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THE USE OF SCALE MODELS TO DETERMINE THERMO-HYDROMECHANICS OF HOT-DRY-ROCK RESERVOIRS

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1. INTRODUCTION

Thermal energy can be extracted from natural hydrothermal reservoirs that contain hot water simply by drilling into the reservoir. Unfortunately, the combination of high temperature, adequate rock permeability, and a good supply of water is fairly rare. Most of the geothermal heat that occurs at accessible depths is in rock formations that either contain little water or have permeabilities so low that the existing water cannot be removed at useful rates. The technical challenge of these "hot dry rock reservoirs" is how to obtain their abundant energy economically in a useful form.

In the early 1970's, several investigators originated the idea of creating artificial fractures in hot dry rock reservoirs, through which a fluid could be circulated, heated, and recovered (for example, Refs. 2, 3, and 9)*. Since then, Los Alamos Scientific Laboratory (LASL) has been conducting field tests to determine the technical and economic feasibility of the idea at a site in the Jemez Plateau in New Mexico (Ref. 9). In summary, LASL's method is:

- drill a well into the rock to a depth where the geothermal temperature is high
- case the well completely except for short lengths at specified depths
- pressurize the well, by pumping down water, until new fractures of the rock are initiated at the uncased locations or existing faults and cracks are opened
- drill a second well to intersect the fractures or faults, thereby forming a low-impedance connection, and
- pump water down one well, through the fractures, and up the other well.

*The references are listed in the Bibliography and are generally arranged by year of issue and, within each year, by the major organization performing the work. All the references contain data or information used in the present study, but not all are cited specifically in this report.

The two wells connected at depth constitute the hydrothermal system, with the heat transfer surface provided mainly by the fractures. Other methods, such as the "huff-puff" technique that can use only one well, have also been proposed (Refs. 2, 31). (In the huff-puff mode, the fracture system is inflated with water which is then heated by the rock; the heated water is recovered by allowing the fracture to deflate.)

LASL's full-scale experiments have shown that their method is technically feasible but have brought to light a number of problems. The problems, briefly, can be put into two categories:

1. lack of knowledge of the down-hole geometry, and
2. scaling up to large sizes

The first category of problem results in difficulty in understanding and predicting the behavior of hot dry rock reservoirs, because the extent of the fracturing and the geometric details of the well-to-well connection are unknown. For example, through the course of the program, the conception of the down-hole geometry described in LASL's reports has changed from a simple circular-like fracture (Figure 1a), to a system where the extraction well is connected to a fracture originating at the injection well by a series of natural faults (Figure 1b), to a system where both wells are directly connected by a series of fractures. The test data (water temperature, flow rates, well pressure, injection and extraction points) can apparently be made to fit either of the last two representations equally well (Ref. 46, 47). Difficulty in scaling-up test results, the second category of problem, means that the useful life of a reservoir may not be estimated reliably. For small reservoirs (e.g., the Phase I LASL program), the thermal drawdown, or the decrease in the rock temperature, is very rapid. For larger reservoirs (e.g., the Phase II LASL program now underway), the drawdown will be so slow that whether thermal-stress cracking or any of the other postulated reservoir-life-enhancement mechanisms will be in fact operable will not be known for five to seven years after start up; if they are not, the useful life will be too short.

a. IDEALIZED CONCEPTION

b. FRACTURE AND NATURAL JOINT SYSTEM

c. THREE FRACTURE SYSTEM

FIGURE 1. VARIOUS CONCEPTIONS OF LASL PHASE I SYSTEM

There are at least three ways by which investigations can be continued, either separately or in combination, to help resolve these problems. First, the large scale program can be continued. This is being done. Such studies are necessarily site-specific, and they require a long time to obtain results. There is also no direct way to study what is happening in the reservoir. Second, better mathematical models can be formulated. They are unlikely to be completely satisfactory, however, unless the reservoir geometry can be specified as an input to the model. It might be noted that a hot dry rock reservoir is not bounded above or below by a medium of distinctly different properties, as is an oil or gas reservoir for which mathematical models have proved to be reliable. Third, the reservoir can be simulated by a geometric scale model in which the important non-dimensional groups of full-scale physical parameters are maintained in the model (Ref. 56). This method also has advantages and disadvantages. It is usually impossible to duplicate exactly every physical effect in the model, a limitation that scale models share with mathematical models. The advantages of the method are many and well-known:

- little understood phenomena can be explored
- a wide range of parameters and reservoir characteristics can be studied
- geometry or physics need not be idealized
- extensive instrumentation can be employed, and the model can be studied visually at any time or at the conclusion of testing
- costs are relatively low

Considering these advantages, it is evident that the use of scale models should be evaluated as a tool for the investigation of hot dry rock reservoirs.

The study reported here had as its main objective a determination of the feasibility of physical scale models as a way to confirm mathematical models and to explore fundamental behavior of hydraulically-fractured

hot dry rock reservoirs. Included in the study are:

- similitude analyses, based on full-scale data, simplified mathematical models, and physical reasoning
- formulation of "scaling laws" from the similitude analyses
- preliminary determination of the processes and phenomena that can be reliably studied in scale model tests
- recommended test program to implement the results of the study

Many of the major studies conducted in hot dry rock reservoir engineering were reviewed and evaluated in the course of meeting the objectives of this study.

2. LITERATURE REVIEW & EVALUATION

Personal contacts and a computerized literature search turned up well over three hundred papers and reports written since 1970 on the subject of hot dry rock geothermal reservoirs. Many of them had little relevance to this study. Of those that did, about fifty representative samples were selected for a detailed review; they are listed in the Bibliography. Evaluations of this literature, organized by topic, are presented below. The reviewed literature, and independent mathematical analyses and physical reasoning, formed the basis of the similitude study presented in Section 3.

2.1 Hydraulic Fracturing

In hydraulic fracturing, water or some other incompressible fluid is pumped at high pressure through an uncased section of a deep well. A fracture is created when the pressure exceeds the sum of (1) the smallest component of the earth stress and (2) the inherent strength of the rock as modified by stress concentrations. The size and shape of such fractures in a homogeneous isotropic medium have been studied theoretically by several investigators through the use of linear elastic fracture mechanics (Refs. 10, 16, 40, 51, 52, for example). The predictions have been qualitatively confirmed by scale model experiments (Refs. 51 and 52, to be discussed later). These studies have demonstrated the relative importance of the parameters that govern hydraulic fracturing, but they should be considered only as approximations to the fracturing of hot dry rock since the important effects of natural faults and cracks are neglected. According to data from many test bores (Ref. 1), at least one fracture at depth should be expected for every three meters of well length. Hydraulic pressure is as likely to open these pre-existing cracks as it is to create new fractures, and the extent of any new fracture is likely to be controlled by its intersections with natural faults and fractures. In fact, it has been shown theoretically that a hydraulically-created fracture containing a pressurized fluid whose density is less than that of the rock will "float" to the surface by extending itself at its upper edge and closing at the lower, unless it intersects a natural

fault. Experience at LASL's hot dry rock field site has also shown the importance of natural fractures in controlling the permeability of the reservoir system, the well-to-well flow impedance, and the water loss rate.

Both linear elastic fracture mechanics and field experience agree in predicting that the hydraulic fracture lies in a vertical plane because the fracture opens against the smallest earth stress, which is horizontal at the depths of interest. For the purposes of this study, the interrelations among the parameters can be shown by Sack's idealized theory (Ref. 58):

$$P = \sigma_3 + [\pi E \gamma / (1 - \nu^2) R]^{1/2} \quad (2.1a)$$

and

$$\delta = 2[(1 - \nu^2) R / \pi E]^{1/2} \quad (2.1b)$$

where

- P = fracturing pressure (newton/m²)
- σ_3 = smallest component of compressive earth stress (newton/m²)
- E = rock elastic modulus (newton/m²)
- γ = surface energy of rock (newton/m)
- ν = rock Poisson ratio (-)
- R = fracture radius (m)
- δ = average fracture width (m)

Since R is large, the fracturing pressure does not have to exceed the compressive stress by a significant amount. It might also be noted that, in practice, the fracture radius would be sensitive to the small difference $P - \sigma_3$, and therefore likely to be poorly predicted in advance.

2.2 Analysis of Transient Pressures and Flows for a Single Fracture

The analysis of flow in a porous compressible medium is well developed. Such analyses have been extended recently, for the purposes of well-testing

and hydraulic fracturing, to encompass flow from a well into an attached fracture and then into a porous medium (Refs. 7, 8, 12, 23, 42, and 54). The rate of change of well pressure under conditions of a constant flow of injected fluid, or the injection rate under conditions of a constant well pressure, is predicted, with the results called "type curves". For the simplest case of an infinitely-extending fracture that does not offer any resistance to the flow, the pressure change ΔP in the well for a constant injection rate Q is:

$$\Delta P = \frac{2Q}{A} \left(\frac{\mu t}{\pi \phi k c} \right)^{1/2} \quad (2.2)$$

where

- A = fracture surface area (m^2)
- k = rock permeability (m^2)*
- c = compressibility of rock-fluid medium (m^2/newton)
- μ = fluid viscosity (newton-sec/ m^2)
- ϕ = porosity of rock (-)
- Q = injection rate of liquid (m^3/sec)
- t = time after start of injection (sec)

If a plot of experimental well pressure against the square root of time is a straight line, a fracture of this type can be inferred. More complicated relations are needed in general, and the exponent on time can change with time. Nonetheless, information about the reservoir size can still be obtained.

These analyses are most applicable to oil and gas reservoirs, where the rock above and below the reservoir is of such different properties that the fracture extent can be estimated separately. They have also helped resolve questions about whether horizontal fractures are possible. For this study, their main use is in helping define important parameters. LASL has used

*The common unit of permeability is the darcy, equal to $0.99 \times 10^{-12} m^2$. For dimensional analysis, the more fundamental dimension used here is needed.

such models to help correlate data from hot dry rock reservoir tests, but there the permeability is due mostly to the natural and hydraulically-created fractures, rather than being a property of the rock. Since the distribution and sizes of the fractures can only be roughly estimated, it is almost impossible to do anything better than place bounds on the reservoir characteristics (Ref. 48). Conceptually, the permeability and the porosity of a series of fractures are related by an adaptation of the Kozeny equation (Ref. 2):

$$k = \frac{4}{5} d^2 \theta^3 \quad (2.3)$$

Here, d is a measure of the distance between cracks. More generally, the permeability of a medium is related to the cube of the porosity and the square of some length scale.

Equation (2.2) does not consider two-phase flow, and the capillary pressure due to surface tension of the fluid in the pores is neglected in comparison to hydrostatic pressure. Neither two-phase flow nor capillarity will be considered in this study, in accordance with their expected lack of importance in hot dry rock reservoirs.

2.3 Heat Transfer and Fluid Flow in Fractured Rock

The energy production rate of the reservoir is governed by the fluid flow through the fractures and the heat transfer to the fluid from the rock. (The cooling of the rock can also lead to further cracking, thereby providing more heat transfer surface. This will be discussed subsequently.) In the literature, two different approaches have been taken in modeling these processes mathematically. In one, the flow is confined to the fractures (generally, a single fracture) and heat is transferred by conduction from the rocks (Refs. 3, 14, 23, 24, and 29 are examples). In the other, the fractures are considered as distributed porosity, and the fluid flow through the pores and the heat transfer are closely coupled; this method seems highly artificial for hot dry rock reservoirs, where there should be a limited number of fluid-bearing fractures, and thus is not considered further until thermal stress cracking is considered. The equations presented here are adopted from Ref. 14.

For the purposes of determining the important parameters, the physical model is simplified. Only a single fracture is considered; the plane of the fracture is vertical, and the fracture shape is assumed circular with a radius R . All the fluid enters the fracture at a single injection point and leaves at a single extraction point. The flow is in the x - y plane of the fracture and is assumed to be laminar, which is reasonable considering that the width is of the order of a centimeter. Thus, the momentum equations for the flow are

$$\frac{\partial P}{\partial x} = - \frac{12\mu}{w^2} u \quad (2.4)$$

$$\frac{\partial P}{\partial y} = - \frac{12\mu}{w^2} v + g\rho\beta(T - T_i) \quad (2.5)$$

Here,

u, v = x, y components of the velocity (m/sec) with y vertical

g = acceleration of gravity (m/sec²)

ρ = fluid density (kg/m³)

β = coefficient of expansion of the fluid (1/°K)

T = fluid temperature (°K)

T_i = inlet fluid temperature (°K)

The other symbols are the same as defined for Eq. (2.2). (The permeability of the fracture is $w^2/12$ in this model.) Flow into the porous rock is neglected, and such a flow could not be recovered in any case. Equations (2.4) and (2.5) are valid when the Reynolds number is small. Nothing is assumed about the magnitude of the Grashof number (ratio of buoyancy to viscous forces), and it can be large or small.

The energy equation for the flow reduces to

$$\rho C_p w \frac{\partial T}{\partial t} + \rho C_p w \left(u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right) = -2\lambda_r \frac{\partial T_r}{\partial z} \Big|_{z=0^+} \quad (2.6)$$

where

$$\begin{aligned} C_p &= \text{specific heat of fluid (watt-sec/kg-}^\circ\text{K)} \\ \lambda_r &= \text{rock conductivity (watt/m-}^\circ\text{K)} \\ T_r &= \text{rock temperature (}^\circ\text{K)} \end{aligned}$$

The width w of the fracture is assumed to be so small that the heat conducted into the fluid from the rock can be evaluated at $z = 0$ rather than $z = \pm w/2$. Equation (2.5) is valid when the Peclet number (ratio of convective heat transfer to conductive heat transfer in the fluid) is large, as it is expected to be for narrow channels.

The conductive heat transfer in the rock is given by

$$\frac{\partial T_r}{\partial t} = \alpha_r \frac{\partial^2 T_r}{\partial z^2} \quad (2.7)$$

where

$$\begin{aligned} T_r &= \text{rock temperature (}^\circ\text{K)} \\ \alpha_r &= \text{thermal diffusivity of rock (m}^2\text{/sec).} \end{aligned}$$

Equation (2.7) is valid when the depth of penetration of the cooled rock, roughly $\sqrt{\alpha_r t}$, is much less than R ; this is the case for hot dry rock reservoirs.

These equations, with suitable boundary and initial conditions, have been used to demonstrate the interaction of buoyancy and viscous forces on the flow streamlines in the fracture. Figure 2 shows some of the results. For the cases shown, it is evident that buoyancy must be considered in the similitude analysis. However, the ratio of fracture width to fracture radius is somewhat larger for these cases than ought to be expected in practice. For a smaller fracture width, the influence of buoyancy is considerably smaller.

NOTE: EACH STREAMLINE
ENCOMPASSES 10%
OF TOTAL FLOW

FIGURE 2. DEMONSTRATION OF BUOYANCY EFFECTS ON FLOW
IN HOT DRY ROCK FRACTURE (FROM REF. 14)

2.4 Thermal Stress Cracking

Because of the low thermal diffusivity of rock, the volume of rock near the fracture has heat removed by the water more rapidly than it can be replaced by conduction from adjacent rock. The cooled volume, thus, forms a thin boundary layer of thickness $(\alpha_r t)^{1/2}$ along the fracture surfaces. The energy in the bulk of the rock outside the boundary layer cannot be extracted at a useful rate, and the reservoir experiences a "thermal drawdown" that will eventually limit the life of the reservoir unless some means is obtained for tapping the energy in the region where the temperature is still high. Thermal stress cracking is believed to be one such means.

As the temperature of the rock decreases, the compressive earth stresses are relieved and, for sufficient cooling, are replaced by shrinkage-induced tensile stresses. New fractures can thereby be created in the cooled rock. Since the plane of these fractures is normal to the first fracture (but probably is still vertical) the fractures may extend into the high temperature region. If water can be made to flow in these cracks, the life of the reservoir can be extended. Since hot dry rock reservoirs may not be economically feasible unless the reservoir size can be augmented, several investigators have studied the dynamics of thermal stress cracking. It has been found that a kind of instability occurs in which the spacing between the thermally-induced fractures remains about equal to the penetration depth. As some of the cracks extend, others either do not grow or even may close, so as to retain the balance between spacing and depth (Refs. 16, 40, 51). This behavior is apparently also dependent on the shape of the temperature profile in the rock. It is not evident, even with thermal stress cracking, that connected flow paths will be formed that can increase the effective volume of rock from which energy can be extracted. Thermal stress cracking is, thus, a phenomenon that will be a major influence on whether hot dry rock reservoirs will ever be economically feasible.

For the purposes of this study, it is only necessary to know the parameters that govern thermal stress cracking in hot dry rock. The relations between these parameters are given by thermoelasticity, as shown in

Ref. 31. Neglecting the effects of stress-concentration (or, alternatively, looking only at the opening of pre-existing fractures), the temperature decrease required to initiate cracking is

$$\Delta T = \frac{3}{2} \left[\frac{1 - \nu}{\beta_r E} \right] [\sigma_2 - P] \quad (2.8)$$

where

- β_r = coefficient of linear expansion of rock ($1/^\circ\text{K}$)
- σ_2 = intermediate component of horizontal earth stress
(newton/m²)

The other symbols have already been defined. When the rock pores contain water, the pore pressure P_0 must be added to σ_2 in Equation (2.8).

Equation (2.8) represents an overly-idealized model from the point of view of quantitative predictions, but it does indicate all the important parameters.

2.5 Previous Scale Model Studies

There have been several studies in the past that have investigated some part of the thermo-hydro-mechanics of hot dry rock reservoirs, or analogous phenomena, by the use of scale models.

The production of geothermal steam has been studied at two different scales, the smaller model being a 7.62 cm diameter by 99.1 cm long steel cylinder filled with unconsolidated sand (Ref. 4) and the larger being a 0.425 m³ pressure vessel filled with granite rocks of about 2.5 cm average dimension (Ref. 13). In both cases, water was used to permeate the pores of the vessel contents, which was then heated and pressurized. The models were instrumented extensively. Tests have been conducted on a variety of subjects, such as in-place boiling, flash front movement, two-phase flow, steam-water stratification, and vapor pressure of steam in small pores. These models, although they are not applicable in their present form to hot dry rock reservoirs, have shown:

- the usefulness of scale model tests to geothermal problems, and
- that data can be obtained by in-situ instrumentation in the model that would be unavailable in full-scale field tests.

The interaction of two hydraulic fractures originating from the same borehole has been studied by hydraulically-fracturing small, clear plastic cubes (15 or 20 cm on a side) under compression on all but two opposite sides (Ref. 51). It was found that there was not any noticeable tendency for the fractures to join and that the planes of the fractures were practically parallel during the major phases of growth. The fractures extended during growth so as not to open against the vertical and horizontal compression applied to the model. These tests also pointed out the strong influence of natural fractures and faults in hot dry rock reservoirs. The vertical compression was 20 bars and the horizontal compression was 15 bars. Since the minimum external compression applied to the model (analogous to the minimum earth stress) was zero, it might be expected that a fracture would be created readily. Yet, a "downhole" pressure of 200 bars was required. At the New Mexico field site, fractures were created (or existing fractures opened) at a downhole hydraulic pressure of about 340 bars where the minimum principal earth stress was 330-340 bars (Ref. 9). The maximum difference, $P - \sigma_3$, of about 10 bars is a power of ten smaller than the model result.

The plastic models were also used to study thermal stress cracking. It was found that a very cold liquid (e.g., liquid nitrogen at about 75°K) was needed to achieve thermal cracking, another indication of the out-of-scale strength of the model. The pattern of thermal cracks emanating from the main fractures was complicated and invariably exhibited curvature in two directions. As the cracks grew in length, the spacing between cracks increased, in agreement with the predictions of theory (Refs. 16, 40, 51). Other tests of cooled granite agree with this conclusion (Ref. 50).

Other scale model experiments have been conducted to investigate the initiation and growth of hydraulically-created fractures in plastics and

rocks, aimed primarily at the behavior of cracks near an interface between two different materials (Ref. 52). The models were 30.5 cm x 30.5 cm x 10.2 cm in size, and the fractures were created by drilling a hole in the model and pressurizing the hole. Since the models were not under external compression for many of the tests, the direction of crack growth was controlled by making pre-cracks at the bottom of the hole. It was found that whether a hydraulic fracture will cross an interface depends on the relative values of the elastic moduli and strength of the two materials as well as on the strength of the interface bond itself.

Other laboratory-scale tests have been conducted to study various aspects of the flow or heat transfer in fractures. Reference 22 describes tests of a large granite cylinder that was split in half and a flow of water forced through the split. It was found that the aperture increased or decreased with flow pressure. The flow losses (i.e., pressure drop) caused by the flow converging towards an exit in a fracture has been studied by an idealized scale model (Refs. 27, 36). A substantial part of the total flow impedance of a hot dry rock reservoir system apparently may be due to this acceleration of the flow at the fracture exit. Other studies (Ref. 52) have used various kinds of fractured rock, with a series of fracture apertures, to determine the relations between pressure drop, velocity, and aperture width for flow in a rock fracture. It was found that the roughness of the fracture surfaces will increase the ordinary, laminar flow friction factor but only by 50% or so even in extreme cases.

2.6 Los Alamos Field Experience

Since about 1975, LASL has conducted many types of field tests on their Phase 1 System, which consists of an injection well, EE-1, and an extraction well, GT-2. For most of the tests, the wells were thought to be connected by a system composed of a vertical hydraulic fracture that emanated from EE-1 at a depth of 2750 m and intersected a series of sloping parallel natural joints near 2700m; the natural joints completed the connection to GT-2, as was shown previously in Figure 1b. In later tests, a larger hydraulic fracture was created from EE-1 at a depth of 2930m; the connection

to GT-2 was again thought to be completed by the natural joints. During the initial pressurization, a hydraulic fracture was assumed to be formed whenever the well began to accept a significant flow, since there was apparently no sharp change in the pressurization to indicate a sudden fracture otherwise. This may indicate that the fractures were the re-opening of existing faults (Ref. 20). The locations of the flow exits and entrances in the wells were inferred primarily by downhole temperature and flow rate surveys of the well bores.

A number of the tests were conducted to try to evaluate the parameter $A\sqrt{k c \phi}$, which according to Equation (2.2) controls transient pressure and flow responses. From theoretical considerations (Ref. 48) the dependency of ϕ , c , and k on the downhole reservoir pressure was determined to be:

$$\phi = \phi_0 / [1 - C_3 P]^{0.6} \quad (2.8a)$$

$$k = k_0 / [1 - C_2 P]^{0.8} \quad (2.8b)$$

and

$$c = d\phi/dP = C_0 / [1 - C_3 P]^{1.6} \quad (2.8c)$$

The last relation implies $\phi_0 = C_0 / (0.6 C_3)$ so ϕ_0 and C_0 are not independent. The constants C_3 and C_2 and the parameters $A\sqrt{k c}$ (after eliminating ϕ) and k/c were determined by data fits from tests such as:

- shut-in GT-2, and pressurize EE-1 in steps, using constant flow and constant pressure phases, and
- shut-in one well and pressurize the other at a constant injection rate

The models could be made to fit the data fairly well, and gave results such as:

$$C_3 = 0.098 \times 10^6 \text{ m}^2/\text{newton} \quad (2.9a)$$

$$C_2 = C_3/5.0 \quad (2.9b)$$

$$A\sqrt{kc} = 4.4 \times 10^{-10} \text{ m}^4/(\text{newton})^{1/2} \quad (2.9c)$$

$$A\mu c/\pi k = 3.5 \times 10^6 \text{ seconds} \quad (2.9d)$$

The implied permeabilities and porosities were large, however, compared to those that were determined experimentally from core samples (Ref. 21). None of the parameters A , k , and c could be determined separately, and such tests could not reveal whether the reservoir permeability was due primarily to fractures and faults or to the rock matrix itself. Further, A , k , and c as determined from data fits to short term tests are probably not appropriate for long-term tests because of the importance of the entire flow field of the reservoir for long-term tests as contrasted with the importance of fracture geometry near the well bore for short term tests. All that can be done is to place rather wide limits on the parameters, as shown in Figure 3. In these graphs, the symbol I represents the observed well-to-well flow impedance as defined by $\Delta P/Q$, B represents a flow-dependent contribution to the permeability of the form $(1 + BQ)^{-1}$, and α is $A\sqrt{kc}$.

An overall heat transfer area was inferred by matching computed thermal draw-down rates to the observed ones. A circular shape was assumed for a single fracture, and point inlets and outlets were located at positions along the fracture vertical diameter corresponding to the observed inlets and outlets along the well bore. For the smaller of the two Phase 1 systems, (Figure 1b), there was sufficient draw-down during a series of heat extraction tests (called Run Segment 3) so that an effective area of 8000 m^2 could be calculated. Note that this is a temperature-weighted area, while those shown in Figure 3 are flow-weighted; thus, the flow and heat transfer area estimates can differ. For the larger of the two Phase 1 systems, very little draw-down had occurred through 1981, but calculations indicated that the new fracture area had to be at least $30,000 \text{ m}^2$ (Ref. 46). Interestingly, these later tests indicated that the smaller fracture now had an area of $15,000 \text{ m}^2$.

Dye tracer experiments have been used to give an indication of the total fracture volume. The smaller fracture system appears to have

- a. Approximate limits on fracture width and height set by short-term injection and flow-through parameters

- b. Limits on permeability and fracture area set by long-term flow parameters.

FIGURE 3. LIMITS ON PHASE 1 RESERVOIR PARAMETERS

an effective flow volume of 25 to 48 m³, thus giving an average fracture width of 0.3 cm to 0.6 cm. The larger fracture system appears to have a volume of 192 to 236 m³, with an average width of 0.6 cm to 0.8 cm (Ref. 46).

The later transient flow tests of the larger system have been interpreted with the aid of several idealized models of the connections to yield the flow impedances (1) between EE-1 and the fracture system, (2) of the fracture system itself, and (3) between GT-2 and the fracture system. The latter impedance turned out to be the largest by a considerable margin. Although the total impedance of the larger system has remained constant (Refs. 46, 47), that of the smaller system decreased significantly in a discontinuous way during Run Segment 3 (Ref. 31). The decreased impedance is assumed to reflect the opening of additional natural faults and to thermal stress cracking; if the primary cause is thermal stress cracking, the lack of a decrease in the impedance of the larger system is understandable, since the rock had cooled little during the tests through 1981.

Recent tests of the larger system have led to some perplexing results. The water loss rate is apparently larger than it should be, based on scaling-up the results from the smaller system (Ref. 47). More importantly, the reservoir is now thought to behave as a series of parallel, interacting fractures (Figure 2c), rather than as a series of independent fractures (Figure 2b). However, all the test data can still be fitted to a model of independent fractures (Figure 2b), as before (Ref. 47). It is evident that the lack of knowledge of what the fracture system geometry is in reality will lead to problems in scaling the results to economically-sized systems and in predicting their performance.

Many other kinds of field and laboratory tests have been conducted at LASL. The summary given above is, however, sufficient for the purposes of similitude analysis.

3. SIMILITUDE ANALYSIS

The hydro-thermo-mechanics of hot dry rock reservoirs were described in Section 2. Idealized mathematical models were formulated as a means of understanding and correlating field test results. In this section, the physical parameters will be listed that govern the hydro-thermo-mechanics of hot dry rock reservoirs, with the selection based on the mathematical models. The parameters will then be arranged into non-dimensional groups, and the relative importance and significance of each group will be assessed.

3.1 Listing of Parameters

Parameters are needed to describe the geometry of the hot dry rock reservoir, the physical properties of the media, the thermal and fluid-flow characteristics, and the responses. For the purposes of devising scale model test procedures, all parameters except the responses can be thought of as being in principle under the control of the experimenter. The response parameters are the desired outputs of the reservoir, and should not appear in the non-dimensional groups of controllable or test parameters (Ref. 56). A system of force (F), length (L), time (T), and temperature (θ) is used to express the fundamental dimensions of each parameter. Table 3.1 gives the parameter list.

The listing is fairly self-explanatory. Poisson's ratio of the rock, ν_r , is the only parameter mentioned in Section 2 that is not in the list. The compressibility c_r , elastic modulus E_r , and porosity ϕ_r determine a unique value of ν_r (or rather c_r can be computed from E_r , ϕ_r , and ν_r) so ν_r is not needed as a separate parameter.

3.2 Non-dimensional Parameter Groups

Since there are thirty-eight parameters in Table 3.1 that are expressed in terms of four fundamental dimensions, thirty-four independent non-dimensional groups can be formed, of which seven are response groups. Many of the groups are merely ratios of lengths or physical properties.

Table 3.1 Scale Modeling Parameters

SYMBOL	DESCRIPTION	DIMENSIONS
● Geometric Parameters		
R	Characteristic fracture length	L
d_o	Well-bore diameter	L
z	Well depth	L
w	Characteristic fracture width	L
d_s	Characteristic spacing between faults, etc.	L
z_f	Other geometric parameters	L
● Physical Property Parameters (Liquid)		
ρ	Density	FT^2/L^4
μ	Viscosity	FT/L^2
λ	Thermal conductivity	$F/T\theta$
C_p	Specific heat	$L^2/T^2\theta$
β	Coefficient of linear expansion	θ^{-1}
C	Compressibility	L^2/F
● Physical Property Parameters (Rock)		
ρ_r	Density	FT^2/L^4
λ_r	Thermal conductivity	$F/T\theta$
C_{pr}	Specific heat	$L^2/T^2\theta$
β_r	Coefficient of linear expansion	θ^{-1}
c_r	Compressibility	L^2/F
E_r	Modulus of elasticity	F/L^2
γ_r	Surface energy	F/L
k_r	Permeability	L^2
ϕ_r	Porosity	--
● Reservoir and Flow Parameters		
$\sigma_1, \sigma_2, \sigma_3$	Principal earth stresses, $\sigma_1 > \sigma_2 > \sigma_3$	F/L^2
T_r	Rock Temperature	θ
T_i	Injection-well inlet temperature	θ
P_i	Injection-well inlet pressure	F/L^2
P_o	Extraction-well outlet pressure	F/L^2
Q	Injection-well flow rate	L^3/T
g	Gravity	L/T^2
t	Time	T
● Response Parameters		
T_o	Extraction-well outlet temperature	θ
e	Thermal energy added to outlet flow	FL/T
ΔQ	Water loss rate	L^3/T
z_c	Thermal-stress crack length	L
w_c	Thermal-stress crack width	L
d_c	Thermal-stress crack spacing	L
ΔT_r	Rock temperature drawdown	θ

The more revealing groups, which express the physics of the fluid and rock interaction, have been arranged so as to imply a relation between two physical effects (forces, energies, etc.). The non-dimensional parameters are arranged finally as a "modeling law".

$$\left. \begin{array}{l}
 (T_o - T_i)/(T_r - T_i) \\
 e/\rho C_p Q(T_r - T_i) \\
 \Delta Q/Q \\
 l_c/R \\
 d_c/R \\
 W_c/R \\
 \Delta T_r/(T_r - T_i)
 \end{array} \right\} f \left[\frac{W}{R}, \frac{d_o}{R}, \frac{l}{R}, \frac{d_s}{R}, \frac{l_i}{R}, \frac{\sigma_3}{\sigma_1}, \frac{\sigma_2}{\sigma_1}, \frac{\sigma_3}{P_o}, \frac{P_i}{P_o}, \right.$$

$$\left. \frac{E}{\sigma_1}, E c_r, \phi_r, \frac{\rho}{\rho_r}, \frac{C_p}{C_{pr}}, \frac{\lambda}{\lambda_r}, \frac{c}{c_r}, \frac{T_r - T_i}{T_r}, \right.$$

$$= \left[\frac{(P_o + \rho g l) k_r c_r}{R_2}, \frac{R^2 \mu c_r}{k_r t}, \frac{\beta_r (T_r - T_i)}{w}, \frac{R \sigma_3^2}{E \gamma}, \right.$$

$$\left. \frac{\rho Q}{\mu R}, \frac{\rho C_p Q}{\lambda R^2}, \frac{\rho^2 g \beta (T_r - T_i) w^3}{\mu^2}, \frac{\lambda_r t}{\rho_r C_{pr} R^2}, \right.$$

$$\left. \frac{Q t}{R^2 w}, \frac{P_o - P_i}{P_i + \rho g l \beta (T_r - T_i)} \right] \quad (3.1)$$

The principles of similarity state that if all non-dimensional groups on the right of Equation (3.1) are the same for model and full scale, then all the non-dimensional responses of the model will be the same as those of full scale. In that case, scale model test results can be reliably scaled up to full scale by inserting full scale values of the test parameters into the non-dimensional response groups and solving for the full scale responses. It is unlikely that all the non-dimensional test groups can be duplicated in the model, however, so the effects of distortions must be estimated. Any test restrictions imposed by the least important non-dimensional groups can be relaxed, if necessary, in order to develop a feasible model as long as the subsequent distortion is not important. This subject is discussed in the next section.

3.3 Implications of Modeling Law

It is clear that at least some geometric distortion must be tolerated; for example, a uniform reduction of fracture extent, well bore diameter, and

well length would result in a microscopic-size diameter for the model well bore. In order to assess the need for possible distortions and their implications for scale modeling, typical numerical values will be assigned the parameters and a nominal reduction of geometric scale of 1/500 will be assumed. A reservoir fracture of moderate size, $R = 200\text{m}$, will also be assumed. Nominal values for the parameters are given in Table 3.2, as taken from various LASL documents and standard references. (The rock porosity is the only parameter for which a numerical value could not be found. It must be considerably less, however, than the 10^{-5} inferred for LASL's reservoir system, which contains several large fractures.)

Geometric implications. As stated earlier, a uniform geometric reduction is not possible. With respect to the well-bore diameter and well depth, the geometric restriction is not serious. The well depth essentially just provides a hydrostatic pressure in addition to P_i ; thus its effects can be simulated in a model scale well of reasonable length by increasing the scaled injection pressure. The additional buoyancy in the extraction well caused by the decreased density of the heated fluid will also be out-of-scale. (The last term in Equation (3.1) models this effect.) Again, the distortion is not serious and can be compensated in scaling-up the model results. There is also no requirement to reduce the well bore diameter in proportion to the geometric reduction; any convenient model size, as long as it is small relative to the crack extent, is satisfactory. The pressure (flow) losses in the well bores will probably be out-of-scale, but the scale model results can be corrected readily.

According to Equation (2.1b), the width of a hydraulic fracture should scale as the square of the crack extent, i.e., as $(500)^{-1/2} = 1/22.4$, if the scale model E and γ are equal to full scale. If this is so, the scale model crack will be of a convenient width, about 0.5mm. Care must be taken in selecting flow and thermal parameters to allow for this different geometric reduction of R and w . For cases when the primary fractures of the system are natural faults and joints, Equation (2.1b) probably does not apply. In that case, the width is controlled both by the value of $P - \sigma_3$ and by the

Table 3.2 Nominal Values of Scale-Modeling Parameters

PARAMETER	NUMERICAL VALUE
Characteristic fracture length, R	200 m
Well-bore diameter, d_o	0.25 m
Well depth, z	3000 m
Characteristic fracture width, w	0.01 m
Characteristic fault spacing, d_s	10 m
Liquid density, ρ	$10^3 \text{ N} \cdot \text{s}^2/\text{m}^4$
Liquid viscosity, μ	$9 \times 10^{-4} \text{ N} \cdot \text{s}/\text{m}^2$
Liquid thermal conductivity, λ	$0.63 \text{ N}/\text{s} \cdot ^\circ\text{K}$
Liquid specific heat, C_p	$4.2 \times 10^3 \text{ m}^2/\text{s}^2 \cdot ^\circ\text{K}$
Liquid coefficient of linear expansion, β	$4 \times 10^{-4} (^\circ\text{K})^{-1}$
Liquid compressibility, c	$5 \times 10^{-4} \text{ m}^2/\text{N}$
Rock density, ρ_r	$2.7 \times 10^3 \text{ N} \cdot \text{s}^2/\text{m}^4$
Rock thermal conductivity, λ_r	$2.6 \text{ N}/\text{s} \cdot ^\circ\text{K}$
Rock specific heat, C_{pr}	$1.05 \times 10^3 \text{ m}^2/\text{s}^2 \cdot ^\circ\text{K}$
Rock coefficient of linear expansion, β_r	$8 \times 10^{-6} (^\circ\text{K})^{-1}$
Rock compressibility, C_r	$2.7 \times 10^{-11} \text{ m}^2/\text{N}$
Rock modulus of elasticity, E_r	$6 \times 10^{10} \text{ N}/\text{m}^2$
Rock surface energy, γ_r	100 N/m
Rock permeability, k_r	10^{-18} m^2
Rock porosity, ϕ_r	$\ll 10^{-5}$
Maximum earth stress, $\sigma_1 = \rho_r g z$	$8.0 \times 10^7 \text{ N}/\text{m}^2$
Minimum earth stress, $\sigma_3 \approx 1/2 \sigma_1$	$4.0 \times 10^7 \text{ N}/\text{m}^2$
Rock temperature, T_r	473°K
Liquid inlet temperature, T_i	300°K
Liquid inlet pressure, P_i	$1.0 \times 10^7 - 1.5 \times 10^7 \text{ N}/\text{m}^2$
Liquid outlet pressure, P_o	$0.1 \times 10^7 - 1.0 \times 10^7 \text{ N}/\text{m}^2$
Inlet flow rate, Q	$0.01 \text{ m}^3/\text{s}$
Gravity, g	$9.8 \text{ m}/\text{s}^2$

volume of liquid contained in the system. Thus, the width can be varied by adjusting the inlet and outlet pressures to obtain a desired scale factor for w .

Material property implications. The scale model reservoir will likely be fabricated from a material weaker than granite or natural rock. On the other hand, the scale model fluid will have physical properties not too much different from water. To work out the implications of the non-dimensional groups, all the strength-related properties of the rock will be assumed to be reduced proportionally while all the thermophysical properties will be assumed to be nearly full scale.

Earth stress implications. It is not practical to duplicate the high full-scale values of σ_1 , σ_2 , and σ_3 in the scale model. Based on dimensional considerations, it would seem, in fact, that the stresses must be reduced in proportion to the strength of the simulated rock, as shown for example by the term E/σ_1 in Equation (3.1). The implications of this will be worked out below.

Implications from physical effect interactions. The last eleven non-dimensional groups on the right of Equation (3.1) represent the essential hydro-thermo-mechanical interactions. These terms fix the scale factors and so must be examined for scaling contradictions or inconsistencies. The procedure used here is, first, assign each parameter a scale factor and, second, determine the scale factor of the parameter in terms of those for material properties and geometry. The symbol ζ will be given to the geometric scale factor; all other factors will have a subscript to indicate the parameter; for example, ζ_Q will represent the ratio of the scale model value of flow rate to the full scale value. All the rock strength parameters will be assigned the common scale factor ζ_E .

The term $(T_r - T_i)/T_r$ implies that

$$\zeta_{\Delta T} = \zeta_{T_r} \quad (3.2)$$

where $\Delta T = T_r - T_i$. In itself, the term is unimportant and ΔT can be scaled differently from T_r if it is convenient or necessary. Scaling of the absolute temperature is required only if one of the fluids is a compressible gas; that is not the case here.

The next term, $(P_o + \rho g l)k_r c_r / R^2$, is one of the factors controlling the permeation rate of fluid into the rock; see Equation (2.2). Since the rock permeability is very low, the flow into the rock is negligible, and k_r should be interpreted as the "permeability" of the fracture, $w^2/12$, and c_r as the compressibility of the fracture. The term then relates the storage of fluid in the fracture under pressure and the flow out of it. It implies that

$$\begin{aligned} \bar{\zeta}_{P_o} &= \zeta^2 / \zeta_{k_r} \zeta_{c_r} = \zeta^2 \zeta_E / \zeta_w^2 \\ &\approx \zeta_E \zeta \end{aligned} \quad (3.3)$$

where $\bar{P}_o = P_o + \rho g l$. The approximation given in Equation (3.3) follows from $\zeta_{c_r} = \zeta_E^{-1}$ and the assumption that $\zeta_w = (\zeta \zeta_Y / \zeta_E)^{1/2}$ as shown from Equation (2.1b). Since the surface energy is not a well-defined parameter, it is not clear that $\zeta_Y \approx \zeta_E$. On the other hand, the scale model tests of fracturing reported in Ref. 51 indicate that crack width is reduced much less proportionally than crack length, so the assumption that $\zeta_w = \zeta^{1/2}$ which follows from $\zeta_Y = \zeta_E$ will be used here as a working hypothesis. Note that pressure is not scaled in the same way as the dimensionally equivalent modulus of elasticity. Yet to satisfy other requirements, such as thermal expansion of the cracks, the pressure should scale similarly as the modulus of elasticity or the earth stresses. In fact, Equation (3.3) is not quite right since c_r is the compressibility of the cracks, not of the rock; and the crack compressibility depends on the fracture width and extent as well as on the modulus of elasticity. Thus, the distortion caused in the transient fluid storage by the requirement that $\bar{\zeta}_{P_o} = \zeta_E$ is much less than implied by inserting this relation directly into Equation (3.3). Testing would be required to evaluate the magnitude of the distortion.

The third term, $\mu c_r R^2 / k_r t$, is a non-dimensional time that indicates when the permeating distance of the flow into the rock becomes about equal to the fracture extent (Ref. 48). If R is interpreted as the distance between the outlet and the inlet of the reservoir, this parameter also scales the time constant for pressurization of the outlet well. It implies that the time scale is

$$\begin{aligned} \tau_t &= \tau \tau_{c_r} \tau^2 / \tau_{k_r} \approx \tau^2 / \tau_w^2 \tau_E \\ &\approx \tau / \tau_E \end{aligned} \quad (3.4)$$

This implied scaling of time is only important for transient effects, and may be ignored for steady flow conditions. (Again, the true scaling may be somewhat different since τ_{c_r} is not exactly τ_E^{-1} .)

The fourth term, $\beta_r (T_r - T_i) / w$ is one of the factors that control the way the fracture width changes as the rock cools. (The other requirements are that E/σ_3 and P/σ_3 be the same in model and full scale.) It implies that

$$\begin{aligned} \tau_{\Delta T} &= \tau_w / \tau_{\beta_r} \\ &\approx \tau_w \approx \tau^{1/2} \end{aligned} \quad (3.5)$$

The next term, $R \sigma_3^2 / E \gamma$ controls the influence of surface energy on hydraulic fracturing; see Equation (2.1). It implies that

$$\begin{aligned} \tau_{\sigma_3} &= (\tau_Y \tau_E / \tau)^{1/2} \\ &\approx (\tau_E^2 / \tau)^{1/2} \end{aligned} \quad (3.6)$$

Since P and σ_3 must be reduced in proportion in order to scale thermal expansion and thermal cracking, Equations (3.3) and 3.6) cannot be made to be

consistent. For practicality, it is desirable to have a much smaller σ_3 in the model than in full scale. The effects of surface energy probably cannot be scaled correctly in the model, and $P - \sigma_3$ will be a bigger fraction of the surface energy effect in model scale than in full scale; an experimental result similar to this was reported in Ref. 51, as was described previously in Section 2. Consequently, the fracturing pressures found in scale model tests cannot be used to predict full scale requirements. When the primary fracturing mechanism is the re-opening of natural cracks and faults, surface energy is an unimportant parameter, and the distortion of surface energy is not serious. Apparently, LASL's field site is of the latter kind, and it might be expected that most real hot dry rock reservoirs will be composed of cracked and faulted rock rather than being an expanse of isotropic, homogeneous, uncracked rock.

The term $\rho Q/\mu R$ is the familiar Reynolds number (noting that the velocity is Q/RW) and is a ratio of fluid inertial forces to viscous forces. It implies that

$$\zeta_Q = \zeta \zeta_\mu / \zeta_\rho = \zeta \quad (3.7)$$

The flow rate through the reservoir must be reduced. The flow velocity must scale correspondingly as

$$\zeta_V = \zeta_Q / \zeta \zeta_w \approx \zeta^{-1/2} \quad (3.8)$$

The velocity must be increased. However, in full scale, the flow is in the low laminar range, where inertial forces are negligible. (From Table 3.2, the nominal Reynolds number is about 50, for a velocity in the fracture of 0.005m/sec.) Therefore, an alternative scaling parameter is the ratio of pressure drop to viscous forces; that is, the flow rate scaling factor should be derived from a term such as $(P_0 - P_1)w^2R/Q$ rather than from the Reynolds number. This kind of scaling gives:

$$\begin{aligned} \zeta_Q &= \zeta_{\Delta p} \zeta \zeta_w^2 \approx \zeta_{\Delta p} \zeta^2 \\ &\approx \zeta^3 / \zeta_E \end{aligned} \quad (3.7a)$$

and

$$\zeta_v \approx \zeta^{3/2} / \zeta_E \quad (3.8a)$$

Both the flow rate and the velocity are reduced. This latter form of scaling is more restrictive than that given by Equations (3.7) and (3.8). It will be seen below that the more general form may be needed to satisfy other scaling requirements.

The next term, $\rho c_p Q / \lambda R$, is the Peclet number, the ratio of convective heat transfer in the flow direction to conduction heat transfer normal to the flow. It evidently requires that $\zeta_Q = \zeta$, in agreement with Equation (3.7). In full scale, the Peclet number is about 300, a value so large that it need not be exactly duplicated in model scale as long as convective heat transfer still dominates convection. This fact would allow some compromises to be made between Equations (3.7) and (3.7a), if needed, without changing the influence of the Peclet number.

The Grashof number, $\rho^2 g \beta (T_r - T_i) w^3 / \mu^2$, relates the effects of buoyancy and viscosity in natural convection. The nominal full scale value is 8.3×10^5 . If there were not a forced flow through the fracture, this value of Grashof number would indicate that buoyancy forces might just be large enough to induce a significant convective flow. But for a combined forced flow and a natural convection flow, a much larger Grashof number is needed to yield a noticeable buoyancy effect (Ref. 57). Thus, although Grashof scaling requires $\zeta_{\Delta T} = \zeta_w^{-3} = \zeta^{-3/2}$, such scaling is not actually needed. It might be noted that scaling $\zeta_{\Delta T}$ as $\zeta^{1/2}$ (see Equation (3.5)) will reduce the significance of buoyancy in the model.

The ninth term, $\lambda_r t / p_r c_{pr} R^2$, relates the depth of penetration of the temperature drawdown normal to the fracture to the extent of the fracture. It requires that

$$\zeta_t = \zeta_{p_r} \zeta_{c_{pr}} \zeta^2 / \zeta_{\lambda_r} \approx \zeta^2 \quad (3.9)$$

Since ζ_E is not equal to ζ^{-1} , this requirement is in conflict with that expressed by Equation (3.4). The liquid will penetrate the fracture system initially at a faster rate, relative to the temperature drawdown, in the model than in full scale. However, the contradiction is not important for two reasons. First, the drawdown is much slower in absolute terms than the liquid penetration rate. Thus, the bulk of the test time is controlled by Equation (3.9). Second, the liquid penetration of the reservoir system is an initial transient, and once steady flow is obtained, Equation (3.9) is again the more important.

The tenth term, Qt/WR^2 , expresses the time required to fill or empty the fracture system. Since from Equation (3.7), $\zeta_Q = \zeta$, the apparent time scale factor is $\zeta_t = \zeta^2 \zeta_w / \zeta_Q \approx \zeta^{3/2}$. Again, this is in conflict with Equation (3.8). Nonetheless, the distortion caused by two different time scales is very small during steady state flow conditions, since (1) filling or emptying the fracture is not needed for such conditions, and (2) the thermal drawdown is much slower in absolute terms than is the filling or emptying time. These conclusions even hold for a "huff-puff" mode of operation, where the reservoir is alternately filled and emptied.

The last term, $\Delta P/[P_i + \rho g \alpha \beta (T_r - T_i)]$, expresses the influence of buoyancy in the extraction well, which in full scale can reduce the pumping power required. The effect cannot be duplicated in the model, since the well depth is not scaled. But, after ΔP is measured in scaled tests, it is a straightforward calculation to correct the data for buoyancy.

3.4 Summary of Modeling Laws

The modeling laws that govern the construction and interpretation of scale-model tests were derived in the preceding discussion. Table 3.3 summarizes these laws. In the table, the scaling of rock strength properties, ζ_E , is taken as $\zeta^{1/2}$. All pressures and stresses are reduced by this

Table 3.3 Summary of Modeling Laws

 ζ = Geometric Scale Factor

PARAMETER	SCALE FACTOR	COMMENTS
1. Fluid properties: $\rho, \mu, \lambda, C_p, \beta, c$	1.0	It is not necessary that the full scale properties be duplicated; it is convenient
2. Rock thermo-physical properties $\rho_r, \lambda_r, C_{pr}, \beta_r$	1.0	Same as (1) above; a change of the scale factor does not affect the feasibility of modeling
3. Rock strength properties: $E_r, \gamma_r, 1/C_r$	$\zeta^{1/2}$	This choice keeps the scale factor for pressures, stresses, and rock strength equal
4. Rock permeability and porosity: k_r, ϕ_r	ζ^2 (permeability) 1.0 (porosity)	Scaling of rock permeability and porosity is unimportant as long as the scale-model values are near zero
5. Earth stresses: $\sigma_1, \sigma_2, \sigma_3$	$\zeta^{1/2}$	Same as (3) above
6. Down-hole fluid pressures: $p_o + \rho g z,$ $p_i^o + \rho g z$	$\zeta^{1/2}$	Same as (3) above
7. Fracture extent and spacing of natural faults and cracks: R, d_s	ζ	Fracture extent and initiation pressure may not scale exactly
8. Fracture width: w	$\zeta^{1/2}$	Theoretically-derived relation
9. Injection-well flow rate: Q	ζ	Reynolds-number scaling
10. Temperature difference between rock and inlet water: ΔT	$\zeta^{1/2}$	Preserves scaling thermal expansion and thermal-stress cracking; distorts buoyancy-viscous interaction
11. Time scale: t	ζ^2	Scales thermal drawdown time; sets time scale for interpreting results
12. Gravity: g	1.0	Laboratory requirement
13. Other geometric lengths: d_o, ℓ	--	Unscaled
14. Responses: $T_o - T_i$ e ΔQ ℓ_c d_c w_c ΔT_r	$\zeta^{1/2}$ $\zeta^{3/2}$ ζ ζ ζ ζ $\zeta^{1/2}$	Derived from scaling laws given above

same scaling factor, for the reasons explained previously. As was shown, this choice tends to distort the compressibility of the fracture system and therefore such transient effects as the inflation and deflation of the system. The magnitude of the distortion is believed to be small, but it cannot be evaluated completely in the absence of test data.

The scaling laws for the response parameters are also shown in Table 3.2. These have been derived from the modeling law, Equation (3.1), by using the scale factors derived for the "test" parameters.

4. FEASIBILITY OF SCALE MODELING

The preceding discussion has shown that the use of scale models as a method of investigating the hydro-thermo-mechanics of hot dry rock reservoirs is feasible although not without some difficulties. It is evident that two general kinds of models might be investigated:

- A simulated mass of homogeneous isotropic rock that does not contain any pre-existing cracks and faults, and
- A simulated mass of rock containing a realistic distribution of "natural" cracks and faults.

Each of these kinds of models will be discussed individually.

4.1 Homogeneous, Isotropic, Uncracked Rock Model

Some of the theoretical models reviewed in Section 2 are applicable primarily to a mass of homogeneous, isotropic, uncracked rock. For that reason, and as a means of obtaining a better understanding of the technology, a scale model study of the hydro-thermo-mechanics of a "perfect" hot dry rock reservoir may be useful, even though such a reservoir would never be found in nature.

A geometric scale of 1/500 or so will give a model that is large enough to allow adequate instrumentation and yet be of a reasonable laboratory size. The model would be perhaps 1m x 1m x 0.33m, with the smaller dimension corresponding to the direction of the least minimum earth stress (horizontal), and the hydraulic fracture would lie in the 1m x 1m plane. To construct a model of this size by casting a block of clear plastic, which would be needed for visual observations of the tests, would not be a simple matter. An easier-to-cast material such as a fine-grain mortar (concrete without aggregate) should probably be used, even though this would make visual observations of a test impossible. The strength and elastic properties of either plastic or mortar could be adjusted to give a suitable reduction in comparison with granite.

After fabrication, electrical strip heaters would be bonded to the surface of the model and covered with a protective layer. The model would then be loaded hydraulically, as shown in Figure 4, in order to simulate the earth stresses. Ordinary hydraulic actuators would be more than adequate to provide the scaled stresses for a pressure scale factor of $(1/500)^{1/2} = 1/22.4$. Although neither the stresses nor the temperatures would be perfectly uniform at the exterior surfaces of the model with this test arrangement, it is expected that they would be nearly uniform within the model at the fracture location. Several cutouts in the loading frames would be provided for instrumentation access.

Conducting tests with the model would be a small-scale version of a full-scale field test. The model would be heated to the appropriately scaled ΔT above the inlet fluid supply, after which the power to the heaters would be reduced to a level sufficient to replace environmental losses. A hole would be drilled in the model and cased with small-diameter metal tubing. Water would be used to create a hydraulic fracture. A second hole would be drilled to intersect the fracture at the desired height, after first locating the fracture with small test holes that could later be filled in or used as instrumentation sites. Other small diameter holes would be drilled in the vicinity of the fracture to serve as access to the primary stress and temperature instrumentation.

Such tests seem completely feasible. According to the discussion given in Section 3, realistic results could be obtained for all the parameters of interest with the exception of fracturing pressure, which is likely to be high in comparison to the correctly-simulated full scale pressures. Pressure drops, flow rates, heating rates, temperature drawdown, and thermal stress cracking could all be investigated. Nonetheless, these kinds of tests are not of first priority for two reasons:

1. Homogeneous, isotropic, uncracked rock reservoirs will never occur in nature,

2. The fracture geometry and well-to-well connection geometry are readily describable. Hence, mathematical modeling would probably be a more satisfactory method of investigating the hydro-thermo-mechanics.

The primary need for such scale model tests of homogeneous, isotropic reservoirs would be to gain an understanding of thermal-stress-cracking, which is perhaps the only area in which the basic physics needed for mathematical modeling is not in hand.

4.2 Naturally Cracked Model

Most hot dry rock reservoirs will contain natural faults and fractures. Thus, the scale model should simulate these features. Since the natural cracks will be distributed more or less at random, it is not practical to try to duplicate in the model any specific hot dry rock reservoir site. Instead, the model of a naturally-cracked reservoir should be used to help understand the anomalous results of LASL's field tests, as described in Section 2, and to create a data base for a system of known geometry and properties that could then serve to validate mathematical models.

Again, the general size of the model would be about $1\text{m} \times 1\text{m} \times 0.33\text{m}$, corresponding to about 1/500 scale. At this scale, a spacing of 10 meters between natural faults is reduced to 2 cm or so in the model. Although the model could be fabricated of an array of bricks whose minimum dimension is 2 cm, a more convenient size would be about 5 cm. The corresponding full scale spacing is 50 meters, which is still significantly smaller than the expected lateral extent of the hydraulic fracture. The bricks could be cast in several sizes, say $5\text{ cm} \times 15\text{ cm} \times 30\text{ cm}$ and $5\text{ cm} \times 10\text{ cm} \times 20\text{ cm}$, so as to provide a system of cracks of various spacing. Each brick would be ground smooth before assembly, since any roughness of the natural fractures should be very small when reduced to 1/500 scale. Clear plastic could be used to simulate rock, since castings of the size described are completely feasible. The bricks would be assembled into a large model by the use of glue or other

bonding agent to form the joints; the strength of the joint must be substantially less than that of the plastic in order to simulate the weak bonding of the natural faults.

The model would be heated and loaded as described previously; see Figure 4. In operation, a hole would be drilled in the model and cased with metal tubing to a pre-selected depth; the uncased part of the hole would intersect one of the "natural" vertical faults as well as of the uncracked plastic. After creating a fracture or opening a fault under hydraulic pressure, the model would be examined and a second hole drilled. The second hole could be made to intersect either the fracture or one or more natural faults that themselves intersected the fracture; the latter method apparently corresponds to the LASL field tests. In addition to the instrumentation positioned beforehand during assembly of the bricks, access holes could be drilled in the vicinity of the fracture system for further instrumentation.

This kind of model could be used for various kinds of short term or long term tests, and the fracture geometry could be altered readily by replacing the affected bricks. According to the discussion given in Section 3, most of the important phenomena of interest can be simulated adequately in tests conducted with this model. Flow distribution and losses, pressure drops, thermal drawdown, and thermal stress cracking could all be studied. Such a scale model might also be the only practical way to study the relative advantages of the "huff-puff" mode of operation compared to a steady flow mode. Further, the effect of different fracture and connection geometries could be investigated. Many of the observations could be made visually as an aid in understanding the responses. Of course, the load frame would have to be removed temporarily to make visual observations. During these observations, the injected liquid could be colored with a series of dyes to study the flow patterns and system volume.

All in all, it appears that scale model tests of a hot dry rock reservoir containing natural faults and cracks are feasible both from the point of view of simulation reliability and laboratory practicality. The tests would also be valuable as an aid in understanding hot dry reservoir behavior and in validating mathematical models.

5. CONCLUSIONS AND RECOMMENDATIONS

Detailed evaluations were conducted of field-test data and mathematical models relevant to energy extraction from hot dry rock reservoirs. In conjunction with independent mathematical modeling and physical reasoning, these evaluations formed the basis of a similitude study aimed at determining the feasibility of scale models as a method of investigating the hydro-thermo-mechanics of hydraulically-fractured hot dry rock reservoirs. The similitude analysis considered the interacting effects of

- fracture and well-bore geometry
- fluid and rock physical properties
- fluid flow rates and fluid storage
- fluid viscous and buoyancy forces
- heat transfer in the rock and fluid
- fracture initiation
- thermal stress cracking
- naturally-occurring faults
- thermal drawdown
- transient and steady state operation

Each non-dimensional similarity parameter was examined in turn, and scale factors were derived for the construction of scale models and the interpretation of test results. Several contradictions in the derived scale factors were noted and resolved by examining the effect of relaxing one or more of the scale factor requirements. It was concluded that an adequate simulation could be obtained for all the important parameters, with the exception that the pressure required to initiate a hydraulic fracture would be higher than indicated by the pressure scale factor. Even this distortion would be negligible in the case of a reservoir containing natural faults and cracks, since then the natural fractures would tend to open under pressure rather than a new fracture being created; the opening of the natural fractures can be simulated realistically.

The main conclusions of the similitude study can be summarized in terms of the geometric scale factor ζ , the ratio of the scale model fracture extent to full scale:

- Fracture extent - ζ
- Fracture width - $\zeta^{1/2}$
- Rock strength properties - $\zeta^{1/2}$
- Earth stresses - $\zeta^{1/2}$
- Applied pressures - $\zeta^{1/2}$
- Fluid flow rates - ζ
- Difference between rock
and fluid temperature - $\zeta^{1/2}$
- Natural fault spacing - ζ

The feasibility of two different kinds of scale models was investigated. One kind of model would simulate a reservoir composed of homogeneous, isotropic rock containing no natural faults. It was shown that such a model would provide an adequate simulation of full scale behavior and is practical in terms of laboratory sizes and procedures. As noted above, fracturing pressures probably could not, however, be simulated. Visual observations of the system behavior also would not be possible because of the impracticality of casting a single plastic block in the size required. More importantly, the model would simulate an overly-idealized hot dry rock reservoir, one, in fact, that might be better analyzed by mathematical models.

The second and more important kind of scale model would simulate a reservoir containing a variety of natural faults and cracks such as is found in nature. This model could be fabricated from a number of smaller, specially cast plastic bricks weakly joined together. Visual observations could be made periodically with such a model. Fluid flow, pressure drops, fluid losses, thermal drawdown, thermal-stress cracking, transient and steady state conditions, and "huff-puff" operation could all be studied with such a model. The effects of different fracture geometry and well-to-well connections could also be readily investigated.

It is recommended that laboratory tests be conducted to verify the conclusions of this feasibility study. The model should simulate a hot dry rock reservoir containing natural cracks and fractures. Small plastic bricks, say of the order of 5 cm x 15 cm x 30 cm, should be used to construct a model of overall size of at least 1 mm x 1 m x 0.33 m, with the smaller dimension corresponding to the direction of minimum earth stress (horizontal). The model should be heated and stressed by a suitable loading frame in order to simulate geothermal temperatures and earth stresses. It should contain adequate instrumentation (temperature, strain, etc.) in the vicinity of the fracture system to resolve the hydro-thermo-mechanics of the reservoir. The model should be operated in several modes in order to study

- opening and closing of natural fractures
- direct and indirect connection to the extraction well
- "huff-puff" operation
- transient and steady state flow
- thermal drawdown
- thermal stress cracking

At the conclusion of the recommended scale model validation phase, there will be available scale modeling techniques, test procedures, and data interpretation methods that can be used as an aid in solving the technical and economic problems of hot dry rock geothermal reservoirs.

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