ELECTRON DIAMAGNETISM AND TOROIDAL COUPLING OF TEARING MODES

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ABSTRACT

Using a simple model for the layer of the tearing mode, we demonstrate that toroidally coupled tearing modes with two rational surfaces are most unstable when the ω^* 's of the electrons at the rational surfaces are equal. The onset of instability may then occur because of the tuning of ω^* rather than the passage of Δ' -like quantities through zero. This mechanism for the onset of instability is sharp since the resonance is narrow. The effect of toroidal rotation is also discussed.

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Tearing modes are observed to rotate with a frequency of order the electron diamagnetic drift frequency ω^* . ω^* has a significant effect on the dynamics within the "layer" surrounding the rational surface but not on the external solutions.¹⁻³ We consider tearing modes with toroidal mode number l and two rational surfaces in the plasma, one at r_m where $q(r_m) = m/l$ and the other at r_{m+1} where $q(r_{m+1}) = (m+1)/l$. Since a toroidal eigenmode contains all the poloidal harmonics, both rational surfaces will have boundary layers.⁴ We demonstrate that when $\omega_m^*(r_m) \neq \omega_{m+1}^*(r_{m+1})$, there are two eigenmodes with frequencies ω_m^* and ω_{m+1}^* . When $\omega_m^* \approx \omega_{m+1}^*$, the two modes are resonant; this resonant mode is more unstable than the nonresonant mode. The width of this resonance is $(\omega_m^* - \omega_{m+1}^*)/\omega_m^* \sim \delta/a$, where δ is the resistive layer width and a the minor radius. Since the width of the resonance is very narrow, we expect that a sharp onset of the mode can occur when $\omega_m^* - \omega_{m+1}^*$ passes through zero. Matching ω^* 's is equivalent to matching the toroidal component of the diamagnetic drift velocities.

The internal "layer" solution matches to the external solution via the quantity Δ' .⁴ (Δ' is the jump in the ratio of small to large Mercier solutions across the rational surface.) For a mode with two rational surfaces there are two jumps: Δ'_m at r_m and Δ'_{m+1} at r_{m+1} . Δ'_m and Δ'_{m+1} are constrained by the external solution to satisfy⁴

$$(\Delta'_m - \alpha)(\Delta'_{m+1} - \beta) - \gamma = 0 \quad (1)$$

α , β , and γ are constants obtained by solving the ideal MHD equations in the external region.⁴ In a large aspect ratio torus $\alpha \sim \Delta'_m$ and $\beta \sim \Delta'_{m+1}$ of an equivalent cylinder and γ is of order the inverse aspect ratio squared.⁴ Layer theory provides the relationships $\Delta'_m = \Delta'_m(\omega)$ and $\Delta'_{m+1} = \Delta'_{m+1}(\omega)$ which when substituted into Eq. (1) yield a dispersion relation.

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Equation (1) describes a hyperbola in the Δ'_m, Δ'_{m+1} plane (See Fig. 1). Most layer theories require $\Delta' > 0$ for instability; therefore, instability is only possible when part of the hyperbola is in the positive quadrant of Δ'_m, Δ'_{m+1} space. If $\alpha < 0$, $\beta < 0$, and $\alpha\beta < \gamma$, then no part of the hyperbola is in the positive quadrant. The layer theory describes another curve in the Δ'_m, Δ'_{m+1} space. (This curve is parameterized by ω .) The intersections of the two curves give Δ'_m and Δ'_{m+1} of the eigenfunction. Often ω , Δ'_m , and Δ'_{m+1} are complex and their graphical representation is in four dimensions!

When $\nu T_e = 0$ at the rational surfaces, the semicollisional and collisionless layer theories yield relations of the form^{2,3}

$$\begin{aligned}\Delta'_m &= \frac{i\beta}{\delta_m} \frac{\rho_m}{\omega_m} \left(\frac{\omega}{\omega_m^*} - 1 \right) \\ \Delta'_{m+1} &= \frac{i\beta}{\delta_{m+1}} \frac{\rho_{m+1}}{\omega_{m+1}} \left(\frac{\omega}{\omega_{m+1}^*} - 1 \right)\end{aligned}\quad (2)$$

The subscript m or $m+1$ means the quantity is evaluated at the m 'th or $m+1$ 'th surface. $\beta_p = 4\pi nT/B^2(l_S^2/l_n^2)$ is the poloidal beta, l_S is the magnetic shear length and l_n is the density scale length. $\omega^* = lcT_e/ne \, dn/d\psi$ where ψ is the poloidal flux. δ is the layer width which depends on the collisionality and geometry. When the collision rate of the electron ν is much less than ω^* (the collisionless mode), $\delta \approx \rho_e (l_S/l_n)$. When $\nu \gg \omega^*$ and $\rho_i > \delta$ (the semicollisional mode), $\delta \approx \rho_e (l_S/l_n) (i\omega^*/\nu)^{1/2}$.

For definiteness we will deal with the collisionless mode (the semicollisional mode yields similar results). Inserting Eq. (2) into Eq. (1), we obtain

$$[i(\omega - \omega_m^*) - \alpha'] [i(\omega - \omega_{m+1}^*) - \beta'] - \gamma' = 0, \quad (3)$$

where $\alpha' = \alpha \omega_m^* \delta_m / \beta_{pm}$, $\beta' = \beta \omega_{m+1}^* \delta_{m+1} / \beta_{pm+1}$, and $\gamma' = \gamma \omega_m^* \omega_{m+1}^* \delta_m \delta_{m+1} / (\beta_{pm} \beta_{pm+1})$.

We expect that except when we are close to ideal marginality δ , $\beta \sim O(a^{-1})$ and $\gamma \sim O(a^{-2})$, where a is the minor radius of the tokamak. α' , β' , and γ' are, therefore, small since δ/a is very small.

We consider two situations: Case I, $\omega_m^* \neq \omega_{m+1}^*$ and Case II, $\omega_m^* = \omega_{m+1}^* + O(\delta/a)$.

Case I $\omega_m^* \neq \omega_{m+1}^*$

In this case the two solutions to Eq. (3) are:

$$\omega = \omega_m^* - i\alpha' + O(\delta^2/a^2)$$

$$a\delta'_{m+1} \sim \frac{a}{\delta} \gg 1 \quad (4)$$

$$\Delta'_m = \alpha$$

and

$$\omega = \omega_{m+1}^* - i\beta' + O(\delta^2/a^2)$$

$$a\delta'_m \sim a/\delta \gg 1 \quad (5)$$

$$\Delta'_{m+1} = \beta$$

Physically, Eq. (4) corresponds to a solution in which the $m+1$ th surface at r_{m+1} is behaving like an ideal MHD layer ($\Delta' \rightarrow \infty$ is the ideal behavior). ω^* is a fast frequency compared to the reconnection rate (which is of or-

der $\omega^* \delta \Delta'$) so that the layer will respond ideally when the frequency seen by the plasma, $\omega - \omega^*$, is of order ω^* . The solution given by Eq. (4) lies on the asymptote of the hyperbola $\Delta'_m = \alpha$ in the Δ'_m, Δ'_{m+1} space (see Fig. 1). Clearly, Eq. (5) describes the analogous solution to Eq. (4) with $m \rightarrow m+1$ and $m+1 \rightarrow m$. Note that when $\alpha < 0$ and $\beta < 0$ both solutions [Eqs. (5) and (6)] are stable.

Case II $\omega_m^* = \omega_{m+1}^* + 0 (\delta/a)$

In this case the two surfaces are resonant and it is possible to pick ω so that neither surface behaves ideally. Let $\omega = 1/2(\omega_m^* + \omega_{m+1}^*) - i\Gamma$ and $\delta\omega^* = 1/2(\omega_{m+1}^* - \omega_m^*)$. (If Γ has a positive real part, the mode grows.) Equation (3) then becomes

$$2\Gamma = \alpha' + \beta' \pm [(\alpha' + \beta')^2 + 4(\gamma' - \alpha'\beta') - 4\delta\omega^{*2} - 4i\delta\omega^*(\alpha' - \beta')]^{1/2}. \quad (6)$$

When $\delta\omega^* \gg \alpha', \beta'$, and $(\gamma')^{1/2}$, then the solutions of Eq. (6) reduce to Eqs. (4) and (5). Clearly, when $\delta\omega^* \sim 0(\alpha', \beta', (\gamma')^{1/2})$, the resonant effects are important.

We assume that γ is positive (there is no proof of this but it seems from experience that below the ideal stability boundary γ is usually positive). The growth rate is a maximum (minimum) for the positive (negative) sign of the square root when $\delta\omega^* = 0$ (see Fig. 2). There is no growing mode for any value of $\delta\omega^*$ when $\alpha < 0$, $\beta < 0$, and $\gamma < \alpha\beta$.

Perhaps the most interesting case is when $\alpha < 0$, $\beta < 0$, but $\gamma > \alpha\beta$ (this is illustrated in Fig. 1) since when $\delta\omega^* \gg \alpha', \beta'$, and $(\gamma')^{1/2}$, both modes are stable and when $\delta\omega^* = 0$ one mode is unstable, and the other mode is stable. One can visualize this case by noting that the solutions

corresponding to $\delta\omega^* = \omega^*$ lie on the stable asymptotes of the hyperbola in Fig. 1. But, when $\delta\omega^* = 0$, one of the solutions lies on the part of the hyperbola in the positive quadrant of Δ_m, Δ_{m+1} space. (This visualization is complicated by the complex nature of the Δ 's but it still provides some insight.)

In this note we discuss the influence of ω^* on tearing mode stability in a torus. For a plasma with two rational surfaces the most unstable mode is when ω^* is the same at both rational surfaces, $\omega_m^* = \omega_{m+1}^*$. We show that situations exist where the tearing mode is stable when $\omega_m^* \neq \omega_{m+1}^*$, but unstable when $\omega_m^* = \omega_{m+1}^*$. In a typical tokamak the current profile is relatively stationary (it evolves on the resistive time scale), but the density profile may change relatively quickly (especially with pellet fueling or during a sawtooth). We expect that as the q profile and p profile evolve in time towards instability, the first appearance of the hyperbola in the positive quadrant is as illustrated in Fig. 1 (where $\alpha < 0$, $\beta < 0$, and $\gamma > \alpha\beta$). One scenario for the onset of tearing modes is as follows: The hyperbola is as illustrated in Fig. 1. Prior to onset $\omega_m^* \neq \omega_{m+1}^*$, but the ω^* 's change with time (the hyperbola is approximately stationary) and when $\omega_m^* = \omega_{m+1}^*$, the mode becomes unstable and starts to grow. Since the resonance is sharp ($\delta\omega^*/\omega^*$ is of order the layer width over the minor radius), this scenario leads to a rapid onset of the tearing mode. A mode with $\omega_m^* = \omega_{m+1}^*$ involves reconnection (islands) at both rational surfaces and, therefore, the eigenfunction will be qualitatively different from modes where $\omega_m^* \neq \omega_{m+1}^*$ and reconnection takes place at only one surface.

We have ignored the temperature gradient in this treatment; however, in the linear regime of the mode the temperature gradient is stabilizing.^{3,5} The temperature gradient is destroyed by an island of width δ , the resistive layer

width. The density gradient, however, is flattened by an island of width $a_i l_s / l_n$.³ Therefore, if the initial perturbation (i.e., prior to instability onset) is such that the initial island width is bigger than δ and smaller than $a_i l_s / l_n$, the temperature gradient may be ignored but the density gradient part of ω^* must be retained. The T-10 group has reported that just prior to the sawtooth crash $d n T / d r$ at $q=1$ always has the same value.⁶ Similar observations have been made on TFTR.⁷ One might speculate that this observation indicates that the plasma has to satisfy a resonance condition (note that ω_2^* is likely to be relatively constant in a sawtooth discharge whereas ω_1^* may change rapidly). In the presence of toroidal rotation ω is replaced by $\omega - l \omega_\phi$, where ω_ϕ is the local toroidal angular velocity of the rational surface. Variation in ω_ϕ across the minor radius leads to a conclusion similar to the one presented here for variation in ω^* .

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REFERENCES

- ¹B. Coppi, Phys. Fluids 7, 1501 (1964).
- ²J.F. Drake and Y.C. Lee, Phys. Fluids 20, 1341 (1977).
- ³S.C. Cowley, R.M. Kulsrud, and T.S. Hahm, Phys. Fluids 29, 3230 (1986).
- ⁴J.W. Connor, S.C. Cowley, R.J. Hastie, T. Hender, A. Hood, and T. Martin, Phys. Fluids (to be published).
- ⁵J.F. Drake, T.M. Antonsen, A.B. Hassam, and N.T. Gladd, Phys. Fluids 26, 2509 (1983).
- ⁶G.A. Bobrovskii et al., Proceedings of the 12th European Conference on Controlled Fusion and Plasma Physics, Budapest, 1985, (European Phys. Soc., Geneva, 22 July 1985) Vol. 1, p. 142.
- ⁷K. McGuire et al., Bull. Am. Phys. Soc. 31, 9 (November 1985).

FIGURES

Fig. 1 Hyperbola in the Δ_m', Δ_{m+1}' plane. There is no intersection with the positive quadrant when $\alpha < 0$, $\beta < 0$, and $\alpha\beta > \gamma$.

Fig. 2 Real part of Γ, Γ_r , versus $\delta\omega^*$ for $\alpha' > \beta'$ and $\gamma' > 0$.

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Fig. 1

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Fig. 2

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