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SU(3) IN SHELL–MODEL CALCULATIONS

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ABSTRACT

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The essential steps in the formalism for performing multi–shell calculations in an SU(3) basis are outlined and examples of applications in which the SU(3) classification aids the physical interpretation of structure calculations are given.

1. Introduction

The SU(3) model introduced by Elliott[1] has the physically intuitive feature that the states of a given SU(3) representation can be projected from an intrinsic state constructed from asymptotic Nilsson orbits[1–4]. Thus, the close connection of the SU(3) shell–model basis to collective models, deformations and rotational spectra is made explicit. The SU(3) symmetry is exact for quadrupole–quadrupole forces and is broken by the one–body spin–orbit force in particular, as is the SU(4) symmetry established by the strong space–exchange component of the central two–body interaction. In practice, the wave functions of states belonging to well established rotational bands in light–nuclei can be obtained by projection from intrinsic states built from Nilsson orbits which include the effects of realistic single–particle $\vec{\ell}\cdot\vec{s}$ and $\vec{\ell}^2$ terms. In SU(3) shell–model terms, these states correspond to a mixture, similar for all angular–momentum states in the band, of a number of SU(3) representations, and possibly, in the case of non–zero intrinsic spin, to the mixture of different L values necessary to make states with a specific projection of total angular momentum $K_J(=K_L+K_S)$[5].

The approximate goodness of SU(3) and SU(4) symmetries for low–lying states in light nuclei[4,6] provides a strong incentive to perform shell–model calculations in a basis which is classified according to these symmetries. The use of such a basis has advantages for the truncation of large shell–model spaces, the elimination of spurious center of mass states and the calculation of spectroscopic amplitudes for multinucleon transfer reactions. A basic requirement for such calculations is to have codes available for the Racah algebra (Wigner and recoupling coefficients) of the groups involved in the basis classification. The techniques for performing such calculations, and the important contributions of Ted Hecht to make them possible, are outlined in the next section. Then follow some examples in which the SU(3) classification is useful to obtain an understanding of the results of large–scale shell–model calculations.

DISCLAIMER

2. Shell–Model Techniques

Equivalent operator techniques[2,4,5] enabled small–basis shell–model calculations with simplified effective interactions to be made using projected SU(3) wave functions. However, to perform large–basis SU(3) shell–model calculations for general effective interactions, it is almost a necessity to use fractional parentage techniques in some form or other. An interest in the low–lying $p^{-2}(sd)^4$ states of ^{18}F and their mixing with $(sd)^2$ states led me to take this approach[7] for LS–coupling basis states of the form

$$\mathcal{A}|p^{n_1}\,(\lambda_1\mu_1)\,\beta_1 T_1 S_1, (sd)^{n_2}\,[f_2]\,\alpha_2\,(\lambda_2\mu_2)\,\beta_2 T_2 S_2; (\lambda\mu)\,\kappa L S J T\rangle, \qquad (1)$$

with different major shells coupled in $(\lambda\mu)$, T and S. Two particles are split off using the appropriate cfps. In the case of the sd shell, the cfps factorize into a weight factor and a product of SU(6) $\supset$ SU(3) and SU(4) $\supset$ SU(2) $\times$ SU(4) Wigner coefficients. The one–particle SU(6) $\supset$ SU(3) coefficients have been tabulated by Akiyama[8] and the SU(4) $\supset$ SU(2) $\times$ SU(2) coefficients by Jahn and van Wieringen[9] and by Hecht and Pang[10]. The next steps, which require SU(3) recoupling and SU(3) $\supset$ R(3) Wigner coefficients respectively, are to isolate the two particles by recoupling in $(\lambda\mu)$, T and S and then to uncouple them from the remaining particles. The calculational technique, although somewhat inelegant and inefficient, was made possible by combining the results of Hecht[11] for SU(3) $\supset$ SU(2) Wigner coefficients and of Asherova and Smirnov[12] for the transformation $\langle(\lambda\mu)\epsilon\Lambda K|(\lambda\mu)KLM\rangle$ between the SU(3) $\supset$ SU(2) and SU(3) $\supset$ R(3) bases, together with the scheme of Vergados[13] for obtaining an orthonormal set of SU(3) $\supset$ R(3) states labelled by κ, to calculate SU(3) $\supset$ R(3) Wigner coefficients for the product $(\lambda_1\mu_1) \times (\lambda_2\mu_2) \rightarrow (\lambda_2\mu_3)$ with $\mu_2 = 0$ or 1 and also the needed SU(3) recoupling coefficients.

The SU(3) codes just described were soon made obsolete by Draayer and Akiyama who provide a complete solution[14] for the Racah algebra of SU(3) in the practical form of computer codes[15] for the Wigner and U coefficients. Now, the most efficient way to perform SU(3) shell–model calculations was to expand the effective interactions into SU(3) tensors[16,17,18]:

$$V = -1/2 \sum_{\alpha\beta\gamma\delta} \langle\alpha\beta|V|\gamma\delta\rangle a^\dagger_\alpha a^\dagger_\beta a_\gamma a_\delta \qquad (2a)$$

$$= \sum_\alpha C(\alpha)\, T^{(\alpha)} \qquad (2b)$$

where α labels the quantum numbers of a coupled product $T^{(\alpha)}$ of creation and annihilation operators and $C(\alpha)$ is a linear combination of two–body matrix elements (given in some initial representation such as jj coupling). The computation of many–body matrix elements for the basis in Eq. (1), or its generalization to more major shells, can then proceed along the lines of the Oak-Ridge–Rochester shell–model code[19] which uses a jj–coupling basis and a spherical tensor expansion of the interaction. The essential steps, specialized to SU(3), are

Wigner–Eckart Theorem

$$\langle(\lambda\mu)\,\kappa L||T^{(\lambda_t\mu_t)\kappa_t L_t}||\,(\lambda'\mu')\,\kappa' L'\rangle$$
$$=\sum_{\rho}\langle(\lambda'\mu')\,\kappa' L'\,(\lambda_t\mu_t)\,\kappa_t L_t||\,(\lambda\mu)\,\kappa L\rangle_{\rho}\;\langle(\lambda\mu)\,|||T^{(\lambda_t\mu_t)}|||\,(\lambda'\mu')\rangle_{\rho} \qquad (3)$$

Coefficients of Fractional Parentage

In a schematic form, for n particles in a shell,

$$\langle n|||a^{\dagger}|||n-1\rangle=\sqrt{n}\langle n-1,1|\}n\rangle \qquad (4)$$

where the cfps are products of weight factors and Wigner coefficients as described previously. The matrix elements of the properly phased annihilation operator, transforming as $(0Q)$, may be simply related to those of the creation operator.

Intermediate State Expansion

For SU(3) tensor operators operating in the same shell

$$\langle\alpha\,(\lambda\mu)\,|||\left[R^{(\lambda_r\mu_r)}\times S^{(\lambda_s\mu_s)}\right]^{(\lambda_t\mu_t)\rho_t}|||\alpha'\,(\lambda'\mu')\rangle_{\rho}$$
$$=(-)^{\lambda_r+\mu_r+\lambda_s+\mu_s+\lambda_t+\mu_t+\rho_t^{\max}-\rho_t}\sum_{\beta(\bar{\lambda}\bar{\mu})\rho_r\rho_s}$$
$$U\left((\lambda\mu)\,(\mu_r\lambda_r)\,(\lambda'\mu')\,(\mu_s\lambda_s),(\bar{\lambda}\bar{\mu})\,\rho_r\rho_s\,(\mu_t\lambda_t)\,\rho_t\rho\right)$$
$$\langle\alpha\,(\lambda\mu)\,|||R^{(\lambda_r\mu_r)}|||\beta\,(\bar{\lambda}\bar{\mu})\rangle_{\rho_r}\;\langle\beta\,(\bar{\lambda}\bar{\mu})\,|||S^{(\lambda_s\mu_s)}|||\alpha'\,(\lambda'\mu')\rangle_{\rho_s} \qquad (5)$$

This relationship is used to get the reduced matrix elements of strings of creation and annihilation operators, known as single–shell matrix elements, which are computed once and for all and stored for future use.

Coupled Products

For SU(3) tensor operators operating in distinct spaces

$$\langle(\lambda_1\mu_1)\,(\lambda_2\mu_2)\,;(\lambda\mu)\,\rho|||\left[R_1^{(\lambda_r\mu_r)}\times S_2^{(\lambda_s\mu_s)}\right]^{(\lambda_t\mu_t)\rho_t}|||\,(\lambda_1'\mu_1')\,(\lambda_2'\mu_2')\,;(\lambda'\mu')\,\rho'\rangle_{\tilde{\rho}}$$

$$=\sum_{\rho_1\rho_2}\begin{pmatrix}(\lambda_1'\mu_1') & (\lambda_r\mu_r) & (\lambda_1\mu_1) & \rho_1\\ (\lambda_2'\mu_2') & (\lambda_s\mu_s) & (\lambda_2\mu_2) & \rho_2\\ (\lambda'\mu') & (\lambda_t\mu_t) & (\lambda\mu) & \tilde{\rho}\\ \rho' & \rho_t & \rho & \end{pmatrix}\langle(\lambda_1\mu_1)\,|||R_1^{(\lambda_r\mu_r)}|||\,(\lambda_1'\mu_1')\rangle_{\rho_1}$$
$$\langle(\lambda_2\mu_2)\,|||S_2^{(\lambda_s\mu_s)}|||\,(\lambda_2'\mu_2')\rangle_{\rho_2} \qquad (6)$$

This relationship is used iteratively to separate the last shell from all the inner shells treated as a single space and so on. The reduced matrix elements in Eq. (6) have been precomputed using Eq. (5). The 9 – $(\lambda\mu)$ coefficients are calculated using an addition[20] to the package of Akiyama and Draayer[15].

A shell–model calculation is defined by the model–space, single–particle energies and two–body matrix elements. The first task is to compute the $C(\alpha)$ coefficients of Eq. (2) for operators in which the creation and annihilation operators are coupled in the same order as the shells on the basis states of a generalized Eq. (1). It is worth noting that for a translationally invariant interaction the $C(\alpha)$ coefficients are non–zero only for self–conjugate tensors $(\lambda\lambda)$ if $\Delta\hbar\omega = 0$. Similarly, if $\Delta\hbar\omega = 2$ only the $(\lambda\lambda - 2)$ tensor coefficients are nonzero. The one–body terms are treated as a special case of a two–body interaction. The SU(2) analog of Eq. (6) is used to separate the space and spin reduced matrix elements. Then Eq. (4) is used to isolate the dependence of the many–body matrix elements on orbital angular momentum κL. The remaining multi–shell reduced matrix elements are then common to the energy matrices for all J.

Spurious Center–of–mass States

The elimination of spurious center–of–mass states, always a problem in extended shell–model spaces, can be handled with relative ease in an SU(3) basis. The center–of–mass Hamiltonian $H_{\rm cm}$ is a scalar with respect to SU(3), spin and isospin so that it does not couple states with different $(\lambda\mu)TS$. This means that spurious center–of–mass states may be eliminated exactly (within an harmonic oscillator framework) from shell–model spaces truncated according to $(\lambda\mu)$.

In studies of $1\hbar\omega$ excited states in p–shell shell nuclei, Millener and Kurath[21] used the method of Hecht[22] to construct the spurious states by operating on $0\hbar\omega$ states with the center–of–mass creation operator $A^\dagger$. Non–spurious states were obtained by Schmidt orthogonalizing to the explicitly constructed spurious states. However, it is much easier, especially for multi–$\hbar\omega$ excited configurations, to diagonalize a linear combination of $H_{\rm CM}$ and the space–exchange operator $\sum_{i<j} P_{ij}$ in the subspaces of states with the same $(\lambda\mu)TS$. Actually, a matrix of multi–shell reduced matrix elements is diagonalized since the process is independent of KLJ. The Hamiltonian matrix is then transformed to the basis of non–spurious states so obtained and diagonalized. The spurious states themselves can be used to study the effects of their elimination on sum rules.

3. Applications

The SU(3) shell–model codes have been used mainly for nuclei in the range $8 \leq A \leq 21$, where the dominant components of low–lying states are of the type exhibited in Eq. (1) although excitations involving the $0s$ and pf shells are included.

3.1. Multi–nucleon Spectroscopic Amplitudes

One early application involved a study[23] of the structure of light nuclei using multi–nucleon transfer reactions with heavy ion beams at about 10 MeV/nucleon. In these reactions, one to four nucleons are transferred onto p–shell targets with some or all of the transferred nucleons entering the sd–shell. The shell–model wave functions were used to calculate multi–nucleon transfer amplitudes for the

case in which the k nucleons transferred to a target with $A-k$ nucleons are in internal $0s$ states with respect to one another. Essentially, a cfp of the form $\langle A-k,\, k(Q0)|\}A\rangle$, where $Q=\sum_{i=1}^{k} Q_i$ is the total number of quanta associated with the orbits that the transferred particles enter, must be calculated; the Q quanta are then associated with the relative motion of the k–particle cluster with respect to the $A-k$ core. As a further example, Hecht and Braunschweig[24] have made a study of few–nucleon SU(3) parentage coefficients for core–excited states with specific SU(3) quantum numbers in sd–shell nuclei. Hecht[25] has also presented a method for the expansion of properly antisymmetrized cluster–model wave functions in terms of SU(3) shell–model wave functions which illustrates the strength of the SU(3) basis when coordinate transformations are to be made using harmonic oscillator wave functions.

3.2. Inelastic Scattering

The inelastic scattering of medium energy electrons, protons and pions from nuclei has, with increasing beam intensity and energy resolution in the past decade or so, provided a probe of nuclear transition densities over a wide range of momentum transfer. The nuclear structure quantities needed for the analysis of such data are the one–body density–matrix elements (OBDME) connecting the initial (ground) state to the final state. Studies have been made for nearly all the stable light nuclei which can be used as targets. The shell–model calculations use at least the full $0\hbar\omega$ and $1\hbar\omega$ model spaces. The $1\hbar\omega$ space consists of $p^{n-1}(sd)$ and $s^{-1}p^{n+1}$ configurations for p–shell nuclei and $p^{-1}(sd)^{n+1}$ and $(sd)^{n-1}(pf)$ configurations for sd–shell nuclei.

The plane–wave Born approximation (PWBA) is quite accurate for the analysis of electron scattering on light nuclei. For harmonic oscillator wave functions, the form factors can be expressed in the form

$$F = y^{a/2} p(y)\, e^{-y}, \tag{7}$$

where $y=(bq/2)^2$, $p(y)$ is a polynomial in y, and a=0 or 1. In this case, it is particularly useful to have the OBDME in the SU(3) scheme with the creation and annihilation operators coupled to $(\lambda\mu)\Delta L\Delta S\Delta J\Delta T$. The LS classification separates the contributions to the charge density or convection current from those to the magnetization current according to ΔS=0 or 1. For a pair of orbits with Q_1 and Q_2 quanta, the number of terms in the polynomial is controlled by $(\lambda\mu)$ and the polynomial is independent of ΔL for a given $(\lambda\mu)$. Very often, in fact, a single SU(3) OBDME controls the model-space contribution to the electromagnetic matrix element. Contributions from outside the model space are very important and must be incorporated (perturbatively) into the effective one–body operators. Dominantly isoscalar longitudinal excitations need to be enhanced in magnitude while excitations with $(\Delta T\,\Delta S)$= (01), (10) or (11) need to be quenched. The core polarization corrections also cause the form factors to fall off faster or slower beyond the main peak in the case of enhancement or quenching respectively, in agreement with the trend needed to fit the data. Examples can be found in a detailed investigation[26] of inelastic electron scattering from ^{13}C.

The SU(3) classification is particularly useful for longitudinal transitions with ΔL=1 (C1 transitions). The $p \rightarrow sd$ transition densities from the $1\hbar\omega$ model space calculations typically underestimate such form factors by an order of magnitude (e.g., see Fig. 12 of Ref. 26 or Fig. 8 of Ref. 27). The $p \rightarrow sd$ OBDME can have $(\lambda\mu)$= (21) or (10) and, if we take $p \rightarrow sdg$ excitations with $(\lambda\mu)$= (30) as an example of core polarization corrections, we can write[27]

$$F_{C1} = \frac{1}{Z}\sqrt{\frac{2}{5}}y^{1/2}e^{-y}\left[Z^{(21)}y - \sqrt{20}Z^{(10)}\left(1-\frac{y}{2}\right) - \sqrt{12}Z^{(30)}\left(y-\frac{y^2}{3}\right)\right], \quad (8)$$

where $Z^{(\lambda\mu)} = e_0 Z_0^{(\lambda\mu)} + e_1 Z_1^{(\lambda\mu)}$ in terms of the OBDME $Z_{\Delta T}^{(\lambda\mu)}$ and charges $e_{\Delta T}$. For low-lying states, the (10) amplitude is small; the total isoscalar amplitude is zero because the shell-model states are nonspurious and the isovector amplitude is small because $E1$ transitions are weak between low-lying levels. The model-space (21) amplitude gives a form factor which is like that of a $C3$ transition but too weak. This amplitude will be enhanced by $2p2h$ correlations in the ground state but the $p \rightarrow sdg$ amplitude is particularly effective because of the large coefficient of $\sqrt{12}$; the $y^{5/2}$ term makes the form factor fall off faster for momentum transfers above the peak and moves the node in the radial transition density to larger radius.

Finally, it is possible[28] to obtain OBDME in a form which is appropriate for use with single-particle radial wave functions (such as Woods-Saxon) which are a function of the relative coordinate, $r_{i,C} = (A/(A-1))(r_i - R_A)$, between the particle and the A-1 core. Then, the standard center-of-mass correction is no longer needed in PWBA for (e,e'). The relative OBDME are also appropriate for use in DWBA codes, which integrate over relative coordinates.

3.3. Multi-$\hbar\omega$ Calaculations

In early applications of the SU(3) model, the leading configurations with high $(\lambda\mu)$ and maximum spatial symmetry were assumed for low-lying multi-particle, multi-hole configurations in ^{16}O and neighboring nuclei. For example, Brown and Green[29] took the $[p^{-4}(04)\times(sd)^4(80)](84)$ and $[p^{-2}(02)\times(sd)^2(40)](42)$ configurations to be the dominant configurations in the excited 0^+ states at 6.05 MeV and 12.05 MeV respectively. SU(3) shell-model calculations with much larger bases confirm that, as expected, the above configurations dominate in the lowest $4\hbar\omega$ and $2\hbar\omega$ 0^+ states and in the band members with higher spin; there is also a $4p4h$ band with K=2 beginning with the 2^+ state at 9.85 MeV. The wave functions of the $4p4h$ states have large overlaps with with states obtained by weak-coupling the lowest p^{-4} and $(sd)^4$ eigenstates[30]. The weak-coupling basis takes into account the strong correlations of particles in the same major shell and this, coupled with the fact that the isospin average of the ph interaction is very weak, makes it a very economical basis to describe low-lying $mpnh$ states; it is not possible, however, to make an exact elimination of spurious center-of-mass states. The wave functions of the $4p4h$ states also have large overlaps with α-cluster states although to describe a realistic cluster-core relative wave function requires very many oscillator quanta of excitation[31] and this is not a practical proposition for conventional shell-model calculations.

If we believe we have a good description of the dominant components of all low–lying states of whatever nominal $\hbar\omega$ excitation energy, the next problem is to describe the mutual mixing of these states. This can be a subtle question for states whose dominant components don't mix directly via a two-body interaction. Again, this can be illustrated using the Brown and Green model for ^{16}O. In the excitation of the first 2^+ state from the ground state, the $2p2h \rightarrow 2p2h$ and $4p4h \rightarrow 4p4h$ amplitudes are in phase and of comparable importance despite the small intensity ($\sim 1.7\%$) of $4p4h$ components in the ground state. The central interaction connecting p^2 to $(sd)^2$ configurations contains (42) and (20) SU(3) tensors. In the Brown and Green model, only the (42) tensor, which is much "weaker" than the (20) tensor is operative. However, in the large basis shell–model calculations which are possible with the current computer facilities, all configurations up to some $n\hbar\omega$ excitation energy may be included. For example, Haxton and Johnson[32] have recently managed to perform a complete $(0+2+4)\hbar\omega$ calculation for ^{16}O. In such calculations, the (20) component of the interaction causes strong mixing between $\Delta\hbar\omega$=2 configurations.

Table 1: Diagonal energies of $2\hbar\omega$ configurations relative to the ^{16}O closed shell and off-diagonal matrix elements with the closed shell. The energies are in units of MeV.

Configuration	Diagonal Energy	$\langle 2\hbar\omega \vert V \vert 0\rangle$
$\vert(4\ 2)\rangle$	14.0	-4.3
$\vert GMR\rangle$	13.3	-2.5
$\vert 2p2h\rangle$	22.4	10.4
$\vert 1p1h\rangle$	31.5	2.3

To illustrate difficulties that the strong $\Delta\hbar\omega$=2 mixing can cause if treated in an inconsistent manner, we look at a model $(0+2)\hbar\omega$ problem for the J=0,T=0 states of ^{16}O. The basis consists of the closed shell and the [f]=[4444] $2\hbar\omega$ configurations with (42) and (20) SU(3) symmmetry which, for a translationally invariant central interaction and harmonic oscillator wave functions, are the only $2\hbar\omega$ configurations which have non-zero matrix elements with the closed shell. There are five (20) states, three of which are $2p2h$ and two $1p1h$. Of these two linear combinations correspond to singly–spurious and doubly–spurious states. The important non-spurious states, for our purposes, are the monopole state $\vert GMR\rangle$ and a purely $2p2h$ state $\vert 2p2h\rangle$. The remaining non-spurious state $\vert 1p1h\rangle$ is mainly $1p1h$ in nature. The diagonal energies, relative to the energy of the closed shell, of the four non-spurious $2\hbar\omega$ states are given in Table I. Also given are the off–diagonal matrix elements with the closed shell. These include only the effect of the $p^2 \rightarrow (sd)^2$

interaction, the matrix elements of $1p1h$ configurations with the closed shell having been set to zero. Details of the calculation and the explicit structure of the (20) states are given elsewhere[33].

From Table I, we note that the $2p2h$ (20) configuration has a large off-diagonal matrix element with the closed shell. If the $0\hbar\omega$ and $2\hbar\omega$ are allowed to mix, the ground state is driven down by about 10 MeV and picks up a roughly 25% admixture of $2\hbar\omega$ configurations which is dominantly (20) in nature. The first excited 0^+ state, which is mainly (42), is now at around 21 MeV in excitation energy. Thus, the spectrum and the wave functions have been distorted by the strong mixing. We need the $4\hbar\omega$ configurations which connect to the (42) configuration via (20) tensors to push on the (42) state and so on (a slowly convergent process).

To understand better the nature of the $2p2h$ correlations in the ^{16}O ground state, we note that the strongly admixed $|2p2h\rangle$ configuration of Table I has a large overlap with the $2\hbar\omega$ state which is formed by operating on the closed shell with two successive E3 operators, the dominance of such correlations having been demonstrated in early RPA calculations.[34] As indicated above, it would be very difficult to include such correlations consistently in all low-lying excited states. However, the concept of an effective charge for E3 transitions, to take into account these correlations and $1p1h$ excitations through $3\hbar\omega$, works well throughout the mass region of interest. The same is true for E2 transitions, where the effective charge takes into account $1p1h$ excitations through $2\hbar\omega$. We would argue that it is best to subsume the influence of high-lying configurations into effective one-body operators and to concentrate on getting a good description of the mutual mixing of low-lying configurations (of whatever nominal $\hbar\omega$ excitation energy).

Another point of note in Table I is the low energy of the monopole state. The excitation energy of the giant monopole resonance (GMR) is directly related to the compressibility of the nucleus, and thus can be determined only by self-consistent calculations using saturating effective interactions. Self-consistent RPA calculations, which generally use density-dependent interactions, predict[35] the GMR to lie above 20 MeV of excitation in ^{16}O. On the other hand, as documented by Kirson,[36] if harmonic oscillator wave functions are used with standard shell model effective interactions the GMR is shifted strongly downwards from its unperturbed position. This is the situation that applies in our example. In self-consistent calculations, self consistency for the single-particle energies and wave functions (the $1p0f$ orbits are unbound), together with density dependence of the effective interaction, play important roles in determing the energy of the monopole state. A further difficulty arises because the matrix element between a $2\hbar\omega$ $1p1h$ state and the $0\hbar\omega$ state generally contains contributions from both the kinetic and potential energy operators. The matrix elements of these two operators are large (roughly 40 MeV in our example) but of opposite sign, so that the resulting values of $\langle 1p1h\,|T+V|\,0\hbar\omega\rangle$ cannot be determined reliably in non-self-consistent calculations.[37] If no constraints are introduced in full $(0+2)\hbar\omega$ calculations to avoid these problems, the GMR is invariably predicted to lie very low in the spectrum

and large percentages of $1p1h$ excitations are predicted in the ground state wave functions.

Haxton and Johnson[32], in their full $(0+2+4)\hbar\omega$ calculation, allowed the effective interaction to operate only within the p–sd shell-model space. Thus, they avoid the problems with $1p1h$ amplitudes referred to above. They do however obtain strong configuration mixing in their ground state wave function, which consists of 42% $0p0h$, 45% $2p2h$ and 13% $4p4h$ configurations.

4. Summary

The formalism in Section 2 for multi–shell calculations in an SU(3) basis is an extension of that developed and used, with an emphasis on single oscillator shells or pseudo shells, by Hecht and his collaborators[16–18]. The technique is applicable to any group classification scheme for which the Wigner and recoupling coefficients are available[16]. In the case of SU(3), the codes of Akiyama and Draayer[15] make the calculations possible. Since SU(4) $\supset$ SU(2) $\times$ SU(2) is, like SU(3), an approximately good symmetry for many states of light nuclei, it is very desirable to couple the shells of Eq. (1) to a total $[\tilde{f}]$ and extend the formalism of Section 2 to SU(4). Although the results of Hecht and Pang[10] are sufficient for a number of problems of physical interest (see e.g. Ref. 18), a full application of the SU(4) tensor algebra awaits the development of codes for the SU(4) $\supset$ SU(2) $\times$ SU(2) Wigner coefficients and SU(4) recoupling coefficients.

The applications in Section 3 were chosen to illustrate the usefulness of SU(3) in understanding the structure of effective operators, such as the Hamiltonian and transition operators, and nuclear states. By implication, the strategy for shell-model calculations should be to restrict bases to energetically favored configurations and to take into account other configurations perturbatively. Certainly, we know that the concept of an effective charge can work well for low-lying states over a range of mass numbers. The trick is to make the separation in an optimum way. For light nuclei, the SU(3) and SU(4) symmetries provide a way to choose energetically favored configurations for each $\hbar\omega$ in excitation energy.

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