COMPARISON OF SOURCE DIVERSITY AND CHANNEL DIVERSITY METHODS ON
SYMMETRIC AND FADING CHANNELS

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Channel diversity techniques are effective ways to combat channel fading and noise in communication systems. In this thesis, I compare the performance of source and channel diversity techniques on fading and symmetric continuous channels. My experiments suggest that when SNR is low, channel diversity performs better, and when SNR is high, source diversity shows better performance than channel diversity.
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CHAPTER 1

INTRODUCTION

Wireless communications is one of the most active areas of technology development in recent years. This development is being driven primarily by the medium for supporting the transmission of voice telephony, video, images, and text through wireless channel.

The performance of wireless communication system is largely influenced by the channel impairments, characterized by fading, path delay, and path loss.

In wireless communications, fading is deviation of the attenuation that a carrier-modulated telecommunication signal experiences over certain propagation media. Fading can be categorized into fast fading and slow fading. Slow fading arises when the channel response changes at a rate much slower than the transmitted baseband signal. Fast fading occurs when the channel response changes at a rate much faster than the transmitted baseband signal [11]. Path loss and path delay happens because of the transmission distance and the noise on wireless channel.

When the channel varies on a time scale longer than the delay constraints of the desired application, such channel fluctuations cause outages [1]. Diversity techniques are typically made to counter the effect of fading. In most scattering environments, antenna diversity is a practical, effective, and widely applied technique for reducing the effect of multipath fading. The classical approach is to use multiple antennas at the receiver and
perform combining or selection and switching in order to improve the quality of the received signal [2].

Compared with the low capacity and reliability of single input and single output (SISO) system, multiple input and multiple output (MIMO) system provides higher capacity, better transmission quality, and larger coverage without increasing the total transmission energy.

In addition to channel diversity techniques, I can also make use of source diversity techniques. For example, in multiple description coding (MDC), source data is separated into two correlated streams. The received streams are combined to reconstruct source with high-quality. When only one stream is received, the source can be reconstructed at a low quality. Typically, source diversity techniques are only used on erasure (on-off) channels. However, they can also be used on continuous channels.

In this thesis, I compare the performance of source and channel diversity techniques on continuous channels. Especially, I investigate the performance of diversity techniques on AWGN and symmetric channel. Channel diversity is implemented with space time block coding (STBC), and source diversity is implemented with MDC.

In order to make a fair comparison between source and channel diversity techniques, I use two models. In the first model, channel diversity is implemented on 2-by-2 MIMO system (Fig 1.1). This model doesn't use any form of source coding. In the second model, source diversity is implemented to generate two streams of data and transmitted over two parallel channels (Fig 1.2). In this model, channel coding is not used in any forms.
The organization of this thesis is as follows. Chapter 2 discusses survey of related research. Chapter 3 presents the fundamentals of source and channel diversity methods, the proposed approach, and the rate-distortion formulas. Chapter 4 outlines the proposed methods for comparison. Summary, conclusion, and directions for future work are discussed in Chapter 5.
CHAPTER 2
LITERATURE SURVEY

I consider the case of transmitting a source through independent channels with random states (e.g. slow fading channels). Trying to minimize the average distortion between transmitters and receivers, I focus on two commonly used coding algorithms, multiple description coding (MDC) and space time block coding (STBC). MDC exploits diversity at the application layer through multiple description coding, and STBC exploits diversity at the physical layer through parallel channel coding.

2.1 Multiple Description Coding

Multiple description coding (MDC) has been proposed for use in packet audio and video transmission systems as a means of combating both packet loss and link failure, in a variety of application scenarios [7]. MDC is an effective framework for robust transmission over channels with transient shutdown characteristics.

The basic idea in MD coding is to generate multiple independent descriptions of the source such that each description independently describes the source with certain fidelity, and when more than one description is available, they can be synergistically combined to enhance the quality [8]. The generalized (n-channel) MD system can decode the delivered signal with different quality levels depending on how many descriptions are correctly received as opposed to a traditional multi-resolution (MR) system for which the quality of decoded signal depends only on the received signal.
2.1.1 Rate-Distortion

Given the average channel rate across both channels $R$, an MDC coder attempts to minimize two kinds of distortion: average distortion of the two-channel reconstruction $D_0(R)$ and average distortion of the one-channel reconstruction given equal-probable loss of either channel $D_1(R)$.

If we consider a standard single description coder (SDC) that is designed for minimizing the distortion $D(\bar{R})$ with the rate $\bar{R}$, where $D(R)$ is the operational rate-distortion, in order to implement the multiple description coder, there are two approaches. The first one is splitting the SDC’s output bitstream into two equal-sized bitstreams. Then, when both descriptions are received at the decoder, a high quality signal is reconstructed with the distortion $D_0 = D(\bar{R})$. However, if only one description is received at the decoder, even though it is the one that contains the most important information of the source, the final distortion $D_1$ is still high and not acceptable. The second approach also splits the SDC’s output bitstream into two bitstreams, but with correlation, instead of equal-sized. In this case, if both descriptions are received at the decoder, the distortion is also $D_0 = D(\bar{R})$. However, since correlation is added to the two bitstreams, which are transmitted on two independent channels, if only one bitstream is received at the decoder, no matter which one, there will be an acceptable distortion. The tradeoff by using the second method is some additional bits are used to describe the correlation between the two bitstreams. I call these extra bits redundancy $\rho$. Then the question of how to improve the efficiency of MDC converts into how to control the relationship between redundancy, rate, and distortion (RRD).
2.1.2 Multiple Description Transform Coding

Transform-based MDC has advantage in coding sources at variable bit rates. By using transform, the required redundancy can be provided at the source to handle the channel impairments. As mentioned before, the transform introduces correlation between source symbols, which helps to reduce the distortion at the decoder side, when the source symbols are not received.

Given two independent input variables A and B, and two output variables C, D. I generate a pairwise MDC transform with the matrix T.
Figure 2.1 Source Diversity using MDC

As shown in Fig 2.1, two source symbol streams after quantization \((\tilde{A}, \tilde{B})\) are sent to the MDC encoder as inputs. The encoder generates two descriptors \((\tilde{C}, \tilde{D})\), which will be sent over two wireless channels. There are three possibilities at the receiver: either first or second descriptor is received or both of them are received. The channel outputs are decoded by one of the three different kinds of decoders. Then, the decoded \((A, B)\) streams have three different forms.

The quantized versions of \(A\) and \(B\) with a quantization step-size \(Q\) is

\[
\tilde{A} = \left\lfloor \frac{A}{Q} \right\rfloor, \quad \tilde{B} = \left\lfloor \frac{B}{Q} \right\rfloor
\]  \hspace{1cm} (1).

Basic structure of transform is

\[
\begin{bmatrix}
\tilde{C} \\
\tilde{D}
\end{bmatrix} = T \begin{bmatrix}
\tilde{A} \\
\tilde{B}
\end{bmatrix}
\]  \hspace{1cm} (2).
The correlation between $\bar{C}$ and $\bar{D}$ is controlled by $T$. The correlation between $\bar{C}$ and $\bar{D}$ defines the redundancy of the MDC coder [4]. Then, at the decoder, we can decode $\hat{A}$ and $\hat{B}$ through

$$\begin{bmatrix} \hat{A} \\ \hat{B} \end{bmatrix} = T^{-1} \begin{bmatrix} \bar{C} \\ \bar{D} \end{bmatrix} \quad \text{(3)}.$$ 

If I assume $T$ is linear, the $T^{-1}$ matrix could be

$$T^{-1} = \begin{bmatrix} r_1 \sin \theta_1 & r_2 \sin \theta_2 \\ r_1 \cos \theta_1 & r_2 \cos \theta_2 \end{bmatrix} = \begin{bmatrix} v_1 & v_2 \end{bmatrix} \quad \text{(4)}.$$ 

This transform replaces the original variables with two nonorthogonal vectors $v_1$ and $v_2$. The parameters $r_1$, $r_2$ control the length of the vectors and $\theta_1$ and $\theta_2$ control the directions of the vectors.

Figure 2.2 Parameterization of the Pairing Transform

For simplicity, if $T$ is an orthogonal transform, I can use one parameter instead of two to generate a multiple description transform.
\[ T = \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix} \quad (5) \]

or

\[ T^{-1} = \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix} \quad (6) \]

where the angle \( \theta \) controls the redundancy.

The two descriptors \( \tilde{C} \) and \( \tilde{D} \) are transmitted over two independent channels. Assuming the channels are perfect, the decoded streams \( A \) and \( B \) can be described by

\[ \hat{A}_0 = \tilde{A}Q \quad \hat{B}_0 = \tilde{B}Q \quad (7). \]

If one descriptor is received, for example \( C \), then the quantized \( C \) is \( \tilde{C} = \tilde{C}Q \). The lost descriptor \( D \) can be recovered from \( \tilde{C} \) using

\[ \tilde{D}(\tilde{C}) = \gamma_{DC} \tilde{C} \quad (8), \]

where \( \gamma_{DC} \) is a linear estimator. Then, \( \hat{A}_1 \) and \( \hat{B}_1 \) can be decoded from \( \tilde{C} \) and \( \tilde{D} \) by using inverse transform. The linear estimator \( \gamma_{DC} \) is obtained from

\[ \gamma_{DC} = \sigma_C / \sigma_{\tilde{C}} \cot \varphi \quad (9) \]

\[ \cot \varphi = \frac{r^2 - 1}{2r} \sin 2\theta \quad (10), \]

where \( \sigma_x \) is standard deviation of stream \( X \) and \( \varphi \) is the correlation angle between \( C \) and \( D \) [4].

Maximum redundancy can be found by choosing \( \theta = \frac{\pi}{4} \), which can also lead to minimum correlation angle. Then the maximum redundancy is

\[ \rho_{max} = -\frac{1}{2} \log_2 \sin \varphi_{min} \quad (11), \]
and the corresponding single-channel distortion is given by

\[ D_{\text{min}} = \frac{r^2}{r^2 + 1} \sigma_B^2 \]  

(12).

The orthogonal transform has an RRD function with multiplicative constant equal to the average of the variances of the two variables. Thus, redundancy is used in a balanced, but suboptimal, way to reduce the contributions of both \( \sigma_A^2 \) and \( \sigma_B^2 \) in \( D_1 \) [4].

2.2 Space Time Block Coding

Modern wireless systems have more features, such as large coverage, better quality, more power and bandwidth, and can be deployed in diverse environments, to meet the market requirements.

The basic problem that makes reliable wireless transmission difficult is time-varying multipath fading [9]. Because of this, it is very hard to increase the quality or reduce the error rate of a wireless system. Compared with wired communication system, wireless communication system needs to increase SNR significantly to get a better performance in error rate. Increasing the SNR is equivalent to increasing the signal transmission power or using additional bandwidth.

In most scattering environments, antenna diversity is a practical, effective and, hence, a widely applied technique for reducing the effect of multipath fading [9]. Antenna diversity, which will increase the reliability of the wireless system, can be created by using multiple antennas at both transmitter and receiver side. The diversity technique effectively reduces the sensitivity of the transmitted signal to the fading environments, and uses high level modulation schemes at the transmitter to improve the
data rate and decrease the distortion. In STBC algorithm, source signal can be transmitted in an effective way without increasing the total transmission power or expanding the transmission bandwidth.
Figure 2.3 Channel Diversity using STBC \((2 \times 1)\)

\(*\) S\(\rightarrow\)P means signal to pulse and P\(\rightarrow\)S means pulse to signal
In Figure 2.3, the top half shows the transmitter and the bottom half shows the receiver of a 2-by-1 STBC system.

For the transmitter, at given time $t$, symbol $S_1$ is transmitted through antenna $a_1$, and $S_2$ is transmitted from antenna $a_2$ simultaneously. In the next time slot $(t + T_s)$, symbol $-S_2^*$ is transmitted through antenna $a_1$, and $S_1^*$ transmitted from antenna $a_2$ simultaneously, where $*$ represents complex conjugate. Here, the channel is slow Rayleigh fading channel with independent complex multiplicative fading coefficients $[h_1 \ h_2]$. The coefficients $h_1$ and $h_2$ corresponding to the two paths $a_1 - r$ and $a_2 - r$ respectively,

$$h_1(t) = h_1(t + T_s) = \alpha_1 e^{j\theta_1} \quad (13),$$

$$h_2(t) = h_2(t + T_s) = \alpha_2 e^{j\theta_2} \quad (14).$$

where $\alpha_i$s represents the magnitude of complex fading coefficients, and $\theta_i$s represents the phase of complex fading coefficients. The symbol period is given by $T_s = \log_2 K T_b$, where $K$ and $T_b$ represent the size of the alphabet and bit interval respectively.

At the receiver, the received signal is

$$\begin{bmatrix} r_1 \\ r_2^* \end{bmatrix} = \sqrt{E_s} \begin{bmatrix} h_1 & h_2^* \\ h_2 & -h_1^* \end{bmatrix} \begin{bmatrix} S_1 \\ S_2 \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2^* \end{bmatrix} \quad (15)$$

where $r_1$ is received in the first time slot, and $r_2^*$, which is the complex conjugate of the symbol $r_2$, is received in the second time slot. The parameter $E_s$ is the symbol energy and $[n_1 \ n_2]$ are complex random variables representing channel noise. The equation above can also be represented as follows,

$$r = HS + n \quad (16),$$
where \( r = \begin{bmatrix} r_1^* \\ r_2^* \end{bmatrix}, S = \begin{bmatrix} S_1 \\ S_2 \end{bmatrix}, \) and \( n = \begin{bmatrix} n_1^* \\ n_2^* \end{bmatrix}. \)

At the linear combiner, the received symbols \( r_1 \) and \( r_2^* \) are properly combined to get \( \tilde{r} = [\tilde{r}_1 \; \tilde{r}_2]^T \) as follows,

\[
\tilde{r} = H^* r \\
= \rho S + \tilde{n} \quad (17),
\]

where \( H^* \) is the Hermitian of the \( H \) matrix, \( \tilde{n} = H^* n \), and \( \rho = (|h_1|^2 + |h_2|^2) \).

The pair of symbols \([\tilde{r}_1 \; \tilde{r}_2]\) are sent to the maximum likelihood decoders, where the following decision rules are used to estimate the source symbols,

\[
\hat{S}_1 = arg \min_{\tilde{S}_1} |\tilde{r}_1 - \rho \hat{S}_1| \\
\hat{S}_2 = arg \min_{\tilde{S}_2} |\tilde{r}_1 - \rho \hat{S}_2| \quad (18),
\]

where \( \hat{S}_1 \) and \( \hat{S}_2 \) are estimated versions of \( S_1 \) and \( S_2 \). Channel impairments are assumed to be known at the receiver.

The theoretical BER of the 2-by-1 STBC [10] is

\[
BER = \frac{1}{2} \left[ 1 - \mu - \frac{\mu}{8} (1 - \mu^2) \right] \quad (20),
\]

where \( \mu \) is given by \( \sqrt{\frac{\rho}{\rho+2}}, \) and \( \rho = E_s/N_0 \) represents the signal to noise ratio per symbol.
CHAPTER 3
PROPOSED MODEL

This chapter presents the experimental models used to implement source and channel diversity techniques. Performance methods used to evaluate the proposed models are also defined in this chapter.

3.1 System Model

3.1.1 Source Diversity Model

In the source diversity technique shown in Fig 3.1, a pair of source symbols \( s \) is encoded into \( s_1 \) and \( s_2 \) and transmitted through two parallel channels. The decoder receives two outputs \( y_1 \) and \( y_2 \), which are used to reconstruct \( s \). If only one of the \( y_i \)s is received, the resulting codeword is used to produce a low fidelity version of source \( s \). If both \( y_1 \) and \( y_2 \) are received, they are combined to form a high fidelity version of \( s \). We discuss the case when both descriptions are received. I assume that the channel state is known at the transmitter as well as at the receiver.
The communication channel is described by

\[ y = \tilde{s} + n \]  

(30)

where \( y \) stands for the channel output vector \([y_1 \ y_2]\), \( \tilde{s} \) stands for the correlated bit streams after source coding \([s_1 \ s_2]\), and \( n \) represents zero-mean white Gaussian noise with variance \( \sigma^2 \) white Gaussian noise, \( n \sim N(0, \sigma^2) \).

\[ \tilde{s} = TS \]  

(31)

where the matrix \( T \) controls the correlation between channel inputs [4]

\[ T = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \]  

(32).

The angle \( \theta \) is set to \( \frac{\pi}{4} \) in my implementation. At the receiver side, I used the inverse of the \( T \) matrix to decode the source data.
3.1.2 Channel Diversity Model

In the channel diversity technique illustrated in Fig 3.2, a pair of source symbols $\mathbf{s}$ is encoded and transmitted through a 2-by-2 multiple input and multiple output (MIMO) channel. The channel decoder attempts to decode $\mathbf{s}$ from its outputs $(y_1, y_2)$ [1].

$$
\mathbf{y} = h\mathbf{X} + \mathbf{n}
$$

where $\mathbf{y}$ stands for the channel output vector $[y_1 \ y_2]$, $h=[h_1 \ h_2]$ represents the channel fading coefficients, and $\mathbf{X}$ stands for channel input matrix,

$$
\mathbf{X} = \begin{bmatrix}
  x_1 & -x_2^* \\
  x_2 & x_1^*
\end{bmatrix}
$$

I assume the channel states are known by the receiver. Maximum likelihood estimation (ML) is used to decode the received symbols.

The structure of channel coding and decoding affects the form of the outage probability expression [3]. The probability of decoding failure is $\Pr [I < R]$, where $I$
stands for mutual information $I(s : \tilde{s})$ and $R$ stands for data rate [1]. The distortion is measured by mean square error (MSE) between $s$ and $\tilde{s}$.

3.2 Diversity and Rate

3.2.1 Source Coding

In MDC, diversity is achieved through correlated descriptions of the same source. As shown in Fig 3.3,

As shown in Fig 3.3,

\[
\{1, 2 \ldots M\} \quad \{1, 2 \ldots M\} \quad \tilde{s}_1^N \times \tilde{s}_2^N
\]

Figure 3.3 Source Encoder

descriptions are transmitted at the same time on two parallel channels. Assuming that there are $M$ pairs of source symbols encoded into $N$ pairs of blocks, each containing $K$ bits of information, the processing gain is $\beta = N/K$, the channel coding rate $R$ is defined as [1]

\[ R = \frac{\log_2 M}{N} \quad (33) \]

in bits per channel use.

I use two equal rate descriptions in my implementation [2]. The rate distortion relationship, assuming lossless communication channel, is
\[ R(D_0, D_1) = \frac{1}{2\beta} \log_2 \frac{1}{D_0} + \frac{1}{2\beta} \log_2 \frac{(1-D_0)^2}{(1-D_0)^2-(1-2D_1+D_0)^2} \]  

(34)

where \( D_0 \) is the distortion when both descriptions are received and \( D_1 \) is the distortion when only a single description is received. I approximate the minimum average distortion for a multiple description system with independent channel coding [1]. I set the probability of channel being off to \( \epsilon \).

\[
D \approx \min \left\{ \min_{0<\mu<1} \epsilon^2 + 2\epsilon(1-\epsilon)(1+SNR/2)^{-(1-\mu)2\beta}
+ \frac{1}{2}(1-\epsilon)^2(1+SNR/2)^{-(1+\mu)2\beta}[1-(1-\epsilon)^2]
+ (1-\epsilon)^2(1+SNR/2)^{-4\beta} \right\}
\]  

(35)

When \( \mu = 1 \), source coding diversity performance reduces to that of channel coding diversity. When \( \mu = 0 \), source coding diversity performance reduces to that of no diversity with half the signal to noise ratio (SNR).

3.2.2 Channel Coding

Each source signal is transmitted in two time slots using half of the symbol transmission power, compared with one transmit antenna and one receive antenna system, as shown in Fig 3.4.
The function that represents the relation between rate and distortion is described by the following equation.

\[
D = \begin{cases} 
(1 - \varepsilon^2) \exp(-2\beta R) + \varepsilon^2 & \text{if } 0 < R \leq \ln(1 + \text{SNR}/2) \\
(1 - \varepsilon)^2 \exp(-2\beta R) + [1 - (1 - \varepsilon)^2] & \text{if } \ln(1 + \text{SNR}/2) < R \leq 2\ln(1 + \text{SNR}/2) \\
1 & \text{otherwise}
\end{cases}
\]

(36)

In equation above, if two channel codewords are independent, the upper bound for \( R \) is \( 2\ln(1 + \text{SNR}/2) \). However, if two channel codewords are identical, the upper bound for \( R \) is \( \ln(1 + \text{SNR}/2) \) [1].

If we choose the largest rate in each case in equation above, the minimum average distortion can be achieved.

\[
D = \min \{(1 - \varepsilon^2)(1 + \text{SNR}/2)^{-2\beta R} + \varepsilon^2, \\
(1 - \varepsilon)^2(1 + \text{SNR}/2)^{-4\beta R} + [1 - (1 - \varepsilon)^2]\}
\]

(37)
3.3 Modulation Scheme

The characteristics, performance and physical realization of a communication system are significantly affected by the choice of digital modulation scheme. The required data rate, acceptable level of latency, available bandwidth, anticipated link budget and target hardware cost and size are the physical characteristics of the channel that should be considered to decide the modulation scheme.

The objective of a digital communication system is to transport digital data between two or more nodes. In radio communications this is usually achieved with a modulator at the transmitting end and a demodulator at the receiving end to detect the resultant modulation on reception [11].

An alternative to imposing the modulation onto the carrier by varying the instantaneous frequency is to modulate the phase. This can be achieved simply by defining a relative phase shift from the carrier, usually equal-distant for each required state. Therefore a two level phase modulated system, such as Binary Phase Shift Keying (BPSK), has two relative phase shifts from the carrier, + or - 90°. Typically, this technique will lead to an improved bit error rate (BER) performance compared to minimum-shift keying (MSK). The resulting signal will, however, probably not be constant amplitude and not be very spectrally efficient due to the rapid phase discontinuities. Some additional filtering will be required to limit the spectral occupancy. Phase modulation requires coherent generation [11].
In this thesis, I use a higher order modulation scheme, quadrature phase-shift keying (QPSK), which is often preferred to BPSK when improved spectral efficiency is required.

Figure 3.5 QPSK Modulator

As shown in Figure 3.5, with four phases, QPSK can encode two bits per symbol, which is twice the rate of BPSK.

In QPSK, the source symbols can be written in constellation form

\[ s_i(t) = \sqrt{\frac{2E_s}{T}} \cos \left( 2\pi f_c t + (2i - 1) \frac{\pi}{4} \right) \quad i = 1, 2, 3, 4 \]

where \( E_s \) represents the symbol energy and \( f_c \) is the frequency of the carrier-wave. This yields the four phases \( \pi/4, 3\pi/4, 5\pi/4 \) and \( 7\pi/4 \) as needed.
The bit error rate of QPSK modulation scheme is same as BPSK, which is

\[ P_b = Q\left( \sqrt{\frac{2E_b}{N_0}} \right) \]  

(39)

where \( E_b \) represents the bit energy and \( N_0 \) represents the noise power.
CHAPTER 4
SIMULATIONS

My simulations use quadrature phase-shift keying (QPSK) modulation scheme. With four phases, QPSK can encode two bits per symbol, which can double the data rate compared to a Binary Phase Shift Keying (BPSK) system while maintaining the bandwidth of the signal or maintaining the data-rate of BPSK while using half the bandwidth.

4.1 Symmetric Channel

A binary symmetric channel is a common communications channel model used in coding with the alphabet consisting of 4 symbols A, B, C, and D, as shown in Fig 4.1. A source symbol is received without error with a probability of \((1 - Pe)\), and it is received as one of the other three symbols with a probability of \(Pe/3\).
The results obtained from my experiments on symmetric channel are shown in Fig 4.2. The numerical values are shown in Table 4.1. The results suggest that space time block coding (STBC) shows better performance at high $Pe$, and multiple description coding (MDC) shows better performance at low $Pe$. STBC and MDC show almost the same performance, when $Pe$ drops really low.
Figure 4.2 Comparison of Diversity Techniques on a Symmetric Channel using QPSK
<table>
<thead>
<tr>
<th>Pe</th>
<th>MSE in MDC</th>
<th>MSE in STBC</th>
</tr>
</thead>
<tbody>
<tr>
<td>$10^{-6}$</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$10^{-5}$</td>
<td>0</td>
<td>$6.0 \times 10^{-6}$</td>
</tr>
<tr>
<td>$10^{-4}$</td>
<td>$4.3 \times 10^{-5}$</td>
<td>$3.4 \times 10^{-5}$</td>
</tr>
<tr>
<td>$10^{-3}$</td>
<td>$5.1 \times 10^{-4}$</td>
<td>$2.8 \times 10^{-4}$</td>
</tr>
</tbody>
</table>

Table 4.1 Performance of MDC and STBC on Symmetric Channel

4.2 Fading Channel

A fading channel is defined by $p(y_i/x_i; h_i)$ and is parameterized by fading coefficients ($h_i$s).

The results of my implement on fading channel are shown in Fig 4.3. The numerical values are shown in Table 4.2. The results suggest that STBC shows better performance at low signal to noise ratio (SNR), and MDC shows better performance at high SNR. When SNR raises really high, STBC and MDC show almost the same performance.
Figure 4.3 Comparison of Diversity Techniques on a Fading Channel using QPSK
### Table 4.2 Performance of MDC and STBC on Fading Channel

<table>
<thead>
<tr>
<th>SNR</th>
<th>MSE in MDC</th>
<th>MSE in STBC</th>
</tr>
</thead>
<tbody>
<tr>
<td>16</td>
<td>$1.535 \times 10^{-4}$</td>
<td>$1.085 \times 10^{-4}$</td>
</tr>
<tr>
<td>17</td>
<td>$4.45 \times 10^{-5}$</td>
<td>$1.55 \times 10^{-5}$</td>
</tr>
<tr>
<td>18</td>
<td>0</td>
<td>$1 \times 10^{-5}$</td>
</tr>
<tr>
<td>19</td>
<td>0</td>
<td>$6.5 \times 10^{-6}$</td>
</tr>
<tr>
<td>20</td>
<td>0</td>
<td>$6 \times 10^{-6}$</td>
</tr>
<tr>
<td>21</td>
<td>0</td>
<td>$4 \times 10^{-6}$</td>
</tr>
<tr>
<td>22</td>
<td>0</td>
<td>$2 \times 10^{-6}$</td>
</tr>
<tr>
<td>23</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

4.3 Error Metrics

On a symmetric channel, the performances of source and channel diversity are measured by plotting MSE with respect to $Pe$. On an AWGN channel, the performances of source and channel diversity are measured by plotting MSE with respect to channel SNR.
The results shown in Figure 4.2 and 4.3 suggest the following: 1) at low SNR values (high $P_e$), STBC performs better than MDC. 2) At high SNR values (low $P_e$), MDC performs better.
In this thesis, the performance of source diversity and channel diversity are compared. Source diversity is implemented using multiple description coding (MDC) and channel diversity is implemented using space time block coding (STBC). I implement both forms of diversity techniques on a symmetric channel as well as on a fading channel. My results suggest that at low signal to noise ratio (SNR) or high error probability ($Pe$), STBC shows better performance and at high SNR or low $Pe$, MDC shows better performance.
BIBLIOGRAPHY


