PPPL-2316

PPPL-2316

UC20-D

①

25

DR-1588-1

CENTRAL IGNITION SCENARIOS FOR TFTR

By

S.J. Zweben, M.H. Redi, and G. Bateman

MARCH 1986

PLASMA
PHYSICS
LABORATORY

PRINCETON UNIVERSITY
PRINCETON, NEW JERSEY

PREPARED FOR THE U.S. DEPARTMENT OF ENERGY,
UNDER CONTRACT DE-AC82-76-CED-3073.

EXTERNAL DISTRIBUTION IN ADDITION TO UC-20

Plasma Res Lab, Austr Nat'l Univ, AUSTRALIA
 Dr. Frank J. Paoloni, Univ of Wollongong, AUSTRALIA
 Prof. I.R. Jones, Flinders Univ., AUSTRALIA
 Prof. M.H. Brennan, Univ Sydney, AUSTRALIA
 Prof. F. Cap, Inst Theo Phys, AUSTRIA
 Prof. Frank Verheest, Inst theoretische, BELGIUM
 Dr. D. Rulumbo, Og XII Fusion Prog, BELGIUM
 Ecole Royale Militaire, Lab de Phys Plasmas, BELGIUM
 Dr. P.H. Sakanaka, Univ Estadual, BRAZIL
 Dr. C.R. James, Univ of Alberta, CANADA
 Prof. J. Teichmann, Univ of Montreal, CANADA
 Dr. H.M. Skarsgard, Univ of Saskatchewan, CANADA
 Prof. S.R. Greenivasan, University of Calgary, CANADA
 Prof. Tudor W. Johnston, INRS-Energie, CANADA
 Dr. Hannes Barnard, Univ British Columbia, CANADA
 Dr. M.P. Bachynski, MPB Technologies, Inc., CANADA
 Chalk River, Nucl Lab, CANADA
 Zhengwu Li, SW Inst Physics, CHINA
 Library, Tsing Hua University, CHINA
 Librarian, Institute of Physics, CHINA
 Inst Plasma Phys, Academia Sinica, CHINA
 Dr. Peter Lukac, Koreskeho Univ, CZECHOSLOVAKIA
 The Librarian, Culham Laboratory, ENGLAND
 Prof. Schatzman, Observatoire de Nice, FRANCE
 J. Radet, CEN-BP6, FRANCE
 AM Dupas Library, AM Dupas Library, FRANCE
 Dr. Tam Mui, Academy Bibliographic, HONG KONG
 Preprint Library, Cent Res Inst Phys, HUNGARY
 Dr. R.K. Chhajlani, Vikram Univ, INDIA
 Dr. G. Dasgupta, Saha Inst, INDIA
 Dr. P. Kaw, Physical Research Lab, INDIA
 Dr. Phillip Rosenau, Israel Inst Tech, ISRAEL
 Prof. S. Cuperman, Tel Aviv University, ISRAEL
 Prof. G. Rostagni, Univ Di Padova, ITALY
 Librarian, Int'l Ctr Theo Phys, ITALY
 Miss Clelia De Palo, Assoc EURATOM-ENEA, ITALY
 Biblioteca, del CNR EURATOM, ITALY
 Dr. H. Yamato, Toshiba Res & Dev, JAPAN
 Direc. Dept. Lg. Tokamak Dev. JAERI, JAPAN
 Prof. Nobuyuki Inoue, University of Tokyo, JAPAN
 Research Info Center, Nagoya University, JAPAN
 Prof. Kyoji Nishikawa, Univ of Hiroshima, JAPAN
 Prof. Sigeru Mori, JAERI, JAPAN
 Prof. S. Tanaka, Kyoto University, JAPAN
 Library, Kyoto University, JAPAN
 Prof. Ichiro Kawakami, Nihon Univ, JAPAN
 Prof. Satoshi Itoh, Kyushu University, JAPAN
 Dr. S.I. Choi, Adv. Inst Sci & Tech, KOREA
 Tech Info Division, KAERI, KOREA
 Bibliotheek, Pom-Inst Voor Plasma, NETHERLANDS
 Prof. A.S. Wiley, University of Waikato, NEW ZEALAND
 Prof. J.A.C. Cabral, Inst Superior Tecn, PORTUGAL
 Dr. Octavian Petrus, ALI OJZA University, ROMANIA
 Prof. M.A. Hellberg, University of Natal, SO AFRICA
 Dr. Johan de Villiers, Plasma Physics, Nucor, SO AFRICA
 Fusion Div. Library, JEN, SPAIN
 Prof. Hans Wilhelmson, Chalmers Univ Tech, SWEDEN
 Dr. Lennart Stenflo, University of UMEA, SWEDEN
 Library, Royal Inst Tech, SWEDEN
 Centre de Recherches, Ecole Polytech Fed, SWITZERLAND
 Dr. V.T. Tolok, Kharkov Phys Tech Ins, USSR
 Dr. D.D. Ryutov, Siberian Acad Sci, USSR
 Dr. G.A. Eliseev, Kurchatov Institute, USSR
 Dr. V.A. Glukhikh, Inst Electro-Physical, USSR
 Institute Gen. Physics, USSR
 Prof. T.J.M. Boyd, Univ College N Wales, WALES
 Dr. K. Schindler, Ruhr Universitat, W. GERMANY
 Nuclear Res Estab, Julich Ltd, W. GERMANY
 Librarian, Max-Planck Institut, W. GERMANY
 Bibliothek, Inst Plasmaforschung, W. GERMANY
 Prof. R.K. Janev, Inst Phys, YUGOSLAVIA

CENTRAL IGNITION SCENARIOS FOR TFTR

MASTER

S. J. Zweben, M. H. Redi, and G. Bateman
Plasma Physics Laboratory, Princeton University
Princeton, NJ 08544

PPPL--2316

DE86 007524

ABSTRACT

The possibility of obtaining ignition in TFTR by means of very centrally peaked density profiles is examined. It is shown that local central alpha heating can be made to exceed local central energy losses ("central ignition") under global conditions for which $Q \geq 1$. Time dependent 1-D transport simulations show that the normal global ignition requirements are substantially relaxed for plasmas with peaked density profiles.

DISCLAIMER

This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency thereof, nor any of their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.

DISTRIBUTION OF THIS DOCUMENT IS UNLIMITED

27

I. INTRODUCTION

Significant alpha particle heating is normally considered beyond the scope of TFTR, since at the nominal goal of $Q = 1$ (where Q is the ratio of fusion power to input power), the total alpha heating power in steady state is only about 20% of the total energy loss rate (since the alpha power is $\approx 20\%$ of the total fusion power). This global limitation, however, does not preclude the possibility that if very centrally peaked alpha heating profiles can be obtained, the local central power balance can become dominated by alpha heating and ignition conditions may be obtained, at least at the center of TFTR.

In fact, as will be shown below, it may be possible with sufficiently centrally peaked density profiles to create within TFTR a "central ignition" region, in which the local central alpha heating exceeds the local central losses, despite the fact that the global plasma conditions may be far from ignition (i.e., $Q \leq 1$). Furthermore, it will be shown using 1-D space-time BALDUR simulations that, once formed, such a central ignition condition can lead through a radially propagating "burn" to global ignition on time scales on the order of ≈ 1 sec, starting with global n_{Te} values up to a factor of three below those usually required for global ignition.

Crucial to obtaining such favorable plasma scenarios is the attainment of very centrally peaked alpha heating profiles. In the present context, this central peaking was simulated by forcing the density profile to become centrally peaked, e.g., with $n(0)/\bar{n} \approx 2.7$, either through an assumed inward particle pinch or an assumed "frozen" density profile. In practice, such density profile control is not easy to achieve, but might well be within the scope of specifically tailored pellet fueling. What we show is that if sufficiently peaked density profiles can be obtained, then the possibility for central ignition improves substantially.

Of course, the magnitude of alpha heating also depends upon the temperature profile, and hence on the assumed form for the thermal conductivity $\chi(r)$ together with the assumed sawtooth model. Unfortunately, no guidelines for specifying a $\chi_e(r)$ profile appropriate for these peaked $n(r)$, high beam power discharges have yet been well established, so that a "realistic" simulation of these scenarios is not possible here. Instead, what we have done at this stage is simply to choose some fairly standard parametric forms for the anomalous $\chi_e(r)$ and a relatively simple sawtooth model in order

to illustrate in a concrete way the basic effects of a density profile variation on the possibility of central ignition.

Previous work along these lines has already pointed out that relaxed ignition requirements can be obtained with peaked profiles [1-3], and the possibility of central ignition and a radially propagating burn has been discussed in a preliminary form [4,5]. However, up to now the standard modeling of TFTR for $Q = 1$ has been done using only relatively flat density profiles, with the consequence that, so far, alpha particle heating effects are predicted to be relatively small [6].

The present paper is mainly motivated by the recent advances in pellet fueling, which have already produced peaked density profiles $n(0)/\bar{n} \geq 2$ in TFTR [7] and D-III [8], and also by the increasing desirability of some near-term evaluation of diagnostics for alpha particle confinement and heating. Although the attainment of centrally alpha-heated discharges is already a main goal of JET [2], there is no reason, in principle, why such discharges cannot be obtained in TFTR as well, since alpha particles should be classically confined for $I \geq 2$ MA (at $R/a = 3$) and should heat on time scales of ≈ 0.5 sec for plasma densities of $n = 10^{14} \text{ cm}^{-3}$ and plasma temperatures $T_e = 10 \text{ keV}$.

The paper is organized as follows: in Sec. II, we discuss in a simplified way the requirements for creating ignition-like conditions in the center of TFTR; in Sec. III, we describe the specific transport assumptions used in 1-D space versus time BALDUR simulations of this phenomenon; in Sec. IV are the results for two BALDUR scenarios which show the effect of density profile shape on the approach to ignition, and in Sec. V is the Discussion and Summary.

II. SIMPLIFIED ESTIMATES

In this section, we describe in a simplified way the essential point of obtaining ignition through centrally peaked density profiles. More detailed results are obtained through 1-D BALDUR transport simulations, as described in the next section.

A. Central ignition

The simplest form of the balance between global plasma energy losses P_{loss} and total alpha particle heating P_α , i.e., $P_\alpha \approx P_{\text{loss}}$, results in a global ignition requirement of the form $\langle n \rangle \tau_E \approx 3 \times 10^{14} \text{ cm}^{-3} \text{ sec}$ at $\langle T_i \rangle \approx 10$

keV, where $\langle n \rangle$, $\langle T_i \rangle$, and τ_E are the global (i.e., volume-averaged) density, ion temperature, and energy confinement time. However, application of the same argument to the central region of a plasma (e.g., $r/a < 0.33$) leads to the conclusion that "central ignition" can be obtained within some minor radius $r = r_{ig}$ if the central plasma parameters $n(0)$, $T_i(0)$, and $\tau_E(0)$ satisfy the same conditions locally, i.e., $P_\alpha(0) = P_{loss}(0)$, or typically,

$$n(0) \tau_E(0) \approx 3 \times 10^{14} \text{ cm}^{-3} \text{ sec}, \quad T_i(0) \approx 10 \text{ keV}, \quad (1)$$

where we have written schematically $n(0)$, $T_i(0)$, and $\tau_E(0)$ for the typical electron density, ion temperature, and energy confinement time within the central region inside $r = r_{ig}$. Thus, central ignition, as defined above, is evidently easier to obtain than global ignition if $n(0) > \langle n \rangle$, $T_i(0) > \langle T_i \rangle$, and $\tau_E(0) > \tau_E$, the last relationship between central and global energy confinement times having been observed in recent experiments of TFTR and JET [9,10].

Of course, such central ignition will not necessarily lead to a globally self-sustaining plasma burn, since the region outside $r = r_{ig}$ is not specified by Eq. (1), and may well cool without auxiliary heating, thus leading eventually to increased losses through r_{ig} and to loss of central ignition. However, Eq. (1) does imply that at the least inside $r = r_{ig}$ fusion reactor-like conditions are obtained, which is of interest for alpha confinement and heating physics. It also implies the possibility that rapid self-heating of this central region may result in a propagating radial "burn" which can globally ignite the plasma even without any additional heating (see below).

More specifically, if we assume profiles of the form:

$$\begin{aligned} n(r) &= n(0) [1 - (r/a)^2]^6, \\ T_i(r) &= T_i(0) [1 - (r/a)^2]^Y, \end{aligned} \quad (2)$$

then the local alpha energy production rate in a thermonuclear plasma (50% D, 50%T) for $T_i = 10$ keV is given by:

$$P_\alpha(r) \approx 0.16 [n(r)/10^{14} \text{ cm}^{-3}]^2 [T_i(r)/10 \text{ keV}]^2 \quad [\text{watts/cm}^3]$$

$$\text{or } P_{\alpha}(r) = n^2 T^2 = P_{\alpha}(0) \left[1 - (r/a)^2 \right]^{2(\gamma+\delta)}, \quad (3)$$

therefore, $P_{\alpha}(r)$ is naturally very peaked and clearly the influence of local alpha production increases with $\gamma+\delta$. Since for $I \geq 2$ MA in TFTR we can assume (at least to a first approximation) that the alpha orbits are confined sufficiently well so that the alpha heating profile is similar to the alpha creation profile [11], we can calculate the central peaking factor for alpha heating, $P_{\alpha}(0)/\langle P_{\alpha} \rangle = 2(\gamma + \delta) + 1$, where $\langle P_{\alpha} \rangle$ is the volume-averaged alpha heating power, and also the fraction of the alpha heating within $r/a = 0.33$. This is done in Table I for various $\gamma + \delta$. The table shows that even for modestly peaked profiles, $\gamma + \delta = 4$, the central alpha heating is about ten times the volume-averaged alpha heating, and that about 65% of the total alpha heating occurs within $r/a = 0.33$. Thus, one would expect that central ignition might be obtained under conditions far from global ignition, i.e., at low Q.

B. Estimate of Q

The conventional figure of merit for TFTR is "Q," the ratio of the total fusion power P_{fus} (alphas plus neutrons) divided by the total external input power P_{ext} (principally from the neutral beams). An approximate Q value for a central ignition scenario using the profiles of Eq. (2) can be obtained by assuming $P_{\text{ext}} = 3\langle n \rangle \langle T \rangle / \tau_E$ in steady state (i.e., $\langle P_{\alpha} \rangle \ll P_{\text{ext}}$), $P_{\text{fus}} = 5 \langle P_{\alpha} \rangle$, $\langle P_{\alpha} \rangle = P_{\alpha}(0) / [2(\gamma + \delta) + 1]$, and $P_{\alpha}(0) = 3n(0) T(0) / \tau_E(0)$ (i.e., central ignition):

$$Q = \frac{5 [\gamma + \delta + 1]}{[2(\gamma + \delta) + 1]} \frac{\tau_E}{\tau_E(0)}. \quad (4)$$

Thus for peaked profiles $(\gamma + \delta) \gg 1$ and for $\tau_E(0) \gg \tau_E$, the global Q values at central ignition can be considerably smaller than the $Q = 5$ for the onset of global ignition for flat profiles $(\gamma + \delta = 0)$. For example, if $(\gamma + \delta) = 4$ and $\tau_E(0)/\tau_E = 2.8$, then $Q \approx 1$ at central ignition, at least at this level of approximation.

C. Beta

The central ignition condition (1) can be written in a slightly more general form using $P_{\alpha}(0)$ from Eq. (3) and $P_{\text{loss}}(0) = 3n(0)T(0)/\tau_E(0)$, assuming

$T = T_i = T_e$, as:

$$n(0) T_i(0) \tau_E(0) \geq 3 \times 10^{18} \text{ cm}^{-3} \text{ eV sec.} \quad (5)$$

Thus, the central energy confinement time determines the central $\beta(0)$ requirement for central ignition:

$$\beta(0) \sim 15\% \left[\frac{1 \text{ sec}}{\tau_E(0)} \right] \left[\frac{50 \text{ kG}}{B_T(0)} \right]^2. \quad (6)$$

For example, with the central ignition parameters of Eq. (1) for $B_T(0) \approx 50$ kG, $\beta(0) \approx 15\%$.

The volume-averaged $\langle \beta \rangle$ is related to central $\beta(0)$ for the profiles of Eq. (2) by:

$$\langle \beta \rangle = \frac{\beta(0)}{1 + \gamma + \delta}. \quad (7)$$

Thus, the volume-averaged beta for the typical central ignition parameters of Eq.(1) can be a reasonably small, e.g., $\langle \beta \rangle \approx 3\%$ if $\gamma + \delta = 4$.

D. Central ignition parameter

Generalizing slightly the central ignition condition for the case when $P_\alpha(0) > P_{\text{loss}}(0)$, we define the central ignition parameter $M_{ig}(0) \equiv P_\alpha(0)/P_{\text{loss}}(0)$ where again we indicate the central region inside r_{ig} schematically by "(0)"; hence, using Eq. (3) for $P_\alpha(0)$:

$$M_{ig}(0) \approx \frac{1}{3} \left[\frac{n(0)}{10^{14} \text{ cm}^{-3}} \right] \left[\frac{T_i(0)}{10 \text{ keV}} \right] \left[\frac{\tau_E(0)}{1 \text{ sec}} \right]. \quad (8)$$

For $M_{ig}(0) = 1$, the central region is marginally ignited, while for $M_{ig}(0) > 1$ the central region should self-heat and thermally run away (if $\tau_E(0)$ is constant in time) since $P_\alpha \sim T_i^2(0)$. Note also that for the profiles of Eq. (2), the relation between $M_{ig}(0)$ and the global ignition margin $\langle M_{ig} \rangle = P_\alpha/P_{\text{loss}}$ (inside $r_{ig} = a$) is:

$$M_{ig}(0) \approx \left[\frac{2(\gamma + \delta) + 1}{\gamma + \delta} \right] \frac{\tau_E(0)}{\tau_E} \langle M_{ig} \rangle. \quad (9)$$

Thus, the central ignition margin is larger than the global ignition by about a factor of two if $\tau_E(0) \approx \tau_E$ and $\gamma + \delta \gg 1$.

E. Central thermal runaway

Assuming for the moment that the central region inside r_{ig} is isolated from the exterior plasma region, we can write the central power balance as:

$$\frac{d}{dt} [3n(0)T(0)] = P_a(0) - \frac{3n(0)T(0)}{\tau_E(0)} . \quad (10)$$

Assuming $n(0)$ and $\tau_E(0)$ are independent of time and $P_a(0)$ is given by Eq. (3), this equation can be rewritten as:

$$\frac{dT(0)}{dt} = \frac{T(0)}{\tau_E(0)} [M_{ig}(0) - 1] . \quad (11)$$

If $M_{ig}^{init}(0)$ and $T^{init}(0)$ are the central ignition margin and central temperature at some initial time, then the initial rate of thermal runaway for the central region is approximately:

$$\frac{dT(0)}{dt} = \frac{T^{init}(0)}{\tau_E(0)} [M_{ig}^{init}(0) - 1] . \quad (12)$$

Therefore, if the central region remains isolated from the external region, its central temperature runs away on the time scale $\tau_{run}(0) \approx \tau_E(0)/(M_{ig}^{init}(0) - 1)$, e.g., $\tau_{run}(0) \approx \tau_E(0)$ for $M_{ig}^{init}(0) = 2$.

F. Transition to global ignition

The rate of energy loss from the central region inside the r_{ig} to the exterior region outside r_{ig} is (assuming dominantly conductive losses):

$$Q_{loss}(0) = \frac{3n(0)T(0)}{\tau_E(0)} V_{ig} , \quad (13)$$

where $V_{ig} = (\pi r_{ig}^2)(2\pi R)$ is the volume of the central ignition region. The rate of energy loss from the external region $r > r_{ig}$ to the wall is approximately

$$Q_{loss}(a) = \frac{3\langle n \rangle \langle T \rangle}{\tau_E} V , \quad (14)$$

where V is the total plasma volume ($V \gg V_{ig}$), and τ_E is the global energy confinement time. The central energy losses to the external region can equal the external region's losses to the wall when:

$$\frac{n(0)}{\langle n \rangle} \frac{T(0)}{\langle T \rangle} \frac{\tau_E}{\tau_E(0)} = \frac{a^2}{2 r_{ig}} \quad (15)$$

At this point a "burning" central region can feed enough energy into an unburning external region to maintain the energy in the external region, assuming τ_E and $\tau_E(0)$ are constant in time.

For the density and temperature profiles of Eq. (2), this is equivalent to:

$$(\delta + 1)(\gamma + 1) = \left(\frac{a}{r_{ig}}\right)^2 \frac{\tau_E}{\tau_E(0)} \quad (16)$$

for such a balance. With $\delta + \gamma = 4$ and $r_{ig}/a = 0.33$, this requires $\tau_E(0) = (1-2) \tau_E$.

G. Radial burn propagation

If $Q_{loss}(0) > Q_{loss}(a)$, the external region will also heat if τ_E is constant in time, thus increasing the ignition radius r_{ig} . The possibility of a radial burn propagation, whereby $r_{ig} \rightarrow a$ due to heating of the unignited external region by the burning central ignition region, is evidently dependent on the details of the 1-D profiles and, so, cannot be evaluated in any simplified way. However, a straightforward test for such a radial burn can be done using a 1-D versus time transport code, as described in Sec. IV.D: Once a central ignition region is formed, the external auxiliary heating power can be turned off, and then either the external losses overcome the central thermal runaway and r_{ig} decreases with time, or else the central thermal runaway overcomes the external losses and r_{ig} increases with time until $r_{ig} = a$, and the plasma is globally ignited.

Thus, the intuitive idea here is to create in the tokamak a situation analogous to the usual way one burns a piece of wood or gasoline; namely, first a small piece is ignited, then this piece heats adjacent fuel to the ignition temperature, and then the burning region propagates spontaneously. Some evidence for behavior similar to this is seen in the 1-D BALDUR transport simulations in Sec. IV. C.

III. TRANSPORT SIMULATION ASSUMPTIONS

Two specific experimental scenarios for two types of auxiliary heating have been considered for 1-D versus time simulations for TFTR. The first scenario assumes adiabatic compression of a 1500-kA, 70-cm-radius TFTR plasma which has been heated with 27 MW of perpendicular neutral injection for either 0.330 sec or 0.530 sec. The second scenario assumes 27 MW of perpendicular neutral beam heating of an uncompressed full-bore (83 cm), high current (2200 kA) TFTR plasma for up to 2 sec. These two scenarios were designed to fit the potential hardware capabilities of TFTR and are intended to reflect typical rather than optimized assumptions.

The BALDUR transport code has been used for these simulation studies. BALDUR [12] is a multifluid one-dimensional transport code which solves coupled diffusion equations for particle density, energy density, and poloidal magnetic field. It includes the nonlinear effects of ohmic heating (using neoclassical resistivity), neutral beam heating and fueling, adiabatic compression, sawtooth mixing, radiation and neutral losses due to charge exchange and recombination, and classical alpha particle confinement and heating.

For both scenarios, the particle and energy balance equations are:

$$\frac{\partial n_a}{\partial t} = \frac{1}{r} \frac{\partial}{\partial r} \left(r D_a \frac{\partial n_a}{\partial r} + r n_a V_a \right) + S_a \quad a = d, t \quad (17)$$

$$\frac{3}{2} \frac{\partial n_j T_j}{\partial t} = \frac{1}{r} \frac{\partial}{\partial r} \left[r n_j \chi_j \frac{dT_j}{dr} \right] + \frac{3}{2} \left(T_j D_j r \frac{\partial n_j}{\partial r} + r n_j v_j T_j \right) + P_j \quad j=i, e. \quad (18)$$

The transport coefficients D , V , and χ are specified by assumed parametric forms, as described below, and the particle source function S is specified by standard neutral gas penetration and neutral beam penetration models. Note that pellet fueling is not explicitly included in the present modelling. The energy source functions P_j are specified by standard ohmic, neutral beam, and adiabatic compression heating models.

The ion heat conduction coefficient χ_i is taken to be three times the neoclassical Chang-Hinton value. In most cases, this choice resulted in an ion contribution to the total power balance which was small compared to the anomalous χ_e assumed below.

Most of the results were obtained with one of two different models for the electron heat conduction coefficient

$$\chi_e(r) = \chi_e(0) [1 + 4 (r/a)^2] , \quad (19)$$

or

$$\chi_e(r) = \chi_e^* [n_0/n_e(r)] , \quad (20)$$

where in the first case the central $\chi_e(0)$ was varied from 2×10^3 cm²/sec to 2×10^4 cm²/sec, and where in the second case the coefficient χ_e^* was varied from 2×10^3 cm²/sec to 2×10^4 cm²/sec, with $n_0 = 3 \times 10^{14}$ cm⁻³. The first choice (19) was motivated by early TFTR beam heating results which showed the anomalous $\chi_e(r)$ to be an increasing function of r [13], while the second choice (20) is similar to the INTOR scaling (but only partially consistent with results from neutral-beam-heated tokamak experiments [14]). Although these two choices result in differing $\chi_e(0)/\chi_e(a)$ and, consequently, differing electron temperature profiles, particularly for the peaked density profile cases, the qualitative results with respect to central ignition are similar, as discussed in Sec. IV.

We have chosen two ad hoc methods of parameterizing the density profile shape; unfortunately, neither of these corresponds to a physically realizable profile control scheme (such as pellet fueling). In particular, the maintenance of extremely peaked density profiles via cold neutral gas puffing (plus some beam-particle fueling) requires some unrealistic assumptions for D and/or V , since such peaked profiles are not normally observed without pellet injection. However, the intent here is to show that if such density profiles can be achieved (i.e., through pellets), then the possibility of ignition in TFTR is substantially improved.

For the first transport particle model, which we will call the "pinch" model, the assumptions were:

$$D(r) = [0.4 + 1.6(r/a)^2] 10^3 \text{ cm}^2/\text{sec} , \quad (21)$$

and

$$V(r) = D(r) \frac{d}{dr} [\ln (n_e^{\text{init}})] , \quad (22)$$

where here $D(r) = 1/5 \chi_e$ (as evaluated at $\chi_e(0) = 2 \times 10^3$ cm²/sec) and $V(r)$ is an anomalous inward pinch velocity designed to maintain an initially chosen density profile; i.e., if $n_e^{\text{init}} = n(0)[1 - (r/a)^2]^\delta$, then V/D is proportional to δ and so an initially peaked profile with large δ is supported by a large

inward pinch velocity. The initial condition here is a full-current ohmically heated plasma, and the particle source function S [Eq. (3)] is cold neutral gas puffing (i.e., edge fueling) plus beam-particle fueling. Typical values for $V(a/2) = \delta D(a/2) \approx 10^3$ cm/sec for $\delta=1$ and $V(a/2) \approx 5 \times 10^3$ cm/sec for $\delta = 5$. Thus, D is in the range of the observed $D \approx 2 \times 10^3$ cm/sec for TFTR, but the V assumed here can be much higher than the observed $V \sim 10^2 - 10^3$ cm/sec in most tokamaks.

The second particle transport model avoids the need to specify specific transport coefficients by arbitrarily "freezing" an initial density profile. This is done by assuming

$$D = V = 0, \quad (23)$$

and also no recycling at the limiter and no gas puffing. Of course, this "frozen" particle assumption is not meant to be realistic, but rather to check the results obtained with the first particle model, wherever possible.

The simulations were run with an initial (i.e., ohmically heated) density profile of $n(r) = n(0)[1 - (r/a)^2]^\delta$, where δ varied for different runs from 1 to 2 to 5, while keeping the initial line-averaged density the same for all δ . For various δ , the ratio of central to line-averaged plasma density ranged from $n(0)/\bar{n} = 1.5$ (for $\delta = 1$) to $n(0)/\bar{n} = 2.7$ (for $\delta = 5$). These initial density profiles were maintained approximately constant versus time through the pinch assumptions [Eqs. (21) and (22)], or through the frozen particle assumption [Eq. (23)]. Beam fueling and sawtoothing modify these profiles in some cases, as discussed in Sec. IV.

The initial temperature profiles were chosen to be rather broad, i.e., $T(r) = T(0)[1 - (r/a)^2]^{4/3}$ for all δ , with $T_i(0) = T_e(0) = 1$ keV. The subsequent temperature evolution was governed by the energy balance equation in the usual way through Eq. (4). The edge boundary conditions were chosen to be $T_e(a) = T_i(a) = 100$ eV and $n(a) = 2.5-5 \times 10^{12}$ cm $^{-3}$, i.e., $n(r) = n(0)[1 - (r/a)^2]^\delta + n(a)$ where $n(a)/n(0) \approx 10^{-2}$ (see also Sec. IV.E.6). We have assumed for simplicity $Z_{\text{eff}} = 1$ throughout, i.e., no impurity radiation was present in any of these runs.

Sawtoothing in these simulations was handled by averaging $n(r)$ and $T(r)$ within an inversion radius near $q = 1$ with an arbitrarily set periodicity (typically 30 msec). The inversion radius is found from the Kadomtsev

poloidal flux function. It was found that if $q(a)$ was too low, e.g., $q(a) \leq 2.5$, the sawtooth significantly reduced the density peaking effect otherwise built into this model; therefore, in all of the simulations, we chose $q(a) \sim 3.0$ with consequently minor sawtooth effects on the alpha particle heating profile (Sec. IV.E.3).

Alpha confinement was calculated from the neoclassical orbit model, which predicts excellent central alpha confinement for the conditions used in these simulations, i.e., $I(MA) \times (R/a) \approx 6.7$. Alpha heating was modelled using the simplifying assumption that the alphas thermalize classically on the flux surface of their birth; this was checked to be negligibly different from results obtained using the more elaborate finite gyroradius options in BALDUR[11].

IV. TRANSPORT SIMULATION RESULTS

The main variable for these simulations was the density profile shape parameter δ , where $n(r) = n(0) [1 - (r/a)^2]^\delta$. The main outputs from these simulations were the alpha heating profiles $P_\alpha(r,t)$, the ion temperature profiles $T_i(r,t)$, and the radial power balance results which determine whether central ignition has been obtained. In general, these results show that increasingly peaked $n(r)$ profiles produce increased central alpha heating, increased central ignition margins, and an increased likelihood of spontaneous propagation toward a self-sustaining global plasma burn. In parts IV.A and IV.B, the relative effects of density profile shape on central ignition are described for the two basic plasma scenarios; in part IV.C, some "frozen" particle model results are compared to the "pinch" model results of Sec. IV.A and IV.B; in part IV.D, the time dependences of central ignition and radial burn are described, in part IV.E is a discussion of the sensitivity of these results to input modelling choices, and part IV.F describes how more realistic modelling of central ignition in TFTR can be approached.

A. Compression scenarios

In Table II-A are listed the basic machine parameters used for the compression scenarios of central ignition. In all cases, an initial ohmically heated plasma with $R = 279$ cm, $a = 70$ cm, $I = 1500$ kA, $\bar{n}_e = 1.2 \times 10^{14} \text{ cm}^{-3}$, 50%D/50%T, and $T_i(0) = T_e(0) \approx 1$ keV is first run for 0.1 sec, then heated by perpendicular neutral beam injection (21.6 MW at 120 keV, 4 MW at 60 keV, 1.4

MW at 40 keV] for either 0.3 or 0.5 sec, and then adiabatically compressed by a factor $C = 1.24$ over the next 30 ns. The beams are turned off immediately after compression in order to look for the development of global ignition over the next ≈ 0.5 sec.

Some typical results for this compression scenario with the first of the two χ_e models [Eq. (19)] are shown in Figs. 1-3. In Fig. 1 are the density, ion temperature, alpha power, and neutral beam power profiles for the three density profile shapes $\delta = 1, 2$, and 5, all for $\chi_e(0) = 2 \times 10^3$ cm²/sec (and otherwise identical assumptions). In Fig. 2 are the time dependences of the central values $n(0)$, $T(0)$, and the volume-averaged $\langle \beta \rangle$ for these same cases, while in Fig. 3 are examples of $T(0)$ versus time for three different $\chi_e(0)$ assumptions for $\delta = 5$ only. Note that the neutral beam is on from 0.1 to 0.43 seconds and compression is on from 0.40 to 0.43 sec for all these cases, and also that the "pinch" particle model assumptions are used for all runs in this section.

A typical result as illustrated in Fig. 1 is that the most peaked $\delta = 5$ case shows global ignition, i.e., $T_i(0)$ increases after beam turnoff, whereas the least peaked $\delta = 1$ case fails to globally ignite, i.e., $T_i(0)$ falls rapidly after beam turnoff. The intermediate $\delta = 2$ case is marginally globally ignited, with $T_i(0)$ after beam turnoff increasing between sawtooth "crashes," but decreasing slowly over ~ 1 sec. These differences are essentially due to the increased central alpha heating power at the higher $n(0)$ for the peaked $n(r)$ cases, e.g., $P_\alpha(0) = 0.46, 0.93$, and 3.3 watts/cm³ just after compression for $\delta = 1, 2$, and 5, respectively, compared with $P_{\text{beam}} \leq 2$ watts/cm², a trend which is at least qualitatively consistent with the $n(0)$ dependence of the simplified estimates described in Sec. II.

The other general characteristic of these results is the sensitivity to the magnitude of $\chi_e(0)$, as illustrated in Fig. 3 for the $\delta = 5$ cases. Evidently, there is a maximum $\chi_e(0)$ which allows global ignition for a given density profile, a trend which is also qualitatively consistent with the τ_E requirement for ignition as discussed in the simplified estimates.

These two influences of δ and $\chi_e(0)$ on the simulation results are shown more explicitly in Tables III and IV. In Table III-A are the central ignition margins $M_{ig}(a/3)$ just after compression for these three δ and three $\chi_e(0)$ choices, and in Table IV-A are the corresponding central ignition rates $dT_i(0)/dt$ for cases which were at or close to global ignition, as measured

during the interval 0.1 to 0.4 sec after neutral beam turnoff (the beam power is negligible 0.1 sec after beam turnoff). Note that the central ignition margin here is defined as $P_a(a/3)/P_{loss}(a/3)$ as evaluated between sawtooth "crashes" (the sawteeth are relatively small, with a radius of $\leq a/3$ in these simulations).

Table III-A shows the general trend for the central ignition margin to increase with the density profile peaking factor δ at fixed $\chi_e(0)$; for example, with $\chi_e(0) = 6 \times 10^3$ cm²/sec, $M_{ig}(a/3) = 0.13, 0.34$, and 2.8 for $\delta = 1, 2$, and 5 , respectively. Table IV-A shows that the cases with $M_{ig}(a/3) > 1$ were also those cases at (or close to) global ignition, and also that the rate of global ignition $dT_i(0)/dt$ increases with the magnitude of $M_{ig}(a/3)$.

Results analogous to those just described, but for the second $\chi_e(r)$ model [Eq. (20)] are shown in Figs. 4-5, here for $\chi_e^* = 2 \times 10^3$ cm²/sec. The main difference between these two sets of results is in the shapes of the calculated $T_e(r)$ profiles, which are sensitive to $\chi_e(0)/\chi_e(a)$ and so are more peaked for the second χ_e model, particularly for $\delta > 1$. This is illustrated in Fig. 6, which shows the $T_e(r)$ profiles just after compression for the $\delta = 2$, $\chi_e(0) = \chi_e^* = 2 \times 10^3$ cm²/sec cases. The second χ_e model has a more peaked $T_e(r)$ profile, corresponding to $\chi_e(0)/\chi_e(a) \approx 50$, compared to the relatively flat electron temperature profile resulting from the first χ_e model.

Otherwise, the relative dependences of $M_{ig}(a/3)$ and $dT_i(0)/dt$ on δ and χ_e are similar to those for the first χ_e model, as seen in Tables III-B and IV-B. For example, for $\chi_e^* = 2 \times 10^3$ cm²/sec, $M_{ig}(a/3) = 0.47, 0.96$, and 1.9 for $\delta = 1, 2$, and 5 , respectively. The most peaked case $\delta = 5$ globally ignited while the less peaked cases did not. Note that since for the second model the χ_e averaged over radius is typically higher for $\chi_e(0) = \chi_e^*$, the neutral beams were kept on for 0.53 sec for the second model instead of the 0.33 sec for the first χ_e model.

In summary, these compression scenario results are generally consistent with the simplified arguments of Sec. II, which predicted improved central and global ignition prospects with peaked density profiles. A more quantitative comparison of these simulation results with the simplified estimates is made in the discussion of Sec. V.A.

B. Full-bore scenarios

In Table II-B are listed the basic machine parameters used for these

full-bore scenarios of TFTR. In all cases, an initially ohmically heated plasma with $R = 255$ cm, $a = 83$ cm, $I = 2200$ kA, $\bar{n}_e = 0.5 \times 10^{14}$ cm $^{-3}$, 50ZD/50ZT and $T_i(0) = T_e(0) = 1$ keV is first run for 0.1 sec, and then heated by 27 MW of perpendicular neutral beam injection (21.6 MW at 120 keV, 4 MW at 60 keV, 14 MW at 40 keV). For the standard cases, the beams are turned off at 2 sec in order to look for development of global ignition over the next 0.5 sec; runs with shorter beam pulses are discussed in Sec. IV.D.

In general, these full-bore runs ignited more easily than compression runs at the same δ and $\chi_e(r)$. This improvement is mainly due to the larger plasma radius (χ_e is assumed to be independent of the normalized radius, hence $\tau_E \sim a^2$), and to the increased beam pulse length (note $dT_i(0)/dt > 0$ at the end of the beam pulse in the compression cases). The other main difference between the full-bore and compression runs is the increased importance of beam-particle fueling for the full-bore cases, due simply to the longer beam pulse. Note also that as for Sec. IV.A all runs in this section assumed the "pinch" particle model.

In Figs. 7-9 are some typical results for the full-bore scenario with the first of the two χ_e models [Eq. (19)], here with $\chi_e(0) = 2 \times 10^4$ cm 2 /sec. The cases with $\delta = 1$ and $\delta = 2$ both showed a decrease in $T_i(0)$ after 0.7 sec; however, the $\delta = 5$ case shows an increase in $T_i(0)$ until 2 sec, at which point the plasma globally ignites after beam turnoff. The decreases in $T_i(0)$ for $\delta=1$ and 2 are due to the increasing $n(0)$ versus time (mainly from beam fueling), while the increasing $T_i(0)$ for $\delta = 5$ is due to the onset of significant alpha heating, i.e., $P_\alpha(0) = 0.23, 0.50$, and 1.6 watts/cm 3 for $\delta = 1, 2$, and 5 , respectively, at $t = 0.65$ sec, compared with $P_{\text{beam}}(0) = 2.5$ watts/cm 3 at this time. As was the case for the compression scenarios, the trend for increased central alpha heating for more peaked density profiles is at least qualitatively consistent with the simplified estimates in Sec. II.

The dependence of these results on the magnitude of $\chi_e(0)$ is illustrated in Fig. 9. As was the case for the compression runs shown in Fig. 5, there is evidently a maximum $\chi_e(0)$ which allows global ignition for a given density profile; for these examples with $\delta=2$, a plasma with $\chi_e(0) \geq 6 \times 10^3$ cm 2 /sec is marginally globally ignited.

The dependence of $M_{ig}(a/3)$ and central ignition rates on δ and $\chi_e(0)$ are shown for these full-bore runs in Tables V-A and VI-A. The relative trends are similar to those seen for the compression runs in Tables III-A and IV-A;

namely, peaked density profiles result in increased central ignition margins for a given $\chi_e(r)$, and the $M_{ig}(a/3) \geq 1$ cases are those at or close to global ignition with $dT_i(0)/dt > 0$ after beam turnoff. Note that all the results in Tables V and VI are evaluated at the fixed beam turnoff time of 2 sec; however, in some cases the plasma was actually ignited well before this time, as discussed in Sec. IV.D.

Results analogous to those just described but for the second $\chi_e(r)$ model are shown in Figs. 10 and 11, here for $\chi_e^* = 2 \times 10^3 \text{ cm}^2/\text{sec}$. In these examples with very low central χ_e , both the $\delta = 2$ and $\delta = 5$ cases globally ignite after 2 sec while the $\delta = 1$ case does not. Note that although the exact location of the central ignition boundary $M_{ig}(0) = 1$ in δ vs. χ_e space for the second χ_e model in Table V-B is different from that for the first χ_e model in Table V-A, the relative results are similar in that the central ignition margin for a fixed χ_e increases with δ (and for a fixed δ decreases with χ_e), just as it did in all previous cases. Also, note that the boundary between globally ignited and nonignited cases as shown in Table VI-B also follows the trend observed previously that global ignition correlates approximately with $M_{ig}(a/3) = 1$.

In summary, the results described so far have shown that peaked density profiles increase the central ignition margins, and that for the most peaked profiles central ignition with $M_{ig}(a/3) \geq 1$ can be obtained at central $\chi_e(0) = 2 \times 10^3 - 2 \times 10^4 \text{ cm}^2$, the exact value depending on the particular χ_e and heating model assumed. More details concerning the sensitivity of these results to the particular modelling choices are given in Sec. IV.D, and a discussion of the extent to which these scenarios are realistic for TFTR is given in Sec. IV.F and also in Sec. V.B

C. Frozen particle model

Several runs were made which were identical to those discussed in the IV-A and B except that the "frozen" particle model was used [Eq. (23)] instead of the "pinch" model [Eq. (21-22)]. The results of these two models were approximately the same for those cases in which they both succeeded in maintaining the initially peaked $n(r)$ profile.

For example, typical results for the compression scenario showed that for $\chi_e(0) = 2 \times 10^3 \text{ cm}^2/\text{sec}$ [with the first χ_e model] the pinch (frozen) model resulted in central ion temperatures at beam turnoff of $T_i(0) = 10.5$ (11.8)

keV for $\delta=5$, $T_i(0) = 9.2$ (10.0) keV for $\delta=2$, and $T_i(0) = 7.9$ (8.2) keV for $\delta=1$. For $\delta=5$ both pinch and frozen model cases ignited after beam turnoff, while for $\delta=1$ both pinch and frozen model cases failed to ignite. For $\delta=2$, however, the pinch model case marginally failed to ignite globally, with $dT_i(0)/dt \approx -1.7$ keV/sec after beam turnoff, while the frozen model case was marginally globally ignited with $dT_i(0)/dt \approx +2.3$ keV/sec. This difference is due to the sum of several small effects, the main one for $r \leq 0.5$ a being an approximately 2 MW reduction in heat conduction losses in the frozen case due to a slightly flatter temperature profile.

Several full-bore scenarios were also run using the frozen particle model. A typical result with $\chi_e(0) = 6 \times 10^3$ cm²/sec [first χ_e model] and $\delta=5$ showed that both models ignited after beam turnoff at 2 sec, but the pinch model ignited considerably faster than the frozen model. This difference was due to the predominance of beam-particle fueling for these long beam pulse length scenarios, for which the pinch model successfully maintained the initially peaked $n(r)$ profile despite off-axis beam fueling, while the frozen particle model did not. The broadening of the $n(r)$ profile in the latter case caused the frozen model simulations to ignite less rapidly, consistent with the general results of the previous section that broadened $n(r)$ profiles reduce ignition margins.

We conclude that the pinch model is more suitable for maintaining an initially peaked density profile than the frozen particle model, but that both models give similar results in cases where they both succeed in maintaining the desired $n(r)$ profile. Of course, a more realistic model would include explicit pellet fueling with appropriate D and V terms (see Sec. IV.F).

D. Time development of central ignition and radial "burn"

The results described so far have concentrated on the relative effects of density profile shape and the magnitude of χ_e on the eventual development of central and/or global ignition. In this section, the time development of central alpha heating, central ignition, and the "burn" toward global ignition are described in more detail for a few typical cases.

Figure 12(a) shows the central energy balance versus time for a compression case with $\delta = 5$ and $\chi_e(0) = 6 \times 10^3$ cm²/sec. The dominant central energy term (i.e., within $r/a \leq 0.33$ a) before compression is neutral beam heating, such that $Q_\alpha/Q_{\text{beam}} \approx 0.2$ at about 0.4 sec (here Q refers to the

volume-integrated power). However, after compression the central ion temperature and density have risen considerably, so that by ≈ 0.45 sec the alpha heating power exceeds that total energy loss Q_{loss} at $r/a = 0.33$, and so the plasma is centrally ignited. The subsequent time evolution shows the central Q_α increasing approximately linearly due to thermal runaway (note that $Q_{\text{beam}} \approx 0$ after 0.5 sec), but with Q_{loss} also increasing approximately linearly versus time. This increased loss rate is due to changes in dT/dr at $r/a = 0.33$ which lead to increasing electron heat conduction losses $Q_{e,\text{cond}}$ ($\Delta Q_{e,\text{cond}} \approx + 2.6$ MW between 0.5 and 1 sec) and ion heat conduction loss ($\Delta Q_{i,\text{cond}} \approx + 1.7$ MW between 0.5 and 1 sec). Thus, the central thermal runaway, which was begun by central ignition, leads to more centrally peaked temperature profiles (see Figs. 1b and 7b) and so to increased central conduction losses, a process which self-consistently slows down (but does not stop) the process of central thermal runaway.

Another example of this process is shown in Fig. 12(b), which is a full-bore case with $\delta = 5$ and $x_e(0) = 2 \times 10^4$ cm²/sec. Here the central alpha heating Q_α increases gradually in time until central ignition with $Q_\alpha = Q_{\text{loss}}$ occurs within $r_{\text{ig}} = 0.33$ a at ≈ 1.2 sec. Subsequently, Q_α increases rapidly versus time but so does Q_{loss} , again due mainly to the increased heat conduction losses caused by increasingly centrally peaked temperature profiles [see Fig. 7(b)]. Thus, the central thermal runaway rate is apparently restrained by increased losses from the center to the exterior region outside r_{ig} , a process which naturally tends to reduce the central ignition margin $M_{\text{ig}}(0)$ and to increase the global ignition margin $\langle M_{\text{ig}} \rangle$.

In Fig. 13(a) and (b) are plots showing the ignition margins $M_{\text{ig}}(0)$ and $\langle M_{\text{ig}} \rangle$ versus time for these two cases, again evaluated for $r = 0.33$ a. For the compression case in Fig. 13(a) the central ignition margin increases to $M_{\text{ig}}(0) \approx 3$ just after compression, indicating that central alpha heating has increased significantly before temperature profile changes increased Q_{loss} ; however, between 0.45 and 0.6 sec $M_{\text{ig}}(0)$ decreased to ≈ 1.5 due to these losses. On the other hand, the global ignition margin increases after compression, mainly due to heating by these same central losses, such that $\langle M_{\text{ig}} \rangle \approx 0.4$ at 0.4 sec but $\langle M_{\text{ig}} \rangle \approx 1.5$ at $t = 0.7$ sec. Subsequent evolution shows $\langle M_{\text{ig}} \rangle > M_{\text{ig}}(0)$, at which time global ignition is established.

Another example of this comparison of the time evolution of central and global ignition is shown in Fig. 13(b), which plots $M_{\text{ig}}(0)$ and $\langle M_{\text{ig}} \rangle$ for the

full-bore case in Fig. 12(b). Here $M_{ig}(0)$ inside $r = 0.33a$ increases slowly in time until central ignition occurs at ≈ 1.2 sec, at which time $\langle M_{ig} \rangle \approx 0.6$. Subsequently, the increase in $M_{ig}(0)$ is restrained by increased central losses, which contribute to increased $\langle M_{ig} \rangle$, such that global ignition $\langle M_{ig} \rangle = 1$ is obtained at ≈ 1.75 sec.

In comparison with these particular $\delta=5$ compression cases, in Fig. 13(c) and 13(d) are shown simulations exactly the same as 13(a) and 13(b) but with a less centrally peaked density profile, i.e., $\delta = 2$. Neither of these cases showed central ignition within $r = 0.33a$, and neither globally ignited. At the other extreme, a strongly ignited full-bore case (for the lowest $\chi_e(0)$ with $\delta = 5$) is shown in Fig. 13(e); again, the central ignition margin $M_{ig}(0)$ is followed closely in time by the global ignition margin $\langle M_{ig} \rangle$. Therefore, taken together the results of Figs. 12 and 13 suggest that central ignition leads to global ignition if the central ignition radius is in the range $r_{ig} = 0.33a$ for these cases.

Note further that the choice of radius $r=0.33a$ for evaluating $M_{ig}(0)$ was arbitrary and, in fact, r_{ig} develops in time as shown in Fig. 14. In 14(a) are shown r_{ig} versus time (inside of which $M_{ig}(r_{ig}) = 1$) for several cases which eventually globally ignited, including the two discussed in Figs. 12(a) and (b). In each case r_{ig} is seen to increase gradually from $r_{ig} = 0$ to $r_{ig} = a$ in the time scale ≤ 1 sec. Thus, the transition from central ignition to global ignition proceeds in two ways: the central ignition margin within a fixed radius increases in time [Fig. 13(a),(b)], but also the radius within which $M_{ig}(r_{ig}) = 1$ increases in time until $r_{ig} = a$ at global ignition, i.e., $\langle M_{ig} \rangle = 1$ (Fig. 14a). This latter process is qualitatively similar to the radially propagating burn as discussed in Sec. II, since the "burning" region inside r_{ig} is defined by the net alpha heating condition $M_{ig}(r_{ig}) \geq 1$.

Note incidentally that the rate of increase of r_{ig} is correlated with increased density profile peakedness δ ; for example, for the full-bore cases with $\delta = 5$ and $\chi_e(0) = 2 \times 10^4$ cm²/sec, r_{ig} increases from $r_{ig} = 0.33a$ to $r_{ig} = a$ in 0.8 sec, whereas for the $\delta = 2$ case r_{ig} increases from $r_{ig} = 0.33a$ to only $r_{ig} \approx 0.7a$ in 0.8 sec. Also, the rate of increase in r_{ig} is correlated with decreased $\chi_e(0)$, as can be seen by comparing the $\delta = 2$ cases in this figure with $\chi_e(0) = 6 \times 10^3$ cm²/sec and 2×10^3 cm²/sec.

Also shown in Fig. 14(b) are cases which had only transient regions of central ignition but which did not evolve into global ignition. This occurs

if δ is too small for a given $\chi_e(0)$, or if χ_e is too large for a given δ . Note that the case with very slowly decreasing r_{ig} ($\delta = 2$, $\chi_e(0) = 2 \times 10^3$ cm²/sec, with compression) was the sub-marginally ignited case described in Sec. IV-A, which was marginally centrally ignited between sawtooth crashes.

In order to see more clearly the time development of this type of radial burn, several full-bore $\delta=5$ simulations were run in which the duration of the usual 1.9-sec long neutral beam pulse was gradually reduced to the point where global ignition was not obtained. With such shorter beam pulse durations the time development of central ignition and the radial burn can be seen without the complication of simultaneous neutral beam heating. Some results from these runs are shown in Fig. 15 for the case $\chi_e(0) = 2 \times 10^4$ cm²/sec which was discussed previously in Figs. 12(b) and 13(b).

Figure 15 shows that for this case global ignition with $dT_i(0)/dt > 0$ after beam turnoff is obtained for a beam turnoff time of ≥ 1.3 sec, but not for ≤ 1.15 sec. The time development of the ignition radius for these cases is shown in Fig. 16(a). Note that at 1.3 sec $r_{ig} = 0.4$ a, which is consistent with the suggestion from Figs. 12 and 13 that $r_{ig} = 0.4$ a is sufficient for the development of global ignition with this χ_e model. The spontaneous radial burn from $r_{ig} = 0.4$ a to $r_{ig} = a$ can be seen to occur within ≈ 0.5 sec after beam turnoff at 1.3 sec.

Another example of this radial burn process is shown in Fig. 16(b), where $\chi_e^* = 2 \times 10^3$ cm²/sec is used with the second $\chi_e(r) \sim 1/n(r)$ model instead of the fixed $\chi_e(r)$ model used for the examples above. In this case the central ignition radius required for a radial burn is slightly larger than that for the first $\chi_e(r)$ model, i.e., a central ignition radius of $r_{ig} = 0.45$ a is obtained without the eventual development of global ignition. This is evidently due to the relatively larger loss rates in the exterior plasma regions implicit in the second χ_e model, given these peaked $n(r)$ profiles.

In summary, these examples of the time development of central ignition have shown that central ignition can develop into global ignition when the central ignition radius is large enough that the central thermal energy losses can sufficiently "feed" the nonignited region exterior to r_{ig} to allow r_{ig} to propagate radially. This process was seen to occur either during or after neutral beam heating. The latter case, in which central ignition spontaneously propagates in a radial burn without auxiliary heating, represents a minimal route to global ignition, as discussed in Sec. V.B.

E. Sensitivity to modelling choices

There are many choices possible for each of the parameters involved with heating, fueling, and transport in a plasma simulation. In this section the typical sensitivity of these results to a few of these choices are illustrated. Note that for most of these sensitivity checks the first of the χ_e models was chosen [Eq.(19)].

1. Beam pulse length

The relatively short beam pulse length of 0.33 sec for the first set of compression runs [Figs. 1-3] was chosen originally for comparison with previous $Q=1$ modelling of TFTR. However, since $dT_i(0)/dt$ is still increasing after 0.33 sec of beams in these simulations (Fig. 1), it is clear that longer beam pulses would further increase the likelihood of central ignition.

Several compression simulations were run with a longer beam pulse length of 0.53 sec instead of the .33 sec used previously for the cases shown in Figs. 1-3 [$\delta=2$, $\chi_e(0) = 2 \times 10^3$ cm²/sec]. The result was that the 0.53 sec beam cases had higher $T_i(0)$ just before compression, e.g., for the $\delta=2$ case $T_i(0)=9.4$ keV instead of 6.4 keV, and so these longer beam cases naturally ignited more readily. Also, the globally unignited $\delta=2$ case with 0.33 sec beams globally ignited with 0.53 sec beams, and the $\delta=5$, $\chi_e(0)=2 \times 10^4$ cm²/sec case, which was not even centrally ignited with 0.33 sec beams (see Table III-A), was close to global ignition with $M_{ig}(a/3) \approx 1$ with 0.53 sec beams. Note that even after 0.53 sec of beams $T_i(0)$ was still rising; thus the results of Sec. IV.A for the compression cases significantly underestimate the central ignition margins possible with long-pulse beams in TFTR.

The 2-sec-long beam pulse used for most of the full-bore runs was chosen to fit the maximum hardware capability of TFTR. However, in many cases, global ignition could be reached with shorter beam pulses; for example, Fig. 15 showed that for $\delta=5$ and $\chi_e(0) = 2 \times 10^4$ cm²/sec a 1.3-sec beam pulse is sufficient for central ignition to develop into global ignition. Furthermore, even shorter beam pulses are enough to obtain significant alpha heating within $r = 0.33$ a, e.g., for this same case, $P_\alpha = 11$ MW and $M_{ig}(0) = 0.75$ for a 1-sec beam turnoff and $P_\alpha = 6$ MW and $M_{ig}(a/3) = 0.5$ for a 0.7-sec beam turnoff.

The short beam pulse cases also show most clearly the minimum requirements for central ignition and radial burn. Some of these cases are discussed in more detail in Sec. IV.F.

2. D/T species mix

The initial 50%D/50%T gas fill used for these simulations is not optimal when significant deuterium beam fueling occurs, as is the case for all of the 2-sec-long full-bore cases, despite the fact that the code automatically adjusts the gas puffing mix to try to maintain the initial plasma D/T mix. For example, an initial species mix of 50%D/50%T becomes 79%D/21%T after 2 sec of beams for the $\delta=5$, $\chi_e(0) = 2 \times 10^4 \text{ cm}^2/\text{sec}$ case discussed in Sec. IV.B.

Additional simulations were therefore made using this same case, but with an initial fill mix of 25%D/75%T or 1%D/99%T in order to optimize the thermonuclear reaction rate. The resulting central ion temperatures were higher for the higher tritium fill percentages; i.e., at 2 sec, $T_i(0) = 18.3 \text{ keV}$, 19.8 keV , and 20.4 keV for 50%, 75%, and 99% initial tritium fill, respectively (all at approximately the same central density of $6.5 \times 10^{14} \text{ cm}^{-3}$). Incidentally, with 100% D and zero alpha heating power, the corresponding ion temperature was $T_i(0) = 8.5 \text{ keV}$ at 2 sec.

Of course, the optimal tritium fill will ultimately depend upon the actual particle confinement physics, which is not realistically modelled in these simulations. However, these results do show that the ignition margins for the full-bore cases of Sec. IV.B. are underestimated somewhat due to nonoptimal D/T mixes.

3. Sawtoothing

In choosing the baseline simulation parameters an attempt was made to keep the sawtoothing radius as small as possible by minimizing the plasma current for a given B_T , consistent with the usual requirement for classical alpha confinement, $I(\text{MA}) \times R/a \gtrsim 6$, with the result that $q(a) \approx 3$ for all cases described so far. The sawtoothing radius was then determined from the initial temperature and current profiles (note that these simulations were all started at full current), and from the self-consistent time evolution of the electron temperature profile and associated neoclassical resistivity. In most cases, the resulting sawtoothing radius in the simulations was either very small or even nonexistent, with typically $q = 1$ at $r \leq 0.17 a$, so that the resulting sawtoothing effects were negligible. For example, in the compression case with $\chi_e(0) = 2 \times 10^3 \text{ cm}^2/\text{sec}$ and $\delta=5$ for which the sawtoothing can be seen to flatten out the alpha heating profile to approximately $r/a=0.25$ [Fig. 1(c)], a simulation without sawtoothing had a

time-averaged $T_i(0)=9.9$ keV instead of the $T_i(0)=9.8$ keV for the sawtooth case.

However, it is by no means clear that the actual sawtooth in full-beam power TFTR discharges will be at such a benign level, since unusually large sawteeth already have been observed after compression, and since the effect of alpha heating itself on sawtooth remains to be seen. If large sawtooth mixing regions do exist, they will directly affect $n(0)$, $T(0)$, and $\tau_E(0)$ and so potentially could strongly inhibit the course of central ignition.

4. χ_i^{NC} multiplier

The effect of ion neoclassical heat conduction on these scenarios is generally small, even with the standard multiplying factor of 3 used in all cases (see Sec. II). One run was made with a multiplier of 1 (full-bore, $\chi_e(0) = 2 \times 10^4$, $\delta=5$, pinch), with the result that the mean ion temperature at 2 sec increased from $\langle T_i \rangle = 18.6$ keV with $3 \times \chi_i^{NC}$ to $\langle T_i \rangle = 18.9$ keV with $1 \times \chi_i^{NC}$. Note, however, that if χ_i was chosen to be anomalous according to the possibility that $\chi_i = \chi_e$, then the effect of χ_i can be significant. For example, the same full-bore simulation was run with $\chi_i(r) = \chi_e(r) = 2 \times 10^4$ cm²/sec $[1+(r/a)^2]$ (i.e., similar to the case shown in Fig. 7 except for the χ_i model); this anomalous χ_i case reached only $T_i(0) = 6$ keV after 2 sec instead of the $T_i(0) = 18.6$ keV reached with the standard ion thermal conduction model (three times ion neoclassical).

5. χ_e radial profile

The shape of the assumed $\chi_e(r)$ profile is important in that it determines the shape of the temperature profiles and also the ratio of the central to global energy confinement times which, as noted in Sec. II, is important in determining the ratio of central to global ignition margins. However, for the two forms of $\chi_e(r)$ used in Secs. IV.A and IV.B, the results with respect to central ignition were qualitatively similar, in that the central ignition margins increased with peaked density profiles and for smaller χ_e .

To see more clearly how the shape of $\chi_e(r)$ affected these results, a set of full-bore, $\delta=5$ simulations was made varying only the shape of the $\chi_e(r)$ profile with fixed $\chi_e(0)$, i.e., $\chi_e(r) = 2 \times 10^4$ cm²/sec $[1 + \lambda(r/a)^2]$ where $\lambda = 0, 1.5, 4$ (the standard choice for this model) and 9. The resulting central ion temperatures after 1.9 sec of beams were $T_i(0) = 32, 30, 18$, and 6 keV for

these four cases, respectively. The first three cases globally ignited after beam turnoff, but the last case did not. Evidently, such changes in χ_e , even though they are mainly in the exterior region $r \gtrsim a/2$, do affect the development of $T_i(0)$ and central ignition. This occurs through changes in dT_e/dr and $Q_e(r)$ caused by the changing exterior χ_e ; for example, in these cases Q_e at $r = 0.33 a$ at 600 msec varied from 5.7 MW for $\lambda = 0$ to 6.6 MW for $\lambda = 1.5$, 7.4 MW for $\lambda = 4$, and 9.2 MW for $\lambda = 9$.

In order to optimize the possibility of central ignition at low values of Q and/or low values of the global ignition margin, one would like to maximize the central $\tau_E(0)$, i.e., minimize the central $\chi_e(0)$, as described in Sec II. The extent to which these simulations do this is described in Sec. IV.F.

6. Exterior density

The magnitude of the exterior density $r \gtrsim a/2$ affects the electron heat flow similarly to the exterior χ_e , since $Q_e \sim n\chi_e$. Thus, one expects that for a given $n(0)$, the exterior $n(r)$ profile also affects central temperatures and central ignition. This is a significant area of sensitivity for these simulations, since $n(r)$ profiles having quite low exterior density were implicit in the parameterization $n(r) = n(0) [1 - (r/a)^2]^\delta + n(a)$, where in the cases discussed so far $n(a) = 2.5 (5) \times 10^{12} \text{ cm}^{-3}$ for the full-bore (compression) profiles.

A set of simulations was run to see the effect of the choice of $n(a)$ on a full-bore, fairly strongly ignited case with $\delta=5$ and $\chi_e(0) = 6 \times 10^3 \text{ cm}^2/\text{sec}$. In these cases the edge density could be increased to $n(a) = 2 \times 10^{13} \text{ cm}^{-3}$ without losing ignition; however, $T_i(0)$ does decrease from $T_i(0) = 15.7 \text{ keV}$ at 600 msec for $n(a) = 2.5 \times 10^{12} \text{ cm}^{-3}$ to $T_i(0) = 11.1 \text{ keV}$ for $n(a) = 1 \times 10^{13} \text{ cm}^{-3}$ and to $T_i(0) = 7.6 \text{ keV}$ for $n(a) = 2 \times 10^{13} \text{ cm}^{-3}$, indicating a significant sensitivity to the exterior $n(r)$ profile within the context of this model. However, in these cases the density profile as a whole also changes with $n(a)$ through the inward pinch process, such that at $n(0)/\bar{n} = 2.5$, 2.2, and 1.9 at 600 msec for $n(a) = 0.25$, 1, and $2 \times 10^{13} \text{ cm}^{-3}$, respectively. Thus this apparent $n(a)$ sensitivity is associated in part to a broadening of the density profile as well as to a local increasing of the edge density.

F. Approaches to more realistic TFTR simulations

The simulation results described here have had several unrealistic aspects: the assumed density profiles were maintained by an ad hoc pinch model, the assumed $\chi_e(r)$ profiles were specified independently of the time-evolving plasma parameters, the constraints imposed by MHD stability were not explicitly applied (i.e., beta and density limits), and the impurity radiation losses were absent. These simplifications are appropriate for preliminary explorations of the possibility of central ignition in TFTR, but eventually more realistic modelling choices need to be made. However, at present the appropriate pellet scenarios, scaling laws, operational limits, and impurity levels are not well known, so further more realistic modelling must be deferred.

On the other hand, a rough assessment of the degree to which the present results are realistic for TFTR can come from a search for those cases which showed central ignition for the minimum required values of τ_E , $\bar{n}$, $\langle\beta\rangle$, and Q , the projected values of which are fairly well known. Several of these "optimized" cases are shown in Table VII.

The first two cases "A" and "B" in Table VII are compression and full-bore simulations using the first of the χ_e models [Eq. (19)], the second two cases "C" and "D" are compression and full-bore simulations using the second χ_e model [Eq. (20)], and the last case "E" is a full-bore simulation using a modified form of the second χ_e scenario in which the shape of $\chi_e(r)$ is multiplied by (r/a) to increase the central energy confinement relative to the global energy confinement time. Note that these cases have varying beam durations of 0.5-1.2 sec and that for all but one of them the most peaked $\delta = 5$ density profile was assumed.

The central ignition margins for the first four cases were all $M_{ig}(a/3) = 1$ at $r/a = 0.33$, indicating central ignition according to the convention previously used for Tables III and V. These cases all have a relatively high global τ_E in the range 0.6-0.9 sec, which at present seems somewhat unrealistically large for high neutral beam power TFTR plasmas. However, note that the ratio of central to global energy confinement times for cases A-D are in the range $\tau_E(a/3)/\tau_E = 1.0-1.1$ for the first χ_e model and 1.1-1.6 for the second χ_e model, thus missing the potential advantages of relatively high central energy confinement time as discussed in Sec. II. The last case "E" with a modified χ_e profile resulted in a relatively low global energy

confinement time of $\tau_E = 0.46$ sec for this case with $M_{ig}(a/3) = 0.7$ and $M_{ig}(a/6) = 1$.

In general, the results shown in Table VII suggest that these central ignition scenarios are marginally realistic for TFTR in terms of the required confinement times, densities, betas, and Q's. This is discussed somewhat more quantitatively in the next section.

V. DISCUSSION

In Sec. V.A is a comparison between the BALDUR simulation results presented in Sec. IV and the simplified estimates discussed in Sec. II, and in Sec. V.B is a brief discussion of the constraints which at present appear to impede the attainment of central ignition in TFTR. The last section (V.C.) is a summary.

A. Comparison of simulation results with simplified estimates

The simplified estimates of central ignition outlined in Sec. II argued that: (i) central peaking of the density profile helps to create a central ignition region with $M_{ig}(0) > 1$, (ii) with peaked profiles central ignition can be reached at relatively low global Q and beta, and (iii) such a central ignition region, once formed, can lead to a radially propagating burn and to global ignition. The BALDUR simulation results, which included more realistic plasma heating and loss physics as discussed in Sec. IV, have confirmed at least qualitatively these simple estimates: in Sec. IV.A and IV.B it was shown that the central ignition margin increases with peaked density profiles, all else being equal; in Table VII the Q and $\langle \beta \rangle$ values for some specific central ignition scenarios were shown to be reasonably small, and in Sec. IV.D the development of a radial burn was shown explicitly for some specific cases.

To examine the first point more quantitatively, refer to Tables III and IV, which show the central ignition margins for simulations which differ only in the shapes of the density profiles (at constant line-averaged density). The improvements in $M_{ig}(a/3)$ are typically a factor of two for the variation from $\delta = 1$ to $\delta = 2$, and another factor of two from $\delta = 2$ to $\delta = 5$, at least for cases with $M_{ig}(a/3) \approx 1$. Also, the improvement with a density profile change of $\delta = 1$ to $\delta = 5$ allows central ignition at typically a factor of three higher x_e . In terms of the simplified estimates, one would expect that an increased $n(0)$ due to peaked profiles would linearly increase the central

ignition margin, at least at fixed $T_i(0)$ and fixed $\tau_E(0)$, and proportionally decrease the requirement for $x_E(0)$ at a fixed $T_i(0)$; therefore the simulated increases in $M_{ig}(0)$ are somewhat larger than expected, since $n(0)$ increases by only a factor of two between $\delta = 1$ and $\delta = 5$. This discrepancy is most likely due to the increases in $T_i(0)$ near central ignition due to alpha heating, which were not accounted for in the simplified estimates.

To further examine the central ignition requirements more quantitatively, refer to Table VII, which contains a list of simulation cases for which central ignition was obtained at "minimal" values of τ_E , Q , $\langle\beta\rangle$. Note that the simulation parameters were evaluated at (or very shortly after) beam turn-off, after which the simulated temperatures gradually fell with time, indicating submarginal global ignition, as shown for example in Fig. 17 for case (D).

From Table VII, it can be seen that the central ignition margins for these cases were approximately $M_{ig}(a/3) \approx 1$, suggesting that the simplified estimate for the central ignition condition [Eq. (5)] should be approximately satisfied. In fact, the simulated central figure of merit $n(0)T_i(0)\tau_E(a/2)$ ranges from $2.3 - 5.0 \times 10^{18} \text{ cm}^{-3} \text{ eV sec}$ for these cases, which is approximately equivalent to the simplified requirement $n(0)T_i(0)\tau_E(0) \geq 3 \times 10^{18} \text{ cm}^{-3} \text{ eV sec}$ from Eq. (5).

With regard to the second point, the associated Q values as calculated at beam turnoff (normalized to the full-beam power) of $Q = 1.5-3.8$ agree roughly with the $Q = 1.5-2.5$ predicted from Eq. (4) for the $\gamma + \delta \gg 1$ and $\tau_E(0) = \tau_E$ of these cases. Although no simulations with central ignition at $Q < 1$ were obtained, in principle this could be achieved if one were to allow a higher τ_E , or a $\tau_E(0) \gg \tau_E$, i.e., through a $x_e(r)$ profile which is even more hollow than that of case E.

The $\langle\beta\rangle$ values obtained just after beam turnoff were $\langle\beta\rangle = 2.2-4.1\%$ with central $\beta(0) = 3-6\%$ $\langle\beta\rangle = 9-17\%$. These are again roughly consistent with the expected $\beta(0) = 15\%$ at $\tau_E(0) = 1 \text{ sec}$ and $\langle\beta\rangle = \beta(0)/(\gamma + \delta + 1) = 3\%$ anticipated from the simplified arguments described by Eqs. (6) and (7) (note that the compression cases have lower β values in part due to the higher toroidal field for the compressed plasma).

With regard to the third point, the time scales for central thermal runaway as estimated by Eqs. (10-12) can be seen for example in Fig. 15, where the temperature approximately doubles within one second after central ignition

has been reached (1.3-2.3 sec). Since the typical central ignition margin for this case is ≈ 1.5 during this time [see Fig. 13(b)], and since the central energy confinement time for this case is typically ≈ 1 sec (see Table VII), this result is approximately consistent with the simplified estimate of a central thermal runaway time of ≈ 2 sec from Eq. (12). However, in general, the thermal runaway rate does not simply increase proportionally to $[M_{ig}(a/3) - 1]$, as implied by Eq. (10), but rather shows a balance whereby central thermal runaway "feeds" global energy losses more similarly to that expressed in Eqs. (14-16). For example, the centrally ignited case shown in Fig. 17 and in Table VII(D) approximately satisfies this balance condition whereby central energy losses support global losses (i.e., Eq. (16) is approximately satisfied with $\delta = 5$, $\gamma = 1/2$, $(a/r_{ig})^2 = 10$, and $\tau_E(0)/\tau_E = 1$). Therefore, although these simulation results show approximate agreement with the simplified estimates of these time scales in Sec. II, they are not really adequate for quantitative estimates of the detailed 1-D versus time development of these scenarios, since they assumed a central region which is decoupled from the exterior region.

In short, we conclude that the simplified arguments on central ignition in Sec. II are at least qualitatively and in many respects quantitatively consistent with the BALDUR simulation results. Some of these simplified arguments will be used in the next section to discuss the potential for realization of central ignition in TFTR.

B. Realization in TFTR

Since TFTR can confine alpha particles at $I > 2$ MA and thermalize them over ≈ 0.5 sec, there is no reason in principle why such central ignition scenarios cannot be realized in TFTR. However, the required plasma performance as outlined in Sec. II and exemplified in Table VII does stretch the existing empirical limits at several points, as outlined below.

The most serious difficulty in obtaining central ignition is probably that associated with the high $\langle \beta \rangle$ and $\beta(0)$ required, typically $\langle \beta \rangle = 2-4\%$ and $\beta(0) = 10-15\%$ for these simulations. The empirical $\langle \beta \rangle$ limit for circular tokamaks like TFTR is presently estimated to be [15] $\langle \beta \rangle = 3.5 I(\text{MA}) / [a(m) B_T(T)] = 1/[5A q(a)]$ (where $A = R/a$), which for TFTR is typically $\langle \beta \rangle \approx 2\%$. The empirical $\beta(0)$ limit is not so well documented; however, $\beta(0) = 6.7\%$ at $\langle \beta \rangle > 2\%$ have been obtained in Doublet III at $T_i(0) = 5.5$ keV [16].

The requirement for high global $\langle \beta \rangle$ at central ignition is actually set by the requirement for a high central beta [Eq. (6)] and by the assumed profiles [Eq. (7)]. Thus, if the central energy confinement times were higher than the simulated $\tau_E(a/2) \approx 0.5\text{--}1.0$ sec (see Table VII), then the associated $\beta(0)$ and $\langle \beta \rangle$ requirements would be relaxed. Recently, very high central energy confinement times of $\tau_E(a/3) \geq 1$ sec with $\chi_e(a/3) \approx 10^3$ cm²/sec were observed in TFTR following pellet injection [7] (although at modest neutral beam powers of $P_b \approx 5$ MW and $T_i(0) \approx 2$ keV, and not the simulated $P_b \approx 27$ MW and $T_i(0) \approx 10$ keV). Thus the beta requirement does not necessarily preclude central ignition in TFTR.

The requirement for centrally peaked density profiles with $n(0) \approx 3 \times 10^{14}$ cm⁻³ and $n(0)/\bar{n} \approx 2.5$ cm can only be obtained with pellet fueling. The best results obtained so far with pellets on TFTR have had $n(0) \approx 2 \times 10^{14}$ cm⁻³ with $n(0)/\bar{n} \approx 2.25$ [for a 4-mm pellet], and $n(0) \leq 1 \times 10^{14}$ cm⁻³ with $n(0)/\bar{n} \approx 1.6$ [with smaller 2.67-mm pellets]. The radial profiles in the former case fit approximately $n(r) \sim [1 - (r/a)^2]^5$ within $r/a \approx 0.5$, but had significantly higher density in the exterior region, i.e., typically, $n(3a/4) \approx 5 \times 10^{13}$ cm⁻³. Peaked density profiles with pellet injection have also been produced in PDX [17] with $n(0) \approx 3 \times 10^{14}$ cm⁻³ and $n(0)/\bar{n} \approx 3$, in Alcator-C [18] with $n(0) \approx 8 \times 10^{14}$ cm⁻³ and $n(0)/\bar{n} \approx 2$, and in D-III [8] with $n(0) = 1.5 \times 10^{14}$ cm⁻³ and $n(0)/\bar{n} \geq 2$. Note that in all these cases the plasmas were relatively cold (≈ 2 keV), i.e., pellet penetration at the required $T_i(0) = 10$ keV may be a problem [8]. Note that the empirical Murakami density limit [15] of $\bar{n} \approx 1.5 \times 10^{14} B_T(T)/q(a)R_0(m) = 1 \times 10^{14}$ cm⁻³ is exceeded by only ≈ 2 in the BALDUR simulations of Table VII.

To reiterate the main point: a high global confinement product $n\tau_E$ is not required for achieving central ignition, which depends only on the central parameters, as discussed in Sec. II. Although the scalings of central plasma performance are not as well documented as those of global plasma performance, a high central confinement product of $n(0)\tau_E(a/3) \approx 1 \times 10^{14}$ cm⁻³ has already been obtained in TFTR at $T_i(0) \approx 2$ keV with peaked profiles and $P_b \approx 5$ MW during pellet injection, giving $n(0) T_i(0) \tau_E(a/3) \approx 0.2 \times 10^{18}$ cm⁻³ eV sec, compared with the value of $\approx 3 \times 10^{18}$ cm⁻³ eV sec required for central ignition. If $T_i(0)$ increases linearly with beam power to $P_b = 27$ MW, and if the central $n(0)\tau_E(0)$ can be increased by $\approx 50\%$, then central ignition conditions will be obtained in TFTR. Note that even if the peaked density

profiles and high central energy confinement times are transient, the relevant time scales needed to observe alpha heating effects are relatively short, e.g., approximately 0.3 sec for the cases in Table VII.

The central and global parameter requirements for global ignition as determined from some marginally globally ignited BALDUR simulation runs are plotted in Fig. 18. The required global $n\tau_E$ are reduced by as much as a factor of three from the usual zero-dimensional ignition condition, even in these simulations for which the central energy confinement time is not significantly larger than the global energy confinement time. It is interesting to note that the results for global $n\tau_E$ versus T_i requirements as obtained from the simulations with peaked density profiles ($\delta=5$) are similar to the predictions of an early analytic estimate for ignition with peaked profiles [1].

C. Summary

We have suggested here that central ignition conditions might be obtained in TFTR if the density profile can be made sufficiently peaked. We have also shown that once formed such a central ignition region can lead to a radially propagating burn and to global ignition, starting with global $n\tau_E$ values as much as a factor of three below those calculated for flat profiles. Although several of the requirements for such scenarios stretch the existing empirical limits on tokamak performance, there is no reason in principle why they cannot be achieved on TFTR, since alpha particles near the center of TFTR should be well confined and quickly thermalized.

If central ignition conditions can be achieved in TFTR, then some of the unresolved issues concerning "burning plasma" phenomena could be assessed. For example, it is not yet clear whether alpha heating would be associated with deteriorated confinement due to new alpha-generated instabilities, or with improved confinement more similar to that usually associated with ohmically heated plasmas.

With respect to diagnosing the existence of central ignition, perhaps the first step would be to observe just the central temperature, which should reflect the central alpha heating through an increased decay time after beam turnoff (see Fig. 17). The testing of appropriate alpha diagnostics could begin on centrally ignited TFTR discharges.

Although the BALDUR simulations described here have been used to explore

the general possibility of central ignition, they can evidently be improved in several respects. Most important would be an improved particle model, which would include internal peller fueling and more realistic $D(r)$ and $V(r)$. Also important to include would be appropriate scaling laws for both the electron and ion $\chi(r)$ for these peaked density profile (and compression scenarios) used here. Of course, more realistic impurity and sawtooth modelling would also be desirable.

Even if the eventual results on TFTR fall short of actual central ignition, the attainment of a significantly alpha heated plasma core would be of interest for future ignition studies. New physics concerning alpha confinement and heating, the effect of alphas on energy confinement and sawtoothing, and alpha-generated instabilities could be obtained well before the next generation of ignition machines, and even possibly at $Q < 1$ given this more relaxed goal.

ACKNOWLEDGMENTS

The authors wish to acknowledge helpful discussions with Dan Jassby, David Mikkelsen, and Cliff Singer.

This work was supported by the U.S. Department of Energy Contract No. DE-AC02-76-CHO-3073.

REFERENCES

- [1] KESNER, J. and CONN, R.W., Nucl. Fusion 16 (1976) 397.
- [2] GIBSON, A., Proceedings of International Conference on Plasma Physics, (Lausanne, 1984) Vol. 1, p. 13.
- [3] MENSE, A.T., HOULBERG, W.A., ATTENBERGER, S.E., and MILORA, S.L., Nucl. Fusion 19 (1980) 1473.
- [4] JASSBY, D., Symposium on Plasma Heating in Toroidal Devices, Varenna, Italy, 1974, 259.
- [5] SINGER, C.E., JASSBY, D.L., and HOVEY, J., Nucl. Fusion 20 (1980) 489.
- [6] SINGER, C.E., Nucl. Eng. Design 74 (1982) 347; also SINGER, C.E., HEIFETZ, D.B., and POST, D.E., J. Vac. Sci., Technol. A 1(2) (1983) 964.
- [7] SCHMIDT, G., MILORA, S.L., BELL, M.G., BITTER, M., BUSH, C.E., et al., Proc. European Physical Society Conf. on Plasma Physics, Budapest (1985).
- [8] SENGOKU, S., NAGAMI, M., ABE, M., HOSHINO, K., KAMEARI, A., et al., Nucl. Fusion 25 (1985) 1475.
- [9] MURAKAMI, M., ARUNASALAM, V., BELL, J.D., BELL, M.G., BITTER, M., et al., Princeton Plasma Physics Laboratory Report PPPL-2285 (1985).
- [10] BICKERTON, R.J., ALLADIO, F., BARTLETT, D.V., BEHRINGER, K., BEHRISCH, R., et al., Proc. European Physical Society Conf. on Plasma Physics, Budapest (1985).
- [11] MIKKELSEN, D. and POST, D.A., Proc. Workshop on Physics of Plasmas Close to Thermonuclear Conditions, Varenna, 1979, DOE-CONF-790866, 41.
- [12] SINGER, C.E., POST, D.E., MIKKELSEN, D.R., REDI, M.H., MCKENNEY, A., et al., Comput. Phys. Commun. (to be published).
- [13] MURAKAMI, M., ARUNASALAM, V., BELL, J.D., BELL, M.G., BITTER, M., et al., Fusion Technol. 8, Part 2A (July, 1985) 657.
- [14] KAYE, S.M., Phys. Fluids 28 (1985) 2327.
- [15] NEILSON, G.H., WOOTTON, A.J., BELL, J.D., BUSH, C.E., CARNEVALI, A., et al., Nucl. Fusion 25 (1985) 825.
- [16] KITSUNEZAKI, A., ABE, M., HIRAYAMA, T., HOSHINO, K., KAMEARI, A., et al., Proc. Tenth Int'l. Conf. on Plasma Physics and Cont. Nucl. Fus. Research, London (1984), paper IAEA-CN-44/A-I-4.
- [17] FONCK, R.J., BELL, M., BOL, K., BUDNY, R., COUTURE, P. et al., J. Nucl. Mater. 128 and 129 (1984) 330.
- [18] GREENWALD, M., GWINN, D., MILORA, S., PARKER, J., PARKER, R., et al., Proc. Tenth Int'l. Conf. on Plasma Phys. and Cont. Nucl. Fus. Research, London (1984) paper IAEA-CN-44/I-A-3.

TABLE 1: CENTRAL PEAKING FACTOR OF ALPHA HEATING

$\gamma + \delta$	$P_\alpha(0)/\langle P_\alpha \rangle$	$\Sigma P_\alpha(a/3)/\Sigma P_\alpha$
1	3	0.29
2	5	0.44
4	9	0.65
6	13	0.77

TABLE II: TFTR Scenarios

COMPRESSION SCENARIO

R = 279 + 225 cm

a = 70 + 63

 I_p = 1500-1860 kA B_T = 5 T $q(a)$ = 2.9 $\bar{n}_e \approx 1 \times 10^{14} \text{ cm}^{-3} \rightarrow 2 \times 10^{14} \text{ cm}^{-3}$ Heated by 0.33 sec 27 MW NBI and fast compression (30 ms). $C \approx 1.24$ FULL-BORE SCENARIO

R = 255 cm

a = 83 cm

 I_p = 2200 kA B_T = 5 T $q(a)$ = 3.1 $\bar{n}_e \approx .5\text{--}2 \times 10^{14} / \text{cm}^3$

Heated by two seconds of 27 MW NBI

TABLE III: CENTRAL IGNITION MARGINS FOR COMPRESSION SCENARIOS

A. First χ_e model: $\chi_e(r) = \chi_e(0) [1 + 4(r/a)^2]$

DENSITY PROFILE EXPONENT

		$\delta=1$	$\delta=2$	$\delta=5$
$\chi_{2E}(0)$ (CM ² /SEC)	2×10^3	1.28	2.3	4.6
	6×10^3	0.41	1.28	2.6
	2×10^4	0.03	0.38	1.55

B. Second χ_e model: $\chi_e(r) = \chi_e^* [3 \times 10^{14} \text{ cm}^{-3} / n(r)]$

DENSITY PROFILE EXPONENT

χ_E^* (CM ² /SEC)		$\delta=1$	$\delta=2$	$\delta=5$
	2×10^3	1.0	1.8	3.0
	6×10^3	0.12	0.22	0.62
	2×10^4	0.02	0.02	0.06

TABLE IV: CENTRAL IGNITION RATES FOR COMPRESSION SCENARIOS

(numbers in keV/sec @ .1-.4 sec after beam turnoff)

A. First χ_e model: $\chi_e(r) = \chi_e(0) [1 + 4(r/a)^2]$

	$\delta=1$	$\delta=2$	$\delta=5$
$\chi_e(0)$ (CM ² /SEC) 2×10^3	-	-1.7	14
6×10^3	-	-	7.3
2×10^4	-	-	-

B. Second χ_e model: $\chi_e(r) = \chi_e^* [3 \times 10^{14} \text{ cm}^{-3} / n(r)]$

	$\delta=1$	$\delta=2$	$\delta=5$
χ_e^* (CM ² /SEC) 2×10^3	-	-2.8	+9.2
6×10^3	-	-	-
2×10^4	-	-	-

TABLE V: CENTRAL IGNITION MARGINS FOR FULL-BORE SCENARIOS

A. First χ_e model: $\chi_e(r) = \chi_e(0) [1 + 4(r/a)^2]$

DENSITY PROFILE EXPONENT

$\chi_E(0)$ (CM ² /SEC)		$\delta=1$	$\delta=2$	$\delta=5$
	2×10^3	0.43	1.05	> 10
	6×10^3	0.13	0.34	2.82
	2×10^4	0.01	0.07	0.76

B. Second χ_e model: $\chi_e(r) = \chi_e^* [3 \times 10^{14} \text{ cm}^{-3} / n(r)]$

DENSITY PROFILE EXPONENT

χ_E^* (CM ² /SEC)		$\delta=1$	$\delta=2$	$\delta=5$
	2×10^3	0.47	0.96	1.9
	6×10^3	0.12	0.18	0.59
	2×10^4	<0.01	<0.01	0.02

TABLE VI: CENTRAL IGNITION RATES FOR FULL-BORE SCENARIOS

(numbers in keV/sec @ .1-.4 sec after beam turnoff)

A. First χ_e model: $\chi_e(r) = \chi_e(0) [1 + 4(r/a)^2]$

DENSITY PROFILE EXPONENT

		$\delta=1$	$\delta=2$	$\delta=5$
$\chi_E^{(0)}$ (CM ² /SEC)	2×10^3	6.7	20	48
	6×10^3	-	2	43
	2×10^4	-	-	13

B. Second χ_e model: $\chi_e(r) = \chi_e^* [3 \times 10^{-14} \text{ cm}^{-3} / n(r)]$

		$\delta=1$	$\delta=2$	$\delta=5$
χ_E^* (CM ² /SEC)	2×10^3	-2.5	+8.6	+30
	6×10^3	-	-	-
	2×10^4	-	-	-

TABLE VII: SOME CENTRAL IGNITION SCENARIOS FOR TFTR

TYPE	A	B	C	D	E
	COMPR.	FULL-BORE	COMPR.	FULL-BORE	FULL-BORE
$\chi_e(r)$ (cm ² /sec)	$2 \times 10^4 \cdot (1+4(r^2/a^2))$	$2 \times 10^4 \cdot (1+4(r^2/a^2))$	$2 \times 10^3 \cdot (3 \times 10^{14}/n(r))$	$2 \times 10^3 \cdot (3 \times 10^{14}/n(r))$	$4 \times 10^3 \cdot (r/a) \times (3 \times 10^{14}/n(r))$
δ	5	5	2	5	5
M_{ig} (a/3)	0.97	1.2	0.96	1.15	0.68
$P_a(a/3)/P_b(a/3)$	8.3	1.7	1.6	0.65	0.70
$n(0)$ (10^{14} cm ⁻³)	6.7	4.3	4.1	3.0	3.4
$T_i(0)$ (KeV)	9.5	13.6	10.6	16.5	11.8
$\tau_E(a/2)$ (sec)	0.59	0.77	0.97	1.0	0.58
$\bar{n}$ (10^{14} cm ⁻³)	3.1	1.9	2.5	1.3	1.5
τ_E (sec)	0.58	0.72	0.60	0.88	0.46
$B(0)$ (%)	12.7	17.2	8.8	14.0	12.6
$\langle \beta \rangle$ (%)	3.2	4.1	3.1	2.6	2.2
Q^*	3.0	3.8	2.3	2.1	1.5
NBI (sec)	0.5	1.2	0.5	0.7	0.9
ACI#	227	182	255	236	270

* P_{fusion}/P_{beam} at time of beam turnoff (Full-Bore cases) or shortly after (compr. cases)

FIGURE CAPTIONS

- FIG. 1. Radial vs. time dependences of (a) density, (b) ion temperature, (c) alpha heating power, and (d) neutral beam heating power for compression simulations with neutral beam heating from 0.1-0.43 sec and adiabatic compression from 0.40-0.43 sec. In each case the top plot is for a density profile shape of $\delta = 1$, the middle plot is for $\delta = 2$, and the bottom plot is for the most peaked $\delta = 5$. In these examples the $\delta = 5$ case globally ignites after beam turnoff, while the less peaked cases do not.
- FIG. 2. Time dependences of the central density, central temperatures, and volume-averaged $\langle \beta \rangle$ for the same compression simulations shown in Fig. 1. Note that the central temperature increases after beam turn-off (0.43 sec) for the $\delta = 5$ case, indicating global ignition.
- FIG. 3. Central temperatures vs. time for compression simulations similar to those in Fig. 2 with $\delta = 5$, but for three different assumptions for $\chi_e(0)$. Global ignition is obtained for approximately $\chi_e(0) \leq 10^4$ cm²/sec in this model.
- FIG. 4. Time dependences of the central density, central temperatures, and volume-averaged $\langle \beta \rangle$ for compression simulations similar to those shown in Fig. 2 but for the second $\chi_e(r)$ model, for which $\chi_e(r) \sim 1/n(r)$. Note that the beam pulse length was 0.53 sec for these cases, instead of the 0.33 sec for Figs. 1-3. Note also that the central temperature increases after beam turnoff for the $\delta = 5$ case, indicating global ignition.
- FIG. 5. Central temperatures vs. time for compression simulations similar to those in Fig. 4 with $\delta = 5$, but for various χ_e^* with the second $\chi_e(r)$ model. The results are at least qualitatively similar to those shown in Fig. 3, i.e., global ignition is obtained for approximately $\chi_e^* \leq 10^4$ cm²/sec.
- FIG. 6. Radial profiles of density and temperature for $\delta = 5$ compression simulations for the first ("a" and "b") and second ("c" and "d") $\chi_e(r)$ models, respectively. The electron temperature profiles in the second model are more peaked, reflecting an increased χ_e near the plasma edge. These profiles are for a time just after compression (0.45 sec for the first χ_e model and 0.65 sec for the second χ_e model).

- FIG. 7. Radial vs time dependences of (a) density, (b) ion temperature, (c) alpha heating power, and (d) neutral beam heating power for full-bore scenario simulations with neutral beam heating applied from 0.1-2.0 sec. In each case the top plot is for a density profile shape of $\delta = 1$, the middle plot is for $\delta = 2$, and the bottom plot is for the most peaked $\delta = 5$ case. In these examples the $\delta = 5$ case globally ignites after beam turnoff, while the less peaked cases do not, which is similar to the compression scenario examples shown in Fig. 1.
- FIG. 8. Time dependences of the central density, central temperatures, and volume-averaged $\langle g \rangle$ for the full-bore simulations shown in Fig. 7. Note that central temperature increases with time for the $\delta = 5$ case after beam turnoff (2 sec), indicating global ignition.
- FIG. 9. Central temperatures vs time for full-bore simulations similar to the $\delta = 5$ case in Fig. 8, but for three different assumptions for $\chi_e(0)$. Global ignition is obtained for $\chi_e(0) \leq 10^4 \text{ cm}^2/\text{sec}$ in this model.
- FIG. 10. Time dependences of central density, temperatures, and volume-averaged $\langle g \rangle$ for full-bore simulations similar to those shown in Fig. 8, but for the second $\chi_e(r)$ model. The results are at least qualitatively similar to those shown in Fig. 8, in that more peaked profiles for a given $\chi_e(r)$ ignite more readily; however, the magnitude of χ_e is different here than for Fig. 8.
- FIG. 11. Central temperatures vs time for full-bore simulations similar to the $\delta = 5$ case in Fig. 10, but for three different assumptions for χ_e^* . Again, as for Figs. 3, 5, and 8, global ignition is obtained for sufficiently low central χ_e within the context of this model.
- FIG. 12. (a) Time dependences of the central energy balance terms for a $\delta = 5$ compression case which is globally ignited after beam turnoff at 0.43 sec (see Fig. 3). The central alpha energy input Q_α increases approximately linearly with time after 0.5 sec, but so does the total central energy loss rate Q_{loss} . (b) Time dependences of the central energy balance terms for a $\delta = 5$ full-bore case which is globally ignited after beam turnoff at 2 sec (see also Figs. 1 and 2). Central ignition with $Q_\alpha = Q_{\text{loss}}$ is reached at approximately 1.2 sec, after which the central alpha energy input Q_α and central energy loss Q_{loss} both increase approximately linearly with time.

FIG. 13. Central and global ignition margins vs time for several simulations. In (a) and (b) are the two simulation cases of Fig. 12 with $\delta = 5$ (note that central ignition is obtained well before global ignition in both cases); in (c) and (d) are simulations similar to those in (a) and (b), except with less peaked $\delta = 2$ density profiles (note that neither central nor global ignition is obtained); in (e) is a rapidly ignited case which shows how the global ignition margin follows the central ignition margin.

FIG. 14. Time dependence of the central ignition radius r_{ig} for various simulations. In each case the first χ_e model and the full neutral beam pulse lengths are used, i.e., for compression the beam pulse ends at 0.43 sec, while for the full-bore cases it ends at 2 sec. In (a) are cases which reached global ignition ($r_{ig} = a$), while in (b) are cases which obtained central ignition ($r_{ig} \leq a$) but which did not globally ignite. Note that the central ignition radius increases monotonically in time for almost all cases in (a), while for (b) the central ignition radius can reach 0.45 a without development of global ignition.

FIG. 15. Time dependences of central temperatures for one full-bore simulation case with a varying neutral beam pulse length. For this case $\chi_e(0) = 2 \times 10^4$ cm²/sec with $\delta = 5$ [see also Figs. 8 and 14(a)]. Note that global ignition is eventually obtained for cases in which the beam pulse length is ≥ 1.3 sec.

FIG. 16. Time dependences of the central ignition radius for full-bore simulations in which the neutral beam pulse length is shortened from its normal 2 sec. In (a) are cases corresponding to those shown in Fig. 15 for the first χ_e model, while in (b) are cases also with $\delta = 5$ but with the second χ_e model, with $\chi_e^* = 2 \times 10^3$ cm²/sec. Note that global ignition is eventually obtained if at some time $r_{ig} \geq 0.5$ a.

FIG. 17. Central temperatures vs time for full-bore simulations in which a centrally ignited 50XD/50XT case is compared to an identical case but with 100XD. For these simulations $\delta = 5$, $\chi_e^* = 2 \times 10^3$ cm²/sec, and the beam turn-off time is 0.8 sec [see also Fig. 16(b)]. Note that the decay time of the central temperatures after beam turnoff is significantly longer with D/T, showing that central alpha heating is

substantial and would be easily observable.

FIG. 18. Requirements for global ignition in TFTR based on some sample BALDUR simulations for peaked density profile cases. For each case the simulations were centrally ignited at beam turnoff, after which global ignition was obtained through a radial burn. The dashed line is from a calculation in Ref. 1 of global ignition requirements with peaked profiles. Cases #1-4 are for the first χ_e model, while cases #4 and 5 are for the second χ_e model; cases #1-3 and 6 are for full-bore scenarios, while cases 4 and 5 are for compression scenarios. All cases are for $\delta = 5$ except for case #3 which is for $\delta = 2$.

Fig. 1(a)

Fig. 1(b)

Fig. 1(c)

Fig. 1(d)

Fig. 2

Fig. 3

Fig. 4

#86X0033

Fig. 5

Fig. 6

Fig. 7(a)

Fig. 7(b)

Fig. 7(c)

Fig. 7(d)

Fig. 8

Fig. 9

Fig. 10

#86X0034

Fig. 11

Fig. 12(a)

Fig. 12(b)

Fig. 13(a)

Fig. 13(b)

Fig. 13(c)

Fig. 13(a)

Fig. 13(a)

Fig. 14(a)

Fig. 14(b)

Fig. 15

#85X1759

Fig. 16(a)

86X0117

Fig. 16(b)

#85X2139

Fig. 17

Fig. 13