**A Theory for Fluidelastic Instability of
Tube-Support-Plate Inactive Modes**

CONF-910602--2

DE91 006001

by

Y. Cai, S. S. Chen

Argonne National Laboratory

Argonne, Illinois

S. Chandra

Northeast Utilities Service Company

Hartford, Connecticut

ABSTRACT

Fluidelastic instability of loosely supported tubes, vibrating in a tube support plate (TSP)-inactive mode, is suspected to be one of the main causes of tube failures in some operating steam generators and heat exchangers. A mathematical model, which incorporates all motion-dependent fluid-forces based on the unsteady flow theory, is presented here for fluidelastic instability of loosely supported tubes exposed to nonuniform crossflow. In the unstable region associated with a TSP-inactive mode, the tube motion can be described by two linear models: TSP-inactive mode when tubes do not contact with the TSP and TSP-active mode when tubes contact the TSP. A bilinear model, consisting of these linear models, is presented in this paper to simulate the characteristics of fluidelastic instability of loosely supported tubes in the stable and unstable region associated with TSP-inactive modes. The analytical results are compared with published experimental data; they agree reasonably well. The prediction

The submitted manuscript has been authored by a contractor of the U. S. Government under contract No. W-31-109-ENG-38. Accordingly, the U. S. Government retains a nonexclusive, royalty-free license to publish or reproduce the published form of this contribution, or allow others to do so, for U. S. Government purposes.

MASTER

DISTRIBUTION OF THIS DOCUMENT IS UNLIMITED

procedure presented for fluidelastic instability response of loosely supported tubes is applicable in the stable and unstable regions of TSP-inactive mode.

NOMENCLATURE

A_{jn}	Coefficients
$a_{jn}, \dot{a}_{jn}$	Generalized tube displacement and velocity in the x direction
B_{jn}	Coefficients
$b_{jn}, \dot{b}_{jn}$	Generalized tube displacement and velocity in the y direction
D	Tube diameter
E	Young's modulus
e	Wall thickness of a tube
e_1, e_2	Tube-support gaps
F_i	Impact force
f	Oscillation frequency
f_n	Natural frequency of the n-th mode
I	Moment of inertia of tube cross section
K_c	Equivalent stiffness
k_{n1}, k_{n2}	Eigenvalues for tube vibration
ℓ	Tube length
m_j	Mass per unit length of the j-th tube
N	Number of tubes in an array or row
n	Number of modes
P	Pitch
R	Radius of a tube
T_c	Contact time of tube/support

t	Time
t_s	The time when tube strikes TSP
t_d	The time when tube leaves TSP
U	Flow velocity
U_m	Mean flow velocity
U_v	Reduced flow velocity ($= U_m/fD$ or U/fD)
$u_j, \dot{u}_j$	Displacement and velocity of the j -th tube in the x direction
$u_1, \dot{u}_1$	Displacement and velocity of the tube for TSP-inactive mode
$u_2, \dot{u}_2$	Displacement and velocity of the tube for TSP-active mode
$v_j, \dot{v}_j$	Displacement and velocity of the j -th tube in the y direction
x, y, z	Cartesian coordinates
W_r	Wear work rate parameters
$\alpha_{jk}, \beta_{jk}, \sigma_{jk}, \tau_{jk}$	Added mass coefficients
$\alpha'_{jk}, \beta'_{jk}, \sigma'_{jk}, \tau'_{jk}$	Fluid damping coefficients
$\alpha''_{jk}, \beta''_{jk}, \sigma''_{jk}, \tau''_{jk}$	Fluid stiffness coefficients
ξ, ξ_1, ξ_2	Dimensionless coordinates
ζ_j, η_j	Damping ratio in vacuum
γ_j	Mass ratio ($= \rho\pi R^2/m_j$) of the j -th tube
μ, ν	Dimensionless distances
ω	Circular frequency
ω_j, Ω_j	Natural frequency in radian of the j -th tube in vacuum
$\varphi_n(z)$	Orthonormal function of the n -th mode
$\varphi(z)$	Flow velocity distribution function

Subscripts

j,k	Tube number j,k ($j,k = 1$ to N)
N	Number of tubes
n	Number of modes
1	For model 1 (TSP-inactive mode)
2	For model 2 (TSP-active mode)

1. INTRODUCTION

Fluidelastic instability associated with a TSP (tube support plate)-inactive mode for loosely held tubes has been demonstrated in laboratory tests and observed in a few heat exchangers [1-6]. It is suspected to be one of the main causes of tube failures in some operating steam generators and heat exchangers. The phenomenon occurs as a result of design inherent clearances between the tubes and their supports, such as baffle plates and anti-vibration bars. When these tubes are subjected to a cross flow, flow-induced vibration can cause tubes to impact and rub against their supports and result in tube wear. If tube vibration is excessive in duration and amplitude, wear can result in sufficient tube wall material loss to cause fatigue cracking and/or tube leaks.

In recent years, extensive experimental and analytical studies have been performed for fluidelastic instability of loosely held tubes and wear. Chen et al. [1] investigated the fluidelastic behavior of loosely held tubes in laboratory. They observed that as the flow velocity is increased to a threshold value, instability in a TSP-inactive mode may occur. Then for a range of flow velocities larger than the threshold flow velocity, the tube vibrates predominantly in a TSP-inactive mode

with the response amplitude limited by the clearance between the tube and the TSP. With a further increase of flow velocity a second threshold, or critical, flow velocity is reached at which instability in a TSP-active mode begins. In this case, large amplitude oscillations occur and, in many cases, tubes may impact with one another.

Additional studies have been published recently to determine the response of loosely supported tubes under some specific flow conditions [7-16]. References [7] through [11] described experiments on tubes vibrating in a TSP-inactive mode including dynamic contact forces between tubes and supports and fretting wear mechanism. Other studies [12-19] considered computer simulations of the fluidelastic instability of loosely held tubes using both analytical and numerical methods. For example, Fisher [13] and Rao [12,15] developed finite element computer codes to simulate tube vibration and fretting wear, and compared these results with experimental measurements. Nonlinear analytical methods for analyzing the fluidelastic instability and impacting behavior of loosely held tubes were presented by Fricker [14] and Axisa [20]. They used quasi-static or quasi-steady flow theories, which are applicable in specific parameter ranges. A study [23] on the basis of the unsteady flow model has also been developed for fluidelastic instability of tubes in nonuniform flow.

This paper presents an analytical model to predict tube response for loosely held tube arrays in crossflow. An analytical/numerical procedure is described to predict the critical flow velocity and tube response in the instability zone associated with a TSP-inactive mode. First, fluid coupling effects among tubes including fluid inertia, fluid damping, and fluid stiffness are described by the unsteady flow theory. At present, there are very limited data available for these fluid-force coefficients. In this paper, we use the coefficients presented in Ref. 23 based on the experimental data by Tanaka and his colleagues. Second, as the first

step, all TSPs are considered as elastic stops and fluid effects are not included. Finally, the coupled tube/flow system is reduced to a bilinear model which can be solved analytically.

Many calculations have been carried out to predict tube displacements including time histories and frequency spectra. RMS tube displacements as a function of flow velocity for various clearances agree reasonably well with experimental data [1]. Impact forces at supports due to tube/support interaction associated with the instability of a TSP-inactive mode are also calculated to correlate with wear work rate parameter. Comparisons of the analytical/numerical results with experimental data show that the unsteady flow theory and bilinear model presented in this paper are adequate to describe the nonlinear behavior of fluidelastic instability associated with TSP-inactive modes for loosely supported tube arrays in crossflow. The model can be used to evaluate tube vibration and wear in steam generators and shell-and-tube heat exchangers.

2. EQUATIONS OF MOTION

An unsteady flow theory for fluidelastic instability of tubes in crossflow has been described in detail by Chen and his colleagues [22,23]. For the sake of completeness, the unsteady flow model for fluidelastic instability of loosely supported tubes is briefly described here.

Consider a row of N tubes oscillating in a flow as shown in Fig. 1. The axes of the tubes are parallel to the z axis. The subscript j is used to denote variables associated with tube j . Tube displacement components in the x and y directions are u_j and v_j respectively. The orthonormal modal function of the tube vibrating in vacuum and in fluid is $\phi(z)_n$;

$$\frac{1}{\ell} \int_0^\ell \varphi_n^2(z) dz = 1 \quad (1)$$

where ℓ is the length of the tubes. Let

$$u_j(z, t) = \sum_{n=1}^{\infty} a_{jn}(t) \varphi_n(z),$$

and (2)

$$v_j(z, t) = \sum_{n=1}^{\infty} v_{jn}(t) \varphi_n(z),$$

where $a_{jn}(t)$ and $b_{jn}(t)$ are functions of time only. Assume that the flow velocity distribution is given by

$$U(z) = U_m \psi(z) \quad (3)$$

where U_m is mean flow velocity; $\psi(z)$ is flow velocity distribution function.

The equations of motion for the tubes are

$$\begin{aligned}
& \frac{d^2 a_{jn}}{dt^2} + 2\zeta_{jn}\omega_{jn} \frac{da_{jn}}{dt} + \omega_{jn}^2 a_{jn} \\
& + \frac{\rho\pi R^2}{m_j} \sum_{k=1}^N \left(\alpha_{jk} \frac{d^2 a_{kn}}{dt^2} + \sigma_{jk} \frac{d^2 b_{kn}}{dt^2} \right) \\
& - \frac{\rho U_m^2}{m_j \omega} \sum_{k=1}^N \left(\alpha_{jkn}^d \frac{da_{kn}}{dt} + \sigma_{jkn}^d \frac{db_{kn}}{dt} \right) \\
& - \frac{\rho U_m^2}{m_j} \sum_{k=1}^N \left(\alpha_{jkn}^e a_{kn} + \sigma_{jkn}^e b_{kn} \right) = 0,
\end{aligned}$$

and

(4)

$$\begin{aligned}
& \frac{d^2 b_{jn}}{dt^2} + 2\eta_{jn}\Omega_{jn} \frac{db_{jn}}{dt} + \Omega_{jn}^2 b_{jn} \\
& + \frac{\rho\pi R}{m_j} \sum_{k=1}^N \left(\tau_{jk} \frac{d^2 a_{kn}}{dt^2} + \beta_{jk} \frac{d^2 b_{kn}}{dt^2} \right) \\
& - \frac{\rho U_m^2}{m_j \omega} \sum_{k=1}^N \left(\tau_{jkn}^d \frac{da_{kn}}{dt} + \beta_{jkn}^d \frac{db_{kn}}{dt} \right) \\
& - \frac{\rho U_m^2}{m_j} \sum_{k=1}^N \left(\tau_{jkn}^e a_{kn} + \beta_{jkn}^e b_{kn} \right) = 0,
\end{aligned}$$

where U_m is mean flow velocity; ρ is fluid density; R is tube radius; ω is the radian frequency; ζ_{jn} and η_{jn} are damping ratios in vacuum; ω_{jn} and Ω_{jn} are natural frequencies in radian of j -th cylinder in vacuum; m_j is cylinder mass per unit length of cylinder j , and

$$\begin{aligned}
\alpha_{jkn}^d &= \frac{1}{\ell} \int_0^\ell \alpha'_{jk} \varphi_n^2(z) \varphi^2(z) dz, & \alpha_{jkn}^e &= \frac{1}{\ell} \int_0^\ell \alpha''_{jk} \varphi_n^2(z) \varphi^2(z) dz, \\
\sigma_{jkn}^d &= \frac{1}{\ell} \int_0^\ell \sigma'_{jk} \varphi_n^2(z) \varphi^2(z) dz, & \sigma_{jkn}^e &= \frac{1}{\ell} \int_0^\ell \sigma''_{jk} \varphi_n^2(z) \varphi^2(z) dz, \\
\tau_{jkn}^d &= \frac{1}{\ell} \int_0^\ell \tau'_{jk} \varphi_n^2(z) \varphi^2(z) dz, & \tau_{jkn}^e &= \frac{1}{\ell} \int_0^\ell \tau''_{jk} \varphi_n^2(z) \varphi^2(z) dz, \\
\beta_{jkn}^d &= \frac{1}{\ell} \int_0^\ell \beta'_{jk} \varphi_n^2(z) \varphi^2(z) dz, & \beta_{jkn}^e &= \frac{1}{\ell} \int_0^\ell \beta''_{jk} \varphi_n^2(z) \varphi^2(z) dz,
\end{aligned} \tag{5}$$

Note that α_{jk} , σ_{jk} , τ_{jk} , and β_{jk} are added-mass coefficients, α'_{jk} , σ'_{jk} , τ'_{jk} , and β'_{jk} are fluid-damping coefficients, and α''_{jk} , σ''_{jk} , τ''_{jk} and β''_{jk} are fluid-stiffness coefficients. All these force coefficients are based on the experimental data by Tanaka [24] and have been compiled and evaluated by Chen and his colleagues [23,25].

Equations (4) and (5) are applicable for tube arrays in which all tubes have the same length and same type of boundary conditions. For a group of N tubes, corresponding to each n -th mode for a single tube, there are $2N$ coupled modes. Based on Eqs. (4), different types of instability can be analyzed; these include coupled-mode flutter and single-mode flutter or velocity-controlled mechanism and displacement-controlled mechanism.

Consider the case of fluidelastic instability in which a velocity-controlled, negative damping mechanism is dominant. The stability of the tube row may be analyzed approximately by considering only one flexible tube among other rigid tubes and neglecting the coupling in the two directions. For this case, Eqs. (4) can be written as

$$\begin{aligned}
& (1 + \gamma_j \alpha_{jj}) \frac{d^2 a_{jn}}{dt^2} + \left(2\zeta_{jn} \omega_{jn} - \frac{\gamma_j U_r^2}{\pi^3} \omega \alpha_{jj}^d \right) \frac{da_{jn}}{dt} \\
& + \left(\omega_{jn}^2 - \frac{\gamma_j}{\pi^3} U_r^2 \alpha_{jj}^e \right) a_{jn} = 0,
\end{aligned} \tag{6}$$

$$\begin{aligned}
& (1 + \gamma_j \beta_{jj}) \frac{d^2 b_{jn}}{dt^2} + \left(2\eta_{jn} \Omega_{jn} - \frac{\gamma_j U_r^2}{\pi^3} \omega \beta_{jj}^d \right) \frac{db_{jn}}{dt} \\
& + \left(\Omega_{jn}^2 - \frac{\gamma_j}{\pi^3} U_r^2 \beta_{jj}^e \right) b_{jn} = 0,
\end{aligned}$$

$$j = 1, 2, 3, \dots, N,$$

$$n = 1, 2, 3, \dots, \infty,$$

where

$$\gamma_j = \frac{\rho \pi R^2}{m_j} \tag{7}$$

$$U_r = \frac{\pi U_m}{\omega R}. \tag{8}$$

and

$$\omega = \sqrt{\frac{\omega_{jn}^2 - \frac{\gamma_j}{\pi^3} U_r^2 \alpha_{jj}^e}{1 + \gamma_j \alpha_{jj}}}. \tag{9}$$

Notice that ω depends on the coefficient α_{jj}^e , which depend on the reduced flow velocity $U_r (= U_m/fD = U_m \cdot 2\pi/\omega D)$. Therefore, an iterated method is required to calculate ω .

3. A BILINEAR MODEL FOR LOOSELY SUPPORTED TUBES

A two-span flexible tube with one intermediate support among a rigid tube row, given in Fig. 2, was tested by Chen et al. [1]. When the right end (C3) of the tube does not contact with the stop, it is a pinned-pinned-free model (model 1, see Fig. 3). When the right end strikes the stop, it becomes a pinned-pinned-spring supported model (model 2, see Fig. 4), where the spring at C3 end is used to represent tube/support interaction when the tube contacts with the stop.

The vibration of the tube can be represented by a bilinear model consisting of model 1 and model 2. In model 1, as the displacement of the tube increases, its right end may strike the stop. Once the right end contacts the stop, it becomes model 2. Due to the increase of system stiffness, the tube motion is stabilized and its right end leaves the stop to return to model 1.

These two models of the tube vibration can be analyzed in two different time regions.

For model 1, assume that during the time interval $0 \leq t \leq t_s$, the tube displacement at C3 are within the stop limits, $-e_2 < u(\ell, t) < e_1$. The normal modes of model 1 are

$$\phi_{1n}^{(1)}(\xi_1) = \sin k_{n1}\xi_1 - \frac{\sin k_{n1}\mu}{\text{sh } k_{n1}\mu} \text{sh } k_{n1}\xi_1, \quad 0 < \xi_1 < \mu \quad (10)$$

$$n_1 = 1, 2, 3, \dots,$$

$$\varphi_{1n}^{(2)}(\xi_1) = (\cos k_{n1}\mu - \sin k_{n2}\mu \operatorname{cth} k_{n1}\mu).$$

$$\frac{[(\sin k_{n1}v + \operatorname{sh} k_{n1}v)(\cos k_{n1}\xi_2 + \operatorname{ch} k_{n1}\xi_2) - (\cos k_{n1}v + \operatorname{ch} k_{n1}v)(\sin k_{n1}\xi_2 + \operatorname{sh} k_{n1}\xi_2)]}{2(1 + \cos k_{n1}v \operatorname{ch} k_{n1}v)},$$

(11)

$$0 < \xi_2 < v,$$

$$n_1 = 1, 2, 3, \dots,$$

where ξ_1 , ξ_2 , μ , and v are dimensionless distances

$$\xi_1 = z_1/\ell, \quad \xi_2 = z_2/\ell,$$

(12)

$$\mu = \ell_1/\ell, \quad v = \ell_2/\ell.$$

k_{n1} are the solution of the characteristic equation

$$\frac{\sin k_{n1}\mu}{\cos k_{n1}\mu + \sin k_{n1}\mu \operatorname{cth} k_{n1}\mu} - \frac{\cos k_{n1}v \operatorname{sh} k_{n1}v - \sin k_{n1}v \operatorname{ch} k_{n1}v}{2(1 + \cos k_{n1}v \operatorname{ch} k_{n1}v)} = 0. \quad (13)$$

Let

$$\varphi_{1n}(\xi) = \begin{cases} C_{1n} \varphi_{1n}^{(1)}(\xi) & 0 < \xi < \mu \\ C_{1n} \varphi_{1n}^{(2)}(1 - \xi) & \mu < \xi < 1 \end{cases},$$

(14)

$$\xi = z/\ell,$$

where

$$C_{1n} = \frac{1}{\sqrt{\int_0^\mu [\varphi_{1n}^{(1)}(\xi_1)]^2 + \int_0^\nu [\varphi_{1n}^{(2)}(\xi_2)]^2 d\xi_2}}. \quad (15)$$

Therefore the solutions of the tube vibration (here only considered in the x direction) by the normal-mode method can be described as

$$\left. \begin{aligned} u(\xi, t) &= \sum_{n=1}^{\infty} a_{1n}(t) \varphi_{1n}(\xi) \\ \dot{u}(\xi, t) &= \sum_{n=1}^{\infty} \dot{a}_{1n}(t) \varphi_{1n}(\xi) \end{aligned} \right\} \quad 0 < t < t_s \quad (16)$$

At the time $t = t_s$, the right end of the tube strikes the stop, it becomes model 2. The normal modes of model 2 are

$$\varphi_{2n}^{(1)}(\xi_1) = \sin k_{n2} \xi_1 - \frac{\sin k_{n2} \mu}{\operatorname{sh} k_{n2} \mu} \operatorname{sh} k_{n2} \xi_1, \quad 0 < \xi_1 < \mu, \quad n_2 = 1, 2, 3, \dots, \quad (17)$$

and

$$\begin{aligned} \varphi_{2n}^{(2)}(\xi_2) &= D_{1n} [D_{2n} (\sin k_{n2} \xi_2 + \operatorname{sh} k_{n2} \xi_2) + D_{3n} (\cos k_{n2} \xi_2 + \operatorname{ch} k_{n2} \xi_2) \\ &\quad + D_{4n} \sin k_{n2} \xi_2 + D_{5n} \operatorname{sh} k_{n2} \xi_2], \quad 0 < \xi_2 < \nu, \quad n_2 = 1, 2, 3, \dots, \end{aligned} \quad (18)$$

where

$$D_{1n} = \frac{\cos k_{n2} \mu - \sin k_{n2} \mu \operatorname{cth} k_{n2} \mu}{2(1 + \cos k_{n2} \nu \operatorname{ch} k_{n2} \nu) + \frac{2K_c}{EI} \frac{1}{k_{n2}^3} (\cos k_{n2} \nu \operatorname{sh} k_{n2} \nu - \sin k_{n2} \nu \operatorname{ch} k_{n2} \nu)}$$

$$D_{2n} = \cos k_{n2}v + \operatorname{ch} k_{n2}v$$

$$D_{3n} = (\sin k_{n2}v + \operatorname{sh} k_{n2}v)$$

$$D_{4n} = -\frac{2K_c}{EI} \frac{1}{k_{n2}^3} \operatorname{sh} k_{n2}v$$

$$D_{5n} = -\frac{2K_c}{EI} \frac{1}{k_{n2}^3} \sin k_{n2}v$$

$$n_2 = 1, 2, 3, \dots,$$

where k_c is the equivalent stiffness, E is the Young's modulus, and I is the moment of inertia. k_{n2} are the solutions of the characteristic equation.

$$\frac{\sin k_{n2}\mu}{\cos k_{n2}\mu - \sin k_{n2}\mu \operatorname{cth} k_{n2}\mu} + \frac{\sin k_{n2}v}{\cos k_{n2}v - \sin k_{n2}v \operatorname{cth} k_{n2}v} = 0. \quad (19)$$

Let

$$\varphi_{2n}(\xi) = \begin{cases} C_{2n} \varphi_{2n}^{(1)}(\xi) & 0 < \xi < \mu \\ C_{2n} \varphi_{2n}^{(2)}(1 - \xi) & \mu < \xi < 1 \end{cases}, \quad (20)$$

$$C_{2n} = \frac{1}{\sqrt{\int_0^\mu [\varphi_{2n}^{(1)}(\xi_1)]^2 d\xi_1 + \int_0^1 [\varphi_{2n}^{(2)}(\xi_2)]^2 d\xi_2}}. \quad (21)$$

The solutions of the tube vibration by the normal-mode method are

$$\left. \begin{aligned} u(\xi, t) &= \sum_{n=1}^{\infty} a_{2n}(t) \varphi_{2n}(\xi) + u_2(\xi) \\ \dot{u}(\xi, t) &= \sum_{n=1}^{\infty} \dot{a}_{2n}(t) \varphi_{2n}(\xi) \end{aligned} \right\} t_s < t < t_d \quad (22)$$

The flow-velocity distribution in Fig. 2 can be defined as

$$\psi(\xi) = \begin{cases} 0 & 0 < \xi < \xi_a \\ 1 & \xi_a < \xi < \xi_b \\ 0 & \xi_b < \xi < 1 \end{cases} \quad (23)$$

$$\xi_a = a/\ell, \quad \xi_b = b/\ell.$$

Therefore, the force coefficients α_{jj}^d and α_{jj}^e will be

$$\alpha_{jj}^d = \int_0^1 \alpha_{jj}' \varphi^2(\xi) \psi^2(\xi) d\xi = \alpha_{jj}' \int_0^1 \varphi^2(\xi) \psi^2(\xi) d\xi \quad (24)$$

$$\alpha_{jj}^e = \int_0^1 \alpha_{jj}'' \varphi^2(\xi) \psi^2(\xi) d\xi = \alpha_{jj}'' \int_0^1 \varphi^2(\xi) \psi^2(\xi) d\xi$$

$$\varphi(\xi) = \begin{cases} \varphi_{1n}(\xi) & 0 < t < t_s \\ \varphi_{2n}(\xi) & t_s < t < t_d \end{cases} \quad (25)$$

where α_{jj}' and α_{jj}'' are fluid-damping and fluid-stiffness coefficients which are functions of the reduced flow velocity U_r .

When the tube vibrates in model 2, the right end is supported with a spring (Fig. 4), and impact forces at the end will be described as

$$F_i = -K_c [u(\xi, t) - u(\xi, t_s)] \Big|_{\xi=1} \quad t_s < t < t_d \quad (26)$$

or

$$\begin{aligned} F_i &= -K_c [u(\xi, t) - u_2(\xi)] \Big|_{\xi=1} \\ &= -K_c \left[\sum_{n=1}^{\infty} a_{2n}(t) \varphi_{2n}(1) \right] \quad t_s < t < t_d \end{aligned} \quad (27)$$

4. SIMULATIONS OF LOOSELY HELD TUBES

The natural frequency of the n -th mode for a uniform tube in air given in Fig. 2 can be expressed as

$$f_n = \frac{k_n^2}{2\pi\ell^2} \sqrt{\frac{EI}{m_j}} \quad (28)$$

where E is the modulus of elasticity (in our case, $E = 15.786 \times 10^{-6}$ Lb/in.²), I is the area moment of inertia of the cross section about the neutral axis (tube parameters, $R = 0.016$ m, $e = 0.00159$ m), ℓ is tube length, m_j is the mass per unit length and k_n is a dimensionless parameter which is related to the mode numbers and boundary conditions. In our case k_n can be calculated from the characteristic equations according to model 1 and model 2 described in Section 3.

If the tube is submerged in water, the added mass of water on the tube should be taken account for, then Eq. (45) becomes

$$f_n = \frac{k_n^2}{2\pi\ell^2} \sqrt{\frac{EI}{m_j(1 + \gamma_j\alpha_{jj})}} \quad (29)$$

where $\gamma_j = \rho\pi R^2/m_j$, ρ is the density of water; α_{jj} is the added-mass coefficient (in our case $P/D = 1.33$, $j = 1$, $\alpha_{11} = 1.113$).

Table 1 shows the comparison of the fundamental frequencies between the calculations and experiments. Note that the values of calculated frequencies are very closed to those measured in the experiments both for model 1 and model 2.

Using the bilinear fluidelastic model described in Sections 2 and 3, we developed a computer program to determine the fluidelastic characteristics of loosely held tubes in the region when a TSP-inactive mode becomes unstable. The parameters used in the calculations are based on those in the experiments [1] (see Fig. 2).

Figure 5 shows the time histories of tube oscillations (displacements and velocity of tube) at the point C3 ($z/\ell = 1.0$) and the point ($z/\ell = 0.35$) of midspan between C1 and C2 for $U_m = 2.0$ m/s and $e_1 = e_2 = 2.54$ mm. At this flow velocity, the modal damping of the TSP-inactive mode is negative and the tube is unstable. Tube oscillation amplitudes increase until the tube hits the TSP at the right end C3. Once the tube contacts the TSPs, its boundary conditions are changed, resulting in the increase of tube frequencies, modal damping and stiffness. The tube is stabilized, the oscillation amplitudes are reduced and the tube loses contact with the stop at the right end C3. Therefore, the tube goes back to the original state associated with the instability of the first TSP-inactive mode. The tube oscillations will go through the same cycles again. When the tube motion changes between model 1 and model 2 there are large changes in the velocity.

Figure 6 shows the time histories of tube displacement at different flow velocities and their phase portraits. When the flow velocity is lower than the critical flow velocity, the damping of the system is positive, the motion is stable, and oscillation amplitudes decrease with time (see Fig. 6(a), $U_m = 1.0$ m/s). When the flow velocity is equal to the critical flow velocity $U_m = 1.5$ m/s, oscillation amplitudes remain the same and the phase portrait is a circle (see Fig. 6(b)). When the flow velocity is larger than the critical flow velocity (see Fig. 6(c), $U_m = 1.8$ m/s), the damping becomes negative, and the tube becomes unstable. Its amplitudes increase until the right end reaches the TSP. Its phase portrait is very complicated.

Figure 7 is the ratio of RMS tube displacement to tube diameter vs. flow velocity with different stop clearances. Tube response characteristics here includes two regions: low-amplitude oscillation of the TSP-inactive mode, and instability of the TSP-inactive mode.

At low-amplitude oscillation region (for flow velocity lower than 1.5 m/s), the tube motion is small and the tube responds in the TSP-inactive mode.

At instability region of the TSP-inactive mode, as the flow velocity increases to the critical flow velocity, the TSP-inactive mode becomes unstable. Once the tube loses its stability, large amplitude oscillation occur and the tube impacts against the TSP. Therefore, the maximum peak-to-peak displacement at the TSP is that of the diametral clearance. In this range of flow velocity, the tube displacement is almost independent of the flow velocity because the motion shifts into a stable TSP-inactive mode when tube impacts against the TSP. From Fig. 7 we note that the results from the simulation by the bilinear fluidelastic model agree with experimental data [1] reasonably well. Figure 8 shows the ratio of RMS tube displacement to tube diameter vs flow velocity at transducer points A and B with

the diametral clearance 1.02 mm. Because the TSP-inactive model is dominant, the RMS displacement value at point A is always larger than that at point B. (If the TSP-active model is dominant, the RMS displacement value at point B will be larger than that at point A, see [1]. The bilinear method cannot predict the nonlinear response in this region).

The PSD of tube displacements at different flow velocities are given in Fig. 9. The fundamental oscillation frequencies are very close to the natural frequencies of model 1 (see Table 1).

In the region of instability of the TSP-inactive mode the tube end (see Fig. 2) will impact against the TSP to produce a series of impact forces which mostly depend on tube/support parameters, such as contact time, contact stiffness and fluid squeeze film. Tube impacting the TSP is a process which affects tube wear. It is known that the detailed dynamics of tube/support interaction is extremely complicated. We will consider additional theoretical and experimental studies to understand this complicated interaction in the further work. In this paper, we only use the bilinear instability method to reasonably simulate the impact force and correlate with work rate parameter.

Figure 10 shows the time histories of impact force vs. various flow velocities with a diametral gap of 2.54 mm. When flow velocity increases, impact force will become larger and more regular; therefore, the average impact force will increase with the flow velocity. Table 2 shows the changes of average impact force and work-rate parameter with flow velocity. Work-rate parameter is the impact force multiplied by number of impacts per unit time; this work-rate parameter is related to work rate of the impact force. The results given in Fig. 10 and Table 2 show that for large gap (2.54 mm) the average impact force and work rate parameter increase with flow velocity, and the number of impacts is about 2 per

cycle of oscillation. Those results are well consistent with the experiments by Chen, et al. in 1983 [1].

5. CONCLUSIONS

A mathematical methodology for fluidelastic instability of loosely supported tubes subjected to nonuniform crossflow is presented based on the unsteady flow theory and linear modal analysis technique. The simulation, when compared with prior experimental data, shows that this methodology is applicable for predicting fluidelastic instability response of tube when TSP-inactive mode becomes unstable. However, it is not applicable for the instability region associated with TSP-active modes because once a TSP-active mode becomes unstable other nonlinear effects become important and those effects are not included in this model.

Tube response characteristics associated with the instability of the TSP-inactive mode are different from other types of excitation and those of the TSP-active modes. When a tube becomes unstable in the TSP-inactive mode, because of the constraints at the TSP, large tube oscillations will not occur. With the increase of flow velocity, tube displacement remains almost constant until the instability of the TSP-active mode occurs. Those are illustrated in RMS displacements of tube motion both in experiments and simulations. But, in this region, the oscillation frequency of tube motion increases with flow velocity.

Reduced flow velocity and oscillation frequency, which play important roles on the critical flow velocity, can be estimated by an iterative procedure in which the relation of fluid-force coefficients to the reduced flow velocity is taken into account. These force coefficients at present can be evaluated only from

experimental data. The present work is based on these available experimental data in the literature.

Impact forces at the TSP associated with the instability of a TSP-inactive mode have been simulated. The results show that impact forces are correlated with wear work rate and are a function of flow velocity, the results agree with the published experimental data reasonably well.

The detailed dynamics of tube/support interaction, which includes the effects of fluid in gap, stiffness of support, impact forces and wear mechanism, is extremely complicated. They are not discussed in this report. Additional experiments and analytical/numerical procedures are needed to understand the interaction process and its effects on tube wear and life prediction in future.

ACKNOWLEDGMENTS

This work was co-sponsored by Northeast Utilities Service Company and the U.S. Department of Energy under Contract W-31-109-Eng-38. The work also represents a U.S. Contribution to the International Energy Agency (IEA) Program of Research and Development on Energy Conservation in Heat Transfer and Heat Exchangers.

REFERENCES

1. Chen, S. S., Jendrzejczyk, J. A., and Wambsganss, M. W. 1984. "Dynamics of Tubes in Fluid with Tube-Baffle Interaction," ASME Symposium on Flow-Induced Vibration, New Orleans, Louisiana, Vol. 2, pp. 285-304.

2. Haslinger, K. H., Martin, M. L., and Steininger, D. A. 1987. "Pressurized Water Reactor Steam Generator Tube Wear Prediction Utilizing Experimental Techniques," International Conference on Flow Induced Vibration, Bowness-on-Windermere, England, Paper K2, pp. 437-448.
3. Antunes, J., Axisa, F., Beaufils, B., and Guilbaud, D. 1988. "Coulomb Friction Modelling in Numerical Simulations of Vibration and Wear Work Rate of Multispan Tube Bundles," International Symposium on Flow Induced Vibration and Noise, Chicago, Illinois, Vol. 5, pp. 157-176.
4. Godon, J. L., and Lebre, J. 1988. "Influence of the Tube-Support Plate Clearance on Flow-Induced Vibration in Large Condensers," International Symposium on Flow Induced Vibration and Noise, Chicago, Illinois, Vol. 5, pp. 177-186.
5. Yeh, Y. S., and Chen, S. S. 1990. "Flow-Induced Vibration of Component Cooling Water Heat Exchangers," Submitted for presentation at the 1990 ASME PVP Conference.
6. Yu, X., "An analysis of Tube Failure in a U-shape Tube Bundle," ASME PVP Conference, Chicago, July 20-24, 1986, PVP, Vol. 104, pp. 187-192.
7. Fisher, N. J., and Ingham, B. 1988. "Measurement of Tube-to-Support Dynamic Force in Fretting-Wear Rigs," International Symposium on Flow Induced Vibration and Noise, Chicago, Illinois, Vol. 5, pp. 137-156.
8. Haslinger, K. H., Martin, M. L., and Steininger, D. A. 1989. "Experimental Characterization of Fluid and Squeeze Film Effects in Heat Exchanger Tube

Supports," Flow-Induced Vibration – 1989, ASME PVP Conference, PVP-Vol. 154, pp. 31-41.

9. Ko, P. L. , and Magel, E. 1989. "Impact and Sliding Wear in Steam Generators and Heat Exchangers: New Experimental Test Rig and Wear Model," Flow-Induced Vibration – 1989, ASME PVP Conference, PVP-Vol. 154, pp. 63-69.
10. Nakamura, T., and Fujita, K. 1987. "A Study on Impact Vibration of Loosely Held Tube by Cross Flow," International Conference on Flow Induced Vibration, Bowness-on-Windermere, England, Paper K1, pp. 427-436.
11. Antunes, J., Axisa, F., and Vento, M. A. "Experiments on Vibro-Impact Dynamics Under Fluidelastic Instability," The 1990 PVP Conference, Nashville, Tennessee, June 17-21, 1990, PVP Vol.189, pp. 127-138.
12. Rao, M. S. M., et al. 1987. "Computer Modelling of Vibration and Wear of Multispan Tubes with Clearances at Tube Supports," International Conference on Flow Induced Vibration, Bowness-on-Windermere, England, Paper K3, pp. 449-466.
13. Fisher, N. J., Olesen, M. J., Rogers, R. J., and Ko, P. L. 1988. "Simulation of Tube-to-Support Dynamic Interaction in Heat Exchange Equipment," International Symposium on Flow Induced Vibration and Noise, Chicago, Illinois, Vol. 5, pp. 121-136.
14. Fricker, A. J. 1988. "Numerical Analysis of the Fluidelastic Vibration of a Steam Generator Tube with Loosely Supports," International Symposium on Flow Induced Vibration and Noise, Chicago, Illinois, Vol. 5, pp. 105-120.

15. Rao, M. S. M., Steininger, D. A., and Eisinger, F. L. 1988. "Numerical Simulation of Fluidelastic Vibration and Wear of Multispan Tubes with Clearances at Supports," International Symposium on Flow Induced Vibration and Noise, Chicago, Illinois, Vol. 5, pp. 235-250.
16. Goyder, N. G. D., and Teh, C. E. 1988. "A Study of the Impact Dynamics of Loosely Supported Heat Exchanger Tubes," International Symposium on Flow Induced Vibration and Noise, Chicago, Illinois, Vol. 5, pp. 87-104.
17. Eisinger, F. L., Rao, M. S. M., and Steininger, D. A. 1989. "Numerical Simulation of Fluid Instability of Multispan Tubes Partially Exposed to Crossflow," Trans. of the 10th SMiRT, Vol. T, pp. 45-56.
18. Andjelic, M., Austermann, R., and Popp, K. "Multiple Stability Boundaries of Tubes in a Normal Triangular Cylinder Array," The 1990 PVP Conference, Nashville, Tennessee, June 17-21, 1990, PVP Vol.189, pp. 87-98.
19. Chen, S. S., and Jendrzejczyk, J. A. 1987. "Characteristics of Fluidelastic Instability of Tube Row In Crossflow," International Conference on Flow Induced Vibration, Bowness-on-Windermere, England, Paper B3, pp. 77-84.
20. Axisa, F., Antunes, J., and Villard, B. 1988. "Overview of Numerical Methods for Predicting Flow-Induced Vibration," Trans. ASME PVP, Vol. 110, pp. 6-14.
21. Andjelic, M., and Popp, K. 1989. "Stability Effects in a Normal Triangular Cylinder Array," J. of Fluid and Structure, Vol. 3, pp. 165-185.

22. Chen, S. S. 1989. "Some Issues Concerning Fluidelastic Instability of a Group of Circular Cylinders in Cross-Flow," Trans. ASME, J. Pressure Vessel Technology, Vol. 111, pp. 507-518.
23. Chen, S. S., and Chandra, S. 1989. "Fluidelastic Instabilities in Tube Bundles Exposed to Nonuniform Crossflow," The 1990 PVP Conference, Nashville, Tennessee, June 17-21, 1990, PVP Vol.189, pp. 65-77.
24. Tanaka, M. 1980. "Study on Fluidelastic Vibrations of Tube Bundle," Japan Society of Mechanical Engineering, Transaction, Section B, 46(408), pp. 1398-1407.
25. Chen, S. S., et al. 1983. "Stability of Tube Array in Crossflow," Nucl. Eng. and Design, 75(3), pp. 351-374.

DISCLAIMER

This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency thereof, nor any of their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.

Table 1. Natural frequencies of the tube (Hz) (calculated and experimental)

Models	In Air		In Water	
	Calculated	Measured	Calculated	Measured
Model 1 (TSP-inactive)	26.48	26.50	22.68	22.70
Model 2 (TSP-active)	44.93	45.40	38.49	38.54

Table 2. Impact force and work rate with various flow velocities

Flow Velocity U_m (m/s)	Impact Force F_i (N)	Contact Time T_c (%)	Impact Number per Cycle	Work-rate Parameter W_r
1.8	6.626	11.18	1.88	12.46
2.0	7.281	10.67	1.88	13.69
2.2	7.484	10.68	1.88	14.09
2.5	7.683	11.19	1.97	15.15

List of Figures

<u>Fig.</u>	<u>Title</u>
1	A tube row in nonuniform crossflow
2	Schematic of a tube and supports in crossflow
3	Schematic of a tube with right end free
4	Schematic of a tube with right end supported by a spring
5	Displacement and velocity of tube motion at points $z/\ell = 1.0$ and $z/\ell = 0.35$
6	The time histories of displacement and portraits of tube motion at various flow velocities (gap = 2.54 mm)
7	RMS tube displacement as a function of flow velocity for various diametral gaps
8	RMS tube displacement as a function of flow velocity with a diametral gap of 1.02 mm
9	PSD of tube displacement for various flow velocities with a diametral gap of 2.54 mm
10	The time histories of impact force with various flow velocities (gap = 2.54 mm)

Fig. 1. A tube row in nonuniform crossflow

$L = 1289\text{mm}$
 $L_1 = 914\text{mm}$
 $L_2 = 375\text{mm}$
 $L_3 = 259\text{mm}$
 $L_4 = 181\text{mm}$
 $L_5 = 332\text{mm}$
 $a = 327.5\text{mm}$
 $b = 586.5\text{mm}$

A, B — Transducers

Fig. 2. Schematic of a tube and supports in crossflow

Fig. 3. Schematic of a tube with right end free

Fig. 4. Schematic of a tube with right end supported by a spring

Fig. 5 . Displacement and velocity of tube motion at points $z/\ell = 1.0$ and $z/\ell = 0.35$
 $(U_m = 2.0 \text{ m/s, gap} = 2.54 \text{ mm})$

(a) $U_m = 1.0$ m/s

(b) $U_m = 1.5$ m/s

(c) $U_m = 1.8$ m/s

Fig. 6 . The time histories of displacement and portraits of tube motion at various flow velocities (gap = 2.54 mm)

Fig. 7. RMS tube displacement as a function of flow velocity for various diametral gaps

Fig. 8. RMS tube displacement as a function of flow velocity with a diametral gap of 1.02 mm

(a) $U_m = 1.6\text{ m/s}$

(b) $U_m = 2.0\text{ m/s}$

(c) $U_m = 2.3\text{ m/s}$

(d) $U_m = 2.5\text{ m/s}$

Fig. 9 . PSD of tube displacement for various flow velocities with a diametral gap of 2.54 mm

(a) $U_m = 1.8 \text{ m/s}$

(b) $U_m = 2.0 \text{ m/s}$

(c) $U_m = 2.2 \text{ m/s}$

(d) $U_m = 2.5 \text{ m/s}$

Fig. 10. The time histories of impact force with various flow velocities (gap = 2.54 mm)

END

DATE FILMED

05 / 08 / 91