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ION-ACOUSTIC DOUBLE LAYERS IN THE PRESENCE OF PLASMA SOURCE

BY

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Ion-Acoustic Double Layers in the Presence of Plasma Source

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ABSTRACT

Steady-state plasma turbulence and formation of negative potential spikes and double layers in the presence of ion acoustic instabilities have been studied by means of one-dimensional particle simulations in which velocities of a small fraction of electrons are replaced by the initial drifting Maxwellian at a constant rate. A steady state is found where negative potential spikes appear randomly in space and time giving rise to an anomalous resistivity much greater than previously found. Comparisons of the simulation results with laboratory and space plasmas are discussed.

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I. INTRODUCTION

One of the most interesting nonlinear manifestations associated with the current driven ion acoustic instability ($v_d < v_{te}$, $T_e \gg T_i$) is the formation of potential double layers in which a coherent nonlinear state is established following the development of the instability.^{1,2} Here v_d is the electron drift speed relative to the ions, and v_{te} is the thermal speed of electrons. T_e and T_i are temperatures of electrons and ions, respectively. This novel nonlinear state, first found by means of numerical simulations, was the subject of subsequent theoretical interpretations as a particular solution to time-independent Vlasov-Poisson system.^{3,4} It was found in the simulations that the current inhibition due to a negative potential spike plays a key role in the formation of a double layer and generation of anomalous resistivity. The enhanced resistivity, η , at the time of double layer formations reached $\eta/\eta_0 \approx 6 \times 10^{-4}$ where $\eta_0 = 4\pi/\omega_{pe}$, which is an order of magnitude greater than the resistivity associated with the one-dimensional ion acoustic instability.^{1,5}

The nonlinear state is, however, unstable with respect to the emission of ion acoustic solitons which carry the energy away from the double layer, so that the life time of double layers is rather short. Once the double layers decay away after the saturation of instability, anomalous resistivity vanishes and so does the plasma turbulence. This is a characteristic feature of initial value simulations in which no energy source is provided externally so that plasma turbulence and anomalous resistivity cannot be maintained in a steady state, and the instability evolves into a damped state.⁶

In this paper, a simulation technique is used to model source and sink of electrons in which velocities of a small fraction of electrons are replaced by an initial drifting Maxwellian.^{7,8} This models a physical process in which

new electrons with the initial drifting Maxwellian velocity distribution constantly replace the old electrons in order to maintain a steady state. The rate of the replacement can be adjusted according to a choice of input rate of new particles. This may also be considered to model ionization and recombination processes or loss of plasmas to an outside system. The replacement or recycling may be done either spatially uniformly across the system or locally corresponding to a closed plasma system where source and sink of a plasma may be located more or less uniformly in the system, or they are localized at one location of the system. This may be the case for the auroral field lines where new electrons enter the auroral zone from the ionosphere and leave toward the higher altitude. In this model, the total number of particles is unchanged which is relevant to a steady-state model. This model has been used successfully for the study of electrostatic ion cyclotron instabilities on auroral field lines.^{7,8} A similar model was also used to study a steady-state drift wave turbulence and anomalous diffusion in three-dimensional models.⁹

While the model is quite simple, it serves to enlighten several important nonlinear physics associated with ion acoustic instabilities. For example, the model reveals the importance of electro. quasilinear diffusion as a nonlinear saturation mechanism. Besides being technically straightforward, this model does not introduce unwanted noises and fluctuations associated with the plasma injection. The results of simulations are presented in the next section. Typically $m_i/m_e = 100$, $T_e/T_i = 20$ (initially), $n\lambda_e = 100$ and $\Delta = \lambda_e$ are used where Δ is the computational mesh size and λ_e is the electron Debye length. The rate of recycling, δ , is typically 10^{-3} , meaning that, at each time step, 0.1% of electrons will have new velocities randomly selected from the initial drifting Maxwellian. Most of the electrons,

therefore, will have new velocities in δ^{-1} time steps so that $\Delta t/\delta$ gives a measure of recycling time.

II. RESULTS OF SIMULATIONS

The first example we consider here is treated previously as the conventional initial value simulations.¹ Here we consider the same example using the new model with a plasma source and emphasize the difference of the results. System length $L = 1024 \lambda_e$, $v_d = 0.6 v_{te}$, $\omega_{pe} \Delta t = 0.25$ and $\delta = 10^{-3}$ are used with the periodic boundary conditions. Recycling was done uniformly across the system using a random number generator. Note the choice of $\delta = 10^{-3}$ means that in one thousand steps which is equal to $\omega_{pe} t = 250$, most of the electrons will have new velocities chosen from the initial drifting Maxwellian. Note the electrons drift $150 \lambda_e$ on the average during the recycling period.

Figure 1 shows electron density, n_e/n_0 , (a), potential $e\psi/T_e$, (b), ion density, n_i/n_0 , (c) and ion phase-space plot, (d), at $\omega_{pe} t = 900$ averaged over one plasma period. It is clearly seen that four negative potential spikes and double layers exist at this moment over $1024 \lambda_e$ system length. This should be compared with the previous initial value simulations where only one double layer formed in $1024 \lambda_e$ system length. Each double layer is similar to the previous case in its structure and amplitude. The distance between double layers is now much shorter than $1024 \lambda_e$, and in fact the closest distance can be as short as one hundred Debye length. When the distance between double layers is longer, the amplitude of double layer is large but the potential jump does not exceed T_e/e in agreement with the previous case.

Since there are four double layers in the system, one can expect the level of electric field and anomalous resistivity are considerably higher than

the previous case. Figure 2 shows the total electric field energy normalized by the initial electron kinetic energy. Compared with the previous initial value simulations for the same parameters, the saturation level is about 30% higher, and, more importantly, a high level of fluctuations is maintained after the initial growth; this is in sharp contrast to the initial value simulations where the level of fluctuations drops to thermal noise following the saturation of the instability.^{1,6} The high level of fluctuations in the present simulations is apparently maintained by the balance between the damping and the growth due to fresh electrons. In the absence of electron recycling, electric field energy starts decaying after the plateau formation on the electron velocity distribution due to the formations of a high-energy ion tail. With recycling, fresh electrons tend to restore the initial distribution contributing to the destabilization of the ion waves.

Figure 3 indicates the time evolution of the electron and ion currents normalized by the initial values measured in a laboratory frame in which ions were initially stationary. Note the behavior of the electron current is almost the same as the previous initial value simulations until $\omega_{pe} t = 1000$, indicating the initial decay due to ion acoustic instability until $\omega_{pe} t = 500$ followed by the more rapid dissipation of current corresponding to the formation of double layers. In the initial value simulations, the electron current held almost constant beyond $\omega_{pe} t = 1200$ indicating the presence of a superconducting phase which corresponds to a Bernstein-Greene-Kruskal type equilibrium. Because of the recycling of particles, the electron current increases beyond $\omega_{pe} t = 1000$ until $\omega_{pe} t = 1400$ where it again starts decaying, and the same pattern seems to repeat in time. This process has the following simple physical interpretation. When the initial double layers are formed, the resistivity drops to nearly zero at $\omega_{pe} t = 1000$, which causes the buildup

of the current due to recycling. When the electron velocity distribution restores the initial distribution sufficiently closely, the instability is triggered again causing the dissipation of the current at $\omega_{pe} t = 1400$. It seems reasonable that the similar pattern repeats itself until the system reaches a point where the ion acoustic instability is stable due to ion heating for a given electron drift.

In order to determine the anomalous resistivity, we shall use the gain in the ion current shown in Fig. 3. Note the ions gain momentum from electrons at roughly a constant rate in time indicating the turbulence is more or less stationary. Using the loss in the electron current is difficult in this model because of the presence of source and sink for the electron current. Assuming the gain in the ion momentum is equal to the loss in the electron momentum, the anomalous resistivity measured in the steady state for $\omega_{pet} > 500$ will be

$$\frac{\eta}{\eta_0} = 6 \times 10^{-4}, \quad (1)$$

which is much larger than the resistivity previously found.¹⁶ Note the instantaneous resistivity at the time of double layer formation, $\omega_{pe} t = 500 \sim 1000$, can be larger than the value given by Eq. (1).

Heating of the ions is shown in Fig. 4 along with the change of electron kinetic energy. Note the ion heating follows closely the change of the ion current shown in Fig. 3. The initial loss of electron energy is followed by the gain at $\omega_{pe} t = 1200$ due to the saturation of the instability so that the gain of the energy overcomes the loss to the electrons. Note the change of the electron energy is small, about one percent of the initial energy due to the recycling of electrons.

Ion heating is mostly in the formation of high energy tail and little in

the bulk distribution as shown in Fig. 5. High energy ions reaching up to $10 v_{ti}$ are seen in the direction of the electron drift. A smaller tail in the other direction is also seen and it is believed to be caused by the acceleration by the large, potential jump at the potential spikes.

Persistence of double layers can be seen from the electron density at $\omega_{pe} t = 1800$ shown in Fig. 6 where several density spikes are present. A close look for the time evolution of double layers reveals that each double layer undergoes growth and decay phase with the time scale of about $\omega_{pe} = 500$ which is equal to the transit time of an ion acoustic soliton to cross the double layer width.² The presence of randomly located double layers gives rise to anomalous resistivity whose value is more or less independent in time. It is observed that double layers can be newly-born as well as revived from an old decayed state. On the average, in a steady state, several double layers are seen on the system length of $1024 \lambda_e$.

The second example studied is a much shorter system with $L = 128 \lambda_e$ and the recycling rate $\delta = 2 \times 10^{-3}$. All the remaining parameters are the same as before. Since the system is much shorter, the evolution of the instability can be followed on a much longer time scale. In the previous initial value simulations, no formation of double layers were observed for the system length $L = 128 \lambda_e$; however, we expect a double layer in the present model. This is because the level of ion wave turbulence is significantly higher in the present model so that more ion trapping can take place.³

Figures 7(a)-7(f) show the instantaneous potential structure at different times. As the ion wave turbulence develops, a potential spike appears at $\omega_{pe} t = 1500$ shown in (a) which then propagates toward downstream while the amplitude decreases, (b), and increases again shown in (c). The propagation speed is subsonic, less than one-half of the sound speed. At $\omega_{pe} t = 4000$,

(d), negative potential profile disappeared, and it reappears in (e) at $\omega_{pe} t = 5500$ and propagates toward the upstream direction. It is clear that in a system of energy sink and source, ion wave turbulence and the potential structure exist in a steady state well beyond the initial saturation. The instability should saturate eventually when the ion heating is large enough so that the ion wave instability is no longer unstable. If we recycle the ions as well so that ions are cooled, then the saturation may not take place at all.

Figure 8 shows the ion phase space plot at $\omega_{pe} t = 1500$ when the initial potential spike is just formed. A hole is clearly seen at the location where the negative potential spike exists consistent with theory.⁴

Long time behavior of the current and the electric field energy are shown in Figs. 9 and 10 where both the current and the field energy are in a steady state. Initial dissipation of the current is recovered due to the recycling at $\omega_{pe} t = 3000$ which subsequently causes the ion wave instability. The anomalous resistivity associated with the instability dissipates the current again, and the whole cycle repeats in time. The period of the cycle depends on the time of recycling, $\Delta t/\delta$, and the diffusion rate of the electron distribution due to ion wave fluctuations.¹⁰

$$D = \frac{e^2}{m} \sum_k \frac{\gamma_k |E_k|^2}{(\omega_k - kv)^2 + \gamma_k^2} \quad (2)$$

It is interesting to note that the system oscillates between stable and unstable cycles and there is no steady state in which the growth rate balances with the damping rate so that the net growth rate stays zero. The observed

period of oscillation is a fraction of the recycling time, and the diffusion time is consistent with the physical picture discussed.

Associated with the ion wave instability is the steady-state anomalous resistivity and ion heating shown in Fig. 11. Both the gain of ion momentum from the electrons and the temperature of the ions increase in time at more or less a constant rate. Ion heating rate in this case is faster than the previous case, and this is believed to be due to the faster recycling rate so that the fluctuation level is also higher. Gain in the ion momentum is also 50% faster in the present example, and therefore the anomalous resistivity in this example is found approximately

$$\frac{\eta}{\eta_0} = 10^{-3} . \quad (3)$$

III. CONCLUSIONS

Steady-state anomalous resistivity and ion acoustic double layers have been studied by means of a one-dimensional particle simulation model in which velocities of a small fraction of particles are continuously replaced by an initial drifting Maxwellian. It is found that the model is capable of producing a long time steady-state ion wave turbulence in which the system repeats stable and unstable cycles with a period determined from the input rate of particles and diffusion rate. Both the heating rate and the anomalous resistivity in a steady state are found to be much larger than previously found.

Negative potential spikes and the associated potential double layers are found to be always accompanied with the ion wave turbulence. It is found that the potential spikes are generated randomly in time and space causing the electrons to lose the momentum to the ions. Potential spikes and double

layers should have profound effects on the nonlinear behavior of ion wave turbulence. The order of magnitude of the anomalous resistivity due to negative potential spikes may be estimated in the following manner. A particle streaming with a velocity v_d may be reflected by the negative potential barrier if

$$\frac{1}{2} m_e v_d^2 < e \phi, \quad (4)$$

where ϕ is the potential depth of the barrier. Otherwise the particle passes through the barrier, and little net change occurs for the particle velocity. Assuming there are N potential barriers in the length of L , the loss of momentum by reflection will be roughly $m_e v_d N$ for the length L . Anomalous collision frequency will then be obtained by equating the loss of momentum to the effective dissipation of the momentum $m_e v_d \nu_e L/v_d$, so that

$$\frac{\nu_e}{\omega_{pe}} \approx N \frac{v_d}{v_{te}} \frac{\lambda_e}{L} \quad (5)$$

for the reflected particles satisfying Eq. (4). Passing particles do not contribute to the resistivity. The macroscopic resistivity will be obtained by averaging Eq. (5) over the particle distribution which will reduce the value of resistivity from Eq. (5). Here we will simply write

$$\frac{\nu_e}{\omega_{pe}} = C N \frac{v_d}{v_{te}} \frac{\lambda_e}{L}, \quad (6)$$

where C is a constant which represents a contribution from reflected particle. Note the resistivity η is given by $\eta = m_e \nu_e / ne^2$ so that $\eta/\eta_0 = \nu_e/\omega_{pe}$ always holds. For the case of $L = 1024 \lambda_e$, Eq. (6) gives

$v_e/\omega_{pe} = 0.002C$ using $v_d/v_{te} = 0.6$, $\lambda_e/L = 1024$ and $N = 4$. From simulation measurements $v_e/\omega_{pe} = 6 \times 10^{-4}$ so that $C = 0.3$. Considering the amplitude of the two potential barriers at the left shown in Fig. 1 is small, $e\phi/T_e \approx 0.4$, $C = 0.3$ is a very reasonable number. It is important to note that the anomalous resistivity v_e increases with the number of double layers and also with the drift speed.

While there are a number of laboratory experiments^{11,12} and observations in space¹³ on the double layers, there are significant differences between the work reported here and the experiments. In the laboratory experiments, it appears that double layers are formed not as a result of ion acoustic instabilities but as a result of new equilibrium after the injection of a high energy beam into a plasma, and no instabilities appear to be involved during the process of double layer formation. The potential jump across the double layer is usually many thermal energy of background particles whereas in the simulation $e\phi/T_e \lesssim 1$ is always satisfied for $v_d < v_{te}$. It appears that the negative potential spikes and double layers discussed in this work are microscopic in nature and are significantly different from the conventional macroscopic double layers. It is, therefore, reasonable to call them ion acoustic double layers to distinguish them from the conventional double layers.^{14,15}

Finally, while the recycling of particles has been carried out uniformly across the system in the present simulations, the behavior of steady-state turbulence will remain the same even if the source of fresh electrons is localized. This is because the ion sound waves and double layers propagate at the sound speed, C_s , or less, so that fresh electrons continuously stream through the waves at the speed of $v_d > C_s$, and therefore the waves and double layers experience the fresh electrons.⁷ This has been confirmed for the case of ion cyclotron waves and is expected to be true for the ion sound waves.

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FIGURE CAPTIONS

Fig. 1. (a) Electron density, (b) electrostatic potential, (c) density, and (d) ion phase-space are shown at $t = 900 \omega_{pe}^{-1}$ for the system length $L = 1024 \lambda_e$.

Fig. 2. Electric field energy with time. Note the field energy stays constant after saturation and does not decay in time as in the case for the initial value simulation. $L = 1024 \lambda_e$.

Fig. 3. Electron and ion currents in time. Note the electron current oscillates after saturation while the ion current increases monotonically in time indicating the presence of steady-state anomalous resistivity. $L = 1024 \lambda_e$.

Fig. 4. Heating of ions and electrons. Electron temperature stays very close to the initial value due to recycling while the ions are heated monotonically in time. $L = 1024 \lambda_e$.

Fig. 5. Ion velocity distribution initially and at $t = 2000 \omega_{pe}^{-1}$. Note the high energy tail at $t = 2000 \omega_{pe}^{-1}$. Tail for the negative velocity is due to trapping.

Fig. 6. Electron density distribution at $t = 1800 \omega_{pe}^{-1}$ is shown to ensure the existence of density spikes at late times. It is observed that the spikes appear and disappear randomly in time and space.

Fig. 7. Potential structure at different times for a small system $L = 128 \lambda_e$. Note the potential structure moves around the plasma at a speed substantially less than the speed of sound. Note also the structure disappears and appears more or less randomly.

Fig. 8. Ion phase space plot at $t = 1500 \omega_{pe}^{-1}$ corresponding to Fig. 7(a). Notice the large hole corresponding to the negative potential structure. $L = 128 \lambda_e$.

Fig. 9. Temporal behavior of electron current. Note the oscillatory behavior of the current corresponding to stable-unstable cycles of ion wave instability. $L = 128 \lambda_e$.

Fig. 10. Electric field energy also indicates oscillations in time as the current oscillates. There is a small time lag between the current and the field. $L = 128 \lambda_e$.

Fig. 11. Heating of ions and generations of ion current due to anomalous resistivity. Note the rate of heating and current rise is almost constant in time suggesting the presence of stationary anomalous resistance in this model. $L = 128 \lambda_e$.

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FIG. 1

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Fig. 2

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Fig. 3

Fig. 4

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Fig. 5

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Fig. 6

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Fig. 7

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Fig. 8

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Fig. 10

Fig. 11