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A CRUDE SCALING LAW FOR LASERTRONS*

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The lasertron is under consideration in several laboratories as a promising new source of high-power microwaves that is competitive with conventional klystrons in its properties — particularly in efficiency. In this note, a crude scaling law for high-power lasertrons is derived based on the observation that the main limiting phenomenon, the phenomenon that curtails the tube's efficiency, is debunching due to longitudinal space-charge forces acting in the electron gun. The scaling law indicates that lasertrons are quite restricted in the power they can generate at high frequencies.

Comparing the lasertron with the conventional klystron, we note immediately the crucial difference as far as longitudinal space-charge effects are concerned: In the lasertron the beam is "born bunched." It leaves the cathode and spends its low energy epoch in the most tightly bunched condition it will ever experience and in which longitudinal space-charge fields are the greatest. At high beam currents these forces will debunch the beam and drive the efficiency down. In the conventional klystron, in contrast, the beam in the gun is unbunched and indifferent to longitudinal fields which are absent in any case. In terms of this kind of debunching, the superiority of the conventional klystron is evident.

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Having identified longitudinal space-charge debunching in the gun as the phenomenon of interest, we need to devise a simplified model of a lasertron gun to use in estimating the debunching effects. We take the cathode to be a disk of radius a which emits a cylindrical electron beam of radius a with a pulse duration τ . This is the duration of the laser light pulse, assumed to be rectangular. In the gun, between the cathode and the anode, we assume a uniform, externally applied electric field, $\mathcal{E}_0$, along the z -axis (the longitudinal direction) and a gap between cathode and anode of g , so the gun voltage, V_0 , is

$$V_0 = \mathcal{E}_0 g.$$

Before taking up the subject of space-charge effects, let us consider the low-current case in which space-charge fields are negligible. During the laser pulse a column of electrons is emitted from the cathode. Let us choose our time scale so that the laser pulse begins at $t = 0$; it ends at $t = \tau$. What is the length of the column in the gun?

Each electron in the absence of space-charge forces obeys the equation of motion,

$$\ddot{z} = \frac{e\mathcal{E}_0}{m}. \quad (1)$$

We shall work in the non-relativistic approximation. The mass, m , is the rest-mass. The motion of an electron at the head of the column is

$$z_h = \frac{e\mathcal{E}_0}{2mc^2} c^2 t^2, \quad (2)$$

and that of an electron at the tail, which was emitted at $t = \tau$, is

$$z_t = \frac{e\mathcal{E}_0}{2mc^2} c^2 (t - \tau)^2. \quad (3)$$

The difference between these two positions is the length of the column.

$$L_0(t) = \frac{e\mathcal{E}_0}{2mc^2} c^2 \tau^2 \left(2\frac{t}{\tau} - 1 \right), \quad (4)$$

where $t \geq \tau$ because the column is not fully formed until the tail leaves the

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cathode. The column lengthens as it crosses the gun because the tail is always moving slower than the head. The initial length — the length just after the tail has departed the cathode — is

$$\ell_0(\tau) = \frac{e\mathcal{E}_0}{2mc^2} c^2 \tau^2. \quad (5)$$

The position of the middle of the column is

$$\frac{z_h + z_t}{2} = \frac{e\mathcal{E}_0}{2mc^2} c^2 t^2 \left(1 - \frac{\tau}{t} + \frac{\tau^2}{2t^2}\right). \quad (6)$$

Let T be the time taken by the middle of the column to reach the anode and suppose that $T \gg \tau$. Then

$$T = \sqrt{\frac{2mc^2}{eV_0}} \frac{g}{c}. \quad (7)$$

To address the high-current case and the question of space-charge effects, we need to elaborate our simple model slightly. We imagine that the electrons are emitted in two bursts, one at $t = 0$ and one at $t = \tau$. Each burst is very brief — much briefer than $\tau = 0$ — and each burst contains half the charge of the pulse, i.e., $I\tau/2$, where I is the peak current emitted during the laser pulse. By this change we have transformed the column of charge into two thin discs of charge separated by ℓ . In the high-current case, these disks, which we shall consider to be rigid structures of charge, repel each other. The space-charge forces act on top of the externally applied field and lengthen the separation of the disks, a process we shall identify with debunching.

We shall approximate the electric field of a disk as the field on the axis. The longitudinal field decreases (quadratically) as we go off the axis and the correct force to use is the average of the field over the other disk, but we shall use the value of the field on the axis.

$$\mathcal{E}_z = \frac{I\tau}{a^2} \left[1 - \frac{b}{\sqrt{a^2 + b^2}}\right], \quad (8)$$

where b is the distance along the axis from the center of the disk to the field

point (the units are c-g-s). But being crude, we shall approximate still further by dropping the second term and writing for our purposes

$$\mathcal{E}_z = \frac{I\tau}{a^2}. \quad (9)$$

This is tantamount to the assumption that $b \ll a$. Taking account of space charge now, the equations of motion for the head disk and the tail disk become

$$\ddot{z}_{h,t} = \frac{e}{m} \left[\mathcal{E}_0 \pm \frac{I\tau}{a^2} \right]. \quad (10)$$

The solutions are

$$z_h = \frac{e}{2mc^2} \left[\mathcal{E}_0 + \frac{I\tau}{a^2} \right] c^2 t^2 \quad (11)$$

$$z_t = \frac{e}{2mc^2} \left[\mathcal{E}_0 - \frac{I\tau}{a^2} \right] c^2 (t - \tau)^2. \quad (12)$$

It is obvious from the outset that the space-charge fields must be small in comparison to the externally applied gun field or the bunching created by the laser pulse will be totally destroyed. (In our model with its approximations, for example, the tail will never leave the cathode if the space-charge fields equal the applied field.) Consequently we can anticipate that space charge will not affect the transit time of the bunch very much, because the space-charge forces push the head forward and the tail backward and their effects, being small to begin with, should approximately cancel. Indeed, we can show that the current-dependent part of the center-of-charge motion

$$\frac{z_h + z_t}{2}$$

is small in comparison with the current-independent part in cases of interest. Thus to the extent that we can neglect τ in comparison with t , we may interpret T as given by Eq.(7) as the time taken by the bunch to cross the gun of the lasertron.

The length of the bunch will vary in the presence of space-charge forces in a different way than it would in their absence. The bunch-length is the difference $\ell = z_h - z_t$.

$$\ell(t) = \ell_0(t) + \frac{e}{2mc^2} \frac{I\tau}{a^2} 2c^2 t^2 \left(1 - \frac{\tau}{t} + \frac{\tau^2}{2t^2}\right). \quad (13)$$

Now assuming that $\tau \ll T$,

$$\ell(T) - \ell_0(T) = \frac{e}{2mc^2} \frac{I\tau}{a^2} 2c^2 T^2 \quad (14)$$

This expression describes the debunching due to space-charge forces. When $\ell(T)$ grows substantially relative to its value in the absence of space charge, the efficiency of the lasertron is compromised. Still assuming $\tau \ll T$,

$$\ell_0(T) = \frac{e\mathcal{E}_0}{2mc^2} 2c^2 \tau T. \quad (15)$$

A high-efficiency lasertron must satisfy the condition,

$$\frac{\ell(T) - \ell_0(T)}{\ell_0(T)} = \frac{IT}{a^2 \mathcal{E}_0} \ll 1; \quad (\text{cgs}). \quad (16)$$

In m-k-s units the condition reads

$$\frac{Z_0 c}{4\pi} \frac{IT}{a^2 \mathcal{E}_0} \ll 1; \quad (\text{mks}), \quad (17)$$

where Z_0 is the impedance of free space. Now making use of Eq.(6), we can eliminate T and get

$$\frac{1}{4\pi} \sqrt{\frac{2mc^2}{eV_0}} \frac{IZ_0}{V_0} \frac{g^2}{a^2} \ll 1. \quad (18)$$

This is the first form of the crude scaling law. We may convert it to another useful form by putting it in terms of the beam power averaged over rf structure,

$$P = V_0 I D,$$

where D is the rf duty cycle, the ratio of the laser pulse width to the radiofrequency period. For efficiency, of course, $D \ll 1$. This merely requires that the

spectrum of the beam current be strong at the operating frequency. In these terms,

$$\frac{1}{4\pi} \sqrt{\frac{2mc^2}{eV_0}} \frac{Z_0 P}{E_0^2 a^2 D} \ll 1. \quad (19)$$

And finally we can observe that the radius of the beam, at least in existing designs, is limited by the radius of the beam aperture in the rf output network (cavity).

$$a \leq c/\omega. \quad (20)$$

This restriction might be ameliorated somewhat by some sort of converging optical system to allow the beam to be larger in the gun than it is in the rf output region. However such a system would have to be isochronous, a difficult condition to achieve, so we shall adopt this restriction. With it we obtain the final form of the crude scaling law.

$$\frac{1}{4\pi} \sqrt{\frac{2mc^2}{eV_0}} \frac{Z_0 P \omega^2}{E_0^2 c^2 D} \ll 1. \quad (21)$$

Speaking practically, the radical will be at most of the order of unity, E_0 is limited by sparking and D is limited as noted above. The result is that the product $P\omega^2$ is strictly limited. As the frequency of operation is increased, the power output to be expected from lasertrons decreases.

This scaling law is admittedly crude. It is based on non-relativistic mechanics, a simple model and pessimistic approximations. Nonetheless it is undoubtedly qualitatively correct, and it emphasizes the point that these disruptive space-charge effects act deleteriously in the lasertron, making the conventional klystron superior at high frequency and high power in this respect at least.

We shall close this note with an example. The following parameters are given for a lasertron by W. B. Herrmannsfeldt in a note dated November 19, 1984. They are typical of those sought in the SLAC design.

$$P = 70 \text{ MW}$$

$$\omega = 2\pi(2856) = 1.8 \times 10^{10} \text{ per second}$$

$$E_0 = 8 \text{ MV/m}$$

$$V_0 = 0.4 \text{ MV}$$

$$D = 0.17$$

From these we find

$$\frac{1}{4\pi} \sqrt{\frac{2mc^2}{eV_0}} \frac{Z_0 P \omega^2}{E_0^2 c^2 D} = 1.1$$

This value suggests that the performance of this lasertron would be seriously degraded by space-charge debunching in the gun, and computer simulations indicate that such is indeed the case.

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