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VERIFICATION OF THE NIKE3D STRUCTURAL ANALYSIS CODE
BY COMPARISON AGAINST THE ANALYTIC SOLUTION FOR A SPHERICAL
CAVITY UNDER A FAR-FIELD UNIAXIAL STRESS

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ABSTRACT

The original scope of this task was to simulate the stresses and displacements of a hard rock tunnel experimental design using a suitable three-dimensional finite element code. NIKE3D was selected as a suitable code for performing these primarily approximate linearly elastic 3D analyses, but it required modifications to include initial stress, shear traction boundary condition and excavation options.

During the summer of 1988, such capabilities were installed in a special version of NIKE3D. Subsequently, we verified both the LLNL's commonly used version of NIKE3D and our private modified version against the analytic solution for a spherical cavity in an elastic material deforming under a far-field uniaxial stress. We find the results produced by the unmodified and modified versions of NIKE3D to be in good agreement with the analytic solutions, except near the cavity, where the errors in the stress field are large. As can be expected from a code based on a displacement finite element formulation, the displacements are much more accurate than the stresses calculated from the 8-noded brick elements. To reduce these errors to acceptable levels, the grid must be refined further near the cavity wall. The level of grid refinement required to simulate accurately tunnelling problems that do not have any spatial symmetry in three dimensions using the current NIKE3D code is likely to exceed the memory capacity of the largest CRAY 1 computers at LLNL.

1.0 TASK BACKGROUND

The original scope of this task [1] was to compute, using a suitable three-dimensional finite element code, the stresses and displacements induced by the excavation of a hard rock mine tunnel. The purpose of such simulations was to assist in the experimental design of an excavation response experiment to be performed at the Underground Research Laboratory (URL) at Lac Du Bonnet, Manitoba, Canada, under an initial agreement between the U. S. Department of Energy (DOE) and the Atomic Energy Commission, Ltd., (AECL), of Canada. However, on July 21, 1988, LLNL received instructions from DOE to stop such simulations until a new agreement with AECL could be negotiated. In response to these new instructions, the task was altered to that of verifying an enhanced version of the NIKE3D code against the analytical solution for a spherical cavity deforming under a uniaxial far-field stress at infinity, [2,3]. The purpose of this new exercise was to determine whether the modified NIKE3D code is a suitable means of simulating the URL experiment in any future agreement with AECL.

During the summer of 1988 we had requested the Mechanical Engineering Department of LLNL to modify NIKE3D to make it suitable for the URL simulations. These modifications were:

- a capability to handle shear boundary traction,
- initialization of in-situ stresses,
- an excavation option.

This report will focus on the verification of the original and modified versions of the code through comparison against the closed form analytic solution for a spherical cavity under uniaxial stress at infinity.

NIKE3D [4], written by John Hallquist, is a fully implicit, three-dimensional finite element code which simulates the static and dynamic finite strain response of elastic and inelastic solids, shells, and beams. Eight node constant-pressure solid elements, 2-node truss elements and beam elements, and 4-node membrane and shell elements are available for the spatial discretization of the problem domain. The solutions are obtained either by the

Quasi-Newton, or by Broyden's nonlinear iterative solution schemes. The code is vectorized for improved performance on CRAY computers.. Among the current capabilities of the unmodified NIKE3D code are:

- o a variety of elastic and inelastic material models,
- o sliding interfaces,
- o body force loads,
- o concentrated nodal loads,
- o pressure boundary conditions,
- o displacement boundary conditions,
- o thermal stresses,
- o large capacity.

NIKE3D uses a sophisticated preprocessor called INGRID [5], which generates complex solid geometries by combining simple parts. In addition, INGRID defines slide-surfaces, boundary surfaces, and internal surfaces. Material models and multiple composition bodies, and symmetry, displacement, and pressure boundary conditions are readily handled by NIKE3D/INGRID.

Although NIKE3D and INGRID possess all of the capabilities mentioned above, they do not have certain other features required for treating problems of geomechanical interest. NIKE3D and INGRID do not have joint-type elements for fracture simulation, shear traction boundary forces, a method for initializing the in-situ stresses, or an excavation option. However, minor enhancements, which permit us to circumvent the above mentioned restrictions with the exception of fracture modeling, were made to NIKE3D during the summer of 1988.

A brief description of the function of each code mentioned in this report is given in Appendix 8.4 to assist readers who are unfamiliar with these LLNL codes.

2.0 NEW FEATURES INSTALLED IN NIKE3D

Bruce Englemann and Robert Ferencz of the Mechanical Engineering Department at LLNL modified NIKE3D to treat in-situ stresses and the shear traction boundary forces. No modifications were made to INGRID since Robert Rainsberger estimated that such work would require an effort that could not be supported with available funds. Robert Ferencz suggested a simple well known method for excavating elements which could be used without any code modification. In this method, excavation would be simulated by permitting the modulus of the excavated rock to drop 3-4 orders of magnitude using the thermal elastic option currently in NIKE3D. It is emphasized here that these modifications made to NIKE3D are only quick fixes implemented by the code developers with the limited funds available.

2.1 Shear Boundary Traction Option

Since INGRID [5] cannot yet treat shear boundary traction, we require a complex treatment of the INGRID, the output file produced by INGRID, which NIKE3D reads to simulate a problem with traction boundary forces. We first present the stages required to prepare an input file for NIKE3D, and then discuss in detail the procedures in each stage.

Suppose there are NSURF surfaces which will receive n distinct shear traction forces (in the global coordinate system), having components t_x , t_y , and t_z along the x , y , and z directions respectively as

$$\underline{t}^m = t_x^m \underline{i} + t_y^m \underline{j} + t_z^m \underline{k}. \quad (m \leq n)$$

on each surface. In order to let NIKE3D know that such distinct forces exist on each boundary surface, we require the following procedure.

We discretize the geometry of the 3D solid body as usual with INGRID, with the usual options and material models. Then we make two copies of the input file to INGRID. To model traction forces, we proceed in these stages:

1. In one copy of the INGRID input file, we flag those surfaces to receive shear traction forces by slide surfaces. This output file from INGRID is called file A.
2. In the other copy of the INGRID input file, we do not flag such surfaces. We run INGRID and save the output file as file B.
3. We edit file A by deleting all lines before the nodal location list, and all lines pertaining to the element and node connectivity list.
4. When we come to the list of slide surfaces which receive shear traction forces, we count the number of nodes on that surface, $\text{surf}\cdot\text{nod}^i$, and insert the following line for the i th surface

$$\text{nsurf}\cdot\text{nod}^i \quad t_x^i \quad t_y^i \quad t_z^i \quad \text{load curve number}^i$$

and the load curve number i corresponds to the loading history i , see [4].

Before the list of nodes and positions, we insert the line

$$m \text{ nodes} \quad m \text{ traction surfaces}$$

5. This modified file is now called INPUT. We then execute Bruce Englemann's code, called TRACT, using the input file, INPUT, to form a new file called OUTPUT which has the traction forces for each surface. The last number in this file refers to the number of follower force cards due to traction forces and is called nf . The listing of TRACT is given in the Appendix.
6. We then edit the output file B from INGRID. On the third card, (see p 12 of the NIKE3D manual), we add nf to the number of concentrated nodal loads. Then we append the file OUTPUT to the end of file B. The resulting file is designated file C.

This completes the setup for NIKE3D to handle shear traction boundary forces.

2.2 Initial Stress Option

Because of limited funds available for this task, the code developers could only install a constant in-situ stress option. To treat constant in-situ stresses, we again edit file C. After the last line of file C, we input the components of the in-situ stress tensor in the following manner:

$$s_{xx} \ s_{yy} \ s_{zz} \ s_{xy} \ s_{xz} \ s_{yz}$$

using the format, 6F10.0, and designate the resulting file as file D.

To execute file D with the shear boundary tractions and the in-situ stresses, we cannot use the public version of NIKE3D. Rather, we execute the modified NIKE3DK (version 63r, compiled on July 29, 1988), with file D as input in the following manner:

```
XNIKE3DK i=D/t v
```

where t is the allowed execution time in minutes and v is the priority value.

2.3 Excavation Option

The excavation option is simulated by the thermo-elastic-plastic model found on page 139 of the INGRID manual [5]. In our simulations, we used only the thermo-elastic option by leaving blank the yield stresses and the tangent moduli. The user should be aware that the NIKE3D manual is incomplete. INGRID permits only 8-noded brick elements, and the 6-noded elements have not been implemented. Note that the capabilities of NIKE2D [7] are different from those of NIKE3D.

In this excavation method, the rock is viewed as being composed of two materials with exactly the same Young's modulus and Poisson's ratio at the initial time, but with the temperature dependent Young's modulus and Poisson's ratio in the region to be excavated. The region to be excavated is then heated, its Young's modulus is reduced by 4 orders of magnitude while the rest of the rock mass has its properties held fixed. The potential drawback of

this method is that the stiffness coefficient matrix can become more ill-conditioned leading to large round-off errors and requiring more non-linear iterations for convergence.

To activate the thermo-elastic reduction of the Young's modulus and Poisson's ratio, we must input an additional load card of the form (Ref. [5], p 24).:

$$\text{LCD } n \ m \ t_1 \ f_1 \ \dots \ t_m \ f_m$$

where

n is the n th load curve,

m is the number of pairs,

t_i is the i -th time,

f_i is the i -th fraction.

Note that in the static analysis mode in NIKE3D, time is a dummy variable to indicate the final time or the completion of the static analysis. To activate the pseudo temperature in material i whose Young's and Poisson's ratio will be reduced, we insert the "FT" command in the data referring to material i , (see p 82 of Ref [5]). This command has the form:

FT (spatial region) (load curve #) (final temperature).

This section completes the information necessary to run the new executable NIKE3DK code. The problem setup for the standard options are found in the NIKE3D and INGRID User Manuals [4,5].

One should also be aware that the 3D post processor, TAURUS [6], lacks many of the features found in ORION, the 2D postprocessor. Specifically, TAURUS does not have the capability of plotting stresses and displacements along any given line. TAURUS can only give contour plots of such quantities. In the

verification runs of NIKE3D for the spherical cavity problem, under a uniaxial stress, with and without the excavation option, we modified the output file by tedious editing to obtain tables of displacements and stresses along the three coordinate axes. We recommend that resources be allocated to improve also the capabilities of the 3D post-processor TAURUS for any serious future 3D studies.

3.0 COMPUTATIONAL RESULTS

We will present here a representative sample of the results obtained using the NIKE3D code for the spherical cavity deforming under a uniform far-field uniaxial stress for which analytic solutions exist. Solutions for the stresses and displacements along the three Cartesian coordinates are given in Ananda Wijesinghe's memoranda [2, 3] that are included in the Appendix. Although the solution for the stress field is standard, the solution for the displacements given in [2] could not be found in the literature. This solution is employed as described in [3] in four different ways to verify independently the unmodified and modified NIKE3D codes.

To investigate the effect of increased grid refinement beyond that which is possible (due to hardware limitations) with fully three dimensional simulations using NIKE3D, we first present an axisymmetric simulation of the cavity problem on a refined grid using the NIKE2D code.

Next, the 3D spherical cavity problem was generated with INGRID. Since this problem has an eight-fold geometric symmetry, only one octant was gridded. To enforce symmetry, we used the "plane" option for symmetry planes found on page 20 of the INGRID manual, [5]. Three symmetry planes along each coordinate axis are sufficient to prevent rigid body motion.

Because we cannot simulate a solid body of infinite extent, we bounded this problem by three planes parallel to the Cartesian coordinate planes located at a distance R_0 which was taken to be 12.5 times the cavity radius. In all problems presented, the cavity radius is 2 m, and R_0 is 25 m.

The cavity is assumed to exist in a linearly elastic rock having the following properties:

E (Young's Modulus) = 70 GPa,

ν (Poisson's Ratio) = 0.25.

At the top boundary, a uniform normal compressive traction force of 20 MPa was imposed at $Y = R_{\infty}$, and at $X = R_{\infty}$ and $Z = R_{\infty}$ zero traction forces were applied.

At each of the far-field boundary planes, we did not impose any restrictions on the displacements. INGRID and NIKE3D only permit either free movement or fixed displacements and/or rotations on boundaries, (see p. 43 of the INGRID manual, [5]).

The basic question regarding NIKE3D is how well can it simulate the spherical cavity problem and how well do the modifications perform, especially the excavation option of reducing the Young's modulus by several orders of magnitude?

3.1 Example 1: Axisymmetric Simulation of a Spherical Cavity Under Uniaxial Stress Using the NIKE2D Code.

In the first example of simulating a spherical cavity under uniaxial stress, we used NIKE2D [7] (version p, compiled on September 18, 1988), using MAZE [8] as the preprocessor. The problem solved is depicted in Figure 1 and also in Figure 1 of reference [3] which shows the discretization of the spherical cavity and the surrounding elastic rock using 1560 nodes with 1440 4-noded elements and ratio zoning. As with all examples to be presented, the cavity radius is 2 m, and the outer boundary is at 25 m. To prevent rigid body motion and to enforce the symmetry condition, the radial displacement along $x = 0$ and the axial displacement along $y = 0$ were constrained to be

equal to zero. In addition, we placed no constraints on the displacements along the cylindrical surface at $x = 25$ m. We will discuss later the results with the constraint of zero displacements at this boundary. Along the circular top boundary at $z = 25$ m, a uniform uniaxial stress of 20 MPa was applied.

Because we used the axially symmetric option in NIKE2D, this problem using 1600 nodes is equivalent to a fully 3D simulation using 64,000 nodes per octant assuming that 40 nodes are used along each edge of the octant. The spherical cavity problem has an eight-fold symmetry, so an equivalent 3D problem without symmetry would require 512,000 nodes. Figs. 2-17 show the analytic and the numerical NIKE2D results for the stress s_{xx} , s_{yy} , s_{zz} , and the displacements u_x and u_y along the x and y axes. The error in the numerical results is defined as the analytical value minus the finite element numerical value. The relative error was not presented because the exact quantities vary over a large range and in some cases are equal to zero. We note that the analytic and numerical results are nearly coincident, except near the cavity wall. There is a large numerical error near the cavity since the exact stresses and displacements have the following behavior:

$$s = \alpha + \beta/r^3 + \gamma/r^5,$$

$$u = \delta r + \epsilon/r^2 + \phi/r^4,$$

where r is the distance from the cavity, and α , β , γ , δ , ϵ and ϕ are constants which depend on the cavity radius, the applied stress, the Young's modulus, and the Poisson's ratio. As expected, the difference between the exact and numerical solutions indicate that the greatest error is near the cavity, requiring an even finer grid to obtain better agreement. The displacements are accurate near the cavity to within 3%, but the stresses are in error up to 50%. The accuracy in the stress components improves away from the steep gradient regions and is within 10% around 4 m (twice the cavity radius). Nevertheless, we see that we can have confidence in NIKE2D's ability to model this problem accurately if sufficiently fine gridding is used.

3.2 Example 2: Three-Dimensional Simulation-Verification of the Unmodified NIKE3D Code.

In this example, we used the public unmodified version (version W, compiled on October 29, 1987), of NIKE3D. The problem solved here is again that given in Figure 1 of reference [3] with the difference that a fully three-dimensional finite element grid is used. The grid is presented in Fig. 18, and its enlargement in Fig. 19. This problem used 5146 nodes with 3734 elements. This small problem still requires about 62% of the CRAY1 two-megaword memory since the stiffness coefficient matrix, even with band-width minimization, requires a considerable amount of machine memory. The total CPU time for such runs was approximately 19 minutes. While running a problem with 9876 nodes, we encountered a failure in execution due to a disk I/O error because of the large amount of disk scratch storage space needed to execute the problem.

The results of these runs are presented in Figs. 20-44. Although the numerical results follow the analytic solutions, as with the NIKE2D results, large errors still appear near the cavity where the stresses vary as r^{-5} . Note that even with this relatively coarse gridding, an equivalent non-symmetric problem without the 8-fold symmetry would require about 35,000 nodes. As with the NIKE2D code, we have confidence that the public version of NIKE3D is capable of providing accurate results for the spherical cavity problem under uniaxial stress if this problem could be very finely discretized. However, we observe that a CRAY1 computer does not have sufficient memory for adequate resolution of a non-symmetric 3D problem.

3.3 Example 3: Three-Dimensional Simulation-Verification of the Modified NIKE3D Code Exercising the In-Situ Stress Option.

In this simulation, we used the modified NIKE3D code (version 63r, compiled on July 29, 1988), on the spherical cavity problem under uniaxial stress to exercise the in-situ stress option. The problem solved here is illustrated in Figure 3 of [3]. We used a grid identical to that presented previously. We repeat the gridding used in Fig. 44; the enlarged view is presented in Fig. 45.

We used the identical INGRID output file, but we appended the in-situ stress tensor having a non-zero contribution from the s_{yy} component which is the negative of the applied pressure. The perturbed stress solution in which the s_{yy} components of stress along each axis vanish at the outer boundaries of the domain, and the perturbed displacements no longer exhibit the linear contribution.

Figs. 46-69 show the perturbed analytic and numerical solutions and the differences between the analytic and NIKE3D results. Without the dominating linear behavior in the displacement solutions, we see the numerical truncation errors more clearly.

3.4 Example 4: Three-Dimensional Simulation-Verification of the Modified NIKE3D Code Exercising the Shear Traction Boundary Condition Option.

In this example, we solve the perturbed problem by applying a shear traction boundary force on the surface of the cavity as shown in Figure 2 of [3]. As with the exercise of the in-situ stress option, the perturbed analytic solution is the same as in Example 3. The grid which was used is the same as in the previous NIKE3D examples and is shown again in Fig. 70. An enlarged view in the immediate neighborhood of the cavity is given in Figure 71.

Our first attempt at applying the traction boundary force option gave disappointingly poor results. We found the average normal of a surface element using the technique in Appendix 3. Our results, which are not shown, basically underestimated the applied traction forces. In spherical geometry, the y-direction component of the normal vector at the cavity surfaces varies very rapidly. Our averaging procedure on a surface with high curvature was very inaccurate, even though it appears that we have sufficient grid resolution in the interior. In our next calculation, we used the exact normal component in the spherical geometry by computing the traction force to be

$-S_{\text{applied}}(y/r_{\text{cavity}})$. The results of these calculations are presented in Figs. 72-95. The accuracy of this calculation is now comparable to the results using the in-situ option. Although we presented a quick fix, we recommend that a more general and more accurate scheme be devised for calculating the traction force components at the nodes for the general surface problem, especially for surfaces with large curvature.

3.5 Example 5: Three-Dimensional Simulation-Verification of the Modified NIKE3D Code Exercising the Excavation Option.

In this simulation, we used the modified NIKE3D code on the spherical cavity problem under uniaxial stress to exercise the excavation option as illustrated in Figure 4 of reference [3]. We used the same grid as before, except that we filled the cavity with rock having the same Young's modulus and Poisson's ratio as the unexcavated rock. Fig. 96 shows the grid used in this problem, and Fig. 97 shows the enlarged view.

To model the process of excavation, the Young's modulus in the excavated part of the rock was reduced by a factor of 5000. These reductions were accomplished by the use of two load curves: one load curve ramped the applied stress from its zero value at $t = 0$ to its full value at time 0.33 after which it was kept constant. Until $t = 0.33$, the temperature was held fixed at zero. From $t = 0.33$ to 0.66, the temperature was ramped to its full value, and then was kept constant at its full value until the final time, at $t = 1.0$.

The results of this code exercise are presented in Figs. 98-121 in which the analytic and NIKE3D excavation option values and their differences are presented. We see that the displacements and the stresses are extremely inaccurate and that the stress remains fairly uniform for the s_{yy} components as if the cube was loaded uniaxially without excavation. We reduced the modulus of excavated rock by the factors of 1000, and 10,000 without any significant improvement in the results. One possible explanation for this behavior is that the large modulus contrast resulted in a poorly conditioned

stiffness coefficient matrix that led to large truncation errors in solution procedure. Another possible explanation is that this option was not correctly installed. We recommend that the excavation option be implemented rigorously through the removal of elements. As stated earlier, because of limited funding, this method, although suggested to the code developers, was not implemented.

4.0 SUMMARY AND CONCLUSIONS

NIKE3D is a 3D structural analysis, finite element code which is used routinely by many institutions to solve linear and non-linear solid mechanics problems. A powerful preprocessor, INGRID, is available to prepare computational grids and other problem data for NIKE3D problems. A less sophisticated post processor, TAURUS, is also available to graphically display the output results.

The present work contains new features which were installed in NIKE3D in order for it to compute 3D linearly elastic idealizations of geomechanics problems encountered in the excavation of underground tunnels. In particular, NIKE3D was modified to allow:

1. an initial stress state
2. shear traction boundary conditions
3. an option for excavating of rock in the solution domain.

The principal objective of this work is to verify computational accuracy of both the unmodified and modified NIKE3D codes and to assess the feasibility of using this code to compute tunnel excavation problems in three spatial dimensions.

Our results show that unmodified NIKE3D code and the newly installed initial stress and shear traction boundary condition options work correctly as intended. Furthermore, NIKE3D, in principle, is capable of producing quantitatively accurate results in rapidly varying stress fields on sufficiently refined grids. However, because of the low order interpolation and numerical integration schemes adopted in NIKE3D, the level of grid refinement needed to compute results to the same accuracy as that of the experimental data, would require much more computer storage and memory than that currently available on the Cray 1 Computers. On the other hand, the excavation option which was installed does not function correctly on our model problem, and is, at

present, unuseable. Because of very limited funds we were unable to undergo a thorough debugging of this option. We recommend that sufficient resources be allotted to implement rigorously an excavation option in which elements are actually removed.

As can be expected from a code based on a displacement finite element formulation, the displacements in the model spherical cavity problem are everywhere more accurate than the stresses. The accuracy of the displacements and the stresses is poor in the region near the cavity where they rapidly vary, but improves several cavity radii away as the rapid variations decay. The NIKE3D stress results follow the exact solution quite well in the far-field regions where the solution varies as r^{-3} . While it is true that accuracy can be improved by further refining the grid near the cavity, we also have the concurrent problem of increasing the problem size and the computational expense. Codes that employ higher order elements can circumvent these inaccuracies while using far fewer finite elements.

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- Fig. 13 The difference between analytic and NIKE2D results for s_{yy} along the y-axis.
- Fig. 14 Analytic and NIKE2D results for hoop stress along the y-axis.
- Fig. 15 The difference between the analytic and NIKE2D results for hoop stress along the y-axis.
- Fig. 16 Analytic and NIKE2D results for the displacement u_y along the y-axis.
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- Fig. 19 An enlarged view of Fig. 18 near the cavity.
- Fig. 20 Analytic and the unmodified NIKE3D results of s_{xx} along the x-axis using the grid of Fig. 18.
- Fig. 21 The difference between the analytic and NIKE3D results of s_{xx} along the x-axis.
- Fig. 22 Analytic and NIKE3D the unmodified results of s_{yy} along the x-axis using the grid of Fig. 18.
- Fig. 23 The difference between the analytic and NIKE3D results of s_{yy} along the x-axis.

- Fig. 24 Analytic and the unmodified NIKE3D results of s_{zz} along the x-axis using the grid of Fig. 18.
- Fig. 25 The difference between the analytic and NIKE3D results of s_{zz} along the x-axis.
- Fig. 26 Analytic and the unmodified NIKE3D results of u_x along the x-axis using the grid of Fig. 18.
- Fig. 27 The difference between the analytic and NIKE3D results of u_x along the x-axis.
- Fig. 28 Analytic and the unmodified NIKE3D results of s_{xx} along the y-axis using the grid of Fig. 18.
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- Fig. 30 Analytic and the unmodified NIKE3D results of s_{yy} along the y-axis using the grid of Fig. 18.
- Fig. 31 The difference between the analytic and NIKE3D results of s_{yy} along the y-axis.
- Fig. 32 Analytic and the unmodified NIKE3D results of s_{zz} along the y-axis using the grid of Fig. 18.
- Fig. 33 The difference between the analytic and NIKE3D results of s_{zz} along the y-axis.
- Fig. 34 Analytic and the unmodified NIKE3D results of u_y along the y-axis using the grid of Fig. 18.

- Fig. 35 The difference between the analytic and NIKE3D results of u_y along the y-axis.
- Fig. 36 Analytic and the unmodified NIKE3D results of s_{xx} along the z-axis using the grid of Fig. 18.
- Fig. 37 The difference between the analytic and NIKE3D results of s_{yx} along the z-axis.
- Fig. 38 Analytic and the unmodified NIKE3D results of s_{yy} along the z-axis using the grid of Fig. 18.
- Fig. 39 The difference between the analytic and NIKE3D results for s_{yy} along the z-axis.
- Fig. 40 Analytic and the unmodified NIKE3D results of s_{zz} along the z-axis using the grid of Fig. 18.
- Fig. 41 The difference between the analytic and NIKE3D results of s_{zz} along the z-axis.
- Fig. 42 Analytic and the unmodified NIKE3D results of u_z along the z-axis using the grid of Fig. 18.
- Fig. 43 The difference between the analytic and NIKE3D results u_z along the z-axis.
- Fig. 44 A 3D grid of the spherical cavity in-situ stress option.
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- Fig. 47 The difference between the analytic and NIKE3D results of s_{xx} along the x-axis.
- Fig. 48 Analytic and NIKE3D in-situ stress option results of s_{yy} along the x-axis.
- Fig. 49. The difference between the analytic and NIKE3D results of s_{yy} along the x-axis.
- Fig. 50 Analytic and NIKE3D in-situ stress option results of s_{zz} along the x-axis.
- Fig. 51 The difference between the analytic and NIKE3D results of s_{zz} along the x-axis.
- Fig. 52 Analytic and NIKE3D in-situ stress option results of u_x along the x-axis.
- Fig. 53 The difference between the analytic and NIKE3D results of u_x along the x-axis.
- Fig. 54 Analytic and NIKE3D in-situ stress option results of s_{xx} along the y-axis.
- Fig. 55 The difference between the analytic and NIKE3D results of s_{xx} along the y-axis.
- Fig. 56 Analytic and NIKE3D in-situ stress option results of s_{yy} along the y-axis.
- Fig. 57 The difference between the analytic and NIKE3D results of s_{yy} along the y-axis.

- Fig. 58 Analytic and NIKE3D in-situ stress option results of s_{zz} along the y-axis.
- Fig. 59 The difference between the analytic and NIKE3D results of s_{zz} along the y-axis.
- Fig. 60 Analytic and NIKE3D in-situ stress option results of u_y along the y-axis.
- Fig. 61 The difference between the analytic and NIKE3D results of u_y along the y-axis.
- Fig. 62 Analytic and NIKE3D in-situ stress option results of s_{xx} along the z-axis.
- Fig. 63 The difference between the analytic and NIKE3D results of s_{xx} along the z-axis.
- Fig. 64 Analytic and NIKE3D in-situ stress option results of s_{yy} along the z-axis.
- Fig. 65 The difference between the analytic and NIKE3D results of s_{yy} along the z-axis.
- Fig. 66 Analytic and NIKE3D in-situ stress option results of s_{zz} along the z-axis.
- Fig. 67 The difference between the analytic and NIKE3D results of s_{zz} along the z-axis.
- Fig. 68 Analytic and NIKE3D in-situ stress option results of u_z along the z-axis.

- Fig. 69 The difference between the analytic and NIKE3D results of the u_z along the z-axis.
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- Fig. 71 An enlarged view of the cavity of Fig. 70.
- Fig. 72 Analytic and NIKE3D traction option results of s_{xx} along the x-axis.
- Fig. 73 The difference between analytic and NIKE3D results of s_{xx} along the x-axis.
- Fig. 74 Analytic and NIKE3D traction option results of s_{yy} along the x-axis.
- Fig. 75 The difference between analytic and NIKE3D results of s_{yy} along the x-axis.
- Fig. 76 Analytic and NIKE3D traction option results of s_{zz} along the x-axis.
- Fig. 77 The difference between the analytic and NIKE3D results of s_{zz} along the x-axis.
- Fig. 78 Analytic and NIKE3D traction option results of u_x along the x-axis.
- Fig. 79 The difference between the analytic and NIKE3D results of u_x along the x-axis.
- Fig. 80 Analytic and NIKE3D traction option results of s_{xx} along the y-axis.

- Fig. 81 The difference between the analytic and NIKE3D results of s_{xx} along the y-axis.
- Fig. 82 Analytic and NIKE3D traction option results of s_{yy} along the y-axis.
- Fig. 83 The difference between the analytic and NIKE3D results of s_{yy} along the y-axis.
- Fig. 84 Analytic and NIKE3D traction option results of s_{zz} along the y-axis.
- Fig. 85 The difference between the analytic and NIKE3D results of s_{zz} along the y-axis.
- Fig. 86 Analytic and NIKE3D traction option results of u_y along the y-axis.
- Fig. 87 The difference between the analytic and NIKE3D results of u_y along the y-axis.
- Fig. 88 Analytic and NIKE3D traction option results of s_{xx} along the z-axis.
- Fig. 89 The difference between the analytic and NIKE3D results of s_{xx} along the z-axis.
- Fig. 90 Analytic and NIKE3D traction option results of s_{yy} along the z-axis.
- Fig. 91 The difference between the analytic and NIKE3D results of s_{yy} along the z-axis.
- Fig. 92 Analytic and NIKE3D traction option results of s_{zz} along the z-axis.

- Fig. 93 The difference between the analytic and NIKE3D results of s_{zz} along the z-axis.
- Fig. 94 Analytic and NIKE3D traction option results of u_z along the z-axis.
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- Fig. 96 A 3D grid of a block of rock for the excavation option in which the spherical cavity is initially filled with rock having the same uniform properties everywhere.
- Fig. 97 An enlarged view of the gridding near the filled cavity.
- Fig. 98 Analytic and NIKE3D excavation option results of s_{xx} along the x-axis.
- Fig. 99 The difference between the analytic and NIKE3D results of s_{xx} along the x-axis.
- Fig. 100 Analytic and NIKE3D excavation option results of s_{yy} along the x-axis.
- Fig. 101 The difference between the analytic and NIKE3D results of s_{yy} along the x-axis.
- Fig. 102 Analytic and NIKE3D excavation option results of s_{zz} along the x-axis.
- Fig. 103 The difference between the analytic and NIKE3D results of s_{zz} along the x-axis.
- Fig. 104 Analytic and NIKE3D excavation option results of u_x along the x-axis.

Fig. 105. The difference between the analytic and NIKE3D results of u_x along the x-axis.

Fig. 106 Analytic and NIKE3D excavation option results of s_{xx} along the y-axis.

Fig. 107. The difference between the analytic and NIKE3D results of s_{xx} along the y-axis.

Fig. 108 Analytic and NIKE3D excavation option results of s_{yy} along the y-axis.

Fig. 109 The difference between the analytic and NIKE3D results of s_{yy} along the y-axis.

Fig. 110 Analytic and NIKE3D excavation option results of s_{zz} along the y-axis.

Fig. 111 The difference between the analytic and NIKE3D results of s_{zz} along the y-axis.

Fig. 112 Analytic and NIKE3D excavation option results of u_y along the y-axis.

Fig. 113 The difference between the analytic and NIKE3D results of u_y along the y-axis.

Fig. 114 Analytic and NIKE3D excavation option results of s_{xx} along the z-axis.

Fig. 115 The difference between the analytic and NIKE3D results of s_{xx} along the z-axis.

Fig. 116 Analytic and NIKE3D excavation option results of s_{yy} along the z-axis.

Fig. 117 The difference between the analytic and NIKE3D results of s_{yy} along the z-axis.

Fig. 118 Analytic and NIKE3D excavation option results of s_{zz} along the z-axis.

Fig. 119 The difference between the analytic and NIKE3D results of s_{zz} along the z-axis.

Fig. 120 Analytic and NIKE3D excavation option results of u_z along the z-axis.

Fig. 121 The difference between the analytic and NIKE3D results of u_z along the z-axis.

7.1 Example 1: Axisymmetric Simulation Using the Public
 NIKE2D Code. Figs. 1-17

Fig. 1 A 2D grid generated by MAZE for the axisymmetric spherical cavity.

Fig. 2 Analytic and NIKE2D results for s_{xx} along the x-axis.

Fig. 3 The difference between analytic and NIKE2D results for s_{xx} along the x-axis.

Fig. 4 Analytic and NIKE2D results for s_{yy} along the x-axis.

Fig. 5 The difference between analytic and NIKE2D results for s_{yy} along the x-axis.

Fig. 6 Analytic and NIKE2D results for hoop stress along the x-axis.

Fig. 7 The difference between analytic and NIKE2D results for hoop stress along the x-axis.

Fig. 8 Analytic and NIKE2D results for the displacement u_x along the x-axis.

Fig. 9 The difference between analytic and NIKE2D results of the displacement, u_x , along the x-axis.

Fig. 10 Analytic and NIKE2D results for s_{xx} along the y -axis.

Fig. 11 The difference between analytic and NIKE2D results for s_{xx} along the y -axis.

Fig. 12 Analytic and NIKE2D results for s_{yy} along the y -axis.

Fig. 13 The difference between analytic and NIKE2D results for s_{yy} along the y -axis.

Fig. 14 Analytic and NIKE2D results for hoop stress along the y-axis.

Fig. 15 The difference between the analytic and NIKE2D results for hoop stress along the y-axis.

Fig. 16 Analytic and NIKE2D results for the displacement u_y along the y -axis.

Fig. 17 The difference between the analytic and NIKE2D results for the displacement, u_y , along the y -axis.

7.2 Example 2: Three-Dimensional Simulation-Verification
 of the Unmodified NIKE3D Code. Figs. 18-43.

Fig. 18 A 3D grid for the spherical cavity problem with 8-fold symmetry for the public NIKE3D simulation.

Fig. 19 An enlarged view of Fig. 18 near the cavity.

Fig. 20 Analytic and the *unmodified* NIKE3D results of s_{xx} along the x-axis using the grid of Fig. 18.

Fig. 21 The difference between the analytic and NIKE3D results of s_{xx} along the x-axis.

Fig. 22 Analytic and NIKE3D the unmodified results of s_{yy} along the x-axis using the grid of Fig. 18.

Fig. 23 The difference between the analytic and NIKE3D results of s_{yy} along the x-axis.

Fig. 24 Analytic and the unmodified NIKE3D results of s_{zz} along the x-axis using the grid of Fig. 18.

Fig. 25 The difference between the analytic and NIKE3D results of s_{zz} along the x-axis.

Fig. 26 Analytic and the unmodified NIKE3D results of u_x along the x-axis using the grid of Fig. 18.

Fig. 27 The difference between the analytic and NIKE3D results of u_x along the x-axis.

Fig. 28 Analytic and the unmodified NIKE3D results of s_{xx} along the y -axis using the grid of Fig. 18.

Fig. 29 The difference between the analytic and NIKE3D results of s_{xx} along the y -axis.

Fig. 30 Analytic and the unmodified NIKE3D results of s_{yy} along the y -axis using the grid of Fig. 18.

Fig. 31 The difference between the analytic and NIKE3D results of s_{yy} along the y -axis.

Fig. 32 Analytic and the unmodified NIKE3D results of s_{zz} along the y-axis using the grid of Fig. 18.

Fig. 33 The difference between the analytic and NIKE3D results of s_{zz} along the y-axis.

Fig. 34 Analytic and the unmodified NIKE3D results of u_y along the y -axis using the grid of Fig. 18.

Fig. 35 The difference between the analytic and NIKE3D results of u_y along the y -axis.

Fig. 36 Analytic and the unmodified NIKE3D results of s_{xx} along the z -axis using the grid of Fig. 18.

Fig. 37 The difference between the analytic and NIKE3D results of s_{xx} along the z -axis.

Fig. 38 Analytic and the unmodified NIKE3D results of s_{yy} along the z -axis using the grid of Fig. 18.

Fig. 39 The difference between the analytic and NIKE3D results for s_{yy} along the z -axis.

Fig. 40 Analytic and the unmodified NIKE3D results of s_{zz} along the z -axis using the grid of Fig. 18.

Fig. 41 The difference between the analytic and NIKE3D results of s_{zz} along the z -axis.

Fig. 42 Analytic and the unmodified NIKE3D results of u_z along the z -axis using the grid of Fig. 18.

Fig. 43 The difference between the analytic and NIKE3D results u_z along the z -axis.

**7.3 Example 3: Three-Dimensional Simulation-Verification of the
Initial Stress Option. Figs. 44-69.**

Fig. 44 A 3D grid of the spherical cavity in-situ stress option.

Fig. 45 An enlarged view of the cavity of Fig. 44.

Fig. 46 Analytic and NIKE3D in-situ stress ratio results of s_{xx} along the x -axis.

Fig. 47 The difference between the analytic and NIKE3D results of s_{xx} along the x -axis.

Fig. 48 Analytic and NIKE3D in-situ stress option results of s_{yy} along the x-axis.

Fig. 49. The difference between the analytic and NIKE3D results of s_{yy} along the x-axis.

Fig. 50 Analytic and NIKE3D in-situ stress option results of s_{zz} along the x-axis.

Fig. 51 The difference between the analytic and NIKE3D results of s_{zz} along the x-axis.

Fig. 52 Analytic and NIKE3D in-situ stress option results of u_x along the x -axis.

Fig. 53 The difference between the analytic and NIKE3D results of u_x along the x -axis.

Fig. 54 Analytic and NIKE3D in-situ stress option results of s_{xx} along the y -axis.

Fig. 55 The difference between the analytic and NIKE3D results of s_{xx} along the y -axis.

Fig. 56 Analytic and NIKE3D in-situ stress option results of s_{yy} along the y -axis.

Fig. 57 The difference between the analytic and NIKE3D results of s_{yy} along the y -axis.

Fig. 58 Analytic and NIKE3D in-situ stress option results of s_{zz} along the y -axis.

Fig. 59 The difference between the analytic and NIKE3D results of s_{zz} along the y -axis.

Fig. 60 Analytic and NIKE3D in-situ stress option results of along the y-axis.

Fig. 61 The difference between the analytic and NIKE3D results u_y along the y-axis.

Fig. 62 Analytic and NIKE3D in-situ stress option results of s_{xx} along the z -axis.

Fig. 63 The difference between the analytic and NIKE3D results of s_{xx} along the z -axis.

Fig. 64 Analytic and NIKE3D in-situ stress option results of s_{yy} along the z -axis.

Fig. 65 The difference between the analytic and NIKE3D results of s_{yy} along the z -axis.

Fig. 66 Analytic and NIKE3D in-situ stress option results of s_{zz} along the z -axis.

Fig. 67 The difference between the analytic and NIKE3D results of s_{zz} along the z -axis.

Fig. 68 Analytic and NIKE3D in-situ stress option results of u_z along the z -axis.

Fig. 69 The difference between the analytic and NIKE3D results of the u_z along the z -axis.

7.4 Example 4: Three-Dimensional Simulation - Verification of the Shear Traction Boundary Condition Option.
Figs. 70-95.

Fig. 70 a 3D grid of the spherical cavity problem for the traction option.

FIG. 71 An enlarged view of the cavity of Fig. 70.

Fig. 72 Analytic and NIKE3D traction option results of s_{xx} along the x-axis.

Fig. 73 The difference between analytic and NIKE3D results of s_{xx} along the x-axis.

Fig. 74 Analytic and NIKE3D traction option results of s_{yy} along the x-axis.

Fig. 75 The difference between analytic and NIKE3D results of s_{yy} along the x-axis.

Fig. 76 Analytic and NIKE3D traction option results of s_{zz} along the x-axis.

Fig. 77 The difference between the analytic and NIKE3D results of s_{zz} along the x-axis.

Fig. 78 Analytic and NIKE3D traction option results of u_x along the x -axis.

Fig. 79 The difference between the analytic and NIKE3D results of u_x along the x -axis.

Fig. 80 Analytic and NIKE3D traction option results of s_{xx} along the y-axis.

Fig. 81 The difference between the analytic and NIKE3D results of s_{xx} along the y-axis.

Fig. 82 Analytic and NIKE3D traction option results of s_{yy} along the y-axis.

Fig. 83 The difference between the analytic and NIKE3D results of s_{yy} along the y-axis.

Fig. 84 Analytic and NIKE3D traction option results of s_{zz} along the y -axis.

Fig. 85 The difference between the analytic and NIKE3D results of s_{zz} along the y -axis.

Fig. 86 Analytic and NIKE3D traction option results of u_y along the y -axis.

Fig. 87 The difference between the analytic and NIKE3D results of u_y along the y -axis.

Fig. 88 Analytic and NIKE3D traction option results of s_{xx} along the z -axis.

Fig. 89 The difference between the analytic and NIKE3D results of s_{xx} along the z -axis.

Fig. 90 Analytic and NIKE3D traction option results of s_{yy} along the z -axis.

Fig. 91 The difference between the analytic and NIKE3D results of s_{yy} along the z -axis.

Fig. 92 Analytic and NIKE3D traction option results of s_{zz} along the z -axis.

Fig. 93 The difference between the analytic and NIKE3D results of s_{zz} along the z -axis

Fig. 94 Analytic and NIKE3D traction option results of u_z along the z -axis.

Fig. 95 The difference between the analytic and NIKE3D results of u_z along the z -axis.

7.5 **Example 5: Three-Dimensional Simulation - Verification
 of the Excavation Option. Figs. 96-121.**

Fig. 96 A 3D grid of a block of rock for the excavation option in which the spherical cavity is initially filled with rock having the same uniform properties.

Fig. 97 An enlarged view of the gridding near the filled cavity.

Fig. 98 Analytic and NIKE3D excavation option results of s_{xx} along the x-axis.

Fig. 99 The difference between the analytic and NIKE3D results of s_{xx} along the x-axis.

Fig. 100 Analytic and NIKE3D excavation option results of s_{yy} along the x-axis.

Fig. 101 The difference between the analytic and NIKE3D results of s_{yy} along the x-axis.

Fig. 102 Analytic and NIKE3D excavation option results of s_{zz} along the x-axis.

Fig. 103 The difference between the analytic and NIKE3D results of s_{zz} along the x-axis.

Fig. 104 Analytic and NIKE3D excavation option results of u_x along the x-axis.

Fig. 105. The difference between the analytic and NIKE3D results of u_x along the x-axis.

Fig. 106 Analytic and NIKE3D excavation option results of s_{xx} along the y-axis.

Fig. 107. The difference between the analytic and NIKE3D results of s_{xx} along the y-axis.

Fig. 108 Analytic and NIKE3D excavation option results of s_{yy} along the y -axis.

Fig. 109 The difference between the analytic and NIKE3D results of s_{yy} along the y -axis.

Fig. 110 Analytic and NIKE3D excavation option results of s_{zz} along the y -axis.

Fig. 111 The difference between the analytic and NIKE3D results of s_{zz} along the y -axis.

Fig. 112 Analytic and NIKE3D excavation option results of u_y along the y -axis.

Fig. 113 The difference between the analytic and NIKE3D results of u_y along the y -axis.

Fig. 114 Analytic and NIKE3D excavation option results of s_{xx} along the z -axis.

Fig. 115 The difference between the analytic and NIKE3D results of s_{xx} along the z -axis.

Fig. 116 Analytic and NIKE3D excavation option results of s_{yy} along the z -axis.

Fig. 117 The difference between the analytic and NIKE3D results of s_{yy} along the z -axis.

Fig. 118 Analytic and NIKE3D excavation option results of s_{zz} along the z -axis.

Fig. 119 The difference between the analytic and NIKE3D results of s_{zz} along the z -axis.

Fig. 120 Analytic and NIKE3D excavation option results of u_z along the z -axis.

Fig. 121 The difference between the analytic and NIKE3D results of u_z along the z -axis.

8.0 APPENDIX

In this Appendix, we include the memoranda of A. Wijesinghe on the analytical solutions for the spherical cavity problem, and a listing of the shear traction force code written by Bruce Englemann to be used in the modified NIKE3D code.

MEMORANDUM

TO: Edward Kansa

FROM: Ananda M. Wijesinghe, L-200, 3-0605

DATE: October 5, 1988

SUBJECT: URL Scoping Calculations: Analytical
Solutions for NIKE3D Code Verification

The linear elastic analytical solutions for the deformation of an infinitely long circular cylindrical cavity, and a spherical cavity, under a uniform far-field uniaxial stress are given below for your use in verifying the NIKE3D code. The stress solutions for these configurations are standard, but I derived the solutions for the displacements.

The results are given for the stresses and displacements along the coordinate axes with reference to the Cartesian coordinate frames shown in Figures 1 & 2. Thus, these results can be directly compared against the finite element results of NIKE3D computed in a Cartesian coordinate frame.

I. Infinitely Long Circular Cylindrical Cavity

Figure 1.

Stresses and displacements along the y-axis:

$$\sigma_{xx} = -S \left\{ \frac{1}{2} \frac{a^2}{y^2} - \frac{3}{2} \frac{a^4}{y^4} \right\}, \quad (I.1)$$

$$\sigma_{yy} = -S \left\{ 1 - \frac{5}{2} \frac{a^2}{y^2} + \frac{3}{2} \frac{a^4}{y^4} \right\}, \quad (I.2)$$

$$\sigma_{zz} = -S \frac{\nu^*}{1+\nu^*} \left\{ 1 - 2 \frac{a^2}{y^2} \right\}, \quad (I.3)$$

$$\sigma_{xy} = 0, \quad (I.4)$$

$$\sigma_{yz} = 0, \quad (I.5)$$

$$\sigma_{zx} = 0, \quad (I.6)$$

$$u_x = 0, \quad (I.7)$$

$$u_y = \frac{Sy}{E^*} \left\{ 1 + \frac{5+\nu^*}{2} \frac{a^2}{y^2} - \frac{1+\nu^*}{2} \frac{a^4}{y^4} \right\}, \quad (I.8)$$

$$u_z = 0, \quad (I.9)$$

where,

$$E^* \equiv \frac{E}{(1-\nu^2)}, \quad (I.10)$$

$$\nu^* \equiv \frac{\nu}{1-\nu}. \quad (I.11)$$

Stresses and displacements along the x-axis:

$$\sigma_{xx} = -S \left\{ \frac{3}{2} \frac{a^2}{x^2} - \frac{3}{2} \frac{a^4}{x^4} \right\}, \quad (I.12)$$

$$\sigma_{yy} = -S \left\{ 1 + \frac{1}{2} \frac{a^2}{x^2} + \frac{3}{2} \frac{a^4}{x^4} \right\}, \quad (I.13)$$

$$\sigma_{zz} = -S \frac{\nu^*}{1+\nu^*} \left\{ 1 + 2 \frac{a^2}{x^2} \right\}, \quad (I.14)$$

$$\sigma_{xy} = 0, \quad (I.15)$$

$$\sigma_{yz} = 0, \quad (I.16)$$

$$\sigma_{zx} = 0, \quad (I.17)$$

$$u_x = \frac{Sx}{E^*} \left\{ \nu^* + \frac{3-\nu^*}{2} \frac{a^2}{x^2} + \frac{1+\nu^*}{2} \frac{a^4}{x^4} \right\}, \quad (I.18)$$

$$u_y = 0, \quad (I.19)$$

$$u_z = 0. \quad (I.20)$$

II. Spherical Cavity

Figure 2.

Stresses and displacements along the y-axis:

$$\sigma_{yy} = -S \left\{ \frac{3(3-5\nu)}{2(7-5\nu)} \frac{a^3}{y^3} - \frac{6}{7-5\nu} \frac{a^5}{y^5} \right\}, \quad (\text{II.1})$$

$$\sigma_{xx} = -S \left\{ 1 - \frac{19-5\nu}{7-5\nu} \frac{a^3}{y^3} + \frac{12}{7-5\nu} \frac{a^5}{y^5} \right\}, \quad (\text{II.2})$$

$$\sigma_{zz} = -S \left\{ \frac{3(3-5\nu)}{2(7-5\nu)} \frac{a^3}{y^3} - \frac{6}{7-5\nu} \frac{a^5}{y^5} \right\}, \quad (\text{II.3})$$

$$\sigma_{xy} = 0, \quad (\text{II.4})$$

$$\sigma_{yz} = 0, \quad (\text{II.5})$$

$$\sigma_{zx} = 0, \quad (\text{II.6})$$

$$u_x = 0 \quad (\text{II.7})$$

$$u_y = -\frac{S}{E} y - \frac{Sa}{G} \left\{ \frac{19-15\nu}{4(7-5\nu)} \frac{a^2}{y^2} - \frac{3}{2(7-5\nu)} \frac{a^4}{y^4} \right\}, \quad (\text{II.8})$$

$$u_z = 0, \quad (\text{II.9})$$

where,

$$G \equiv \frac{E}{2(1+\nu)}. \quad (\text{II.10})$$

Stresses and displacements along the x -axis:

$$\sigma_{xx} = -S \left\{ \frac{6}{7-5\nu} \frac{a^2}{x^2} - \frac{6}{7-5\nu} \frac{a^5}{x^5} \right\}, \quad (\text{II.11})$$

$$\sigma_{yy} = -S \left\{ 1 + \frac{4-5\nu}{2(7-5\nu)} \frac{a^3}{x^3} + \frac{9}{2(7-5\nu)} \frac{a^5}{x^5} \right\}, \quad (\text{II.12})$$

$$\sigma_{zz} = -S \left\{ -\frac{6-15\nu}{2(7-5\nu)} \frac{a^3}{x^3} + \frac{3}{2(7-5\nu)} \frac{a^5}{x^5} \right\}, \quad (\text{II.13})$$

$$\sigma_{xy} = 0, \quad (\text{II.14})$$

$$\sigma_{yz} = 0, \quad (\text{II.15})$$

$$\sigma_{zx} = 0, \quad (\text{II.16})$$

$$u_x = \frac{\nu S}{E} z + \frac{S a}{G} \left\{ \frac{6-5\nu}{4(7-5\nu)} \frac{a^2}{x^2} - \frac{3}{4(7-5\nu)} \frac{a^4}{x^4} \right\}, \quad (\text{II.17})$$

$$u_y = 0, \quad (\text{II.18})$$

$$u_z = 0. \quad (\text{II.19})$$

Stresses and displacements along the z -axis:

$$\sigma_{xx} = -S \left\{ -\frac{6-15\nu}{2(7-5\nu)} \frac{a^3}{z^3} + \frac{3}{2(7-5\nu)} \frac{a^5}{z^5} \right\}, \quad (\text{II.20})$$

$$\sigma_{yy} = -S \left\{ 1 + \frac{4-5\nu}{2(7-5\nu)} \frac{a^3}{z^3} + \frac{9}{2(7-5\nu)} \frac{a^5}{z^5} \right\}, \quad (\text{II.21})$$

$$\sigma_{zz} = -S \left\{ \frac{6}{7-5\nu} \frac{a^2}{z^2} - \frac{6}{7-5\nu} \frac{a^5}{z^5} \right\}, \quad (\text{II.22})$$

$$\sigma_{xy} = 0, \quad (\text{II.23})$$

$$\sigma_{yz} = 0, \quad (\text{II.24})$$

$$\sigma_{zx} = 0, \quad (\text{II.25})$$

$$u_x = 0, \quad (\text{II.26})$$

$$u_y = 0, \quad (\text{II.27})$$

$$u_z = \frac{\nu S}{E} z + \frac{S a}{G} \left\{ \frac{6-5\nu}{4(7-5\nu)} \frac{a^2}{z^2} - \frac{3}{4(7-5\nu)} \frac{a^4}{z^4} \right\}. \quad (\text{II.28})$$

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MEMORANDUM

TO: Edward Kansa

FROM: Ananda M. Wijesinghe, L-200, 3-0605

A.M.W.

DATE: October 19, 1988

SUBJECT: URL Scoping Calculations: Use of the Solution to
the Spherical Cavity Problem for Verifying
Different Modified NIKE3D Code Options

In my memorandum of October 5, 1988, for the purpose of verifying the NIKE3D code, I provided you the linear elastic analytical solution for a spherical cavity deforming under a uniform far-field uniaxial stress. In my memorandum of June 29, 1988 I outlined how this one solution can be used to verify the 1) shear traction boundary condition, 2) initial stress, and 3) excavation options incorporated in the modified NIKE3D code. I am writing this memorandum to explain in greater detail how this can be done systematically to independently verify each new option.

CASE (1). Verification of unmodified NIKE3D code without shear traction boundary condition, initial stress and excavation options.

Figure 1.

In this case we directly solve the problem of a spherical cavity deforming under a uniaxial far-field stress with the boundary conditions shown in Figure (1) without applying an initial stress. The analytical solution to the problem is given by the stresses σ_{ij} and displacements u_i given in my memorandum of October 5, 1988.

CASE (2). Verification of the shear traction boundary condition option of the modified NIKE3D code.

Figure 2.

In this case we solve the problem shown Figure (2) for the stress σ_{ij} and displacement u_i without applying an initial stress. The tractions are applied as shown at the surface of the cavity parallel to the x_2 -axis. The magnitude of this traction varies around the cavity according to:

$$\vec{t} \equiv 0 \vec{i}_1 + S n_2 \vec{i}_2 + 0 \vec{i}_3, \quad (1)$$

where, n_2 is the direction cosine of the *outnormal* to the cavity surface with respect to the direction of the applied uniaxial stress. Because we want to exercise the shear traction option, we input this boundary condition as the resultant of a traction t_n normal to the cavity surface and two shear traction components t_r, t_s that are tangential to the cavity surface. Thus, the traction vector on the spherical cavity surface must be input as:

$$\vec{t} = t_n \vec{n} + t_r \vec{r} + t_s \vec{s}, \quad (2)$$

where,

$$\vec{r} = \frac{\vec{x} - \vec{x}_c}{r}$$

$$t_1 \equiv S n_1 r_1, \quad (4)$$

$$t_2 \equiv S n_1 s_1, \quad (5)$$

and, $\bar{n}, \bar{r}, \bar{s}$ are a set of orthogonal unit vectors that define the *outnormal* and the tangent plane of the cavity surface. The quantities n_1, r_1, s_1 are the direction cosines of the unit vector $\bar{i}_1$ with respect to this local orthogonal coordinate frame. A procedure for computing all of these direction cosines from the finite element grid nodal coordinate data is given in the Appendix.

The analytical solution to this problem is given by:

$$\delta_{ij} \equiv \sigma_{ij} - \sigma_{ij}^0, \quad (6)$$

$$\hat{u}_i \equiv u_i - u_i^0, \quad (7)$$

where, the stress tensor $\sigma_{ij}^0 = -S \delta_{ij} \delta_{j1}$. The displacement components u_i^0 along each coordinate axis in directions normal to that coordinate axis are equal to zero. The displacement component u_1^0 along each coordinate axis parallel to the direction of that coordinate axis is equal to the part that is linear in the coordinates x, y, z in the corresponding solution for the displacements in CASE (1).

CASE (3). Verification of the initial stress option of the modified NIKE3D code.

Figure 3.

In this case we solve the problem shown Figure (3) for the stress σ_{ij} and the displacement u_i while imposing an initial stress tensor given by $\sigma_{ij}^0 = -S \delta_{ij}$. The boundary conditions for this problem are the same as for CASE (1). The analytical solution for the stresses and displacements of this problem is the same as for CASE (2).

CASE (4). Verification of the excavation option of the modified NIKE3D code.

In this case we solve the problem shown Figure (4) for the stress $\hat{\sigma}_{ij}$ and displacement $\hat{u}_i$, *without imposing an initial stress*. Initially, the spherical cavity contains material having the same properties as the region external to the cavity. The uniaxial stress is then applied at the far-field boundary as shown and the displacements are computed. This initial solution should be equal to σ_{ij}^0 and u_i^0 defined in CASE (2). Then, the excavation option is activated to simulate excavation of the material in the cavity region. The analytical solution for the post-excavation stresses and displacements is the same as for CASE (1).

APPENDIX

Procedure for computing the direction cosines at the cavity surface

Figure 5.

For the 8-noded brick elements employed in NIKE3D, the cavity surface would be composed of a mesh of 4-noded plane quadrilateral elements as shown in the above Figure. The geometry of this surface element is described in terms of the local coordinates (r,s) that vary over the range $(-1,1)$. We set up our system of orthogonal direction vectors $\vec{n}, \vec{r}, \vec{s}$ such that $\vec{n}$ is normal to the surface element, $\vec{r}$ lies in the plane of the element and is parallel to the r local coordinate axis, and $\vec{s}$ lies in the plane of the element and is normal to the r local coordinate axis. This, however, implies that $\vec{s}$ is generally *not* parallel to the s local coordinate axis. The equation to the plane surface element can be written as:

$$F(\vec{x}) = A_1x_1 + A_2x_2 + A_3x_3 - B = 0, \quad (A.1)$$

where $x_1 = x, x_2 = y,$ and $x_3 = z$.

The coefficients (A_i, B) defining the surface can be found by requiring any three of the four corner nodes to lie on the surface, and analytically solving the system of simultaneous linear equations:

$$z_{i(I)} A_i = B, \quad I = 1, 3; \quad i = 1, 3, \quad (\text{A.2})$$

for A_i/B . Here, $z_{i(I)}$ is the i -th coordinate of the I -th corner node of the surface element. The normal vector can then be computed from the gradient of F with respect to the spatial coordinates, and is given by:

$$n_i = \frac{A_i}{\sqrt{A_1^2 + A_2^2 + A_3^2}}, \quad i = 1, 3. \quad (\text{A.3})$$

The direction cosines of $\vec{F}$ can be computed by noting that the r coordinate axis is a straight line that bisects the sides 2-3 and 4-1. Therefore, the direction cosines of $\vec{F}$ are given by:

$$r_i = \frac{\hat{r}_i}{\sqrt{\hat{r}_1^2 + \hat{r}_2^2 + \hat{r}_3^2}}, \quad i = 1, 3, \quad (\text{A.4})$$

$$\hat{r}_i \equiv z_{i(2)} + z_{i(3)} - z_{i(4)} - z_{i(1)}; \quad i = 1, 3. \quad (\text{A.5})$$

Finally, the direction cosines of $\vec{s}$ can be computed from the vector cross product of the unit vectors $\vec{n}$ and $\vec{r}$, and are given by:

$$s_i = \begin{pmatrix} n_2 r_3 - n_3 r_2 \\ n_3 r_1 - n_1 r_3 \\ n_1 r_2 - n_2 r_1 \end{pmatrix}; \quad i = 1, 3. \quad (\text{A.6})$$

Equations (3), (4), and (6) provide the direction cosines n_i , r_i , and s_i required to compute the traction components for the CASE (2) verification problem.

```

cft i=1,nr,i=trct1,0,1,0,0
cft i=trct1,x=trct1
program trct
c
c.... this program will read in the nodal coordinate data and the
c surface definition and will print out the correct nodal force
c data to be used with nike3d
c
c input: all input is unformatted
c
c 1. the first card: the number of nodes (numnp),
c the number of surfaces (nsurf).
c
c 2. the next numnp cards: contain the nodal coordinate data exactly
c as outputted by ingrid
c
c for each surface
c
c 1. the first card: for each surface-
c the number of cards used to define the
c surface (ncard), the x component of the
c applied traction (tx), the y component
c (ty), the z component (tz), and the
c load curve number lc
c
c 2. the next ncard cards: the surface definitions one right
c after the other
c exactly as outputted by ingrid
c
c output: formatted concentrated nodal loads for nike3d
c
c 1. the first nload cards: the concentrated nodal load data
c
c 2. the last card: number of nodal loads (nload)
c
c common/heap/buffers(20000b)
c dimension x(3,1000),xc(4),yc(4),len(4),rdis(4)
c
c fac=1/sqrt(3.)
c nload=0
c
c.... set device numbers
c
c iin=15
c iout=16
c call link ("unit55.tty,
c i=input,unit55=(i,text,open),
c o=output,unit55=(o,text,create)"/)
c
c.... read in control info
c
c read(iin,*)numnp,nsurf
c
c.... read in nodal coordinates
c
c do 10 i=1,numnp
c 10 read(iin,*)id,rd1,x(1,i),x(2,i),x(3,i),rd2
c
c do 50 i=1,nsurf
c

```

```

      read(1:n,*)ncard,fx,fy,fz,lc
c
      do 60 lcard=1,ncard
c
        read(1:n,*)ldum,ldum1,len(1),len(2),len(3),len(4)
c
        x21=x(1,len(2))-x(1,len(1))
        y21=x(2,len(2))-x(2,len(1))
        z21=x(3,len(2))-x(3,len(1))
        x1=sqrt(x21**2+y21**2+z21**2)
        x21=x21/x1
        y21=y21/x1
        z21=z21/x1
c
        x31=x(1,len(3))-x(1,len(1))
        y31=x(2,len(3))-x(2,len(1))
        z31=x(3,len(3))-x(3,len(1))
c
        xn=y21*z31-z21*y31
        yn=z21*x31-x21*z31
        zn=x21*y31-y21*x31
        x1=sqrt(xn**2+yn**2+zn**2)
        xn=xn/x1
        yn=yn/x1
        zn=zn/x1
c
        xt=yn*z21-zn*y21
        yt=zn*x21-xn*z21
        zt=xn*y21-yn*x21
        x1=sqrt(xt**2+yt**2+zt**2)
        xt=xt/x1
        yt=yt/x1
        zt=zt/x1
c
        do 70 i=1,4
          xc(i)=x21*x(1,len(i))+y21*x(2,len(i))+z21*x(3,len(i))
70      yc(i)=xt*x(1,len(i))+yt*x(2,len(i))+zt*x(3,len(i))
c
        call dist(xc,yc,rdis,fac)
c
        do 80 i=1,4
c
          nload=nload+1
          f1=fx*rdis(i)
          idir=1
          write(iout,40)len(i),idir,lc,f1
          nload=nload+1
          f2=fy*rdis(i)
          idir=2
          write(iout,40)len(i),idir,lc,f2
          nload=nload+1
          f3=fz*rdis(i)
          idir=3
          write(iout,40)len(i),idir,lc,f3
c
          80 continue
          60 continue
          50 continue
c
        write(iout,*)nload

```

```

c      call exit
c
c      stop
40 format(15,15,15,e10.2)
end
subroutine dist(xc,yc,rdis,fac)
c
c      dimension xc(4),yc(4),rdis(4),h(4),xi(4),et(4)
c
c      h(1) = 1.,   $ h(2) = -1. $ h(3) = 1.,   $ h(4) = -1.
c      xi(1) = -1. $ xi(2) = 1.  $ xi(3) = 1.  $ xi(4) = -1.
c      et(1) = -1. $ et(2) = -1. $ et(3) = 1.  $ et(4) = 1.
c
c      c1=0.
c      c2=0.
c      c3=0.
c      c4=0.
c      c5=0.
c      c6=0.
c      do 10 i=1,4
c      c1=c1+et(i)*yc(i)
c      c2=c2+xi(i)*xc(i)
c      c3=c3+xi(i)*yc(i)
c      c4=c4+et(i)*xc(i)
c      c5=c5+h(i)*yc(i)
c      c6=c6+h(i)*xc(i)
10  rdis(i)=0.
c      rj=(c1*c2-c3*c4)/16.
c      rj1=(c5*c2-c3*c6)/16.
c      rj2=(c1*c5-c5*c4)/16.
c
c      do 20 n=1,4
c      rxi=xi(n)*fac
c      ret=et(n)*fac
c      rjacob=rj+rj1*rxi+rj2*ret
c      do 20 i=1,4
c      rdis(i)=rdis(i)+.25*(1.+rxi*xi(i))*(1.+ret*et(i))*rjacob
20  continue
c
c      return
end

```

8.4 Table of LLNL codes cited in this report.

1. NIKE3D - Three dimensional structural analysis code.
2. INGRID - Three-dimensional grid generator and import data pre-processor for NIKE3D.
3. TAURUS - Three-dimensional post processor for tabular and graphical display and printing of NIKE3D output results.
4. NIKE2D - Two-dimensional structural analysis code corresponding to NIKE3D. This code is used more often and has more structural features than NIKE3D.
5. MAZE - Two-dimensional grid generator and input data pre-processor for NIKE2D. Corresponding to INGRID.
6. ORION - Two-dimensional post-processor for NIKE2D. Corresponding to TAURUS.
7. TRACT - The code segment for implementing the boundary traction force option that is listed in Appendix 8.3.