when } m_1 = m_2 \equiv m \quad (\text{H.17 b}) \\
 s - P^2 &= (m_1^2 x + m_2^2 y)(1-x-y) - (M_1^2 x + M_2^2 y) \\
 &\quad - t(1-x-y) + q^2 xy \quad (\text{H.18})
 \end{aligned}$$

Specializing to the case $m_1 = m_2 \equiv m$ and $M_1 = M_2 \equiv M$, for the purposes of this demonstration, we then have

$$s - P^2 = m^2(x+y)(1-x-y) - M^2(x+y) - t(1-x-y) + q^2 xy \quad (\text{H.19 a})$$

which is symmetric in x and y . Using (H.10a) we may write this as

$$s - P^2 = m^2 x(1-x) - M^2 x - t(1-x) + \frac{q^2}{4}(x^2 - \bar{x}^2) \quad (\text{H.19 b})$$

Since we are able to interchange $x \leftrightarrow y$ in the integrand and obtain an identical expression, we are similarly able to let $\bar{x} \leftrightarrow -\bar{x}$. We may then average these two expressions. In either case

$$\begin{aligned}
 x &\leftrightarrow y \\
 \bar{x} &\leftrightarrow -\bar{x}
 \end{aligned} \quad (\text{H.20})$$

We will refer to this as the " $b \leftrightarrow c$ trick". Since the denominator, $s - P^2$, is even in $\bar{x}$, when we perform this averaging we will eliminate all terms in the

numerator which are odd in $\bar{x}$. Inserting (H.10 b) into (H.15) yields

$$\begin{aligned}
 \mathcal{M}^\mu &\sim - \int_{M_1^2}^{\Lambda^2} dt \oint_0^1 dx \int_0^{1-x} dy \frac{i}{96\pi^2} (1-x) \\
 &\quad \frac{\frac{1}{2}[x(\not{p}_2 - \not{p}_1) - \bar{x}(\not{p}_1 + \not{p}_2) + 2\not{p}_1 \gamma^\mu x(\not{p}_1 - \not{p}_2) + \bar{x}(\not{p}_1 + \not{p}_2) - 2\not{p}_2]}{(s - P^2)^2}
 \end{aligned} \quad (\text{H.21})$$

Expanding this and keeping only even powers of $\bar{x}$ gives a numerator term

$$-\frac{1}{4} [x^2 \not{p}_1 \gamma^\mu \not{p}_1 - \bar{x}^2 \not{p}_2 \gamma^\mu \not{p}_2 + 2(\not{p}_1 \gamma^\mu \not{p}_1 - \not{p}_2 \gamma^\mu \not{p}_2) + 4\not{p}_1 \gamma^\mu \not{p}_2]$$

where here

$$\begin{aligned}
 \not{p}_1 &= \not{p}_1^\mu + \not{p}_2^\mu \\
 \not{p}_2 &= \not{p}_1^\mu - \not{p}_2^\mu.
 \end{aligned} \quad (\text{H.22})$$

The above expression will be sandwiched between $\bar{u}(p_1)$ and $v(p_2)$ so that any $\not{p}_1$ to the left of γ^μ will generate a factor of m_1 via the Dirac equation and any $\not{p}_2$ to the right of γ^μ will generate an m_2 . If $m_1 = m_2 = m \ll M$ or q^2 (which appears in terms not shown) then we can ignore these terms and (H.19 b) becomes

$$s - P^2 \approx - \left[t(1-x) + M^2 x + \frac{1}{4} q^2 (\bar{x}^2 - x^2) \right] \quad (\text{H.23})$$

which, when used in (H.21), gives us

$$\begin{aligned}
 \mathcal{M}^\mu &\sim - \int_{M_1^2}^{\Lambda^2} dt \oint_0^1 dx \int_0^{1-x} dy \frac{i}{96\pi^2} \\
 &\quad - \frac{\frac{1}{2}(1-x) [x^2 \not{p}_1 \gamma^\mu \not{p}_1 - \bar{x}^2 \not{p}_2 \gamma^\mu \not{p}_2]}{(t(1-x) + M^2 x + \frac{1}{4} q^2 (\bar{x}^2 - x^2))^2}.
 \end{aligned} \quad (\text{H.24})$$

In point of fact, since we are ignoring the external fermion mass terms,

$$\bar{q}\gamma^\mu q \approx q\gamma^\mu q$$

and (H.24) may be written as

$$M^\mu \sim - \int_{M_1^2}^{\Lambda^2} dt \int_0^1 dx \int_0^{1-x} dy \frac{i}{96\pi^2} \frac{\frac{1}{2}(1-x)[(\bar{z}^2 - z^2) q\gamma^\mu q]}{(t(1-x) + M^2 x + \frac{1}{2}y^2(\bar{z}^2 - z^2))^{\frac{3}{2}}} \quad (\text{H.25})$$

H.2 THE GORDON DECOMPOSITION

The Gordon Decomposition is a useful identity³⁷ relating currents of even and odd numbers of sigma (or gamma) matrices. In this sense it is merely a restatement of the Dirac equation recast into a form which includes a $\sigma^{\mu\nu}$ term. Since the statement and proof of this identity may be found in any number of standard references, it may seem strange to devote a major portion of an appendix to it. Applications of this result are to be found throughout this paper, often in generalized or atypical forms which we will proceed to derive.

Four-Component Formulations

Working with Dirac spinors the Gordon Decomposition of the Dirac vector current is generally expressed as

$$\bar{\psi}(p', s') \gamma^\mu \psi(p, s) = \frac{1}{2m} \{ \bar{\psi}(p', s') [(p' + p)^\mu + i\sigma^{\mu\nu}(p' - p)_\nu] \psi(p, s) \} \quad (\text{H.26})$$

or

$$\bar{\psi}(p', s') \gamma^\mu \psi(p, s) = \frac{1}{2m} \{ \bar{\psi}(p', s') [\tilde{q}^\mu + i\sigma^{\mu\nu} q_\nu] \psi(p, s) \} \quad (\text{H.27})$$

where, for symmetry, we have let

$$q^\mu = p'^\mu - p^\mu \quad \tilde{q}^\mu = p'^\mu + p^\mu. \quad (\text{H.28})$$

We use the basic identities

$$\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu} \quad (\text{H.29})$$

$$[\gamma^\mu, \gamma^\nu] = -2i\sigma^{\mu\nu} \quad (\text{H.30})$$

$$\not{A} \not{B} = a_\mu b_\nu \left(\frac{1}{2} \{ \gamma^\mu, \gamma^\nu \} + \frac{1}{2} [\gamma^\mu, \gamma^\nu] \right) = \frac{1}{2} a_\mu b_\nu \{ g^{\mu\nu} - 2i\sigma^{\mu\nu} \} \quad (\text{H.31})$$

$$= a \cdot b - ia^\mu b^\nu \sigma_{\mu\nu}.$$

The standard identity in (H.27) considers only the instance of a vector current (γ^μ) where the initial and final spinorial states are of the same type. We wish to extend this to admit the possibility of admixtures of vector and pseudo vector currents connecting states of different species. We will refer to these as ψ^i and ψ^j and consider the vertex $[A + B\gamma_5]\gamma^\mu$.

We will require a few identities involving γ_5 in addition to (H.29)-(H.31).

$$[\gamma_5, \gamma_\mu] = 0 \quad [\gamma_5, \sigma^{\mu\nu}] = 0 \quad \gamma_5^2 = \gamma_5 \quad (\text{H.32})$$

$$\gamma^5 \not{A} \not{B} = \gamma^5 [a \cdot b - ia^\mu b^\nu \sigma_{\mu\nu}] \quad \not{A} \not{B} \gamma^5 = [a \cdot b - ia^\mu b^\nu \sigma_{\mu\nu}] \gamma^5$$

First consider the case where $A = 1$ and $B = 0$. Let a^μ be any four-vector.

$$\bar{\psi}^i(p', s') (\not{p} - m_i) \not{A} \psi^j(p, s) + \bar{\psi}^j(p', s') \not{A} (\not{p} - m_j) \psi^j(p, s) = 0$$

by the Dirac equation. The arrows atop the momentum operators indicate in which direction we intend to operate. This is true for arbitrary a . Using (H.31) we find that this is equal to

$$-(m_i + m_j) \bar{\psi}^i(p', s') \not{A} \psi^j(p, s) + \bar{\psi}^j(p', s') [(\not{p}^\dagger)^\mu a_\mu + i \not{p}^\dagger^\mu a^\nu \sigma_{\mu\nu} + a^\mu \not{p}_\mu - ia^\mu \not{p}^\nu \sigma_{\mu\nu}] \psi^j(p, s)$$

thus

$$(m_i + m_j) \bar{\psi}^i(p', s') \gamma^\mu a_\mu \psi^j(p, s) = \bar{\psi}^j(p', s') [(\not{p}^\dagger)^\mu + \not{p}^\mu] a_\mu - i\sigma^{\mu\nu} (\not{p}_\nu - \not{p}^\dagger_\nu) \psi^j(p, s)$$

since $\sigma^{\mu\nu} = -\sigma^{\nu\mu}$. As this is true for any a_μ we may write

$$\bar{\psi}^i(p', s') \gamma^\mu \psi^j(p, s) = \frac{1}{m_i + m_j} \left\{ \bar{\psi}^i(p', s') [(\not{p}^\dagger)^\mu + i\sigma^{\mu\nu} \not{p}_\nu] \psi^j(p, s) \right\} \quad (\text{H.33})$$

When $i = j$ we obtain (H.27), of course. We now consider the case where $A = 0$ and $B = 1$ and repeat the above analysis

$$\begin{aligned} \bar{\psi}^i(p', s') (\not{p} - m_i) \gamma^5 \not{A} \psi^j(p, s) &\pm \bar{\psi}^j(p', s') \gamma^5 \not{A} (\not{p} - m_j) \psi^j(p, s) = 0 \quad \forall a \\ &= -(m_i \pm m_j) \bar{\psi}^i(p', s') \gamma^5 \not{A} \psi^j(p, s) \\ &+ \bar{\psi}^j(p', s') [(\not{p}^\dagger)^\mu a_\mu \gamma^5 + i \not{p}^\dagger^\mu a^\nu \sigma_{\mu\nu} \gamma^5 \\ &\pm \gamma^5 a^\mu \not{p}_\mu \mp ia^\mu \not{p}^\nu \sigma_{\mu\nu}] \psi^j(p, s) \end{aligned}$$

The two different signs give rise to the equivalent identities:

$$\bar{\psi}^i(p', s') \gamma^5 \gamma^\mu \psi^j(p, s) = -\frac{1}{m_j + m_i} \bar{\psi}^i(p', s') \gamma^5 [(\not{p}^\dagger)^\mu + i\sigma^{\mu\nu} \not{p}_\nu] \psi^j(p, s) \quad (\text{H.34})$$

$$\begin{aligned} \bar{\psi}^i(p', s') \gamma^5 \gamma^\mu \psi^j(p, s) &= +\frac{1}{m_j - m_i} \bar{\psi}^i(p', s') \gamma^5 [(\not{p}^\dagger)^\mu + i\sigma^{\mu\nu} \not{p}_\nu] \psi^j(p, s) \\ &\quad (m_i \neq m_j). \quad (\text{H.35}) \end{aligned}$$

These can now be combined with (H.33) to obtain the more general results

$$\begin{aligned} \bar{\psi}^i(p', s') [A + B\gamma_5] \psi^j(p, s) \\ = \frac{1}{m_i + m_j} \bar{\psi}^i(p', s') \{ (A \not{p}^\dagger - B \not{p}^\mu \gamma^5) + i(A \not{p}_\nu - B \not{p}_\nu \gamma^5) \sigma^{\mu\nu} \} \psi^j(p, s) \end{aligned} \quad (\text{H.36})$$

$$\begin{aligned} & \bar{\psi}^i(p', s') [A + B\gamma^5] \psi^j(p, s) \\ &= \bar{\psi}^i(p', s') \left[\frac{A}{m_j + m_i} + \frac{B\gamma_5}{m_j - m_i} \right] (\bar{q}^\mu + i\sigma^{\mu\nu} q_\nu) \psi^j(p, s) \quad (m_i \neq m_j) \end{aligned} \quad (\text{H.37})$$

In either version we recover (H.27) when $A = 1$, $B = 0$ and $m_i = m_j$. Writing

$$\gamma_\pm = \frac{1}{2} (1 \pm \gamma_5) \quad \gamma_+ = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \quad \gamma_- = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \quad (\text{H.38})$$

and using (H.36) we find that

$$\begin{aligned} (m_i + m_j) \bar{\psi}^i(p') \gamma_\pm \gamma^\mu \psi^j(p) = \\ \bar{\psi}^i(p') \{ \gamma_\mp p'^\mu + \gamma_\pm p^\mu + i\sigma^{\mu\nu} [\gamma_\mp p'^\nu - \gamma_\pm p^\nu] \} \psi^j(p). \end{aligned} \quad (\text{H.39})$$

Two-Component Formulation

As always, the Weyl spinor formulation of an identity derived for Dirac spinors is more involved. We now have a pair of spinors and their conjugates which are related via the Dirac equation. One possibility is to take the results of the previous section and expand them out in two-component form. By choosing $A + B\gamma_5$ to be $\frac{1}{2} (1 \pm \gamma_5)$ we will project out the left- and right-pieces of the vector current.

Expanding some Dirac spinorial currents into their two-component subdivi-

sion (see Appendix A)

$$\begin{aligned} \bar{\psi} \gamma_+ \psi &= \bar{\psi}_- \psi_+ \\ \bar{\psi} \gamma_- \psi &= \psi_+ \bar{\psi}_- \\ \bar{\psi} \gamma_+ \gamma^\mu \psi &= \bar{\psi}_- \sigma^\mu \psi_- \\ \bar{\psi} \gamma_- \gamma^\mu \psi &= \psi_+ \sigma^\mu \bar{\psi}_+ \\ \bar{\psi} \gamma_+ \sigma^{\mu\nu} \psi &= \bar{\psi}_- \sigma^{\mu\nu} \bar{\psi}_+ \\ \bar{\psi} \gamma_- \sigma^{\mu\nu} \psi &= \psi_+ \sigma^{\mu\nu} \psi_- \end{aligned} \quad (\text{H.40})$$

Using these with (H.39) results in

$$\begin{aligned} \psi_+^i(p') \sigma^\mu \bar{\psi}_+^j(p) = \\ \frac{1}{m_i + m_j} \{ \psi_+^i(p') (p'^\mu + i\sigma^{\mu\nu} p'_\nu) \bar{\psi}_-^j(p) + \bar{\psi}_-^i(p') (p^\mu - i\sigma^{\mu\nu} p_\nu) \bar{\psi}_+^j(p) \} \end{aligned} \quad (\text{H.41})$$

$$\begin{aligned} \bar{\psi}_-^i(p') \sigma^\mu \psi_-^j(p) = \\ \frac{1}{m_i + m_j} \{ \psi_+^i(p') (p^\mu - i\sigma^{\mu\nu} p_\nu) \bar{\psi}_-^j(p) + \bar{\psi}_-^i(p') (p'^\mu + i\sigma^{\mu\nu} p'_\nu) \bar{\psi}_+^j(p) \}. \end{aligned} \quad (\text{H.42})$$

These could equally well have been obtained by working with the Weyl spinors directly. For instance, starting with

$$\psi_+^i(p') (\sigma^\nu)_{\alpha\dot{\alpha}} \bar{\psi}_-^j(p') (\bar{\sigma}^\nu)^{\dot{\alpha}\beta} \psi_-^k(p') + 3 \text{ similar terms} = 0$$

and repeating the steps leading to (H.33) would lead us to (H.42) directly. It is often convenient to use the sum and difference of (H.41) and (H.42) which correspond to decomposing the full γ^μ and $\gamma^5 \gamma^\mu$ currents.

The sum term is

$$\begin{aligned} & \psi_+^i \sigma^{\mu\nu} \psi_+^j + \bar{\psi}_-^i \bar{\sigma}^{\mu\nu} \psi_-^j \\ &= \frac{1}{m_i + m_j} \left\{ \psi_+^i [\bar{\sigma}^{\mu\nu} + i\sigma^{\mu\nu} \not{q}_\nu] \psi_-^j + \bar{\psi}_-^i [\bar{\sigma}^{\mu\nu} + i\bar{\sigma}^{\mu\nu} \not{q}_\nu] \bar{\psi}_+^j \right\} \end{aligned} \quad (\text{H.43})$$

which is, of course, merely the expansion of (H.33). We can use the Dirac equation (section B.1) to put this entirely in terms of $\bar{\psi}_-$ and ψ_- ,

$$\begin{aligned} & \bar{\psi}_-^i(p') \left[\frac{\not{q} \cdot \not{p}'}{m_i} + \frac{\not{\sigma} \cdot \not{p}}{m_j} \right] \psi_-^j(p) \not{q}^\nu \\ &= -i \bar{\psi}_-^i(p') \left[\frac{\not{\sigma} \cdot \not{p}'}{m_i} \sigma^{\mu\nu} + \not{\sigma}^{\mu\nu} \frac{\not{\sigma} \cdot \not{p}}{m_j} \right] q_\nu \psi_-^j(p) \\ &+ (m_i + m_j) \bar{\psi}_-^i(p') \left[\not{\sigma}^\mu + \frac{\not{\sigma} \cdot \not{p}'}{m_i} \not{\sigma}^\mu \frac{\not{\sigma} \cdot \not{p}}{m_j} \right] \psi_-^j(p). \end{aligned} \quad (\text{H.44})$$

A similar expression exists for ψ_+ and $\bar{\psi}_+$. (On-shell we may cancel the spinors from these expressions.)

APPENDIX I

The Supersymmetric Lagrangian

1.1 BASICS

Superspace and Superfields

We remind the reader of some fundamental aspects of supersymmetric theories. We extend normal space-time to "super-space" which includes a number of complex anti-commuting (Grassman) coordinates $\theta_a, \bar{\theta}_a$ ($\bar{\theta} \equiv \theta^\dagger$). Superfields are functions of these superspace coordinates

$$\phi = \phi(x^\mu; \theta_a, \bar{\theta}_a) \quad \mu = 1, \dots, D \quad a = 1, \dots, K$$

where D is the dimensionality of space-time and we refer to this as an " $n = K$ " supersymmetry. In this paper we will restrict ourselves to $D = 4$ and $K = 1$ which corresponds to the simplest supersymmetry.

We shall take θ^a and $\bar{\theta}^a$ to be two-component anti-commuting objects (akin to Weyl spinors):

$$\theta = \begin{pmatrix} \theta_1 \\ \theta_2 \end{pmatrix} \quad \bar{\theta} = \begin{pmatrix} \bar{\theta}_1 \\ \bar{\theta}_2 \end{pmatrix}. \quad (1.1)$$

Thus

$$\{\theta_i, \theta_j\} = \{\bar{\theta}_i, \bar{\theta}_j\} = \{\theta_i, \bar{\theta}_j\} = 0 \quad i, j = 1, 2. \quad (1.2)$$

In particular

$$\theta_1^2 = \theta_2^2 = \bar{\theta}_1^2 = \bar{\theta}_2^2 = 0. \quad (1.3)$$

For short we write

$$\theta^2 = \theta\theta = \frac{1}{2} \epsilon^{\alpha\beta} \theta_\alpha \theta_\beta = \theta_1 \theta_2. \quad (1.4)$$

Note that $\theta^3 = 0 = \bar{\theta}^3$ and likewise for higher powers of θ .

Supertransformations and the Supersymmetry Generators

We examine a superfield under the action of an infinitesimal translation in superspace ("supertransformation")

$$\theta \rightarrow \theta + \epsilon \quad \bar{\theta} \rightarrow \bar{\theta} + \bar{\tau} \quad x^\mu \rightarrow x^\mu + \delta x^\mu \quad (1.5)$$

where ϵ and $\bar{\tau}$ are infinitesimal, two-component, anti-commuting c-numbers. We desire to construct δx^μ from the anti-commuting coordinates θ and $\bar{\theta}$ subject to the constraint that x^μ must be a real coordinate which commutes, not anti-commutes. Since $\chi\psi$, the product of two Grassman objects, is a commuting object we must construct δx^μ from products of ϵ 's and θ 's.

The two simplest transformations which retain the Lorentz structure are

$$\begin{aligned} x^\mu &\rightarrow x^\mu + \epsilon^\alpha \sigma^\mu_{\alpha\beta} \bar{\theta}^\beta + \theta^\alpha \sigma^\mu_{\alpha\beta} \bar{\tau}^\beta \\ x^\mu &\rightarrow x^\mu + i \left(\theta^\alpha \sigma^\mu_{\alpha\beta} \bar{\tau}^\beta - \epsilon^\alpha \sigma^\mu_{\alpha\beta} \bar{\theta}^\beta \right). \end{aligned} \quad (1.6)$$

Note that using $\epsilon_{\alpha\beta}$ and $\epsilon^{\dot{\alpha}\dot{\beta}}$ these may be recast into equivalent forms. The second equation in (1.6) is the traditional choice. These forms follow because σ^μ (and $\bar{\sigma}^\mu$) are the only dimensionless constants with a single Lorentz index. Note that here $\epsilon, \bar{\tau}$ are c-numbers and we are dealing with rigid (global) supersymmetry. In a local theory (i.e. supergravity) $\epsilon = \epsilon(x)$ and $\bar{\tau} = \bar{\tau}(x)$ but $\epsilon \neq \epsilon(\theta_i)$.

Representing these supertransformations by (ϕ is a superfield)

$$\phi \rightarrow \phi + \delta_\epsilon \phi \quad (1.7)$$

where

$$\delta_\epsilon \equiv \epsilon Q + \bar{\tau} \bar{Q} \quad (1.8)$$

we can find the supersymmetry generators Q and $\bar{Q}$.

$$\begin{aligned} \phi(x^\mu, \theta, \bar{\theta}) &\rightarrow \phi + \delta_\epsilon \phi = \phi(\theta + \epsilon, \bar{\theta} + \bar{\tau}, x^\mu + i(\theta \sigma^\mu \bar{\tau} - \epsilon \sigma^\mu \bar{\theta})) \\ &= \phi + \epsilon \frac{\partial \phi}{\partial \theta} + \bar{\tau} \frac{\partial \phi}{\partial \bar{\theta}} + i \frac{\partial \phi}{\partial x^\mu} (\theta \sigma^\mu \bar{\tau} - \epsilon \sigma^\mu \bar{\theta}) \\ &= \phi + \epsilon \left(\frac{\partial}{\partial \theta} - i \sigma^\mu \bar{\theta} \frac{\partial}{\partial x^\mu} \right) + \bar{\tau} \left(\frac{\partial}{\partial \bar{\theta}} - i \theta \sigma^\mu \frac{\partial}{\partial x^\mu} \right) \end{aligned} \quad (1.9)$$

where the final minus sign comes from anticommuting $\bar{\tau}$ through $\theta \sigma^\mu$. Note that $\partial/\partial\theta$ and $\partial/\partial\bar{\theta}$ are two-component anticommuting operators. Comparing (1.7), (1.8) and (1.9) we have

$$\begin{aligned} Q &= \frac{\partial}{\partial \theta} - i \sigma^\mu \bar{\theta} \frac{\partial}{\partial x^\mu} \\ \bar{Q} &= \frac{\partial}{\partial \bar{\theta}} - i \theta \sigma^\mu \frac{\partial}{\partial x^\mu}. \end{aligned} \quad (1.10)$$

Note that the definition of $\bar{Q}$ differs by a sign when compared with those of Wess and Bagger.¹ This arises because they use $\bar{\theta}_\alpha$ and we use $\bar{\theta}^{\dot{\alpha}} = \epsilon^{\dot{\alpha}\beta} \theta_\beta$ "=-" $\bar{\theta}_\alpha$ where "=" means "which has the numerical magnitude".

Summarizing, under the supersymmetry transformation δ_ϵ

$$\begin{aligned} \theta &\rightarrow \theta + \epsilon \\ \bar{\theta} &\rightarrow \bar{\theta} + \bar{\tau} \\ x^\mu &\rightarrow x^\mu + i(\theta \sigma^\mu \bar{\tau} - \epsilon \sigma^\mu \bar{\theta}) \end{aligned} \quad (1.11)$$

the superfield $\phi(x^\mu, \theta, \bar{\theta})$ transforms as

$$\begin{aligned}\phi &\rightarrow \phi + \delta_\epsilon \phi \\ \delta_\epsilon &= \epsilon Q + i\bar{\epsilon}\bar{Q} \\ Q &= \frac{\partial}{\partial \theta} - i\sigma^\mu \bar{\theta} \frac{\partial}{\partial x^\mu} \\ \bar{Q} &= \frac{\partial}{\partial \bar{\theta}} - i\theta \sigma^\mu \frac{\partial}{\partial x^\mu}.\end{aligned}\quad (1.12)$$

Relationship with Hamiltonian

It can be shown that $[\delta_\epsilon, \delta_{\bar{\epsilon}}] \sim (\epsilon\sigma^\mu \bar{\epsilon} - \bar{\epsilon}\sigma^\mu \epsilon)\partial_\mu$ and so two successive supersymmetry transformations lead to a Lorentz transformation (in a local theory this will necessitate a torsional term). In a sense the supersymmetry transformations are "square roots" of Lorentz transformations. In particular $(Q_\alpha, \bar{Q}_{\dot{\beta}}) = 2\sigma^\mu_{\alpha\dot{\beta}} P_\mu$. Multiplying by $(\bar{\sigma}^0)^{\dot{\alpha}\alpha}$ (i.e. taking the trace) leads to

$$Q_1 \bar{Q}_1 + Q_2 \bar{Q}_2 + \text{h.c.} = P_0 = H \quad (1.13)$$

and so the Hamiltonian is sum of the squares of the supersymmetry generators and is always positive semi-definite. This Q is like the "square root of H ".

Covariant Derivative

We desire a supersymmetric derivative which commutes with ϵQ and $i\bar{\epsilon}\bar{Q}$ so that if $\phi \rightarrow \phi + \delta_\epsilon \phi$ then $D\phi \rightarrow D\phi + \delta_\epsilon D\phi$ (i.e. if ϕ transforms as a superfield then so does $D\phi$). It is easily checked that if we let

$$\begin{aligned}D &= \frac{\partial}{\partial \theta} + i\sigma^\mu \bar{\theta} \frac{\partial}{\partial x^\mu} \\ \bar{D} &= \frac{\partial}{\partial \bar{\theta}} + i\theta \sigma^\mu \frac{\partial}{\partial x^\mu}\end{aligned}\quad (1.14)$$

then $\{\epsilon Q, \xi D\} = \{\bar{\epsilon}\bar{Q}, \xi D\} = 0$ and similarly

$$\{Q, \bar{D}\} = \{\bar{Q}, D\} = \{\bar{Q}, D\} = 0. \quad (1.15)$$

Note that D is like Q with a change of sign. (1.15) implies that $\delta_\epsilon D\phi = D\delta_\epsilon \phi$ using the definition of δ_ϵ in (1.12).

Differentiation and Integration

Differentiation and integration over Grassman variables is largely a matter of definition. We define (ξ is a single component)

$$\frac{\partial}{\partial \xi} A = 0 \quad \frac{\partial}{\partial \xi} \xi = 1 \quad (1.16)$$

when A is independent of ξ and could be commuting, anti-commuting, or a combination.

Note that

$$\frac{\partial}{\partial \xi} A\xi = \begin{cases} A & \text{if } A \text{ is commuting} \\ -A & \text{if } A \text{ is anti-commuting} \end{cases} \quad (1.17)$$

For integrations we invoke three axioms

(i) Linearity

$$\int (AF(\xi) + BG(\xi))d\xi = A \int F(\xi)d\xi + B \int G(\xi)d\xi. \quad (1.18)$$

Note that $\int d\xi$ is a definite integral over "all values of ξ " and that $\int Ad\xi = -\int d\xi A$ if A is anti-commuting.

(ii) Shifting variable of integration

$$\int F(\xi + \chi)d\xi = \int F(\xi)d\xi. \quad (1.19)$$

(iii) If $\chi\xi = 0 \quad \forall \quad \chi$ then $\xi = 0$.

Consider:

$$\begin{aligned}
 \int (\xi + \chi) d\xi &= \int \xi d\xi + \int \chi d\xi \quad \text{by linearity} \\
 \int (\xi + \chi) d\xi &= \int \xi d\xi \quad \text{by shifting} \\
 \Rightarrow \int \chi d\xi &= 0 \quad (1.20) \\
 &= \chi \int d\xi \quad \forall \chi \\
 \Rightarrow \int d\xi &= 0.
 \end{aligned}$$

Since $\xi^2 = 0$ (ξ a single-component object) any function of ξ has a trivial expansion of the form

$$F(\xi) = A + B\xi \quad (1.21)$$

and the most general function is a linear function.

Then

$$\int F(\xi) d\xi = \int A d\xi + B \int \xi d\xi = B \int \xi d\xi$$

and unless we want integration to be totally trivial (all integrals vanish) we must have $\int \xi d\xi \neq 0$. For simplicity we select

$$\int \xi d\xi \equiv 1. \quad (1.22)$$

We see that the rules for differentiation and integration are identical with $\partial_\xi = \int d\xi$ acting like projection operators, projecting out only those terms with a ξ .

Multiple integrals work the same way (keeping track of anti-commuting $d\xi d\chi$, etc.). Thus for a two-component object $\theta = (\theta_1 \quad \theta_2)^T$

$$\int \theta^2 d^2\theta = \int \theta_1 d\theta_1 \int \theta_2 d\theta_2 = 1 \quad (1.23)$$

and all other combinations vanish. Note again that $\partial F / \partial \theta^2 \equiv \int F d\theta^2$.

Component Fields

Let us consider the expansion of a superfield $\phi(x, \theta, \bar{\theta})$ where $\theta, \bar{\theta}$ are two-component anti-commuting (coordinates) parameters

$$\begin{aligned}
 \phi(x, \theta, \bar{\theta}) &= \phi(x) + (\bar{\theta}\psi_1 + \bar{\psi}_2\theta) + (a_1\theta\theta + a_2\bar{\theta}\bar{\theta}) \\
 &+ A_\mu \bar{\theta}\sigma^\mu\theta + \lambda_1\bar{\theta}\theta\theta + \bar{\theta}\bar{\theta}\bar{\lambda}_2 + D\theta\theta\bar{\theta}\bar{\theta}. \quad (1.24)
 \end{aligned}$$

All other Lorentz invariant combinations, such as $B_{\mu\nu}\theta\sigma^{\mu\nu}\theta$ vanish due to $\theta_i^2 = 0$. $\bar{\theta}\sigma^\mu\theta\bar{\theta}\sigma_\mu\theta$ is proportional to $\theta\theta\bar{\theta}\bar{\theta}$ and so is not included. If ϕ is to be real then a, A_μ and D are real and $\psi_1 = \psi_2, a_1 = a_2$ and $\lambda_1 = \lambda_2$.

This represents many degrees of freedom. Just as a massless gauge field, A^μ , has two, not four, physical degrees of freedom. To reduce the multitude of non-physical degrees of freedom a number of "supergauge-like" constraints are used. Although it is not readily apparent, a hallmark of supersymmetric theories is that the number of physical fermionic and bosonic degrees of freedom are equal. First it should be realized from (1.24) that any function of a superfield is another superfield.

This can be explicitly checked by testing that it transforms as a superfield under δ_ϵ . For instance, since $D\phi$ and $\bar{D}\phi$ are superfields, $\phi^2\bar{D}\phi e^{i\theta}D\phi$ would also be a superfield. For our constraint we could choose $f(\phi) = 0$ which would be

a manifestly supersymmetric constraint. In practice the two which are usually chosen are $D\bar{\phi} = 0$ and $\phi = \phi^*$.

Scalar Superfields

We call superfields satisfying

$$\bar{D}\phi = 0 \quad (1.25)$$

chiral (or scalar) superfields and their conjugates

$$D\bar{\phi}^\dagger = 0 \quad (1.26)$$

anti-chiral superfields, or superfields of the opposite chirality. These equations may be trivially solved. Recalling that

$$\bar{D}_\alpha = \frac{\partial}{\partial \bar{\theta}^\alpha} + i \theta^\mu \sigma^\mu_{\alpha\dot{\alpha}} \frac{\partial}{\partial x^\mu}$$

we note that

$$\bar{D}_\alpha(x^\mu + i\theta\sigma^\mu\bar{\theta}) = \bar{D}_\alpha\theta^\alpha = 0 \quad (1.27)$$

and therefore

$$\phi = \phi(x^\mu + i\theta\sigma^\mu\bar{\theta}, \theta) \quad (1.28)$$

is a solution of $\bar{D}\phi = 0$. This follows since

$$\frac{\partial}{\partial \bar{\theta}} \phi(x^\mu + i\theta\sigma^\mu\bar{\theta}, \theta) = -i\theta\sigma^\mu \frac{\partial \phi}{\partial x^\mu}$$

(remember to anti-commute $\partial/\partial\bar{\theta}$ past θ) and

$$\begin{aligned} \frac{\partial}{\partial x^\mu} \phi(x^\mu + i\theta\sigma^\mu\bar{\theta}, \theta) &= \phi' \\ \left(\frac{\partial}{\partial \bar{\theta}} + i\theta\sigma^\mu \frac{\partial}{\partial x^\mu} \right) \phi &= 0 = \bar{D}\phi. \end{aligned}$$

Similarly $D\bar{\phi} = 0$ has solution

$$\bar{\phi} = \bar{\phi}(x^\mu - i\theta\sigma^\mu\bar{\theta}, \bar{\theta}) \quad (1.29)$$

is an "anti-chiral" field.

Since we can write

$$\begin{aligned} \phi &= \phi(y, \theta) \\ \bar{\phi} &= \bar{\phi}(y^\dagger, \bar{\theta}) \end{aligned} \quad (1.30)$$

$$y^\mu = x^\mu + i\theta\sigma^\mu\bar{\theta}$$

we see that ϕ is not an explicit function of $\bar{\theta}$ and that $\bar{\phi}$ is not an explicit function of θ . This is the principle reason that these particular constraints were chosen.

The expansion of $\phi(y, \theta)$ is now simplified considerably

$$\phi = A(y) + \sqrt{2}\theta\psi(y) + \theta^2 F(y) \quad (1.31)$$

where the $\sqrt{2}$ is chosen to make the closure of the algebra simple. Since $\phi(y, \theta)$ is independent of $\bar{\theta}$ we see that any function of $\phi(y, \theta)$ will also be a chiral superfield, assuming that there is no $\bar{\theta}$ explicitly in the function definition, because $f(\phi)$ will also be independent of $\bar{\theta}$. Note that $A(y)$ is a scalar field and $\psi(y)$ a spinor. This closure is easily seen by considering the effect of the supersymmetry generators Q and $\bar{Q}$ on A , ψ and F which are called the "component fields" of the superfield ϕ .

Transformation of Scalar Superfield Components

$$\begin{aligned}\delta_\epsilon A &= (\epsilon Q + \bar{\epsilon} \bar{Q}) A \\ \delta_\epsilon \psi &= (\epsilon Q + \bar{\epsilon} \bar{Q}) \psi \\ \delta_\epsilon F &= (\epsilon Q + \bar{\epsilon} \bar{Q}) F.\end{aligned}\quad (1.32)$$

Now consider the dimensionality of the quantities involved. Since $Q \sim \sqrt{H} \sim \sqrt{E} \Rightarrow$

$$|Q| = m^{1/2}$$

Similarly

$$|\theta| = m^{-1/2} \quad |d\theta| = m^{1/2} \quad |A| = m \quad |\psi| = m^{3/2} \quad |F| = m^2 \quad |\bar{Q}| = m^{1/2}.\quad (1.33)$$

In constructing a supermultiplet with a left-handed Weyl spinor ψ we might try $QA \sim \psi$ on dimensional grounds and since $|Q\psi| = m^2$ we could invent a state with dimension m^2 or we can have $Q\psi \sim \partial_\mu A$. In general when we operate with Q we will obtain a state of higher dimension by $m^{1/2}$ or the derivative of a state of lower dimension. It can be shown that $\bar{Q}$ adds $\frac{1}{2}$ a unit of right-handed helicity while Q adds $\frac{1}{2}$ a unit of left-handed helicity (helicity here means s_z). As always adding $j = \frac{1}{2}$ may result in states of $j_z \pm \frac{1}{2}$ corresponding to adding $|\frac{1}{2}, \frac{1}{2}\rangle$ or $|\frac{1}{2}, -\frac{1}{2}\rangle$. Thus operating with δ_ϵ results in both states which have been increased and decreased by $j = \frac{1}{2}$. Since we are considering a multiplet with a left-handed spin we take $\bar{Q}A = 0$ (since it would give a right-handed Weyl spinor). Take

$$QA \sim \sqrt{2}\psi.\quad (1.34)$$

(The $\sqrt{2}$ is conventional.) Similarly

$$Q\psi = \sqrt{2}F \quad \bar{Q}\psi = i\sqrt{2}\sigma^\mu \partial_\mu A.\quad (1.35)$$

Since there is no field with θ^3 coefficient F had better transform entirely into ψ 's. (Also $F \sim Q^2 A$ and so $QF = 0$ since $Q^3 = 0$ being anti-commuting) thus

$$QF = 0 \quad \bar{Q}F = i\sqrt{2}\partial_\mu \sigma^\mu \psi\quad (1.36)$$

or

$$\begin{aligned}\delta_\epsilon A &= \sqrt{2}\epsilon\psi \\ \delta_\epsilon \psi &= i\sqrt{2}\sigma^\mu \epsilon \partial_\mu A + \sqrt{2}\epsilon F \\ \delta_\epsilon F &= i\sqrt{2}\epsilon \sigma^\mu \partial_\mu \psi = \partial_\mu \{i\sqrt{2}\epsilon \sigma^\mu \psi\}.\end{aligned}\quad (1.37)$$

Note that $\delta_\epsilon F$ is a total divergence. This is of critical significance and is always true of the "top-most" component field of a superfield (i.e. that which is multiplied by the most powers of θ 's and $\bar{\theta}$'s). The reason is that if we use the F -term as a Lagrangian density then $L = \int dx F$ will transform under a supersymmetric shift δ_ϵ by $\sim \int \partial_\mu \psi \sim \int \psi \cdot dS$ which is a surface term at infinity and vanishes for topologically trivial systems (such as all that will be considered here). Thus $\delta_\epsilon L = 0$ and the Lagrangian will be invariant under supersymmetry transformations. More on this in Section 1.2.

Vector Superfields

The previous sections introduced the scalar or chiral superfield which contains a Weyl spinor, a scalar and an object " F " called an auxiliary field. This was subject to the constraint $\bar{D}\phi = 0$. Here we introduce the multiplet which contains

gauge fields. The constraint used is that the superfield is real. We saw from (I.24) and the subsequent discussion that the expansion of such a superfield into its component fields is

$$V = \frac{1}{2}(x) + \bar{\theta}\psi + \bar{\psi}\theta + \theta^2\alpha + \bar{\theta}^2\alpha^* + \theta\sigma^\mu\bar{\theta}A_\mu + \lambda\bar{\theta}\theta\theta + \bar{\theta}\bar{\theta}\bar{\theta}\bar{\lambda} + D\bar{\theta}\bar{\theta}\bar{\theta} \quad (I.38)$$

Any superfield of the form $\bar{\phi}\phi$ is a real superfield ($\bar{\phi} = \phi^\dagger$) and similarly any terms like $\phi + \bar{\phi}$ in a Lagrangian will be real. In particular ϕ could be a scalar superfield.

Super Gauge Transformations

In order to expose the physical degrees of freedom of the vector superfield, V , we will have to foreshadow our discussion of Lagrangian construction of the next section.

The kinetic term of a supersymmetric Lagrangian can be written as ($d^4\theta \equiv d^2\theta d^2\bar{\theta}$)

$$\mathcal{L} = \int d^4\theta \bar{\phi}(y^1, \bar{\theta})\phi(y, \theta) \quad (I.39)$$

where

$$y^\mu = x^\mu + i\theta\sigma^\mu\bar{\theta} \quad y^{\mu\dagger} = x^\mu - i\theta\sigma^\mu\bar{\theta} \quad (I.40)$$

recalling that

$$\bar{D}\phi = 0 \quad D\bar{\phi} = 0. \quad (I.41)$$

This may seem like a strange kinetic term since it has no derivatives in it! (One might expect $\int d^4\theta \bar{\phi} D\phi$ or some such similar expression.) Remember that ϕ

and $\bar{\phi}$ are at *different points in superspace* ($x^\mu \pm i\theta\sigma^\mu\bar{\theta}$) and so we have a *difference* instead of a derivative.

Consider an Abelian theory for now. In a standard theory matter fields transform under the group rotation

$$\psi \rightarrow e^{i\Lambda(z)}\psi \quad (I.42)$$

where we have absorbed the charge into the definition of $\Lambda(z)$. When $\Lambda(z) = \Lambda$, i.e. the rotation is rigid (global), the kinetic term $\mathcal{L}_{KIN} = \bar{\psi}\not{\partial}\psi$ is invariant under (I.42). When $\Lambda(z)$ is spatially dependent

$$\bar{\psi}\not{\partial}\psi \rightarrow \bar{\psi}e^{-i\Lambda(z)}\not{\partial}[e^{i\Lambda(z)}\psi] = \bar{\psi}\not{\partial}\psi + i\bar{\psi}(\not{\partial}\Lambda(z))\psi.$$

To compensate we make the standard "minimal substitution"

$$\partial^\mu \rightarrow D^\mu = \partial^\mu + ieA^\mu \quad (I.43)$$

where A^μ is a new field, called a gauge field, subject to the transformation

$$A^\mu \rightarrow A^\mu - \partial^\mu\Lambda(z) \quad (I.44)$$

when ψ is subject to (I.42). " D " is the "covariant derivative". The "covariant" Lagrangian (i.e. invariant under the gauge transformation (I.44)) is

$$\mathcal{L}_{KIN} = \bar{\psi}\not{D}\psi = \bar{\psi}(\not{\partial} + ie\not{A})\psi$$

transforms as

$$\begin{aligned} \bar{\psi}\not{D}\psi &\rightarrow \bar{\psi}e^{-i\Lambda(z)}(\not{\partial} + ie\not{A} + ie\not{\partial}\Lambda(z))e^{i\Lambda(z)}\psi \\ &= \bar{\psi}(\not{\partial} + ie\not{A})\psi \\ &= \bar{\psi}\not{D}\psi. \end{aligned} \quad (I.45)$$

We desire a similar realization for a supersymmetric version of an Abelian gauge theory. Let us promote $\Delta(x)$ to be a chiral (scalar) superfield

$$\bar{D}\Delta(x, \theta, \bar{\theta}) = 0 \quad D\bar{\Delta}(x, \theta, \bar{\theta}) = 0. \quad (1.46)$$

The matter field is also a scalar superfield, i.e. ψ is replaced by $\phi(x, \theta, \bar{\theta})$ and $\bar{\psi}$ by $\bar{\phi}(x, \theta, \bar{\theta})$. The kinetic term (1.39) becomes

$$\bar{\phi}\phi \rightarrow \bar{\phi}(y^\dagger, \bar{\theta}) e^{-i\bar{\Delta}(y, \bar{\theta})} e^{i\Delta(y, \theta)} \phi(y, \theta) \neq \bar{\phi}\phi \quad (1.47)$$

where we have used (1.30). We can amend this in the same fashion as before by introducing a (vector) gauge superfield:

$$\mathcal{L}_{KIN} \rightarrow \bar{\phi} e^V \phi \quad (1.48)$$

where

$$V \rightarrow V + i(\bar{\Delta} - \Delta) \quad (1.49)$$

and so

$$\begin{aligned} \bar{\phi} e^V \phi &\rightarrow \bar{\phi} e^{-i\bar{\Delta}} e^{i(\bar{\Delta} + V - i\Delta)} e^{i\Delta} \phi \\ &= \bar{\phi} e^V \phi. \end{aligned} \quad (1.50)$$

The non-Abelian case is similar except that

$$\phi \rightarrow e^{iT^a \Lambda_a} \phi \quad \bar{\phi} \rightarrow e^{-i\bar{\Lambda}_a T^{a\dagger}} \bar{\phi} \quad (1.51)$$

and the gauge transformation take the highly non-linear form

$$e^V \rightarrow e^{+i\bar{\Lambda}_a T^{a\dagger}} e^V e^{-iT^a \Lambda_a}. \quad (1.52)$$

Note that in such a transformation we must maintain the constraint on the superfields, that is chiral superfield remain chiral and vector (real) superfields

remain so. Recalling from the section on chiral superfields that any function of chiral superfields is itself a chiral superfield (and similarly for anti-chiral fields) we see that $e^{i\Delta(x)}$ is chiral and so is $\phi e^{i\Delta(x)}$ so that the scalar nature is retained under this transformation. From (1.49), since $\bar{\Delta} = \Delta^\dagger$, we see that the vector nature of V is also retained.

Functions of chiral fields, ϕ_i with $\bar{D}\phi_i = 0$, are chiral because

$$\bar{D}F(\phi_i) = \frac{\partial F}{\partial \phi_i} \bar{D}\phi_i = 0 \quad (1.53)$$

and similarly $DF(\bar{\phi}_i) = 0$. Products of chiral and anti-chiral superfields are, in general, neither chiral nor anti-chiral.

Wess-Zumino Gauge

We are going to make use of our gauge freedom in the selection of a particular $\Lambda(y, \theta)$ to reduce the degrees of freedom (i.e. component fields) in V as much as possible. From (1.30) and (1.31)

$$\phi = \tilde{\psi}(y) + \sqrt{2}\theta\psi(y) + \theta^2 F(y)$$

$$y = x + i\theta\sigma\bar{\theta}$$

substituting and expanding about $x = y$ as a power series allows us to express ϕ in terms of x^μ :

$$\begin{aligned} \phi(x, \theta, \bar{\theta}) &= \tilde{\psi}(x) + \sqrt{2}\theta\psi(x) + \theta^2 F(x) + i\theta\sigma^\mu\bar{\theta}\partial_\mu\tilde{\psi}(x) \\ &\quad - \frac{i}{\sqrt{2}}\theta^2\partial_\mu\psi(x)\sigma^\mu\bar{\theta} + \frac{1}{4}\theta^2\bar{\theta}^2\partial_\mu\partial^\mu\tilde{\psi}(x) \end{aligned} \quad (1.54)$$

with a similar expression for $\bar{\phi}$ (the $\tilde{\psi}$ is a bosonic field [we formerly called it

$A(x))$ whereas, from (1.38)

$$V(x, \theta, \bar{\theta}) = s(x) + \bar{\theta}\chi + \bar{\chi}\theta + \theta^2\alpha + \bar{\theta}^2\alpha^* + \theta\sigma^\mu\bar{\theta}A_\mu + \lambda\bar{\theta}\theta\theta + \bar{\theta}\theta\bar{\theta}\bar{\lambda} + D\theta\theta\theta\bar{\theta}. \quad (1.55)$$

Thus for the gauge transformation $V \rightarrow V + i(\bar{\Lambda} - \Lambda)$ we can select $\tilde{\psi}_\Lambda + \bar{\tilde{\psi}}_\Lambda$ (i.e. $Re \tilde{\psi}_\Lambda$) to cancel $s(x)$; kill α using F_Λ and α^* using $F_\Lambda^\dagger$ (from $\bar{\Lambda}$); and cancel off χ with ψ_Λ (the " Λ " means "from expanding $\Lambda(x, \theta, \bar{\theta})$ as we did $\phi(x, \theta, \bar{\theta})$ in (1.54)"). Thus we are free to set $s, \chi, \bar{\chi}, \alpha$ and α^* to zero in (1.55). This is akin to using the gauge $A_0 = 0$ in QED. We are left with the relatively simple set of component fields:

$$V(x, \theta, \bar{\theta}) = -\theta\sigma^\mu\bar{\theta}A_\mu(x) + i(\theta^2\bar{\theta}\bar{\lambda}(x) - \bar{\theta}^2\theta\lambda(x)) + \frac{1}{2}\theta^2\bar{\theta}^2D(x). \quad (1.56)$$

We have altered the coefficients to conform with the definitions used in Wess and Bagger. This cancellation can be seen from writing out the transformations of the components explicitly. For instance

$$s(x) \rightarrow s(x) + \tilde{\psi}_\Lambda(x) + \bar{\tilde{\psi}}_\Lambda(x)$$

which leads to our choice of setting $s(x)$ to zero.

This special choice of gauge is called the **Wess-Zumino Gauge**. We shall use it henceforth at all times. By working in a particular gauge our calculations will lose manifest supersymmetry invariance.

We have not exhausted our gauge freedom yet. Since $Re \tilde{\psi}_\Lambda$ was used to cancel $s(x)$ we still have $Im \tilde{\psi}_\Lambda$ to play with. This implements the standard $U(1)$

gauge transformations (this is all for the Abelian case):

$$A_\mu \rightarrow A_\mu + \partial_\mu(Im \tilde{\psi}_\Lambda). \quad (1.57)$$

We are left with a gauge vector boson A_μ , an associated Weyl gauge fermion (λ) and an auxiliary field which transforms into a total derivative (just as F did) under a supersymmetry transformation δ_λ .

Aside from eliminating so many non-physical fields, the Wess-Zumino gauge also possesses the useful property of being of quadratic or higher powers in $\theta, \bar{\theta}$ in all of its components. For this reason $V^3 = 0$ and any function of V terminates after only 3 terms. In particular

$$e^V = 1 + V + \frac{1}{2}V^2 \quad (1.58)$$

a welcome simplification in the kinetic term $\mathcal{L} = \bar{\phi}e^V\phi$.

1.2 ANATOMY OF A LAGRANGIAN

In the previous section we have established the machinery to construct supersymmetric Lagrangians.

Supersymmetry Invariance

In addition to all the invariance properties we normally expect from a Lagrangian (gauge, Lorentz) we additionally require that $\mathcal{L}$ be invariant under supersymmetry transformations. If $\mathcal{L}$ is a Lagrangian density then the full Lagrangian, L , is obtained by integrating $\mathcal{L}$ over all non-timelike coordinates

$$L = \int d^3x d^4\theta \mathcal{L}(x^\mu, \theta, \bar{\theta}) . \quad (1.59)$$

Since $\int d\theta_i = 0$ and $\int \theta_i d\theta_i = 1$ we see that (1.59) has no anti-commuting character left ($L = L(t)$) and that (1.59) merely projects out the $\theta^2\bar{\theta}^2$ ($\equiv \theta^4$) component.

When we have only scalar fields, $\phi(y, \theta)$ and $\bar{\phi}(y^\dagger, \bar{\theta})$, as terms in a Lagrangian (and no terms involving both) we see that

$$L = \int d^3y \left[\int d^2\theta F(\phi) + \int d^2\bar{\theta} F^*(\bar{\phi}) \right] \quad (1.60)$$

where (since $\bar{\theta} = \theta^\dagger$) L is now manifestly real and we have used the assumed translational property of the differentials $d^3x = d^3y$. For vector superfields we must use (1.59)

$$L = \int d^3x \int d^4\theta [F(V) + F^*(V)] \quad (1.61)$$

and no conjugation of V is needed since V is real.

Superpotentials

The term which corresponds to the potential in an ordinary field theory is the superpotential. Denoted $W[\phi_i]$ it is a function of the chiral (but not anti-chiral) superfields in the theory and is thus itself a chiral superfield. It appears in $\mathcal{L}$ as

$$\mathcal{L}_{POT} = \int W[\phi] d^2\theta + h.c. \quad (1.62)$$

where $h.c. = \int W^*[\bar{\theta}] d^2\bar{\theta}$ ensures a real Lagrangian. Such terms are responsible for masses and Yukawa couplings. This is known as the "F" sector since the highest (most θ 's) component fields is of the "F" type (see (1.31)).

From (1.33) we have $[\phi] = m$ and $[d^2\theta] = m$ so that the most general renormalizable term of the type (1.62) is

$$W[\phi] = C + \lambda_i \phi_i + m_{ij} \phi_i \phi_j + g_{ijk} \phi_i \phi_j \phi_k . \quad (1.63)$$

We will ignore C which could only play a role in supergravity. Note that there is no quartic term so one might suppose that four boson couplings cannot occur. Since the "F" term always appears without a derivative in $\mathcal{L}$ (once it has been expanded into its component fields) we can trivially obtain its equation of motion via Lagrange's equations

$$\frac{\delta \mathcal{L}}{\delta F} = \partial_\mu \frac{\delta \mathcal{L}}{\delta (\partial F / \partial x^\mu)} = 0 . \quad (1.64)$$

When F is solved for and resubstituted into $\mathcal{L}$ we might obtain quartic scalar component field couplings (once the other terms in $\mathcal{L}$ have been added). Note that in (1.63) there may be many different scalar fields.

Why we need more Higgs bosons

The Higgs-scalar interactions which generate quark masses originate in the cubic terms. It is because we can only use ϕ , and not $\bar{\phi}$, that the supersymmetric standard model requires two Higgs doublets. In the standard model Lagrangian we have

$$\mathcal{L} = \bar{C}^{ij} H \ell_L \bar{\ell}_{Rj} + d^{ij} H q_L \bar{d}_{Rj} + u^{ij} H' q_L \bar{u}_{Rj} + h.c. \quad (1.65)$$

where $\bar{C}^{ij}$ is diagonal and $\langle H \rangle = v$. The supersymmetric version would be

$$\mathcal{L} = \bar{C}^{ij} \hat{H} \hat{L}_i \hat{R}_j + d^{ij} \hat{H} \hat{Q}_i \hat{D}_j + u^{ij} \hat{H}' \hat{Q}_i \hat{U}_j \quad (1.66)$$

and since $\hat{H}^\dagger = \hat{H}^T$ cannot appear in W we must define a second $\hat{H}$, $\hat{H}_2$, with the electroweak quantum numbers of $\hat{H}^\dagger$.

Components of the Superpotential

In terms of component fields ($\tilde{\psi}$ scalar, ψ a left-handed Weyl spinor, F a bosonic auxiliary):

$$\phi_i(y, \theta) = \tilde{\psi}_i(y) + \sqrt{2}\theta\psi_i(y) + \theta\theta F_i(y)$$

$$\phi_i\phi_j = \tilde{\psi}_i(y)\tilde{\psi}_j(y) + \sqrt{2}\theta[\tilde{\psi}_i\tilde{\psi}_j + \tilde{\psi}_i\psi_j] + \theta\theta[\tilde{\psi}_i F_j + \tilde{\psi}_j F_i - \psi_i\psi_j]$$

$$\phi_i\phi_j\phi_k = \tilde{\psi}_i\tilde{\psi}_j\tilde{\psi}_k + \sqrt{2}\theta[\tilde{\psi}_i\tilde{\psi}_j\tilde{\psi}_k + \tilde{\psi}_i\tilde{\psi}_j\psi_k + \tilde{\psi}_i\tilde{\psi}_j\psi_k]$$

$$+ \theta\theta[F_i\tilde{\psi}_j\tilde{\psi}_k + \tilde{\psi}_i F_j\tilde{\psi}_k + \tilde{\psi}_i\tilde{\psi}_j F_k - \psi_i\tilde{\psi}_j\tilde{\psi}_k - \psi_j\tilde{\psi}_i\tilde{\psi}_k - \psi_k\tilde{\psi}_i\tilde{\psi}_j] \quad (1.67)$$

obtained by simple multiplication ignoring θ^3 or higher terms. When we take $\int d^2\theta$ of these we project out the $\theta\theta$ ("F") term. The $\sqrt{2}$ is in keeping with Wess

and Bagger notation. For

$$W[\phi_i] = \lambda_i\phi_i + m_{ij}\phi_i\phi_j + g_{ijk}\phi_i\phi_j\phi_k$$

$$\begin{aligned} \mathcal{L}_{TOT} &= \int W[\phi_i] d^2\theta = \lambda_i\phi_i|_{\theta=0} + m_{ij}\phi_i\phi_j|_{\theta=0} + g_{ijk}\phi_i\phi_j\phi_k|_{\theta=0} \\ &= \lambda_i F_i + m_{ij}[\tilde{\psi}_i F_j + \tilde{\psi}_j F_i - \psi_i\psi_j] \\ &\quad + g_{ijk}[F_i\tilde{\psi}_j\tilde{\psi}_k + \tilde{\psi}_i F_j\tilde{\psi}_k + \tilde{\psi}_i\tilde{\psi}_j F_k - \psi_i\tilde{\psi}_j\tilde{\psi}_k - \psi_j\tilde{\psi}_i\tilde{\psi}_k - \psi_k\tilde{\psi}_i\tilde{\psi}_j] \end{aligned} \quad (1.68)$$

If m_{ij} and g_{ijk} are symmetric then we can write this as

$$\mathcal{L} = \lambda_i F_i + \frac{1}{2} m_{ij} \{ \tilde{\psi}_i F_j - \psi_i \psi_j \} + \frac{1}{6} g_{ijk} \{ F_i \tilde{\psi}_j \tilde{\psi}_k - \psi_i \psi_j \tilde{\psi}_k \} \quad (1.69)$$

Note that, for the term $W[\phi] = \phi_1\phi_2\cdots\phi_N$, that the θ^3 (i.e. "F") coefficient will be

$$\begin{aligned} &\tilde{\psi}_1\tilde{\psi}_2\cdots\tilde{\psi}_{N-1}F_N + \tilde{\psi}_1\tilde{\psi}_2\cdots\tilde{\psi}_{N-2}F_{N-1}\tilde{\psi}_N + \cdots + F_1\tilde{\psi}_2\tilde{\psi}_3\cdots\tilde{\psi}_N \\ &\quad + \psi_1\tilde{\psi}_2\tilde{\psi}_3\tilde{\psi}_4\cdots\tilde{\psi}_N + \psi_1\tilde{\psi}_2\tilde{\psi}_3\tilde{\psi}_4\cdots\tilde{\psi}_N \\ &\quad + \cdots + \tilde{\psi}_1\tilde{\psi}_2\tilde{\psi}_3\cdots\tilde{\psi}_{N-2}\psi_{N-1}\psi_N \end{aligned} \quad (1.70)$$

which can be written as

$$\left. \frac{\partial W[\phi]}{\partial \phi_i} \right|_{\phi_i=\tilde{\psi}_i} F_i + \left. \frac{\partial^2 W[\phi]}{\partial \phi_i \partial \phi_j} \right|_{\phi_i=\tilde{\psi}_i} \psi_i \psi_j \quad (1.71)$$

In particular for $\phi_i = \phi_j \forall i, j$, i.e. $W[\phi] = \phi^N$ we have θ^3 coefficient

$$\begin{aligned} &N\tilde{\psi}^{N-1}F + N(N-1)\tilde{\psi}^{N-1}\psi^2 \\ &= W'F + W''\psi\psi \end{aligned} \quad (1.72)$$

evaluated at $\phi = \tilde{\psi}$.

A more convenient notation might be

$$\mathcal{L}_{POT}(V) = W_a(\tilde{\psi})F_i + W_{ij}(\tilde{\psi})\psi_i\psi_j \quad (1.73)$$

where $W_a(\tilde{\psi})$ means $\partial W[\phi_1, \dots, \phi_N]/\partial \phi_i$ evaluated at $\phi_1 = \tilde{\psi}_1, \dots, \phi_N = \tilde{\psi}_N$.

Solving for F

We may expand the kinetic term in (1.40) by using (1.54) and its conjugate. We find that the θ^4 coefficient is $-\frac{1}{4}\partial_i\partial_j\phi_j(x, \theta, \bar{\theta})$ is

$$\begin{aligned} \partial_i(x, \theta, \bar{\theta})\phi_j(x, \theta, \bar{\theta}) \Big|_{\theta\bar{\theta}} &= F_i^*F_j + \frac{1}{4} \left[\tilde{\psi}_i^* \square \tilde{\psi}_j + \tilde{\psi}_j \square \tilde{\psi}_i^* \right] \\ &\quad - \frac{1}{2} (\partial_\mu \tilde{\psi}_i^*) (\partial^\mu \tilde{\psi}_j) + \frac{i}{2} \left[(\partial_\mu \tilde{\psi}) \sigma^\mu \psi_j - \tilde{\psi}_i \sigma^\mu \partial_\mu \psi_j \right] \end{aligned} \quad (1.74)$$

writing (thus far)

$$\mathcal{L} = \mathcal{L}_{KIN} + \mathcal{L}_{POT} = \int \bar{\partial}_i \phi_i d^4\theta + \int W[\phi_i] d^2\theta + \int W^*[\bar{\phi}_i] d^2\bar{\theta} \quad (1.75)$$

and integrating $\mathcal{L} = \int d^4x \mathcal{L}$ by parts to convert the $(\partial_\mu \tilde{\psi}_i^*)(\partial^\mu \tilde{\psi}_i)$ term in (1.74) to the form $\tilde{\psi}_i^* \square \tilde{\psi}_i$, we obtain (using (1.69))

$$\begin{aligned} \mathcal{L} &= i(\partial_\mu \tilde{\psi}_i) \sigma^\mu \psi_i + \tilde{\psi}_i^* \square \tilde{\psi}_i + F_i^* F_i \\ &\quad + \left[m_{ij}(\tilde{\psi}_i F_j - \frac{1}{2} \psi_i \psi_j) + g_{ijk}(\tilde{\psi}_i \tilde{\psi}_j F_k - \psi_i \psi_j \tilde{\psi}_k) + \lambda_i F_i + h.c. \right]. \end{aligned} \quad (1.76)$$

By using (1.64) (or its conjugate) for the equation of motion of F

$$\frac{\partial \mathcal{L}}{\partial F_i} = F_i^* + \lambda_i + m_{ik} \tilde{\psi}_i + g_{ijk} \tilde{\psi}_i \tilde{\psi}_j = 0 \quad (1.77)$$

or

$$F_i^* = -(\lambda_i + m_{ik} \tilde{\psi}_i + g_{ijk} \tilde{\psi}_i \tilde{\psi}_j). \quad (1.78)$$

Now examining (1.63) (with $C=0$) we see that

$$F_i^* = - \frac{\partial W[\phi]}{\partial \phi_i} \Big|_{\phi_i = \tilde{\psi}_i}. \quad (1.79)$$

Consequently from (1.71)-(1.73)

$$\begin{aligned} \mathcal{L}_{POT} &= - \sum_i \left| W_a(\tilde{\psi}) \right|^2 + \sum_{ij} W_{ij}(\tilde{\psi}) + h.c. \\ &= - \sum_i |F_i|^2 + \sum_{ij} W_{ij}(\tilde{\psi}) + h.c. \end{aligned} \quad (1.80)$$

By using (1.78) in (1.76) we can eliminate F and F^* and place $\mathcal{L}$ in the form

$$\mathcal{L} = i\partial_\mu \psi_i^* \sigma^\mu \psi_i + \tilde{\psi}_i^* \square \tilde{\psi}_i \quad (1.81)$$

$$- \left(\frac{1}{2} m_{ik} \psi_i \psi_k + g_{ijk} \psi_i \psi_j \tilde{\psi}_k + h.c. \right) - \mathcal{V}(\tilde{\psi}, \tilde{\psi}^*)$$

where the potential $\mathcal{V}$ is

$$\mathcal{V} = F_i^* F_i. \quad (1.82)$$

From (1.78) we see that $\mathcal{V}$ will provide a mass term for $\tilde{\psi}_i$ (a scalar) of

$$m_{\tilde{\psi}} = \sum_i m_{ik} m_{ik}^* \tilde{\psi}_i^* \tilde{\psi}_i + \left(\sum_{jk} g_{ijk} \lambda_i^* \tilde{\psi}_i \tilde{\psi}_j + h.c. \right) \quad (1.83)$$

$$m_{\tilde{\psi}} = \tilde{\psi}^T m m^T \tilde{\psi} + \left(\tilde{\psi}^T [g_k \lambda_k^*] \tilde{\psi} + h.c. \right)$$

where the second line was written in matrix form with $(\tilde{\psi}_i)$ a column vector, $\tilde{\psi}^T = (\tilde{\psi}_1 \tilde{\psi}_2 \dots \tilde{\psi}_n)$ and m, g_k matrices.

Note that since $D\alpha = \bar{D}\alpha = 0$, where α is a constant, that $\bar{D}(\phi + \alpha) = 0$ is still a scalar superfield. Thus shifting $\phi \rightarrow \phi + \alpha$, $\bar{\phi} \rightarrow \bar{\phi} + \alpha^*$ in a Lagrangian yields another perfectly valid Lagrangian.¹ For this reason it is often possible to eliminate the $\lambda_i \phi_i$ term from $W[\phi_i]$. If that is done then (I.83) becomes

$$m_{\tilde{j}} = \tilde{\psi}^\dagger [m] \tilde{\psi} \quad (\lambda_i = 0) \quad (I.84)$$

and from (I.81)

$$m_{\phi} = \frac{1}{2} \psi^T m \psi + \text{h.c.} \quad (I.85)$$

and we see an explicit symmetry between the fermion (ψ, ψ^*) and boson $(\tilde{\phi}, \tilde{\phi}^*)$ masses. In particular when there is only one scalar field, ϕ , present with components $\tilde{\psi}$ and ψ

$$-\mathcal{L}_{\text{mass}} = m^2 \tilde{\psi}^* \tilde{\psi} + \frac{1}{2} m \psi \psi + \frac{1}{2} m \psi^* \psi^*. \quad (I.86)$$

When there are multiple ϕ_i , (I.84) and (I.85) lead to non-diagonal mass terms in $\mathcal{L}$. Both may be simultaneously diagonalized by an orthogonal rotation $\psi_i \rightarrow O_{ij} \psi_j$; $\tilde{\psi}_i \rightarrow O_{ij} \tilde{\psi}_j$, i.e. the superfields may be mixed $\phi_i \rightarrow O_{ij} \phi_j$ with

$M_D = O^T m O$. Note that the fermion mass terms are Majorana. Additional Dirac mass terms can be obtained via the Higgs mechanism in the usual way or via supersymmetry-breaking.

R-Parity

R-parity is an additive quantum number that we frequently impose to further restrict our class of theories. It may formerly be written as a global $U(1)$

symmetry of $\mathcal{L}$ (see references 1, 3 and 39.) It may also be written as (in lesser generality)¹⁴

$$R = (-1)^{2J+2B+L} \quad (I.87)$$

where J , B , and L are the spin, baryon number and lepton number of a particle. It is also true that

$$R = -1 \quad \text{for superpartners} \quad (I.88)$$

$$R = +1 \quad \text{for ordinary matter.}$$

In a renormalizable Lagrangian, invariance under R may be broken by the terms⁴⁰

$$\phi_i \quad \text{or} \quad \phi_i \phi_j \quad (i \neq j). \quad (I.89)$$

This is not surprising as $\mathcal{L} = \int \lambda \phi_i \theta^2 \theta$ contains a term which would require that a single supersymmetric state be created. Conservation of R -parity implies that supersymmetric states must be created in pairs and consequently the lightest supersymmetric particle is stable. For most scalar superfields the spinor term (ψ) is mundane matter and the scalar $(\tilde{\psi})$ is the superpartner. The sole exception is the Higgs superfield where scalar component field is the normal Higgs boson H (i.e. $\tilde{\psi} \equiv H$) and whose fermionic partner has odd R -parity $(\psi \equiv \tilde{H})$. This explains how a term such as $g_{ijk} \psi_i \psi_j \tilde{\psi}_k$ can be invariant under R -parity. If $\phi_k = \tilde{H}_k$, so that $\tilde{\psi}_k = H_k$, and $\psi_i = Q_i = \begin{pmatrix} u \\ d \end{pmatrix}_i$ while $\psi_j = \bar{u}_j$ or $\bar{d}_j$ we have a normal Yukawa-type Higgs term where each field has $R = +1$. If only a single scalar field of the ϕ_i is present in the theory then a term like $g \phi_i^2$ would violate R -parity (as well as lepton number and charge).

Comments on the Auxiliary Fields

We have seen that the component field of highest dimensionality (F for scalar superfields; D for vector superfields) is an auxiliary field, that is appears with no derivative couplings in $\mathcal{L}$. Such fields are non-dynamic (i.e. do not propagate) and, via their equations of motion ($\partial\mathcal{L}/\partial F = 0$), provide a constraint on the other fields. The question arises as to why we bother including such fields at all. (This is somewhat akin to asking why we bother including a sigma field in the linear sigma model when we are going to solve for it in producing the non-linear sigma model).

We have seen that F is extremely convenient in characterizing the potential energy term (V) when a scalar superpotential is added. A more fundamental reason lies in the fact that an unbroken supersymmetric theory must have an equal number of fermionic and bosonic degrees of freedom. Indeed the supersymmetry algebra can be derived from this statement¹⁸ (plus a few assumptions and requirements such as closure). Looking back at Eqs. (1.32), (1.34) and (1.35) we may ask why, given the superfield components A (a scalar, called $\tilde{\psi}$ elsewhere) and ψ (a left-handed Weyl spinor), we could not simply close the algebra with $Q\psi = 0$? (i.e. make θ and $\bar{\theta}$ one-component objects so that $\phi(y, \theta) = A(y) + \sqrt{2}\theta\psi(y)$ and $Q, \bar{Q}$ would also be one-component anti-commuting operators with $Q^2 = \bar{Q}^2 = 0$.)

Indeed in supersymmetric ($N = 1$) quantum mechanics this can be done (corresponding to an " $N = \frac{1}{2}$ " field theory) but not in a complete field theory. Consider a free field theory of massless particles. On-shell, with A and ψ obeying their equations of motion ($\square A = 0$; $\sigma^\mu\partial_\mu\psi = 0$), ψ has one degree of freedom (one helicity state), as does A , and all is right with the universe. In a field theory A and ψ must be able to propagate off-shell. Then ψ would have two degrees

of freedom (just as if it were massive) but A still only has one. The solution is to introduce an additional bosonic degree of freedom, F , off-shell which has no on-shell degree of freedom. Such a state (remember there is no superpotential term W in this free theory) must have the equation of motion $F = 0$. To obtain this from $\delta\mathcal{L}/\delta F = (\partial/\partial_\mu)(\delta\mathcal{L}/\delta(\partial^\mu F))$ requires that $\partial_\mu F$ not appear and that F^n appear with $n > 1$. Assuming only terms of quadratic order or less in the field for our free Lagrangian we see that F appears only in the term $\frac{1}{2}F^2$ (and so cannot propagate or interact.) By dimensional counting we see that $[F] = m^2$. The invariant action now assumes the form

$$S = \int d^4x \left\{ -\frac{1}{2}(\partial_\mu A)^2 - \frac{1}{2}\psi^\dagger\sigma^\mu\partial_\mu\psi + \frac{1}{2}F^2 \right\}.$$

Another way of looking at this is that without F (1.37) would become (dropping the i and $\sqrt{2}$):

$$\delta_\epsilon A = \epsilon\psi \quad \delta_\epsilon\psi = \sigma^\mu\epsilon\partial_\mu A$$

then

$$[\delta_{\epsilon_1}, \delta_{\epsilon_2}] A = 2\epsilon_2\sigma^\mu\epsilon_1\partial_\mu A \quad (1.90)$$

$$[\delta_{\epsilon_1}, \delta_{\epsilon_2}] \psi = 2\epsilon_2\sigma^\mu\epsilon_1\partial_\mu\psi - \epsilon_2\sigma^\nu\epsilon_1\partial_\nu\sigma^\mu\partial_\mu\psi$$

and, on-shell, $\sigma^\mu\partial_\mu\psi = 0$ so that we have a closed complete super-algebra. Off shell we need F with

$$\delta_\epsilon\psi = \sigma^\mu\epsilon\partial_\mu A + F \quad \delta_\epsilon F = \epsilon\sigma^\mu\partial_\mu\psi$$

which will cancel the $\sigma^\mu\partial_\mu\psi$ term in (1.90).

The same arguments may be applied to vector superfields. In a free, massless theory we have a gauge boson with two degrees of freedom (transverse polarizations) and a Majorana gauge fermion with two degrees of freedom. On shell the vector boson gains a longitudinal polarization and the fermion has four degrees of freedom, so, again, another bosonic degree of freedom is required. Thus the D -field term $\frac{1}{2} D^2$ is born. D , as F , transforms into a total derivative of the fermion field λ .

$$\delta_i D = i \bar{\sigma}^\mu \partial_\mu \lambda \quad \delta_i \bar{D} = i \sigma^\mu \partial_\mu \bar{\lambda}.$$

Super Field Strength

Moving on to the gauge sector we desire to find an analogue of the field strength tensor, $F_{\mu\nu}$, of a standard field theory. We shall again start with Abelian theories.

To motivate our construction we consider that, in a standard field theory

$$F_{\mu\nu} = -i[\partial_\mu, \partial_\nu] = \partial_\mu A_\nu - \partial_\nu A_\mu + i[A_\mu, A_\nu] \quad (1.91)$$

where the final term is absent in the Abelian case under discussion. In a supersymmetric theory we might try to construct a field strength out of the real vector gauge superfield, V , and covariant (super-)derivatives D and $\bar{D}$. Under a gauge transformation $F_{\mu\nu} \rightarrow F_{\mu\nu}$ because it is constructed from covariant (gauge) derivatives. This new object must be invariant under a supergauge transformation as found in (1.49): $V \rightarrow V + i(\bar{\Lambda} - \Lambda)$. Writing the field strength as $\hat{O}V$, a differential operator $\hat{O}$ acting on V , we demand

$$\hat{O}V \rightarrow \hat{O}V \quad \text{or} \quad \hat{O}(\bar{\Lambda} - \Lambda) = 0 \quad (1.92)$$

under the $U(1)$ gauge transformation where we recall that Λ and $\bar{\Lambda}$ are scalar superfields ($D\bar{\Lambda} = 0$; $\bar{D}\Lambda = 0$). Choosing $\hat{O} = D$ or $\hat{O} = \bar{D}$ will not work because $D\Lambda \neq 0$ and $\bar{D}\bar{\Lambda} \neq 0$ (and $D\Lambda \neq \bar{D}\bar{\Lambda}$). Trying $\hat{O} = D\bar{D}$ we note that $D\bar{D}\bar{\Lambda} = -\bar{D}D\bar{\Lambda} + \{D, \bar{D}\}\bar{\Lambda} = \{D, \bar{D}\}\bar{\Lambda}$. Now

$$\{D_\alpha, \bar{D}_\beta\} = -2i\sigma^\mu_{\alpha\beta} \partial_\mu \quad (1.93)$$

and

$$[\partial_\mu, D_\alpha] = \{\partial_\mu, \bar{D}_\beta\} = 0. \quad (1.94)$$

Therefore $D\bar{D}\bar{\Lambda} = -2i\sigma^\mu \partial_\mu \bar{\Lambda} \neq 0$. However, trying the next order

$$\hat{O} = D^2 \bar{D} \quad (1.95)$$

does work

$$\begin{aligned} \hat{O}\Lambda &= D^2 \bar{D}\Lambda = 0 \quad \text{since} \quad \bar{D}\Lambda = 0 \\ \hat{O}\bar{\Lambda} &= D^2 \bar{D}\bar{\Lambda} = -D\bar{D}D\bar{\Lambda} + D\{D, \bar{D}\}\bar{\Lambda} \\ &= -2iD\sigma^\mu \partial_\mu \bar{\Lambda} \quad \text{by (1.93)} \\ &= -2i\sigma^\mu \partial_\mu D\bar{\Lambda} \quad \text{by (1.94)} \\ &= 0. \end{aligned}$$

In the literature $\hat{O}V$ is called W_Δ which is unfortunate since the superpotential is also called W . Its conjugate, W_α , also satisfies the above criteria and is a field strength

$$W_\Delta = -\frac{1}{4} D\bar{D}\bar{D}_\alpha V \quad W_\alpha = \frac{1}{4} \bar{D}\bar{D}D_\alpha V. \quad (1.96)$$

We note some immediately obvious properties of W_α .

1. Since D and $\bar{D}$ are anti-commuting, W_α is an anti-commuting object like θ_α and $\bar{W}_{\dot{\alpha}}$ is like $\bar{\theta}_{\dot{\alpha}}$.
2. Since $\bar{D}^2 = 0$, $\bar{D}W_\alpha = 0$ and W_α is a chiral superfield.
3. From part 2 we can write $W_\alpha = W_\alpha(y, \theta)$ (just as for any chiral field) and $\bar{W}_{\dot{\alpha}} = \bar{W}_{\dot{\alpha}}(y^\dagger, \bar{\theta})$. We can then use the explicit forms of D , $\bar{D}$ and V (in Wess-Zumino gauge, of course!) to expand W_α into its component fields

$$W_\alpha = -i\lambda_\alpha(x) + \{D(y)\theta_\alpha - (\sigma^{\mu\nu})_\alpha{}^\beta \theta_\beta F_{\mu\nu}\} + \theta^2 (\sigma^\mu)_\alpha{}^\beta \partial_\mu \bar{\lambda}^{\dot{\beta}}(y). \quad (1.97)$$

Note that (on-shell) the final term would vanish by the Dirac equation leaving a gauge fermion, auxiliary field (D) and $\sigma^{\mu\nu}F_{\mu\nu}$ term. Since W_α is fermion it cannot appear alone in $\mathcal{L}$ but must appear in a combination such as $\theta^\alpha W_\alpha$ or $W^\alpha W_\alpha$. Also note that the further relation $D^\alpha W_\alpha = \bar{D}_{\dot{\alpha}} \bar{W}^{\dot{\alpha}}$ is satisfied. Taking $\mathcal{L} = \int \theta^\alpha W_\alpha d^2\theta$ would yield (since this projects out the θ^2 term) a convective $\sigma^{\mu\nu}F_{\mu\nu}$ term in $\mathcal{L}$ which we don't want in a QED-like theory. We desire a term akin to $F_{\mu\nu}F^{\mu\nu}$ so we will try $W^\alpha W_\alpha = W_\alpha \epsilon^{\alpha\beta} W_\beta$.

$$\int W^\alpha W_\alpha d^2\theta = W^\alpha W_\alpha|_{\theta\theta} = -2i\lambda^\alpha \partial_\mu \bar{\lambda}^{\dot{\alpha}} - \frac{1}{2} F^{\mu\nu} F_{\mu\nu} + D^2 + \frac{i}{4} F^{\mu\nu} \tilde{F}_{\mu\nu} \quad (1.98)$$

where $\tilde{F}_{\mu\nu} = F^{\alpha\beta} \epsilon_{\alpha\beta\mu\nu}$. This final term is known to be the source of much pain in anomalous theories. In this paper we will always be concerned with topologically trivial sol. s. Then, after some integration by parts, (and discarding the, assumed trivial, surface terms) we have the real supersymmetric version of the Lagrangian for a free vector field

$$\mathcal{L}_{\text{Gauge}} = \int W^\alpha W_\alpha d^2\theta + \int \bar{W}_{\dot{\alpha}} \bar{W}^{\dot{\alpha}} d^2\bar{\theta} = \frac{1}{2} D^2 - \frac{1}{4} F^{\mu\nu} F_{\mu\nu} - i\lambda^\alpha \partial_\mu \bar{\lambda}^{\dot{\alpha}}. \quad (1.99)$$

(The $F^{\mu\nu}F_{\mu\nu}$ is like $E^2 - B^2$ and $F^{\mu\nu}\tilde{F}_{\mu\nu}$ is like $E \cdot B$). In the non-Abelian

case, just as we had the gauge transformation change from $V \rightarrow V - i(\Lambda - \Lambda)$ to $e^V \rightarrow e^{-i\Lambda} e^V e^{i\Lambda}$, the superfield strength becomes

$$W_\alpha = -\frac{1}{4} \bar{D}^2 e^{-V} D_\alpha e^V. \quad (1.100)$$

Constructing Supersymmetric Lagrangia

We now have all of the ingredients necessary for constructing a realistic Lagrangian which is supersymmetrically invariant:

(1) The "Kinetic Term" $\int \bar{\phi} e^V \phi d^4\theta$

This contains kinetic terms for the component fields of ϕ and the matter-gauge coupling terms along with all of the corresponding supersymmetric vertices. In the W-Z gauge these terms conserve R -parity automatically and all vertices may (qualitatively) be obtained by replacing fields from standard vertices in pairs by their supersymmetry counterparts, except when this would result in a new vertex of dimension greater than four. For example, using (the $\sqrt{2}$ is Wess and Bagger notation)

$$\phi(y) = \tilde{\psi}(y) + \sqrt{2}\theta\psi(y) + \theta^2 F(y)$$

$$y^\mu = x^\mu + i\theta\sigma^\mu\bar{\theta}$$

$$\phi(x) = [\tilde{\psi}(x) + i\theta\sigma^\mu\bar{\theta}\partial_\mu A(x) + \frac{1}{2}\theta^2\bar{\theta}^2\Box A(x)] \quad (1.101)$$

$$+ i\sqrt{2}\theta\psi(x) - \frac{i}{\sqrt{2}}\theta^2\partial_\mu\psi(x)\sigma^\mu\bar{\theta} + \theta^2 F(x)$$

$$V = -\theta\sigma^\mu A_\mu - i\bar{\theta}^2\theta\lambda + h.c. + \frac{1}{2}\theta^2\bar{\theta}^2 D$$

$$e^V = 1 + \theta V + \frac{1}{2}\theta^2 V^2$$

yields (remembering that $d^4\theta$ project the $\theta^2\bar{\theta}^2$ term out of the expression)

$$\begin{aligned}\mathcal{L}_{KIN} = & F^* F + \tilde{\psi} \square \tilde{\psi}^* + i \psi^\dagger \bar{\sigma}^\mu \partial_\mu \psi \\ & + \frac{g}{2} \left(\psi^\dagger \bar{\sigma}^\mu A_\mu \psi + i \tilde{\psi}^* A_\mu \partial^\mu \tilde{\psi} - i \tilde{\psi} A_\mu \partial^\mu \tilde{\psi}^* - i \sqrt{2} [\tilde{\psi}^\dagger \lambda^\dagger \psi^\dagger - \tilde{\psi}^* \lambda \psi] \right) \\ & + \frac{1}{2} \tilde{\psi}^* \tilde{\psi} (gD - \frac{1}{2} g^2 A_\mu A^\mu)\end{aligned}\quad (1.102)$$

where this is for a simple scalar matter superfield and a single Abelian vector superfield. This contains the $\frac{ig}{\sqrt{2}} \tilde{\psi}^* \psi \lambda$ "electron-selectron-photino" type of term, the $\frac{ig}{2} \tilde{\psi}^* A \tilde{\psi}$ derivative scalar electrodynamics term (for selections) and the final term, which is a "seagull" term for the selections. There is no order five $\psi^* A_\mu A^\mu \psi$ term unless it is buried in $(g/2) \tilde{\psi}^* \tilde{\psi} D$. Normally the only other term with a " D " in it will be $\frac{1}{2} D^2$ coming from the field-strength term below. Thus, solving for the auxiliary field D from its equations of motion, will lead to a $[\tilde{\psi}^* \tilde{\psi}]^2$ term (i.e. a four-selection vertex) whose coefficient is proportional to g^2 . (In general this will appear as $|\tilde{\psi}^* T^a \tilde{\psi}|^2$ where T^a is the group generator.) This term is instrumental in initiating spontaneous supersymmetry breaking (SSB) (see point four below). Note that if multiple fields are involved in $\mathcal{L}$ that mixing terms will naturally result.

(2) The Superpotential term or " F -sector"

$$\int W[\phi] d^2\theta + h.c. = \int [\lambda_i \phi_i + m_{ij} \phi_i \phi_j + g_{ijk} \phi_i \phi_j \phi_k] \delta^2\theta + h.c.$$

This term encompasses the mass terms, Yukawa couplings and potential energy term for the matter fields,¹¹ plus the various supersymmetric variations.

¹¹ These arise in part from FF^* in the "Kinetic Term" but not until a superpotential is added.

R -parity must be imposed. The Higgs structure of a theory is found in this sector. Recall from (1.67) and (1.69) that the individual terms are

$$\begin{aligned}\int \lambda_i \phi_i d^2\theta &= \lambda_i F_i(y) \text{ (often suppressed by } R\text{-parity)} \\ \int m_{ij} \phi_i \phi_j d^2\theta &= 2m_{ij} \tilde{\psi}_i \tilde{\psi}_j - m_{ij} (-)^p \psi_i \psi_j \\ \int g_{ijk} \phi_i \phi_j \phi_k d^2\theta &= 3g_{ijk} [F_i \tilde{\psi}_j \tilde{\psi}_k - (-)^p \psi_i \psi_j \tilde{\psi}_k]\end{aligned}\quad (1.103)$$

where the $(-)^p$ is to remind us that ψ is anti-commuting and so $\psi_1 \psi_2$ and $\psi_2 \psi_1$ enter with opposite signs (positive for $\psi_1 \psi_2$) and we have assumed that m_{ij} and g_{ijk} are symmetric ($\psi_i \psi_j \tilde{\psi}_k$ enter with positive sign for ijk cyclic permutations of 123)

We found that this could be written as

$$\begin{aligned}\mathcal{L} = & W^* F + W'' \psi \psi \text{ (evaluated at } \phi = \tilde{\psi}) + h.c. \\ = & \frac{\partial W[\phi]}{\partial \phi_i} \Big|_{\phi_i = \tilde{\psi}_i} F_i + \frac{\partial^2 W[\phi]}{\partial \phi_i \partial \phi_j} \Big|_{\phi_i = \tilde{\psi}_i} \psi_i \psi_j + h.c.\end{aligned}$$

and when the kinetic term was added yields

$$\begin{aligned}\mathcal{L} = & \text{Kinetic terms} + \text{gauge-matter couplings} - \left[\frac{1}{2} m_{ij} \psi_i \psi_j + g_{ijk} \psi_i \psi_j \tilde{\psi}_k + h.c. \right] \\ & - \mathcal{V}_{F,E}(\tilde{\psi}, \tilde{\psi}^*)\end{aligned}\quad (1.104)$$

where $\mathcal{V} = F^* F$ is a potential energy term and, since the equation of motion for F is $F = -W^*$, $\mathcal{V} = |W'|^2$.

(3) The Super Field Strength term or " D -sector" $\frac{1}{2} \int W^a W_a d^2\theta + h.c.$

This term includes the free gauge field sector and its supersymmetric counterparts. We find the vector and gaugino kinetic terms and the analogue of the field strength $F_{\mu\nu}^a F^{\mu\nu}$ which, in the non-Abelian case, includes all of the trilinear and quadrilinear gauge self-interactions (and gaugino in the trilinear case). Finally we have an auxiliary D_i field for each vector super-field V_i present

$$\mathcal{L} = -i\lambda_i \sigma^\mu \partial_\mu \bar{\lambda}_i - \frac{1}{4} F_{\mu\nu}^i F^{\mu\nu} + \frac{1}{2} D_i^2. \quad (1.105)$$

(4) Fayet-Iliopoulos Progenitor $\int V d^4\theta$

The only term which has not yet been discussed is a function of the vector superfields, V_i , just as $W[\phi_i]$ is a function of the scalar superfields (there is also a gravity superfield but we will not concern ourselves with it in this paper).

We know that, in the W-Z gauge, the terms V and V^2 arise. The term $\int V^2 d^4\theta$ would introduce vector and gaugino mass terms. Just as such terms are not gauge invariant in ordinary gauge theories, they are not supergauge-invariant here. Under (the Abelian case) $V \rightarrow V - i(\bar{\lambda} - \lambda)$ the term $\int V^2 d^4\theta$ changes by

$$\int V^2 d^4\theta \rightarrow \int V^2 d^4\theta + 2 \int (\bar{\lambda}\lambda) d^4\theta \quad (1.106)$$

and may be excluded.

The linear term, $\int V d^4\theta$, is gauge invariant, however. The reason is that $\int d^4\theta$ is, like d^4/d^4 , a projection operator for the $\theta^2\bar{\theta}^2$ term. Therefore

$$\begin{aligned} \int V(x, \theta, \bar{\theta}) d^2\theta d^2\bar{\theta} &\rightarrow \int V(x, \theta, \bar{\theta}) d^2\theta d^2\bar{\theta} + i(\bar{\lambda}(y, \bar{\theta}) - \lambda(y, \theta)) d^2\theta d^2\bar{\theta} \\ &= \int V(x, \theta, \bar{\theta}) d^2\theta d^2\bar{\theta}. \end{aligned} \quad (1.107)$$

So this term cannot be so easily excluded. If V is not a $U(1)$ gauge superfield

(i.e. mediates a non-Abelian force), $V \rightarrow V_{ij} = T_{ij}^a V^a$, then V carries quantum numbers and $\int V d^4\theta$ would explicitly break the ordinary gauge symmetry and would also be excluded. We are left with the sole possible term

$$\mathcal{L}_{F.I.} = \int c_i V_i d^4\theta = c_i D_i \quad (1.108)$$

where V_i are $U(1)$ vector superfields and the c_i are constants. The $c_i D_i$ are called "Fayet-Iliopoulos" terms. Their sole function in life is to alter the auxiliary D field constraint by a constant and thereby initiate spontaneous supersymmetry-breaking. How this is accomplished will be outlined in the following subsection. Since the theory we will be dealing with will be broken by explicit terms we will assume that no F-I type terms arise. This term (for obvious reasons) is frequently included in the "D-sector" along with the superfield strength term.

Spontaneous Supersymmetry Breaking

I intend to gloss over this very briefly since it is not used elsewhere in this paper. Supersymmetry may be spontaneously broken both in the "F" sector and the "D" sector. Supersymmetry may also be dynamically broken (e.g. "super-color").

Let us first examine the "F" sector. We recall from (1.13) that

$$H \sim \sum_i |Q_i|^2 \geq 0.$$

Thus, if the vacuum state has non-zero energy it must be true that $Q_i|0\rangle \neq 0$ for at least one supersymmetry generator Q_i . The vacuum state is therefore not annihilated by such a generator and supersymmetry is spontaneously broken. For supersymmetry to be unbroken we require $H = 0$ in the vacuum. H achieves its

minimum when the potential energy, $V = F_i^\dagger F_i = \sum_i \left| \frac{\partial W}{\partial \phi_i} \right|^2$, vanishes. Therefore we require $F_i = 0$; $\left. \frac{\partial W}{\partial \phi_i} \right|_{\phi_i = \tilde{\psi}_i} = 0 \forall_i$ in the vacuum. That $F_i = 0$ for unbroken supersymmetry may be understood from the transformations in (1.37). In the vacuum, since there can be no derivative term, we have $(A \approx \tilde{\psi})$

$$\delta_\epsilon A = \sqrt{2} \epsilon (\psi) \quad \delta_\epsilon \psi = \sqrt{2} \epsilon (F) \quad \delta_\epsilon F = 0. \quad (1.109)$$

Hence when we vary the scalar field we obtain zero unless $\langle \psi \rangle \neq 0$. This it would be a disaster since it would imply that the vacuum was not Lorentz invariant! (Actually the vacuum can consist of a *single*, massless, momentum-zero fermion.) Ignoring this possibility we see that if $F_i = 0$ then the variation of all of the component fields under a supersymmetric transformation vanishes.

The other formulation of the condition for unbroken supersymmetry, $\left. \frac{\partial W}{\partial \phi_i} \right|_{\phi_i = \tilde{\psi}_i} = 0 \forall \phi_i$ for some set of $\tilde{\psi}_i$ values (i.e. the vacuum), means that supersymmetry is spontaneously broken if the set of equations, $\frac{\partial W}{\partial \phi_i} = 0$ has no consistent solution.

Examples:

$$1) \quad W(\phi) = \phi^5$$

$$\left. \frac{\partial W}{\partial \phi} \right|_{\phi = \tilde{\psi}} = 0 \Rightarrow 5\tilde{\psi}^4 = 0 \Rightarrow \tilde{\psi} = 0 \text{ is a solution.}$$

$$2) \quad W(\phi) = g\phi^3 + m\phi^2$$

$$\left. \frac{\partial W}{\partial \phi} \right|_{\phi = \tilde{\psi}} = 0 \Rightarrow 3g\tilde{\psi}^2 + 2m\tilde{\psi} = 0 \Rightarrow \tilde{\psi} = 0 \text{ is a solution.}$$

(1.110)

$$3) \quad W(\phi_1, \phi_2, \phi_3) = \lambda\phi_1 + m\phi_2\phi_3 + g\phi_1\phi_2\phi_3$$

$$\left. \frac{\partial W}{\partial \phi_1} \right| = 0 \Rightarrow \lambda + g\phi_2^2 = 0$$

$$\left. \frac{\partial W}{\partial \phi_2} \right| = 0 \Rightarrow m\phi_3 + 2g\phi_1\phi_3 = 0$$

$$\left. \frac{\partial W}{\partial \phi_3} \right| = 0 \Rightarrow m\phi_2 = 0.$$

The first and third equations are clearly inconsistent when λ and m are non-zero and we have spontaneous supersymmetry-breaking. This is called the O'Raifeartaigh Potential.⁴¹ Unlike internal symmetry breaking, in which a scalar field acquires a vacuum expectation value, spontaneous supersymmetry-breaking occurs when a constraint (auxiliary) field acquires a VEV. (Note how the "dangerous" single superfield term $\lambda\phi$ is again involved. $W = \lambda\phi$ would break supersymmetry by itself but could violate R -parity.)

In the " D " sector the Fayet-Iliopoulos term, $\mathcal{L}_{FI} = \epsilon \int V d^4\theta = cD$, provides the breaking. From (1.102), (1.105) and (1.108) we see that (for one Abelian gauge superfield and one scalar superfield), when all of the parts of $\mathcal{L}$ are included, the terms with " D " in them are

$$\mathcal{L}_D = \frac{g}{2} \tilde{\psi}^* \tilde{\psi} D + \frac{1}{2} D^2 + cD \quad (1.111)$$

so $\frac{\partial \mathcal{L}}{\partial D} = 0 = \frac{1}{2} g \tilde{\psi} \cdot \tilde{\psi} + c + D$ or

$$D = -\left(c + \frac{g}{2} \tilde{\psi} \cdot \tilde{\psi}\right) \quad \mathcal{L}_D = -\frac{1}{2} \left(c + \frac{g}{2} \tilde{\psi} \cdot \tilde{\psi}\right)^2. \quad (1.112)$$

This is a potential term; $\mathcal{V}_D$ is to be added to $\mathcal{V}_F$, which replaces the $|\phi|^4$ term discussed earlier. ($\frac{1}{2} \sum_a g_a |\tilde{\psi}^\dagger T^a \tilde{\psi}|^2$ for a non-Abelian gauge group, introducing Clebsch-Gordon coefficients into the relative coupling strengths.) When $c < 0$ the minimum is achieved when $\langle \tilde{\psi} \rangle = \sqrt{\frac{2|c|}{g}}$ and the gauge symmetry is spontaneously broken. Supersymmetry is unbroken because, in the vacuum, when $\langle \tilde{\psi} \rangle = \sqrt{\frac{2|c|}{g}}$ we have a vanishing potential and $H = 0$ is yet possible (unless broken in the F -sector). When $c > 0$, $\mathcal{H}_D > 0$ and we have spontaneous supersymmetry-breaking. In this case the gauge symmetry is unbroken. Note that

$$\begin{aligned} \mathcal{L}_D &= -\frac{1}{2} D^2 = -\mathcal{V}_D \\ \mathcal{V}_D &= \frac{1}{2} D^2 \end{aligned} \quad (1.113)$$

so we may write the total potential.

$$\mathcal{V} = \mathcal{V}_F + \mathcal{V}_D = F_i^\dagger F_i + \frac{1}{2} D_i D_i. \quad (1.114)$$

Again we have spontaneous supersymmetry-breaking when the auxiliary field (D) gets a VEV and gauge breaking when the scalar field does. In general gauge and supersymmetries may be independently conserved or spontaneously broken. In Fig. 1.1 we summarize this breaking picture by showing potential terms of each of the four possibilities (gauge, supersymmetry either preserved or spontaneously broken). Finally spontaneous supersymmetry-breaking results in a Goldstone fermion ("Goldstino") just as spontaneous gauge symmetry breaking results in a Goldstone boson.

The approach actually utilized is simply to add "soft" breaking terms directly into $\mathcal{L}$. Such terms could arise from spontaneous supersymmetry-breaking as discussed (at least *a priori*) at some greater energy scale (specifically from local supersymmetry breaking) but we simply do not worry about such specific origins. In point of fact global spontaneous supersymmetry-breaking has proven singularly unsuccessful in providing realistic models. Supergravity-inspired softly-broken theories appear much more promising.⁴²

One of the nicest aspects of supersymmetry-breaking is that it cannot be induced radiatively (due to improved divergence cancellations; there are still non-perturbative instanton-like effects which can break supersymmetry). There is a "set it and forget it" attitude. If supersymmetry is unbroken at the tree level it will remain so to any number of loops. Masses and Yukawa couplings are not renormalized at all in unbroken supersymmetry (and typically receive only finite corrections in spontaneously or softly broken theories). Only the wavefunctions need be renormalized. The advantage of using only "soft" breaking terms is that these impressive renormalization properties are not destroyed.

How does this supersymmetry breaking manifest itself? Note that both " D " and " F " spontaneous supersymmetry-breaking give masses to the bosons, but not the fermions, thus lifting the mass degeneracy. The fermions gain mass via the traditional Higgs mechanism. Prior to breaking supersymmetry forces the bosonic superpartners to attain identical masses.

Assembling the Lagrangian: SQED

We are now ready to put all the ingredients we have discussed together to produce a realistic theory. Before going to the supersymmetric standard model we present the much simpler case of unbroken supersymmetric QED. Since it

is unbroken we will not require a Fayet-Iliopoulos term (linear in V) or a superpotential term linear in either of the scalar superfields ϕ_L and ϕ_R . Since there are also no Yukawa terms we may do away with trilinear terms in these superfields. V is the photon superfield, ϕ_L contains e_L and its superpartner $\tilde{e}_L$ (the "left-selection") while ϕ_R contains e_R and $\tilde{e}_R$. The subscripts on $\tilde{e}_L$ and $\tilde{e}_R$ convey no angular momentum information (they are scalars after all!) but indicate which of the spinor states the boson is associated with. Elsewhere ϕ_L is called L and ϕ_R is called R . This is convenient when we consider the extension to multiple generations (L_i, R_i) and nontrivial (doublet) structure under other groups ($SU(2)$). Note that since R is a chiral (left-handed) superfield (as is L) then $\bar{R}$ must contain $e_R^\dagger$ while R contains $e_L^\dagger$. This accounts for the differences in charge signs in the kinetic terms.

$$\text{Kinetic Term : } \bar{L}e^{-eV}L + \bar{R}e^{eV}R \quad (1.115)$$

Since we desire a Dirac mass, not a Majorana mass, for the electron, we require a term such as $m(\tilde{e}_L e_R + \tilde{e}_R e_L)$ in the Lagrangian.

$$\text{Superpotential ("F"-term) : } m_e(RL + \bar{L}\bar{R}) . \quad (1.116)$$

As usual the (Abelian) superfield strength is given by $W^a = -\frac{1}{4}\bar{D}^2 D^a V$.

$$\text{Gauge Field ("D"-term) : } \frac{1}{4}WW + \frac{1}{4}\bar{W}\bar{W} . \quad (1.117)$$

Together we have

$$\begin{aligned} \mathcal{L}_{SQED} = & \int \left\{ \bar{L}e^{-eV}L + \bar{R}e^{eV}R \right\} d^4\theta + \int \frac{1}{4}W^2 d^2\theta + \int \frac{1}{4}\bar{W}^2 d^2\bar{\theta} \\ & + m_e \left\{ \int RL d^2\theta + \int \bar{L}\bar{R} d^2\bar{\theta} \right\} . \end{aligned} \quad (1.118)$$

This was presented in Appendix E (Eq. E3). There we used the fact that $\delta(\theta_i) = \theta_i$ for any Grassman variable θ_i

$$\left(\int \theta_i d\theta_i = \int \delta(\theta_i) d\theta_i = 1; \quad \int \theta_i^2 d\theta_i = \int \theta_i \delta(\theta_i) d\theta_i = 0 \right)$$

to bring the final terms into the form $d^4\theta = d^2\theta d^2\bar{\theta}$. The expansion into component fields, using the rules assiduously developed in this chapter, were given in Eq. (E4).

1.3 THE SUPERSYMMETRIC STANDARD LAGRANGIAN

We now use the results of the previous sections to construct the supersymmetric version of the "standard model" (as of 1987) consisting of 3 generations and an $SU(3) \times SU(2) \times U(1)$ symmetry group which breaks spontaneously, via the Higgs mechanism, to $SU(3)_{\text{color}} \times U(1)_{EM}$.

Superfield Content

Using the notation we have established, the superfields required are given in table 1.1.

The Higgs singlet, $\hat{N}$, may be included to implement the standard Higgs mechanism while retaining an unbroken supersymmetry. Note that $\hat{Q}$, $\hat{U}$ and $\hat{D}$ are color triplets while $\hat{G}$ is a color octet.

Kinetic Term

The general form for this term was found to be

$$\mathcal{L}_{KIN} = \int \sum_{\hat{a}} \bar{\phi}_i e^{g^{\hat{a}} T_{i(a)} V_a} \phi_a d^4\theta \quad (1.119)$$

where "i" sums over all of the gauge groups that ϕ transforms non-trivially under. For instance, if $\phi = \hat{Q}$, which experiences $SU(3)$, $SU(2)$, and $U(1)$ forces, then

$$(g_i T_a^i V_a)_{jk} = g_{\text{strong}} \lambda_{jk}^a \hat{G}_a + g_{\text{weak}} \tau_{jk}^a \hat{V}_a + g_Y y \delta_{jk} \hat{V}' \quad (1.120)$$

Since $\hat{R}$ is a singlet under $SU(3)$ and $SU(2)$ only the final term would be required when $\phi = \hat{R}$. Here λ^a are the $SU(3)$ generators.

We shall not be concerned with the action of gluons or gluinos in this paper. Ignoring all but electroweak contributions we can expand $\mathcal{L}_{KIN}$ into its

component field terms. These consist of standard kinetic terms for all of the leptons, quarks and Higgses plus the gauge-coupling terms and their supersymmetric counterparts. Letting¹²

$$\hat{V} \text{ coupling be } g \quad (SU(2)) \quad \hat{V}' \text{ coupling be } g' \quad (U(1)_Y) \quad (1.121)$$

$$\mathcal{L}_{qqV,V'} = -\frac{1}{2} (g \tau_{ij}^a V_a^\mu + Y_Q g' V'^\mu \delta_{ij}) q^i \bar{\sigma}_\mu q^j \quad (1.122)$$

$$- \frac{1}{2} g' Y_U V'^\mu (u_R)^i \bar{\sigma}_\mu u_R - \frac{1}{2} g' Y_D V'^\mu d_R^i \bar{\sigma}_\mu d_R$$

where $q^i = (u_L \ d_L)$ is a weak doublet and $\psi_Q^i \tau_{ij} \psi_Q^j$ is an $SU(2)$ singlet (Lorentz spinor indices have been suppressed). (1.122) must also be summed over generations. Of course we may write $r^\pm = \frac{1}{\sqrt{2}} (r^1 \pm i r^2)$ and $W^\pm = \frac{1}{\sqrt{2}} (V^1 \pm i V^2)$ in order to put (1.122) in a more conventional form. For the leptons we have

$$\mathcal{L}_{\ell\ell V,V'} = -\frac{1}{2} (g \tau_{ij}^a V_a^\mu + Y_L g' V'^\mu \delta_{ij}) \ell^i \bar{\sigma}_\mu \ell^j \quad (1.123)$$

$$- \frac{1}{2} g' Y_{\ell^c} V'^\mu (\ell_R^c)^i \bar{\sigma}_\mu (\ell_R^c)^i$$

since there is no ν_R term. Other terms which arise are (see (1.102))

$$\begin{aligned} \mathcal{L}_{\tilde{q}\tilde{q}V,V'} &= -\frac{i}{2} \tilde{\ell}^T (g \tau_{ij}^a V_a^\mu + Y_L g' V'^\mu \delta_{ij}) \partial_\mu \tilde{\ell}^i \\ &\quad - \frac{i}{2} g' Y_{\ell^c} \tilde{\ell}_R^c V'^\mu \partial_\mu \tilde{\ell}_R \end{aligned} \quad (1.124)$$

with a similar expression for quarks with $\tilde{\ell} \rightarrow \tilde{q}$, $\tilde{\ell}_R^c \rightarrow \tilde{u}_R$ and $\tilde{d}_R$. Once again generations are summed over. A related term is

$$\begin{aligned} \mathcal{L}_{\tilde{q}\tilde{q}\lambda} &= \frac{i}{\sqrt{2}} \tilde{q}^i (g \tau_{ij}^a \lambda_a q^j + g' Y_Q \lambda' q^j) \\ &\quad - \frac{i}{\sqrt{2}} \tilde{u}_R g' Y_u \lambda' u_R - \frac{i}{\sqrt{2}} \tilde{d}_R g' Y_D \lambda' d_R \end{aligned} \quad (1.125)$$

and similarly for leptons. In addition to these there are seagull terms, quartic slepton and squark terms and gauge-Higgs terms. For these terms, re-expressed

in terms of more familiar forms ($W^\pm$ instead of $W^{1,2}$, etc.), Refs. 12 and 13 are recommended. We already see that the kinetic term alone is enormously complicated. This is only the beginning, however, since many of these states mix (especially when $\langle H_{1,2}^0 \rangle$, $\langle N \rangle \neq 0$). Furthermore we have auxiliary terms in $\mathcal{L}_{KIN}$ which can only be disentangled once the remainder of the Lagrangian is added. Into this we plan to add soft supersymmetry-breaking terms. The result is a prodigiously tangled Lagrangian. For this reason we will not attempt to simplify each term as it is presented. We might as well wait until the entire Imbroglio has been accounted for.

Superpotential

The superpotential term, $W[\phi_i]$, may be divided into the pure Higgs potential and the mass-generation terms (all trilinear)

$$W[\phi_i] = g_L^{\bar{ij}} H_1 L_i R_j + g_D^{\bar{ij}} H_1 Q_i D_j + g_U^{\bar{ij}} H_2 Q_i U_j + W_N[H_1, H_2, N]. \quad (1.126)$$

In (1.126) i and j are generation indices. The weak isospin indices have been suppressed, however it is assumed that each term is contracted into an isosinglet (e.g. $g_L^{\bar{ij}} \epsilon_{\alpha\beta} H_1^\alpha L_i^\beta R_j$). It is clear that these terms include the normal Higgs-induced lepton and quark masses in $\mathcal{L}_{FOT} = \int W d^3\theta + \int W d^3\bar{\theta}$ once the neutral Higgs bosons gain a VEV. It is also clear that quadrilinear terms such as $H_0 H_0 \tilde{L}_i^2$ will induce identical masses in these sleptonic partners. As discussed previously, two Higgs doublets are required for such a scheme. Note that all of the other interactions which involve Higgsinos will couple with the same Yukawa strengths (g_U, g_D, g_L) which will not require radiative renormalizations. Note also that no bilinear potential terms, which would constitute explicit mass terms, have been included.

In order to have unbroken supersymmetry spontaneously break the gauge symmetry $SU(2)_W \times U(1)_Y \rightarrow U(1)_{EM}$ in the Higgs sector it is usual to take

$$W_N[\tilde{H}_1, \tilde{H}_2, \tilde{N}] = g_N[\tilde{H}_1 \tilde{H}_2 - \frac{1}{2} v^2] \tilde{N}. \quad (1.127)$$

The equation of motion for the F -term of N is

$$F_N = g_N[\epsilon_{ij} H_1^i H_2^j - \frac{1}{2} v^2] = g_N[H_1^0 H_2^0 + H_1^- H_2^+ - \frac{1}{2} v^2] \quad (1.128)$$

where H_i^i etc. are scalar field components.

Now for supersymmetry to be unbroken we required (previous section) that $\langle F_i \rangle = 0 \forall i$. Since $\langle H_1^- \rangle = \langle H_2^+ \rangle = 0$ from charge conservation we require $\langle H_1^0 \rangle \langle H_2^0 \rangle \neq 0$. To determine the condition accurately we need to find the potential term, $\mathcal{V} = \sum_F F F^* + \frac{1}{2} \sum_D D^2$, from the entire Lagrangian, including the field-strength sector. This has been done in Ref. 12. The above term (1.128) is the only inhomogeneous term in $\mathcal{V}$ so we may set the VEV of all other terms to zero. Doing so the remaining terms are

$$\begin{aligned} \langle \mathcal{V} \rangle = & \left\langle g_N^2 [\epsilon_{ij} H_1^i H_2^j - \frac{1}{2} v^2]^2 + \frac{1}{8} g^2 \left[|H_1^i H_2^j|^2 - \frac{1}{2} (H_1^i H_1^j)(H_2^i H_2^j) \right] \right. \\ & \left. + \frac{1}{8} g'^2 \left[|H_2^i H_1^j - H_1^j H_2^i|^2 \right] \right\rangle \\ = & g_N^2 |v_1 v_2 - \frac{1}{2} v^2|^2 + \frac{1}{8} g'^2 |v_1^2 - v_2^2|^2 \end{aligned} \quad (1.129)$$

where

$$v_1 = \langle H_1^0 \rangle \quad v_2 = \langle H_2^0 \rangle.$$

We see that the solution to $(V) = 0$ is

$$v_1 = v_2 = \frac{1}{\sqrt{2}} v. \quad (1.130)$$

We further observe that if several singlet superfields $N_1, \dots, N_K$ are employed that (1.130) need not be true. In general it is merely required that

$$v_1^2 + v_2^2 = v^2 \quad v_1 = \langle H_1^0 \rangle \quad v_2 = \langle H_2^0 \rangle. \quad (1.131)$$

We will, in fact, permit this more general VEV assignment. We will not include any scalar Higgs superfields since our breaking will be accomplished via soft explicit terms. The most general Higgs structure with two weak doublets, one singlet and soft (bilinear and certain trilinear) breaking terms has been analyzed.¹³ The presence of such soft terms relieves us of the necessity of employing a Higgs singlet. (See, in particular, Chapter 3, case 2 of the quoted reference.) Knowledge of the precise details underlying the final Higgs structure, which will assume the form $\theta_H [H_1^0 H_1^{0*} + H_2^0 H_2^{0*} - v^2]^2$, need not concern us since they will affect the Higgs sector which we will not be dealing with directly. It is sufficient for our purposes to know that such a soft-breaking scheme exists.

Super Field Strength

Recall that this contribution was of the form

$$\mathcal{L}_{\text{gauge}} = \sum_i \left\{ \int W_i^a W_{a,i} d^2\theta + \int \bar{W}_i^{\dot{a}} \bar{W}_{\dot{a},i} d^2\bar{\theta} \right\} \quad (1.132)$$

where "i" represents each gauge group in the theory and W^a was given by (1.95)

and (1.100). Using the notation of Table 1.1 we have

$$\begin{aligned} W_a^{\hat{V}} &= -\frac{1}{4} \bar{D}^2 D_a \hat{V} \\ W_a^{\hat{\psi}} &= -\frac{1}{4} \bar{D}^2 e^{-i\hat{\psi}} D_a e^{i\hat{\psi}} \\ W_a^{\hat{G}} &= -\frac{1}{4} \bar{D}^2 e^{-i\hat{G}} D_a e^{i\hat{G}} \end{aligned} \quad (1.133)$$

These contain all of the gauge and gaugino kinetic terms and self-interactions. When spontaneous gauge symmetry breaking occurs, resulting in $SU(2) \times U(1) \rightarrow U(1)_{em}$, the $\hat{V}$ superfield components becomes (equally) massive. Since gaugino mass terms added to $\mathcal{L}$ will break supersymmetry only softly (in fact they are one of the only spin- $\frac{1}{2}$ fields in an $N = 1$ supersymmetry for which this is true)¹⁹ we will do so. Since this, and the fermion mixing to be discussed shortly, will alter what we would expect from (1.133) alone we will refrain from expanding (1.133) and into its myriad terms.

1.4 THE FULL LAGRANGIAN

Into the unsullied, fully supersymmetric Lagrangian of the previous section we now add certain terms which break this symmetry softly.

Soft Breaking Terms

"Soft" terms are those, which when included as explicit terms in the Lagrangian at three level, do not introduce new divergences. This is a desirable attribute since it means that the attractive non-renormalization theorems remain valid. An exhaustive tabulation of soft terms has been made.¹⁹ These include all dimension-two operators which would normally arise in $\mathcal{L}$ (once it had been expanded into its component fields) and a few dimension three operators. Specifically included in the latter are gaugino mass terms but not normal fermion masses. Trilinear scalar combinations which arise from the unbroken superpotential are also included. Note that vector mass terms would break ordinary gauge invariance and are not included. Higgsino bilinears of the form $\epsilon^{ij} \tilde{H}_i \tilde{H}_j$ are also acceptable.

The soft terms which we will add to $\mathcal{L}_{SCWS}$, the leptonic sector of "standard" supersymmetric Lagrangian, are

$$\begin{aligned} \mathcal{L}_{Br} = & -\frac{1}{2} M \tilde{W}^0 \tilde{W}^0 - \frac{1}{2} M' \tilde{B}^0 \tilde{B}^0 - \mu \tilde{\psi}_{H_1}^0 \tilde{\psi}_{H_2}^0 - M \tilde{W}^+ \tilde{W}^- + \mu \tilde{\psi}_{H_1}^- \tilde{\psi}_{H_2}^+ \\ & + m_{ij} \tilde{L}_i \tilde{L}_j + \tilde{m}_{ij} \tilde{\nu}_i \tilde{\nu}_j + \text{Higgs boson terms} + h.c. \end{aligned} \quad (1.134)$$

The Higgs boson terms will be responsible for electroweak spontaneous symmetry breaking as discussed previously. There is no $\tilde{N}$ superfield and only H_1^0

and H_2^0 incur non-zero vacuum expectation values

$$\begin{aligned} \langle H_1^0 \rangle &= v_1 & \langle H_2^0 \rangle &= v_2 \\ v_1^2 + v_2^2 &= v^2 = \left(\frac{2m_w}{g} \right)^2 \approx (250 \text{ GeV})^2. \end{aligned} \quad (1.135)$$

It is convenient to define

$$\tan \theta_v = v_1/v_2. \quad (1.136)$$

(This is called $\cot \beta$ in Ref. 13.)

Slepton Mass Lagrangian

As discussed in Chapter 3 we may generally ignore the contribution to the scalar lepton (slepton) masses which arise via the unbroken supersymmetric Higgs mechanism, from the lepton masses, and instead consider the most general mass matrices induced by soft terms:

$$\begin{aligned} \mathcal{L} = & \frac{1}{2} (\tilde{\nu}_L \tilde{\nu}_R \tilde{\mu}_L \tilde{\mu}_R \tilde{e}_L \tilde{e}_R)^* (M_\nu) (\tilde{\nu}_L \tilde{\nu}_R \tilde{\mu}_L \tilde{\mu}_R \tilde{e}_L \tilde{e}_R)^T \\ & + \frac{1}{2} (\tilde{\nu}_L^* \tilde{\nu}_L^* \tilde{\nu}_L^* \tilde{\nu}_L^*)^* (M_\nu) (\tilde{\nu}_L^* \tilde{\nu}_L^* \tilde{\nu}_L^* \tilde{\nu}_L^*)^T. \end{aligned} \quad (1.137)$$

Here we have ignored right-handed sneutrinos. The complexities of this term in general are alluded to in Chapter 3. For our purposes it is sufficient to simplify the mixing to

$$\begin{aligned} \tilde{\mu}_L \text{ and } \tilde{\nu}_L \text{ with angle } \theta_L \\ \tilde{\mu}_R \text{ and } \tilde{\nu}_R \text{ with angle } \theta_R \\ \tilde{\nu}_\mu \text{ and } \tilde{\nu}_\nu \text{ with angle } \theta_\nu \end{aligned} \quad (1.138)$$

in which case we may write

$$\begin{aligned}
\tilde{L}_1 &= \tilde{\mu}_L \cos \theta_L + \tilde{\tau}_L \sin \theta_L \\
\tilde{L}_2 &= -\tilde{\mu}_L \sin \theta_L + \tilde{\tau}_L \cos \theta_L \\
\tilde{R}_1 &= \tilde{\mu}_R \cos \theta_R + \tilde{\tau}_R \sin \theta_R \\
\tilde{R}_2 &= -\tilde{\mu}_R \sin \theta_R + \tilde{\tau}_R \cos \theta_R \\
\tilde{\nu}_1 &= \tilde{\nu}_\mu \cos \theta_\nu + \tilde{\nu}_\tau \sin \theta_\nu \\
\tilde{\nu}_2 &= -\tilde{\nu}_\mu \sin \theta_\nu + \tilde{\nu}_\tau \cos \theta_\nu .
\end{aligned} \tag{1.139}$$

Note that $\tilde{L}_{1,2}$, $\tilde{R}_{1,2}$ and $\tilde{\nu}_{1,2}$ are the physical mass eigenstates. Conserved global family number is the only artifact which would prevent such a construction.

The absence of $\tilde{L}_L - \tilde{L}_R$ mixing is a great simplification. The importance of such mixing is clear from Chapter 2.

Charginos

The supersymmetric partners of the charged Higgs and vector bosons (Higgsinos and gauginos) are collectively referred to as *charginos*.

The supersymmetric counterparts of the $W^+ H_i^- H_j^0$ and $W^- H_i^+ H_j^0$ terms in the unbroken standard $\mathcal{L}_{SGWS}$ include

$$\mathcal{C} = -\frac{ig}{\sqrt{2}} \left[H_1^0 \tilde{W}^+ \tilde{\psi}_{H_1}^- + H_2^0 \tilde{W}^- \tilde{\psi}_{H_2}^+ \right] + h.c.$$

once $SU(2) \times U(1)$ is broken down to $U(1)_{em}$ and $\langle H_{1,2}^0 \rangle = v_{1,2}$

$$\mathcal{C} = -\frac{ig}{\sqrt{2}} \left[v_1 \tilde{W}^+ \tilde{\psi}_{H_1}^- + v_2 \tilde{W}^- \tilde{\psi}_{H_2}^+ \right] + h.c. \tag{1.140}$$

Note that these are Weyl spinors and we have switched to the basis $\tilde{W}^\pm = \frac{1}{\sqrt{2}} (\tilde{W}^1 \pm i\tilde{W}^2)$. These terms will induce mixing between the charged Higgsinos

and gauginos. A similar miscibility occurs in the neutral sector. In this sense the Higgs and gauge superfields have become "united". Fayet⁴² has taken this idea one step further. It is known that gauge and Higgs bosons can be unified in a single superfield in an $N = 2$ supersymmetry. Fayet showed that this is also possible in our $N = 1$ supersymmetry if one is prepared to abandon the Wess-Zumino gauge. If a different set of component fields is eliminated from a vector superfield via the supergauge mechanism described in section one, is possible to be left with a gauge boson, a Weyl gaugino, a Higgs boson and a Weyl Higgsino and no auxiliary fields.

To these fermion mass terms are added the soft breaking terms of (1.134) to yield the chargino mass Lagrangian

$$\mathcal{L}_{Ch} = M \tilde{W}^+ \tilde{W}^- - \mu \tilde{\psi}_{H_1}^- \tilde{\psi}_{H_2}^+ + \sqrt{2}ig(v_1 \tilde{W}^+ \tilde{\psi}_{H_1}^- + v_2 \tilde{W}^- \tilde{\psi}_{H_2}^+) + h.c. \tag{1.141}$$

Following Haber and Kane¹³ we note that if we gather these Weyl fields into four-component fermions of the form

$$\psi^+ = \begin{pmatrix} -i\tilde{W}^+ \\ \tilde{\psi}_{H_2}^+ \end{pmatrix} \quad \psi^- = \begin{pmatrix} -i\tilde{W}^- \\ \tilde{\psi}_{H_1}^- \end{pmatrix} \tag{1.142}$$

(where we note that $\tilde{\psi}^+ \neq \psi^-$) and write

$$\mathcal{L}_{Ch} = -\frac{1}{2} \left[(\psi^+)^T X^T \psi^- + (\psi^-)^T X \psi^+ \right] + h.c. \tag{1.143}$$

(note the use of ordinary transposes) then we find that

$$X = \begin{pmatrix} M & \sqrt{2}m_W \cos \theta_v \\ \sqrt{2}m_W \sin \theta_v & \mu \end{pmatrix} \tag{1.144}$$

will reproduce (1.141) to wit

$$\begin{aligned} \mathcal{L} = & -\frac{1}{2} \left\{ (-i\widetilde{W}^+ \quad \tilde{\psi}_{H_2}^+) \begin{pmatrix} M & \sqrt{2}m_W s_w \\ \sqrt{2}m_W c_w & \mu \end{pmatrix} \begin{pmatrix} -i\widetilde{W}^- \\ \tilde{\psi}_{H_1}^- \end{pmatrix} \right. \\ & \left. + (-i\widetilde{W}^- \quad \tilde{\psi}_{H_1}^-) \begin{pmatrix} M & \sqrt{2}m_W c_w \\ \sqrt{2}m_W s_w & \mu \end{pmatrix} \begin{pmatrix} -i\widetilde{W}^+ \\ \tilde{\psi}_{H_2}^+ \end{pmatrix} \right\} + h.c. \\ = & -\frac{1}{2} \left\{ -M\widetilde{W}^+\widetilde{W}^- - i\sqrt{2}m_W (s_w \widetilde{W}^+ \tilde{\psi}_{H_1}^- + c_w \tilde{\psi}_{H_2}^+ \widetilde{W}^-) + \mu \tilde{\psi}_{H_1}^- \tilde{\psi}_{H_2}^+ \right. \\ & \left. - M\widetilde{W}^- \widetilde{W}^+ - i\sqrt{2}m_W (c_w \tilde{\psi}_{H_1}^- \widetilde{W}^+ + s_w \widetilde{W}^- \tilde{\psi}_{H_2}^+) + \mu \tilde{\psi}_{H_1}^- \tilde{\psi}_{H_2}^+ \right\} \\ & + h.c. \end{aligned}$$

Now, since

$$s_w = \sin \theta_w = \frac{v_1}{v} \quad c_w = \cos \theta_w = \frac{v_2}{v} \quad v^2 = v_1^2 + v_2^2 \quad g = \frac{2m_W}{v} \quad (1.145)$$

$$\begin{aligned} \mathcal{L}_{CA} = & M\widetilde{W}^0\widetilde{W}^0 + i\sqrt{2} \frac{m_W v_1}{v} \widetilde{W}^+ \tilde{\psi}_{H_1}^- + i\sqrt{2} \frac{m_W v_2}{v} \widetilde{W}^- \tilde{\psi}_{H_2}^+ - \mu \tilde{\psi}_{H_1}^- \tilde{\psi}_{H_2}^+ + h.c. \\ = & M(\widetilde{W}^+ \widetilde{W}^- - \mu \tilde{\psi}_{H_1}^- \tilde{\psi}_{H_2}^+ + \sqrt{2}ig(v_1 \widetilde{W}^+ \tilde{\psi}_{H_1}^- + v_2 \widetilde{W}^- \tilde{\psi}_{H_2}^+)) + h.c. \end{aligned}$$

once we have substituted for "g". In the next appendix we will diagonalize this to obtain the true chargino mass eigenstates which we denote $\tilde{\chi}_i^+$ ($i = 1, 2$) and $\tilde{\chi}_i^-$. When this is diagonalized via

$$\tilde{\chi}_i^+ = V_{ij} \psi_j^+ \quad \tilde{\chi}_i^- = U_{ij} \psi_j^- \quad (1.146)$$

we will have

$$\mathcal{L}_{CA} = -\frac{1}{2} \left[m_{\tilde{\chi}_1^+} \tilde{\chi}_1^+ \tilde{\chi}_1^+ + m_{\tilde{\chi}_2^+} \tilde{\chi}_2^+ \tilde{\chi}_2^+ \right] + h.c. \quad (1.147)$$

The matrices U and V (and combinations thereof) will be proliferated through

the Feynman rules. It is also possible to express $\tilde{\chi}^\pm$ in terms of the Dirac spinors

$$\widetilde{W} = \begin{pmatrix} -i\widetilde{W}^+ \\ i\widetilde{W}^- \end{pmatrix} \quad \tilde{H} = \begin{pmatrix} \tilde{\psi}_{H_2}^+ \\ \tilde{\psi}_{H_1}^- \end{pmatrix}. \quad (1.148)$$

Neutrals

This situation is similar here but we have more states. Both Higgs doublets have a neutral state, H_1^0 and H_2^0 , with corresponding Higgsinos $\tilde{\psi}_{H_1}^0$ and $\tilde{\psi}_{H_2}^0$. The two neutral gauginos are $\tilde{B}^0$ and $\tilde{W}^0$, related to the basis $\tilde{\gamma}$ and $\tilde{Z}^0$ by the same ratios as their bosonic counterparts

$$\begin{pmatrix} \tilde{B}^0 \\ \tilde{W}^0 \end{pmatrix} = \begin{pmatrix} c_w & s_w \\ -s_w & c_w \end{pmatrix} \begin{pmatrix} \tilde{\gamma} \\ \tilde{Z}^0 \end{pmatrix} \quad \begin{pmatrix} \tilde{\gamma} \\ \tilde{Z}^0 \end{pmatrix} = \begin{pmatrix} c_w & -s_w \\ s_w & c_w \end{pmatrix} \begin{pmatrix} \tilde{B}^0 \\ \tilde{W}^0 \end{pmatrix} \quad (1.149)$$

where we henceforth establish that

$$\begin{aligned} c_w &= \cos \theta_w & s_w &= \sin \theta_w \\ c_\theta &= \cos \theta_\theta & s_\theta &= \sin \theta_\theta \\ c_{2\theta} &= \cos 2\theta_\theta & s_{2\theta} &= \sin 2\theta_\theta \\ c_\phi &= \cos \phi & s_\phi &= \sin \phi. \end{aligned} \quad (1.150)$$

From this point forth we use the traditional notation

$$\tilde{\gamma} \equiv \lambda. \quad (1.151)$$

The terms derived from the supersymmetrized $W^0 H_1^0 H_2^0$ and $B^0 H_1^0 H_2^0$ vertices yields the mass terms

$$\frac{i}{2} (g' \tilde{B}^0 - g \tilde{W}^0) (v_1 \tilde{\psi}_{H_1}^0 - v_2 \tilde{\psi}_{H_2}^0) + h.c.$$

which again mixes the neutral Higgsinos and gauge bosons. To this we add the

breaking terms

$$-\frac{1}{2} M \tilde{V}^0 \tilde{V}^0 - \frac{1}{2} M' \tilde{B}^0 \tilde{B}^0 - \mu \tilde{\psi}_{H_1}^0 \tilde{\psi}_{H_2}^0 + \text{h.c.}$$

to yield the general "neutralino" Lagrangian

$$\mathcal{L}_0 = -\frac{1}{2} \left[M \tilde{V}^0 \tilde{V}^0 + M' \tilde{B}^0 \tilde{B}^0 \right] - \mu \tilde{\psi}_{H_1}^0 \tilde{\psi}_{H_2}^0 \quad (1.152)$$

$$+ \frac{1}{2} (g' \tilde{B}^0 - g \tilde{V}^0) (v_1 \tilde{\psi}_{H_1}^0 - v_2 \tilde{\psi}_{H_2}^0) + \text{h.c.}$$

In terms of $\tilde{Z}^0$ and λ this is

$$\begin{aligned} \mathcal{L}_0 = & \frac{1}{2} (M' \cos^2 \theta_w + M \sin^2 \theta_w) \lambda \lambda + (M - M') \sin \theta_w \cos \theta_w \lambda \tilde{Z}^0 \\ & + \frac{1}{2} (M \cos^2 \theta_w + M' \sin^2 \theta_w) \tilde{Z}^0 \tilde{Z}^0 + \frac{i}{2} \frac{g}{\cos \theta_w} \tilde{Z}^0 (v_1 \tilde{\psi}_{H_1}^0 - v_2 \tilde{\psi}_{H_2}^0) \\ & + \mu \tilde{\psi}_{H_1}^0 \tilde{\psi}_{H_2}^0 + \text{h.c.} \end{aligned} \quad (1.153)$$

Haber and Kane¹² establish the notation

$$\begin{aligned} \mathcal{L}_0 = & -\frac{1}{2} \psi^{0T} Y \psi^0 \\ = & -\frac{1}{2} \psi^{0'T} Y' \psi^{0'} \end{aligned} \quad (1.154)$$

where two possible bases we work with are

$$\psi^0 = \begin{pmatrix} -i \tilde{B}^0 \\ -i \tilde{V}^0 \\ \tilde{\psi}_{H_1}^0 \\ \tilde{\psi}_{H_2}^0 \end{pmatrix} \quad \psi^{0'} = \begin{pmatrix} -i \lambda \\ -i \tilde{Z}^0 \\ s_w \tilde{\psi}_{H_1}^0 - c_w \tilde{\psi}_{H_2}^0 \\ c_w \tilde{\psi}_{H_1}^0 + s_w \tilde{\psi}_{H_2}^0 \end{pmatrix} \quad (1.155)$$

These consist of four Weyl spinors each. Explicit calculation shows that

$$Y = \begin{pmatrix} M' & 0 & -m_X \sin \theta_w \sin \theta_w & m_X \cos \theta_w \sin \theta_w \\ 0 & M & m_X \sin \theta_w \cos \theta_w & -m_X \cos \theta_w \cos \theta_w \\ -m_X \sin \theta_w \sin \theta_w & m_X \sin \theta_w \cos \theta_w & 0 & -\mu \\ m_X \cos \theta_w \sin \theta_w & -m_X \cos \theta_w \cos \theta_w & -\mu & 0 \end{pmatrix} \quad (1.156)$$

and

$$Y' = \begin{pmatrix} M' c_w^2 + M s_w^2 & (M - M') s_w c_w & 0 & 0 \\ (M - M') s_w c_w & M c_w^2 + M' s_w^2 & m_X & 0 \\ 0 & m_X & \mu s_{2v} & \mu c_{2v} \\ 0 & 0 & \mu c_{2v} & -\mu s_{2v} \end{pmatrix} \quad (1.157)$$

These are related via

$$\psi^{0'}_j = B_{ij} \psi^0_j, \quad B B^\dagger = 1 \quad (1.158)$$

$$B_{ij} = \begin{pmatrix} \cos \theta_w & \sin \theta_w & 0 & 0 \\ -\sin \theta_w & \cos \theta_w & 0 & 0 \\ 0 & 0 & \sin \theta_w & -\cos \theta_w \\ 0 & 0 & \cos \theta_w & \sin \theta_w \end{pmatrix} \quad (1.159)$$

The physical mass eigenstates will be

$$\begin{aligned} \tilde{\chi}_i^0 &= N_{ij} \psi_j^0 \\ \tilde{\chi}_i^{0'} &= N'_{ij} \psi_j^{0'} \end{aligned} \quad (1.160)$$

where

$$N_{ij} = N'_{ik} B_{kj}. \quad (1.161)$$

In correspondence to (1.149) we will frequently express the unmixed neutrali-

ness in the form of four-component Majorana spinors

$$\lambda = \begin{pmatrix} -i\lambda \\ i\lambda \end{pmatrix} \quad \tilde{\chi}^0 = \begin{pmatrix} -i\tilde{\chi}^0 \\ i\tilde{\chi}^0 \end{pmatrix} \quad \tilde{H}_1 = \begin{pmatrix} \tilde{\psi}_{H_1}^0 \\ \tilde{\psi}_{H_1}^0 \end{pmatrix} \quad \tilde{H}_2 = \begin{pmatrix} \tilde{\psi}_{H_1}^0 \\ \tilde{\psi}_{H_1}^0 \end{pmatrix} \quad (1.162)$$

and the eigenstates as

$$\tilde{X}_i^0 = \begin{pmatrix} \tilde{X}_1^0 \\ \tilde{X}_2^0 \end{pmatrix} \quad i = 1, 2, 3, 4. \quad (1.163)$$

Gauge Couplings to Leptons

These are the same as in the standard electroweak Lagrangian (at least at the tree level) and will not be discussed in detail. (Reference 12 expresses this part of the Lagrangian in notation consistent with the supersymmetric terms.) The general form for $\mathcal{L}_{\psi_L \psi_L V}$ (ψ_L = left-handed spin- $\frac{1}{2}$ Weyl fermions; group generators are T^a) is

$$\mathcal{L}_{\psi \psi V} = -\frac{1}{2} g_V T_{ij}^a V_\mu^a \psi_L^i \bar{\sigma}^\mu \psi_L^j. \quad (1.164)$$

In our case (for one generation)

$$\begin{aligned} \mathcal{L}_{\ell \ell V} = & -\frac{1}{2} (g_V^e V_\mu^0 + y_{\ell a} B_\mu^0 \delta_{ij}) \ell^i \bar{\sigma}^\mu \ell^j \\ & -\frac{1}{2} g' y_{\ell a} B_\mu^0 \bar{\sigma}^\mu e_R \quad (a = 1, 2, 3; i = 1, 2) \end{aligned} \quad (1.165)$$

where $\ell^T = (e_L, \nu_L)$, τ^a are Pauli matrices and y is hypercharge ($y_e = -1$; $y_{\ell a} = +2$). In terms of $A_\mu, Z_\mu, W^\pm, \tan \theta_w = g'/g$ and $e = g \sin \theta_w = g' \cos \theta_w$

$$\begin{aligned} \mathcal{L}_{\ell \nu W} = & -\frac{g}{\sqrt{2}} [W_\mu^+ \nu^1 \bar{\sigma}^\mu \ell^- + W_\mu^- \ell^+ \bar{\sigma}^\mu \nu] \\ \mathcal{L}_{12, \ell^+ \gamma} = & +e \ell^+ \bar{\sigma}^\mu A_\mu \ell^- \end{aligned} \quad (1.166)$$

and so forth. In terms of more conventional Dirac spinors, with

$$\gamma_\pm = \frac{1}{2} (1 \pm \gamma_5) \quad (1.167)$$

$$\begin{aligned} \mathcal{L}_{\ell \nu} = & \frac{-g}{\sqrt{2}} [W_\mu^+ \bar{\nu} \gamma^\mu \gamma_- \ell + W_\mu^- \bar{\ell} \gamma^\mu \gamma_- \nu] \\ & - \frac{g}{2 \cos \theta_w} Z_\mu [\bar{\nu} \gamma^\mu \gamma_- \nu - \cos 2\theta_w \bar{\ell} \gamma^\mu \gamma_- \ell - \sin^2 \theta_w \bar{\ell} \gamma^\mu \gamma_+ \ell] \\ & + e A_\mu \bar{\ell} \gamma^\mu \ell. \end{aligned} \quad (1.168)$$

Gauge Couplings to Sleptons

Following the procedure used when we derived the SQED lagrangian we find that the analogue of (1.164) is

$$\mathcal{L}_{\tilde{\psi} \tilde{\psi} V} = -ig_V T_{ij}^a V_\mu^a \tilde{\psi}_L^i \bar{\tilde{\sigma}}^\mu \tilde{\psi}_L^j \quad (1.169)$$

from which $\mathcal{L}_{\tilde{\ell} \tilde{\ell} Z}$, etc. follows just as in the previous section.

Gaugino Coupling to Leptons

The corresponding general term for $\mathcal{L}_{\tilde{\psi} \tilde{\psi} \tilde{V}}$ is

$$\mathcal{L}_{\tilde{\psi} \tilde{\psi} \tilde{V}} = \sqrt{2} ig [\tilde{\psi}_L^i T_{ij}^a \psi_L^j \tilde{V}^a - \tilde{\psi}_L^i T_{ij}^a \psi_L^j \tilde{V}^a] \quad (1.170)$$

where $\tilde{\psi}_L^i \tilde{V}^a$ has the (spinorial) structure $\tilde{\psi}_L^i \bar{\tilde{V}}^a \gamma^{\mu} \gamma_5$, etc. We may again expand this out for $\tilde{W}^\pm$ and $\tilde{B}^0$ and then switch to the $\tilde{W}^\pm, \tilde{Z}^0, \lambda$ basis. For instance (in two-component form)

$$\mathcal{L}_{\tilde{\ell}_L \nu_L \tilde{W}} = ig [\tilde{W}^- \nu_L \tilde{\ell}_L - \tilde{W}^- \nu_L \tilde{\ell}_L + \tilde{W}^+ \ell_L \tilde{\nu} - \tilde{W}^+ \ell_L \tilde{\nu}]. \quad (1.171)$$

To write this in terms of chargino mass eigenstates we would have to use (1.142) and (1.146) (or the equivalent expression for (1.146) in terms of (1.148)) and

obtain $\mathcal{L}_{\tilde{e}_L \nu_e \tilde{\chi}_1^0}$ which would have U and V matrices in the coefficients. An example would be

$$\mathcal{L}_{\tilde{e}_L \nu_e \tilde{\chi}_1^0} = ig[U_{11}\tilde{\chi}_1^+ \nu_e \tilde{e}_L - U_{11}^* \tilde{\chi}_1^+ \tilde{\nu}_e \tilde{e}_L] . \quad (1.172)$$

If we were considering $\mathcal{L}_{\tilde{\nu}_\mu \nu_e \tilde{\chi}_1^0}$ where we have allowed $\tilde{\nu}_\mu$ and $\tilde{\nu}_\tau$ to mix, as in (1.139), then we would find

$$\mathcal{L}_{\tilde{\nu}_\mu \nu_e \tilde{\chi}_1^0} = -ig[V_{j1}\tilde{\chi}_1^+ \tau_L \tilde{\nu}_1^* - V_{j1}^* \tilde{\chi}_1^+ \tau_L \tilde{\nu}_1] \sin \theta_\nu . \quad (1.173)$$

There are obviously many terms and it would be exceedingly tedious to list them all. We will restrict our attention to deriving Feynman rules for those vertices which will be of special importance to the calculations in this work (Appendix K). The same is true of the neutralino vertices.

1.5 GAUGINO COUPLING TO GAUGE BOSONS

These are the analogues of the non-Abelian trilinear gauge couplings. Once the Higgsinos and Gauginos mix to the physical eigenstates $\tilde{\chi}_i$, the vertices $\tilde{\chi}_i \tilde{\chi}_j G_k$ and $\tilde{\chi}_i \tilde{\chi}_j H_k$ will involve two mixing matrices.

Just as the $G_i G_j G_k$ gauge term is

$$\mathcal{L}_{GGG} = ig f^{ijk} G_i^\nu G_j^\mu \partial_\mu G_{\nu k} \quad (1.174)$$

the $\tilde{G}_i \tilde{G}_j G_k$ term simplifies to (Weyl notation)

$$\mathcal{L}_{\tilde{G}\tilde{G}G} = ig f^{ijk} \tilde{G}_i^\nu \sigma^\mu \tilde{G}_j G_{k\mu} \quad (1.175)$$

where f^{ijk} are the group structure constants. We will be interested in (1.175) for the cases $G = \gamma, Z^0$.

We will also need the Higgsino-Gauge-Gaugino terms. The form of the $H\tilde{H}G$, $H\tilde{G}G$ and $H\tilde{H}G\tilde{G}$ terms arises in the usual manner from GGG and $GGGG$ once we implement spontaneous $SU(2) \times U(1)$ breaking. Since $\tilde{H}_1$ and $\tilde{H}_2$ are scalar superfields the Lagrangian for $\tilde{G}\tilde{H}\tilde{H}$ and $\tilde{G}\tilde{H}\tilde{H}$ arises in the kinetic terms in the same way as $\tilde{G}\tilde{e}\tilde{e}$ and $\tilde{G}\tilde{\tau}\tilde{\tau}$ did. A $\tilde{G}\tilde{G}\tilde{H}$ term arises from the $G\tilde{G}H\tilde{H}$ scalar-vector interaction from the kinetic term when one of the Higgs gains a VEV.

Chargino Coupling to Z^0 and γ : Weak Eigenbasis

We will use the notation of Haber and Kane¹²

$$\mathcal{L}_{Z^0 \tilde{\chi}_i^+ \tilde{\chi}_j^-} \equiv \frac{g}{\cos \theta_W} Z_\mu^\nu \left[\frac{1}{2} \tilde{\chi}_i^+ \gamma^\mu (O_{ij}^L \gamma_- + O_{ij}^R \gamma_+) \tilde{\chi}_j^- \right] + \text{h.c.} \quad (1.176)$$

to define the mixing matrices O_{ij}^α from the matrices U and V defined in (1.146). We will use the 4-component spinors $\tilde{W}$ and $\tilde{H}^+$ in (1.148) and the Majorana spinors from (1.8) and (1.9).

The total $\gamma\tilde{X}\tilde{X} + Z^0\tilde{X}\tilde{X}$ term is, in two-component form (using (1.175)):

$$\begin{aligned} \mathcal{L}_{\tilde{X}\tilde{X}Z} = & -gV_\mu^0 \left(\tilde{W}^- \sigma^\mu \tilde{W}^+ - \tilde{W}^+ \sigma^\mu \tilde{W}^- \right) \\ & - \frac{1}{2} (g' y_k W_\mu^0 + g' y_k \delta_{ij} B_\mu^0) \left(\tilde{\psi}_{H_k} \right)^\dagger \sigma^\mu \left(\tilde{\psi}_{H_k} \right)^j + \text{h.c.} \end{aligned} \quad (1.177)$$

where $y_k = \pm 1$ can be read off of Table 1.1. The $\gamma_2 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ terms include both chargino and neutralino parts.

Using (A.1) and (A.3) we may assemble this into Majorana spinors

$$\begin{aligned} \mathcal{L} = & -gW_\mu^0 (\tilde{W}^+) \gamma^\mu \tilde{W}^+ - \frac{1}{2} (gW_\mu^0 + g'E_\mu^0) (\tilde{H}^+) \gamma^\mu \tilde{H}^+ \\ & + \frac{1}{4} (g\tilde{W}_\mu^0 - g'E_\mu^0) (\tilde{H}_1 \gamma^\mu \gamma_5 \tilde{H}_1 - \tilde{H}_2 \gamma^\mu \gamma_5 \tilde{H}_2) + \text{h.c.} \end{aligned} \quad (1.178)$$

The first two terms constitute the chargino contribution. Using

$$\begin{aligned} Z_\mu &= \cos \theta_W W_\mu^0 - \sin \theta_W B_\mu^0 \\ A_\mu &= \sin \theta_W W_\mu^0 + \cos \theta_W B_\mu^0 \end{aligned} \quad (1.179)$$

results in the chargino part:

$$\begin{aligned} \mathcal{L}_{G^0\text{-chargino}} = & -\frac{g}{\cos \theta_W} Z_\mu \left[\cos^2 \theta_W (\tilde{W}^+) \gamma^\mu \tilde{W}^+ + \frac{1}{2} \cos 2\theta_W (\tilde{H}^+) \gamma^\mu \tilde{H}^+ \right] \\ & - e A_\mu \left[(\tilde{W}^+) \gamma^\mu \tilde{W}^+ + (\tilde{H}^+) \gamma^\mu \tilde{H}^+ \right] + \text{h.c.} \end{aligned} \quad (1.180)$$

Relating Bases

To rewrite this in terms of the physical states $\tilde{\chi}_i^\pm$ we need to relate the bases

of states discussed. In summary:

$$\begin{aligned} \psi_j^+ &= \begin{pmatrix} -i\tilde{W}^+ \\ \tilde{\psi}_{H_1}^+ \end{pmatrix} & \psi_j^- &= \begin{pmatrix} -i\tilde{W}^- \\ \tilde{\psi}_{H_1}^- \end{pmatrix} & \text{from (1.142)} \\ \tilde{W}_j^+ &= \begin{pmatrix} -i\tilde{W}^+ \\ i(\tilde{W}^-) \end{pmatrix} & \tilde{H}_j^+ &= \begin{pmatrix} \tilde{\psi}_{H_1}^+ \\ (\tilde{\psi}_{H_1}^-) \end{pmatrix} & \text{from (1.148)} \\ \tilde{\chi}_{1j}^+ &= \begin{pmatrix} \tilde{\chi}_1^+ \\ \tilde{\chi}_1^- \end{pmatrix} & \tilde{\chi}_{2j}^+ &= \begin{pmatrix} \tilde{\chi}_2^+ \\ \tilde{\chi}_2^- \end{pmatrix} & \text{from (1.148)} \end{aligned} \quad (1.181)$$

where $j = 1, 2$. Recall that

$$\tilde{\chi}_i^+ = V_{ij} \psi_j^+ \quad \tilde{\chi}_i^- = U_{ij} \psi_j^-$$

($i, j = 1, 2$) so

$$\psi_j^+ = (V^\dagger)_{ji} \tilde{\chi}_i^+ \quad \psi_j^- = (U^\dagger)_{ji} \tilde{\chi}_i^- \quad (1.182)$$

Now $\gamma_2 = \frac{1}{2}(1 \pm \gamma_5)$

$$\gamma_5 = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \quad \gamma_- = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \quad \gamma_+ = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \quad (1.183)$$

Hence

$$\begin{aligned} \gamma_- \tilde{W}^+ &= \begin{pmatrix} -i\tilde{W}^+ \\ 0 \end{pmatrix} & \gamma_+ \tilde{W}^+ &= \begin{pmatrix} 0 \\ i\tilde{W}^- \end{pmatrix} \\ \gamma_- \tilde{H}^+ &= \begin{pmatrix} \tilde{\psi}_{H_1}^+ \\ 0 \end{pmatrix} & \gamma_+ \tilde{H}^+ &= \begin{pmatrix} 0 \\ \tilde{\psi}_{H_1}^- \end{pmatrix} \\ \gamma_- \tilde{\chi}_i^+ &= \begin{pmatrix} \tilde{\chi}_i^+ \\ 0 \end{pmatrix} & \gamma_+ \tilde{\chi}_i^+ &= \begin{pmatrix} 0 \\ \tilde{\chi}_i^- \end{pmatrix} \end{aligned} \quad (1.184)$$

Now, in anticipation of future needs we introduce the conjugate spinors $\tilde{W}^c$

and $\tilde{\chi}_i^c$ (see also sections A.4 and M.1). In general if

$$\psi = \begin{pmatrix} \xi_a \\ \eta^{\dot{a}} \end{pmatrix}$$

then

$$\bar{\psi} = (\eta^a \quad \xi_{\dot{a}}) \quad \psi^c = \begin{pmatrix} \eta_a \\ \xi^{\dot{a}} \end{pmatrix} \quad (1.185)$$

so

$$\tilde{W}^c = \begin{pmatrix} -i\tilde{W}^- \\ i\tilde{W}^+ \end{pmatrix} \quad \tilde{\chi}_1^- = \begin{pmatrix} \tilde{\chi}_1^- \\ \tilde{\chi}_1^+ \end{pmatrix} \quad \tilde{\chi}_2^- = \begin{pmatrix} \tilde{\chi}_2^- \\ \tilde{\chi}_2^+ \end{pmatrix} \quad (1.186)$$

and

$$\gamma_- \tilde{W}^c = \begin{pmatrix} -i\tilde{W}^- \\ 0 \end{pmatrix} \quad \gamma_+ \tilde{W}^c = \begin{pmatrix} 0 \\ i\tilde{W}^+ \end{pmatrix} \quad (1.187)$$

From (1.182) and (1.184)

$$\begin{aligned} \gamma_- \tilde{W}_j^+ (4 - \text{comp.}) &= -i\tilde{W}^+ (2 - \text{comp.}) = \psi_1^+ = (V^\dagger)_{11} \tilde{\chi}_1^+ + (V^\dagger)_{12} \tilde{\chi}_2^+ \\ &= V_{11}^+ \tilde{\chi}_1^+ + V_{21}^+ \tilde{\chi}_2^+ (2 - \text{comp.}) \\ &= \gamma_- (V_{11}^+ \tilde{\chi}_1^+ + V_{21}^+ \tilde{\chi}_2^+) (4 - \text{comp.}) \end{aligned} \quad (1.188)$$

Using (1.184) again, so

$$\gamma_- \tilde{W}^+ = \gamma_- (V_{11}^+ \tilde{\chi}_1^+ + V_{21}^+ \tilde{\chi}_2^+).$$

Similarly, from (1.181) and (1.182)

$$\begin{aligned} -i\tilde{W}^- &= \psi_1^- = U_{11}^\dagger \tilde{\chi}_1^- + U_{12}^\dagger \tilde{\chi}_2^- \\ &= U_{11}^\dagger \tilde{\chi}_1^- + U_{21}^\dagger \tilde{\chi}_2^- \end{aligned}$$

thus

$$\begin{aligned} i\tilde{W}^- &= U_{11} \tilde{\chi}_1^- + U_{21} \tilde{\chi}_2^- (2 - \text{comp.}) \\ &= U_{11} \gamma_+ \tilde{\chi}_1^- + U_{21} \gamma_+ \tilde{\chi}_2^- (4 - \text{comp.}) \end{aligned}$$

and since

$$\gamma_+ \tilde{W}^+ (4 - \text{comp.}) = i\tilde{W}^- (2 - \text{comp.}) \text{ from (1.181)} \quad (1.189)$$

$$\gamma_+ \tilde{W}^+ = \gamma_+ (U_{11} \tilde{\chi}_1^- + U_{21} \tilde{\chi}_2^-)$$

For the Higgsinos

$$\begin{aligned} \tilde{\psi}_{H_2}^+ &= \psi_2^+ = (V^\dagger)_{21} \tilde{\chi}_1^+ + (V^\dagger)_{22} \tilde{\chi}_2^+ \\ &= \gamma_- (V_{12}^+ \tilde{\chi}_1^+ + V_{22}^+ \tilde{\chi}_2^+) \end{aligned}$$

and

$$\begin{aligned} \gamma_- \tilde{H}^+ &= \gamma_- (V_{12}^+ \tilde{\chi}_1^+ + V_{22}^+ \tilde{\chi}_2^+) \\ \tilde{\psi}_{H_2}^- &= \psi_2^- = (U^\dagger)_{21} \tilde{\chi}_1^- + (U^\dagger)_{22} \tilde{\chi}_2^- \\ \Rightarrow \quad \tilde{\psi}_{H_1}^+ &= (U^T)_{12} \tilde{\chi}_1^+ + (U^T)_{11} \tilde{\chi}_2^+ \\ &= \gamma_+ (U_{12} \tilde{\chi}_1^+ + U_{22} \tilde{\chi}_2^+) \end{aligned}$$

so

$$\gamma_+ \tilde{H}^+ = \gamma_+ (U_{12} \tilde{\chi}_1^+ + U_{22} \tilde{\chi}_2^+)$$

as in (1.188)

$$\begin{aligned} -i\tilde{W}^+ &= \psi_1^+ = (V^\dagger)_{11} \tilde{\chi}_1^+ + (V^\dagger)_{12} \tilde{\chi}_2^+ \\ \Rightarrow \quad i\tilde{W}^+ &= (V^T)_{11} \tilde{\chi}_1^+ + (V^T)_{12} \tilde{\chi}_2^+ \\ \Rightarrow \quad \gamma_+ \tilde{W}^+ &= \gamma_+ (V_{11} \tilde{\chi}_1^+ + V_{12} \tilde{\chi}_2^+) \end{aligned}$$

Summarizing these results

$$\begin{aligned}
 \gamma_+ \widetilde{W}^+ &= \gamma_+ (U_{11} \widetilde{X}_1 + U_{21} \widetilde{X}_2) \\
 \gamma_- \widetilde{W}^+ &= \gamma_- (V_{11}^* \widetilde{X}_1^+ + V_{21}^* \widetilde{X}_2^+) \\
 \gamma_+ \widetilde{H}^+ &= \gamma_+ (U_{12} \widetilde{X}_1^+ + U_{22} \widetilde{X}_2^+) \\
 \gamma_- \widetilde{H}^+ &= \gamma_- (V_{12}^* \widetilde{X}_1^+ + V_{22}^* \widetilde{X}_2^+) \\
 \gamma_+ \widetilde{W}^c &= \gamma_+ (V_{11} \widetilde{X}_1 + V_{21} \widetilde{X}_2)
 \end{aligned} \tag{1.190}$$

Taking the normal Hermitian conjugate in (1.185) ($\gamma_+ = \gamma_-$)

$$\begin{aligned}
 \widetilde{W}^c \gamma_+ &= (V_{11} \widetilde{X}_1 + V_{21} \widetilde{X}_2) \gamma_+ \\
 \widetilde{W}^c \gamma_- &= (U_{11}^* \widetilde{X}_1 + U_{21}^* \widetilde{X}_2) \gamma_- \\
 \widetilde{H}^c \gamma_+ &= (V_{12} \widetilde{X}_1 + V_{22} \widetilde{X}_2) \gamma_+ \\
 \widetilde{H}^c \gamma_- &= (U_{12}^* \widetilde{X}_1 + U_{22}^* \widetilde{X}_2) \gamma_-
 \end{aligned} \tag{1.191}$$

Charged Coupling to Z^0 and γ II: Mass Eigenbasis

We now substitute (1.190) and (1.191) into (1.180). We use

$$\{\gamma_{\pm}, \gamma^{\mu}\} = 0 \quad \gamma_+ + \gamma_- = 1 \quad \gamma_{\pm}^2 = 1 \tag{1.192}$$

to note that

$$\begin{aligned}
 \gamma^{\mu} &= (\gamma_+ + \gamma_-) \gamma^{\mu} (\gamma_+ + \gamma_-) \\
 &= \gamma_+ \gamma^{\mu} \gamma_- + \gamma_- \gamma^{\mu} \gamma_+ .
 \end{aligned} \tag{1.193}$$

Then

$$\begin{aligned}
 \mathcal{L}_{Z^0 \widetilde{X}_i^+ \widetilde{X}_i^-} &= \frac{g}{\cos \theta_W} Z_{\mu} \left[-\cos^2 \theta_W \widetilde{W}^c (\gamma_- \gamma^{\mu} \gamma_+ + \gamma_+ \gamma^{\mu} \gamma_-) \widetilde{W}^+ \right. \\
 &\quad \left. - \cos 2\theta_W \widetilde{H}^c (\gamma_- \gamma^{\mu} \gamma_+ + \gamma_+ \gamma^{\mu} \gamma_-) \widetilde{W}^- \right] + \text{h.c.} \\
 &= \frac{g}{\cos \theta_W} Z_{\mu} \left\{ -\cos^2 \theta_W \left[(U_{11}^* \widetilde{X}_1 + U_{21}^* \widetilde{X}_2) \gamma_- \gamma^{\mu} \gamma_+ (U_{11} \widetilde{X}_1^+ + U_{21} \widetilde{X}_2^+) \right. \right. \\
 &\quad \left. \left. + (V_{11} \widetilde{X}_1 + V_{21} \widetilde{X}_2) \gamma_+ \gamma^{\mu} \gamma_- (V_{11}^* \widetilde{X}_1^+ + V_{21}^* \widetilde{X}_2^+) \right] \right. \\
 &\quad \left. - \frac{1}{2} \cos 2\theta_W \left[(U_{12}^* \widetilde{X}_1 + U_{22}^* \widetilde{X}_2) \gamma_- \gamma^{\mu} \gamma_+ (U_{12} \widetilde{X}_1^+ + U_{22} \widetilde{X}_2^+) \right. \right. \\
 &\quad \left. \left. + (V_{12} \widetilde{X}_1 + V_{22} \widetilde{X}_2) \gamma_+ \gamma^{\mu} \gamma_- (V_{12}^* \widetilde{X}_1^+ + V_{22}^* \widetilde{X}_2^+) \right] \right\} + \text{h.c.}
 \end{aligned}$$

Expanding out and collecting terms into the form of (1.176) results in

$$\begin{aligned}
 O_{ij}^{L_1} &= -\cos^2 \theta_W V_{i1} V_{j1}^* - \frac{1}{2} \cos 2\theta_W V_{i2} V_{j2}^* \\
 O_{ij}^{R_1} &= -\cos^2 \theta_W U_{i1}^* U_{j1} - \frac{1}{2} \cos 2\theta_W U_{i2}^* U_{j2}
 \end{aligned} \tag{1.194}$$

Therefore

$$\begin{aligned}
 O_{ij}^{L_2} &= (\sin^2 \theta_W - 1) V_{i1} V_{j1}^* - \frac{1}{2} (1 - 2 \sin^2 \theta_W) V_{i2} V_{j2}^* \\
 &= \sin^2 \theta_W (V_{i1} V_{j1}^* + V_{i2} V_{j2}^*) - V_{i1} V_{j1}^* - \frac{1}{2} V_{i2} V_{j2}^*
 \end{aligned}$$

The term in brackets is $(VV^+)_{ij} = \delta_{ij}$ since U and V are unitary. So finally

$$\begin{aligned}
 O_{ij}^{L_1} &= \delta_{ij} \sin^2 \theta_W - V_{i1} V_{j1}^* - \frac{1}{2} V_{i2} V_{j2}^* \\
 O_{ij}^{R_1} &= \delta_{ij} \sin^2 \theta_W - U_{i1}^* V_{j1} - \frac{1}{2} U_{i2}^* V_{j2}
 \end{aligned} \tag{1.195}$$

$$\mathcal{L}_{Z^0 \widetilde{X}_i^+ \widetilde{X}_i^-} = \frac{g}{\cos \theta_W} Z_{\mu} \left[\frac{1}{2} \widetilde{X}_i^+ \gamma^{\mu} (O_{ij}^{L_1} \gamma_- + O_{ij}^{R_1} \gamma_+) \widetilde{X}_j^- \right] + \text{h.c.}$$

The photon term comes out directly

$$\mathcal{L}_{\gamma\tilde{\chi}^+\tilde{\chi}^-} = -ie\gamma^\mu \quad (1.196)$$

as one might expect.

Neutralino Coupling to Z^0 and γ^μ

From (1.178) we see that

$$\begin{aligned} \mathcal{L}_{G^0\text{-neutralino}} &= \frac{1}{4} (gW_\mu^0 - g'B_\mu^0) (\tilde{H}_1 \gamma^\mu \gamma_5 \tilde{H}_1 - \tilde{H}_2 \gamma^\mu \gamma_5 \tilde{H}_2) + \text{h.c.} \\ &= \frac{g}{\cos\theta_W} Z_\mu \cdot \frac{1}{4} (\tilde{H}_1 \gamma^\mu \gamma_5 \tilde{H}_1 - \tilde{H}_2 \gamma^\mu \gamma_5 \tilde{H}_2) + \text{h.c.} \end{aligned} \quad (1.197)$$

where the second line follows from (1.179). Since there is normally no Z^3 or $\gamma H^0 H^0$ vertex it is not surprising that the only vertex is the supersymmetric analogue of $Z H^0 H^0$. The terms like $\tilde{H}_i \gamma^\mu \tilde{H}_i$ vanish because the neutralinos are Majorana fermions ($\tilde{\chi}^{0\alpha} = \tilde{\chi}^0$). By analogy with the chargino case we define

$$\mathcal{L}_{Z^0\tilde{\chi}_i^+\tilde{\chi}_j^0} = \frac{g}{\cos\theta_W} Z_\mu \left[\frac{1}{2} \tilde{\chi}_i^0 \gamma^\mu \left(O_{ij}^{nL} \gamma_- + O_{ij}^{nR} \gamma_+ \right) \tilde{\chi}_j^0 \right] + \text{h.c.} \quad (1.198)$$

Recalling that $\tilde{\chi}_i^0 = N_{ij} \psi_j^0$ we may derive expressions similar to (1.190) and

(1.191):

$$\begin{aligned} \gamma_- \tilde{B}^0 &= \gamma_- N_{j1}^* \tilde{\chi}_j^0 \\ \gamma_+ \tilde{B}^0 &= \gamma_+ N_{j1} \tilde{\chi}_j^0 \\ \gamma_- \tilde{W}^0 &= \gamma_- N_{j2}^* \tilde{\chi}_j^0 \\ \gamma_+ \tilde{W}^0 &= \gamma_+ N_{j2} \tilde{\chi}_j^0 \\ \gamma_- \tilde{H}_1 &= \gamma_- N_{j3}^* \tilde{\chi}_j^0 \\ \gamma_+ \tilde{H}_1 &= \gamma_+ N_{j3} \tilde{\chi}_j^0 \\ \gamma_- \tilde{H}_2 &= \gamma_- N_{j4}^* \tilde{\chi}_j^0 \\ \gamma_+ \tilde{H}_2 &= \gamma_+ N_{j4} \tilde{\chi}_j^0 \end{aligned} \quad (1.199)$$

where j is summed over ($j = 1, 2, 3, 4$). The remarkable symmetry is due to the fact that $\tilde{\chi}_i^0$ is a Majorana fermion. Substituting (1.199) into (1.197) and using

$$\gamma_5 = \gamma_+ - \gamma_- \quad (1.200)$$

with (1.192) and (1.193), and then placing the result in the form of (1.198) gives:

$$\begin{aligned} O_{ij}^{nL} &= -\frac{1}{2} \{ N_{i3} N_{j3}^* - N_{i4} N_{j4}^* \} \\ O_{ij}^{nR} &= \frac{1}{2} \{ N_{i3}^* N_{j3} - N_{i4}^* N_{j4} \} \end{aligned} \quad (1.201)$$

Note that

$$O_{ij}^{nR} = -O_{ij}^{nL*} \quad (1.202)$$

and that photons do not couple to the neutralinos.

Table 1.1. Fields Appearing in the Minimal Model

Superfield	Ordinary Matter	Superpartners	Weak Isospin	y
<i>Vector (Gauge) Multiplets</i>				
$\hat{G}$	$g^i, i = 1, \dots, 8$	$\tilde{g}^i, i = 1, \dots, 8$	Singlet	0
$\hat{V}$	$W^{\pm,0}$	$\tilde{W}^{\pm,0}$	Triplet	0
$\hat{V}'$	B^0	$\tilde{B}^0$	Singlet	0
<i>Scalar Multiplets</i>				
$\hat{Q}_i$	u_{Li}, d_{Li}	$\tilde{u}_{Li}, \tilde{d}_{Li}$	Doublet	1/3
$\hat{U}_i$	$(\bar{u}_i)_L = (u_{Ri})^*$	$(\tilde{u}_{Ri})^*$	Singlet	-4/3
$\hat{D}_i$	$(\bar{d}_i)_L = (d_{Ri})^*$	$(\tilde{d}_{Ri})^*$	Singlet	2/3
$\hat{L}_i$	ν_i, ℓ_{Li}	$\tilde{\nu}_i, \tilde{\ell}_{Li}$	Doublet	-1
$\hat{E}_i$	$(\bar{e}_i)_L = (\ell_{Ri})^*$	$(\tilde{e}_{Ri})^*$	Singlet	2
$\hat{H}_1$	H_1^0, H_1^-	$\tilde{\psi}_{H_1}^0, \tilde{\psi}_{H_1}^-$	Doublet	-1
$\hat{H}_2$	H_2^+, H_2^0	$\tilde{\psi}_{H_2}^+, \tilde{\psi}_{H_2}^0$	Doublet	1
<i>Optional Higgs Singlet</i>				
$\hat{N}$	N	$\tilde{N}_L$	Singlet	0

FIGURE CAPTIONS

1. Patterns of Gauge and Supersymmetry Breaking.

- (a) Supersymmetry Unbroken and Gauge Symmetry Unbroken.
- (b) Supersymmetry Broken and Gauge Symmetry Unbroken.
- (c) Supersymmetry Unbroken and Gauge Symmetry Broken.
- (d) Supersymmetry Broken and Gauge Symmetry Broken.

Fig. 1.1

APPENDIX J

Mixing Matrices and Mass Eigenvalues

J.1 CHARGINOS

As we saw in the previous appendix (eqns 1.142-1.148) the chargino mass terms may be written as

$$\mathcal{L}_{Ch} = -\frac{1}{2} \left[(\psi^+)^T X^T \psi^- + (\psi^-)^T X \psi^+ \right] + h.c. \quad (J.1)$$

where $\psi^\pm$ are four-component fermions of the form

$$\psi^+ = \begin{pmatrix} -i\tilde{V}^+ \\ \tilde{\psi}_{H_2}^+ \end{pmatrix} \quad \psi^- = \begin{pmatrix} -i\tilde{V}^- \\ \tilde{\psi}_{H_1}^- \end{pmatrix} \quad (J.2)$$

and

$$X = \begin{pmatrix} M & \sqrt{2}m_W \cos \theta_e \\ \sqrt{2}m_W \sin \theta_e & \mu \end{pmatrix}. \quad (J.3)$$

We may diagonalize this to obtain the true chargino mass eigenstates which we denote $\tilde{\chi}_i^+$ ($i = 1, 2$) and $\tilde{\chi}_i^-$. When this is diagonalized via

$$\tilde{\chi}_i^+ = V_{ij} \psi_j^+ \quad \tilde{\chi}_i^- = U_{ij} \psi_j^- \quad (J.4)$$

we will have

$$\mathcal{L}_{Ch} = -\frac{1}{2} \left[m_{\tilde{\chi}_1^+} \tilde{\chi}_1^- \tilde{\chi}_1^+ + m_{\tilde{\chi}_2^+} \tilde{\chi}_2^- \tilde{\chi}_2^+ \right] + h.c. \quad (J.5)$$

The matrices U and V are unitary 2×2 matrices which diagonalize the mass matrix via

$$M_{\text{Diag}} = U^* X V^{-1}. \quad (J.6)$$

Diagonalization of General 2×2 Matrix

In general to diagonalize a regular 2×2 matrix

$$\begin{pmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{pmatrix}$$

by $M_D = U^* M V^†$, U and V unitary, to

$$M_D = \begin{pmatrix} M_+ & 0 \\ 0 & M_- \end{pmatrix}$$

We use

$$U = \begin{pmatrix} \cos \phi_- & \sin \phi_- \\ -\sin \phi_- & \cos \phi_- \end{pmatrix} \quad V = \begin{pmatrix} \cos \phi_+ & \sin \phi_+ \\ -\sin \phi_+ & \cos \phi_+ \end{pmatrix} \quad D > 0$$

$$U = \begin{pmatrix} \cos \phi_- & \sin \phi_- \\ -i \sin \phi_- & i \cos \phi_- \end{pmatrix} \quad V = \begin{pmatrix} \cos \phi_+ & \sin \phi_+ \\ -i \sin \phi_+ & i \cos \phi_+ \end{pmatrix} \quad D < 0$$

where

$$D = \det M = M_{11}M_{22} - M_{12}M_{21}$$

yielding the positive definite mass eigenvalues

$$M_{\pm} = \frac{1}{2} \{M_+ \pm M_-\} \quad D > 0 \quad (J.7)$$

$$M_{\pm} = \frac{1}{2} \{M_- \pm M_+\} \quad D < 0.$$

Here

$$M_{\pm} = \sqrt{(M_{11} \mp M_{22})^2 + (M_{12} \pm M_{21})^2}. \quad (J.8)$$

This is an appropriate time to interject a word concerning the positivity of the eigenmasses. Normally, when diagonalizing a matrix one cannot be so cavalier

about altering the sign of an eigenvalue. When the determinant of a real 2×2 matrix is negative it indicates one positive and one negative eigenvalue. It has been noted (and can be easily demonstrated) that negating the eigenmass of a spinor is physically equivalent to retaining the original sign and performing the substitution $\psi \rightarrow \gamma_5 \psi$ (or equivalently $\psi \rightarrow -\gamma_5 \psi$). Here we have accomplished the same end by multiplying a given row of both U and V by a factor of i . This negates the corresponding eigenvalue because we have used (J.6) instead of the purely unitary $M_D = U M V^†$. In particular we note that

$$U U^† = V V^† = 1 \quad D > 0$$

$$U U^† = V V^† = -\gamma_5 \quad D < 0.$$

We may similarly negate neutralino masses. The angular entries satisfy

$$\tan(\phi_+ \pm \phi_-) = \frac{M_{12} \pm M_{21}}{M_{11} \mp M_{22}}$$

$$\cos(\phi_+ \pm \phi_-) = \frac{M_{11} \mp M_{22}}{M_{\pm}}$$

$$\sin(\phi_+ \pm \phi_-) = \frac{M_{12} \pm M_{21}}{M_{\pm}}$$

$$\tan 2\phi_{\pm} = \frac{B_{\pm}}{A_{\pm}}$$

(J.9)

$$A_{\pm} = (M_{11}^2 - M_{22}^2) \pm (M_{21}^2 - M_{12}^2)$$

$$B_+ = 2[M_{11}M_{12} + M_{21}M_{22}]$$

$$B_- = 2[M_{11}M_{21} + M_{12}M_{22}]$$

$$\tan \phi_{\pm} = \frac{\sqrt{A_{\pm}^2 + B_{\pm}^2} - A_{\pm}}{B_{\pm}}.$$

Charginos: General Case

We proceed to use the above results for the general chargino pattern given by (J.3). The masses of the charginos ($\tilde{\chi}^\pm$) are given in terms of the supersymmetry breaking terms of M , M' and μ of eqn (3.4) and the Higgs VEV angle, θ_v , of eqn (3.6) (see section 1.4 also) by

$$m_{\tilde{\chi}_{1,2}^\pm} = \frac{1}{2} \{M_\pm \pm M_-\} \eta_\pm \quad M_\pm = \sqrt{(M \pm \mu)^2 + 2M_W^2(1 \mp \sin 2\theta_v)} \quad (\text{J.10})$$

with $\eta_+ = 1$ and $\eta_- = \text{sign}[M\mu - M_W^2 \sin 2\theta_v]$

The masses $m_{\tilde{\chi}_i^\pm}$ have been plotted, for typical parameter values, in Fig. J.1a.

The chargino vertex matrices are given by⁴⁴

$$U_{ij} = \begin{pmatrix} \cos \phi_- & \sin \phi_- \\ -\sin \phi_- & \cos \phi_- \end{pmatrix} \quad V_{ij} = \begin{pmatrix} \cos \phi_+ & \sin \phi_+ \\ -\sin \phi_+ & \cos \phi_+ \end{pmatrix} \quad D > 0$$

$$U_{ij} = \begin{pmatrix} \cos \phi_- & \sin \phi_- \\ -i \sin \phi_- & i \cos \phi_- \end{pmatrix} \quad V_{ij} = \begin{pmatrix} \cos \phi_+ & \sin \phi_+ \\ -i \sin \phi_+ & i \cos \phi_+ \end{pmatrix} \quad D < 0 \quad (\text{J.11})$$

where

$$D = M\mu - M_W^2 \sin 2\theta_v \quad (\text{J.12})$$

$$\tan \phi_\pm = \frac{\sqrt{A_\pm^2 + B_\pm^2} - A_\pm}{B_\pm} \quad (\text{J.13})$$

$$A_\pm = (M^2 - \mu^2) \mp 2M_W^2 \cos 2\theta_v$$

$$B_+ = 2\sqrt{2}M_W(M \cos \theta_v + \mu \sin \theta_v) \quad (\text{J.14})$$

$$B_- = 2\sqrt{2}M_W(M \sin \theta_v + \mu \cos \theta_v).$$

Recall from Appendix I that the general form of the non-Abelian supersymmetric

vertex was given by

$$O_{ij}^{L'} = -V_{i1}V_{j1}^* - \frac{1}{2}V_{i2}V_{j2}^* + \delta_{ij}\sin^2\theta_w \quad (\text{J.15})$$

$$O_{ij}^{R'} = -U_{i1}^*U_{j1} - \frac{1}{2}U_{i2}^*U_{j2} + \delta_{ij}\sin^2\theta_w$$

resulting in the general expressions

$$O_{ij}^{L'} = \begin{pmatrix} \frac{1}{2}\sin^2\phi_+ - \cos^2\theta_w & \frac{1}{2}\sin\phi_+\cos\phi_+ \\ \frac{1}{2}\sin\phi_+\cos\phi_+ & \frac{1}{2}\cos^2\phi_+ - \cos^2\theta_w \end{pmatrix}$$

$$O_{ij}^{R'} = \begin{pmatrix} \frac{1}{2}\sin^2\phi_- - \cos^2\theta_w & \frac{1}{2}\sin\phi_-\cos\phi_- \\ \frac{1}{2}\sin\phi_-\cos\phi_- & \frac{1}{2}\cos^2\phi_- - \cos^2\theta_w \end{pmatrix} \quad (\text{J.16})$$

for $D > 0$

$$O_{ij}^{L'} = \begin{pmatrix} \frac{1}{2}\sin^2\phi_+ - \cos^2\theta_w & -\frac{1}{2}\sin\phi_+\cos\phi_+ \\ \frac{1}{2}\sin\phi_+\cos\phi_+ & \frac{1}{2}\cos^2\phi_+ - \cos^2\theta_w \end{pmatrix}$$

$$O_{ij}^{R'} = \begin{pmatrix} \frac{1}{2}\sin^2\phi_- - \cos^2\theta_w & \frac{1}{2}\sin\phi_-\cos\phi_- \\ -\frac{1}{2}\sin\phi_-\cos\phi_- & \frac{1}{2}\cos^2\phi_- - \cos^2\theta_w \end{pmatrix} \quad (\text{J.17})$$

for $D < 0$.

Charginos: Special Case

Here we examine a few specific subcases of interest.

1) $M_{12} = M_{21}$ ($v_1 = v_2$)

When diagonalizing the general 2×2 symmetric matrix ($M_{12} = M_{21}$) we find

$$U = V = \begin{pmatrix} \cos \phi & \sin \phi \\ -\sin \phi & \cos \phi \end{pmatrix} \quad D > 0$$

$$U = V = \begin{pmatrix} \cos \phi & \sin \phi \\ -i \sin \phi & i \cos \phi \end{pmatrix} \quad D < 0$$

where

$$D = \det M = M_{11}M_{22} - M_{12}^2$$

and in this instance

$$\phi_+ = \phi_- = \phi$$

which is given by

$$\tan \phi = \frac{\sqrt{A^2 + B^2} - A}{B}$$

$$A = M_{11}^2 - M_{22}^2$$

$$B = 2M_{12}(M_{11} + M_{22})$$

The positive eigenvalues are

$$M_{\pm} = \frac{1}{2} \left\{ \sqrt{(M_{11} - M_{22})^2 + 4M_{12}^2} \pm (M_{11} + M_{22}) \right\}. \quad (J.18)$$

For charginos the symmetric condition implies that $\tan \theta_v = 1$. Restricting our-

selves to $-\pi \leq \theta_v \leq \pi$ we conclude that $v_1 = v_2$. Then

$$\tan \phi = \frac{\sqrt{A^2 + B^2} - A}{B} \quad (J.19)$$

$$A = M^2 - \mu^2 \quad (J.20)$$

$$B = 2M_W(M + \mu)$$

and

$$M_{\pm} = \frac{1}{2} \left\{ \sqrt{(M - \mu)^2 + M_W^2} \pm (M + \mu) \right\} \quad (J.21)$$

are the masses.

2) $M \rightarrow 0$, $\mu \rightarrow 0$ $v_1 \neq v_2$

This corresponds to the instance of the general 2×2 matrix in which it is completely off-diagonal, i.e. $M_{11} = M_{22} = 0$ but $M_{12} \neq M_{21}$. In this case some care is required when using the general expressions. There are two equally valid solutions:

$$\phi_- = 0, \phi_+ = \frac{\pi}{2} \text{ leading to } M_D = \begin{pmatrix} M_{12} & 0 \\ 0 & M_{21} \end{pmatrix} \quad (J.22)$$

$$\phi_- = \frac{\pi}{2}, \phi_+ = 0 \text{ leading to } M_D = \begin{pmatrix} M_{21} & 0 \\ 0 & M_{12} \end{pmatrix}.$$

For charginos $M_{12} = \sqrt{2}m_w \cos \theta_v$ and $M_{21} = \sqrt{2}m_w \sin \theta_v$. The corresponding

U, V and O' matrices are, for the first case,

$$U = \begin{pmatrix} 1 & 0 \\ 0 & i \end{pmatrix} \quad \begin{pmatrix} 0 & 1 \\ -i & 0 \end{pmatrix} \quad (J.23)$$

$$O_{ij}^{L'} = \begin{pmatrix} \frac{1}{2} - \cos^2 \theta_w & 0 \\ 0 & -\cos^2 \theta_w \end{pmatrix} \quad O_{ij}^{R'} = \begin{pmatrix} -\cos^2 \theta_w & 0 \\ 0 & \frac{1}{2} - \cos^2 \theta_w \end{pmatrix}. \quad (J.24)$$

3) "SUSY" Limit: $M = \mu = 0$, $v_1 = v_2$

As above, the mass matrix is completely anti-diagonal however, in this case, the off-diagonal terms are equal as well. The solutions $\phi_{\pm} = 0, \frac{\pi}{2}$ apply as before but now there are additional possibilities. Any ϕ_-, ϕ_+ subject to $\phi_- + \phi_+ = \frac{\pi}{2}$ is a valid solution. For aesthetic reasons we will choose to use the symmetric result

$$\phi_- = \phi_+ = \frac{\pi}{4}.$$

The resulting mixing matrices are

$$U = V = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ -i & i \end{pmatrix}$$

$$O_{ij}^{L'} = \frac{1}{4} \begin{pmatrix} 4\sin^2 \theta_w - 3 & -i \\ i & 4\sin^2 \theta_w - 3 \end{pmatrix} \quad (J.25)$$

$$O_{ij}^{R'} = \frac{1}{4} \begin{pmatrix} 4\sin^2 \theta_w - 3 & i \\ -i & 4\sin^2 \theta_w - 3 \end{pmatrix}$$

4) "Unmixed Limit" $M \rightarrow \infty$, $\mu = 0$

Assuming that we set $\mu = 0$ prior to taking the limit $M \rightarrow \infty$, we see that $D = \det X$ is negative. We find

$$M_{\pm} = M \rightarrow \infty$$

$$M_{\pm} \rightarrow \gamma \text{ as } M_W^2/M \quad (J.26)$$

ergo we have one very massive and one massless particle. We also determine that

$$\tan \phi_{\pm} \rightarrow 0 \text{ as } M_W/M$$

so that

$$U = V = \begin{pmatrix} 1 & 0 \\ 0 & i \end{pmatrix} \quad (J.27)$$

$$O_{ij}^{L'} = O_{ij}^{R'} = \begin{pmatrix} -\cos^2 \theta_w & 0 \\ 0 & \frac{1}{2} - \cos^2 \theta_w \end{pmatrix}$$

J.2 NEUTRALINOS

From $\mathcal{L}_g + \mathcal{L}_{Mix}$ in eqns (3.4) and (3.6) we can write the neutralino mass terms in the form:¹³

$$\mathcal{L}_{mass} = -\frac{1}{2} (\psi^0)^T Y \psi^0 + h.c. \quad (J.28)$$

where Y was given by eqn (3.8) or eqn (I.156) and

$$(\psi_j^0)^T = (-i\tilde{B}^0, -i\tilde{V}^0, \tilde{\psi}_{H_1}^0, \tilde{\psi}_{H_2}^0). \quad (J.29)$$

Y is diagonalized by the unitary matrix N via $M_D = N^* Y N^{\dagger}$ (we can choose N^* instead of N because we use $(\psi^0)^T$ instead of $(\psi^0)^{\dagger}$.) The mass eigenstates

then satisfy $\tilde{\chi}_i^0 = N_{ij}\psi_j^0$ ($i = 1, 2, 3, 4$). The masses $m_{\tilde{\chi}_1^0}$ and $m_{\tilde{\chi}_2^0}$ have been plotted, for typical parameter values, in Fig. J.1b. The neutralino vertex matrices are given by N and by¹³

$$O^{L''}_{ij} = \frac{1}{2} (N_{i4}N_{j4}^* - N_{i3}N_{j3}^*) \quad (\text{J.30})$$

$$O^{R''}_{ij} = -O^{L''*}_{ij} \quad (\text{J.31})$$

The General Case

The general 4×4 mass matrix may be diagonalized much as in the chargino case. We now must solve a quartic equation. The general roots are sufficiently involved that it is almost always more expedient to diagonalize specific cases numerically. Furthermore if a fifth neutralino, such as N , is added the resulting quintic is typically unsolvable in closed form. There are a number of special cases which arise which are tractable analytically.

A Special Case

An example of an instance in which the 4×4 Y matrix may be readily diagonalized occurs when $v_1 = v_2$ and $M = M'$. The various general forms following eqn (J156) then become:

$$N' = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos \omega & \sin \omega & 0 \\ 0 & -\sin \omega & \cos \omega & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

where

$$\tan \omega = \frac{M + \mu - M}{2M_X} \quad \tan 2\omega = \frac{2M_X}{M - \mu} \quad (\text{J.32})$$

Here

$$M = \sqrt{(M - \mu)^2 + 4M_X^2} \quad (\text{J.33})$$

Since

$$B = \begin{pmatrix} \cos \theta_\omega & \sin \theta_\omega & 0 & 0 \\ -\sin \theta_\omega & \cos \theta_\omega & 0 & 0 \\ 0 & 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ 0 & 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

we obtain

$$N = \begin{pmatrix} \cos \theta_\omega & \sin \theta_\omega & 0 & 0 \\ -\cos \omega \sin \theta_\omega & \cos \omega \cos \theta_\omega & \frac{1}{\sqrt{2}} \sin \omega & -\frac{1}{\sqrt{2}} \sin \omega \\ \sin \omega \sin \theta_\omega & -\sin \omega \cos \theta_\omega & \frac{1}{\sqrt{2}} \cos \omega & -\frac{1}{\sqrt{2}} \cos \omega \\ 0 & 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} \quad (\text{J.34})$$

The mass eigenvalues are

$$\begin{aligned} m_{\tilde{\chi}_1^0} &= M \\ m_{\tilde{\chi}_2^0} &= \frac{1}{2} (M + \mu + M) \\ m_{\tilde{\chi}_3^0} &= \frac{1}{2} (M + \mu - M) \\ m_{\tilde{\chi}_4^0} &= -\mu \end{aligned} \quad (\text{J.35})$$

To negate $m_{\tilde{\chi}_i^0}$ multiply the i^{th} row of N by $\pm i$ as discussed in the section on charginos.

FIGURE CAPTIONS

1. Masses for the indicated supersymmetry-breaking parameters.

a) Charginos

b) Neutralinos

Fig. J.1

APPENDIX K

Feynman Rules

K.1 INTRODUCTION

Given the Lagrangian $\mathcal{L}$ it is a straightforward process, in principle, to generate the Feynman rules for a theory. The generating functional is

$$W[J] = \int [d\theta] e^{i\{S + \int J(x)\theta(x, \bar{\theta})d^4x\}} \quad (\text{K.1})$$

where the action is given by

$$S = \int \mathcal{L}(x, \theta, \bar{\theta}) d^4x d^4\theta \quad (\text{K.2})$$

for a supersymmetric theory. We then generate connected Green's functions via

$$G^{(n)}(x_1, \dots, x_N) = \langle 0 | T(\phi(x_1) \dots \phi(x_N)) | 0 \rangle = i^n \frac{\delta^n \ln W[J]}{\delta J(x_1) \dots \delta J(x_N)}. \quad (\text{K.3})$$

This can be done with the superfields directly leading to the so-called "supergraph" approach. Such a procedure has proved useful in proving non-renormalization theorems, deriving super-Slavnov-Taylor identities and so forth but has limited utility in performing practical calculations when the supersymmetry has been explicitly broken. In practice it is far easier to read the Feynman rules directly from $\mathcal{L}$ once it has been fully expanded (remembering the factor of "i" from (K.1) and any symmetry factors). There is yet another complication in the gaugino sector. Because we have *Majorana fermions*, which carry no conserved internal quantum number, the direction which a spinor line points is somewhat arbitrary. Using the procedure and construction of refs. 12 and 13 we assign the direction conventionally and establish the corresponding rules.

K.2 MAJORANA FERMIONS

Charge Conjugation

The positive and negative-energy solutions to the Dirac equation

$$(i \not{\partial} - e \not{A} - m)\psi = 0$$

are $u(p, s)$ and $v(p, s)$ respectively. It is convenient to define the anti-fermion, ψ^c , which is a positive-energy solution to $(i \not{\partial} + e \not{A} - m)\psi = 0$.

Given (since $\bar{\psi} = \psi^\dagger \gamma^0$)

$$\psi = \begin{pmatrix} \xi_a \\ \bar{\eta}^{\dot{a}} \end{pmatrix} \quad \bar{\psi} = (\eta^a \bar{\xi}_{\dot{a}}) \quad \psi^c = \begin{pmatrix} \eta_{\dot{a}} \\ \bar{\xi}^a \end{pmatrix} \quad (\text{K.4})$$

$$\xi^a = \epsilon^{ab} \xi_b \quad C = i\gamma^2 \gamma^0 = \begin{pmatrix} \epsilon_{ab} & 0 \\ 0 & \epsilon^{\dot{a}\dot{b}} \end{pmatrix} \quad (\text{K.5})$$

we see that

$$\psi^C = C \bar{\psi}^T. \quad (\text{K.6})$$

Note the following useful relations:

$$C^\dagger = C^{-1} \quad C = \text{charge conjugation matrix} \quad (\text{K.7})$$

$$C^T = -C \quad (\text{K.8})$$

$$C^{-1} \gamma_\mu C = (\gamma_\mu)^T \quad C^{-1} \gamma^\mu C = -(\gamma^\mu)^T \quad (\text{K.9})$$

where in our notation

$$\gamma_{\pm} = \frac{1}{2} (1 \pm \gamma_5) \quad \gamma_5 = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \quad (\text{K.10})$$

For any matrix M , $(M^T)^{-1} = (M^{-1})^T$, in particular

$$(C^T)^{-1} = (C^{-1})^T \quad (\text{K.11})$$

$$u^T(k, s) = v(k, s) C^T \quad v^T(k, s) = u(k, s) C^T$$

$$\bar{u}^T(k, s) = C^{-1} v(k, s) \quad \bar{v}^T(k, s) = C^{-1} u(k, s) \quad (\text{K.12})$$

$$\gamma^\mu \gamma_{\pm} = \gamma_{\mp} \gamma^\mu \quad (\gamma_{\pm})^2 = \gamma_{\pm} \quad \gamma_{\pm} \gamma_{\mp} = 0$$

In any process we attach spinor legs according to

Initial State

Final State

$u(p)u(p)$

$v(p)v(p)$

$u(p)v(p)$

$v(p)u(p)$

(K.13)

Majorana Propagators:

	$\left(\frac{i(\not{p} + m)}{p^2 - m^2 + i\epsilon} \right)_{\alpha\beta}$
	$\left(\frac{iC^{-1}(\not{p} + m)}{p^2 - m^2 + i\epsilon} \right)_{\alpha\beta}$
	$\left(\frac{-i(\not{p} + m)C}{p^2 - m^2 + i\epsilon} \right)_{\alpha\beta}$
	$\left(\frac{i(-\not{p} + m)}{p^2 - m^2 + i\epsilon} \right)_{\alpha\beta}$

Vertex Manipulation

Let λ, ψ, c be a general Majorana fermion, Dirac fermion and boson field respectively, converging at a single vertex. Then¹² the various permutations of the direction arrows and corresponding rules are illustrated in Fig. K.1. The ambiguities arise because Majorana fermions carry no conserved quantum number.

K.3 CHARGING VERTICES

Let e, ν indicate a representative lepton doublet with no family mixing (arrows indicate the flow of positive charge).¹³ We ignore m_e since in the matrix element it is negligible relative to m_τ . The vertex is

$$[-i\partial V_{j1}\gamma + C]_{\alpha\beta}$$

(K.14)

$$[igC^{-1}V_{j1}^*\gamma -]_{\beta\alpha}$$

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Where V_{ij} was defined in Appendices I and J.

When we go to the case where $\bar{\nu}$ is $\bar{\nu}_1$ or $\bar{\nu}_2$ instead of $\bar{\nu}_e$ (i.e. a mixture in flavor space) we will in addition have mixing angles ($\sin\theta_\nu, \cos\theta_\nu$) in the vertex (in general when all 3 generators mix, we will have a mixing matrix element).

To see how some of the above rules work we will derive two new effective rules for the previous vertices when the Dirac lepton (e) is an external particle and the corresponding spinor has not been truncated. We will go directly to the case of $\mu - \tau$ mixing,

$$\tilde{\nu}_1 = \tilde{\nu}_\mu \cos \theta_\nu + \tilde{\nu}_\tau \sin \theta_\nu \quad \tilde{\nu}_2 = -\tilde{\nu}_\mu \sin \theta_\nu + \tilde{\nu}_\tau \cos \theta_\nu \quad (\text{K.15})$$

$$[igC^{-1}V_{\alpha 1}^* \gamma_-]_{\ell_1 \ell_2} \cos \theta_\nu \quad (\text{K.16})$$

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This part of the matrix element is

$$\begin{aligned} & |u^\mu(p_1')|_{\ell_1 \ell_2} [igC^{-1}V_{\alpha 1}^* \gamma_-]_{\ell_1 \ell_2} \cos \theta_\nu \\ &= ig \cos \theta_\nu \left(|u^\mu(p_1')|^T \right)_{\ell_1 \ell_2} V_{\alpha 1}^* \left([C^{-1} \gamma_-]^T \right)_{\ell_1 \ell_2} \\ &= ig V_{\alpha 1}^* \cos \theta_\nu \left(|u^\mu(p_1')|^T [C^{-1} \gamma_-]^T \right)_{\ell_1 \ell_2} \\ &= ig V_{\alpha 1}^* \cos \theta_\nu \left(\bar{u}^\mu(p_1') C^T [(\gamma_-)^T C^{-1}]^T \right)_{\ell_1 \ell_2} \text{ using (K.9) and (K.12)} \quad (\text{K.17}) \\ &= ig V_{\alpha 1}^* \cos \theta_\nu \left(\bar{u}^\mu(p_1') C^T (C^{-1})^T \gamma_- \right)_{\ell_1 \ell_2} \\ &= ig V_{\alpha 1}^* \cos \theta_\nu \left(\bar{u}^\mu(p_1') C^T (C^{-1})^T \gamma_- \right)_{\ell_1 \ell_2} \text{ using (K.11)} \\ &= ig V_{\alpha 1}^* \cos \theta_\nu \left(\bar{u}^\mu(p_1') \gamma_- \right)_{\ell_1 \ell_2} \end{aligned}$$

Now regarding Fig. K.1 we can graphically depict this as

$$[u^\mu(p_1')]_{\ell_1 \ell_2} [igV_{\alpha 1}^* \gamma_-]_{\ell_1 \ell_2} \cos \theta_\nu \quad (\text{K.18})$$

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Thus (K.18) is a new effective non-truncated vertex (which will prove useful). Similarly, from (K.14) the conjugate of the previous example gives us

$$[-igV_{\alpha 1} \gamma_+ C]_{\beta_1 \beta_2} \sin \theta_\nu \quad (\text{K.19})$$

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$$\begin{aligned} &= [-igV_{\alpha 1} \gamma_+ C]_{\beta_1 \beta_2} \sin \theta_\nu [\bar{u}^\mu(p_2')]_{\beta_2 \beta_1} \\ &= -igV_{\alpha 1} \sin \theta_\nu [\gamma_+ C]_{\beta_2 \beta_1}^T [\bar{u}^\mu(p_2')]_{\beta_2 \beta_1}^T \\ &= -igV_{\alpha 1} \sin \theta_\nu \left([C \gamma_+^T]^T [C^{-1} \bar{u}^\mu(p_2')] \right)_{\beta_2 \beta_1} \\ &= -igV_{\alpha 1} \sin \theta_\nu \left(\gamma_+ C^T C^{-1} \bar{u}^\mu(p_2') \right)_{\beta_2 \beta_1} \\ &= -igV_{\alpha 1} \sin \theta_\nu \left(\gamma_+ (-C) C^{-1} \bar{u}^\mu(p_2') \right)_{\beta_2 \beta_1} \\ &= [igV_{\alpha 1} \sin \theta_\nu \gamma_+ \bar{u}^\mu(p_2')]_{\beta_2 \beta_1} \quad (\text{K.20}) \end{aligned}$$

Depicted graphically as

$$(ig V_{ln} \gamma_+ \rho_{2, B1} \sin \theta_w [v^T (O_2^l)]_{B1, B2})$$

(K.21)

It must be emphasized that (K.18) and (K.21) are valid *only* for the specific case where the lepton (μ or τ) is (1) external and (2) in the final state (as in our problem). In general (K.16) and (K.19) must be used. We will refer to (K.18) and (K.21) as "derived" Feynman rules and use them to great effect in the next appendix.

Other "chargino" vertices

$$\frac{ig}{\cos \theta_w} [\gamma^V (O_{ij}^L \gamma_- + O_{ij}^R \gamma_+)]_{\alpha_1 \alpha_2}$$

(K.22)

$$\frac{-ig}{2 \cos \theta_w} \{k_1 + k_2\}^\mu$$

(K.23)

And the normal Z^0 -lepton vertex

$$\frac{ig}{2 \cos \theta_w} [\gamma^V (\cos 2\theta_w \gamma_- - 2 \sin^2 \theta_w \gamma_+)]_{\alpha_1 \alpha_2}$$

(K.24)

$$= ie [\gamma^V (\cot 2\theta_w \gamma_- - \tan \theta_w \gamma_+)]_{\alpha_1 \alpha_2}$$

K.4 NEUTRALINO VERTICES

$$\frac{ig}{\sqrt{2} \cos \theta_w} [\sin 2\theta_w N'_{j1} + \cos 2\theta_w N'_{j2}] \gamma_+ \quad (\text{K.25})$$

With no family mixing.

We wish to express our results in terms of N_{ij} (N and N' were defined in Appendix I).

$$N'_{j1} = N_{j1} \cos \theta_w + N_{j2} \sin \theta_w \quad N'_{j2} = -N_{j1} \sin \theta_w + N_{j2} \cos \theta_w \quad (\text{K.26})$$

The above vertex becomes

$$\begin{aligned} V_{\tilde{\chi}_1^0, l} &= \frac{ig}{\sqrt{2} \cos \theta_w} \left\{ 2 \sin \theta_w \cos \theta_w (N_{j1} \cos \theta_w + N_{j2} \sin \theta_w) \right. \\ &\quad \left. + (\cos^2 \theta_w - \sin^2 \theta_w) (-N_{j1} \sin \theta_w + N_{j2} \cos \theta_w) \right\} \gamma_+ \\ &= \left\{ \frac{ig}{\sqrt{2}} [\tan \theta_w N_{j1} + N_{j2}] \gamma_+ \right\}_{\alpha\beta} \end{aligned} \quad (\text{K.27})$$

Therefore using Fig. K.1

Reversing all arrows in (K.28) yields

$$\left\{ \frac{ig}{\sqrt{2}} [\tan \theta_w N_{j1} + N_{j2}] \gamma_+ C \right\}_{\alpha\beta} \quad (\text{K.28})$$

$$\overline{\left\{ \frac{ig}{\sqrt{2}} [\tan \theta_w N_{j1} + N_{j2}] \gamma_+ C \right\}_{\alpha\beta}} \quad (\text{K.29})$$

$$= \left\{ \frac{-ig}{\sqrt{2}} [\tan \theta_w N'_{j1} + N'_{j2}] C^{-1} \gamma_- \right\}_{\beta\alpha}$$

Again from Fig. K.1

$$= \left\{ \frac{-ig}{\sqrt{2}} [\tan \theta_w N'_{j1} + N'_{j2}] C^{-1} \gamma_- \right\}_{\beta\alpha} \quad (\text{K.30})$$

Similarly for the $\tilde{X}_j^0 \tilde{e}_R$ vertex

$$\begin{aligned} & \frac{i}{\sqrt{2}} \left[g \sin \theta_w N_{j1}^* - \frac{g \sin^2 \theta_w}{\cos \theta_w} N_{j2}^* \right] (1 - \gamma_5) \\ &= \frac{2}{\cos \theta_w} \left(\frac{-ig}{\sqrt{2}} \right) [\sin \theta_w \cos \theta_w (N_{j1}^* \cos \theta_w + N_{j2}^* \sin \theta_w) \\ & \quad - \sin^2 \theta_w (-N_{j1}^* \sin \theta_w + N_{j2}^* \cos \theta_w) \gamma_-] \\ &= \left\{ -\frac{ie\sqrt{2}}{\cos \theta_w} N_{j1}^* \gamma_- \right\}_{\alpha\beta} \end{aligned}$$

so the vertex is

$$\left\{ -\frac{ie\sqrt{2}}{\cos \theta_w} N_{j1}^* \gamma_- \right\}_{\alpha\beta} \quad (K.31)$$

Using Fig. K.1 we obtain these equivalent vertices:

$$\left\{ -ig\sqrt{2} \tan \theta_w N_{j1}^* \gamma_- C \right\}_{\alpha\beta} \quad (K.32)$$

$$\left\{ ig\sqrt{2} \tan \theta_w N_{j1} \gamma_+ \right\}_{\beta\alpha} \quad (K.33)$$

$$\left\{ ig\sqrt{2} \tan \theta_w N_{j2} C^{-1} \gamma_+ \right\}_{\beta\alpha} \quad (K.34)$$

In all cases above the rules have been demonstrated for *no* flavor mixing. (i.e. for $\tilde{e}_L, \tilde{e}_R$ only.) By analogy with (K.15) we have sleptons mixing:

$$\tilde{e}_{1L} = \tilde{\mu}_L \cos \theta_L + \tilde{\tau}_L \sin \theta_L$$

$$\tilde{e}_{2L} = -\tilde{\mu}_L \sin \theta_L + \tilde{\tau}_L \cos \theta_L$$

$$\tilde{e}_{1R} = \tilde{\mu}_R \cos \theta_R + \tilde{\tau}_R \sin \theta_R$$

$$\tilde{e}_{2R} = -\tilde{\mu}_R \sin \theta_R + \tilde{\tau}_R \cos \theta_R$$

(K.35)

In the simplest case

$$\theta_\nu = \theta_L = \theta_R \equiv \theta$$

(K.36)

In the most general case $\tilde{e}_L, \tilde{\mu}_L, \tilde{\tau}_L, \tilde{e}_R, \tilde{\mu}_R$ and $\tilde{\tau}_R$ all mix together as do $\tilde{\nu}_e, \tilde{\nu}_\mu$ and $\tilde{\nu}_\tau$. This leads to rules like:

$$[ig\sqrt{2}\tan\theta_w N_{j1}\gamma_+]\beta_1\beta_2\sin\theta_R$$

(K.37)

$$\left\{ \frac{-ig}{\sqrt{2}} [\tan\theta_w N'_{j1} + N'_{j2}] C^{-1}\gamma_- \right\} \beta_1\beta_2 \cos\theta_L$$

Other "Neutralino" Vertices

$$\frac{ig}{\cos\theta_w} [\gamma^\mu (O_{ij}^{\prime\prime L}\gamma_- + O_{ij}^{\prime\prime R}\gamma_-)]\alpha_1\alpha_2$$

(K.38)

$$\frac{ig}{2\cos\theta_w} \cos 2\theta_w (k_1 + k_2)^\mu$$

(K.39)

$$\frac{-ig}{2\cos\theta_w} \cdot 2\sin^2\theta_w (k_1 + k_2)^\mu$$

(K.40)

These are all of the Feynman rules required for the process $Z^0 \rightarrow \mu^- \tau^+$.

FIGURE CAPTIONS

1. The various permutations of the direction arrows and corresponding rules for a vertex consisting of λ, ψ, c which are a general Majorana fermion, Dirac fermion and boson field respectively.

Here

$$\Gamma_i = 1, i\gamma_5, \gamma_\mu \gamma_5, \gamma_\mu, \sigma_{\mu\nu}.$$

Fig. K.1

$$iK_{abc}^i \Gamma_{ia\beta}$$

$$-iK_{abc}^i (C^{-1} \Gamma_i)_{a\beta}$$

$$iK_{abc}^{ii} \Gamma_{i\beta a}$$

$$iK_{abc}^{ii} (\Gamma_i C)_{\beta a}$$

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APPENDIX L

Computation of $Z^0 \rightarrow \tau\mu$ Matrix Elements

L.1 CHARGINO DIAGRAMS

Diagram 1

We demonstrate the use of the "manipulated" Feynman rules (K.18) and (K.21) here. Diagram one is illustrated in Fig. 1. From the Feynman rules of Appendix K we may read off the matrix element:

$$\begin{aligned}
 M_1^\nu &= \int \frac{d^4 k}{(2\pi)^4} \{ \bar{u}^\mu(p_1') \}_{\alpha\beta} [i g V_{\alpha\beta} \gamma_-]_{\delta\epsilon} \cos \theta_\nu \\
 &\otimes \left\{ \frac{i \left[(p_1' - k) + m_{\tilde{\chi}_1^+} \right]}{(p_1' - k)^2 - m_{\tilde{\chi}_1^+}^2} \right\}_{\delta\epsilon} \frac{i g}{\cos \theta_\nu} \left[\gamma^\nu (O_{ij}^{L'} \gamma_- + O_{ij}^{R'} \gamma_+) \right]_{\alpha\beta} \\
 &\otimes \left\{ \frac{i \left[-(p_2' + k) + m_{\tilde{\chi}_j^+} \right]}{(p_2' + k)^2 - m_{\tilde{\chi}_j^+}^2} \right\}_{\kappa\lambda} [i g V_{j\lambda} \gamma_+]_{\beta\delta} \\
 &\otimes \sin \theta_\nu \frac{i}{k^2 - m_{\tilde{\nu}_1}^2} \{ v^\nu(p_2') \}_{\beta\alpha} \\
 &= \frac{-g^3 \sin \theta_\nu \cos \theta_\nu V_{j1} V_{\alpha\beta}}{\cos \theta_\nu} \int \frac{d^4 k}{(2\pi)^4} \\
 &\times \bar{u}^\mu(p_1') \frac{\gamma_-}{k^2 - m_{\tilde{\nu}_1}^2} \frac{\left[(p_1' - k) + m_{\tilde{\chi}_1^+} \right]}{(p_1' - k)^2 - m_{\tilde{\chi}_1^+}^2} \\
 &\times \gamma^\nu \left[O_{ij}^{L'} \gamma_- + O_{ij}^{R'} \gamma_+ \right] \frac{\left[-(p_2' + k) + m_{\tilde{\chi}_j^+} \right]}{(p_2' + k)^2 - m_{\tilde{\chi}_j^+}^2} \gamma_+ v^\nu(p_2')
 \end{aligned} \tag{L.1}$$

Notice how simple the matrix index structure is. This is a result of using the manipulated rules (K.18) and (K.21). If these had not been utilized we would

essentially have derived them when simplifying the expression which would have resulted.

$$\begin{aligned}
 M_1^\nu &= \frac{g^3 \sin \theta_\nu \cos \theta_\nu V_{j1} V_{\alpha\beta}}{\cos \theta_\nu} \int \frac{d^4 k}{(2\pi)^4} \\
 &\times \bar{u}^\mu(p_1') \frac{\gamma_- \left[(p_1' - k) \gamma^\nu O_{ij}^{L'} (p_2' + k) - m_{\tilde{\chi}_1^+} m_{\tilde{\chi}_j^+} O_{ij}^{R'} \right] \gamma_+}{\left[(p_1' - k)^2 - m_{\tilde{\chi}_1^+}^2 \right] \left[(p_2' + k)^2 - m_{\tilde{\chi}_j^+}^2 \right] \left[k^2 - m_{\tilde{\nu}_1}^2 \right]} v^\nu(p_2')
 \end{aligned} \tag{L.2}$$

where we have used

$$\gamma^\mu \gamma_\pm = \gamma_\mp \gamma^\mu \quad (\gamma_\pm)^2 = \gamma_\pm \quad \gamma_\pm \gamma_\mp = 0$$

several times. Using

$$\frac{1}{k^2 - m_{\tilde{\nu}_1}^2} - \frac{1}{k^2 - m_{\tilde{\nu}_2}^2} = - \int_{m_{\tilde{\nu}_1}^2}^{m_{\tilde{\nu}_2}^2} \frac{dt}{(k^2 - t)^2}$$

we obtain

$$\begin{aligned}
 M_1^\nu &= - \frac{2e^3 \sin \theta_\nu \cos \theta_\nu V_{j1} V_{\alpha\beta}}{\sin^2 \theta_\nu \sin 2\theta_\nu} \int \frac{d^4 k}{(2\pi)^4} \int_{m_{\tilde{\nu}_1}^2}^{m_{\tilde{\nu}_2}^2} dt \\
 &\times \frac{\bar{u}^\mu(p_1') \gamma_- \left[(p_1' - k) \gamma^\nu O_{ij}^{L'} (p_2' + k) - m_{\tilde{\chi}_1^+} m_{\tilde{\chi}_j^+} O_{ij}^{R'} \right] \gamma_+ v^\nu(p_2')}{[k^2 - t]^2 \left[k^2 - 2p_1' \cdot k + m_\mu^2 - m_{\tilde{\chi}_1^+}^2 \right] \left[k^2 + 2p_1' \cdot k + m_\tau^2 - m_{\tilde{\chi}_j^+}^2 \right]}
 \end{aligned} \tag{L.3}$$

We proceed to ignore all m_μ^2, m_τ^2 terms (relative to $q^2, m_{\tilde{\nu}_1}^2, m_{\tilde{\chi}_j^+}^2$ etc. terms).

From equation (H.5)

$$\frac{1}{a^2bc} = 6 \int_0^1 dx \int_0^{1-x} dy \frac{1-x-y}{[a+(b-a)x+(c-a)y]^4}$$

Therefore

$$\begin{aligned} M_1^\nu = & -\frac{2e^2 \sin \theta_\nu \cos \theta_\nu V_{j1} V_{i1}^*}{\sin^2 \theta_\nu \sin 2\theta_\nu} \int \frac{d^4 k}{(2\pi)^4} \int_{m_{\tilde{\chi}_i^+}^2}^{m_{\tilde{\chi}_j^+}^2} dt \ 6 \int_0^1 dx \int_0^{1-x} dy (1-x-y) \\ & \times \frac{\bar{u}^\mu(p_1') \gamma_- \left[(\not{y}_1' - \not{k}) \gamma^\nu O_{ij}^{L'} (\not{y}_2' + \not{k}) - m_{\tilde{\chi}_i^+} m_{\tilde{\chi}_j^+} O_{ij}^{R'} \right] \gamma_+ v^\nu(p_2')}{\left[k^2 - t + (t - 2p_1' \cdot k - m_{\tilde{\chi}_i^+}^2) x + (t + 2p_2' \cdot k - m_{\tilde{\chi}_j^+}^2) y \right]^4} \quad (\text{L.4}) \end{aligned}$$

From (H.10) we can switch variables of integration.

$$x \rightleftharpoons x+y \quad \bar{z} \rightleftharpoons x-y$$

so that

$$xy = \frac{1}{4} (x^2 - \bar{z}^2)$$

$$\int_0^1 dx \int_0^{1-x} dy = \frac{1}{2} \int_0^1 dx \int_{-x}^{\bar{z}}$$

We now employ the formulae of (H.7)

$$\begin{aligned} \int \frac{d^4 k}{(2\pi)^4} \frac{l_i k^\mu}{(k^2 - 2p \cdot k + s - i\epsilon)^4} &= \frac{i[l_i p^\mu]}{96\pi^2 (s - p^2)^3} \\ \int \frac{d^4 k}{(2\pi)^4} \frac{k_\mu k^\nu}{(k^2 - 2p \cdot k + s - i\epsilon)^4} &= \frac{i[p_\mu p^\nu + \frac{1}{2}(s - p^2)\delta_{\mu\nu}]}{96\pi^2 (s - p^2)^3} \end{aligned}$$

to re-cast our integral into the form

$$\begin{aligned} M_1^\nu = & -\frac{2e^2 \sin \theta_\nu \cos \theta_\nu V_{j1} V_{i1}^*}{\sin^2 \theta_\nu \sin 2\theta_\nu} \int_{m_{\tilde{\chi}_i^+}^2}^{m_{\tilde{\chi}_j^+}^2} dt \ \frac{i}{32\pi^3} \int_0^1 dx \int_{-x}^{\bar{z}} d\bar{z} (1-x) \\ & \times \bar{u}^\mu(p_1') \gamma_- \left\{ \frac{\left[(\not{y}_1' - \not{P}) \gamma^\nu O_{ij}^{L'} (\not{y}_2' + \not{P}) - m_{\tilde{\chi}_i^+} m_{\tilde{\chi}_j^+} O_{ij}^{R'} \right]}{(s - p^2)^3} \right. \\ & \left. - \frac{\frac{1}{2} \gamma^\mu \gamma^\nu \gamma_\mu \delta_{\mu\nu} O_{ij}^{L'}}{s - p^2} \right\} \gamma_+ v^\nu(p_2') \quad (\text{L.6}) \end{aligned}$$

with

$$P = p_1' x - p_2' y$$

$$\begin{aligned} s &= -t(1-x-y) - \left[m_{\tilde{\chi}_i^+}^2 x + m_{\tilde{\chi}_j^+}^2 y \right] \\ &= -\left\{ t(1-x) + \frac{1}{2} \left[\left(m_{\tilde{\chi}_i^+}^2 + m_{\tilde{\chi}_j^+}^2 \right) x + \left(m_{\tilde{\chi}_i^+}^2 - m_{\tilde{\chi}_j^+}^2 \right) \bar{z} \right] \right\} \quad (\text{L.6}) \end{aligned}$$

As in (H.17 a) we see that

$$P^2 \doteq \frac{q^2}{4} (\bar{z}^2 - x^2)$$

and so

$$\begin{aligned} s - P^2 &\equiv -D(t) \\ &= -\left\{ t(1-x) + \frac{1}{2} \left[\left(m_{\tilde{\chi}_i^+}^2 + m_{\tilde{\chi}_j^+}^2 \right) x + \left(m_{\tilde{\chi}_i^+}^2 - m_{\tilde{\chi}_j^+}^2 \right) \bar{z} \right] \right. \\ &\quad \left. + \frac{q^2}{4} (\bar{z}^2 - x^2) \right\}. \quad (\text{L.7}) \end{aligned}$$

From (L.5)

$$\mathcal{M}_1^\nu = -\frac{2e^3 \sin \theta_\nu \cos \theta_\nu V_{j1} V_{i1}^*}{\sin^3 \theta_\nu \sin 2\theta_\nu} \cdot \frac{i}{32\pi^3} \int_{m_{\tilde{\nu}_1}^2}^{m_{\tilde{\nu}_2}^2} dt \int_0^1 dx \int_{-x}^x dz (1-x) \quad (\text{L.8})$$

$$\times \bar{u}^\mu(p'_1) \gamma_- \left\{ \frac{T - m_{\tilde{\chi}_i^+}^2 - m_{\tilde{\chi}_j^+}^2 + O_{ij}^{\mu\nu}}{[D(t)]^2} + \frac{\gamma^\nu O_{ij}^{\mu\nu}}{-D(t)} \right\} \gamma_+ v^\nu(p'_2)$$

since $\gamma^\mu \gamma^\nu \gamma_\mu = -2\gamma^\nu$. In (L.8)

$$\bar{u}^\mu(p'_1) T v^\nu(p'_2) = \bar{u}^\mu(p'_1) \gamma_- \left\{ (p'_1[1-x] + p'_2 y) \gamma^\nu O_{ij}^{\mu\nu} (p'_2[1-y] + p'_1 z) \right\} \gamma_+ v^\nu(p'_2) \quad (\text{L.9})$$

Using the Dirac equation

$$\bar{u}^\mu(p'_1) p'_1 \approx 0 \quad p'_2 v^\nu(p'_2) \approx 0$$

this becomes

$$\begin{aligned} \bar{u}^\mu(p'_1) T v^\nu(p'_2) &\approx \bar{u}^\mu(p'_1) \gamma_- p'_2 y \gamma^\nu p'_1 z \gamma_+ v^\nu(p'_2) \\ &\approx -2xy p'_1 \cdot p'_2 \bar{u}^\mu(p'_1) \gamma_- \gamma^\nu \gamma_+ v^\nu(p'_2) \\ &\approx \frac{1}{4} q^2 (\bar{z}^2 - z^2) \xi^\nu \end{aligned}$$

where we recall from (3.9 a) that

$$\xi^\nu_{L,R} = \bar{u}^\mu(p'_1) \gamma_\mp \gamma^\nu \gamma_\pm v^\nu(p'_2) \quad (\text{L.10})$$

Also recall (from (3.9 d) that

$$K_\nu = \frac{i}{16\pi^2} e^3 \sin \theta_\nu \cos \theta_\nu \quad (\text{L.11})$$

and

$$\frac{2e^3 \sin \theta_\nu \cos \theta_\nu V_{j1} V_{i1}^*}{\sin^3 \theta_\nu \sin 2\theta_\nu} \cdot \frac{i}{32\pi^3} = -\frac{K_\nu V_{j1} V_{i1}^*}{\sin^3 \theta_\nu \sin 2\theta_\nu}.$$

Performing the t integration and substituting for T in (L.8)

$$\begin{aligned} \mathcal{M}_1^\nu &= \sum_{ij} \xi^\nu_L \frac{K_\nu V_{j1} V_{i1}^*}{\sin^3 \theta_\nu \sin 2\theta_\nu} \int_0^1 dx \int_{-x}^x dz \\ &\times \left\{ \frac{m_{\tilde{\chi}_i^+}^2 - m_{\tilde{\chi}_j^+}^2 + O_{ij}^{\mu\nu} - \frac{1}{4} q^2 (\bar{z}^2 - z^2) O_{ij}^{\mu\nu}}{D(m_{\tilde{\nu}_1}^2)} - m_{\tilde{\nu}_1} \rightarrow m_{\tilde{\nu}_2} \right. \\ &\quad \left. + O_{ij}^{\mu\nu} \ln \left[\frac{D(m_{\tilde{\nu}_2}^2)}{D(m_{\tilde{\nu}_1}^2)} \right] \right\} \quad (\text{L.12}) \end{aligned}$$

where

$$\begin{aligned} D(m_{\tilde{\nu}_1}^2) &= m_{\tilde{\nu}_1}^2 (1-x) \\ &+ \frac{1}{4} \left[\left(m_{\tilde{\chi}_i^+}^2 + m_{\tilde{\chi}_j^+}^2 \right) x + \left(m_{\tilde{\chi}_i^+}^2 - m_{\tilde{\chi}_j^+}^2 \right) \bar{z} \right] + \frac{1}{4} q^2 (\bar{z}^2 - z^2) \end{aligned}$$

If we had employed a traditional Pauli-Villars regularization scheme we would have obtained

$$\begin{aligned} \mathcal{M}_1^\nu &= \sum_{ij} \xi^\nu_L \frac{K_\nu V_{j1} V_{i1}^*}{\sin^3 \theta_\nu \sin 2\theta_\nu} \int_0^1 dx \int_{-x}^x dz \\ &\times \left\{ \frac{m_{\tilde{\chi}_i^+}^2 - m_{\tilde{\chi}_j^+}^2 + O_{ij}^{\mu\nu} - \frac{1}{4} q^2 (\bar{z}^2 - z^2) O_{ij}^{\mu\nu}}{D(m_{\tilde{\nu}_1}^2)} \right. \\ &+ O_{ij}^{\mu\nu} \ln \left[\frac{\Lambda^2 (1-x) + m_{\tilde{\chi}_i^+}^2 - m_{\tilde{\chi}_j^+}^2 + \frac{1}{4} q^2 (\bar{z}^2 - z^2)}{D(m_{\tilde{\nu}_1}^2)} \right] \\ &\left. - m_{\tilde{\nu}_1} \rightarrow m_{\tilde{\nu}_2} \right\} \end{aligned}$$

Here $\Lambda^2 \rightarrow \infty$ is the Pauli-Villars regulator which is superfluous in this instance

since we have equally well written the log term in (L.12) as a ratio of "D"s" where the numerator "D" is equal to the denominator with $\tilde{\nu}_1$ replaced by $\tilde{\nu}_2$. Dividing (L.12) through by $m_{\tilde{\chi}_1^+}^2$ and using the definitions of the ratios R, S and T from eqn. (3.10) and the integral functions in the table at the conclusion of the appendix (Table L.1) gives us the final form

$$\begin{aligned} \mathcal{M}_1^\nu = & \sum_{ij} \xi_{ij}^\nu \frac{2K_\nu V_{ji} V_{ji}^*}{\sin^2 \theta_w \sin 2\theta_w} \\ & \times \left\{ \sqrt{S_{ij}^*} O_{ij}^{\nu'} \Delta I_{ij}(T_i^+, S_{ij}^+, R_i^+) \right. \\ & \left. - R_i^+ O_{ij}^{\nu'} \Delta I_{ij}(T_i^+, S_{ij}^+, R_i^+) + O_{ij}^{\nu'} \Delta G(T_i^+, S_{ij}^+, R_i^+) \right\} \end{aligned} \quad (\text{L.13})$$

Diagram 2

Diagram two is illustrated in Fig. 2. From the Feynman rules of Appendix K we may read off the matrix element:

$$\begin{aligned} \mathcal{M}_2^\nu = & \int \frac{d^4 k}{(2\pi)^4} [\bar{u}^\mu(p_1')]_{\alpha\beta} [igV_{11}^* \gamma_-]_{\beta\alpha} \cos \theta_\nu \\ & \otimes \left\{ \frac{i \left[(k + m_{\tilde{\chi}_1^+}) \right]}{k^2 - m_{\tilde{\chi}_1^+}^2} \right\}_{\beta\beta_1} [igV_{11} \gamma_+]_{\beta_1\beta} \sin \theta_\nu [v^\nu(p_2')]_{\beta_1\alpha} \\ & \otimes \frac{i}{(p_1' - k)^2 - m_{\tilde{\nu}_1}^2} \cdot \frac{-ig}{2 \cos \theta_w} [p_1' - p_2' - 2k]^\nu \cdot \frac{i}{(p_2' + k)^2 - m_{\tilde{\nu}_1}^2} \\ = & \frac{g^3 \sin \theta_\nu \cos \theta_\nu V_{11} V_{11}^*}{2 \cos \theta_w} \int \frac{d^4 k}{(2\pi)^4} \\ & \times \frac{\gamma_- (k + m_{\tilde{\chi}_1^+}) \gamma_+ (2k + p_2' - p_1')^\nu}{\left[k^2 - m_{\tilde{\chi}_1^+}^2 \right] \left[(p_1' - k)^2 - m_{\tilde{\nu}_1}^2 \right] \left[(p_2' + k)^2 - m_{\tilde{\nu}_1}^2 \right]} v^\nu(p_2') \end{aligned} \quad (\text{L.14})$$

We see that use of (K.18) and (K.21) have again simplified the algebra. In this

case we use a Pauli-Villars cut-off. Introducing

$$\frac{1}{k^2 - m_{\tilde{\chi}_1^+}^2} = - \int_0^1 \frac{dt}{(k^2 - t)^2}$$

Following the techniques used above we obtain

$$\begin{aligned} \mathcal{M}_2^\nu = & \frac{g^3 \sin \theta_\nu \cos \theta_\nu V_{11} V_{11}^*}{\sin^2 \theta_w \sin 2\theta_w} \int \frac{d^4 k}{(2\pi)^4} \int_0^1 dt \int_0^{1-t} dx \int_0^{1-x} dy (1-x-y) \\ & \times \frac{\bar{u}^\mu(p_1') \gamma_+ \not{k} \gamma_- (2k + \not{q})^\nu v^\nu(p_2')}{\left[k^2 - t + (1-2p_1' \cdot k - m_{\tilde{\nu}_1}^2)x + (t+2p_2' \cdot k - m_{\tilde{\nu}_1}^2)y \right]^4} \\ & - \tilde{\nu}_1 \rightarrow \tilde{\nu}_2 \end{aligned}$$

where

$$q^\nu = (p_1' + p_2')^\nu \quad \tilde{q}^\nu = (p_2' - p_1')^\nu. \quad (\text{L.15})$$

This leads to

$$\begin{aligned} \mathcal{M}_2^\nu = & \frac{K_\nu |V_{11}|^2}{\sin^2 \theta_w \sin 2\theta_w} \frac{1}{2} \int_0^1 dx \int_0^{1-x} dy \int \frac{d^4 k}{m_{\tilde{\chi}_1^+}^2} (1-x-y) \\ & \times \frac{\bar{u}^\mu(p_1') \gamma_+ \{ (p_1' x - p_2' y) (p_1' [2x-1] + p_2' [1-2y]) + \gamma^\nu (s - P^2) \} v^\nu(p_2')}{(s - P^2)^3} \\ & - \tilde{\nu}_1 \rightarrow \tilde{\nu}_2 \end{aligned}$$

where

$$s - P^2 \approx -t(1-x) - m_{\tilde{\nu}_1}^2 x - \frac{q^2}{4} (\tilde{z}^2 - x^2).$$

The γ terms will introduce factors of m_μ, m_τ , and so will be down by

$$O(m_r/m_{SUSY})$$

relative to the γ^μ term. Thus

$$\begin{aligned} M_2^\mu &= \sum_{ij} \frac{\xi^\mu_L K_\nu |V_{ij}|^2}{\sin^2 \theta_w \sin 2\theta_w} \int_0^1 dz \int_{-z}^z d\bar{z} \\ &\times O_{ij}^{L'} \ln \left[\frac{\Delta^2(1-z) + m_{\tilde{\nu}_1}^2 z + \frac{1}{4} q^2 (\bar{z}^2 - z^2)}{m_{\tilde{\chi}_i^+}^2 (1-z) + m_{\tilde{\nu}_1}^2 z + \frac{1}{4} q^2 (\bar{z}^2 - z^2)} \right] \\ &- m_{\tilde{\nu}_1} \rightarrow m_{\tilde{\nu}_3} + O\left(\frac{m_r}{M_{SUSY}}\right) \end{aligned} \quad (L.16)$$

and because, as we shall see the total matrix element is finite as $\Delta^2 \rightarrow \infty$ we may write this sans regularization as

$$\begin{aligned} M_2^\mu &= \sum_i \frac{\xi^\mu_L K_\nu |V_{ii}|^2}{\sin^2 \theta_w \sin 2\theta_w} \int_0^1 dz \int_{-z}^z d\bar{z} \\ &\times O_{ii}^{L'} \ln \left[\frac{m_{\tilde{\chi}_i^+}^2 (1-z) + m_{\tilde{\nu}_3}^2 z + \frac{1}{4} q^2 (\bar{z}^2 - z^2)}{m_{\tilde{\chi}_i^+}^2 (1-z) + m_{\tilde{\nu}_1}^2 z + \frac{1}{4} q^2 (\bar{z}^2 - z^2)} \right] \\ &+ O\left(\frac{m_r}{M_{SUSY}}\right) \end{aligned} \quad (L.17)$$

Where K_ν is given by (L.11) and ξ^μ_L by (L.10). Dividing through by $m_{\tilde{\chi}_i^+}^2$ and using the definition of the integral functions in the table at the conclusion of this appendix yields

$$M_2^\mu = \sum_i \frac{\xi^\mu_L K_\nu |V_{ii}|^2}{\sin^2 \theta_w \sin 2\theta_w} \Delta G(T_i^+, R_i^+) \quad (L.18)$$

Diagram Three

Diagrams three and four are leg corrections. For these two sets of diagrams we will not use (K.18) and (K.21) in an effort to demonstrate some of the C-matrix manipulations which occur. From Fig. 3 we see that

$$\begin{aligned} M_3^\mu &= \int \frac{d^4 k}{(2\pi)^4} \{v^\mu(p_1')\}_{\alpha\beta} \{igC^{-1}V_{ii}^* \gamma_-\}_{\delta\gamma} \cos \theta_w \\ &\otimes \left\{ \frac{i \left\{ (p_1' - k) + m_{\tilde{\chi}_i^+} \right\}}{(p_1' - k)^2 - m_{\tilde{\chi}_i^+}^2} \right\}_{\delta\beta} [-igV_{ii}\gamma_+ C]_{\alpha\gamma} \\ &\otimes \sin \theta_w \frac{i}{k^2 - m_{\tilde{\nu}_1}^2} \left\{ \frac{i(-p_1' + m_\tau)}{(p_1')^2 - m_\tau^2} \right\}_{\alpha\beta} \\ &\otimes \frac{ig}{2 \cos \theta_w} [\gamma^\mu (\cos 2\theta_w \gamma_- - \sin^2 \theta_w \gamma_+)]_{\alpha\delta\gamma} [u^\tau(p_2')]_{\alpha\alpha} \end{aligned} \quad (L.19)$$

Now we take a few transposes in order to place this in the form of a matrix product in the spin indices.

$$\begin{aligned} M_3^\mu &= \frac{g^2 \sin \theta_w \cos \theta_w V_{ii}^* V_{ii}}{2 \cos \theta_w (m_\tau^2 - m_{\tilde{\chi}_i^+}^2)} \int \frac{d^4 k}{(2\pi)^4} \\ &\times [v^\mu(p_1')]^T [C^{-1} \gamma_-]^T \left\{ \frac{(p_1' - k) + m_{\tilde{\chi}_i^+}}{(p_1' - k)^2 - m_{\tilde{\chi}_i^+}^2} \right\} \\ &\times [\gamma_+ C]^T [-p_1' + m_\tau]^T \frac{1}{k^2 - m_{\tilde{\nu}_1}^2} \\ &\times [\gamma^\mu (\cos 2\theta_w \gamma_- - \sin^2 \theta_w \gamma_+)]^T [u^\tau(p_2')]^T \end{aligned} \quad (L.20)$$

We now proceed to use the various identities given at the beginning of Appendix K.

Since $C^{-1}\gamma_- = \gamma_-^T C^{-1}$ and $\gamma_+ C = C\gamma_+^T$ using (K.12)

$$\begin{aligned} \mathcal{M}_3^\nu &= \frac{g^2 \sin \theta_\nu \cos \theta_\nu V_{\bar{u}1} V_{\bar{u}1}^*}{2 \cos \theta_w (m_\mu^2 - m_\tau^2)} \int \frac{d^4 k}{(2\pi)^4} \\ &\times \left[\bar{u}^\mu(p_1') C^T \right] \left[\gamma_-^T C^{-1} \right]^T \frac{\left[(\not{p}_1' - \not{k}) + m_{\tilde{\chi}_1^+} \right]}{(\not{p}_1' - \not{k})^2 - m_{\tilde{\chi}_1^+}^2} \\ &\times \left[C \gamma_+^T \right]^T \left[-\not{p}_1' + m_\tau \right]^T \frac{1}{k^2 - m_{\tilde{\nu}_1}^2} \\ &\times \left[(\cos 2\theta_w \gamma_-^T - \sin^2 \theta_w \gamma_+^T)(\gamma^\nu)^T \right] \left[C^{-1} v^\nu(p_2') \right] \end{aligned} \quad (\text{L.21})$$

Now using (K.11)

$$(\gamma_-^T C^{-1})^T = (C^T)^{-1} \gamma_-$$

and using (K.8)

$$(C \gamma_+^T)^T = -\gamma_+ C$$

so

$$\begin{aligned} \mathcal{M}_3^\nu &= \frac{g^2 \sin \theta_\nu \cos \theta_\nu V_{\bar{u}1} V_{\bar{u}1}^*}{2 \cos \theta_w (m_\mu^2 - m_\tau^2)} \int \frac{d^4 k}{(2\pi)^4} \\ &\times \bar{u}^\mu(p_1') C^T (C^T)^{-1} \gamma_- \frac{\left[(\not{p}_1' - \not{k}) + m_{\tilde{\chi}_1^+} \right]}{(\not{p}_1' - \not{k})^2 - m_{\tilde{\chi}_1^+}^2} \\ &\times [-\gamma_+ C] [-\not{p}_1' + m_\tau]^T \frac{1}{k^2 - m_{\tilde{\nu}_1}^2} \\ &\times (\cos 2\theta_w \gamma_-^T - \sin^2 \theta_w \gamma_+^T)(\gamma^\nu)^T C^{-1} v^\nu(p_2') \end{aligned} \quad (\text{L.22})$$

We now "slide" the C^{-1} near the end of (L.22) until it combines with a C via

$CC^{-1} = 1$ and repeatedly use (K.11) to obtain

$$\begin{aligned} \mathcal{M}_3^\nu &= \frac{g^2 \sin \theta_\nu \cos \theta_\nu V_{\bar{u}1} V_{\bar{u}1}^*}{2 \cos \theta_w (m_\mu^2 - m_\tau^2)} \int \frac{d^4 k}{(2\pi)^4} \\ &\times \bar{u}^\mu(p_1') \gamma_- \frac{\left[(\not{p}_1' - \not{k}) + m_{\tilde{\chi}_1^+} \right]}{(\not{p}_1' - \not{k})^2 - m_{\tilde{\chi}_1^+}^2} \\ &\times (-\gamma_+) \left[\not{p}_1' + m_\tau \right] \frac{1}{k^2 - m_{\tilde{\nu}_1}^2} \\ &\times (\cos 2\theta_w \gamma_- - 2 \sin^2 \theta_w \gamma_+)(-\gamma^\nu) v^\nu(p_2') \end{aligned} \quad (\text{L.23})$$

and since $\gamma_\pm \gamma_\mp = 0$ and $\gamma_\pm \gamma_\pm = \gamma_\pm$

$$\begin{aligned} \mathcal{M}_3^\nu &= \frac{g^2 \sin \theta_\nu \cos \theta_\nu V_{\bar{u}1} V_{\bar{u}1}^*}{2 \cos \theta_w (m_\mu^2 - m_\tau^2)} \int \frac{d^4 k}{(2\pi)^4} \\ &\times \bar{u}^\mu(p_1') \frac{\gamma_- (\not{p}_1' - \not{k}) \gamma_+}{(\not{p}_1' - \not{k})^2 - m_{\tilde{\chi}_1^+}^2} \frac{(\not{p}_1' \cos 2\theta_w \gamma_- - 2m_\tau \sin^2 \theta_w \gamma_+)(\gamma^\nu)}{k^2 - m_{\tilde{\nu}_1}^2} v^\nu(p_2') \end{aligned} \quad (\text{L.24})$$

We proceed as we did for the previous diagram, introducing a Pauli-Villars regulator via $\int dt$, but use (H.3) instead of (H.5). We find (after eliminating the regulator)

$$\begin{aligned} \mathcal{M}_3^\nu &= - \sum_i \frac{K_\nu |V_{\bar{u}1}|^2}{\sin^2 \theta_w \sin 2\theta_\nu} \int_{m_{\tilde{\nu}_1}^2}^{m_{\tilde{\chi}_1^+}^2} dt \int_0^1 \frac{x^2 dx}{ix + m_{\tilde{\chi}_1^+}^2 (1-x)} \\ &\times \frac{\cos 2\theta_w m_\mu^2 \xi_L^\nu - 2 \sin^2 \theta_w m_\mu m_\tau \xi_R^\nu}{m_\mu^2 - m_\tau^2} \end{aligned} \quad (\text{L.25})$$

From Table L.1 we see that this integral is like $G(T, R)$ when $R = 0$ (i.e. $g^2 = 0$).

Therefore we can write this as

$$M_3^\nu = \sum_i \frac{K_\nu |V_{i1}|^2}{\sin^2 \theta_w \sin 2\theta_w} \frac{\cos 2\theta_w m_i^2 \xi^\nu_L - 2 \sin^2 \theta_w m_\mu m_i \xi^\nu_R}{m_\mu^2 - m_i^2} \Delta G(T, R=0) \quad (\text{L.26})$$

Diagram Four

Diagram four is computed in precisely the same manner as the previous contribution. From Fig. 4 we may write

$$\begin{aligned} M_4^\nu &= \int \frac{d^4 k}{(2\pi)^4} \left[v^\mu(p_1') \right]_{\alpha, \epsilon} \frac{i g}{2 \cos \theta_w} \\ &\otimes \left[\gamma^\nu (\cos 2\theta_w \gamma_- - \sin^2 \theta_w \gamma_+) \right]_{\alpha, \epsilon} \left\{ \frac{i (p_2^2 + m_\mu)}{(p_2^2)^2 - m_\mu^2} \right\}_{\beta, \alpha_1} \\ &\otimes \left[i g C^{-1} V_{i1} \gamma_- \right]_{\delta, \epsilon} \cos \theta_w \left\{ \frac{i \left[-(p_3^2 - k) + m_{\tilde{\chi}_i^+} \right]}{(p_3^2 - k)^2 - m_{\tilde{\chi}_i^+}^2} \right\}_{\delta, \beta_1} \\ &\otimes \left[-i g V_{i1} \gamma_+ C \right]_{\beta, \delta} \sin \theta_w \frac{i}{k^2 - m_{\tilde{\chi}_i^+}^2} \left[u^\nu(p_2') \right]_{\alpha, \beta_1} \end{aligned} \quad (\text{L.27})$$

Following the same steps leads us to

$$M_4^\nu = - \sum_i \frac{K_\nu |V_{i1}|^2}{\sin^2 \theta_w \sin 2\theta_w} \frac{\cos 2\theta_w m_i^2 \xi^\nu_L - 2 \sin^2 \theta_w m_\mu m_i \xi^\nu_R}{m_\mu^2 - m_i^2} \Delta G(T, R=0) \quad (\text{L.28})$$

From (L.26) and (L.28) we conclude

$$(M_3 + M_4)^\nu = K_\nu \sum_{i=1}^3 \frac{\xi^\nu_L |V_{i1}|^2}{\sin^2 \theta_w \sin 2\theta_w} \cos 2\theta_w \Delta G(T, R=0). \quad (\text{L.29})$$

Note that the ξ^ν_R term has completely cancelled between the two leg correction diagrams.

Total Chargino Matrix Element

We may now gather together the individual contributions and present the complete chargino matrix element. From (L.13), (L.18) and (L.29)

$$\begin{aligned} M_{\pm}^\nu &= 2 K_\nu \xi^\nu_L \sum_{ij=1}^3 \frac{V_{j1} V_{i1}^*}{\sin^2 \theta_w \sin 2\theta_w} \\ &\times \left\{ \sqrt{S_{ij}^+} O_{ij}^{R+} \Delta I_{[1]}(T_i^+, S_{ij}^+, R_i^+) \right. \\ &- \left. R_i^+ O_{ij}^{L+} \Delta I_{[2, -2]}(T_i^+, S_{ij}^+, R_i^+) + O_{ij}^{L+} \Delta \hat{G}(T_i^+, S_{ij}^+, R_i^+) \right\} \\ &+ K_\nu \xi^\nu_L \sum_{i=1}^3 \frac{|V_{i1}|^2}{\sin^2 \theta_w \sin 2\theta_w} [\Delta G(T_i^+, R_i^+) + \cos 2\theta_w \Delta G(T, R=0)]. \end{aligned} \quad (\text{L.30})$$

Large Mass Limits

Various limiting forms of the integral functions are presented in section M.7 of the following appendix. We assert that (L.30) is well-behaved in the limit that any of the masses involved or q^2 or any of the defined ratios thereof (T, S and R) becomes large. In this limit the decoupling theorem enters with full force and contributions from very heavy states become negligible. This is readily evident for most of the limits. When we take T large (i.e. m_i large) this requires a bit of work. As a reminder:

$$T_i = \frac{m_i^2}{m_{\tilde{\chi}_i}^2}, \quad S_{ij} = \frac{m_{\tilde{\chi}_i}^2}{m_{\tilde{\chi}_j}^2}, \quad R_i = \frac{q^2}{4 m_{\tilde{\chi}_i}^2}$$

$$\begin{aligned}
\lim_{T \rightarrow \infty} J_{|N(z, \frac{1}{2})|}(T, S, R) &\rightarrow 0 \\
\lim_{T \rightarrow \infty} \tilde{G}(T, S, R) &\rightarrow 0 \\
\lim_{T \rightarrow \infty} G(T, S, R) &\rightarrow -\frac{1}{2} \ln T \\
\lim_{T \rightarrow \infty} \tilde{G}(T, S, R) &\rightarrow -\frac{1}{2} \ln T
\end{aligned} \tag{L.31}$$

Suppose that we consider what happens if $m_{\tilde{\nu}_1}, m_{\tilde{\nu}_2} \rightarrow \infty$. We find that

$$\Delta G, \Delta \tilde{G} \rightarrow -\frac{1}{2} \ln \frac{m_{\tilde{\nu}_1}^2}{m_{\tilde{\nu}_2}^2} \tag{L.32}$$

and the chargino matrix element becomes

$$\begin{aligned}
\mathcal{M}_{\pm}^{\nu} &\rightarrow \frac{K_{\nu} \xi^{\nu}_L}{\sin^2 \theta_w \sin 2\theta_w} \left(-\frac{1}{2} \ln \frac{m_{\tilde{\nu}_1}^2}{m_{\tilde{\nu}_2}^2} \right) \\
&\times \left\{ \sum_{i=1}^2 |V_{i1}|^2 (1 + \cos 2\theta_w) + 2 \sum_{ij=1}^2 V_{j1} V_{i1}^* O_{ij}^{L^i} \right\}
\end{aligned} \tag{L.33}$$

and we recall that

$$O_{ij}^{L^i} = \delta_{ij} \sin^2 \theta_w - V_{i1} V_{j1}^* - \frac{1}{2} V_{i2} V_{j2}^*$$

and that V is unitary so that $(V^\dagger V)_{ij} = \delta_{ij}$. Using these it is simple to show that

$$\begin{aligned}
\sum_{i=1}^2 |V_{i1}|^2 (1 + \cos 2\theta_w) &= 1 + \cos 2\theta_w \\
\sum_{ij=1}^2 V_{j1} V_{i1}^* O_{ij}^{L^i} &= \sin^2 \theta_w - 1
\end{aligned}$$

and so

$$\begin{aligned}
\mathcal{M}_{\pm}^{\nu} &\rightarrow \frac{K_{\nu} \xi^{\nu}_L}{\sin^2 \theta_w \sin 2\theta_w} \left(-\frac{1}{2} \ln \frac{m_{\tilde{\nu}_1}^2}{m_{\tilde{\nu}_2}^2} \right) \\
&\times \left\{ (1 + \cos 2\theta_w) + 2(1 - \sin^2 \theta_w) \right\} \\
&= 0
\end{aligned} \tag{L.34}$$

Therefore $\mathcal{M}_{\pm} \rightarrow 0$ as $m_{\tilde{\nu}_{1,2}} \rightarrow \infty$.

L.2 NEUTRALINO DIAGRAMS

The neutralino contributions may be obtained from those of the charginos by making appropriate substitutions. There are two sectors to consider. The left and right sleptons behave somewhat differently. Rather than present all of the crossing relations, which may be obtained by examining the Feynman rules of the previous appendix, we will simply present the final result.

$$\mathcal{M}^{\nu} = \mathcal{M}_L \xi^{\nu}_L + \mathcal{M}_R \xi^{\nu}_R$$

$$\xi^{\nu}_{L,R} = \bar{u}^{\nu}(p_1') \gamma_7 \gamma^{\nu} \gamma_{\pm} \nu'(p_2')$$

$$M_L^0 = -\frac{K_L}{2 \sin^3 \theta_w \sin 2\theta_w} \sum_{i=1}^4 \left\{ |\tan \theta_w N_{i1} + N_{i3}|^2 \cos 2\theta_w \Delta \tilde{G}(T_L^0, R_i^0) \right. \\ \left. - 2 \sum_{j=1}^4 (\tan \theta_w N_{ji} + N_{j3}) (\tan \theta_w N_{j1} + N_{j3}) \right.$$

$$\left[\sqrt{S_{ij}^0} O^{R''}_{ij} \Delta I_{[1]}(T_L^0, S_{ij}^0, R_i^0) \right. \\ \left. - R_i^0 O^{L''}_{ij} \Delta I_{[\tilde{2}, -2]}(T_L^0, S_{ij}^0, R_i^0) \right. \\ \left. + O^{L''}_{ij} \Delta \tilde{G}(T_L^0, S_{ij}^0, R_i^0) \right] \Bigg\}$$

$$M_R^0 = \frac{2K_R}{\sin^3 \theta_w \sin 2\theta_w} \sum_{i=1}^4 \left\{ |\tan \theta_w N_{i1} + N_{i3}|^2 2 \sin^2 \theta_w \Delta \tilde{G}(T_R^0, R_i^0) \right. \\ \left. + 2 \sum_{j=1}^4 (\tan \theta_w N_{ji}) (\tan \theta_w N_{j1}) \right.$$

$$\left[\sqrt{S_{ij}^0} O^{L''}_{ij} \Delta I_{[1]}(T_R^0, S_{ij}^0, R_i^0) \right. \\ \left. - R_i^0 O^{R''}_{ij} \Delta I_{[\tilde{2}, -2]}(T_R^0, S_{ij}^0, R_i^0) \right. \\ \left. + O^{R''}_{ij} \Delta \tilde{G}(T_R^0, S_{ij}^0, R_i^0) \right] \Bigg\}$$

$$K_\nu = \frac{i}{16\pi^2} e^3 \sin \theta_\nu \cos \theta_\nu$$

$$K_L = \frac{i}{16\pi^2} e^3 \sin \theta_L \cos \theta_L$$

$$K_R = \frac{i}{16\pi^2} e^3 \sin \theta_R \cos \theta_R$$

R , S , and T are ratios of masses:

$$T_{L,R}^0 = \frac{m_{L_{R_i}}^2}{m_{\tilde{X}_1^0}^2}$$

$$S_{ij}^0 = \frac{m_{\tilde{X}_j^0}^2}{m_{\tilde{X}_i^0}^2}$$

$$R_i^0 = \frac{q^2}{4m_{\tilde{X}_i^0}^2}$$

where m_{L_i} and m_{R_i} are the slepton masses.

A typical substitution would involve going from the $r^-\ell_i\tilde{\chi}_j^+$ to the $r^-\ell_{R_i}\tilde{\chi}_j^0$ vertex. We would make the substitutions

$$-igV_{j1} \rightarrow -i\sqrt{2}g \tan \theta_w N_{j1}^+ \quad \gamma_+ \rightarrow \gamma_- \quad \theta_\nu \rightarrow \theta_R.$$

L.3 COMPLETE MATRIX ELEMENT

Summarizing, the complete matrix element is the total of the chargino and neutralino contributions. It is given by

$$\mathcal{M}^\nu = \mathcal{M}_L \xi^\nu_L + \mathcal{M}_R \xi^\nu_R \quad (\text{L.35})$$

$$\xi^\nu_{L,R} = u^\mu(p_1') \gamma_\mp \gamma^\nu \gamma_\pm \nu^\mu(p_2') \quad \gamma_\pm = \frac{1}{2} (1 \pm \gamma_5)$$

$$K_\nu = \frac{i}{16\pi^2} e^3 \sin \theta_\nu \cos \theta_\nu$$

$$K_L = \frac{i}{16\pi^2} e^3 \sin \theta_L \cos \theta_L$$

$$K_R = \frac{i}{16\pi^2} e^3 \sin \theta_R \cos \theta_R$$

$$\begin{aligned}
M_L = & \frac{K_L}{\sin^2 \theta_w \sin 2\theta_w} \sum_{i=1}^2 \left\{ |V_{ii}|^2 [\Delta G(T_i^+, R_i^+) + \cos 2\theta_w \Delta G(T_i^+, 0)] \right. \\
& + 2 \sum_{j=1}^2 V_{ji}^* V_{ji} \left[\sqrt{S_{ij}^2} O_{ij}^{R^+} \Delta I_{[1]}(T_i^+, S_{ij}^+, R_i^+) \right. \\
& \quad - R_i^+ O_{ij}^{L^+} \Delta I_{[\tilde{2}^2 - \tilde{2}^2]}(T_i^+, S_{ij}^+, R_i^+) \\
& \quad \left. \left. + O_{ij}^{L^+} \Delta \tilde{G}(T_i^+, S_{ij}^+, R_i^+) \right] \right\} \\
& - \frac{K_L}{2 \sin^2 \theta_w \sin 2\theta_w} \sum_{i=1}^4 \left\{ |\tan \theta_w N_{i1} + N_{i2}|^2 \cos 2\theta_w \Delta \tilde{G}(T_L^0, R_i^0) \right. \\
& \quad - 2 \sum_{j=1}^4 (\tan \theta_w N^{*i}{}_{i2} + N^{*i}{}_{i2}) (\tan \theta_w N_{j1} + N_{j2}) \\
& \quad \left[\sqrt{S_{ij}^0} O_{ij}^{R''} \Delta I_{[1]}(T_L^0, S_{ij}^0, R_i^0) \right. \\
& \quad - R_i^0 O_{ij}^{L''} \Delta I_{[\tilde{2}^2 - \tilde{2}^2]}(T_L^0, S_{ij}^0, R_i^0) \\
& \quad \left. \left. + O_{ij}^{L''} \Delta \tilde{G}(T_L^0, S_{ij}^0, R_i^0) \right] \right\} \\
M_R = & \frac{2K_R}{\sin^2 \theta_w \sin 2\theta_w} \sum_{i=1}^4 \left\{ |\tan \theta_w N^{*i}{}_{i1}|^2 2 \sin^2 \theta_w \Delta \tilde{G}(T_R^0, R_i^0) \right. \\
& + 2 \sum_{j=1}^4 (\tan \theta_w N_{i1}) (\tan \theta_w N^{*j}{}_{j1}) \\
& \left[\sqrt{S_{ij}^0} O_{ij}^{L''} \Delta I_{[1]}(T_R^0, S_{ij}^0, R_i^0) \right. \\
& - R_i^0 O_{ij}^{R''} \Delta I_{[\tilde{2}^2 - \tilde{2}^2]}(T_R^0, S_{ij}^0, R_i^0) \\
& \left. \left. + O_{ij}^{R''} \Delta \tilde{G}(T_R^0, S_{ij}^0, R_i^0) \right] \right\}
\end{aligned}$$

R , S , and T are ratios of masses:

$$\begin{aligned}
T_i^+ &= \frac{m_{\tilde{\nu}_i}^2}{m_{\tilde{\chi}_i^0}^2} & T_{L,R}^0 &= \frac{m_{\tilde{\nu}_{L,R}}^2}{m_{\tilde{\chi}_i^0}^2} \\
S_{ij}^+ &= \frac{m_{\tilde{\chi}_i^+}^2}{m_{\tilde{\chi}_j^+}^2} & S_{ij}^0 &= \frac{m_{\tilde{\chi}_i^0}^2}{m_{\tilde{\chi}_j^0}^2} \\
R_i^+ &= \frac{g^2}{4m_{\tilde{\chi}_i^+}^2} & R_i^0 &= \frac{g^2}{4m_{\tilde{\chi}_i^0}^2}
\end{aligned} \tag{L.36}$$

where $m_{\tilde{\nu}_{L1}}$ and $m_{\tilde{\nu}_{R1}}$ are the masses of the charged slepton of the first generation mixture (see eqn. 3a) and $m_{\tilde{\nu}_1}$ is the mass of the first sneutrino ($\tilde{\nu}_1 = \tilde{\nu}_\mu \cos \theta_\nu + \tilde{\nu}_\tau \sin \theta_\nu$). The neutralino masses are $m_{\tilde{\chi}_i^0}$ ($i = 1, 2, 3, 4$) and chargino masses are $m_{\tilde{\chi}_i^\pm}$ ($i = 1, 2$).

Table L.1 Integral Functions

$$I_{|N|}(T, R) = \frac{1}{2} \int_0^1 dx \int_{-x}^x d\tilde{z} \frac{N(x, \tilde{z})}{x(1-T) + 1 + R(\tilde{z}^2 - x^2)}$$

$$I_{|N|}(T, R) = \frac{1}{2} \int_0^1 dx \int_{-x}^x d\tilde{z} \frac{N(x, \tilde{z})}{(1-x)T + x + R(\tilde{z}^2 - x^2)}$$

$$I_{|N|}(T, S, R) = \frac{1}{2} \int_0^1 dx \int_{-x}^x d\tilde{z} \frac{N(x, \tilde{z})}{(1-x)T + \frac{1}{2}[(1+S)x + (1-S)\tilde{z}] + R(\tilde{z}^2 - x^2)}$$

$$\tilde{G}(T, S, R) = -\frac{1}{2} \int_0^1 dx \int_{-x}^x d\tilde{z} \ln \left\{ [(1-x)T + \frac{1}{2}[(1+S)x + (1-S)\tilde{z}] + R(\tilde{z}^2 - x^2)] \right\}^{-3/4}$$

$$\tilde{G}(T, R) = -\frac{1}{2} \int_0^1 dx \int_{-x}^x d\tilde{z} \ln \left\{ [(1-x)T + x + R(\tilde{z}^2 - x^2)] \right\}^{-3/4}$$

$$G(T, R) = -\frac{1}{2} \int_0^1 dx \int_{-x}^x d\tilde{z} \ln \left\{ x(1-T) + 1 + R(\tilde{z}^2 - x^2) \right\}^{-1/4}$$

$$\tilde{G}(T, R) \equiv G(T, R) - G(T, R \equiv 0)$$

Table L.1 Integral Functions (Continued)

Note that

$$\tilde{G}(T, R) = \tilde{G}(T, S = 1, R)$$

$$I_{|N|}(T, R) = I_{|N|}(T, S = 1, R)$$

$$I_{|N|}(T, R) = \frac{1}{T} I_{|N|}\left(\frac{1}{T}, \frac{R}{T}\right)$$

$$I_{|N|}(T, R) = \frac{1}{T} I_{|N|}\left(\frac{1}{T}, \frac{R}{T}\right)$$

$$\tilde{G}(T, R) = G\left(\frac{1}{T}, \frac{R}{T}\right) + \frac{1}{2} \ln T$$

$$I_{|N|}(T, R) = \frac{1}{2R} \tilde{G}(T, R)$$

For any function, F , of $T_i^{0,+} L, R$

$$\Delta F \equiv F|_{l_1, \theta_1} - F|_{l_2, \theta_2}$$

Table L.1 Integral Functions (Continued)

$$I'_{[N]}(T, R) = \frac{T}{2} \int_0^1 dx \int_{-x}^x d\bar{z} \frac{xN(x, \bar{z})}{[x(1-T) + 1 + R(\bar{z}^2 - x^2)]^2}$$

$$I'_{[N]}(T, R) = \frac{T}{2} \int_0^1 dx \int_{-x}^x d\bar{z} \frac{N(x, \bar{z})}{[(1-x)T + x + R(\bar{z}^2 - x^2)]^2}$$

$$I'_{[N]}(T, S, R) = \frac{T}{2} \int_0^1 dx \int_{-x}^x d\bar{z} \frac{N(x, \bar{z})}{[(1-x)T + \frac{1}{2}\{(1+S)x + (1-S)\bar{z}\} + R(\bar{z}^2 - x^2)]^2}$$

$$\hat{G}'(T, S, R) \approx -T I'_{[1-S]}(T, S, R)$$

$$\hat{G}'(T, R) = -T I'_{[1-S]}(T, R)$$

$$G'(T, R) \approx -T I'_{[1-S]}(T, R)$$

$$\hat{G}'(T, R) \approx -T [I_{[0]}(T, R) - I_{[0]}(T, R \approx 0)]$$

$$\hat{G}'(T, R) = \frac{1}{2} - G' \left(\frac{1}{T}, \frac{R}{T} \right)$$

$$I'_{[0]}(T, R=0) = \frac{2T \ln T + 1 - T^2}{(T-1)^2}$$

FIGURE CAPTIONS

1. Diagram One.
2. Diagram Two.
3. Diagram Three.
4. Diagram Four.

Fig. L.1

Fig. L.2

Fig. L.3

Fig. L.4

APPENDIX M

Analysis of Integral Functions

M.1 INVARIANT FORM OF I AND G INTEGRALS: S_{ij} VERSUS S_{ji}

We know from physical intuition that when we interchange $\tilde{x}_i$ and $\tilde{x}_j$ in the vertex shown in Fig. 1 that this should not affect our answer.

Here $\tilde{x}_i$ is $\tilde{x}_i^{z,0}$, $m_{\tilde{\tau}} = m_{\tilde{t},R}$, etc. Using these abbreviated forms we may also shorten the mass ratios originally defined in eqn. 3.10 to

$$T_{i,j} = \frac{m_{\tilde{\tau}}^2}{(m_{\tilde{x}_{i,j}})^2}, \quad S_{ij} = \frac{m_{\tilde{t}}^2}{m_{\tilde{x}_i}^2}, \quad S_{ji} = \frac{m_{\tilde{x}_i}^2}{m_{\tilde{t}}^2}, \quad R_{i,j} = \frac{q^2}{4(m_{\tilde{x}_{i,j}})^2} \quad (M.1)$$

$$S_{ij} = \frac{1}{S_{ji}} \quad (M.2)$$

Then

$$I_{[N]}(T_i, S_{ij}, R_i) = \frac{1}{2} \int_0^1 dx \int_{-x}^x d\tilde{z} \times \frac{N(x, \tilde{z})}{(1-x)T_i + \frac{1}{2}[(1+S_{ij})x + (1-S_{ij})\tilde{z}] + R_i(\tilde{z}^2 - x^2)} \quad (M.3)$$

Now multiply by S_{ji}/S_{ji} to use (M.2)

$$I_{[N]}(T_i, S_{ij}, R_i) = \frac{1}{2} \int_0^1 dx \int_{-x}^x d\tilde{z} \frac{S_{ji}N(x, \tilde{z})}{(1-x)T_i S_{ji} + \frac{1}{2}[(1+S_{ji})x - (1-S_{ji})\tilde{z}] + R_i S_{ji}(\tilde{z}^2 - x^2)} \quad (M.4)$$

Now

$$T_i S_{ji} = T_j \quad \text{and} \quad R_i S_{ji} = R_j \quad (M.5)$$

If $N(x, \tilde{z}) = N(x, -\tilde{z})$ we let $\tilde{z} \rightarrow -\tilde{z}$ in (M.4) and using (M.5) get

$$I_{[N]}(T_i, S_{ij}, R_i) = (S_{ji})^{\frac{1}{2}} \int_0^1 dx \int_{-x}^x d\tilde{z} \times \frac{N(x, \tilde{z})}{(1-x)T_j + \frac{1}{2}[(1+S_{ji})x + (1-S_{ji})\tilde{z}] + R_j(\tilde{z}^2 - x^2)} = S_{ji} I_{[N]}(T_j, S_{ji}, R_j) \quad (M.6)$$

or

$$\sqrt{S_{ij}} I_{[N]}(T_i, S_{ij}, R_i) = \sqrt{S_{ji}} I_{[N]}(T_j, S_{ji}, R_j) \quad (M.7)$$

Similarly from (M.5) and (M.6)

$$T_i I_{[N]}(T_i, S_{ij}, R_i) = T_j I_{[N]}(T_j, S_{ji}, R_j) \quad (M.8)$$

$$R_i I_{[N]}(T_i, S_{ij}, R_i) = R_j I_{[N]}(T_j, S_{ji}, R_j) \quad (M.9)$$

In particular, of the functions which appear in (3.9), $\sqrt{S} I_{[1]}$ and $R I_{[1,-s]}$,

$$\sum_{ij} \sqrt{S} I_{[1]} = 2 \sum_{-s_i, s_j = -s_i} \sqrt{S} I_{[1]} + \sum_i I_{[1]} \quad (M.10)$$

The latter term being the $i = j$ term since $I = J$ with $S \equiv 1$. Likewise

$$\sum_{ij} R_i I_{[1,-s]}(T_i, S_{ij}, R_i) = 2 \sum_{-s_i, s_j = -s_i} R_i I_{[1,-s]}(T_i, S_{ij}, R_i) + \sum_i R_i I_{[1,-s]}(T_i, R_i) \quad (M.11)$$

The sums

$$\sum_{\substack{i,j \\ -i_j^2 = -i_i}}$$

can also be written

$$\sum_{\substack{i,j \\ i \neq j}} \quad (M.10)$$

A similar result holds for the $\hat{G}$ functions. From Table L.1, letting $\bar{z} \rightarrow -\bar{z}$ if $N(z, \bar{z})$ is even in $\bar{z}$, yields

$$\hat{G}(T_i, S_{ij}, R_i) = \hat{G}(T_j, S_{ji}, R_j) + \frac{1}{2} \ell n S_{ji} \quad (M.13)$$

and the "invariant" form is

$$\hat{G}(T_i, S_{ij}, R_i) + \frac{1}{4} \ell n S_{ij} = \hat{G}(T_j, S_{ji}, R_j) + \frac{1}{4} \ell n S_{ji} \quad (M.14)$$

and similarly

$$\hat{G}(T_i, S_{ij}, R_i) + \frac{1}{2} \ell n R_i = \hat{G}(T_j, S_{ji}, R_j) + \frac{1}{2} \ell n R_j \quad (M.15)$$

$$\hat{G}(T_i, S_{ij}, R_i) + \frac{1}{2} \ell n T_i = \hat{G}(T_j, S_{ji}, R_j) + \frac{1}{2} \ell n T_j \quad (M.16)$$

From Table L.1

$$\hat{G}'(T_i, S_{ij}, R_i) = -T_i \hat{I}_{1 \rightarrow i}(T_i, S_{ij}, R_i) \quad (M.17)$$

so that

$$\hat{G}'(T_i, S_{ij}, R_i) = \hat{G}'(T_j, S_{ji}, R_j) \quad (M.18)$$

Thus in all of the terms in M^w , both large and small mass-splitting cases considered, we may restrict our attention to the $S_{ij} \geq 1$ contributions. This will prove convenient for numerical evaluation.

M.2 $\hat{I}_{[N](z, \bar{z})}$ INTEGRALS: IMAGINARY PART

Some fun with complex analysis

The propagator for the i^{th} boson [fermion] is

$$\Delta_i \text{ propagator} = \frac{i[(p_i + m_i)]}{p_i^2 - m_i^2 + i\epsilon_i} \quad (M.19)$$

so we just absorb the $i\epsilon$ into the m^2 term

$$m_i^2 \rightarrow m_i^2 - i\epsilon_i \quad (M.20)$$

The $\hat{I}_{[N]}$ terms came from diagrams like those of Fig. 2 involving loops of three propagators, each with their own, arbitrary $i\epsilon$. From eqn. (L.3) we are lead to integrals like

$$I \sim \int_0^1 dz \int_{-z}^z d\bar{z} \times \frac{m_{X_i}^2}{m_i^2(1-z) + \frac{1}{2}[(m_{X_i}^2 + m_{X_i}^2)z + (m_{X_i}^2 - m_{X_i}^2)\bar{z}] + \frac{1}{4}q^2(\bar{z}^2 - z^2) - i\bar{\epsilon}} \quad (M.21)$$

where

$$\bar{\epsilon} = \bar{\epsilon}_i(1-z) + \frac{1}{2}[(\bar{\epsilon}_{X_i} + \bar{\epsilon}_{X_i})z + (\bar{\epsilon}_{X_i} - \bar{\epsilon}_{X_i})\bar{z}] \quad (M.22)$$

Since each ϵ_i is of arbitrary relative magnitude the overall sign of $\bar{\epsilon}$ might appear to be parameter-dependent in a way which could render the relative signs of different contributions ambiguous. Our task is to prove that this is not the case and to derive the correct sign.

Divide through by $m_{\tilde{L}_i}^2$,

$$I \sim \int_0^1 dz \int_{-\bar{z}}^{\bar{z}} d\bar{z} \frac{1}{T_i(1-z) + \frac{1}{2}(1+S_{ij})z + (1-S_{ij})\bar{z} + R_i(\bar{z}^2 - z^2) - i\epsilon} \quad (\text{M.23})$$

where

$$T_i \equiv \frac{m_{\tilde{L}_i}^2}{m_{\tilde{L}_i}^2} \quad S_{ij} \equiv \frac{m_{\tilde{L}_i}^2}{m_{\tilde{L}_i}^2} \quad R_i \equiv \frac{q^2}{4m_{\tilde{L}_i}^2} \equiv \frac{m_{\tilde{L}_i}^2}{4\tilde{L}_i^2} \quad (\text{M.24})$$

here (as in (M.1)). We re-define

$$\epsilon \equiv \frac{i}{m_{\tilde{L}_i}^2}$$

and, in general,

$$\epsilon_j \equiv \frac{i}{m_{\tilde{L}_i}^2} \quad (\text{M.25})$$

(i subscript understood).

The slepton masses, $m_{\tilde{L}_i}$, are $(m_{\tilde{L}_{L_1}}, m_{\tilde{L}_{L_2}}, m_{\tilde{L}_{L_3}})$ and $m_{\tilde{L}_i}$ is i^{th} gaugino/higgsino mass. We can choose $S_{ij} \geq 1$ as discussed above and so we have

$$q^2 = m_{\tilde{L}_i}^2 > 0 \quad T_i \geq 0 \quad R_i \geq 0 \quad S_{ij} \geq 1 \quad 0 \leq z \leq 1 \quad -s \leq \bar{z} \leq s \quad (\text{M.26})$$

and all $\epsilon > 0$.

Now look at the most general such integral of the type in (M.23). Pulling " R_i " out of (M.23) we may write (making the expression exact)

$$I_{|N(z,\bar{z})|} = \frac{1}{2R} \int_0^1 dz \int_{-\bar{z}}^{\bar{z}} d\bar{z} \frac{N(z,\bar{z})}{\bar{z}^2 + (b + i\delta_b)\bar{z} + (c + i\delta_c)} \quad (\text{M.27})$$

Comparing with (M.23), (M.24), and (M.25)

$$b = \frac{1-S}{2R} < 0 \quad (\text{M.28})$$

$$\delta_b = -\frac{2}{q^2} (\epsilon_{X_i} - \epsilon_{X_i})$$

which is of no fixed sign since ϵ_i, ϵ_j are arbitrary and independent

$$c = -\left[z^2 - \frac{T(1-z) + \frac{1}{2}(1+S)z}{R} \right] \quad (\text{M.29})$$

$$\delta_c = -\frac{4}{q^2} \{ \epsilon_{\tilde{L}}(1-z) + \frac{1}{2}(\epsilon_{X_i} + \epsilon_{X_i})z \} < 0$$

where the inequality follows from (M.26) since all the bracketed terms are positive. Since $|\epsilon_{X_i}^- - \epsilon_{X_i}^+| < \epsilon_{X_i}^- + \epsilon_{X_i}^+$ and $0 \leq z \leq 1$ we can say that

$$0 < z|\delta_b| < -\delta_c \quad (\text{M.30})$$

For (M.27) write

$$I_{|N(z,\bar{z})|} = \frac{1}{2R} \int_0^1 dz \int_{-\bar{z}}^{\bar{z}} d\bar{z} \frac{N(z,\bar{z})}{(\bar{z} - \bar{z}_+)(\bar{z} - \bar{z}_-)} \quad (\text{M.31})$$

where

$$\bar{z}_{\pm} \equiv \frac{1}{2} \left[-(b + i\delta_b) \pm \sqrt{b^2 + 2ib\delta_b - 4c - 4i\delta_c} \right] \quad (\text{M.32})$$

Letting

$$\Delta = b^2 - 4c \equiv 4Q = 4 \left[\left(z + \frac{b}{2} \right)^2 - \frac{(1-z)T + z}{R} \right] \quad (\text{M.33})$$

defines Q as

$$Q = \left(z + \frac{b}{2} \right)^2 - \frac{(1-z)T + z}{R} \geq 0 \quad (\text{M.34})$$

since $\Delta \geq 0$ or else there is no imaginary part. This follows since Δ is, essentially

the discriminant of the denominator of (M.22). If $\Delta < 0$ then the denominator has no real roots and is always positive or negative definite over the range of integration. The result is a real integral. Note that since $(1-z), x, T$ and $R \geq 0$ that $\frac{(1-z)T+x}{R} \geq 0$ and since $Q \geq 0$ for $Jm \bar{I}$ to exist it follows that that

$$Q < (z + b/2)^2. \quad (M.35)$$

Define

$$\begin{aligned} r_{\pm} &= \operatorname{Re}[\bar{z}_{\pm}] \\ &= -\frac{b}{2} \pm \sqrt{Q} \end{aligned} \quad (M.36)$$

so $r_+ > r_-$ ($Q \geq 0$)

$$\bar{z}_{\pm} = r_{\pm} + \frac{1}{2}i \left[\delta_b \pm i \frac{(b\delta_b - 2\delta_c)}{2\sqrt{Q}} \right]. \quad (M.37)$$

Writing

$$\bar{z}_{\pm}(z) = r_{\pm}(z) + i\epsilon_{\pm}(z) \quad (M.38)$$

our analysis will reduce to determining the signs of $\epsilon_{\pm}$.

$$\begin{aligned} \epsilon_{\pm} &= - \left[\delta_b \mp \frac{b\delta_b}{2\sqrt{Q}} \pm \frac{2\delta_c}{2\sqrt{Q}} \right] = -\frac{1}{2\sqrt{Q}} \left[\delta_b \left(\sqrt{Q} \mp \frac{b}{2} \right) \pm \delta_c \right] \\ &= \mp \frac{1}{2\sqrt{Q}} [r_{\pm}\delta_b + \delta_c] \end{aligned} \quad (M.39)$$

Define

$$v_{\pm} = -\frac{b}{2} \pm \sqrt{\frac{(1-z)T+x}{R}} \quad (M.40)$$

and note that

$$v_+ > v_-.$$

Then

$$Q = \left(z + \frac{b}{2}\right)^2 - \frac{(1-z)T+x}{R} = (z - r_+)(z - r_-). \quad (M.41)$$

Of course from (M.35)

$$(z - r_+)(z - r_-) = Q > 0. \quad (M.42)$$

So the cases are

$$(i) \quad z - r_+ > 0 \quad \text{and} \quad z - r_- > 0 \quad (M.43)$$

$$(ii) \quad z - r_+ < 0 \quad \text{and} \quad z - r_- < 0 \quad (M.44)$$

Likewise

$$(z - r_+)(z - r_-) = \frac{(1-z)T+x}{R} > 0 \quad (M.45)$$

Cases

$$(i) \quad z - r_+ > 0 \quad \text{and} \quad z - r_- > 0 \quad (M.46)$$

$$(ii) \quad z - r_+ < 0 \quad \text{and} \quad z - r_- < 0 \quad (M.47)$$

and in all cases $0 \leq z \leq 1$. From (M.31) and (M.38)

$$I_{|N(z, \bar{z})|} = \frac{1}{2R} \int_0^1 dz \int_{-x}^z d\bar{z} \frac{N(z, \bar{z})}{(\bar{z} - r_+ - i\epsilon_+)(\bar{z} - r_- - i\epsilon_-)} \quad (M.48)$$

$$= \frac{1}{2R} \int_0^1 dz \int_{-x}^z d\bar{z} \frac{N(z, \bar{z})}{(r_+ - r_-)} \left[\frac{1}{\bar{z} - r_+ - i\epsilon_+} - \frac{1}{\bar{z} - r_- - i\epsilon_-} \right]$$

with

$$r_{\pm} = -\frac{b}{2} \pm \sqrt{Q}$$

and therefore

$$I_{|N(z)|} = \frac{1}{4R} \int_0^1 \int_{-z}^z d\bar{z} \frac{N(z, \bar{z})}{\sqrt{Q}} \left[\frac{1}{\bar{z} - r_+ - i\epsilon_+} - \frac{1}{\bar{z} - r_- - i\epsilon_-} \right] \quad (M.49)$$

Principal Value Formula

Under $\int dz$:

$$\frac{1}{z - \alpha \pm i\epsilon} \rightarrow \frac{1}{z - \alpha} \mp i\pi \delta(z - \alpha) \quad (M.50)$$

$$Im I_{|N(z, \bar{z})|} = \frac{i\pi}{4R} \int_0^1 \int_{-z}^z d\bar{z} \quad (M.51)$$

$$\times \frac{\theta(Q)}{\sqrt{Q}} N(z, \bar{z}) \{ \text{sign}(\epsilon_+) \delta(\bar{z} - r_+) - \text{sign}(\epsilon_-) \delta(\bar{z} - r_-) \}$$

since $Q > 0$ for an Im part to exist. The signum function, $\text{sign}(X)$, is defined to be:

$$\text{sign}(z) = \begin{cases} -1 & z < 0 \\ 0 & z = 0 \\ 1 & z > 0 \end{cases} \quad (M.52)$$

$$Im I_{|N(z, \bar{z})|} = \frac{i\pi}{4R} \int_0^1 dz \frac{\theta(Q)}{\sqrt{Q}} \left\{ \text{sign}(\epsilon_+) N(z, r_+) \{ \theta(z - r_+) - \theta(-z - r_+) \} \right. \\ \left. - \text{sign}(\epsilon_-) N(z, r_-) \{ \theta(z - r_-) - \theta(-z - r_-) \} \right\} \quad (M.53)$$

Now from (M.35)

$$\sqrt{Q} < \left| z + \frac{b}{2} \right| \leq \left| z - \frac{b}{2} \right| \quad (z \geq 0 \quad b \leq 0) \quad (M.54)$$

and since

$$z - \frac{b}{2} > 0 \quad \sqrt{Q} < z - \frac{b}{2} \quad (M.55)$$

we find

$$-\left(z - \frac{b}{2}\right) \pm \sqrt{Q} < 0 \quad (M.56)$$

and therefore

$$\theta(-z - r_{\pm}) = \theta(-z + \frac{b}{2} \mp \sqrt{Q}) = \theta\left(-\left[z - \frac{b}{2}\right] \mp \sqrt{Q}\right) = 0 \quad (M.57)$$

with the result that (M.53) reduces to

$$Im I_{|N(z, \bar{z})|} = \frac{i\pi}{4R} \int_0^1 dz \quad (M.58)$$

$$\frac{\theta(Q)}{\sqrt{Q}} \{ N(z, r_+) \text{sign}(\epsilon_+) \theta(z - r_+) - N(z, r_-) \text{sign}(\epsilon_-) \theta(z - r_-) \} .$$

Since the two terms are proportional to $\theta(z - r_+)$ and $\theta(z - r_-)$ we require $z - r_+$ and/or $z - r_-$ to be positive in order to get a contribution. Looking at (M.46) and (M.47) we see that this rules out case (ii).

We must have

$$z - r_{\pm} > 0 \quad (M.59)$$

This can be understood in the following way:

Since $\frac{(1-z)T+z}{R} > 0$ we could write a factor of

$$\theta\left(\frac{(1-z)T+z}{R}\right) = 1 \quad (\text{M.60})$$

in $I_{[N]}$ just like $\theta(Q)$

$$\theta\left(\frac{(1-z)T+z}{R}\right) = \theta(|z-r_+||z-r_-|) \quad (\text{M.61})$$

so from (M.58)

$$\begin{aligned} \text{Im } I_{[N(z, \bar{z})]} &= \frac{i\pi}{4R} \int_0^1 dz \frac{\theta(Q)}{\sqrt{Q}} \theta(|z-r_+||z-r_-|) \\ &\quad \times [\theta(z-r_+)\text{sign}(\epsilon_+)N(z, r_+) - \theta(z-r_-)\text{sign}(\epsilon_-)N(z, r_-)] \end{aligned} \quad (\text{M.62})$$

but

$$\theta(|z-r_+||z-r_-|)\theta(z-r_{\pm}) = \theta(z-r_+)\theta(z-r_-) = \theta(z-r_+) \quad (\text{M.63})$$

since $r_+ > r_-$, i.e.

$$0 < r_+ < z \leq 1 \quad \text{also} \quad r_- < r_+ < z \leq 1 \quad (\text{M.64})$$

the net result being

$$\text{Im } I_{[N(z, \bar{z})]} = \frac{i\pi}{4R} \int_0^1 dz \frac{\theta(Q)\theta(z-r_+)}{\sqrt{Q}} [\text{sign}(\epsilon_+)N(z, r_+) - \text{sign}(\epsilon_-)N(z, r_-)] \quad (\text{M.65})$$

From (M.57) $-z-r_- < 0$ and therefore $-z < r_-$. With this and from (M.64)

$$1 \leq -z < r_- < r_+ < z \leq 1 \quad \text{with} \quad 0 < r_+ < z \leq 1 \quad (\text{M.66})$$

From (M.36) and (M.40)

$$\begin{aligned} \bar{r}_+ - r_- &= \sqrt{\frac{(1-z)T+z}{R}} + \sqrt{Q} > 0 \\ r_+ - \bar{r}_- &= \sqrt{\frac{(1-z)T+z}{R}} + \sqrt{Q} > 0 \end{aligned} \quad (\text{M.67})$$

so

$$\bar{r}_- < r_+ < z \leq 1 \quad (\text{M.68})$$

and hence

$$z - \bar{r}_- > 0 \quad (\text{M.69})$$

From (M.43) and (M.44) case (ii) has just been eliminated leading to the inescapable result that

$$z - \bar{r}_{\pm} > 0. \quad (\text{M.70})$$

Thus (because $r_+ > r_-$)

$$\theta(Q) = \theta(|z-r_+||z-\bar{r}_-|) = \theta(z-r_+)\theta(z-\bar{r}_-) = \theta(z-r_+). \quad (\text{M.71})$$

Also

$$\bar{r}_+ - z < 0 \Rightarrow -z - \frac{b}{2} + \sqrt{\frac{(1-z)T+z}{R}} < 0. \quad (\text{M.72})$$

Now

$$-z - \bar{r}_- = -z + \frac{b}{2} + \sqrt{\frac{(1-z)T+z}{R}} < -z - \frac{b}{2} + \sqrt{\frac{(1-z)T+z}{R}} < 0 \quad (\text{M.73})$$

from (M.72) since $b < 0$.

Therefore

$$r_- > -z \quad (\text{M.74})$$

Also note that from (M.72)

$$-1 \leq -z < r_-, r_- < r, r_+ < z \leq 1 \quad r_+ > 0 \quad (\text{M.75})$$

$$0 < \sqrt{\frac{(1-z)T+z}{R}} < z + \frac{b}{2} \quad (\text{M.76})$$

i.e.

$$z + \frac{b}{2} > 0 \quad (\text{M.77})$$

so using (M.71) and (M.65)

$$\text{Im } I_{|N(z,s)|} = \frac{i\pi}{4R} \int_0^1 dz \frac{\theta(z-r_+)\theta(z-r_-)}{\sqrt{Q}} \{ \text{sign}(\epsilon_+) N(z, r_+) - \text{sign}(\epsilon_-) N(z, r_-) \}. \quad (\text{M.78})$$

Note that

$$\theta(z-r_+)\theta(z-r_-) = \theta(z - \max\{r_+, r_-\}) \quad (\text{M.79})$$

From (M.75)

$$|r_{\pm}| < z \quad (\text{M.80})$$

Recall from (M.30) and (M.39) that

$$\epsilon_{\pm} = \mp \frac{1}{2\sqrt{Q}} [r_{\pm}\delta_b + \delta_c] \quad \text{and} \quad z|\delta_b| < \delta_c < 0. \quad (\text{M.81})$$

Since

$$|r_-| < z \Rightarrow |r_{\pm}\delta_b| < z|\delta_b| < |\delta_c| \quad (\text{M.82})$$

we see that

$$\text{sign}[r_{\pm}\delta_b + \delta_c] = \text{sign}[\delta_c] = -1 \quad (\text{M.83})$$

and since $\frac{1}{2\sqrt{Q}} > 0$

$$\text{sign}(\epsilon_{\pm}) = \pm 1 \quad (\text{M.84})$$

we may finally conclude that

$$\text{Im } I_{|N(z,s)|} = \frac{i\pi}{4R} \int_0^1 dz \frac{\theta(z-r_+)\theta(z-r_-)}{\sqrt{Q}} \{ N(z, r_+) + N(z, r_-) \} \quad (\text{M.85})$$

where we recall that

$$\begin{aligned} r_{\pm} &= -\frac{b}{2} \pm \sqrt{\frac{(1-z)T+z}{R}} \\ r_{\pm} &= -\frac{z}{2} \pm \sqrt{Q} \\ Q &= \left(z + \frac{b}{2}\right)^2 - \frac{(1-z)T+z}{R} \\ b &= \frac{(1-S)}{2R} \end{aligned} \quad (\text{M.86})$$

T, S, and R are as in (M.24).

M.3 $I_{|N|}$ AND $I_{|N|}$ INTEGRALS: IMAGINARY PART

We may also obtain $I_{|N(z,s)|}$ directly from (M.85) by letting $s \rightarrow 1$. Then

$b = 0$ and

$$r_{\pm} \rightarrow \pm \sqrt{\frac{(1-z)T+z}{R}} \quad (\text{M.87})$$

$$r_{\pm} \rightarrow \sqrt{Q} \quad (\text{M.88})$$

with

$$\bar{Q} = z^2 - \frac{(1-z)T+z}{R} \quad (\text{M.89})$$

$$\begin{aligned} \text{Im } \bar{I}_{[N(z, \bar{z})]} \\ = \frac{i\pi}{4R} \int_0^1 dz \frac{\theta\left(z - \sqrt{\frac{(1-z)T+z}{R}}\right) \theta(z - \sqrt{Q})}{\sqrt{Q}} \left[N(z, \sqrt{Q}) + N(z, -\sqrt{Q}) \right]. \end{aligned} \quad (\text{M.90})$$

Note that if $N(z, \bar{z})$ is even in $\bar{z}$, i.e. $N(z, \bar{z}) = N(z, -\bar{z})$ then

$$\text{Im } \bar{I}_{[N(z, \bar{z})=N(z, -\bar{z})]} = \frac{i\pi}{2R} \int_0^1 dz \frac{\theta\left(z - \sqrt{\frac{(1-z)T+z}{R}}\right) \theta(z - \sqrt{Q})}{\sqrt{Q}} N(z, \sqrt{Q}). \quad (\text{M.91})$$

If $N(z, \bar{z})$ is odd in $\bar{z}$, i.e. $N(z, -\bar{z}) = -N(z, \bar{z})$ then

$$\text{Im } I_{[N(z, \bar{z}) \text{ odd in } \bar{z}]} = 0. \quad (\text{M.92})$$

We compare I and $\bar{I}$ in Table L.1.

We see that the above analysis goes through for $I_{[N]}$ if we make the changes

$$r_{\pm} \rightarrow r'_{\pm} = \pm \sqrt{\frac{(1-z)+Tz}{R}} \quad (\text{M.93})$$

$$r_{\pm} \rightarrow r'_{\pm} = \pm \sqrt{W} \quad (\text{M.94})$$

with

$$W = z^2 - \frac{(1-z)+Tz}{R} \quad (\text{M.95})$$

Therefore

$$\text{Im } I_{[N]} = \frac{i\pi}{4R} \int_0^1 dz \frac{\theta\left(z - \frac{(1-z)+Tz}{R}\right) \theta(z - \sqrt{W})}{\sqrt{W}} \left[N(z, \sqrt{W}) + N(z, -\sqrt{W}) \right]. \quad (\text{M.96})$$

If $N(z, \bar{z}) = N(z, -\bar{z})$ is even in $\bar{z}$

$$\text{Im } I_{[N]} = \frac{i\pi}{2R} \int_0^1 dz \frac{\theta\left(z - \frac{(1-z)+Tz}{R}\right) \theta(z - \sqrt{W})}{\sqrt{W}} N(z, \sqrt{W}). \quad (\text{M.97})$$

If $N(z, \bar{z}) = N(z, -\bar{z})$ is odd in $\bar{z}$

$$\text{Im } I_{[N]} = 0. \quad (\text{M.98})$$

M.4 $I_{[N]}$, $\bar{I}_{[N]}$ AND $\bar{I}_{[N]}$ INTEGRALS: REAL PARTS

$$\text{Re } I_{[N]} = \frac{1}{2R} \int_0^1 dz \int_{-z}^z d\bar{z} \frac{N(z, \bar{z})}{\bar{z}^2 - W(z)} \quad (\text{M.99})$$

where W is given by (M.95). We will assume for the moment that $N = N'(z)$. If $N'(z, \bar{z}) = \bar{z}^2 - z^2$ then we express it in terms of the above (see the next section).

Then

$$\begin{aligned} \text{Re } I_{[N]} &= \frac{1}{R} \int_0^1 dz N(z) \int_0^z d\bar{z} \\ &\times \left\{ \left[\frac{1}{\bar{z} - \sqrt{W}} - \frac{1}{\bar{z} + \sqrt{W}} \right] \frac{1}{2\sqrt{W}} \theta(W) + \frac{1}{\bar{z}^2 + |W'(z)|} \theta(-W) \right\} \end{aligned}$$

and from the Principal Value Theorem,

$$= \frac{1}{R} \int_0^1 dz N(z) \left\{ \frac{1}{2\sqrt{W}} \left[\ln(z - \sqrt{W}) - \ln(z + \sqrt{W}) \right] \theta(W) \right. \\ \left. + \frac{1}{\sqrt{-W}} \tan^{-1} \frac{z}{\sqrt{-W}} \theta(-W) \right\} \Big|_{z=0}^{z=x}$$

ergo

$$Re I_{[N]} \\ = \frac{1}{R} \int_0^1 dz \frac{N(z)}{\sqrt{|W|}} \left\{ \frac{1}{2} \ln \left[\frac{(\sqrt{W} - z)}{(\sqrt{W} + z)} \right] \theta(W) + \tan^{-1} \frac{z}{\sqrt{-W}} \theta(-W) \right\}. \quad (M.100)$$

The procedure for $I_{[N]}$ is similar

$$Re I_{[N(z)]} = \frac{1}{2R} \int_{-c}^c d\tilde{z} \frac{N(z)}{\tilde{z}^2 + b\tilde{z} + c} \quad (M.101)$$

with

$$b = \frac{1-S}{2R} \quad c = \frac{1}{R} \left\{ -Rx^2 + \frac{1+S}{2}x + (1-x)T \right\}. \quad (M.102)$$

As in (M.33) and (M.34) let

$$Q = \frac{1}{4} (b^2 - 4c) = \left(z + \frac{b}{2} \right)^2 - \frac{z(1-T)+R}{R} \quad (M.103)$$

then

$$Re I_{[N]} = \frac{1}{4R} \int_0^1 \frac{N(z)}{\sqrt{|Q|}} \\ \times \left\{ 2 \tan^{-1} \frac{z+b/2}{\sqrt{-Q}} \theta(-Q) + \frac{1}{2} \ln \frac{z+b/2-\sqrt{Q}}{z+b/2+\sqrt{Q}} \theta(Q) \right\}$$

or

$$Re I_{[N]} = \frac{1}{2R} \int_0^1 \frac{N(z)}{\sqrt{|Q|}} \left\{ \left[\tan^{-1} \left(\frac{z+b/2}{\sqrt{-Q}} \right) - \tan^{-1} \left(\frac{-z+b/2}{\sqrt{-Q}} \right) \right] \theta(-Q) \right. \\ \left. + \frac{1}{2} \ln \left[\frac{\left(\frac{b}{2} \right)^2 - (z-\sqrt{Q})^2}{\left(\frac{b}{2} \right)^2 - (z+\sqrt{Q})^2} \right] \theta(Q) \right\}. \quad (M.104)$$

For $Re I_{[N]}$ we let $b \rightarrow 0$ in (M.104) above giving us back (M.100) with " W " replaced by Q as given in (M.89).

A question arises as to the behavior of $Re I$ at $Q = 0$. Using the asymptotic form $(x \rightarrow \infty) \tan^{-1} x = \pi/2 - 1/x + 1/3x^3 - \dots$ valid in the arctan sheet used

$$Re I(Q \rightarrow 0) \rightarrow \frac{1}{2R} \int_0^1 dz \frac{N(z)}{\sqrt{Q}} \\ \times \left\{ \left(-\frac{\sqrt{Q}}{z+b/2} + \frac{\sqrt{Q}}{-z+b/2} \right) \theta(-Q) + \frac{1}{2} \ln \left[\frac{1 + \frac{2z\sqrt{Q}}{(b/2)^2 - z^2}}{1 - \frac{2z\sqrt{Q}}{(b/2)^2 - z^2}} \right] \theta(Q) \right\}. \quad (M.105)$$

Accordingly

$$Re I(Q \rightarrow 0) \approx \frac{1}{2R} \int_0^1 dz N(z) \left\{ \frac{2z}{(b/2)^2 - z^2} \theta(Q) + \frac{2z}{(b/2)^2 - z^2} \theta(-Q) \right\} \quad (M.106)$$

so the integrand is continuous at $Q = 0$. In fact at $Q = 0$ since, $\theta(Q) + \theta(-Q) = 1$ we have

$$Re I_{[N(z)]}(Q = 0) \rightarrow -\frac{1}{R} \int_0^1 dz \frac{zN(z)}{z^2 - (b/2)^2}. \quad (M.107)$$

Note that when $b = 0$ in the I case

$$\operatorname{Re} I_{|N(z)|}(\bar{Q}(z) \rightarrow 0) \rightarrow -\frac{1}{R} \int_0^1 dz \frac{N(z)}{z}. \quad (\text{M.108})$$

This causes no problems unless $\bar{Q}(z=0)$ happens to be zero, i.e. unless the spot where $\bar{Q}$ vanishes just happens to be $z=0$. But

$$\bar{Q} = z^2 - \frac{(1-z)T+z}{R} \quad (\text{M.109})$$

from (M.89) so

$$\bar{Q}(z=0) = \frac{T}{R} > 0 \quad (\text{M.110})$$

since we have taken $T, R > 0$. So (M.108) is good everywhere ($T/R = 4m_{\tilde{L}}^2/s^2$ so there is a possible problem when $m_{\tilde{L}}$, the slepton mass, vanishes.)

$$\text{M.5} \quad I_{|z^2-z|}$$

The formula for $\operatorname{Im} I_{|N|}$ works well for $N = N(z, \bar{z})$, but (M.104) was restricted to $N = N(z)$. Recall from (M.27) that we may write

$$I_{|N|}(T, S, R) = \frac{1}{2R} \int_0^1 dz \int_{-a}^{\bar{a}} d\bar{z} \frac{N(z, \bar{z})}{\bar{z}^2 + b\bar{z} + c + i\epsilon \text{ terms}} \quad (\text{M.111})$$

$$b = \frac{1-S}{2R} \quad c = \left\{ -z^2 + \left(\frac{1}{R} - b \right) z + \frac{(1-z)T}{R} \right\}. \quad (\text{M.112})$$

From

$$\int d\bar{z} \frac{\bar{z}^2}{\bar{z}^2 + b\bar{z} + c} = \bar{z} - \frac{b}{2} \ln(\bar{z}^2 + b\bar{z} + c) + \frac{b^2 - 2c}{2} \int \frac{d\bar{z}}{\bar{z}^2 + b\bar{z} + c} \quad (\text{M.113})$$

we have

$$I_{|z^2-z|}(T, S, R) = \frac{1}{2R} \int_0^1 dz \times \left\{ 2z - \frac{b}{2} \ln \frac{z^2 + bz + c}{z^2 - bz + c} + \frac{b^2 - 2c - 2z^2}{2} \int_{-a}^{\bar{a}} \frac{d\bar{z}}{\bar{z}^2 + b\bar{z} + c} \right\}. \quad (\text{M.114})$$

Therefore

$$\operatorname{Re} I_{|z^2-z|}(T, S, R) = \frac{1}{2R} \left\{ 1 - \frac{b}{2} \int_0^1 dz \ln \left[\frac{z^2 + bz + c}{z^2 - bz + c} \right] \right\} + \operatorname{Re} I_{|\frac{1}{2}b^2 - c - z^2|}(T, S, R) \quad (\text{M.115})$$

and

$$\begin{aligned} \frac{1}{2}b^2 - c - z^2 &= \frac{1}{2}b^2 + \left(b - \frac{1}{R}\right)z - (1-z)\frac{T}{R} \\ &= \frac{1}{R} [z(T-1+bR) + (\frac{1}{2}b^2R - T)] \end{aligned} \quad (\text{M.116})$$

as well as

$$z^2 + bz + c = \frac{1}{R} \left\{ z(1-T) + T \right\} \quad z^2 - bz + c = \frac{1}{R} \left\{ z(1-T-2bR) + T \right\} \quad (\text{M.117})$$

so

$$\frac{z^2 + bz + c}{z^2 - bz + c} = \frac{z(1-T) + T}{z(1-T-2bR) + T}. \quad (\text{M.118})$$

Let

$$N'(z) \equiv 2 [z(T-1+bR) + (\frac{1}{2}b^2R - T)] \quad (\text{M.119})$$

then from (M.115), (M.116), and (M.117)

$$\begin{aligned} \operatorname{Re} l_{[1^{12}, -s]}(T, S, R) = & \frac{1}{2R} \left\{ 1 - \frac{b}{2} \int_0^1 dx \ln \left| \frac{x(1-T)+T}{x(1-T-2bR)+T} \right| \right. \\ & \left. + \operatorname{Re} l_{[N'(s)]}(T, S, R) \right\}. \end{aligned} \quad (\text{M.120})$$

Now both the numerator and denominator of $\frac{x(1-T)+T}{x(1-T-2bR)+T}$ are always positive ($b < 0$) so we can easily integrate out this term. Since

$$\int_0^1 dx \ln(\alpha x + \beta) = \ln(\alpha + \beta) - 1 + \frac{\beta}{\alpha} \ln\left(\frac{\alpha}{\beta} + 1\right) \quad (\text{M.121})$$

in general we have

$$\int_0^1 dx \ln\left(\frac{\alpha x + \beta}{\alpha' x + \beta'}\right) = \ln\left(\frac{\alpha + \beta}{\alpha' + \beta'}\right) + \frac{\beta}{\alpha} \ln\left(\frac{\alpha}{\beta} + 1\right) - \frac{\beta'}{\alpha'} \ln\left(\frac{\alpha'}{\beta'} + 1\right). \quad (\text{M.122})$$

with

$$\alpha = 1 - T \quad \alpha' = 1 - T - 2bR \quad \beta = \beta' = T \quad (\text{M.123})$$

for the integral under consideration. Since

$$\alpha + \beta = 1 \quad \text{and} \quad \alpha' + \beta' = 1 - 2bR \quad (\text{M.124})$$

we may write

$$\begin{aligned} & \int_0^1 dx \ln \left(\frac{x(1-T)+T}{x(1-T-2bR)+T} \right) \\ &= \frac{T}{1-T} \ln\left(\frac{1}{T}\right) - \ln(1-2bR) - \frac{T}{1-T-2bR} \ln\left(\frac{1-2bR}{T}\right) \\ &= \frac{2bRT \ln T}{(1-T)(1-T-2bR)} - \frac{1-2bR}{1-T-2bR} \ln(1-2bR) \end{aligned} \quad (\text{M.125})$$

or letting

$$B \equiv bR = \frac{1-S}{2} \quad (\text{M.126})$$

$$\int_0^1 dx \ln \left(\frac{x(1-T)+T}{x(1-T-2B)+T} \right) \quad (\text{M.127})$$

$$= \frac{1}{1-T-2B} \left\{ \frac{2BT \ln T}{1-T} - (1-2B) \ln(1-2B) \right\}$$

and (M.120) becomes

$$\begin{aligned} \operatorname{Re} l_{[1^{12}, -s]}(T, S, R) \\ = \frac{1}{2R} \left\{ 1 - \frac{1}{2R} \cdot \frac{B}{1-T-2B} \left[\frac{2BT \ln T}{1-T} - (1-2B) \ln(1-2B) \right] \right. \\ \left. + \operatorname{Re} l_{[N'(s)]}(T, S, R) \right\} \end{aligned} \quad (\text{M.128})$$

or

$$\begin{aligned} \operatorname{Re} l_{[1^{12}, -s]}(T, S, R) \\ = \frac{1}{2R} \left\{ 1 - \frac{1}{2R} \cdot \frac{B}{S-T} \left[\frac{1-S}{1-T} T \ln T - S \ln S \right] \right. \\ \left. + \operatorname{Re} l_{[N'(s)]}(T, S, R) \right\} \end{aligned} \quad (\text{M.129})$$

since

$$B = \frac{1-S}{2} \quad b = \frac{B}{R} \quad 1-2B=S \quad (\text{M.130})$$

$$N'(z) = 2 \{z(T-1+B) + (\frac{1}{2}b^2R-T)\} . \quad (\text{M.131})$$

When $S=1$ ($B=b=0$) this becomes

$$Re \bar{I}_{[1,1-s]}(T, S=1, R) = \frac{1}{2R} \left\{ 1 - 2Re \bar{I}_{[1,1-T]+T]}(T, S=1, R) \right\} . \quad (\text{M.132})$$

M.6 G AND $\bar{G}$

G comes from, for instance, the γ^w term of M_2^w in Appendix L.

$$M_2^w \sim \gamma^w \int_0^1 dx \frac{1}{2} \int_{-x}^x d\bar{z} \int_{m_{X_i}^2}^{\Lambda^2} dt \quad (\text{M.133})$$

$$\times \frac{1-z}{(t-i\epsilon_{X_i})(1-z) + (m_{X_i}^2 - i\epsilon_{X_i})z + \frac{1}{4}q^2(\bar{z}^2 - z^2)} .$$

Comparing the definition of G from Table L.1 or the next section we see that

$$G(T_i, R) = \frac{1}{2} \int_0^1 dx \frac{1}{2} \int_{-x}^x d\bar{z} \int_{m_{X_i}^2}^{\Lambda^2} dt \quad (\text{M.134})$$

$$\times \frac{1-z}{(t-i\epsilon_{X_i})(1-z) + (m_{X_i}^2 - i\epsilon_{X_i})z + \frac{1}{4}q^2(\bar{z}^2 - z^2)}$$

and since $(1-z), z > 0$, when we extract $(1-z)$ we obtain

$$G = \frac{1}{2} \int_0^1 dx \frac{1}{2} \int_{-x}^x d\bar{z} \int_{m_{X_i}^2}^{\Lambda^2} dt \quad (\text{M.135})$$

$$\times \frac{1}{t + \frac{1}{1-z}[m_{X_i}^2 z + \frac{1}{4}q^2(\bar{z}^2 - z^2)] - i\epsilon}$$

where $\epsilon > 0$. Now use the Principal Value Theorem (M.50)

$$\frac{1}{z-a \pm i\epsilon} \rightarrow \frac{1}{z-a} \mp i\pi \delta(z-a)$$

under the integral sign to give

$$Im G = \frac{1}{2} \int_0^1 dx \frac{1}{2} \int_{-x}^x d\bar{z} \int_{m_{X_i}^2}^{\Lambda^2} dt i\pi \delta \left(t + \frac{1}{1-z} \left[m_{X_i}^2 z + \frac{q^2}{4} (\bar{z}^2 - z^2) \right] \right)$$

$$= i\pi \int_0^1 dx \frac{1}{2} \int_{-x}^x d\bar{z} \theta \left(\Lambda^2 + \frac{1}{1-z} \left[m_{X_i}^2 z + \frac{q^2}{4} (\bar{z}^2 - z^2) \right] \right) - (\Lambda^2 \rightarrow m_{X_i}^2) .$$

Since $\Lambda^2 \rightarrow \infty$ $\theta(\Lambda^2 + \dots) = 1$ always and

$$Im G = i\pi \int_0^1 dx \frac{1}{2} \int_{-x}^x d\bar{z} \left[1 - \theta(m_{X_i}^2[1-z] + m_{X_i}^2 z + \frac{q^2}{4} [\bar{z}^2 - z^2]) \right] . \quad (\text{M.136})$$

Now

$$1 - \theta(x-a) = \theta(a-x) \quad (\text{M.137})$$

hence

$$Im G = i\pi \int_0^1 dx \frac{1}{2} \int_{-x}^x d\bar{z} \theta \left[- (R|\bar{z}^2 - z^2| + (1-z) + Tz) \right] \quad (\text{M.138})$$

where we have divided through by $m_{X_i}^2 > 0$ (i.e. $\theta(f(z)) = \theta(af(z))$ if $a > 0$).

From (M.95)

$$W(z) = z^2 - \frac{(1-z) + Tz}{R} > 0. \quad (\text{M.139})$$

Dividing through by R ($R > 0$ if $q^2 > 0$), (M.139) becomes

$$\text{Im } G = i\pi \int_0^1 dz \frac{1}{2} \int_{-s}^s d\bar{z} \theta(W(z) - \bar{z}^2). \quad (\text{M.140})$$

This could be solved analytically (by cases). From (M.135) and (M.50)

$$\text{Re } G = -\frac{1}{2} \int_0^1 dz \int_{-s}^s d\bar{z} \ln |R(\bar{z}^2 - W(z))| \quad (\text{M.141})$$

$$T = \frac{m_{\tilde{\chi}_1^0}^2}{m_{\tilde{\chi}_1^\pm}^2}, \quad R = \frac{q^2}{4m_{\tilde{\chi}_1^\pm}^2} \quad (\text{M.142})$$

$\tilde{G}$ is evaluated in a similar manner.

$$\text{Im } \tilde{G} = i\pi \int_0^1 dz \frac{1}{2} \int_{-s}^s d\bar{z} \theta \left(-\left[\bar{z}^2 + b\bar{z} - z^2 + \frac{1+S}{2R}z + (1-z)\frac{T}{R} \right] \right) \quad (\text{M.143})$$

$$\text{Re } \tilde{G} = -\frac{1}{2} \int_0^1 dz \int_{-s}^s d\bar{z} \ln |R(\bar{z}^2 - z^2) + \frac{1}{2} \{ (1+S)z + (1-S)\bar{z} \} + T(1-z)| \quad (\text{M.144})$$

$$S = S_{ij} = m_{\tilde{\chi}_i^\pm}^2 / m_{\tilde{\chi}_j^\pm}^2, \quad T, R \text{ as in (M.142)} \quad b \equiv \frac{1-S}{2R}. \quad (\text{M.145})$$

Note that in $\text{Re } G$ and $\text{Re } \tilde{G}$ that as $m_{\tilde{\chi}_1^\pm}^2 \rightarrow \infty$ (i.e. $T \rightarrow \infty$) that

$$\text{Re } \tilde{G}, \text{Re } G \rightarrow \sim \ln m_{\tilde{\chi}_1^\pm}^2$$

which would seem unnatural for a term in a cross section.

M.7 G AND $\tilde{G}$ "RE-DEFINED". LIMITING FORMS.

All portions of the matrix elements are finite as $m_{\tilde{\chi}_1^\pm} \rightarrow 0$ or $q^2 \rightarrow \infty$ i.e. as $R \rightarrow \infty$ or $S_{ij} \rightarrow \infty$. There is a troublesome point in the chargino sector as $m_{\tilde{\chi}_1^\pm} \rightarrow \infty$, i.e. as $T \rightarrow \infty$. We desire the infinite $-\frac{1}{2} \ln T$ contributions from G and $\tilde{G}$ to cancel, at least in the SUSY limit.

If we define

$$G(T, R) = -\frac{1}{2} \int_0^1 dz \int_{-s}^s d\bar{z} \ln |z(T-1) + 1 + R(\bar{z}^2 - z^2)| \quad (\text{M.146})$$

$$\tilde{G}(T, S, R) = -\frac{1}{2} \int_0^1 dz \int_{-s}^s d\bar{z} \ln |T(1-z) + \frac{1}{2} \{ (1+S)z + (1-S)\bar{z} \} + R(\bar{z}^2 - z^2)| \quad (\text{M.147})$$

then

$$G(T \rightarrow \infty, R) \rightarrow -\frac{1}{2} \int_0^1 dz \int_{-s}^s d\bar{z} \ln |zT| = -\frac{1}{2} \ln T + \frac{1}{4} \quad (\text{M.148})$$

$$\tilde{G}(T \rightarrow \infty, S, R) \rightarrow -\frac{1}{2} \int_0^1 dz \int_{-s}^s d\bar{z} \ln |T(1-z)| = -\frac{1}{2} \ln T + \frac{3}{4} \quad (\text{M.149})$$

Since all "real" processes will involve ΔG , $\Delta \tilde{G}$ or $\Delta \tilde{G}$ where

$$\Delta F \equiv F|_{\tilde{t}_1, \tilde{b}_1} - F|_{\tilde{t}_2, \tilde{b}_2}, \quad \tilde{F}(T, R) \equiv F(T, R) - F(T, R=0) \quad (\text{M.150})$$

for any function F , the constants will always cancel. It is only in numerical calculations where, perhaps, the $\tilde{t}_1$ (or $\tilde{t}_2$) sector is being artificially examined by itself, that a re-definition of these integral functions becomes desirable. The

$J_{[N]}, J_{[N]}$ functions need not be re-defined, nor the $G', \bar{G}'$ functions (since the derivative of a constant vanishes). Thus we choose the redefinition

$$G(T, R) = -\frac{1}{2} \int_0^1 dx \int_{-x}^x d\bar{x} \ln \{x(T-1) + 1 + R(\bar{x}^2 - x^2)\} - \frac{1}{4} \quad (M.151)$$

$$\bar{G}(T, S, R) = -\frac{1}{2} \int_0^1 dx \int_{-x}^x d\bar{x} \quad (M.152)$$

$$\times \ln \left[T(1-x) + \frac{1}{2} \{ (1+S)x + (1-S)\bar{x} \} + R(\bar{x}^2 - x^2) \right] - \frac{3}{4}.$$

For numerical investigations, if we use the definitions in (M.146)

$$G(T, R \rightarrow \infty) \rightarrow -\frac{1}{2} \ln R + C_{GR}$$

$$\bar{G}(T, R \rightarrow \infty) \rightarrow -\frac{1}{2} \ln R + C_{GR} \quad (M.153)$$

$$\bar{G}(T, S, R \rightarrow \infty) \rightarrow -\frac{1}{2} \ln R + C_{GR}$$

where

$$C_{GR} = 0.80684 - 1.57079570257i \quad (M.154)$$

and

$$\bar{G}(T, S \rightarrow \infty, R) \rightarrow -\ln S + C_{GS} \quad (M.155)$$

where

$$C_{GS} = 0.75. \quad (M.156)$$

Note that when we use the definition (M.152) that

$$\bar{G}(T, S, R \rightarrow \infty) \rightarrow -\frac{1}{2} \ln R + 0.05684 - 1.57079i \quad (M.157)$$

$$\bar{G}(T, S \rightarrow \infty, R) \rightarrow -\ln S. \quad (M.158)$$

In all of the $R \rightarrow \infty, S \rightarrow \infty$ limits $\Delta G \rightarrow 0$ and $\Delta \bar{G} \rightarrow 0$ of course. Note that as $m_{\bar{x}_i} \rightarrow \infty$ that $T_i, R_i, S_{ij} \rightarrow 0$. In this limit the quondam definitions of (M.146) and (M.147) would lead to

$$G(R \rightarrow 0, T \rightarrow 0) \rightarrow -\frac{1}{2} \int_0^1 dx \int_{-x}^x d\bar{x} \ln(1-x) = \frac{3}{4}$$

$$\bar{G}(R \rightarrow 0, T \rightarrow 0) \rightarrow -\frac{1}{2} \int_0^1 dx \int_{-x}^x d\bar{x} \ln x = \frac{1}{4} \quad (M.159)$$

$$\bar{G}(R \rightarrow 0, S > 1, T \rightarrow 0) \rightarrow \frac{1}{4} \frac{2S \ln S + 3(1-S)}{1-S}$$

$\bar{G}$ need not be considered for $S < 1$ since this is not used.

Note that

$$\lim_{S \rightarrow 1} \bar{G}(R \rightarrow 0, S, T \rightarrow 0) = \bar{G}(T \rightarrow 0, R \rightarrow 0) = \frac{1}{4} \quad (M.160)$$

as desired.

If the $m_{\bar{x}_i} \rightarrow \infty, m_{\bar{x}_i} \rightarrow \infty$ limits were thought to be of greater import than the $m_i \rightarrow \infty$ then a different re-definition could have been made in which the constants $-1/4, -3/4$ in (M.151) and (M.152) could be interchanged to bring the limiting forms to zero. Using our revised definitions in (M.152)

$$G(T \rightarrow 0, R \rightarrow 0) = \frac{1}{4}$$

$$\bar{G}(T \rightarrow 0, R \rightarrow 0) = -\frac{1}{4} \quad (M.161)$$

$$\bar{G}(T \rightarrow 0, S > 0, R \rightarrow 0) = \frac{1}{4} \frac{S \ln S}{1-S}.$$

Which are much more symmetric than was the case with the functions as they

were previously defined.

In conclusion we present certain $q^2 = 0$ limiting cases of interest:

$$G(T, 0) = -\frac{1}{4} \left[\ln T - \frac{\ln T^2 + (3-T)(1-T)}{2(1-T)^2} \right] - \frac{1}{4} \quad G(T \rightarrow 1, 0) \rightarrow -\frac{1}{4}$$

$$\bar{G}(T, 0) = -\frac{T^2 \ln T^2 + (3T-1)(T-1)}{4(T-1)^2} - \frac{3}{4} \quad \bar{G}(T \rightarrow 1, 0) \rightarrow -\frac{7}{4}$$

$$I_{[1]}(T, 0) = \frac{T \ln T + 1 - T}{(1-T)^2} \quad I_{[1]}(T \rightarrow 1, 0) \rightarrow \frac{1}{4} \quad (M.162)$$

The formulae developed in this chapter have been used to plot the more important of the integral functions over a range of parameter regimes. It should be mentioned that the Feynman diagrams may be equivalently expressed in terms of what may be becoming a standardized set of integral functions originally put forth by Passarino and Veltman.⁴⁵

FIGURE CAPTIONS

1. Tree Level Vertex with more than one Neutralino.

2. Typical diagram contributing to $I_{[N]}$ terms.

3. $G(T, R)$ vs. $m_{\tilde{L}}$ for $m_{\tilde{X}} = 0.1, 20 \text{ GeV}$.

$$T = \frac{m_{\tilde{L}}^2}{m_{\tilde{X}}^2} \quad R = \frac{q^2}{4m_{\tilde{X}}^2} \quad q^2 = m_{\tilde{Z}}^2 = (90 \text{ GeV})^2$$

4. $G'(T, R)$ vs. $m_{\tilde{L}}$ for $m_{\tilde{X}} = 0.1, 20, 45, 50, 100 \text{ GeV}$.

$$T = \frac{m_{\tilde{L}}^2}{m_{\tilde{X}}^2} \quad R = \frac{q^2}{4m_{\tilde{X}}^2} \quad q^2 = m_{\tilde{Z}}^2 = (90 \text{ GeV})^2$$

5. $\bar{G}(T, S, R)$ vs. $m_{\tilde{L}}$ for $m_{\tilde{X}} = 0.1 \text{ GeV}$ and $S = 1, 10000$.

$$T = \frac{m_{\tilde{L}}^2}{m_{\tilde{X}}^2} \quad R = \frac{q^2}{4m_{\tilde{X}}^2} \quad S = \frac{m_{\tilde{X}}^2}{m_{\tilde{Z}}^2} \quad q^2 = m_{\tilde{Z}}^2 = (90 \text{ GeV})^2$$

6. $\bar{G}'(T, S, R)$ vs. $m_{\tilde{L}}$ for $m_{\tilde{X}} = 0.1 \text{ GeV}$ and $S = 1, 2$

(Indistinguishable).

$$T = \frac{m_{\tilde{L}}^2}{m_{\tilde{X}}^2} \quad R = \frac{q^2}{4m_{\tilde{X}}^2} \quad S = \frac{m_{\tilde{X}}^2}{m_{\tilde{Z}}^2} \quad q^2 = m_{\tilde{Z}}^2 = (90 \text{ GeV})^2$$

7. $\bar{G}(T, R)$ vs. $m_{\tilde{L}}$ for $m_{\tilde{X}} = 1.0 \text{ MeV}$.

$$T = \frac{m_{\tilde{L}}^2}{m_{\tilde{X}}^2} \quad R = \frac{q^2}{4m_{\tilde{X}}^2} \quad q^2 = m_{\tilde{Z}}^2 = (92.9 \text{ GeV})^2$$

8. $R \times I_{[2, -2, 1]}(T, R)$ vs. $m_{\tilde{X}}$ for $m_{\tilde{L}} = 20, 40, 60, 80, 100 \text{ GeV}$.

$$T = \frac{m_{\tilde{L}}^2}{m_{\tilde{X}}^2} \quad R = \frac{q^2}{4m_{\tilde{X}}^2} \quad q^2 = m_{\tilde{Z}}^2 = (92.9 \text{ GeV})^2$$

9. $I_{[1]}(T, S, R)$ vs. $m_{\tilde{L}}$ for $m_{\tilde{X}} = 0.1 \text{ GeV}$ and $S = 1, 6 \times 10^5, 8 \times 10^5, 10^6$.

$$T = \frac{m_{\tilde{L}}^2}{m_{\tilde{X}}^2} \quad R = \frac{q^2}{4m_{\tilde{X}}^2} \quad S = \frac{m_{\tilde{X}}^2}{m_{\tilde{Z}}^2} \quad q^2 = m_{\tilde{Z}}^2 = (90 \text{ GeV})^2$$

10. $I'_{[1]}(T, S, R)$ vs. $m_{\tilde{L}}$ for $m_{\tilde{X}} = 0.1 \text{ GeV}$ and $S = 1.4, 1.6$.

$$T = \frac{m_{\tilde{L}}^2}{m_{\tilde{X}}^2} \quad R = \frac{q^2}{4m_{\tilde{X}}^2} \quad S = \frac{m_{\tilde{X}}^2}{m_{\tilde{Z}}^2} \quad q^2 = m_{\tilde{Z}}^2 = (90 \text{ GeV})^2$$

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Fig. M.1

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Fig. M.2

Fig. M.3

Fig. M.4

Fig. M.5

Fig. M.6

Fig. M.7

Fig. M.8

Fig. M.9

Fig. M.10

APPENDIX N

Background

We wish to consider the difficulties involved in observing $e^+e^- \rightarrow \mu\tau$ at SLC (or LEP) while operating at a Z^0 -resonance whereat $(p_{\tau^-} + p_{e^+})^2 = m_Z^2$ which, for the purposes of numerical estimates, we will take as $(90 \text{ GeV})^2$ in this appendix. The detector signal is a μ^+ and a τ^+ exiting back-to-back, each with essentially half of the total energy and with no missing mass (from ν 's, γ 's, $\bar{\gamma}$'s, etc.)

This signal may be mimicked by the far more common decay mode $Z^0 \rightarrow \tau\tau$ with the subsequent decay of one or both of the τ 's into a muon and neutrinos. Since the τ will travel $\sim 0.25 \text{ cm}$ prior to decay, and such a kink would be unobservable if the decay products of the τ do not come out at a large angle with respect to the τ 's direction, the event will resemble a muon and the decay products of a τ . In either process the τ 's must emerge back-to-back, both with $E = 45 \text{ GeV}$ (to within a few MeV), but since all decay modes of the tau involve at least one undetectable particle (the tau neutrino), and many ($\tau \rightarrow \mu\nu\nu, e\nu\nu$, etc.) give rise to several such states, accurate energy and momentum reconstruction is difficult. There is no problem with the modes $\tau \rightarrow \text{HADRONS}$ or $\tau^+ \rightarrow e^+\nu_e\bar{\nu}_\tau$, but $\tau^+ \rightarrow \mu^+\nu_\mu\bar{\nu}_\tau$ provides a substantial background. Some of the μ^+ will have virtually all of the τ^+ momentum (soft neutrinos) and so the event will appear as $e^+e^- \rightarrow \tau^+\mu^-$ instead of $\tau^+\tau^-$.

One method of reducing the background would be to eliminate all signals which have an opening angle which is discernibly different than 180 degrees. The difficulty in distinguishing the two decay modes $Z^0 \rightarrow \tau\mu$ and $Z^0 \rightarrow \tau\tau \rightarrow \tau\mu + \text{soft neutrinos}$ is that both involve at least one tau reconstruction. Since such a reconstruction will necessarily introduce an error in the inferred tau momentum

angle (which varies over the range of the detector) there will be some $\Delta\theta$ (for the MARK II detector this is actually $\Delta\phi$, the error in the 'acoplanarity') below which it will be impossible to distinguish a back-to-back event from one whose opening angle is $\pi - \Delta\theta$. For a background event the opening angle will range up to approximately twice what it was for a genuine $\mu\tau$ event. Thus we will be able to experimentally eliminate those events with the largest opening angles. But these are precisely the events which will be most easily differentiated on the basis of calorimetric information (since events with larger opening angles will tend to be those with stiffer neutrinos). Thus it appears that angular information will not be useful in discriminating the background from the actual signal.⁴⁶ An analogous argument applies to using the impact parameter.

It appears that we must rely upon energy and momentum considerations to exclude the background contribution. The difficulty of eliminating the spurious component depends crucially upon the energy resolution of the detector and what the tail end of the energy distribution of $\tau(45 \text{ GeV}) \rightarrow \mu\nu\nu$ ($p_\mu = 45 \text{ GeV}$) looks like. When $p_\mu \doteq p_\tau$, the angular kink is negligible (the $\tau \rightarrow \mu$ decay point is 2.5 mm from the interaction region) and will be indiscernible. The time of flight (delay because $m_\tau > m_\mu$) difference is $\sim 0.25 \text{ cm}/(1-\beta_\tau)c$ with $\beta_\tau = \sqrt{1-(1/\gamma_\tau^2)}$ where

$$\gamma_\tau \approx 4.5 \text{ GeV}/(m_\tau \approx 1.784 \text{ GeV}) \approx 25.2. \quad (\text{N.1})$$

Therefore

$$\beta_\tau \approx 1 - \frac{1}{2\gamma_\tau^2} \quad (\text{N.2})$$

and

$$1 - \beta_r \approx \frac{1}{2\gamma_r^2} \quad (\text{N.3})$$

yielding

$$\Delta t = \frac{2.5 \times 10^{-3} m}{3 \times 10^{10} m/s} \cdot \frac{1}{2(25.2)^2} \approx 6.6 \times 10^{-16} \text{ sec.} \quad (\text{N.4})$$

So the delay time is about 6.6 femtoseconds and experimentally unobservable.

We need to calculate the energy spectrum of the $\mu^\pm$ produced from the decay of a $\tau^\pm$. We will perform an idealized background calculation. We will consider only events which have passed cuts imposed by the detector topology and ignore

1. Beam width (i.e. assume that the beam energy is precisely known).
2. Bremsstrahlung. The μ and τ will bremsstrahlung in the strong magnetic field of the detector (even over the 2.5 mm decay distance).
3. Chance of particle interaction (imperfect vacuum) in detector (secondary scattering).
4. Imperfect particle identification.

At SLC the e^- beam will be 100% polarized (say in the $\hat{Z}$ direction). Thus to make a (spin = 1) Z^0 only the e^+ component with spin in the $+\hat{Z}$ direction also (almost all of it) will be used. As a result nearly all of the Z^0 will be created polarized in the $+\hat{Z}$ direction and at rest. Examining the helicity states of μ^- , τ^+ in $e^+e^- \rightarrow Z^0 \rightarrow \mu^- \tau^+$, as illustrated in Fig. N.1, we see that polarization effects are expected to be important.

Let us consider the τ decaying at rest and then "boost" this to the lab ($p_r = 45 \text{ GeV/c}$) frame as in Fig. N.2. We know that the τ will likely be polarized. The normalized number distribution of the muons in the tau rest frame⁴⁷

(using the 4-Fermi Lagrangian, which is an adequate approximation here, and $m_\mu \ll m_\tau$) is

$$d^2 N_\mu = x^2 [3 - 2x \pm \alpha(1 - 2x) \cos \theta_{\text{Boost}}] dx d\cos \theta \quad (\text{N.5})$$

+ for $\tau^- \rightarrow \mu^-$

- for $\tau^+ \rightarrow \mu^+$

$\theta = \theta_{\text{Boost}}$ here

α = degree of polarization

($\alpha = 0$ unpolarized) (N.6)

$\alpha = 1$ 100% polarized)

$$x = \frac{E_i}{\frac{1}{2} m_\tau} \quad (\text{N.7})$$

$E_i = E_\mu$ in τ rest frame, prior to boosting by $\frac{1}{2} m_\tau$.

$\frac{1}{2} m_\tau = \max |E_i|$ in rest frame so $0 \leq x \leq 1$.

Therefore

$$\begin{aligned} \frac{dN_\mu}{dE_i} &= \frac{dN_\mu}{dx} \left(\frac{x}{E_i} \right) = \frac{2}{m_\tau} \left[\frac{2E_i}{m_\tau} \right]^2 \\ &\times \left[3 - 2 \left(\frac{2E_i}{m_\tau} \right) \pm \alpha \left(\frac{\cos \theta}{\cos(\theta + \pi)} \right) \left(1 - 2 \left[\frac{2E_i}{m_\tau} \right] \right) \right] d\cos \theta \quad (\text{N.8}) \\ &= \frac{16E_i^2}{m_\tau^3} [3m_\tau - 4E_i - \alpha(m_\tau - 4E_i) \cos \theta] d\cos \theta. \end{aligned}$$

Note that the "+" sign holds for both μ^+ and μ^- because for μ^+ the polarization is in the momentum direction (same as boost) whereas for μ^- it is in the opposite direction. This just compensates for the sign difference in (N.5).

The (normalized) number of μ 's in the range $\{\theta, \theta + d\theta\}$ (i.e. τ rest frame) with energy E_i is

$$dN_\mu(E_i, \theta) = \frac{16E_i^2}{m_\mu^2} [3m_r - 4E_i - \alpha(m_r - 4E_i) \cos \theta] \frac{2\pi d \cos \theta}{4\pi} dE_i \quad (\text{N.9})$$

(since there is no ϕ dependence and $\int d\Omega = 4\pi$).

We now boost (in $\hat{z}$ direction say) up to $E_f = m_x/2$. In general⁴⁰

$$\gamma_f = \gamma_\mu[\tau\text{-rest}]\gamma_i[\tau\text{-rest}] \left(1 + \frac{\vec{V}_\mu \cdot \vec{V}_i}{c^2} \right) \quad (\text{N.10})$$

and since $\beta_\mu \approx \beta_i \approx 1$

$$\gamma_f = \frac{E_i}{m_\mu} \cdot \frac{\frac{1}{2}m_x}{m_r} (1 + \beta_\mu \beta_i \cos \theta) \approx \frac{E_i E_x}{m_\mu m_r} \frac{(1 + \cos \theta)}{2}. \quad (\text{N.11})$$

Let E_f be the final energy of the μ in the lab (Z^0 at rest) frame

$$E_f = m_\mu \gamma_f = \frac{1}{2} E_i \frac{m_x}{m_r} (1 + \cos \theta) \quad (\text{N.12})$$

(remembering that θ is in the τ rest frame). The probability of the final state having energy E_f in the lab frame, given energy E_i and angle θ in the τ rest frame, is

$$E_f = \int (\text{Prob. of } E_i)(\text{Prob. of } \theta) \quad (\text{N.13})$$

up to a normalization, where $E_f = \frac{1}{2} E_i \frac{m_x}{m_r} (1 + \cos \theta)$ from (N.12). Letting

$$y \equiv \frac{2E_f m_r}{m_x(1 + \cos \theta)} \quad (\text{N.14})$$

this condition is $E_i = y$.

The probability of E_f being the final μ energy is, therefore, (N is a normalization factor)

$$N \int_{E_{i \min}}^{E_{i \max}} dE_i \int_{-1}^1 d \cos \theta \frac{16E_i^2}{m_\mu^2} [m_r(3 - \alpha \cos \theta) - 4E_i(1 - \alpha \cos \theta)] \delta(E_i - y) \quad (\text{N.15})$$

with $E_{i \min} = 0$ and $E_{i \max} = m_r/2$. Letting $z = \cos \theta$ The energy constraint equation becomes

$$y = \frac{2E_f m_r}{m_x(1 + z)}$$

and the number of muons expected at a given lab energy is

$$\begin{aligned} \frac{dN_\mu}{dE_f} &= N \int_0^{m_r/2} dE_i \int_{-1}^1 dz \frac{16E_i^2}{m_\mu^2} [m_r(3 - \alpha z) - 4E_i(1 - \alpha z)] \delta(E_i - y) \\ &= N \int_{-y}^{m_r/2-y} d(E_i - y) \int_{-1}^1 dz \frac{16E_i^2}{m_\mu^2} [m_r(3 - \alpha z) - 4E_i(1 - \alpha z)] \delta(E_i - y) \\ &= N \int_{-1}^1 \frac{1}{2} dz \frac{16}{m_\mu^2} \left[\frac{2E_f m_r}{m_x(1 + z)} \right]^2 \\ &\quad \times \left[m_r(3 - \alpha z) - 4 \cdot \frac{2E_f m_r}{m_x(1 + z)} (1 - \alpha z) \right] \theta \left(\frac{m_r}{2} - \frac{2E_f m_r}{m_x(1 + z)} \right). \end{aligned} \quad (\text{N.16})$$

The argument of the theta-function must exceed zero requiring that

$$\frac{m_r}{2} - \frac{2E_f m_r}{m_x(1 + z)} > 0 \Rightarrow z > \frac{4E_f}{m_x} - 1.$$

Therefore

$$\frac{dN_\mu}{dE_f} = N \int_{(4E_f/m_x)-1}^1 dz \frac{E_f^2}{m_\mu^2 m_x^2} \left[\frac{m_r(3 - \alpha z)}{(1 + z)^2} - \frac{8E_f m_r(1 - \alpha z)}{m_x(1 + z)^2} \right] \quad (\text{N.17})$$

and letting $\omega = 1 + \epsilon$

$$\begin{aligned}
 \frac{dN_\mu}{dE_f} &= N \frac{E_f^2}{m_z m_z^2} \int_{E_f/m_z}^1 d\omega \left[\frac{3 - \alpha(\omega - 1)}{\omega^3} - \frac{8E_f}{m_z} \frac{1 - \alpha(\omega - 1)}{\omega^3} \right] \\
 &= N \frac{E_f^2}{m_z m_z^2} \int_{E_f/m_z}^1 d\omega \left[-\frac{m_z \alpha}{\omega} + \frac{(3 + \alpha)m_z + 8\alpha E_f}{\omega^3} - \frac{8(1 + \alpha)E_f}{\omega^3} \right] \\
 &= N \frac{E_f}{4m_z m_z^2} \left[4\alpha m_z E_f \ln \frac{2E_f}{m_z} + 6(\alpha - 1)m_z E_f \right. \\
 &\quad \left. + 4(1 - 3\alpha)E_f^2 + 2m_z^2 \right]
 \end{aligned} \tag{N.18}$$

Let

$$\eta = \frac{N}{4m_z m_z^2}.$$

The normalization is established by requiring that

$$\int_0^{m_z/2} \left(\frac{dN_\mu}{dE_f} \right) dE_f = 1$$

and so

$$\begin{aligned}
 \eta \int_0^{m_z/2} dE_f E_f \\
 \times \left\{ 4\alpha m_z E_f \ln \frac{2E_f}{m_z} + 6(\alpha - 1)m_z E_f + 4(1 - 3\alpha)E_f^2 + 2m_z^2 \right\} = 1.
 \end{aligned} \tag{N.19}$$

Performing the integration yields the result

$$1 = \eta \left\{ \frac{m_z^2}{144} (9 + \alpha) \right\}$$

or

$$\eta = \frac{144}{m_z^2 (9 + \alpha)}$$

which, when used in (N.18), gives

$$\frac{dN_\mu}{dE_f} = \frac{288E_f}{(9 + \alpha)m_z^2} \left\{ m_z^2 + 3(\alpha - 1)m_z E_f + 2(1 - 3\alpha)E_f^2 + 2\alpha m_z E_f \ln \frac{2E_f}{m_z} \right\} \tag{N.20}$$

Where $\alpha = 0$ for the unpolarized case and $\alpha = 1$ in the completely polarized case.

For $\alpha = 0$:

$$\frac{dN_\mu}{dE_f} = \frac{32E_f(E_f - m_z)(2E_f - m_z)}{m_z^2} \quad (\text{unpolarized case}). \tag{N.21}$$

For $\alpha = 1$:

$$\frac{dN_\mu}{dE_f} = \frac{144}{5} \frac{E_f}{m_z^2} \left\{ (m_z^2 - 4E_f^2) + 2m_z E_f \ln \frac{2E_f}{m_z} \right\} \quad (\text{polarized case}). \tag{N.22}$$

These distributions are depicted in Fig. N.3.

We are interested in those μ 's with $E_\mu \doteq E_r = m_z/2$. The number, $N(\Delta\epsilon)$, of μ 's within energy $\Delta\epsilon$ of $m_z/2$ is

$$N(\Delta\epsilon) = \int_{(m_z/2) - \Delta\epsilon}^{m_z/2} dE_f \frac{dN_\mu}{dE_f}. \tag{N.23}$$

Therefore

$$\begin{aligned}
 N(\Delta\epsilon) &= 1 - \frac{288}{9+\alpha} \int_0^{(m_z/2)-\Delta\epsilon} \frac{dE_f E_f}{m_z^2} \\
 &\times \left\{ m_z^2 + 3(\alpha-1)m_z E_f + 2(1-3\alpha)E_f^2 + 2\alpha m_z E_f \ln \frac{2E_f}{m_z} \right\} \\
 &= 1 - \frac{288}{9+\alpha} \frac{1}{m_z^2} \\
 &\times \left\{ \frac{1}{2} m_z^2 E_f^2 + (\alpha-1)m_z E_f^3 + \frac{1}{2} (1-3\alpha)E_f^4 \right. \\
 &\left. + \left(\frac{2}{3} \alpha m_z \ln \frac{2}{m_z} E_f^2 + 2\alpha m_z E_f^2 \left(\frac{1}{3} \ln E_f - \frac{1}{9} \right) \right) \right\} \Bigg|_0^{(m_z/2)-\Delta\epsilon}
 \end{aligned}$$

and expanding all terms to order $(\Delta\epsilon)^2$ yields (eventually)

$$N(\Delta\epsilon) = \frac{3-\alpha}{9+\alpha} \cdot 24 \frac{\Delta\epsilon^2}{m_z^2} \quad (\text{N.24})$$

$$\alpha = 0 \quad \text{gives} \quad N(\Delta\epsilon) = 8 \frac{(\Delta\epsilon)^2}{m_z^2} \quad (\text{N.25})$$

$$\alpha = 1 \quad \text{gives} \quad N(\Delta\epsilon) = \frac{24}{5} \frac{(\Delta\epsilon)^2}{m_z^2} . \quad (\text{N.26})$$

Thus polarization effects decrease the background by a factor of 5/3.

The energy-momentum resolution for stiff muons for the Mark II detector at the SLC will be, at best, (once the close vertex detector has been installed)²⁸

$$\frac{\Delta p}{p} \gtrsim 0.1\% \text{ per GeV}. \quad (\text{N.27})$$

For muons in the 45 GeV range this implies that

$$\frac{\Delta p}{p} \gtrsim 5\%. \quad (\text{N.28})$$

(There is an additional 1.4% error anticipated from multiple scatterings, within the drift chamber alone, which must be added in quadrature.) Taking the "super-optimal" momentum resolution, $\Delta\epsilon_{OPT}$, at SLC to be 5% then

$$\Delta\epsilon_{OPT} = 5\% \left(\frac{m_z}{2} \right) = 0.05 \left(\frac{m_z}{2} \right) \quad (\text{N.29})$$

and we observe from (N.24) that

$$N(\Delta\epsilon_{OPT}) = \frac{3-\alpha}{9+\alpha} \cdot 24 \left(\frac{0.05 \cdot \frac{1}{2} m_z}{m_z} \right)^2 = \left(\frac{3-\alpha}{9+\alpha} \right) 1.5 \times 10^{-3} . \quad (\text{N.30})$$

$$\text{For } \alpha = 0 \quad N(\Delta\epsilon_{OPT}) = 5 \times 10^{-4} \quad (\text{N.31})$$

$$\text{For } \alpha = 1 \quad N(\Delta\epsilon_{OPT}) = 3 \times 10^{-3} . \quad (\text{N.32})$$

Note that the difference between E_μ and $m_z/2$ (due to $m_e \neq m_\mu$) is only about 17 MeV, far below the energy resolution.

Now

$$BR(Z \rightarrow \tau\tau) \simeq 3\% \quad BR(Z \rightarrow \mu\mu) \simeq 3\% \quad (\text{N.33})$$

$$BR(\tau \rightarrow \mu\nu\nu) \simeq 17.5\% \quad BR(\tau \rightarrow e\nu\nu) \simeq 17.5\% \quad (\text{N.34})$$

and therefore an expected sample (ultimate fiducial yearly yield at SLC) of

$10^8 Z^0$'s yields 30,000 $\tau\tau$ events. We expect roughly

$$10^8 \times 0.03 \times 2 \times 0.175 \times (3 \times 10^{-3}) \sim 30 \quad (\text{N.35})$$

misidentified stiff μ 's from $e^+e^- \rightarrow \tau\tau$ events (many of which will go undetected due to their decay configuration). We also expect $\sim 30,000 \mu\mu$ events. From chapter three recall that

$$\frac{\sigma_{\mu\tau}}{\sigma_{\mu\mu}} \lesssim 3 \times 10^{-4} \quad (\text{N.36})$$

and so we would expect fewer than

$$10^8 \times 0.03 \times 2 \times 3 \times 10^{-4} \approx 2 \times 10^{-1} \text{ events per year.} \quad (\text{N.37})$$

We may simultaneously search for both $\tau^+\mu^-$ and $\tau^-\mu^+$ events, accounting for the additional factor of two. Again some of this signal will be lost.

The question arises as to whether we could improve these results if the principal mixing arose in the $\tilde{e} - \tilde{\mu}$ sector rather than the $\tilde{\mu} - \tilde{\tau}$ sector. Since factors of the lepton masses have cancelled from (N.24), the same expression is valid for $\tau \rightarrow e\bar{\nu}\nu$. Similarly $e^+e^- \rightarrow Z^0 \rightarrow \mu^+e^-$ will have the same rate as $e^+e^- \rightarrow Z^0 \rightarrow \tau^+\mu^-$ however the primary background will now arise from

$$\begin{aligned} e^+e^- &\rightarrow Z^0 \rightarrow \tau^+\tau^- \\ \tau^+ &\rightarrow \mu^+\bar{\nu}\nu \\ \tau^- &\rightarrow e^-\bar{\nu}\nu. \end{aligned} \quad (\text{N.38})$$

We expect that (P means 'probability of')

$$P\{\tau \rightarrow \mu(\Delta\epsilon)\bar{\nu}\nu; \tau \rightarrow e(\Delta\epsilon)\bar{\nu}\nu\} = FP\{\tau \rightarrow \mu(\Delta\epsilon)\bar{\nu}\nu\} \cdot P\{\tau \rightarrow e(\Delta\epsilon)\bar{\nu}\nu\} \quad (\text{N.39})$$

where F is a factor of order unity. This describes events in which the taus emerge back-to-back and decay into an electron and muon, both of which retain almost all of the momentum of their parent taus, and appear to continue diametrically back-to-back with no apparent energy loss or deviation. From (N.24) we know that

$$N(\Delta\epsilon)_{(\mu\tau)} = \frac{3-\alpha}{9+\alpha} \cdot 6 \frac{(\Delta p)^2}{p^2} \quad (\text{N.40})$$

therefore

$$N(\Delta\epsilon)_{(\mu\tau)} \approx \left(\frac{3-\alpha}{9+\alpha}\right)^2 \cdot 36 \left(\frac{\Delta p}{p}\right)^2. \quad (\text{N.41})$$

For $\alpha = 1$ this becomes

$$N(\Delta\epsilon)_{(\mu\tau)} \approx 1.4 \left(\frac{\Delta p}{p}\right)^2. \quad (\text{N.42})$$

which, for a 5% error gives

$$N(\Delta p) \approx 9 \times 10^{-6}.$$

From the branching ratios in (N.34) we expect about

$$10^8 \times 0.03 \times 2 \times 0.175 \times 0.175 \times (9 \times 10^{-6}) \sim 2 \times 10^{-2} \quad (\text{N.43})$$

misidentified $e^+e^- \rightarrow e^+\mu^+$ events per sample of $10^8 Z^0$'s. Comparing with (N.37) we see that this could, in theory, be as much as an order of magnitude

less than the signal rate. This conclusion is, of course, based upon somewhat idealized ("super-optimal") background estimates and production rates. It should be noted that, in partial compensation, it will be substantially easier to eliminate background on the basis of angular information in this system since no r -track momentum reconstruction will be required.

FIGURE CAPTIONS

1. The decay $Z^0 \rightarrow r\mu$ in the laboratory rest frame illustrating the configurations which are and are not helicity-suppressed.
2. The decay $r \rightarrow \mu\nu\nu$ in r rest frame.
 θ = the angle of the μ momentum with respect to r polarization factor.
 θ is also the boost direction angle for the μ^+ case.
 $\theta + \pi$ is boost direction angle for the $r^- \rightarrow \mu^-$ case.
3. The distribution of emergent muons, $\frac{dN_\mu}{dE_\mu}$, from $r \rightarrow \mu\nu\nu$ versus E_μ , the muon energy in the laboratory frame.
 The degree of polarization of the $r^\pm$ is α .

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Fig. N.1

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Fig. N.2

Fig. N.3

APPENDIX O

Matrix Element for $\mu \rightarrow e \gamma$

O.1 INTRODUCTION

A one loop calculation of the $\mu e \gamma$ vertex has been performed. Unlike the pretermision in the $Z \rightarrow \mu e$ computations, here we shall not neglect the masses of the external fermions and assume the photon to be on-shell. The result may be added to the effective Lagrangian of the theory and is valid whenever the leptons are on-shell (when all three states are off-shell the result is somewhat more involved since some portion of the m_l^2 terms will be replaced by p_l^2 and gauge invariance will be more difficult to employ as a check on the final form of the result). The calculation was performed with the two-component formalism for a purely SQED theory. We know from considerations of Lorents and gauge invariance²⁹ that the results may be placed in the form:

$$M_{\mu e \gamma}(q^2) = K \bar{\psi}^e(p', s') \left\{ [F_1(q^2) + F_1^S(q^2) \gamma_5] \frac{\not{q} \not{p} \not{q}}{m_l} + [F_2(q^2) + F_2^S(q^2) \gamma_5] \sigma^{\mu\nu} q_\nu \right\} \psi^\mu(p, s) \quad (O.1)$$

$$K = -\frac{\alpha^{3/2} \sin \theta \cos \theta}{144 \sqrt{\pi}} \quad q_\nu = p_\nu - p'_\nu \quad (O.2)$$

where θ is the $\hat{e} - \hat{\mu}$ mixing angle.

There are twenty Weyl-spinor diagrams in all, falling into the catagories of triangle and leg-correction contributions. Each contributes a piece to both the F_1 and F_2 terms as well as other pieces which would be disallowed by our general considerations. These can be shown explicitly (and rather laboriously) to cancel

among themselves. To facilitate this it was found useful develop a projection technique, similar to that employed by Brodsky and Sullivan,⁴⁰ to project out of each two-component expression that portion which contributed to each of the terms in (O.1). The tangle of algebra which followed was made tractable by judicious (i.e. extensive) use of the symbolic manipulation routines REDUCE and MACSYMA.

O.2 SQED FORM FACTORS

In the SQED case we use the integral functions of Table L.1 but the mass ratios of eqn. (3.10) become

$$T_{L,R} = \frac{m_{\ell}^2}{m_\lambda^2}, \quad R = \frac{q^2}{4m_\lambda^2}. \quad (O.3)$$

We also define

$$m_+ \equiv \frac{1}{2}(m_\mu + m_e), \quad m_- \equiv \frac{1}{2}(m_\mu - m_e). \quad (O.4)$$

It is then found that

$$\begin{aligned} F_1(q^2) &= -g \frac{\Delta \tilde{G}(T_R, R)}{R} \\ &\quad - \frac{36[3m_+^2 + m_-^2][q^2 - 2(2m_+^2 - 3m_+m_- + 3m_-^2)]}{[q^2 - 4m_+^2][q^2 - 4m_-^2]} \\ &\quad \times [\Delta I_{[z(1-z)]}(T_R, R) - \Delta I_{[z(1-z)]}(T_R, R=0)] + R \rightarrow L \\ F_1^A(q^2) &= -g \frac{\Delta \tilde{G}(T_R, R)}{R} \\ &\quad - \frac{36[3m_-^2 + m_+^2][q^2 - 2(2m_-^2 - 3m_-m_+ + 3m_+^2)]}{[q^2 - 4m_+^2][q^2 - 4m_-^2]} \\ &\quad \times [\Delta I_{[z(1-z)]}(T_R, R) - \Delta I_{[z(1-z)]}(T_R, R=0)] - R \rightarrow L \\ F_2(q^2) &= -\frac{36}{m_\lambda^2[q^2 - 4m_+^2][q^2 - 4m_-^2]} \left\{ N_2 \quad \Delta I_{[z(1-z)]}(T_R, R) \right. \\ &\quad \left. - 2m_-[3m_+^2 + m_-^2][q^2 + 2m_+(m_+ - 3m_-)] \quad \Delta I_{[z(1-z)]}(T_R, R=0) \right\} \\ &\quad + R \rightarrow L \\ N_2 &= m_+q^4 - 2(m_+ - m_-)(2m_+^2 - m_+m_- + m_-^2)q^2 \\ &\quad + 4m_+m_-(3m_+^2 - 5m_+^2m_- + m_+m_-^2 - 3m_-^2) \\ F_2^A(q^2) &= -\frac{36}{m_\lambda^2[q^2 - 4m_+^2][q^2 - 4m_-^2]} \left\{ N_2^A \quad \Delta I_{[z(1-z)]}(T_R, R) \right. \\ &\quad \left. - 2m_+[3m_-^2 + m_+^2][q^2 + 2m_-(m_- - 3m_+)] \quad \Delta I_{[z(1-z)]}(T_R, R=0) \right\} \\ &\quad - R \rightarrow L \\ N_2^A &= m_-q^4 + 2(m_+ - m_-)(2m_-^2 - m_-m_+ + m_+^2)q^2 \\ &\quad + 4m_+m_-(3m_-^2 - 5m_-^2m_+ + m_-m_+^2 - 3m_+^2) \end{aligned} \quad (O.5)$$

These expressions are valid for all q^2 . Note that F_1^2 and F_2^2 are the same as F_1 and F_2 respectively except for $m_+ \leftrightarrow m_-$ and the relative signs of the left and right sectors. In deriving these form factors algebraic simplification was facilitated by use of the equation

$$J_{1/2}(T, R) = \frac{1}{2R} \hat{G}(T, R) \quad (0.6)$$

as found in Table L.1 of Appendix L. Since $R = \frac{q^2}{4m_1^2}$, as $q^2 \rightarrow 0$ (photon on shell) we have $R \rightarrow 0$ and we see that the $\Delta I_{1/2}(x-z)$ terms in F_1 and F_1^2 explicitly vanish. It is also true that $\hat{G}(T, R)$ vanishes as $R \rightarrow 0$ but

$$\lim_{R \rightarrow 0} \frac{\hat{G}(T, R)}{R} \rightarrow \frac{(1-T)(2T^2 - 7T + 11)}{(1-T)^4}. \quad (0.7)$$

This expression is regular at $T=1$.

$$\lim_{T \rightarrow 1} \lim_{R \rightarrow 0} \frac{\hat{G}(T, R)}{R} = -\frac{3}{2}. \quad (0.8)$$

Similar considerations enable us to evaluate (0.5) in various regimes of momentum space. Relatively simple closed-form expressions have been derived for the form factors in the large and small $|q|^2$ limits. For example

$$F_1(q^2 \rightarrow 0) \rightarrow \Delta f_1(T_R) - \frac{R}{5} \Delta f_1^0(T_R) + (R \rightarrow L)$$

where

$$f_1(x) \equiv \frac{(1-x)(2x^4 - 7x + 11) + 6\pi x}{(1-x)^4} \quad f_1(1) = -\frac{3}{2}$$

$$f_1^0(x) \equiv \frac{(x-1)(3x^4 - 17x^3 + 43x^2 - 77x - 12) + 6\pi x \ln x}{x(1-x)^3} \quad f_1^0(1) = 2$$

0.3 EXACT ANALYTIC EXPRESSIONS

The various integral functions may be expressed in closed form although, in practice, little is gained thereby. As an example

$$\hat{G}(T, R) = 1 - \{G_+(U_+) - G_+(L) + G_-(U_-) - G_-(L)\}$$

with the following tedious definitions

$$G_{\pm}(z) \equiv \frac{1}{4} \left(\frac{A_{\pm}}{B_{\pm}} - C_{\pm} - D_{\pm} \right)$$

$$A_{\pm} = 2 \ln z [z^4 \rho^2 \pm 4z^3 \rho^3 + z^2 (\chi^4 + 5\rho^4) \mp 2z\rho (\chi^4 + \rho^4)] \\ + \{-z^4 \rho^2 \pm 6z^3 \rho^3 - z^2 \rho^4 \pm 2z\rho (2\rho^4 - \chi^4) + 2\rho^2 \chi^4\}$$

$$B_{\pm} = 4\rho^2 |z \pm \rho|^3$$

$$C_{\pm} = \frac{\chi^4}{2\rho^3} \ln(\mp z - \rho)$$

$$D_{\pm} = \chi^2 [2 \ln z \cdot \ln(1 \pm z/\rho) - \text{li}_2(\pm z/\rho)]$$

with li_2 is the dilogarithmic function defined by

$$\text{li}_2(x) \equiv - \int_0^x \frac{\ln(1-z)}{z} dz. \quad (0.9)$$

Note that $\text{li}_2(z) = - \int \ln(1-z) d(\ln z)$ and that

$$\int \frac{\ln x}{x+1} dx = \ln x \ln(1+x) + \text{li}_2(-x).$$

Continuing with the definitions:

$$\rho \equiv \frac{1-T}{2R}$$

$$\chi \equiv \sqrt{\rho^2 - 1/R}$$

$$L \equiv \sqrt{\rho^2 - \chi^2}$$

$$= \frac{1}{\sqrt{R}}$$

$$U_{\pm} \equiv \sqrt{(1+\rho)^2 - \chi^2} \pm 1$$

Similar expressions exist for the other integral functions.

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