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INTEGRABILITY CONDITIONS FOR THE EXISTENCE OF A LAGRANGIAN IN NEWTONIAN MECHANICS AND FIELD THEORY

Annual Progress Report

for the period

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by

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DISCLAIMER

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Research Project
DOE No. ER-78-S-02-4742.A000

Sternberg and Santilli
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ANNUAL PROGRESS REPORT

ABSTRACT

This research project was conducted under DOE grant No. ER-78-S-02-4742.A000 for the period March 1, 1978 through May 31, 1979. The investigations undertaken are the following.

Sternberg continued his studies of the principle of general covariance and its application to particle motion and continuum mechanics (See refs. 1 and 2 attached). He also developed, jointly with Professor Guillemin, a new method in microlocal analysis which has applications to integral geometry, geometrical quantization and the fine structure of certain types of spectra (see ref. 3).

Santilli studied first the classical aspect of his program, by conducting a comprehensive analysis of the integrability conditions for the existence of a Lagrangian or, independently, of a Hamiltonian for the representation of given Newtonian systems with forces nonderivable from a potential, as well as the methods for the computation of these functions from the equations of motion. These studies resulted in a monograph published by Springer-Verlag, New York-Heidelberg, in 1978 (see ref. 4). The study of a classical, complementary, methodological approach to the same class of systems was also initiated. It consists of the representation of systems with forces nonderivable from a potential via a generalization of Hamilton's equations possessing a Lie-admissible algebraic structure. This complimentary approach resulted in a second monograph published by the Hadronic Press in 1978 (ref. 5), as well as in a number of papers (refs. 6 through 10).

The problem of the quantization of forces nonderivable from a potential was then studied via the use of these complementary methods. The use of the integrability conditions for the existence of a Hamiltonian representation (the Inverse Problem) yielded, under certain restrictions, the conventional Heisenberg's equations, but expressed in terms of a generalized Hamiltonian structure. The use of the Lie-admissible formulations yielded a generalization of Heisenberg's equations possessing a generalized (Lie-admissible) algebraic structure, but expressed in terms of a conventional Hamiltonian structure.

These preliminary studies were then applied to the investigation of the old idea that the strong interactions are of the type considered, local and nonderivable from a potential, as an approximation of expected nonlocal settings. In particular, the experimental verification of the validity or invalidity of Pauli's exclusion principle and other basic physical laws for the nuclear and the hadronic structure was proposed (see ref.s 8 and 9).

1. PROGRESS REPORT BY STERNBERG

Sternberg continued his studies of the principle of general covariance and its application to particle motion and continuum mechanics (see ref.s 1 and 2 attached). He also developed, jointly with Professor Guillemin, a new method in microlocal analysis which has applications to integral geometry, geometrical quantization and the fine structure of certain types of spectra (see ref. 3).

Sternberg worked on the project under the current DOE grant 5 % of his time.

2. PROGRESS REPORT BY SANTILLI

Santilli's program under the current DOE grant contempleted the study of methods for the classical and quantum mechanical treatment of local forces nonderivable from a potential, and their applications, with particular reference to high energy physics.

The following results were obtained from March 1, 1978 up to February 28, 1979.

(A) Studies on the Inverse Problem

A first methodological approach for the classical treatment of systems with forces nonderivable from a potential is offered by the conventional canonical, Lie and symplectic formulations.

To study this approach, Santilli entered into a comprehensive analysis of a body of mathematical tools known under the name of the Inverse Problem. For this task, a considerable library search was completed to the effect of identifying all pertinent literature since the beginning of the past century. Second, the major methodological approaches to the problem were identified. They consisted of the use of (a) variational techniques; (b) functional techniques and (c) cohomology techniques, that all resulted to be equivalent in the final analysis. The variational approach was then selected because most appropriate for bringing these techniques to a level understandable and applicable by a broader audience of applied mathematicians, physicists and engineers. The functional and the cohomology approaches were then used as complementary tools for the more rigorous treatment of individual aspects, whenever needed, as well as for further studies by the interested researchers.

The project was then implemented according to the following three major aspects.

(A.1) Review of the elemental mathematical tools of the analysis, that consisted of the existence theory for ordinary differential equations, the calculus of differential forms, and the calculus of variations.

(A.2) Detailed analysis of the integrability conditions for the existence of an analytic representation of given systems of differential equations in terms of the conventional Lagrange's and Hamilton's equations. These conditions are the so-called conditions of variational selfadjointness, and were individually studied for the most important forms of writing a system of differential equations.

(A.3) Proof of the so-called Fundamental Analytic Theorems of the Inverse Problem and identification of the methods for the construction of a Lagrangian or a Hamiltonian from the given equations of motion. These theorems were independently proved for Lagrangian and Hamiltonian formulations and illustrated in details. A number of additional aspects were also studied in details, such as the algebraic and geometrical meaning of the conditions of variational selfadjointness, the generalization of variational principles for the inclusion of the integrability conditions for their existence, etc.

These studies were presented in the monograph Foundations of Theoretical Mechanics, Volume I, published by Springer-Verlag, New York-Heidelberg, in December 1978 (ref. 4).

Finally, Santilli continued his studies on the methods for transforming a given nonselfadjoint system into an equivalent selfadjoint form capable of verifying the integrability conditions for the applicability of conventional canonical, Lie and symplectic formulations. The completion of this second phase is projected for the second year of the current DOE grant (see the Application for Renewal for more details), and it is expected to result in Volume II of Santilli's monographs on the Inverse Problem.

(B) <u>Studies on the Lie-admissible problem</u>

A second methodological approach to systems with forces nonderivable from a potential is offered by the equations originally conceived by Lagrange's and Hamilton's, those with external terms. This alternative approach is however different, although complementary to that of the Inverse Problem, because Lagrange's and Hamilton's equations with (without) external terms are variationally nonselfadjoint (selfadjoint) and, as such, they do not (do) verify the integrability conditions for the applicability of conventional Lie and symplectic formulations.

Owing to the intriguing possibilities of physical applications (see below), Santilli entered into an initial, but extended study of the conceivable generalizations of conventional canonical, Lie and symplectic formulations which appear to be suggested by the presence of the external terms (representing the nonselfadjoint forces).

A most crucial result was the identification of the property that Hamilton's equations with external terms can be readily written in a form which characterizes, via the brackets of the time evolution law, a generalization of the Lie algebras known under the name of the Lie-admissible algebras. For this reason, this second complementary approach to systems with forces nonderivable from a potential was called "the Lie-admissible problem".

A second crucial result was the identification of the so-called direct universality of this approach, that is, the capability of representing all systems of the class considered (local, regular and of at least class C^1) without equivalence transformations, that is, in the coordinate frame of their experimental detection.

A third result was the identification that such direct universality occurs for contravariant Lie-admissible tensors whose covariant versions characterize symplectic-admissible two-forms, that is, two forms whose antisymmetric part is symplectic.

This appeared to confirm the hope for a possible, future, generalization of the conventional canonical, Lie and symplectic formulations which is Lie-admissible, rather than Lie, in algebraic character. The conventional formulations were recovered identically at the limit of null nonselfadjoint forces.

These studies were presented in the monograph Lie-admissible approach to the hadronic structure, Volume I, published by the Hadronic Press, Nonantum, Massachusetts, on November 1978,ref. 4, as well as in the papers of refs. 5 through 10.

(C) Studies on the relativity for nonconservative systems.

The first motivation for Santilli's undertaking of his studies on the Inverse Problem and the Lie-admissible Problem was the intent of being able to initiate the study of the relativity which is applicable to Newtonian nonconservative (open) systems. The argument is that the conventional Galilei's relativity, as currently known, is not applicable to the systems considered because they violate, in general, all Galilei's conservation laws, as well as they are, in general, not invariant under the Galilei group.

The previously studied methods of the Inverse Problem and of the Lie-admissible problem resulted to be significat for the problem, although on different grounds. The Inverse Problem resulted to be effective in the study of the mechanism of the breaking of the Galilei relativity by nonselfadjoint forces, but it exibited no apparent, direct, constructive role for a possible generalized relativity. The situation for the Lie-admissible problem was complementary to that. This latter approach exibited no effective capability for the study of the mechanism of Galilei's relativity breaking, but emerged as possessing intriguing possibility for the conceivable construction of a generalized relativity.

These studies resulted in Santilli's proposal for a Lie-admissible generalization of the Galilei relativity which is capable of providing (1) the form invariant description of the systems considered; (2) the characterization of the nonconservation of the energy, total linear momentum, total angular momentum, as well as the lack of uniform motion of the center of mass (the systems being nonconservative by assumption), and (3) the capability of recovering the conventional Galilei's relativity identically, at the limit of null value of the nonselfadjoint (Galilei breaking) forces.

The proposal for this generalization of the Galilei relativity was presented in paper 6 and subsequently subjected to further study in papers 7 and 9. It should be stressed here that such a proposal is of strict conjectural character at this moment.

Jointly, Santilli initiated his studies for a conceivable extension of Einstein's special relativity to Minkowski nonconservative forces which are not Lorentz invariant, or equivalent relativistic extensions of Newtonian nonconservative and Galilei form noninvariant forces. See the proposal for grant renewal.

(D) Studies on the quantization of nonconservative forces.

Santilli then entered into an initial study of the quantization of nonconservative systems, via the use of the classical methods of the Inverse Problem and of the Lie-admissible problem. As it had been the case for the relativity profile, these two complementary methodological approaches resulted to possess different capabilities and permit different insights.

The primary function of the Inverse Problem was that of producting a representation of the nonconservative systems considered via the conventional Hamilton-Jacobi theory. This allowed the use of conventional quantization techniques. The emerging Schrödinger's and Heisenberg's equations resulted to be of the conventional type, although expressed in terms of generalized Hamiltonian operators, that is, other than $H = T + V$, as necessary to account for the presence of forces nonderivable from a potential. Nevertheless, this quantum mechanical approach was not readily effective to identify possible implications of the nonselfadjoint forces for basic quantun mechanical laws, primarily due to the preservation of the underlying Lie algebra structure.

An initial study of the quantization of nonconservative systems via the Lie-admissible formulations was then attempted. A conceivable generalization of Heisenberg's equations of Lie-admissible (rather than Lie) character was then proposed for subsequent detailed study. This latter approach did allow some initial information on the implications for basic quantum mechanical laws of the nonselfadjoint forces. It essentially emerged that, in the transition from quantum mechanical conservative systems represented via conventional Hamiltonians $H = T + V$, to systems with local forces nonderivable from a potential, the need for suitable implementations or generalizations of Pauli's exclusion principle, Heisenberg's indeterminacy principle and other quantum mechanical laws becomes conceivable. This situation opens new, intriguing, experimental perspectives, as indicated below.

The studies under consideration were presented by Santilli in reference 8 and 9.

(E) Applications to nuclear and hadron physics.

Santilli finally initiated the applications of the classical and quantum mechanical versions of the available techniques for the treatment of nonselfadjoint systems to particles interactions.

It was first identified that the electromagnetic interactions are variationally selfadjoint at all levels of study, that is, classical nonrelativistic, classical relativistic, field theoretical and gravitational. It was further identified that the same property is shared by the Abelian and non-Abelian unified gauge theories of weak and electromagnetic interactions.

This analysis was conducted in ref. 5. Ref. 10 presented additional insights for gauge theories offered by the techniques of the Inverse Problem, with particular reference to a new form of gauge symmetry breaking without affecting the conserved charge current, called isotopic breaking.

Secondly, the Inverse Problem allowed the identification of a class of interactions which, even though still local, is structurally more general than the electromagnetic and weak interactions. This class was called of variational nonselfadjoint type.

A detailed analysis was then conducted by santilli to the effect of identifying the possible physical relevance of these broader nonselfadjoint interactions in high energy physics. It essentially emerged that, when the particles are well approximated by a pointlike structure under long range interactions, the forces are derivable from a potential and conventional techniques apply in full. This is typically the case for the peripheral electron of the hydrogen atom.

When the extended, experimentally established size of (massive, charged) particles is nonignorable and the interactions are short range (strong), the situation is different. This is typically the case of both, the nuclear and the hadronic structure.

By keeping into account that the problem of both, the nuclear and the hadronic forces is still fundamentally open at this moment, Santilli's studies essentially indicated that these forces may indeed be of nonseladjoint character, as an approximation of expected nonlocal settings. In particular, the nonselfadjoint component of the nuclear forces is expected to be very small, while that of the hadronic forces emerged as being potentially large. This differentiation can be visually represented via the "geometry" of penetration of the charge radii of the constituents among themselves, which is very small for the nuclear structure (according to available experimental data), while it is conceivably large for the hadronic structure, because all experimentally established strongly interacting particles have approximately the same charge radius ($\sim$ 1F) which is then expected to be preserved by their constituents.

The following intriguing experimental openings emerged by the analysis. By recalling that the validity of Pauli's principle and other quantum mechanical laws is unequivocal for the atomic structure (for which these laws were ultimately conceived), the same laws might conceivably admit very small deviations at the nuclear level, and larger deviations at the level of the structure of hadrons.

This situation is created by the fact that the mechanism of breaking of the Galilei relativity (both, classical and quantum mechanically) by nonselfadjoint forces appears to imply the breaking of its fundamental part, the SU(2)-spin, angular momentum part. According to this view, two nucleons, while member of a nuclear structure and in a state of partial penetration of their charge radii, are no longer exact fermions.A very small departure from the applicability of Pauli's principle (and other laws) is then consequential, and so is the quantitatively larger expectation for the hadronic structure. It should be stressed here that these are dynamical effects at the level of each individual constituent of a nuclear or hadronic state, while no departure from conventional laws and principle is detectable at the level of the total states under at most electromagnetic interactions.

Owing to this possibility, Santilli concluded his studies with a call for the experimental verification of the validity or invalidity for the strong interactions of Pauli's exclusion principle, the spin-statistics theorem and other basic physical laws.

This call was presented in reference 8, with the understanding that a long labor of studies is needed to achieve the maturity of formulation of the problem in a form readily testable with the current technology.

Santilli worked on the studies outlined here under the current DOE grant on a full time basis.

No additional publication by Santilli is contemplated from now (end of February) until the completion of the current contract (end of May 1979), because he will be working full time on the studies for Volume II of the "<u>Foundations of Theoretical Mechanics</u> with

Springer-Verlag, and this project is expected to be completed during the sec ond year of the current contract.

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