A major purpose of the Technical Information Center is to provide the broadest dissemination possible of information contained in DOE's Research and Development Reports to business, industry, the academic community, and federal, state and local governments.

Although a small portion of this report is not reproducible, it is being made available to expedite the availability of information on the research discussed herein.

MAR 10 1986

TITLE INTERSTITIAL DISLOCATION LOOP NUCLEATION AND GROWTH AND
SWELLING PRODUCED BY HIGH-ENERGY CASCADES

LA-UR--86-679

DE86 007374

AUTHOR(S) Jinnan Yu, W. F. Sommer and J. N. Bradbury

SUBMITTED TO The Proceedings of the 13th International Symposium
on Effects of Radiation on Materials, Seattle, WA June 23-
25, 1986.

DISCLAIMER

This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency thereof, nor any of their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.

MASTER

By acceptance of this article the publisher recognizes that the U.S. Government retains a nonexclusive, royalty-free license to publish or reproduce the published form of this contribution or to allow others to do so for U.S. Government purposes.

The Los Alamos National Laboratory requests that the publisher identify this article as work performed under the auspices of the U.S. Department of Energy.

**INTERSTITIAL DISLOCATION LOOP NUCLEATION AND GROWTH AND SWELLING PRODUCED
BY HIGH-ENERGY CASCADES**

Jinnan Yu*

W. F. Sommer and J. N. Bradbury**

***Institute of Atomic Energy, China**

****Los Alamos National Laboratory**

ABSTRACT

Structural materials for advanced energy sources such as magnetically-and inertially-confined thermonuclear reactors are subjected to a radiation environment that produces high-energy displacement cascades and transmutation products that include helium. We address these characteristics and formulate their effect on the microstructural evolution of a material.

We consider that, immediately after a high energy cascade event, a vacancy-rich region exists near the primary event site and that an interstitial-rich zone, formed by collision chains, exists some distance from the primary event site. Transmutation-product helium can diffuse into the vacancy-rich zone and stabilize bubble nuclei that will later grow if sufficient vacancies and helium atoms diffuse to the nucleus. These bubbles are the sinks for excess radiation-produced vacancies. The excess radiation-produced interstitials migrate and bond; if the binding energy is high enough, a di-interstitial is considered to be a stable dislocation-loop nucleus. The loop nuclei grow if they receive more interstitials than vacancies; this represents material swelling since the bubbles do not cause a lattice contraction to offset the dilation caused by the growing dislocation loops.

Consider that the interstitial-loop growth process can be represented by a Langevin equation. If the growth process at time t is assumed to be dependent only on the instantaneous interstitial concentration and instantaneous microstructural state (dislocation, bubbles, and grain boundaries) and independent of previous history, the growth process can be considered to be a Markov process. Application of an equivalent Fokker-Plank equation allows deduction of an interstitial-loop nucleation and growth equation, the solution of which yields an interstitial-loop distribution function.

Use of appropriate material parameters gives the incubation and growth regimes of swelling and also allows a determination of the maximum swelling temperature. These results predict a maximum swelling temperature of 117°C for aluminum, 550°C for stainless steel and 1100°C for molybdenum, in good agreement with experimental results.

Basically, the maximum swelling temperature is found to be a function of only s_v , c_v^f/c_v^m , $(\rho_N + \rho_l)$, and $\dot{N}_p$. The maximum swelling temperature increases as $(\rho_N + \rho_l)$, $\dot{N}_p$ and c_v^f/c_v^m increase and decreases as s_v increases. Here, s_v is the vacancy formation entropy, c_v^f is the vacancy formation energy, c_v^m is the vacancy mobility energy, ρ_N is the network dislocation density, ρ_l is the dislocation loop density and $\dot{N}_p$ is the production rate of vacancies and interstitials per unit volume under irradiation.

It is found further that the re-resolution characteristics for interstitial dislocation loops have a strong bearing on their evolution since the threshold for re-resolution of an interstitial loop is lower than the value for helium in the bubbles. As vacancies also condense at interstitial loops the re-resolution is enhanced.

I. Introduction

Radiation damage to structural material in a fusion reactor environment consists of high-energy displacement cascades of atoms and the simultaneous introduction of impurity helium atoms due to (n,α) reactions. This introduction of point defects, which results from the cascades and impurity atoms, places the material in a non-equilibrium state. The kinetic processes governing the departure from and then the attempt to return to equilibrium are considered to consist of two steps. First, in a time frame of $\sim 10^{-13}$ s, the interaction of an incoming fast neutron with a lattice atom produces a primary knock-on-atom (PKA), which in turn can produce point defects and defect clusters by cascade collisions. The second step is a diffusional process during which the radiation-produced point defects diffuse to sinks such as each other (annihilation by vacancy-interstitial interaction), dislocations and clusters such as void or bubble nuclei, voids or bubbles and dislocation loops. During this second step, the helium atoms also diffuse to sinks such as the voids or bubbles.

The departure from equilibrium in the material, due to fast-neutron bombardment, leads to microstructural changes: this process is generally described in terms of the nucleation and growth of both interstitial loops and voids or bubbles. Figure 1 is a flow diagram that shows the interrelationship of the total process. The task here is to quantify these relationships. Relative to the position of a PKA in the lattice, interstitials can be formed at distances of several atomic spacings by collision chains[1]. The region near the PKA is thus depleted of atoms (depleted zone). Helium atoms produced by transmutation may diffuse to these depleted zones and stabilize the vacancy clusters[2]; this process leads to the nucleation of bubbles. Bubble nucleation is generally thought to consist of two steps. First, the PKA initiates a

cascade region that produces a depleted zone. Second, mobile helium atoms diffuse to the depleted zone and stabilize the vacancy clusters. Another possible process for bubble nucleation occurs when mobile helium atoms combine and form helium-atom clusters. The material properties as well as the number of vacancies and helium atoms in a cluster or depleted zone govern whether the cluster will grow or shrink.

Under irradiation, point-defect concentrations are far in excess of thermal equilibrium levels[3]. The depleted zones are a collection of excess vacancies; interstitials formed far from the depleted zone may condense to form interstitial dislocation loops (IDL). The IDL are an extra plane of atoms, thus their formation and growth causes a crystalline solid to swell. If vacancies were to condense and form vacancy dislocation loops equal to the IDL in number and size, the contraction in the solid would balance the dilatation due to the IDL and no net macroscopic changes in the solid would be evident. However, if and when vacancy clusters in the depleted zones are stabilized into bubbles, no contraction of the lattice due to vacancies occurs to cancel the dilatation due to the IDL and a net volume increase of the solid ensues. We use this picture in developing the formulation below.

II. Interstitial Dislocation-Loop Evolution Formulation

Interstitial atoms, formed some distance from the cascade region, lead to an interstitial concentration far in excess of the thermal equilibrium level. The interstitials may migrate and form clusters in the form of IDL. Even di-interstitials may be considered to be stable interstitial dislocation loop nuclei since the binding energy is higher, -125 kJ/mole in stainless steel, relative to mobile activation energy of interstitials in the matrix, -19.3 kJ/mole.

We assume here that the IDL growth process satisfies a Langevin equation of the form:

$$\dot{r} = K_Q(r, t) + f_Q(r, t), \quad (1)$$

where r is the interstitial-loop radius, $\dot{r}$ is the interstitial-loop growth rate, $K_Q(r, t)$ is the interstitial-loop growth function, and $f_Q(r, t)$ is fluctuating growth function. The interstitial-loop growth at time t depends only on the instantaneous interstitial concentration and instantaneous state of microstructure (dislocation density, bubble distribution and grain and sub-grain boundary structure) at the time t and is independent of its previous history. Thus the IDL growth can be described as a Markov process. The function, $f_Q(r, t)$, cannot generally be represented, so we assume that $f_Q(r, t) = g(r)f_Q(t)$ with

$$\begin{aligned} \langle f_Q(t) \rangle &= 0 \\ \langle f_Q(t)f_Q(t') \rangle &= D\delta(t-t'), \end{aligned} \quad (2)$$

where D is a fluctuating growth coefficient and $\delta(t-t')$ is the delta function. The function $f_Q(t)$ is assumed to have a Gaussian distribution with a mean value of zero. It is difficult to find solutions for nonlinear cases. It has also been shown[4] that unless $g(r)$ is constant, the product, $g(r)f_Q(t)$, is meaningless because of the singular nature of $f_Q(t)$. Equation (1) is then written as ;

$$\dot{r} = K_Q(r, t) + f_Q(t).$$

A stochastic process characterized by Markov process has an equivalent Fokker-Plank equation [4,5] written as;

$$\frac{\partial W_q(r,t;r_0)}{\partial t} = - \frac{\partial K_q(r,t)W_q(r,t;r_0)}{\partial r} + \frac{D\partial^2 W_q(r,t;r_0)}{2\partial r^2}, \quad (3)$$

where $W_q(r,t;r_0)$ is the probability density function such that $W_q(r,t;r_0)dr$ is the probability density that an IDL radius lies in range $[r, r+dr]$ at the time t , given that $r=r_0$ at $t=0$. When $W_q(r,t;r_0)dr$ is multiplied by the number of nuclei, we obtain $N_q(r,t)dr$ the IDL concentration with radius in the range $[r, r+dr]$ at time t . Then we add two terms to equation (3) one to represent the nucleation rate of IDL [6] and one to represent the re-solution of interstitial atoms from the IDL [12]. The IDL evolution equation is then written as:

$$\frac{\partial N_q(r,t)}{\partial t} = - \frac{\partial (K_q(r,t)N_q(r,t))}{\partial r} + \frac{D\partial^2 N_q(r,t)}{2\partial r^2} + \frac{Z_{q1}QD_1C_1^2}{a_0}\delta(r-r_0) - \frac{\pi r^2}{A_0}bN_q(r,t), \quad (4)$$

where A_0 is the area per atom in the interstitial loop and b is the probability per second for ejection of an interstitial atom from the loop into the matrix. The third term on the right side

$$\frac{Z_{q1}QD_1C_1^2}{a_0}\delta(r-r_0), \quad (4')$$

is the IDL nucleation rate by interstitial-interstitial reaction and combination. The coefficient Z_{q1} accounts for the number of sites from which a

di-interstitial can be formed in a single jump as well as the fact that both atoms that may eventually combine are mobile. It has been determined that [6]

$$100 \leq Z_{q1} \leq 200. \quad (4'')$$

Here, ϖ is the matrix atomic volume, D_i is the diffusion coefficient for interstitials, C_i is the concentration of interstitials, a_0 is the lattice atom distance, r_0 is the radius of interstitial loop nucleus. When the interstitial loop growth function, $K_q(r, t)$ is known, equation (4) can be solved to obtain the IDL concentration, subject to the boundary condition;

$$\begin{aligned} N_q(r, t=0) &= 0 \\ N_q(r \rightarrow \infty, t) &= 0. \end{aligned} \quad (5)$$

III. Interstitial Dislocation Loop Growth Function, $K_q(r, t)$

The IDL growth function is a quasi-steady-state growth rate which neglects only the fluctuations. The point-defect absorption rates by loops are [6];

$$\begin{aligned} Q_v^q(r, t) &= Z_v D_v \rho_q(r, t) \left[C_v - C_v^{eq} \exp \left(- \frac{d\epsilon / dm_i}{kT} \right) \right] \\ Q_i^q(r, t) &= Z_i D_i \rho_q(r, t) \left[C_i - C_i^{eq} \exp \left(- \frac{d\epsilon / dm_i}{kT} \right) \right]. \end{aligned} \quad (6)$$

Here, $Q_v^q(r, t)$ and $Q_i^q(r, t)$ are the vacancy and interstitial absorption rate by loops, respectively. Z_v and Z_i are the vacancy and the interstitial capture coefficient by loops, respectively. D_v and D_i are the vacancy and the interstitial diffusion coefficient, respectively. C_v and C_i are the vacancy and

the interstitial concentration, respectively. C_v^{eq} and C_i^{eq} are respectively the thermodynamic equilibrium concentration of vacancies and interstitials. $\rho_l(r, t)$ is the dislocation density, which can also be written as $2\pi r N_l(r, t) dr$, and ϵ is the energy of a faulted dislocation loop composed of m_i interstitials in a disk of radius r . T is the absolute temperature, and k is Boltzmann's constant.

The difference between the interstitial and vacancy absorption rate by IDL is the IDL growth rate, which at equilibrium is simply written as

$$Q_i^l(r, t) - Q_v^l(r, t) = N_l(r, t) \frac{d}{dt} \left(\frac{4\pi r^2}{A_0} \right) l \quad (7)$$

Combination of equations (6) and (7) yields;

$$\left(\frac{dr}{dt} \right) l = A_0 \left[Z_i D_i C_i - Z_i D_i C_i^{eq} \exp \frac{d\epsilon/dm_i}{kT} - Z_v D_v C_v + Z_v D_v C_v^{eq} \exp \frac{d\epsilon/dm_i}{kT} \right] \quad (8)$$

The energy of a faulted IDL of m_i interstitials in a disk of radius r is

$$\epsilon = (2\pi r) \tau_d \cdot \xi + \pi r^2 \gamma_{sf}, \quad (9)$$

where τ_d is the energy per unit length (line tension) of the dislocation comprising the periphery of a loop, generally written also as $\tau_d = G b_i^2$. Here G is the shear modulus of the material, and b_i is the Burgers vector of the faulted loop. The term γ_{sf} is the energy per unit area of the stacking fault enclosed by the loop. Because the energy difference is due to second-order atomic arrangements, γ_{sf} is relatively small (typical values being ~ 10 dynes/cm). The elastic energy of an unfaulted dislocation loop composed of m_i interstitial's in a disk of radius r is

$$\epsilon^P = 2\pi G b_2^2 r \cdot \xi, \quad (10)$$

where b_2 is the Burger's vector of the unfaulted loop and ξ is a parameter which adjusts the elastic energy of an interstitial loop to match the energy of an interstitial loop for large r , the details of which are shown in Appendix I. The radius of an IDL consisting of m_i atoms on a slip plane is

$$r = \left(\frac{A_0 m_i}{\pi} \right)^{1/2}. \quad (11)$$

Since γ_{sf} is small and $\epsilon > \epsilon^P > 2\pi G b_1^2 r \cdot \xi$, we use ϵ^P as an approximation for ϵ . Then,

$$\frac{d\epsilon}{dm_i} = \frac{G b_2^2 A_0}{r} \cdot \xi. \quad (12)$$

Since $\frac{C_1^{eq}}{C_1}, \frac{C_v^{eq}}{C_v} \ll 1$ and $\frac{G b_2^2 A_0}{r k T} \cdot \xi \ll 1$, equation (8) can be approximated as;

$$\left(\frac{dr}{dt} \right)_i = A_0 Z_1 D_i C_i \left[1 - \frac{C_1^{eq}}{C_1} - \frac{Z_v D_v (C_v - C_v^{eq})}{Z_1 D_1 C_1} - \left(\frac{C_1^{eq}}{C_1} + \frac{Z_v D_v C_v^{eq}}{Z_1 D_1 C_1} \right) \frac{G b_2^2 A_0}{r k T} \cdot \xi \right]$$

or

$$K_i(r, t) = A - \frac{B}{r}, \quad (8')$$

where

$$A = A_0 \left[Z_i D_i (C_i - C_i^{eq}) - Z_v D_v (C_v - C_v^{eq}) \right], \text{ and} \quad (8'')$$

$$B = A_0^2 (Z_i D_i C_i^{eq} + Z_v D_v C_v^{eq}) \frac{Gb_2^2}{kT} \cdot \xi. \quad (8''')$$

The parameters A_0 , b_1 , b_2 are given in Table 1. A represents the interstitial dislocation-loop growth rate induced by the supersaturation of irradiation-produced defects. This effect is often referred to as a chemical stress because it produces interstitial-loop growth like a mechanical force.

Table 1 : Values for lattice constants

	f.c.c crystal	b.c.c crystal
Slip plane	{111}	{110}
Unfaulted slip plane	{110}	{111}
Burger's vector b_2 (unfaulted loop)	$b_2 = a_0 / \sqrt{2}$	$\frac{\sqrt{3}}{2} a_0$
Burger's vector b_1 (faulted loop)	$b_1 = a_0 / \sqrt{3}$	$b_1 = a_0 / \sqrt{2}$
Area per atom of the slip plane	$A_0 = \sqrt{3} a_0^2 / 4$	$A_0 = \sqrt{2} a_0^2 / 2$
Matrix atomic volume Ω	$\Omega = a_0^3 / 4$	$\Omega = a_0^3 / 2$
Distance between slip plane d_1	$d_1 = a_0 / \sqrt{3}$	$d_1 = a_0 / \sqrt{2}$

In order to obtain a solution for equation (4), it is first necessary to determine the interstitial concentration. We consider a system that consists of bubble nuclei, bubbles, a dislocation network, and IDL at time t and further consider that the material is under irradiation. An IDL must increase its line length during interstitial absorption or vacancy emission, but a network dislocation may not if the nodal points of the network are free to climb. The total dislocation density includes an initial random dislocation network, ρ_N , and irradiation-produced interstitial loops, with a dislocation density of ρ_l [7].

If the system at time t is in thermal equilibrium, the vacancy and interstitial rate equations are

$$\begin{aligned} \frac{dC_v^{eq}}{dt} = 0 = & \dot{n}_{pv}^{th} - k_{iv}C_v^{eq}C_i^{eq} - 4\pi D_v C_v^{eq} \int_{r_{ol}}^{r_{max}} r N(r, t) \left[1 - \exp\left(-\left(p - \frac{2\gamma}{r}\right) \frac{\Omega}{kT}\right) \right] dr \\ & - Z_v \rho_N D_v C_v^{eq} - Z_v D_v \int 2\pi r N_l(r, t) (C_v^{eq} - C_v^{eq} \exp\left(-\frac{Gb_2^2 A_0}{rkT} \cdot \xi\right)) dr \\ & - 4\pi R_p N_p D_v C_v^{eq}, \text{ and} \end{aligned} \quad (13)$$

$$\begin{aligned} \frac{dC_i^{eq}}{dt} = 0 = & \dot{n}_{pi}^{th} - k_{iv}C_v^{eq}C_i^{eq} - 4\pi D_i C_i^{eq} \int_{r_{ob}}^{r_{max}} r N(r, t) \left[1 - \exp\left(-\left(p - \frac{2\gamma}{r}\right) \frac{\Omega}{kT}\right) \right] dr \\ & - Z_i \rho_N D_i C_i^{eq} - Z_i D_i \int 2\pi r N_l(r, t) (C_i^{eq} - C_i^{eq} \exp\left(-\frac{Gb_2^2 A_0}{rkT} \cdot \xi\right)) dr \\ & - 4\pi R_p N_p D_i C_i^{eq}, \end{aligned} \quad (14)$$

where $\dot{n}_{pv}^{th}$ and $\dot{n}_{pi}^{th}$ are the vacancy and interstitial production rate in thermal equilibrium, respectively; k_{iv} is a rate constant for vacancy-interstitial recombination in units of m^3/sec , and $N(r, t)$ is the concentration of bubbles with radius in the range, $[r, r+dr]$. p and γ are the bubble internal pressure and surface tension, respectively, and r_{ob} is the bubble nuclei radius. R_p and N_p are the radius and concentration of coherent precipitate particles.

If the system at time t is under irradiation, the vacancy and interstitial rate equations are

$$\begin{aligned} \frac{dC_v}{dt} = \dot{N}_p + \dot{n}_{pv}^{th} - k_{iv}C_vC_i - 4\pi D_v \int_{r_{ob}}^{r_{max}} rN(r,t) \left[C_v - C_v^{eq} \exp\left(-\left(p - \frac{2\gamma}{r}\right) \frac{Q}{kT}\right) \right] dr \\ - Z_v \rho_N D_v C_v - Z_v D_v \int 2\pi r N_l(r,t) \left[C_v - C_v^{eq} \exp\left(-\frac{Gb_l^2 A_0}{r k T} \cdot \xi\right) \right] dr - 4\pi R_p N_p D_v C_v, \end{aligned} \quad (15)$$

and

$$\begin{aligned} \frac{dC_i}{dt} = \dot{N}_p + \dot{n}_{pi}^{th} - k_{iv}C_vC_i - 4\pi D_i \int_{r_{ob}}^{r_{max}} rN(r,t) \left[C_i - C_i^{eq} \exp\left(-\left(p - \frac{2\gamma}{r}\right) \frac{Q}{kT}\right) \right] dr - Z_i \rho_N D_i C_i \\ - Z_i D_i \int 2\pi r N_l(r,t) \left[C_i - C_i^{eq} \exp\left(-\frac{Gb_l^2 A_0}{r k T} \cdot \xi\right) \right] dr - 4\pi R_p N_p D_i C_i. \end{aligned} \quad (16)$$

Substituting equations (13) and (14) into equations (15) and (16) respectively, we obtain;

$$\begin{aligned} \frac{dC_v}{dt} = \dot{N}_p - k_{iv}(C_vC_i - C_v^{eq}C_i^{eq}) - 4\pi D_v(C_v - C_v^{eq}) \int_{r_{ob}}^{r_{max}} rN(r,t) dr \\ - Z_v(\rho_N + \rho_l)D_v(C_v - C_v^{eq}) - 4\pi R_p N_p D_v(C_v - C_v^{eq}) \end{aligned} \quad (15')$$

and

$$\begin{aligned} \frac{dC_i}{dt} = \dot{N}_p - k_{iv}(C_vC_i - C_v^{eq}C_i^{eq}) - 4\pi D_i(C_i - C_i^{eq}) \int_{r_{ob}}^{r_{max}} rN(r,t) dr \\ - Z_i D_i(C_i - C_i^{eq})(\rho_l + \rho_N) - 4\pi R_p N_p D_i(C_i - C_i^{eq}), \end{aligned} \quad (16')$$

where $\dot{N}_p$ is the vacancy and interstitial production rate per unit volume by the radiation and $\int 2\pi r N_l(r,t) dr$ is the dislocation-loop density. If the system

evolution is in a quasi-steady state, the vacancy change rate should be equal to the interstitial change rate, and $\frac{dC_v}{dt}$ and $\frac{dC_i}{dt}$ can be neglected.* Using the form for point defect absorption characteristics of coherent precipitates [6]; ($4\pi R_p N_p D_i (C_i - C_i^{eq}) = 4\pi R_p N_p D_v (C_v - C_v^{eq})$), and assuming a quasi-steady state condition, we can obtain the following relationship;

$$4\pi \int_{r_{ob}}^{r_{max}} r N(r, t) dr \left[D_v (C_v - C_v^{eq}) - D_i (C_i - C_i^{eq}) \right] = (\rho_N + \rho_I) \left[Z_i D_i (C_i - C_i^{eq}) - Z_v D_v (C_v - C_v^{eq}) \right].$$

Thus, the sum of the number of vacancies and interstitials that diffuse to hubbles is equal to the sum of the number of interstitials and vacancies absorbed by dislocations. We rewrite this expression as

$$\left[\int_{r_{ob}}^{r_{max}} 4\pi D_v r N(r, t) dr + D_v Z_v (\rho_N + \rho_I) \right] (C_v - C_v^{eq}) = \left[\int_{r_{ob}}^{r_{max}} 4\pi D_i r N(r, t) dr + D_i Z_i (\rho_N + \rho_I) \right] (C_i - C_i^{eq}). \quad (17)$$

Substituting equation (17) into equation (16') and neglecting $\frac{dC_i}{dt}$ (quasi-steady state condition) yields

$$C_i - C_i^{eq} = \frac{C'}{2} \left\{ \left(1 + \frac{\dot{N}_p H}{C'^2} \right)^{1/2} - 1 \right\}, \quad (18)$$

where C' and H are as follows:

$$C' = C_i^{eq} + \frac{4\pi D_v N(t) \langle r \rangle + Z_v D_v (\rho_N + \rho_I)}{4\pi D_i N(t) \langle r \rangle + Z_i D_i (\rho_N + \rho_I)} C_v^{eq} + \frac{4\pi D_v N(t) \langle r \rangle + Z_v D_v (\rho_N + \rho_I)}{k_{iv}}, \quad (19)$$

*This approximation is justifiable on the grounds that changes in the sink strengths (and hence the rates of point defect removal) due to the evolution of the microstructure of the metal during irradiation are very slow compared with the time required for the point-defect population to respond to such changes.

and

$$H = 4 \frac{4\pi D_v N(t) \langle r \rangle + Z_v D_v (\rho_N + \rho_I)}{k_{1v} [4\pi D_i N(t) \langle r \rangle + Z_i D_i (\rho_N + \rho_I)]}, \quad (20)$$

with

$$\langle r \rangle = \int_{r_0}^{r_{\max}} r N(r, t) dr / \int_{r_0}^{r_{\max}} N(r, t) dr, \quad (21)$$

and

$$N(t) = \int_{r_0}^{r_{\max}} N(r, t) dr. \quad (22)$$

Substituting equation (18) into equation (17) gives

$$C_v - C_v^{\text{eq}} = \frac{\rho_N + \rho_I + \frac{4\pi}{Z_i}}{\rho_N + \rho_I + \frac{4\pi}{Z_v} \langle r \rangle N(t)} \frac{Z_i D_i}{Z_v D_v} \frac{C'}{2} \left\{ \left(1 + \frac{\dot{N}_{D^H}}{C'} \right)^{1/2} - 1 \right\}. \quad (23)$$

Substituting equation (17) into equation (8") yields

$$A = A_0 \left(\frac{Z_i}{Z_v} - 1 \right) \frac{4\pi \langle r \rangle N(t) / (\rho_N + \rho_I)}{1 + 4\pi \langle r \rangle N(t) / (\rho_N + \rho_I)} D_i \frac{C'}{2} \left\{ \left(1 + \frac{\dot{N}_{D^H}}{C'} \right)^{1/2} - 1 \right\}. \quad (24)$$

IV. Interstitial Loop Concentration and Swelling

If equation (4) is solved to obtain $N_0(r, t)$, irradiation swelling is given by

$$\frac{\Delta V}{V} = \int_{r_0}^{\infty} \pi r^2 N_q(r, t) dr \cdot d_1, \quad (25)$$

where d_1 is the distance between slip planes, as shown in Table 1.

Substituting equation (8') into equation (4), we get

$$\frac{\partial N_q(r, t)}{\partial t} = - \frac{\partial}{\partial r} \left[N_q(r, t) \left(A - \frac{B}{r} \right) \right] + \frac{D}{2} \frac{\partial^2}{\partial r^2} N_q(r, t) + C \delta(r - r_0) - \frac{\pi b r^2}{A_0} N_q(r, t), \quad (4')$$

where $C = \frac{Z_{q1} Q D_1 C_1^2}{a_0}$. We now define $N_q(r, t) = n_q(r, t) \exp\left(\frac{A}{D}r - \frac{B}{D} \ln r\right)$ and substitute it into equation (4'), to obtain;

$$\begin{aligned} \frac{\partial n_q(r, t)}{\partial t} &= \frac{D}{2} \frac{\partial^2 n_q(r, t)}{\partial r^2} + C \delta(r - r_0) \exp\left(-\frac{A}{D}r + \frac{B}{D} \ln r\right) \\ &\quad - \left(\frac{1}{2D} \left(A - \frac{B}{r} \right)^2 - \frac{B}{2r^2} + \frac{\pi b}{A_0} r^2 \right) n_q(r, t). \end{aligned} \quad (4'')$$

The Fourier transform of $n_q(r, t)$ is

$$\begin{cases} \tilde{n}_q(\lambda, t) = \int_R n_q(r, t) e^{-i\lambda r} dr \\ n_q(r, t) = \frac{1}{2\pi R} \int \tilde{n}_q(\lambda, t) e^{i\lambda r} d\lambda \end{cases}, \quad (26)$$

where R is the defined region of $n_q(r, t)$. Substituting $\tilde{n}_q(\lambda, t)$ into equation (4'') yields

$$\frac{d\tilde{n}_q(\lambda, t)}{dt} = - \frac{D}{2} \lambda^2 \tilde{n}_q(\lambda, t) + \tilde{f}(\lambda, t), \quad (27)$$

where

$$f(r, t) = C\delta(r-r_0)\exp(-\frac{A}{D}r + \frac{B}{D}\ln r) - (\frac{1}{2D}(A - \frac{B}{r})^2 - \frac{\pi b}{A_0}r^2)n_q(r, t).$$

According to the definition, $N_q(r, t) = \frac{e^{Ar/D}}{(r)^{B/D}}n_q(r, t)$, and boundary condition (5), the boundary conditions for $n_q(r, t)$ and $\tilde{n}_q(\lambda, t)$ become;

$$\begin{cases} n_q(r, t) = 0 & \text{when } t=0 \\ n_q(r, t) = 0 & \text{as } r \rightarrow \infty \end{cases} \quad (5')$$

and

$$\begin{cases} \ddot{n}_q(\lambda, 0) = 0 \\ \ddot{n}_q(\infty, t) = 0. \end{cases} \quad (5'')$$

The solution of equation (27) with condition (5'') is as follows:

$$\tilde{n}_q(\lambda, \tau) = \int_0^t \tilde{f}(\lambda, \tau)\exp(-\frac{D}{2}\lambda^2(t-\tau))d\tau. \quad (28)$$

The inverse transform of $\exp[-\frac{D}{2}\lambda^2(t-\tau)]$ is

$$F^{-1}[\exp(-\frac{D}{2}\lambda^2(t-\tau))] = \frac{\exp[-\frac{r^2}{2D(t-\tau)}]}{\sqrt{2\pi D(t-\tau)}}. \quad (29)$$

Thus

$$n_q(r, t) = \frac{1}{i2\pi D} \int_0^t \int_R \frac{f(\xi, \tau)}{\sqrt{t-\tau}} \exp[-\frac{(r-\xi)^2}{2D(t-\tau)}] d\xi d\tau. \quad (30)$$

Substituting $f(r, t)$ into equation (30), we get

$$n_q(r, t) = \frac{e^{-Ar_0/D}(r_0)^{B/D}}{\sqrt{2\pi D}} \int_0^t \frac{t_q(\tau)}{\sqrt{t-\tau}} \exp\left[-\frac{(r-r_0)^2}{2D(t-\tau)}\right] d\tau$$

$$= \frac{1}{\sqrt{2\pi D}} \int_0^t \int_R \frac{\exp\left[-\frac{(r-\xi)^2}{2D(t-\tau)}\right]}{\sqrt{t-\tau}} \left\{ \frac{1}{2D} \left(A - \frac{B}{\xi}\right)^2 - \frac{B}{2\xi} + \frac{\pi b \xi^2}{A_0} \right\} n_q(\xi, \tau) d\tau d\xi. \quad (30')$$

During the initial irradiation period, $Z_1(\rho_N + \rho_q)$ and $Z_v(\rho_N + \rho_q) \gg 4\pi N(t)\langle r \rangle$, and equation (19) and (20) can be simplified as

$$C_1' = \frac{Z_v D_v}{Z_1 D_1} C_v^{eq} + \frac{Z_v \mu_v}{k_{1v}} (\rho_N + \rho_q), \quad (19')$$

and

$$H_1 = \frac{4Z_v D_v}{k_{1v} Z_1 D_1}, \quad (20')$$

and since $\rho_q \ll \rho_N$, C_1^2 is independent of time. If $\frac{Z_v D_v}{Z_1 D_1} C_v^{eq} \gg \frac{Z_v D_v}{k_{1v}} (\rho_N + \rho_q)$,

$$C_1^2 = \frac{1}{4} \left(\frac{Z_v D_v}{Z_1 D_1} C_v^{eq} \right)^2 \left\{ 2 + \frac{\dot{N}_p H}{(C_v^{eq})^2} - 2 \left(1 + \frac{\dot{N}_p H}{(C_v^{eq})^2} \right)^{1/2} \right\}, \quad (31)$$

but if the inverse is true $\frac{Z_v D_v}{k_{1v}} (\rho_N + \rho_q) \gg \frac{D_v Z_v}{D_1 Z_1} C_v^{eq}$,

$$C_1^2 = \frac{1}{4} \left[\frac{Z_v D_v}{k_{1v}} (\rho_N + \rho_q) \right]^2 \left\{ 2 + \dot{N}_p \frac{k_{1v}}{Z_1 D_1 Z_v D_v (\rho_N + \rho_q)} - 2 \left(1 + \dot{N}_p \frac{k_{1v}}{Z_1 D_1 Z_v D_v (\rho_N + \rho_q)} \right)^{1/2} \right\}. \quad (32)$$

At high temperature, C_1^2 is independent of time. At low temperature, C_1^2 is small. Thus, we can assume that A is independent of time during the initial

irradiation period, and that A is comparatively small. The changes are very slow and the term $(\frac{1}{2D}(A - \frac{B}{r})^2 - \frac{B}{2r^2} + \frac{\pi r^2}{A_0}b)n_0(r,t)$ can be treated as a perturbation. The solution of equation (27) is

$$\begin{aligned} n_0(r,t) = & C e^{-Ar_0/D} (r_0)^{B/D} \left[\sqrt{2t/\pi D} \exp\left[-\frac{(r-r_0)^2}{2Dt}\right] - \frac{r-r_0}{D} \left[1 - \Phi\left(\sqrt{(r-r_0)^2/2Dt}\right)\right] \right] \\ & - C e^{-Ar_0/D} (r_0)^{B/D} \frac{\exp\left[-\frac{(r-r_0)^2}{2Dt}\right]}{\sqrt{2\pi D}} \int_0^t \sqrt{2\tau/\pi D(t-\tau)} d\tau \left\{ \left[\frac{\pi b(r_0+a)}{A_0 2\mu} + \left(\frac{B^2}{2D} + \frac{B}{2}\right) \frac{1}{r_0} \right] \exp\left[-\frac{\tau}{2D(t-\tau)} (r-r_0)^2\right] \right. \\ & + \left[\frac{A^2}{4D} - \left(\frac{B^2}{2D} + \frac{B}{2}\right)\mu + \frac{\pi b}{A_0} \left(\frac{1}{4\mu} + \frac{a^2}{2}\right) \right] \sqrt{\pi/\mu} \left[1 - \Phi\left(\sqrt{\tau(r-r_0)^2/2Dt(t-\tau)}\right)\right] + \left(\frac{r}{D(t-\tau)} + \frac{r_0}{2D\tau} - \frac{AB}{D}\right) f_1(a) \\ & \left. + \left[\frac{1}{2D} \left(A - \frac{B}{r}\right)^2 + \frac{B}{2r^2} + \frac{\pi b}{A_0} r^2 \right] f_2(\tau, r, t) \right\}, \end{aligned} \quad (33)$$

where $\mu = \frac{t}{2D\tau(t-\tau)}$, $a = \left[r_0 + \frac{\tau}{t}(r-r_0)\right]$, and $f_1(a)$ and $f_2(\tau, r, t)$ is described by

$$f_1(a) = \int_{r_0}^{\infty} \frac{1}{\xi} \exp[-\mu(\xi-a)^2] d\xi, \quad (34)$$

and

$$f_2(\tau, r, t) = \int_0^{\infty} \frac{x}{D} \left[1 - \Phi\left(\sqrt{1/2D\tau} x\right) \right] \exp\left[-\frac{(x-(r-r_0))^2}{2D(t-\tau)}\right] dx. \quad (35)$$

The expression for $N_0(r,t)$ is

$$N_0(r,t) = \frac{C \exp\left[\frac{A}{D}(r-r_0)\right]}{\left(\frac{r}{r_0}\right)^{B/D}} \exp\left[-\frac{(r-r_0)^2}{2Dt}\right] f'(t, r) \quad (36)$$

where

$$\begin{aligned}
 f'(t, r) = & \left\{ \sqrt{2t/\pi D} - \frac{r-r_0}{D} \left[1 - \Phi(\sqrt{(r-r_0)^2/2Dt}) \right] \exp\left[-\frac{(r-r_0)^2}{2Dt}\right] - \frac{b}{A_0} t^2 \left[r A_3 + r_0 (A_2 - A_4) \right] \right. \\
 & - \frac{t}{\pi D} \left(\frac{B^2}{2D} + \frac{B}{2} \right) \frac{1}{r_0} A_1 - \frac{t^{3/2}}{\sqrt{2\pi D}} \left[\left(\frac{A^2}{2D} + \frac{\pi b r_0^2}{A_0} \right) B_2 + \frac{2\pi b r_0 (r-r_0)}{A_0} B_3 \right. \\
 & \left. \left. + \frac{\pi b (r-r_0)^2}{A_0} B_4 \right] - \frac{t^{1/2}}{D\sqrt{2\pi D}} \left(\frac{B^2}{2D} + \frac{B}{2} \right) B_1 + t^{3/2} \frac{b}{A_0} \sqrt{\pi D/2} B_5 - \frac{F}{\sqrt{2\pi D}} \right\}. \quad (36')
 \end{aligned}$$

Here the A_i and B_i ($i = 1, 2, 3, 4, 5$) are degenerate hypergeometric functions, see Appendix II. F is

$$\begin{aligned}
 F = & \sqrt{2/\pi D} \, t \left[\int_0^\infty \left(\frac{r}{Dt} \frac{\sqrt{x}}{1+x} + \frac{r_0}{2Dt} \frac{1}{(1+x)\sqrt{x}} - \frac{Ab}{D} \right) f_1 \left(r_0 + \frac{x}{1+x} (r-r_0) \right) dx \right. \\
 & \left. + \int_0^\infty \frac{\sqrt{y}}{(1+y)^2} dy \int_0^\infty \frac{x}{D} \left[1 - \Phi(\sqrt{(1+y)/2\pi t y} x) \right] e^{-\frac{(x-(r-r_0))^2}{2Dt}} (1+y) dx \left(\frac{1}{2D} \left(A - \frac{B}{r} \right)^2 + \frac{B}{2r^2} + \frac{\pi b t^2}{A_0} \right) \right].
 \end{aligned}$$

Neglecting smaller terms, $N_q(r, t)$ is approximately

$$\begin{aligned}
 N_q(r, t) = & \frac{C e^{\frac{A}{D}(r-r_0)}}{\left(\frac{r}{r_0} \right)^{B/D}} e^{-\frac{(r-r_0)^2}{2Dt}} \left\{ \sqrt{2t/\pi D} - \frac{r-r_0}{D} \left[1 - \Phi(\sqrt{(r-r_0)^2/2Dt}) \right] \exp\left[-\frac{(r-r_0)^2}{2Dt}\right] \right. \\
 & \left. - \frac{b}{A_0} t^2 \left[r A_3 + \sqrt{\pi/2Dt} (2r_0(r-r_0) B_3 + (r-r_0)^2 B_4) + \sqrt{\pi Dt/2} B_5 \right] - \frac{t}{\pi D} \left(\frac{B^2}{2D} + \frac{B}{2} \right) \frac{A_1}{r_0} \right\}, \quad (37)
 \end{aligned}$$

with

$$\rho_d(r, t) = 2\pi r N_d(r, t), \quad \rho_d(t) = \int_{r_0}^{\infty} \rho_d(r, t) dr, \quad (38)$$

$$\frac{\Delta V}{V} = \int_{r_0}^{\infty} F(r, t) dr, \text{ and } F(r, t) = \pi r^2 d_1 N_d(r, t), \quad (39)$$

A complete numerical analysis using equation (37) requires a computer-based calculation. However, the form of the expression for the IDL distribution function is contained in the term;

$$\frac{C \exp\left[\frac{A}{D}(r-r_0) - \frac{(r-r_0)^2}{2Dt}\right]}{\left(\frac{r}{r_0}\right)^{B/D}}, \quad (37')$$

Use of this expression gives some fundamental results.

(1) Equation (37') is plotted in Figure 2. The extreme value is at $r_m^0 = \frac{1}{2}(At+r_0)(1 + \sqrt{1+4(D-B)t/(At+r_0)^2})$, and this extreme value increases with At . This is similar to a void distribution relationship as a function of void radius [8]. These curves will be significantly lowered by the re-solution parameter b . The irradiation re-solution efficiency of IDL is higher than for bubbles; since the threshold of re-solution of IDL is lower than that for helium in a bubble. The re-solution parameter b for IDL is more important than in the case of bubbles; it will significantly lower the peak of the curves and decrease the swelling. Development of the re-solution parameter, b , is presently being pursued.

(2) If the bubble density increases to a quantity such that the sink strength for bubbles is comparable to that of the total dislocation density ($\rho_N + \rho_d$), the At value increases more rapidly; then, the number of IDL increases faster. The swelling will obviously also increase. Thus, the swelling process can be

divided into two stages. The first stage occurs when $4\pi\langle r \rangle N(t) \ll (\rho_N + \rho_I)$, the IDL and bubble volumes are very slowly increasing, and swelling is too small to be detected. This stage is the incubation period. The second stage is when $4\pi\langle r \rangle N(t) \sim (\rho_N + \rho_I)$; then the interstitial loops and bubbles increase quickly, and swelling proceeds faster. This is the swelling period; the results here agree with experimental results[9].

(3) The swelling strongly depends on the extreme point of the curves as shown in Figure 3. The extreme point is at

$$r_m^V = \frac{1}{2}(At + r_0)(1 + \sqrt{1 + 4(2D - B)t/(At + r_0)^2}); \quad (40)$$

If the irradiation time is long, r_m^V is approximately $(At + r_0)$. The relationship between swelling and temperature is mainly dependent on the temperature relationship to A as shown in Figure 4. The parameters for Figure 4 are shown in Table 2.

Table 2 The value of various parameters

Parameter	Unit	Stainless steel	Aluminum	Molybdenum
A_0	$\text{\AA}^2$	5.728	7.099	7.002
G	kg/cm^2	$0.787 \cdot 10^6$	$2.7 \cdot 10^3$	$1.6 \cdot 10^6$
b_2	$\text{\AA}$	2.572	2.863	2.725
Z_1		1.08	1.08	1.08
Z_v		1	1	1
D_v^0	cm^2/s	1.058	0.04	0.03
ϵ_v^m	eV	1.4	1.569	0.5292
D_i^0	cm^2/s	0.001	0.08	0.16
ϵ_i^m	eV	0.20	0.10	0.085
ϵ_v^f	eV	1.60	0.662	3.0
$\exp(\epsilon_v/k)$		8	1	7.75
$\rho_N + \rho_l$	cm^{-2}	10^9	10^9	10^9
$\dot{N}_p$	$\text{cm}^{-3} \cdot \text{s}^{-1}$	10^{15}	10^{15}	10^{15}

These curves exhibit a maximum point in temperature, T_{em} . This reflects the maximum swelling temperature. The value for T_{em} is calculated to be 550°C for stainless steel, 116.5°C for aluminum and 1100°C for molybdenum. These values agree with experimental results[10].

(4) The extreme value, T_{em} , results from the following equation,*

$$\frac{C_0}{2D_v(T_{em})(C_v^{eq}(T_{em})+B_0) \left[\left(1 + \frac{\epsilon_v^f}{\epsilon_v^m}\right) C_v^{eq}(T_{em}) + B_0 \right]} + 1 = \left(1 + \frac{C_0}{D_v(T_{em})[C_v^{eq}(T_{em})+B_0]}\right)^{1/2}, (41)$$

* This equation is derived using the definition $\frac{dA}{dT}|_{T=T_{em}} = 0$

where $C_0 = \dot{N}_p \frac{4a_0^2}{500g} \frac{Z_1}{Z_v}$, $R_c = \frac{Z_1 a_0^2}{500g} (\rho_N + \rho_l)$. T_{em} increases as $(\rho_N + \rho_l)$ increases. This relation explains the maximum swelling temperature increase in irradiated 20% cold-worked 316 stainless steel[11].

(5) The equilibrium concentration for vacancies, C_v^{eq} , is

$$C_v^{eq} = \frac{1}{g} \left(\exp \frac{s_v}{k} \right) \exp \left(-\frac{\epsilon_v^f}{kT} \right),$$

where s_v and ϵ_v^f are the entropy and formation energy for vacancies. For different types of stainless steel, there are different values for s_v and ϵ_v^f , and T_{em} is correspondingly different. As s_v decreases, T_{em} increases and as ϵ_v^f increases, T_{em} increases.

(6) As D becomes larger, the curves for the IFL distribution function spread; the number of large dislocation loops increases.

V. Conclusions:

(1) The results for interstitial-loop nucleation, growth, and swelling obtained by a non-equilibrium statistics method can reflect the characteristics of interstitial-loop nucleation and growth and swelling as observed by experiment. The results give an interstitial-loop distribution function, predict the incubation and growth period of swelling and the maximum swelling temperature, and predict the influence of a cold-worked state on the maximum swelling temperature.

(2) The maximum swelling temperature calculated for stainless steel is 550°C, for aluminium is 116.5°C, and for molybdenum is 1100°C. The values agree with experimental results. The maximum swelling temperature is found to be a function of s_v , $\frac{\epsilon_v^f}{\epsilon_v^m}$, $(\rho_N + \rho_l)$, and $\dot{N}_p$. The maximum swelling temperature increases

as $(\rho_N + \rho_I)$, $\dot{N}_p$ and $\frac{\epsilon_v^f}{\epsilon_v^m}$ increase and decrease as s_v increases (here ϵ_v^m is the mobile energy of vacancies).

(3) The irradiation re-solution parameter for IDL can play an important role in the evolution of IDL and swelling.

VI. Acknowledgements: The authors express their appreciation to Professor Monroe Wechsler and Dr. Robert Brown for helpful discussions.

Appendix I: Energy Associated with an Interstitial Dislocation Loop

The dislocation-loop energy consists of elastic energy, stacking fault energy, and the additional energy of the dislocation core. If the radius of a dislocation loop is large enough, the elastic energy is

$$\epsilon_P^D = \xi \cdot 2\pi G b_2^2 r = \xi \cdot \xi_p G b_2^2 \left(\frac{A_0}{\pi}\right)^{1/2} \sqrt{m}, \quad (A-1)$$

where m is the number of interstitials enclosed by the loop. In this condition, the stacking fault energy and the additional energy of the dislocation core can be neglected, and the elastic energy of the dislocation loop is approximately equal to the dislocation loop energy. This can be written as

$$\epsilon_P(m) = m\epsilon_1^f - f(m)B', \quad (A-2)$$

where ϵ_1^f is the formation energy of an interstitial atom B' is the binding energy between interstitials per pair and the interstitials in the close-packed lattice in the interstitial loop plane and $f(m)$ is the number of bond pairs between interstitials for a total of m atoms enclosed in the interstitial loop. The value of $\epsilon_e^P(m)$ should approach $\epsilon_P(m)$ if the number of interstitials enclosed in loop (m) is large enough. If we take $m > 100$, $\epsilon_e^P(m)$ approaches $\epsilon_P(m)$. ϵ_e^P is about

$$\epsilon_e^P = 320.0759\sqrt{m} \quad (\text{kJ/mole} \cdot m) \quad (A-3)$$

for stainless steel, where ϵ_1^f is 420 kJ/mole, $B'=144.62$ kJ/mole. If m is below 100, use of equation (A-3), shows that $\epsilon_e^P(m)$ is smaller than $\epsilon_P(m)$. The difference is about 13-35%. This is because in this condition the additional

energy of dislocation-core and stacking-fault energy cannot be neglected; they also contain a fraction of dislocation-loop energy. (A-3) is a reasonable approximation, as shown in the Fig. 5, and equation (A-3) is relatively convenient for use, while equation (A-2) is relatively complex for use. The disadvantage of (A-3) is that the elastic energy increases at a rate faster than in (A-2). If $m > 299$, ϵ_e^P is higher than ϵ^P ; the difference is about 10%. This is not reasonable. Therefore the use of equation (A-3) has some limitation's. Only in some ranges can equation (A-3) be used as an approximation.

The interstitial-loop-energy change rate is

$$\frac{d\epsilon_e^P}{dm} = 1.6584/\sqrt{m} \quad (\text{eV/atom}). \quad (\text{A-4})$$

If $m > 10^4$, equation (8') is correct; if $m < 10^4$, equation (8) is approximated as

$$\left(\frac{dr}{dt}\right)_l = A - A_0 Z_1 D_1 C_1^{\text{eq}} \exp \frac{1.6584(A_0/\pi)^{1/2}}{rkT} \quad (\text{A-5})$$

Then, the approximate solution of equation (4) is

$$N_l(r, t) = \left[\exp\left(\frac{A}{D}(r-r_0) - \frac{B^0}{D}(re^{E/r} - r_0 e^{E/r_0}) + \frac{B^0 E}{D}(Ei(E/r) - Ei(E/r_0)) - \frac{(r-r_0)^2}{2Dt}\right) \right. \\ \left. \left\{ \sqrt{2t/\pi D} - \frac{r-r_0}{D} (1 - \Phi(\sqrt{(r-r_0)^2/2Dt})) \exp \frac{(r-r_0)^2}{2Dt} \right. \right. \\ \left. \left. - t^2 \frac{b}{A_0} [(A_1 r + r_0 (A_2 - A_4)) + \sqrt{\pi/2Dt} ((\frac{A_2}{2\pi D} \frac{A_0}{b} + r_0^2) B_2 + 2r_0(r-r_0) B_1) \right] \right\} \right]$$

$$+ (r-r_0)^2 B_4 + \sqrt{\pi D t / 2 B_5}] - F \}, \quad (A-6)$$

where $B^0 = A_0 Z_1 D_1 C_1^{eq}$, $E = \frac{1.6584 (\frac{A_0}{\pi})^{1/2}}{kT}$, and F is

$$\frac{1}{\sqrt{2\pi D}} \int_0^t \frac{\sqrt{2\tau}}{\sqrt{\pi D(t-\tau)}} d\tau \int_R e^{E/x} \left(\frac{B^0 E}{2x^2} + \frac{B^0}{2D} e^{E/x} - \frac{AB^0}{D} \right) e^{-\mu(x-a)^2} dx -$$

$$- \int_0^t \frac{d\tau}{\sqrt{t-\tau}} \int_R \left[\frac{\pi b^2}{A_0} x^2 + \frac{B^0 E}{2x^2} e^{E/x} + \frac{1}{2D} (A - B^0 e^{E/x})^2 \right] \frac{x-r_0}{D} (1 - \Phi(\sqrt{(x-r_0)^2 / 2D\tau})) e^{-\frac{(r-x)^2}{2D(t-\tau)}} dx,$$

with $\mu = \frac{t}{2D\tau(t-\tau)}$, $a = r_0 + \frac{\tau}{t}(r-r_0)$.

Appendix II The Degenerate Hypergeometric Function

The $A_1, A_2, A_3, A_4; B_1, B_2, B_3, B_4, B_5$ are as follows:

$$A_1(\mu) = \int_0^{\infty} \frac{\sqrt{x}}{(1+x)^2} e^{-\mu x} dx = \Gamma\left(\frac{3}{2}\right) \Psi\left(\frac{3}{2}, \frac{1}{2}; \mu\right)$$

$$A_2(\mu) = \int_0^{\infty} \frac{\sqrt{x}}{(1+x)^3} e^{-\mu x} dx = \Gamma\left(\frac{3}{2}\right) \Psi\left(\frac{3}{2}, -\frac{1}{2}; \mu\right)$$

$$A_3(\mu) = \int_0^{\infty} \frac{x^{5/2}}{(1+x)^5} e^{-\mu x} dx = \Gamma\left(\frac{7}{2}\right) \Psi\left(\frac{7}{2}, -\frac{1}{2}; \mu\right)$$

$$A_4(\mu) = \int_0^{\infty} \frac{x^{1/2}}{(1+x)^5} e^{-\mu x} dx = \Gamma\left(\frac{3}{2}\right) \Psi\left(\frac{3}{2}, -\frac{5}{2}; \mu\right)$$

$$B_1(\mu) = \int_0^{\infty} \frac{dx}{1+x} (1 - \phi(\sqrt{\mu x}))$$

$$B_2(\mu) = \int_0^{\infty} \frac{x dx}{(1+x)^3} (1 - \phi(\sqrt{\mu x}))$$

$$B_3(\mu) = \int_0^{\infty} \frac{x^2 dx}{(1+x)^4} (1 - \phi(\sqrt{\mu x}))$$

$$B_4(\mu) = \int_0^{\infty} \frac{x^3 dx}{(1+x)^5} (1 - \phi(\sqrt{\mu x}))$$

$$B_5(\mu) = \int_0^{\infty} \frac{x^2 dx}{(1+x)^5} (1 - \phi(\sqrt{\mu x}))$$

where $\mu = \frac{(r-r_0)^2}{2Dt}$.

References

1. H. L. Heinisch, J. Nucl. Mater. 103 & 104 (1981) 1325-1330.
2. V. A. Coghlan and L. K. Mansur, J. Nucl. Mater. 122 & 123 (1984) 495-501.
3. H. Wiedersich, Radiat. Effect. 12 (1972) 111.
4. L. Garrido, et al., "Stochastic Processes in Nonequilibrium Systems" Springer-Verlag, Berlin, Heidelberg, New York (1978) pp. 9, 287.
5. D. A. Macquarrie "Statistical Mechanics," Harper & Row, Publishers, New York (1976) pp. 457.
6. Donald R. Olander, TID 26711(1976) pp. 209, 481, 486.
7. A. D. Brailsford and R. Bullough, J. Nucl. Mater. 48 (1973), pp. 87-106; 44 (1972) pp. 121-135.
8. J. O. Stiegler, Radiation-induced Voids in Metals, AEC Symposium 26. U. S. Atomic Energy Commission Office of Information Services (1972) pp. 292.
9. P. J. Maziasz, J. Nucl. Mater., 122 & 123 (1984) 236-241; 472-486.
10. S. D. Harkness and Che-Yu Li, Met. Trans. 2 (1971) 1457; H. R. Bager and J. L. Straalsund, J. Nucl. Mater. 46 (1973) 134-158; Y. Adda, Radiation-induced Void in Metals AEC Symposium 26; U. S. Atomic Energy Commission Office of Information Services (1972) pp. 31.
11. J. L. Straalsund, H. R. Bager & J. J. Holmes, Radiation-Induced Voids in Metals, AEC Symposium 26, U. S. Atomic Energy Commission Office of Information Services (1972) pp. 142.

12. Jinan Yu, W. F. Sommer and J. N. Bradbury, Radiation Re-resolution, Los Alamos National Laboratory Report, LA-UR- , April 1986.

Symbols

Interstitial Dislocation Loop Nucleation and Growth and Swelling Produced by High-Energy Cascades

	Units
$K_Q(r, t)$ - interstitial-loop growth function	m/s
$f_Q(r, t)$ - fluctuating function for interstitial-loop growth	m/s
$W_Q(r, t; r_0)$ - probability density function	
$N_Q(r, t)$ - interstitial-dislocation-loop distribution function	m^{-4}
$Q_I^0(r, t), Q_V^0(r, t)$ - interstitial-vacancy absorption rate by IDL	$m^{-3}s^{-1}$
D_I, D_V - interstitial-vacancy diffusion coefficient	m^2s^{-1}
Z_I, Z_V - interstitial-vacancy capture coefficient by loop	
C_I, C_V - interstitial-vacancy concentration	m^{-3}
C_I^{eq}, C_V^{eq} - interstitial-vacancy concentration at thermal equilibrium	m^{-3}
Z_{QI} - combinatorial number that includes the number of sites from which a di-interstitial can be formed in a single jump of one interstitial atom to an adjacent one	
a_0 - lattice atom distance	m
r_0 - IDL nucleus radius	m
b - probability per second of ejection of an interstitial atom from the IDL into matrix	s^{-1}
m_I - the number of interstitial's enclosed in IDL of radius r	
ϵ - internal energy for a faulted IDL of radius r	J
ϵ^P - internal energy for an unfaulted IDL of radius r	J
$\rho_Q(r, t)$ - IDL density in radius range $[r, r+dr]$, $2\pi r N_Q(r, t) dr$	m^{-2}
τ_d - energy per unit length of the dislocation comprising the periphery of loop	J/m
G - shear modulus of the material	Pa

b_1, b_2 - Burgers vector of faulted, unfaulted loop	m
γ_{sf} - stacking fault energy	J/m ²
ξ - parameter which adjusts the elastic energy of interstitial dislocation loop to suit the energy of interstitial dislocation loop for large r	
A_0 - area per atom on slip plane	m ²
Ω - matrix atomic volume	m ³
d_1 - distance between slip planes	m
A - interstitial dislocation loop growth rate induced by the supersaturation of irradiation-produced defects on growth	m/s
ρ_N - network dislocation density	m/m ³
$\dot{n}_{pi}^{th}, \dot{n}_{pv}^{th}$ - interstitial, vacancy production rate in thermal equilibrium	1/m ³ s
k_{iv} - rate constant for vacancy-interstitial recombination	m ³ /s
R_p, N_p - radius, concentration of coherent precipitate particle	m, m ⁻³
r_{cb} - bubble nucleus radius	m
r_o - IDL nucleus radius	m
$N(r,t)$ - bubble distribution function	1/m ⁴
p - bubble internal pressure	Pa
γ - bubble surface tension	J/m ²
$N(t)$ - total bubble concentration	1/m ³
$\langle r \rangle$ - average radius of bubble	m
$\dot{N}_p$ - Frenkel pairs (vacancy and interstitial) production rate	1/m ³ s
T_{em} - temperature at which maximum swelling occurs	K
s_v - vacancy formation entropy	eV/K
c_v^f - vacancy formation energy	eV
c_v^m - vacancy migration energy	eV
c_i^m - interstitial migration energy	eV

CAPTIONS

Figure 1: Flow diagram showing the interrelationship of the various processes that occur in metals during irradiation.

Figure 2: Interstitial-loop distribution. The parameter A is the interstitial dislocation-loop growth rate due to a supersaturation of interstitials induced by radiation and is constant for all four curves. The irradiation time, t, is different for each curve and is the time required for a loop nucleus of radius r_0 to grow to $5r_0$, $4r_0$, $3r_0$ and $2r_0$ due only to the effect of the supersaturation of interstitials. Also, C is the interstitial dislocation loop nucleation rate.

Figure 3: The swelling produced in a metal due to the interstitial dislocation-loop distribution shown in Figure 2.

Figure 4: The swelling relationship to temperature for three materials.

$$\Delta'_0 = A_0 \frac{4\pi \langle r \rangle N(t) / (\rho_N + \rho_Q)}{1 + 4\pi \langle r \rangle N(t) / (\rho_N + \rho_Q)} \frac{1}{2} \frac{Z_v}{Z_i} \left(\frac{Z_i}{Z_v} - 1 \right)$$

Figure 5: The energy associated with interstitial dislocation loops as a function of the number of atoms in the loop showing the error introduced by making the necessary assumption that the elastic energy approximates the total energy.

Figure 1: Flow diagram showing the interrelationship of the various processes that occur in metals during irradiation.

Figure 2: Interstitial-loop distribution. The parameter A is the interstitial dislocation-loop growth rate due to a supersaturation of interstitials induced by radiation and is constant for all four curves. The irradiation time, t , is different for each curve and is the time required for a loop nucleus of radius r_0 to grow to $5r_0$, $4r_0$, $3r_0$ and $2r_0$ due only to the effect of the supersaturation of interstitials. Also, C is the interstitial dislocation loop nucleation rate.

Figure 3: The swelling produced in a metal due to the interstitial dislocation loop distribution shown in Figure 2.

Figure 4: The swelling relationship to temperature for three materials.

$$A_0' = A_0 \frac{4\pi \langle r \rangle N(t) / (\rho_N + \rho_Q)}{1 + 4\pi \langle r \rangle N(t) / (\rho_N + \rho_Q)} \frac{1}{2} \frac{Z_v}{Z_1} \left(\frac{Z_1}{Z_v} - 1 \right)$$

Figure 5: The energy associated with interstitial dislocation loops as a function of the number of atoms in the loop showing the error introduced by making the necessary assumption that the elastic energy approximates the total energy.