ASYMMETRIC B-FACTORY COLLIDER NOTE

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1. Introduction

The purpose of this note is to document some work done as part of the effort directed at designing and simulating a bunch-by-bunch feedback system to control longitudinal coupled bunch instabilities in the B-factory. In particular, I discuss the ring model used in the simulation program developed to study this feedback system. Basically the simulation is a simple tracking program in which the rf drive voltage, the wakefields due to all the bunches, the synchrotron radiation losses, and the kicks applied to the bunches by the bunch-by-bunch feedback are all modelled as voltages applied at a single, discrete point in the ring. The computation of the bunch-by-bunch feedback voltages may of course be done by any desired algorithms (for a general discussion of the algorithms simulated to date, see the B-factory Conceptual Design Report, January 1991). An example and discussion of the general behavior *without* bunch-by-bunch feedback is given at the end of this report.

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2. Equations of motion and discrete-kick implementation

The bunch phases φ_i are taken with respect to the zero crossings of the RF voltage. We choose the sign such that $\varphi_i > 0$ for a bunch arriving ahead of its synchronous phase $\varphi_{0,i}$ (above transition). The RF generator voltage (excluding the cavity feedback component required to cancel beam loading in the fundamental mode) is given by

$$V_g = \hat{V}_g \sin \varphi_i$$

Let ϵ_i be the energy deviation of bunch i , that is, its deviation from the energy it would have if it were riding at its synchronous phase. The equations of motion for φ_i and ϵ_i are:

$$\dot{\varphi}_i = -\frac{\alpha\omega_{rf}}{E_0}\epsilon_i$$

$$\dot{\epsilon}_i = \frac{[\text{Energy change of bunch } i \text{ in one turn}]}{T_0}$$

Here α is the momentum compaction factor, E_0 is the nominal energy for a particle on the design orbit, ω_{rf} is the RF frequency, and T_0 is the revolution period.

Putting the last two equations together, and using the fact that the synchrotron period is much longer than revolution period, we have

$$\begin{aligned} \Delta\varphi_i &\approx T_0\dot{\varphi}_i = \\ &= -\frac{\alpha\omega_{rf}}{E_0}[\text{Energy change of bunch } i \text{ in one turn}] \end{aligned}$$

for the kick imparted to bunch i on one turn.

We assume this kick is given to the bunch at a single point in the ring. Take the starting point of a turn around the ring to be right after the kick point. The motion of bunch i on one turn is divided into two parts, namely

1. Propagation around the ring, outside the kick point:

$$\begin{aligned} \varphi_i &\rightarrow \varphi_i + T_0\dot{\varphi}_i \\ \dot{\varphi}_i &\rightarrow \dot{\varphi}_i \end{aligned}$$

2. The kick given to the bunch at the kick point:

$$\begin{aligned}\varphi_i &\rightarrow \varphi_i \\ \dot{\varphi}_i &\rightarrow \dot{\varphi}_i + \Delta\dot{\varphi}_i\end{aligned}$$

The kick given to bunch i is comprised of several pieces, arising from the various contributions to the energy change of bunch i on one turn:

1. The RF generator voltage in the cavity at the time bunch i passes through. This is just $\hat{V}_g \sin \varphi_i$, plus a voltage $V^{cav.fbk.}$ representing the RF cavity feedback needed to control beam loading in the fundamental.
2. The wake fields (both the fundamental mode and HOM's) ringing in the cavity.
3. The synchrotron radiation loss (which really occurs all around the ring, but is lumped into the kick in the simulation).
4. The voltage V_i^{fdbk} applied to bunch i by the bunch-by-bunch feedback system.

That is,

$$\Delta\dot{\varphi}_i = -\frac{\alpha\omega_{rf}}{E_0/e} \left[\hat{V}_g \sin \varphi_i + V^{cav.fbk.} - \frac{U_i^{rad}}{e} + V^{wake} + V_i^{fdbk} \right]$$

Synchrotron radiation damping comes from the variation of synchrotron radiation loss with energy:

$$\begin{aligned}U_i^{rad} &= U_0 + \epsilon_i \left[\frac{dU^{rad}}{dE} \right]_{E_0} \\ &= U_0 - \frac{E_0}{\alpha\omega_{rf}} \left[\frac{dU^{rad}}{dE} \right]_{E_0},\end{aligned}$$

where the relation to the synchrotron radiation damping time τ_R is

$$\frac{1}{\tau_R} = \frac{1}{2T_0} \left[\frac{dU^{rad}}{dE} \right]_{E_0}$$

Thus, the equation for the kick becomes:

$$\Delta\dot{\varphi}_i = -\frac{\alpha\omega_{rf}}{E_0/e} \left[\hat{V}_g \sin \varphi_i + V^{cav.fdbk.} - \frac{U_0}{e} + V^{wake} + V_i^{fdbk.} \right] - \frac{2T_0}{\tau_R} \varphi_i .$$

3. The wake field in the cavity

We keep track of the (complex) wake voltage in each mode m . This voltage gets a discontinuous increment each time a bunch passes through cavity, and it rotates in phase (and attenuates since ω_m is also complex) between bunch passages. Note that according to the fundamental theorem of beam loading, a bunch sees half of the wake voltage it induces during current passage.

The wake voltage in mode m just after bunch $i-1$ has passed through cavity is:

$$V^{cz}(m)'_{i-1} ,$$

where the prime denotes the fact that this is a value of the wake voltage immediately after a bunch has passed through the cavity.

Then, the value of the complex wake voltage in mode m during the passage of bunch i through the cavity is:

$$V^{cz}(m)_i = V^{cz}(m)'_{i-1} \exp \left[i\omega_m \left(\frac{L_{i,i-1}}{c} + \frac{\varphi_{i-1}}{\omega_{rf}} - \frac{\varphi_i}{\omega_{rf}} \right) - \frac{1}{2} W_m e N_i \right] .$$

The wake voltage in mode m just after bunch i passes through cavity is:

$$V^{cz}(m)'_i = V^{cz}(m)_i - \frac{1}{2} W_m e N_i .$$

The total real voltage seen by bunch i , from all modes is:

$$V^{wake} = \text{Re} \sum_m V^{cz}(m)_i .$$

4. Cavity feedback $V^{cav.fbk}$.

Since the beam loading in the fundamental cavity mode is very heavy, rf cavity feedback will be necessary to compensate it. In simulations to date, we have assumed the compensation is perfect, i.e., that $V^{cav.fbk}$ is such as to exactly cancel the fundamental mode of the wake field in the cavity. In reality, the situation is complicated (cavity phase loop, amplitude loop, tuning loop, etc.), and the compensation will not be perfect. A better representation, taking into account the finite response time of cavity and the effect of the gap in the bunch train, is under study.

5. Initial conditions

We start a given simulation run with specified initial conditions on φ_i and $\dot{\varphi}_i$ for each bunch. A typical initial condition studied is to have one bunch with a relatively large offset from the average synchronous phase, while the remaining bunches are close to the synchronous phase.

Eventually, it will be desirable to simulate addition of charge to one bunch at a time, with appropriate increments to φ and $\dot{\varphi}$ of that bunch. This fits easily into the present program structure.

6. Program output

One can make the following plots for any of the bunches:

1. φ_i vs turn number.
2. $\dot{\varphi}_i$ vs turn number, where

$$\dot{\varphi}_i \equiv [(\varphi_i - \varphi_{0,i})^2 + T_0(\varphi_i - \varphi_{0,i})\dot{\varphi}_i + (\dot{\varphi}_i/\omega_{s,0})^2]^{1/2} .$$

is the approximate invariant amplitude for the motion defined in the two pieces shown earlier. Here

$$\omega_{s,0}^2 \equiv \frac{\alpha\omega_r\hat{V}_g}{E_0/eT_0} \sqrt{1 - \left(\frac{U_0}{e\hat{V}_g}\right)^2} .$$

3. The program also keeps track of $\varphi_i^{\min}$, $\varphi_i^{\max}$, and φ_i^{avg} for each of the bunches. Note that after sufficiently many turns, φ_i^{avg} gives a good approximation to

the actual synchronous phase of bunch i . Any of these three quantities may conveniently be plotted as a function of bunch number.

We show an example, *with no bunch-by-bunch feedback* in Figure 1. This is a plot of maximum bunch offset reached in 2000 turns, versus bunch number, for the first 100 bunches in the high energy ring filled with 1658 bunches (harmonic number 3492, with every other bucket filled, except in the gap). This is with a 5% gap, however the result is insensitive to the size of the gap (provided the gap is long enough that the wake field completely damps during that interval). Other parameters are: $\alpha = 0.00241$, $\hat{V}_g = 18.5$ MV, $U_0 = 3.52$ MV. There is a single longitudinal HOM, with $R/Q = 41\Omega$ per cell and $Q = 70$, and there are 20 RF cells. The initial conditions are to start bunch 20 at an offset $\varphi = 0.2915$ radian (0.1 radian away from the synchronous phase near 0.1915 radian), and the remaining bunches at 0.1915 radian, i.e., close to their synchronous phases.

Note that the time between injections, 1/60 second, corresponds to about 2300 turns. The disturbance due to the offset of a given bunch does not propagate very far back in the train during this time, even without a feedback system. However, for more turns the disturbance rapidly gets out of control. In Figure 2, we show the same example after 3000 turns. Here, some of the bunches have reached an amplitude larger than that of the originally offset bunch 20. Finally, in Figure 3 we show what has happened after 6000 turns (note the change in vertical scale!).

The slight disturbance visible in bunches 1 through 19 is due to the fact that they were not started at exactly the synchronous phase, and not to any wake field effects reaching across the gap.

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Figure 1

Figure 2

Figure 3