**SATDSK: A Numerical Simulation of the
Magnetic Field Due to Saturated Iron
in Cyclotron Poletips**

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Gregory S. McNeilly

Abstract

SATDSK is a computer program, written in FORTRAN, which calculates the median plane magnetic field due to fully saturated iron poletips. Optionally, SATDSK calculates the magnetic field due to disks of magnetic "charge", which can simulate the effect of holes in the iron poletip, or circular trim rods embedded in the poletip. SATDSK is intended for poletip geometries that are both symmetric about the median plane, and have azimuthal sector symmetry. Thus, the program is primarily designed to simulate the magnetic field due to iron poletips in superconducting cyclotrons.

I. Introduction

Numerical simulation, in three dimensions, of the magnetic field due to iron pole tips and current carrying conductors is, in general, quite complex. GFUN3D¹ is an example of a state-of-the-art code of this genre. Design studies requiring the use of codes of this type are seriously impeded by the triple threat of large core size, long running time, and insufficient accuracy. Often, in fact, these calculational deficiencies require the construction of scale-model magnets, and subsequent physical measurements to provide the accelerator physicist with the magnetic field data necessary to analyze the beam dynamics of a proposed cyclotron design.

Fortunately, in the case of high-field, superconducting cyclotron design studies, the physical situation permits a considerable simplification of the calculation required to adequately simulate the magnetic field. In sufficiently high magnetic fields, the iron pole tip "saturates", i.e., the magnetic dipoles within the iron are completely aligned by the imposed field throughout the iron volume. This results in uniform volume magnetization, or equivalently, zero volume magnetic pole density, leaving only the surface density of magnetic pole strength, or surface magnet : "charge" producing the field due to the pole tip. This simplification (see Ref. 2 and references therein) reduces a volume integral to a surface integral, and the above triple threat is diminished by about an order of magnitude. SATDSK will permit a convenient and more rapid method to study a variety of pole tip geometries than does the general three-dimensional method. The underlying yoke structure with non-uniform magnetization should still be simulated with,

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e.g., TRIM³ (in the azimuthally symmetric case) or, e.g., GFUN3D¹ (in the azimuthally varying case).

II. Methods

A. Poletip Calculation

The general expression for the magnetostatic field (in MKS units) is⁴

$$\begin{aligned} \vec{B}(\vec{r}) = & \frac{\mu_0}{4\pi} \int_{V_0} \frac{\vec{r}-\vec{r}'}{|\vec{r}-\vec{r}'|^3} dv' \\ & + \frac{\mu_0}{4\pi} \int_{S_0} \frac{\vec{r}-\vec{r}'}{|\vec{r}-\vec{r}'|^3} da' + \mu_0 \vec{M}(\vec{r}), \end{aligned} \quad (1)$$

where μ_0 is the vacuum magnetic permeability,

$$\rho_M(\vec{r}') = -\text{div } \vec{M}(\vec{r}'),$$

$$\sigma_M(\vec{r}') = \vec{M}(\vec{r}') \cdot \vec{n}, \quad (\vec{n} \text{ unit normal to the surface}),$$

$\vec{M}(\vec{r}')$ is the magnetization or magnetic dipole moment per unit volume, and the integrals are a volume integration $-V_0$ and a surface integration $-S_0$ over the magnetized material. For points outside the magnetized material $\vec{M}(\vec{r}) = 0$, and with a uniform magnetization within the magnetized material $\text{div } \vec{M}(\vec{r}') = 0$, thus reducing Eq. 1 to its second term, with a constant $\sigma_M(\vec{r}') \equiv \sigma_M$. (Note: σ_M is an empirical constant, set to yield 2.14 T from a pair of infinite flat poletips.)

If the surface S_0 of the poletip consists of N azimuthally symmetric sectors and is composed of a set of horizontal surfaces z_i (e.g. $2N$ hills and valleys), then the remaining term of Eq. 1 in cylindrical coordinates is

$$\vec{B}^i(\vec{r}) = \frac{\mu_0}{4\pi} \sigma_M \int_{R_0^i}^{R_f^i} \int_{\theta_0(r')}^{\theta_f(r')} \frac{|\vec{r} - \vec{r}'|}{\Delta r_i^3} r' dr' d\theta', \quad (2)$$

for the i th surface at z_i with radial and azimuthal extents as indicated, and

$$\Delta r_i^3 = [r^2 + r'^2 - 2rr'(\cos\theta \cos\theta' + \sin\theta \sin\theta') + (z-z_i)^2]^{3/2}. \quad (3)$$

Expressed in $\hat{x}$, $\hat{y}$, and $\hat{z}$ components, Eq. 2 becomes

$$\begin{aligned} \vec{B}^i(\vec{r}) = \frac{\mu_0}{4\pi} \sigma_M \times \\ \left\{ \hat{x} \left[r \cos\theta \int r' dr' \int \frac{d\theta'}{\Delta r_i^3} - \int r'^2 dr' \int \frac{\cos\theta' d\theta'}{\Delta r_i^3} \right] \right. \\ + \hat{y} \left[r \sin\theta \int r' dr' \int \frac{d\theta'}{\Delta r_i^3} - \int r'^2 dr' \int \frac{\sin\theta' d\theta'}{\Delta r_i^3} \right] \\ \left. + \hat{z} \left[(z-z_i) \int r' dr' \int \frac{d\theta'}{\Delta r_i^3} \right] \right\} \quad (4) \end{aligned}$$

with the limits of integration as in Eq. 2.

Equation 4, when summed over the set of $2N$ surfaces, yields the field

due to one poletip. The other poletip is symmetric with respect to the median plane ($z = 0$), and so its contribution to the field is obtained from Eq. 4 with the transformations $z_i^j \rightarrow -z_i^j$ and $\sigma_M \rightarrow -\sigma_M$. Thus to obtain the magnetic field at a particular point, $2N$ times 2 two-dimensional integrals must be determined. (If the pole gaps vary with radius, the above integrals may, of course, be broken into radial sub-integrals with constant z_i^j and then summed.)

SATDSK is specialized to the case of median plane fields only. When $z = 0$, the $\hat{z}$ magnetic field due to both poletips is merely two times Eq. 4, and the $\hat{x}$ and $\hat{y}$ components are zero, yielding

$$B_z(r, \theta, z=0) \equiv B(r, \theta) = \sum_{i=1}^{2N} \sum_{j=1}^{N_R} B_z^{ij}(r, \theta, z=0)$$

$$= -\frac{\mu_0}{2\pi} \sigma_M \sum_{i=1}^{2N} \sum_{j=1}^{N_R} z_{ij}^j \int_{R_0^j}^{R_f^j} r' dr \int_{\theta_0}^{\theta_f} \frac{d\theta'}{\Delta r_{ij}^3}, \quad (5)$$

where the possibility of N_R radial steps in the pole gaps is indicated explicitly by the j index, and the r' dependence of the azimuthal limits is suppressed for conciseness.

Equation 5 can be calculated by two-fold numerical quadrature, and SATDSK provides this option. However, with further reduction, Eq. 5 can be more efficiently determined. With the substitution $\theta'' = \theta' - \theta$, Eq. 5 becomes

$$B(r, \theta) = -\frac{\mu_0 \sigma_M}{2\pi} \sum_{i=1}^{2N} \sum_{j=1}^{N_R} z'_{ij} \int_{R_0^j}^{R_f^j} r' dr' \int_{\theta_0 - \theta}^{\theta_f - \theta} \frac{d\theta''}{(a_{ij} - b \cos \theta'')^{3/2}}, \quad (6)$$

where $a_{ij} = r^2 + r'^2 + z_{ij}^2$, and $b = 2rr'$.

The azimuthal integral in Eq. 6 can be evaluated to yield⁵

$$B(r, \theta) = -\frac{\mu_0 \sigma_M}{\pi} \sum_{i=1}^{2N} \sum_{j=1}^{N_R} z'_{ij} \int_{R_0^j}^{R_f^j} r' \frac{E(\delta, \eta)}{(a_{ij} - b) \sqrt{a_{ij} + b}} \bigg|_{\theta_0 - \theta}^{\theta_f - \theta} dr', \quad (7)$$

with $\delta = \arcsin \left[\frac{(a_{ij} + b)(1 - \cos \theta'')}{2(a_{ij} - b \cos \theta'')} \right]^{1/2}$, and

$$\eta = \sqrt{\frac{2b}{a_{ij} + b}}, \text{ where } E(\delta, \eta) \text{ is}$$

the elliptic integral of the second kind. Furthermore, $a_{ij} > b > 0$ and $\pi \geq \theta'' \geq 0$ must be true. The numerical evaluation of Eq. 7 is about six times faster than the evaluation of Eq. 5, with equivalent accuracy. Both methods are available in SATDSK in order to provide independent methods for checking test cases with new geometries, but the faster method should be used for production runs.

B. Disk Calculation

For the special case of a disk pair centered at $r' = 0$, $z' = \pm z_j$, and radius R_j , Eq. 7 reduces to

$$B_j(r) = -\frac{2\nu_0\sigma_M}{\pi} z_j \int_0^{R_j} r' \frac{E(\eta)}{(a-b)\sqrt{a+b}} dr' , \quad (8)$$

with $a = r^2 + r'^2 + z_j^2$, $b = 2rr'$,

$\eta = \sqrt{\frac{2b}{a+b}}$, and $E(\eta)$ being the complete elliptic integral of the

second kind. Eq. 8 is generalized to the case of N_D disks placed with sector symmetry N around the poletip to

$$B_T^D(r, \theta) = \sum_{j=1}^{N_D} \sum_{i=1}^N B_j(d_{ij}), \quad (9)$$

with $d_{ij} = \left\{ r^2 + R_j^2 - 2rR_j \cos \left[\theta - \theta_j - \frac{2(i-1)\pi}{N} \right] \right\}^{1/2}$.

By superimposing the fields due to multiple disks of magnetic "charge", with appropriate sign, upon the poletip field, one may simulate the presence of circular holes in the poletip, or circular rods extending from the surface of the poletip. This is illustrated in the example in the following section.

III. Usage

The sample data set in this section is intended only to illustrate the use of SATDSK, and does not represent any particular cyclotron design. Figure 1 is a plan view of a poletip, and Figure 2 is an elevation view of the same poletip, sectioned along the line indicated in Fig. 1. Table 1 is the data set which describes this geometry for SATDSK, as well as defining the field points to be calculated, the numerical method to be used, and the desired accuracy. Table 1 also indicates the program variable names for the input quantities, and their input format. Further documentation is internal to SATDSK, a listing of which appears in the appendix.⁶ A card-by-card explanation of the sample data set is given below. Note that input lengths are in meters and angles in degrees.

CARD 1 (RI, DR, NR, TI, DT, NT, NMAX) defines the initial value, step value, and number of steps for the radial and azimuthal field points to be calculated, and the number of points to be stored in the OUT array before output. The usual choice is for NMAX = NT, so the calculated points are output in groups for each radius.

CARD 2 (N, NRP, IPUNCH, METHOD, DRP, RIT, EP) defines the sector number, the number of points in the poletip arrays, punch control, numerical method control, step size of poletip arrays, initial radius of poletip, and the quadrature accuracy parameter. When studying a new poletip geometry, one should investigate variation of calculated results with variation of poletip step size, accuracy parameter, and numerical method. After acceptable results are obtained, production runs should use METHOD = 1.

CARDS 3-13 (TIP array) define NRP values for the initial hill angle versus radius.

CARDS 14-24 (TH array) define NRP values for the hill width versus radius.

CARDS 25-35 (TV array) define NRP values for the valley width versus radius.

CARD 26 (NPSTEP) defines the number of poletip gap steps versus radius.

CARDS 37-38 (RIP, RP, GH, GV arrays) define NPSTEP initial and final radii, and NPSTEP hill and valley poletip gaps.

CARD 39 (ZDISK(1), NDISK, NRDISK, DRDISK) defines the yoke height, the number of magnetic charge disks, the number of points in the disk arrays, and the step size of the disk arrays. The array BDISK contains the results of the magnetic charge disks calculation, and will be interpolated and summed, with sector symmetry, into the poletip field. The stepsize DRDISK should be adjusted to provide acceptable results.

CARDS 40-45 (SIGN, RIDISK, TIDISK, RDISK, ZDISK arrays) define the charge sign and geometry of the various disks. RIDISK and TIDISK define the various centers of the disks in polar coordinates, and RDISK and ZDISK define the various radii and disk pair separations. The six cards in this example define a 10 cm diameter rod pair located at $R_r = 0.254$ m, $\theta_r = 17.049^\circ$ extending 1 cm toward the median plane from the poletip, and a 20 cm diameter hole pair located at $R_h = 0.508$ m, $\theta_h = 26.598^\circ$. These two objects are repeated with sector symmetry N around the poletip.

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Fig. 1. Plan View of Poletip Geometry Defined in Table 1.

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Fig. 2. Elevation View of Poletip Defined in Table 1.

COPY OF INPUT DATA

CARD

PROGRAM VARIABLE NAMES

INPUT FORMAT

COPY OF INPUT DATA									CARD	PROGRAM VARIABLE NAMES	INPUT FORMAT
0.0000	0.0000	15	22.50	45.00	9	9			1	RI, DR, NR, TI, DT, NT, NMAX	2(2F10.4,15),15
3	81	1	0.01270	0.	0.0001000				2	N, NRP, IPUNCH, METHOD, DRP, RIT, EP	415,3F10.5
-22.5000	-22.0226	-21.5451	-21.0677	-20.5902	-20.1128	-19.6353	-19.1579		3	TIP ARRAY	8F10.4
-18.6504	-18.2330	-17.7255	-17.2481	-16.7706	-16.2932	-15.8157	-15.3383		4	.	.
-14.8538	-14.3334	-13.9059	-13.4285	-12.9510	-12.4736	-11.9961	-11.5187		5	.	.
-11.0412	-10.5638	-10.0863	-9.6089	-9.1314	-8.6540	-8.1765	-7.6991		6	.	.
-7.2216	-6.7442	-6.2667	-5.7893	-5.3118	-4.8344	-4.3569	-3.8795		7	.	.
-3.4020	-2.9246	-2.4471	-1.9697	-1.4922	-1.0148	-0.5373	-0.0599		8	.	.
0.4176	0.8950	1.3725	1.8499	2.3274	2.8048	3.2823	3.7597		9	.	.
4.2372	4.7146	5.1921	5.6695	6.1470	6.6244	7.1019	7.5793		10	.	.
8.0569	8.5342	9.0117	9.4891	9.9666	10.4440	10.9215	11.3989		11	.	.
11.8764	12.3538	12.8313	13.3087	13.7862	14.2636	14.7411	15.2185		12	.	.
15.6963									13	.	.
60.	60.	60.	60.	60.	60.	60.	60.		14	TH ARRAY	8F10.4
60.	60.	60.	60.	60.	60.	60.	60.		15	.	.
60.	60.	60.	60.	60.	60.	60.	60.		16	.	.
60.	60.	60.	60.	60.	60.	60.	60.		17	.	.
60.	60.	60.	60.	60.	60.	60.	60.		18	.	.
60.	60.	60.	60.	60.	60.	60.	60.		19	.	.
60.	60.	60.	60.	60.	60.	60.	60.		20	.	.
60.	60.	60.	60.	60.	60.	60.	60.		21	.	.
60.	60.	60.	60.	60.	60.	60.	60.		22	.	.
60.	60.	60.	60.	60.	60.	60.	60.		23	.	.
60.	60.	60.	60.	60.	60.	60.	60.		24	.	.
60.	60.	60.	60.	60.	60.	60.	60.		25	TV ARRAY	8F10.4
60.	60.	60.	60.	60.	60.	60.	60.		26	.	.
60.	60.	60.	60.	60.	60.	60.	60.		27	.	.
60.	60.	60.	60.	60.	60.	60.	60.		28	.	.
60.	60.	60.	60.	60.	60.	60.	60.		29	.	.
60.	60.	60.	60.	60.	60.	60.	60.		30	.	.
60.	60.	60.	60.	60.	60.	60.	60.		31	.	.
60.	60.	60.	60.	60.	60.	60.	60.		32	.	.
60.	60.	60.	60.	60.	60.	60.	60.		33	.	.
60.	60.	60.	60.	60.	60.	60.	60.		34	.	.
60.	60.	60.	60.	60.	60.	60.	60.		35	.	.
60.	60.	60.	60.	60.	60.	60.	60.		36	NPSTEP	15
60.	60.	60.	60.	60.	60.	60.	60.		37	RIP, RP, GH, GV ARRAYS	4F10.5
60.	60.	60.	60.	60.	60.	60.	60.		38	.	.
60.	60.	60.	60.	60.	60.	60.	60.		39	ZDISK(1), NDISK, NRISK, DRDISK	F10.4,215,F10.4
60.	60.	60.	60.	60.	60.	60.	60.		40	SIGN, RIDISK, TIDISK, RDISK, ZDISK	5F10.4
60.	60.	60.	60.	60.	60.	60.	60.		41	ARRAYS	.
60.	60.	60.	60.	60.	60.	60.	60.		42	.	.
60.	60.	60.	60.	60.	60.	60.	60.		43	.	.
60.	60.	60.	60.	60.	60.	60.	60.		44	.	.
60.	60.	60.	60.	60.	60.	60.	60.		45	.	.

Table 1

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