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MASTER
ARCHSIM: A PROTON-SYNCHROTRON TRACKING PROGRAM,
INCLUDING LONGITUDINAL SPACE CHARGE*

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Summary

A particle-tracking program has been written for simulating the acceleration cycle of a rapid-cycling proton synchrotron. A lattice can consist of up to 100 cells and rf cavities. Transport of the beam in six dimensions includes all second-order optical terms. The rf field and proton velocity are treated exactly. Longitudinal space charge is handled in a self-consistent manner. The fluctuations due to the finite number of particles are handled by a Gaussian smoothing algorithm. The program runs on a VAX 11/780 and can track 100 particles without space charge through the full acceleration cycle from 0.8 to 32 GeV in 49 minutes. A thousand particles with space charge takes about ten hours of computer time.

Introduction

The motivation for writing this tracking program was the need to explore the effect of various rf accelerator-cavity parameters on the beam dynamics and stability of a proposed 32-GeV rapid-cycling synchrotron called LAMPF II. LAMPF II is being designed for 100-μA average current. Initially there was some concern that space-charge forces might cause a strong blowup as the beam passed through the negative-mass phase transition at ~12-GeV. Simulation using ARCHSIM has demonstrated that probably there will not be a problem at transition.

The reference design for the accelerator has a superperiodicity of 5. The straight sections are dispersion-free to reduce the possibility of synchrotron-betatron coupling effects. It is proposed that there be 60 rf cavities to provide a peak acceleration voltage of 14-MV/turn.

This paper will describe the organization of the program and the special features of the main subroutines. This will be followed by a discussion of runtime experience on the VAX 11/780.

Program Description

Figure 1 shows a simplified flow chart. INPUT reads in parameters characterizing the accelerator. It calls three subroutines whose purposes are to calculate...
transition gamma \( \gamma \), to initialize certain arrays for the space-charge calculation and to check (and if necessary correct) the transport matrices to ensure that they are equivalent to symplectic matrices. These transport matrices are obtained from programs such as DIMAT3 or TRANSPORT and were not intended to be used the 60 000 times necessary to simulate an acceleration cycle—small inaccuracies are magnified by such cycling. Making the matrices symplectic to the accuracy of the computer ensures that the phase-space volume of the bunches is not artificially increased during transport between rf cavities.

The main DO-loop on \( N \) is not intended to be terminated by the 10 000 turn maximum, but rather by the time, \( T_{MAX} \), that it takes to achieve the maximum energy. Particles are injected during the first \( N \) turns. Particles are represented in the program by 9-D vectors. The first six components are the usual TRANSPORT variables \((x, y, \beta, \gamma, \phi, \lambda)\). The other three components are the difference between the time taken by the particle to reach a certain location on the ring and the time that a synchronously accelerated particle would have taken; the difference in energy between the particle and the synchronous particle; and a tag (0 or 1) to determine if the particle has been lost, that is, if at some time its transverse \((x,y)\) coordinates have exceeded the apertures of the accelerator. This tag is changed in subroutine \( LOST \).

The particles are transported from node to node around the ring by second-order transport matrices in subroutine \( ADV \). This subroutine also calls subroutine \( PFUN \), which changes the synchronous momentum. At each node several things may occur.
depending on control parameters: The particles may have their longitudinal momentum and energy changed by a simulated rf cavity, in subroutine ACC. They may receive a linear change in transverse momentum in a thin-lens, time-dependent quadrupole (TDQ) or a skew quadrupole (SKQ). They may be given a nonlinear kick in an octupole (OCT). The longitudinal space-charge force is represented as a nonlinear kick (SCK) at the nodes. The size of the kick depends on the charge distribution, which is updated at a node (SCD). Because the longitudinal charge distribution changes slowly, it is not necessary to update it more than once per turn. At each node, one has the option of printing out various kinematic parameters such as synchronous momentum, rf-voltage and phase, synchrotron tune, relative changes in bucket area, Laslett tune shift, etc. One can also make plots of cross sections of phase space; for example, x-\sigma, y-\sigma, x-y, l-\ell, etc. Because there is a special concern about the phase-space behavior near phase transition, special provision has been made for extra plots and printouts near \gamma_\pi. The width of the region of interest near transition is controlled by an input parameter DEL. Finally, printing and plotting occur at the end of the run when \text{TIME} \geq \text{TMAX}.

\textbf{INPUT.} The input is organized into 13 sections dealing with (1) global parameters and universal constants, (2) injection, (3) longitudinal space charge, (4) machine apertures for subroutine LOST, (5) rf acceleration, (6) time-dependent quadrupoles, (7) skew-quadrupoles, (8) octupoles, (9) transport matrices, (10) node parameters (IPRT, IPLT, ..., IOCT, NADV) that define the sequence of operations at nodes and the choice of transport sections between nodes, (11) synchronous momentum, (12) miscellaneous initializations, and (13) (for the future) restoration of arrays and variables in the restart mode. When running on a small time-share computer, it is impractical to simulate during one continuous run a complete acceleration cycle for 1000 pseudoparticles with space-charge effects. Soon the code will have a provision to store arrays and other parameters at \text{TIME} = \text{TMAX} for restart at a later time.

The need to check that transport matrices are equivalent to symplectic matrices has been mentioned above. First- and second-order matrices derived from TRANSPORT or DIMAT are not symplectic because transport variables are not canonical, that is, the variables \sigma, \omega, and \delta are not the canonically conjugate momenta corresponding to x, y, and \ell. For first-order transport, the symplectic conditions are equivalent to requiring that certain subdeterminants, for example, R_{11}R_{22} - R_{12}R_{21}, be unity. There are five such conditions in first order; second-order conditions are more complicated. The process of symplectification is described in a forthcoming Los Alamos internal report.\textsuperscript{9}

\textbf{INJECTION.} This subroutine has three sections. The first section generates a random point in a 6-D, tilted, phase-space ellipsoid to simulate a particle in the injected bunched beam. The emittance of the bunches from LAMPF are quite small compared to the desired emittance for LAMPF II. To avoid the transverse space-charge-force limit, steering magnets will be used to transversely offset injected bunches
from the designed closed orbit. This is simulated in INJ by adding a sinusoidally varying offset to the particle's \((x, \phi, y, \omega)\)-coordinates, where \(x\) and \(y\) can have a different sweep frequency. The longitudinal phase-space enlargement is handled in two parts. The variable \(\eta\) can be given a constant offset in INJ; then, the synchronous phase \(\phi_{\text{RF}}\) can be varied sinusoidally in subroutine ACC during the injection period. The third section of INJ corrects \(\delta\) for the fact that the synchronous momentum is changing from turn to turn, even during injection.

**ADVANCE.** The main purpose of this subroutine is to multiply the first six components of the ray vectors describing particles by the first-order (6 by 6) and second-order (6 by 36) transport matrices. This is the most time-consuming part of the program, and special effort has gone into making the multiplication efficient. There are only 13 nonzero, nontrivial elements in the first-order matrix and 39 in the second-order matrix. They are put into nonarray variables to speed look-ups. Instead of doing matrix multiplications with DO-loops, explicit formulas are written out that use only the nonzero elements of the transport matrices. When desired, one can skip the second-order transport matrices for more speed. ADV calls PFUN, which updates the synchronous momentum \(p_{\text{syn}}\) and hence changes \(\delta = (p - p_{\text{syn}})/p_{\text{syn}}\) for each particle. The variables \(\omega\) and \(\phi\) also are renormalized by the change in synchronous momentum.

**PFUN.** The momentum rises as a function of time according to the formula

\[
PFUN = (P_{\text{MAX}} + P_{\text{MIN}})/2 - (P_{\text{MAX}} - P_{\text{MIN}}) \times \cos \left( \frac{\text{TWOPI} \times \text{FREQ} \times \text{TIME}}{2} \right),
\]

where FREQ is read in and corresponds to the inverse of the time for a complete acceleration cycle. Of course the purpose of ARCHSIM is to make changes in PFUN and ACC to see what effect they have on beam dynamics.

**ACCCELERATE.** The current version of this routine attempts to keep the bucket area constant until the synchronous phase rises to \(60^\circ\). The change in synchronous energy required to keep up with the synchronous momentum change is

\[
DW = \sqrt{(P^2 + M^2)} - \sqrt{(P_{\text{LAST}}^2 + M^2)} = V \sin (\phi),
\]

where \(P\) is the synchronous momentum at the present node and \(P_{\text{LAST}}\) is the momentum at the previous node where ACC was called; \(V\) is the maximum voltage and \(\phi\) the phase of the rf power. From Bovet et al., the relative bucket area is

\[
A/A_0 = \alpha(\phi) \left( \frac{B/B_0}{W/W_0} \right) \sqrt{\frac{W}{V_0}} \left( \frac{W}{W_0} \right)^{\frac{n}{\eta}},
\]

where \(\alpha(\phi)\) is a tabular function, \(W\) is the proton's energy, \(B\) is the usual relativistic proton velocity and

\[
\eta = 1 - \frac{1}{y_T^2} - \frac{1}{y_T^4}.
\]
The subscript o means these quantities are evaluated at the beginning of the acceleration cycle. Setting $A/A_o = 1$, gives an expression for $V$ as a function of $\phi$ that can be substituted into Eq. (1). This transcendental equation is solved numerically for $\phi$. As $\gamma$ approaches $\gamma_T$, $\phi$ will increase toward 90°. The rise is cut off at $\sim 60°$ and $\phi$ is held constant until transition; after transition, $\phi$ is always changed to $180°-\phi$. $V$ is allowed to rise to the maximum required ($DW/\sin\phi$) and is not allowed to decrease. Toward the end of the cycle when $DW$ gets small, this has the effect of bunching the beam in longitudinal space to produce the short pulses that will be required for some of the physics experiments.

**SCK.** The effect of longitudinal space-charge forces is included in the kick approximation. The energy (and hence $\lambda$) of each particle is changed at the nodes according to the formula

$$\Delta \lambda = \frac{r_p mc^2}{\gamma^2} \left( \frac{\hbar}{R} \right)^2 \left[ 1 + 2 \pi \left( \frac{b/a}{a^2} \right) \right] \frac{d\lambda(\phi)}{d\phi} L,$$

where $r_p$ is the classical proton radius, $mc^2$ the rest mass energy, $mc^2\gamma$ the proton energy, $\hbar$ the harmonic number, $2\pi R$ the machine circumference, $b$ the beam-pipe radius, $a$ the beam radius, $\lambda(\phi)$ the linear density of charge along the beam, and $L$ the distance along the ring since the last kick. The variable $\phi$ is proportional to the difference in the time the particle arrives at the node and the synchronous particle arrives at the node. The charge density and its derivative are calculated in subroutine SCD. With only 1000 particles, the density would have troublesome statistical fluctuations that would make $d\lambda/d\phi$ unrealistic. We get around this by making a Fourier transform of $\lambda(\phi)$, multiplying by a Gaussian smoothing function, taking the derivative of the function by multiplying the Fourier transform by the Fourier-transform variable, and then taking the inverse Fourier transform. The amount of Gaussian smoothing is controlled by an empirically adjusted parameter.

**PRPTPLT.** The phase-space plots are done on the line printer using subroutines adapted from a high-energy physics program. This makes the program more transportable but produces somewhat low-quality output as shown in Fig. 2. Our plans are to augment this plot routine with one that can produce plots for graphics terminals.

**Run Time Experience**

By varying the number of particles and choice of options for second-order transport and space charge, one can vary the run time from minutes to many hours. To examine characteristics of a certain rf-voltage and phase program, one can run using only one particle and no phase-space plots, except near transition and at the end. An acceleration cycle of 6700 turns with 5 nodes runs in about 2.5 min. One can get a better feel for the phase-space behavior by running with 100 particles and no space-charge effects. This takes about 49 min. A full 1000 particles with no space
charge takes 7.4 hours. Space-charge calculations add considerably to the run time. At present, one can run 1000 particles through 2300 turns with space charge and full second-order transport in about 3.4 hours.

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References