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Three-Dimensional Equilibrium in Quadrupole Symmetric Tandem Mirrors in the Paraxial Limit (Reduced MHD)

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ABSTRACT

Equilibrium in quadrupole symmetric mirrors is fully three dimensional; however, because axial scale lengths are long compared with radial scale lengths (equivalently weak curvature) it is possible to reduce the complexity of the equations by expanding in the appropriate smallness parameter. Such a procedure leads to set of reduced MHD equations. The general theory will be presented, numerical results discussed, modifications due to finite Larmor radius will be added, and an analytic solution for sharp boundary pressure models will be developed.

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INTRODUCTION

In quadrupole tandem mirrors the main field is in the axial direction (z) and the perpendicular field components are generally small, of order λ, the ratio of radial to axial scale lengths. The parallel current, which is equivalent to the Pfirsch-Schlüter current in toroidal devices, is of order λ compared to the perpendicular current. In addition, the radius of a curvature of a field line is large, of order λ^{-2} compared with the radius. This separation of scale lengths permits the "long-thin" or paraxial approximation.[1,2,3] Equivalently time scales due to compressional wave propagation perpendicular to the field are of order λ^0, whereas time scales due to shear Alfvén wave propagation parallel to the field are of order λ^{-1}. These disparate time scales permit the separation of solution of the MHD equation into two stages.[4] First perpendicular and parallel pressure balance are established during the fast compressional phase and then the parallel current constraint and interchange/ ballooning MHD stability is established in the incompressible shear Alfvén stage.[1,4] The approximation utilizing this separation is referred to as reduced MHD in toroidal problems. In this paper I will follow Newcomb[1] and write down an appropriately ordered energy principle to extract the equations of motion. I will also include modifications due to finite Larmor radius (FLR).[5] I will only sketch the derivation; details, if desired, can be found in the aforementioned references. Our final equations differ from Strauss[4] in that flux is used as the independent variable.

I will then show some equilibrium results from direct numerical integration of the equation and compare them to analytic results (reduced to quadratures) from a beta expanded equilibrium.[3] I will then finish by presenting an analytic solution using a sharp boundary pressure profile. Such equilibrium are now being used to ascertain MHD stability using the rigid "m=1" formalism.[6]

GENERAL EQUATIONS

The vacuum field geometry for quadrupole symmetric tandem mirrors are completely specified within the paraxial limit by two functions of z;[7] vis (subscript v refers to vacuum quantities)

$$x_v = x_0\sigma_v(z) \tag{1}$$

$$y_v = y_0 \tau_v(z)$$

$$\sigma_v \tau_v = 1/B_v(z) \quad ,$$

such that

$$\sigma_v(z) = \tau_v(-z) \tag{2}$$

and

$$2\psi = x_0^2 + y_0^2$$

$$\theta = \tan^{-1}\left(y_0/x_0\right) \tag{3}$$

where x_0 and y_0 are the position of the field lines at the midplane and ψ and θ are the flux variables with the constraint that

$$B^{-1}(\psi,\theta) = \frac{\partial(x,y)}{\partial(\psi,\theta)} \quad . \tag{4}$$

We are to determine $x,y(\psi,\theta,z)$ for general beta. Also we need only know pressure balance to lowest order, which is given by

$$B^2 + 2p_\perp(\psi,B) = B_v^2(z) \qquad \text{(perpendicular)}$$

$$\left(\frac{Q}{B}\right)' = \frac{B_v B_v'}{B} \qquad \text{(parallel)}$$

$$Q = B^2 + p_\perp - p_\parallel \quad . \tag{5}$$

In general, pressures are functions of both flux variables; here we consider the simpler case and choose it to be a function of a single flux variable. In particular, we choose $p_\parallel = \overline{\omega}(\psi)\ \hat{p}_\perp(B,R(\psi))$; R is the mirror ratio where the trapped plasma goes to zero.

At this point we could proceed by systematically expanding the equations of motion in powers of λ;[4] rather, we write down an energy integral with the first variation generating the suitably ordered term.[1] For the potential energy we have

$$V = \frac{1}{2}\int Q\, \underline{x}' \cdot \underline{x}' \,\frac{d\psi d\theta dz}{B} - \frac{1}{2} B(L)C\!:\!\int \underline{x}\underline{x}\; d\psi d\theta\Big|_{z=-L}^{L} \quad , \tag{6}$$

where primes refer to partials with respect to z. Also remember that only mod-B appears since the perpendicular components are small, similarly $s \to z$. In Eq. (6)

$$C = \begin{pmatrix} \sigma_v'/\sigma_v & 0 \\ 0 & \tau_v'/\tau_v \end{pmatrix} \quad . \tag{7}$$

The integration is up to some lateral boundary ψ_m which is assumed to be a perfect conductor; then with $\Delta\underline{x} = \underline{\xi} = (\xi,\eta)$ we have (subscripts refer to partials) $(\xi\, y_\theta - \eta x_\theta)_{\psi=\psi} = 0$ and the first variation with the constraint Eq. (4) becomes

$$\Delta V - \int \kappa \Delta \frac{\partial(x,y)}{\partial(\psi,\theta)}\, d\psi d\theta dz \;\; = 0 \quad .$$

This produces

$$\begin{aligned} &\underline{\nabla}\kappa + B\underline{\nabla}\underline{x} \cdot \left(\frac{Q\underline{x}'}{B}\right)' \\ &\quad = Q\underline{k} + B_v B_v'\, \underline{x}' \\ &\quad = Q\underline{k} + \frac{B_v B_v'}{B}\, \underline{B}_\perp \quad , \end{aligned} \tag{8}$$

where the curvature in the paraxial limit is given by

$$\begin{aligned} &\underline{k} = \nabla\psi \mathcal{A} + \nabla\theta \mathcal{W} \\ &\mathcal{A} = x'' x_\psi + y'' y_\psi \\ &\mathcal{W} = x'' x_\theta + y'' y_\theta \\ &\underline{x} = (x,y) \quad . \end{aligned} \tag{9}$$

Next the Lagrange multiplier, κ, is eliminated by taking the appropriate partial derivatives and combining; this leads to the parallel current constraint

$$S \equiv B \,|\Box\; Q_\psi + B^2 \left(\frac{iQ}{B^2}\right)' = 0 \quad , \tag{10}$$

with the parallel current per unit flux given by

$$i = B\left[\frac{\partial(x',x)}{\partial(\psi,\theta)} + (x \rightarrow y)\right] \; . \tag{11}$$

Also to arrive at Eq. (10) we have made use of Eqs. (5). To close the equation we must pick a suitable form for $p_{\parallel}(\psi,B)$. This we do by assuming a single weighting factor $\overline{\omega}(\psi)$ that goes to zero at $\psi = \psi_p$ (the plasma boundary), and by choosing separate mod-B dependences for the various regions (central cell and plug for TMX-U). The Eqs. (4), (5), (9)-(11), along with the boundary conditions of zero parallel current at the ends, and a perfect conductor at the lateral boundary completely specify the equilibrium.

A direct solution of these equations is a formidable numerical task since it involves a direct solution of a 3-D partial differential equation. The standard procedure for solving is to follow the 3-D time dependent equations[8] removing energy artificially forcing the equations to relax to equilibrium.

Thus our next task is to add time dependence. Thus, we must add to the energy integral (6) the kinetic energy,

$$T = \frac{1}{2}\int \rho \, \underset{\sim}{x}_t \cdot \underset{\sim}{x}_t \, \frac{d\psi d\theta dz}{B} \quad . \tag{12}$$

A similar variation produces[1]

$$\rho \, \underset{\sim}{x}_\psi \cdot \underset{\sim}{x}_{tt} = B\underset{\sim}{x}_\psi \cdot \left(\frac{Q\underset{\sim}{x}'}{B}\right)' - \kappa_\psi \tag{13}$$

and similarly for θ. The subscript t denotes partial differentiation with respect to time. Now since the motion is incompressible at this stage, we have in terms of a stream function U

$$x_t = B(U_\theta x_\psi - U_\psi x_\theta)$$

$$y_t = B(U_\theta y_\psi - U_\psi y_\theta) \quad . \tag{14}$$

where U satisfies

$$\rho(z)\ \nabla_\perp^2 U_t = S \quad . \tag{15}$$

Once again κ has been eliminated by taking suitable linear combinations. Since the equilibrium is independent of ρ it can be chosen arbitrarily so as to simplify the computation. Thus we can set $\rho(z) = B_v^2(z)$, i.e., constant Alfvén speed to relax the Courant constraint. Also we need $\nabla_\perp^2$ in flux coordinates, viz,

$$\rho\ \nabla_\perp^2 = B\left(\frac{\partial}{\partial\psi} x_\theta - \frac{\partial}{\partial\theta} x_\psi\right) \rho B \left(x_\theta \frac{\partial}{\partial\psi} - x_\psi \frac{\partial}{\partial\theta}\right) + \ (x \rightarrow y) \quad . \tag{16}$$

Initial conditions for this stage of the calculation are $U = 0$ and $x = x_0(\psi,\theta,z)$ etc., which satisfy the Jacobian constraint; a simple choice is

$$x_0 = g(\psi,z)\ \sigma_v(z)\ \cos\theta$$

$$y_0 = g(\psi,z)\ \ \tau_v(z)\ \sin\theta \tag{17}$$

where,

$$g^2(\psi,z) = 2\ B_v(z) \int_0^\psi d\psi\ B^{-1}(\psi,z) \quad . \tag{17a}$$

The advantage of this method is that by treating the rhs of Eq. (16) as a known function at the previous time step we need only invert a 2-D elliptic operator; a disadvantage is that the equations have an explicit time differencing which generally produces stringent time step constraints because of numerical stability. As previously mentioned to reach equilibrium energy must be dissipated. There are many ways of accomplishing this, e.g., friction of some form can be added to Eq. (15).

In this work we use the method suggested by Strauss,[4] namely set $U = 0$ every time the volume averaged kinetic energy reaches a maximum thereby converting potential to kinetic energy and removing it. Such a procedure generally takes 5 to 10 thousand time steps (hundreds of Alfvén transit times) to converge, and it is the developing octupole distortion of the flux surfaces, first described by Stupakov,[2] which must be followed carefully.

A comparison of results from the 3D code and the beta expanded equilibrium[3] is shown in Fig. 1 where we depict the parallel current at $\theta = \pi/10$ and $\psi = 1/2\ \psi_p$. As a comparison we show the lowest order (linear in β) current (the dashed line). The calculations were performed with 25% peak beta in plug and central cell. At the bottom of this figure we show the time history of the surface average octupole distortion and the residual of the rhs of Eq. (15). Also shown are a comparison of the pressure surfaces (flux surfaces) at the midplane; the dots label the same surfaces for the vacuum field.

For completeness, we now write down the modifications due to FLR. Since both the FLR and shear Alfvén motion are perturbations, both can be computed ignoring the other and then added appropriately.[5,9] Thus, FLR can be computed for a cylinder (straight theta pinch geometry) and added to the Lagrangian such the Lagrangian density becomes (we use the Lagrangian rather than the energy to obtain the linear time derivative),

$$L = \frac{1}{2}\rho\, \underset{\sim}{x}_t^2 + X\, \underset{\sim}{x}_t \cdot \underset{\sim}{x}_\theta - Y\, \underset{\sim}{x}_\theta^2 - Q\, \underset{\sim}{x}'^2 \tag{18}$$

where for the additional terms

$$X = 2\rho\phi_\psi - \frac{1}{2B}(MB^2)_\psi$$

$$Y = -\rho\phi_\psi^2 + \frac{1}{2B}(MB^2)_\psi\phi_\psi - \frac{1}{B}K_\psi B_\psi$$

$$M = -8\pi B \sum \frac{1}{e}\int f\mu\, \frac{d\mu d\varepsilon}{v_\parallel}$$

$$K = 2\pi B^3 \sum \frac{1}{e^2}\int f\mu^2\, \frac{d\mu d\varepsilon}{v_\parallel} \tag{19}$$

with ϕ the electrostatic potential and μ the magnetic moment. These equations are in the most general form; if however our interest is only in equilibrium, we can discard the linear time derivative term and recover MHD-like equations. Hence only S is modified and the same numerical procedure as before can be used. In this limit $S = S_{MHD} + S_{FLR}$ where,

$$S_{FLR} = \frac{\partial}{\partial\theta} Bx_\psi Y\, x_{\theta\theta} - \frac{\partial}{\partial\psi} Bx_\theta Y\, x_{\theta\theta} + (x \rightarrow y) \quad , \tag{20}$$

and for a Maxwellian isothermal plasma

$$Y = -\rho\,(\omega_{EXB} - \bar{\omega}_{\nabla B})\,(\omega_{EXB} + \omega_{*i}) \quad . \tag{21}$$

To extract the FLR corrections to the octupole distortion of the flux, we follow the method of Ref. 3 using the total source (S). In the limit in which L_c the central length is large, we obtain (see Eq. (71) of Ref. 3)

$$\int_0^L \frac{dz}{B}\left[\left(4\frac{d}{d\psi}\,\psi^2 Y\,\frac{d}{d\psi} - 15\,Y\right) + \psi\,\frac{\partial p}{\partial\psi}\,(\sigma_v\sigma_v'' + \tau_v''\tau_v)\right]\phi_0\,(\psi,z)$$

$$= -\frac{\psi^3}{12}\,B_0\int_0^L \frac{dz}{B}\left[\int_z^L \frac{dz'}{B}\,\frac{\partial p}{\partial\psi}\,(\sigma_v''\sigma_v - \tau_v''\tau_v)\right]^2 \quad . \tag{22}$$

The lhs is the standard expression determining the m=4, ω = 0 mode in a cylinder;[9] the differential operator is the additional term. Without this term we find the octupole distortion growing with central length;[2] with it, saturation occurs since the new term is also linearly proportional to the central cell length. Given that the second term is negative (for interchange stability), the distortion is always reduced if $Y > 0$.[10] On the other hand if $Y < 0$ there are bands where the homogeneous equation can be satisfied; these are bifurcation points of the equilibrium. Presumably, there is a nearby equilibrium not satisfying the symmetry of the vacuum field, Eq. (2).

SHARP BOUNDARY EQUILIBRIUM

Consider a square-pressure profile, constant out to ψ_p and zero beyond. We force the plasma boundary to remain elliptical to all orders in beta and find the shape which minimizes the energy. This procedure generates an ordinary differential equation for the ellipticity of a field line. It can be shown that constraining the shape to be an ellipse is exact only to order β; consequently, the equilibrium derived is only variational. Now κ, see Eq. (8), plays the same role as the pressure does for incompressible hydrodynamics; thus we seek a virtual displacement of the plasma surface at constant total flux, such that the work done is zero. Consequently the jump in $\kappa\xi_n$ across the plasma surface must be zero where ξ_n is the normal displacement of the surface. It is easy to show that $\xi_n \sim \cos 2\theta\ (x^2/\sigma^4 + y^2/\tau^4)^{-1/2}$ and that $d\ell\xi_n \sim d\theta \cos 2\theta$. Hence, $\int[\kappa] \cos 2\theta\, d\theta = 0$ or, integrating by parts

$$\int d\theta \sin 2\theta\ ([Q\square] + B_v B_v' [b_\theta]) = 0 \qquad (23)$$

which is the constraint set by Ampere's law and where $\hat{b} = \underline{B}/|B|$. Note that the equations of the equilibrium field line inside the plasma satisfy the same functional relation as the vacuum field see Eqs. (1)-(3). To close, we force the normal component of $\hat{b}$ to be zero at the surface. Now the field outside the plasma can be written

$$\underset{\sim}{B}_{out} = B_v \left(\hat{z} + \hat{x}x \frac{\sigma_v'}{\sigma_v} + \hat{y}y \frac{\tau_v'}{\tau_v}\right) + \nabla_\perp \Phi + O(\lambda^2) \qquad (24)$$

where Φ is a scalar potential which would be zero in the absence of plasma. And,

$$b_n = \underset{\sim}{\nabla}\psi \cdot \hat{b}_{out}\Big)_{\psi=\psi_p}$$

$$= 2\psi B_v \left[\cos^2\theta \left(\frac{\sigma_v'}{\sigma_v} - \frac{\sigma'}{\sigma}\right) + \sin^2\theta \left(\frac{\tau_v'}{\tau_v} - \frac{\tau'}{\tau}\right)\right] + \nabla\psi \cdot \nabla_\perp \theta \quad . \qquad (25)$$

In the above we have used

$$2\psi = \frac{x^2}{\sigma^2} + \frac{y^2}{\tau^2} \qquad (26)$$

$$\nabla\psi = \frac{x\hat{x}}{\sigma^2} + \frac{y\hat{y}}{\tau^2} - \hat{z}\left(\frac{x^2}{\sigma^2}\frac{\sigma'}{\sigma} + \frac{y^2}{\tau^2}\frac{\tau'}{\tau}\right) \quad . \qquad (26a)$$

Inside $b_n = 0$ by construction since we are using flux coordinates for the interior. Outside the plasma we use the confocal elliptic coordinate system

$$x = \sqrt{2\psi_p(\sigma^2 - \tau^2)}\ \cosh\mu \cos\xi$$

$$y = \sqrt{2\psi_p(\sigma^2 - \tau^2)}\ \sinh\mu \sin\xi \qquad (27)$$

with well-behaved solutions at infinity

$$\Phi = \psi_p \frac{B_v}{B} \left(\Phi_0 \mu + \Phi_2 \cos 2\xi \, e^{-2(\mu-\mu_p)}\right) , \tag{28}$$

and from Eq. (25)

$$\Phi_0 = \left(\frac{p_\perp}{B^2}\right)' \frac{B^2}{B_v^2}$$

$$\Phi_2 = c_v' - c'$$

$$\sigma = e^c \, B^{-1/2} , \tag{29}$$

Next we must evaluate the constraint Eq. (23). First we have

$$[b_\theta] = \psi_p \sin 2\theta \left(\Phi_0 \frac{\sigma^2 - \tau^2}{2} - \Phi_2 (\sigma + \tau)^2\right) \tag{30}$$

and

$$\text{H}_{in} = \sin 2\theta \, (\sigma\sigma'' - \tau\tau'')$$

$$\text{H}_{out} = \psi_B \left(\tau^2 \frac{\sigma_v''}{\tau_v} - \sigma^2 \frac{\sigma_v''}{\sigma_v}\right) \sin 2\theta + \left.\frac{\partial I}{\partial \theta}\right|_{\psi_p} , \tag{31}$$

where

$$I = \left(\frac{\Phi}{B_v}\right)' + \frac{1}{2B_v^2} (\phi_x^2 + \phi_y^2) + \frac{1}{B_v}\left(\frac{\sigma_v'}{\sigma_v} x\phi_x + \frac{\tau_v'}{\tau_v} x\phi_y\right) \tag{32}$$

and all derivatives are evaluated in (x,y,z) space. Substituting Eqs. (26) and (27) we obtain after some algebraic reduction

$$I = \cos 2\theta \left(\frac{\Phi_2}{B}\right)'_{\psi,\theta} + (\sigma^2 \sin^2\theta + \tau^2 \cos^2\theta)^{-1} \frac{1}{4B^2} [- \Phi_0^2$$

$$+ 4\Phi_2^2 (1 - \cos^2 2\theta + B(\sigma^2 + \tau^2) \sin^2 2\theta) + \Phi_0\Phi_2(4 \cos 2\theta - 2B(\sigma^2 - \tau^2) \sin^2 2\theta)] \tag{33}$$

It is this term which prevents the surface from being an exact ellipse, and as mentioned earlier, the departure is of order β^2. Substituting into Eq. (23) produces the equation for equilibrium

$$\left[\frac{Q}{2B_v}\Phi_0(\sigma^2+\tau^2)-\Phi_2\left(\frac{Q}{B_v}(\sigma^2+\tau^2)+\frac{2B_v}{B}\right)\right]'$$

$$=\frac{2p}{B_v}\left[\sigma^2\frac{\sigma_v''}{\sigma_v}-\tau^2\frac{\tau_v''}{\tau_v}+(\tau^2-\sigma^2)\left(\Phi_2^2-\frac{1}{4}\Phi_0^2\right)\right]$$

$$-\frac{B_v}{B}\left[B(\tau^2-\sigma^2)\left(\Phi_2^2-\frac{1}{4}\Phi_0^2\right)+\frac{\sigma-\tau}{\sigma+\tau}\Phi_0^2-2\Phi_0\Phi_2\right] \qquad (34)$$

This along with the boundary conditions

$$\Phi_2(\pm L)=0;\ (B_\perp \to B_{\perp vac}) \qquad (35)$$

and the constraint

$$\Phi_0(\pm L)=0;\ (p_\perp \text{ goes to zero quadratically}) \qquad (36)$$

determines the equilibrium. Note that Eq. (34) is a 2nd order non-linear ODE.

These equations are routinely solved numerically. Results are shown for a MARS (double-fan configuration) in Fig. 2 where we plot mod-B, vacuum and finite β curvatures (κ) and flux surfaces at the high field coil; the various average β values are shown on the mod-B plot. At these extreme values of β we find a strong elliptical distortion of the flux surface at the high field circular coil. To what extent this distortion is exacerbated by the sharp boundary model is now under study using the general β equilibrium code. However, within the framework of this model, it is useful to show the origin of the distortion and a potential cure.

We linearize Eq. (34) about the vacuum field and keep curvatures only in the double fan region where they are large. For simplicity, neglect diamagnetism in the central cell.

In this limit Eq. (34) has the simple form

$$\eta'' + \sigma^- = 2\eta\sigma^+ \tag{34a}$$

where

$$\eta = c - c_v$$

$$\sigma^\pm = \frac{2p}{BB_0}(\tau_v'' \tau_v \pm \sigma_v''\sigma_v) \tag{37}$$

Now the rhs is large only in a small region, moreover, the central cell is long and η''/η is small over this region. In this limit Eq. (34a) can be solved by treating the rhs as a delta function at $z = L_0$ (see Fig. 2) so that

$$\eta' = -\int_z^\infty \sigma^- \, dz' \qquad z > L_0$$

$$= -\int_z^\infty \sigma^- \, dz' + 2\eta(L_0)\,\Sigma^+ \qquad z < L_0 \tag{38}$$

where we have further defined

$$\Sigma^\pm = \int_0^\infty \sigma^\pm(z')dz' \tag{39}$$

Integrating - ` produces

$$\eta(z) = 2z\eta(L_0)\,\Sigma^+ - \int_0^z dz' \int_{z'}^\infty \sigma^- \, dz''$$

and integrating by parts results in

$$\eta(z) = -2z\eta(L_0)\,\Sigma^+ - z\,\Sigma^- - \int_0^z dz'\,(z' - z)\,\sigma^- \tag{40}$$

Again make use of the localized properties of σ to obtain

$$\eta(z) = -\,2L_0\eta(L_0)\,\Sigma^+ - L_0\,\Sigma^- - \begin{bmatrix} 0 \\ \Lambda \\ 2\Lambda \end{bmatrix} \begin{matrix} ,\; z = L^- \\ ,\; z = L_0 \\ ,\; z = L^+ \end{matrix} \tag{41}$$

and

$$\eta(L_0) = -\frac{\Lambda + L_0\Sigma^-}{1 + 2L_0\Sigma^+} \quad , \tag{42}$$

where

$$\Lambda = \frac{1}{2}\int_{L^-}^{L^+} dz(L_0 - z)\sigma^- \quad . \tag{43}$$

The position of the various L's are marked on the graph of Ω in Fig. 2. Finally in the limit of a long central cell

$$\eta(L^\pm) = -\frac{\Sigma^-}{2\Sigma^+} \pm \Lambda \quad . \tag{44}$$

The general behavior of this approximate solution is in good qualitative agreement with the full numerical solution.

Our design criterion[11] is to make Σ^- as small as possible thus minimizing the lowest order parallel current flowing through the central cell. Unfortunately the higher order current no longer vanishes, recall $\eta'(0)$ is in fact proportional to the parallel current flowing through the central cell. Thus to improve the configuration we should aim towards $\eta(L^-) = 0$.

It should be emphasized that to obtain true quantitative criteria it would appear that we must solve the full 3-D general β equilibrium, probably with the FLR modifications.

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FIGURE CAPTIONS

Fig. 1 Comparison between beta expanded equilibrium (TEBASCO) and full 3-D general beta code.

Fig. 2 Sharp boundary equilibrium.

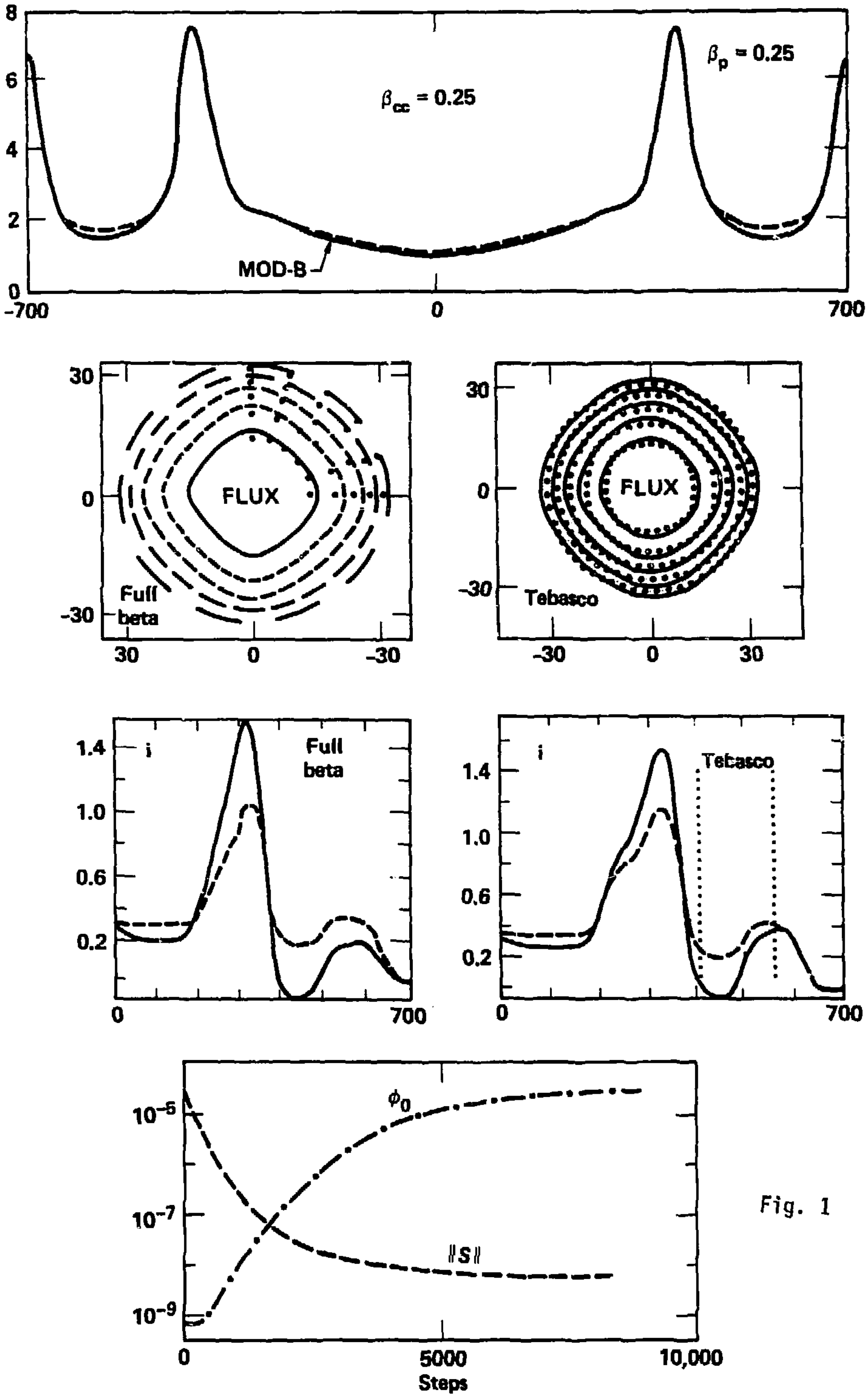
8
6
4
2
0
-700
0
700
β_{cc} = 0.25
β_p = 0.25
MOD-B
30
0
-30
FLUX
Full
beta
Tebasco
1.4
1.0
0.6
0.2
i
Full
beta
Tebasco
10^{-5}
10^{-7}
10^{-9}
ϕ_0
$\|S\|$
5000
10,000
Steps

Fig. 1

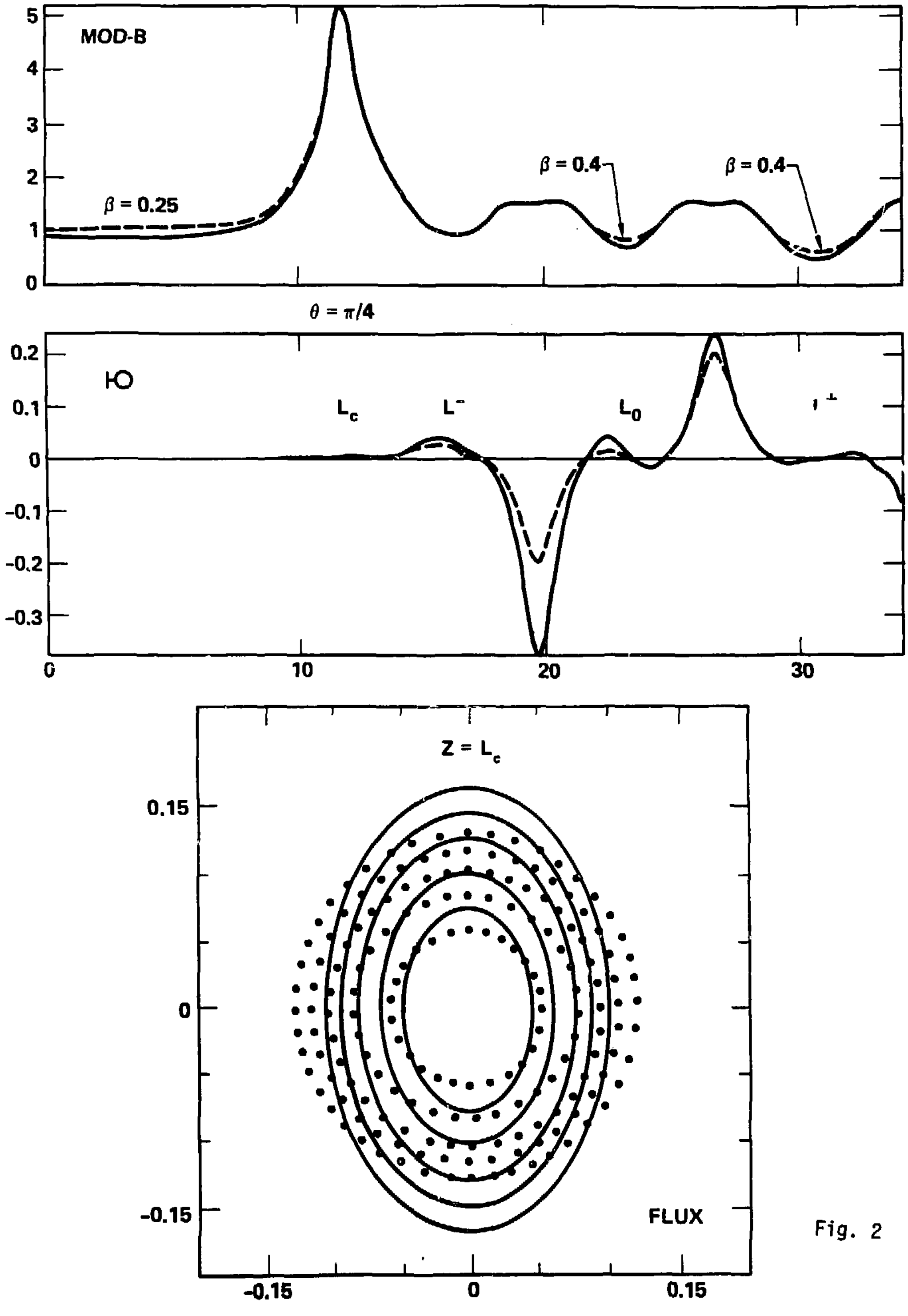

Fig. 2