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THE DETECTION OF A NORMAL ZONE IN
THE MFTF MAGNETS

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A method is described for the electrical detection of a normal zone in inductively coupled superconducting coils. Measurements are made with two kinds of bridges, mutual inductance bridges and self-inductance bridges. The bridge outputs are combined with other measured voltages to form a detector that can be realized with either analog circuits or a computer algorithm. The detection of a normal zone in a pair of coupled coils, each with taps, is discussed in detail. It is also shown that the method applies to a pair of coils when one has no taps and to a pair when one coil is superconducting and the other is not. The method is extended, in principle, to a number of coils. A description is given of a technique for balancing the bridges at near the operating currents of the coils.

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INTRODUCTION

Large superconducting magnets costing millions of dollars are now being built for the fusion program, and larger still will be needed when reactors become a reality.⁽¹⁾ The size of the investment and the critical nature of the application call for the development of safety systems comparable to those used for nuclear reactors.⁽²⁾ In the past, insufficient attention to the protection of the magnet has occasionally resulted in failure.⁽³⁾

Failure can take place when a normal zone develops in the superconductor and an excessive amount of heat is produced. A number of protection methods have been devised, some passive and some requiring the active intervention of protective equipment.⁽⁴⁾ This study considers the problem of detecting the formation of a normal zone. The information can then be applied to whatever mode of protection is selected.

The electrical detection of a normal zone has been described in the past.^(5,6) However, the use of inductively coupled coils poses a new problem. A pair of coupled coils are used in the Mirror Fusion Test Facility (MFTF)⁽⁷⁾ at the Lawrence Livermore Laboratory and proposed mirror fusion reactors contain a number of coupled coils. When the coils are inductively coupled it is necessary to distinguish between the small voltages due to a normal zone and the large voltages due to mutual and self-inductance. The problem is particularly serious when the current in one of the coils is being changed.

The normal zone itself has been the subject of theoretical⁽⁴⁾ and experimental⁽⁸⁾ investigations. It is found that when ohmic heating exceeds the locally available cooling even a minute normal zone grows in size. In order to protect a coil it is necessary to detect a normal zone in its infancy.

OVERVIEW OF THE METHOD

A normal zone in a set of coupled coils is detected by measuring a number of voltages and combining these voltages in such a way that the large mutual and self-induced voltages are eliminated. The most vital of the measured voltages are the outputs of bridges constructed by placing resistor pairs across the coils. There are two kinds of bridges, mutual inductance bridges and self-inductance bridges. The mutual inductance bridges are balanced so that the mutually induced voltages are cancelled. Therefore, their output consists of the self-induced voltages and the normal zone voltage. The self-inductance bridges have an output consisting of the mutually induced voltages and the normal zone voltage.

In general, it is desirable to measure other voltages in addition to the bridge voltages. These voltages are combined with the bridge voltages to form a detector equation, or a detector as it is usually called in the body of the paper. The detector equation is a linear combination of the measured voltages. The coefficients of the voltages, or the detector gains, depend on the mutual and self-inductances of the coils.

The detector can be realized either as an analog circuit or as a digital computation. An analog circuit consists of a number of summing operational amplifiers. The digital algorithm is a sequence of multiply by a constant and add operations. The paper describes the theory of the method but does not detail these implementations.

A normal zone is most difficult to detect during a period when the current in a coil is being changed from one level to another. During this time the mutual and self-induced voltages are large compared with the normal

zone voltage. The method effectively deals with this difficulty. The bridges are balanced and the detector gains tuned during a period of change, thereby eliminating the unwanted voltages in a manner that is direct and practical.

Detectors can be built using bridges that are not purely mutual or self-inductance bridges but which balance at some intermediate set of values. A simpler detector equation is possible, but the balancing is awkward. Since they lack practicality they are not discussed further.

TWO SUPERCONDUCTING COILS WITH TAPS

The main body of the paper describes in detail the detection of a normal zone in a pair of superconducting coils, each with at least one tap. In later sections the results are extended to more than two coils, a pair of coils in which one has no taps, and a pair of coils, one of which is not superconducting.

The equivalent circuit of a pair of coils and the bridges used to detect the normal zone is shown in Figure 1. The two superconducting coils have self-inductances L_1 and L_2 and a mutual inductance M . L_t and L_b are the self-inductances of the upper and lower parts of the coil. The mutual inductances M_b and M_t are the mutual inductances of portions of one coil to the entire second coil. The normal zone resistance r has arbitrarily been placed in the lower, left coil.

The normal zone detection circuit consists of resistors R_{b1} , R_{t1} , R_{b2} , R_{t2} connected across the coils. Together with the coils these resistors form two bridges with outputs v_L and v_R .

The bridge resistors are adjusted so that either the mutual or

self-inductance voltage are balanced out of the bridge's output. This is the essential feature of the detection method. The input to the detector consists of the bridge outputs and a combination of some of the other voltages shown in Figure 1. A number of detectors with useful properties are discussed in the following sections.

ANALYSIS OF A SINGLE BRIDGE

As a preliminary to describing the detectors, it is convenient to analyze the bridge formed by a single coil and two resistors. A bridge of this type balanced for the self-inductance of the coils, has been used for the detection of a normal zone in a single, uncoupled, superconducting coil.⁽⁵⁾

For the moment, only the left coil in Figure 1 is of interest. The voltage v_1 across the coil is

$$v_1 = L_1 \frac{di_1}{dt} + M \frac{di_2}{dt} + r i_1$$

The voltage v_{bc} across the bridge resistor R_{b1} is

$$v_{bc} = \frac{R_{b1}}{R_{B1}} v_1, \text{ where } R_{B1} = R_{b1} + R_{t1}$$

Therefore, the bridge output is

$$v_L = L_{b1} \frac{di_1}{dt} + M_{b1} \frac{di_2}{dt} + i_1 r - \frac{R_{b1}}{R_{B1}} \left(L_1 \frac{di_1}{dt} + M \frac{di_2}{dt} + i_1 r \right)$$

$$\text{Let } p_1 = \frac{L_{b1}}{L_1}, \quad p_{m1} = \frac{M_{b1}}{M}$$

Then v_L can be written as

$$v_L = L_1 \left(p_1 - \frac{R_{b1}}{R_{B1}} \right) \frac{di_1}{dt} + M \left(p_{m1} - \frac{R_{b1}}{R_{B1}} \right) \frac{di_2}{dt} + i_1 r \left(1 - \frac{R_{b1}}{R_{B1}} \right).$$

A self-inductive bridge is formed when the resistors are adjusted so that

$$p_1 = \frac{L_{b1}}{L_1} = \frac{R_{b1}}{R_{B1}}.$$

This selection of resistor values balances out the self-induced voltages so that the bridge output depends only on the mutually induced voltages and the voltage across the normal zone. Therefore, the following expression can be written for the rate of change of current in the second coil.

$$\frac{di_2}{dt} = \frac{1}{M(p_{m1} - p_1)} \left[v_L - i_1 r (1 - p_1) \right], \text{ for } p_1 = \frac{L_{b1}}{L_1} = \frac{R_{b1}}{R_{B1}}. \quad (1)$$

A mutual inductance bridge is formed when the bridge resistors are balanced so that

$$p_{m1} = \frac{M}{L} \frac{b1}{B1} = \frac{R_{b1}}{R_{B1}}.$$

This selection of resistor values balances out the mutually induced voltage, causing the bridge output to depend only on the rate of change of current in the coil associated with the bridge. Therefore,

$$\frac{di_1}{dt} = \frac{1}{L_1(p_1 - p_{m1})} \left[v_L - i_1 r (1 - p_{m1}) \right], \quad p_{m1} = \frac{M_{b1}}{M} = \frac{R_{b1}}{R_{B1}} \quad (2)$$

A similar pair of equations for the rates of change of current can be written for the right coil and bridge. Since the right coil is assumed to have no normal zone

$$\frac{di_1}{dt} = \frac{v_R}{M(p_{m2} - p_2)} \quad , \quad \text{for} \quad p_2 = \frac{L_{b2}}{L_2} = \frac{R_{b1}}{R_{B1}} \quad , \quad (3)$$

and

$$\frac{di_2}{dt} = \frac{v_R}{L_2(p_2 - p_{m2})} \quad , \quad \text{for} \quad p_{m2} = \frac{M_{b2}}{M} = \frac{R_{b1}}{R_{B1}} \quad . \quad (4)$$

A MUTUAL INDUCTANCE BRIDGE ON ONE COIL AND A SELF-INDUCTANCE BRIDGE ON THE OTHER

A particularly simple detection scheme uses a self-inductance bridge on one coil and a mutual inductance bridge on the other. The method is simple because only the bridge outputs need be measured, the computation is short, and the bridges are conveniently balanced. However, a serious disadvantage of the detector is that it is much more sensitive to a normal zone on the self-inductance side than it is to one on the mutual inductance side.

The left coil is arbitrarily selected for the self-inductance bridge, the right for the mutual inductance bridge. That is,

$$\frac{R_{b1}}{R_{B1}} = \frac{L_{b1}}{L_1} = p_1 ,$$

and $\frac{R_{b2}}{R_{B2}} = \frac{M_{b2}}{M} = p_{m2} .$

In addition, let

$$\frac{L_{b2}}{L_2} = p_2 , \quad \text{and} \quad \frac{M_{b1}}{M} = p_{m1} .$$

The output of each bridge is proportional to the rate of change of current in the right coil. From the equations of the previous section,

$$v_L = M(p_{m1} - p_1) \frac{di_2}{dt} + i_1 r (1 - p_1) .$$

$$v_K = L_2 (p_2 - p_{m2}) \frac{di_2}{dt} .$$

Eliminating the derivatives produces an expression for the voltage across the normal zone,

$$i_1 r = K_1 v_L + K_2 v_R ,$$

where $K_1 = \frac{1}{1 - p_1} ,$

$$K_2 = K_1 \frac{M}{L_2} \frac{(p_1 - p_{m1})}{(p_2 - p_{m2})} .$$

If the normal zone is in the left bottom coil, as assumed, the sum $K_1 v_L + K_2 v_R$ yields the voltage across the normal zone. If the normal zone is elsewhere, the output is proportional to the normal zone voltage but not necessarily equal to it. To make this distinction, we write

$$v_o = K_1 v_L + K_2 v_R \quad , \quad (5)$$

where v_o = the output of the detector.

Other detectors, that produce exactly the voltage across a normal zone in another quadrant, or an intermediate voltage, can be designed. However, the ratio of the outputs produced by normal zones in different quadrants remains equal.

The output of the detector defined by Equation 5, when the normal zone is in the other three quadrants of the coils, can now be determined. If it is in the lower right side

$$v_L = M(p_{m1} - p_1) \frac{di_2}{dt} \quad ,$$

$$v_R = L_2 (p_2 - p_{m2}) \frac{di_2}{dt} + i_2 r (1 - p_{m2}) \quad .$$

When these values are substituted into Equation 5 the derivatives cancel and

$$v_o = \frac{M}{L_2} \frac{(p_1 - p_{m1}) (1 - p_{m2})}{(p_2 - p_{m2}) (1 - p_1)} i_2 r \quad .$$

If the normal zone is in the top left, the outputs of the bridge are

$$v_L = M(p_{m1} - p_1) \frac{di_2}{dt} - p_1 i_1 r$$

$$v_R = L_2 (p_2 - p_{m2}) \frac{di_2}{dt}$$

Substituting into Equation 5 yields

$$v_0 = \frac{-p_1}{1 - p_1} i_1 r = -\frac{L_{b1}}{L_{t1}} i_1 r$$

A similar procedure for a normal zone in the right upper coil yields

$$v_0 = -\frac{M_{b2}}{L_{t1}} \frac{L_1}{L_2} \frac{(p_1 - p_{m1})}{(p_2 - p_{m2})} i_2 r$$

In general, a normal zone in one quadrant causes a much different output than a normal zone in another quadrant. However, if the coils are similar, with taps in the same place, there is a simple relationship between the outputs. Similar coil imply equal inductances.

$$L_1 = L_2 = L, \quad L_{b1} = L_{b2} = L_b, \quad L_{t1} = L_{t2} = L_t,$$

$$M_{b1} = M_{b2} = M_b, \quad M_{t1} = M_{t2} = M_t \quad \left. \vphantom{\begin{matrix} L_1 = L_2 = L, \\ L_{b1} = L_{b2} = L_b, \\ L_{t1} = L_{t2} = L_t, \end{matrix}} \right\} (6)$$

$$\text{and } p_1 = p_2, \quad p_{m1} = p_{m2}.$$

In addition, the ratio of a partial self-inductance to the total inductance is in most cases close in value to the ratio of a partial mutual inductance to the total inductance

$$P \approx P_m.$$

The two ratios are not exactly equal because the orientation of the two coils may cause a portion to be more closely coupled than the remainder. In some cases the approximation is definitely not true. However, for the special case in which it is true, the outputs can be simplified.

Table 1, row 2, shows a tabulation of the results for the four quadrants. The ratio M/L , which appears in the output for the right side, is small relative to one. For air core coils the coupling coefficient K , which is equal to M/L for similar coils, is never greater than .8 and is usually much smaller.

A few conclusions can now be made. The detector is more sensitive to a normal zone in the coil associated with the self-inductance bridge than it is to a normal zone in the coil with the mutual inductance bridge. The sensitivity also depends on where the tap is located. If the coil is center-tapped the detector is equally sensitive to normal zones in the top and bottom. However, if the tap is off-center the detector is more sensitive to a normal zone in the shorter portion of the coil.

A BRIDGE OF THE SAME KIND ON EACH OF THE TWO COILS

A detector for a pair of coils based on two similar bridges is equally sensitive to a normal zone in either coil. However, two additional voltages must be measured and the calculation is longer.

If both coils have a self-inductance bridge,

$$v_1 = M(p_{m1} - p_1) \frac{di_2}{dt} + i_1 r (1 - p_1)$$

$$v_p = M(p_{m2} - p_2) \frac{di_1}{dt}$$

The two additional measured voltages are v_C and v_B or v_C and v_T . Since these voltages are symmetrical to each coil, the result is a detector which is equally sensitive to each coil.

Selecting v_C and v_B ,

$$\begin{aligned} v_C - v_B &= L_{b1} \frac{di_1}{dt} + M_{b1} \frac{di_2}{dt} + i_1 r - L_{b2} \frac{di_2}{dt} - M_{b2} \frac{di_1}{dt} \\ &= L_{b1} \left(1 - \frac{M_{b2}}{L_{b1}} \frac{di_1}{dt} \right) - L_{b2} \left(1 - \frac{M_{b1}}{L_{b2}} \right) \frac{di_2}{dt} - i_1 r \end{aligned}$$

Expressions for the derivatives obtained from Equations 1 and 3 are now substituted into this equation

$$\frac{di_2}{dt} = \frac{v_L - i_1 r (1 - p_1)}{M(p_{m1} - p_1)}$$

$$\frac{di_1}{dt} = \frac{v_p}{M(p_{m2} - p_2)}$$

The result is

$$v_0 = v_3 - v_4 = (v_C - v_B) + (v_L - v_R) \quad (7)$$

where

$$v_3 = \left[1 + \frac{L_b Z}{M(p_m + p_1)} \right]^{-1}$$

$$v_4 = v_3 \frac{L_b Z}{M(p_m + p_1)}$$

$$v_5 = -v_3 \frac{L_{b1}}{M(p_m + p_2)}$$

$$q_1 = \frac{M_b Z}{L_{b1}}, \quad q_2 = \frac{M_{b1}}{L_{b2}}$$

For the special case where the coils are similar, a simplified expression can be written. Specifically, if the conditions of Equation 6 are true,

$$v_0 = K_6 (v_C - v_B) + K_7 (v_L - v_R) \quad (8)$$

$$K_6 = \frac{M_b}{L_b} \left(\frac{1 - \frac{L_b}{L} \frac{M}{M_b}}{1 - \frac{M_t}{L} - \frac{L_b}{L}} \right)$$

$$K_7 = \frac{1 - \frac{M_b}{L_b}}{1 - \frac{M_t}{L} - \frac{L_b}{L}}$$

When the normal zone is in the lower left coil the output of the detector is the voltage across the normal zone. The output for other locations of the normal zone is found in a manner similar to that employed in the previous section. For similar coils:

The output for a normal zone in the upper left is

$$v_o = -\frac{L_b}{L} \frac{1 - \frac{M_b}{L_b}}{1 - \frac{M_t}{L} - \frac{L_b}{L}} i_1 r .$$

The output for a normal zone in the lower right is

$$v_o = -i_2 r .$$

The output for a normal zone in the upper right is the same as for one in the upper left, but has the opposite sign.

If the approximations described in the previous section are made, these expressions can be simplified further. The results of these approximations are shown in Table 1.

The outputs for two additional detectors are also shown in Table 1. One of these is a detector based on two mutual inductance bridges and measurement of $v_C - v_B$ and the other is a detector based on two self-inductance bridges and a measurement of v_2 .

The detector using two mutual inductance bridges and $v_C - v_B$ is similar to the one just discussed. An almost identical analysis yields

$$v_o = K_8 (v_C - v_B) + K_9 v_L + K_{10} v_R \quad , \quad (9)$$

$$K_8 = \left[1 - \frac{(1 - q_1)(1 - p_{m1})}{1 - \frac{p_{m1}}{p_1}} \right]^{-1} ,$$

$$K_9 = -K_8 \frac{(1 - q_1)}{\left(1 - \frac{p_{m1}}{p_1}\right)} ,$$

$$K_{10} = K_8 \frac{(1 - q_2)}{\left(1 - \frac{p_{m2}}{p_2}\right)} .$$

A pair of detectors, containing either mutual or self-inductance bridges can also be constructed using $v_C - v_T$ instead of $v_C - v_B$. Because of the symmetry of the circuit, the detector equations are similar to those already derived. It is simply necessary to interchange the lower and upper quantities.

An alternative to measuring $v_C - v_B$ or $v_C - v_T$ is to measure a single voltage, v_2 or v_1 . A detector equation, using v_2 , that produces an output equal to the normal zone when the normal zone is in the left bottom coil is

$$v_o = K_{11}v_L + K_{12}v_Z + K_{13}v_P \quad ,$$

$$K_{11} = \frac{1}{1 - p_1} \quad , \quad 12$$

$$K_{12} = -K_{11} \frac{M}{L_2} (p_{m1} - p_1) \quad ,$$

$$K_{13} = \frac{-K_{12}}{p_{m2} - p_2} \quad .$$

A similar detector uses v_1 instead of v_2 .

Some conclusions can now be drawn. If the coils are similar, detectors based on measuring v_C and v_B have the same magnitude output regardless of which coil contains the normal zone. The outputs to two other detectors, the one that uses a mutual and self-inductance bridge and the one that uses v_Z , depend on which coil contains the normal zone. This can be attributed to the lack of symmetry in these detectors. However, all of the detectors are sensitive to asymmetries in a coil itself. If the coil is not center-tapped the output when a normal zone in the top portion is different from what it is when it is in the bottom. The detectors with self-inductance bridges have a greater output when the normal zone is in the shorter portion of the coil. The other two have a greater output when it is in the longer portion.

In general, since the bridge using $v_C - v_B$ or $v_C - v_T$ are equally sensitive to both coils, they are to be preferred over the other two. When a center tapped coil is used, a normal zone in any quadrant causes the same magnitude of output.

Determining the Location of the Normal Zone

As a study of Table 1 shows, no single detector is capable of fixing the location of the normal zone. However, if two detectors are used, the quadrant containing the normal zone can be determined. A pair of self-inductance or mutual inductance bridges using the measured voltages $v_C - v_B$ determine whether the zone is in the upper or lower part of the coils. To determine if the normal zone is in the left or the right coil, it is necessary to use one of the other detectors. Either a detector using v_1 or v_2 or a detector with a mutual inductance bridge on one coil and a self-inductance bridge on the other coil can be employed.

USING MORE THAN ONE TAP

A normal zone in a pair of coils can be detected and located when there is only a center tap on each coil. However, there are several reasons why more than one tap is desirable. First, the cost of the magnet may warrant redundant protection. In addition, there is the possibility that a normal zone may form at the tap and propagate equally in each direction. This would place a normal zone in two adjacent branches of the bridges. A bridge is ineffective in detecting this event.

Another advantage of multiple taps is that the sensitivity to a normal zone can be increased. Reducing the extent of the coil encompassed by a measurement, reduces the mutual and self-induced voltages. However, the normal zone voltage remains unchanged. Therefore, a normal zone is more easily detected.

A bridge associated with an off-center tap can be connected across the entire coil, as in the previous section, or across part of the coil. If it is connected across the coil, the detector is more sensitive to a normal zone in one part of the coil than the other. Since a threshold must be set, the detector is effective for only part of the coil.

The alternative is to connect resistors so as to form a bridge with only part of the coil. The resulting detector does not detect a normal zone in the part of the coil not spanned by the bridge. However, it is more effective in detecting a normal zone within its span than a detector associated with a center tapped coil.

TWO SUPERCONDUCTING COILS, ONE WITHOUT TAPS

A normal zone can be detected in a pair of inductively coupled superconducting coils, even though one of the coils has no taps. However, a price is paid for the lack of taps on one coil. The detector is relatively ineffective in detecting the normal zone in the untapped coil.

The detector circuit consists of two bridges associated with the tapped coil, a mutual inductance bridge and a self-inductance bridge, as shown in Figure 2. The inputs to the detector are the bridge voltages v_L and v_{LM} and the voltage across the untapped coil v_2 .

As in the previous analyses, the bridges provide expressions for the time derivatives of the two currents,

$$\frac{di_2}{dt} = \frac{v_L - i_1 r (1 - p)}{M(p_m - p)} \quad , \quad (11)$$

$$\frac{di_1}{dt} = \frac{v_{LM} - i_1 r (1 - p_m)}{L_1 (p - p_m)} \quad (12)$$

where $p = \frac{L_b}{L_1}$ and $p_m = \frac{M_b}{M}$.

These expressions are substituted into the equation for the voltage across the second coil

$$v_2 = L_2 \frac{di_2}{dt} + M \frac{di_1}{dt} \quad . \quad (13)$$

The resultant detector equation is

$$v_o = K_{14} v_2 + K_{15} v_L + K_{16} v_{LM} \quad , \quad (14)$$

$$K_{14} = (p - p_m) \left[\frac{M}{L_1} (p_m - 1) - \frac{L_2}{M} (p - 1) \right]^{-1} \quad ,$$

$$K_{15} = \frac{K_{14} L_2}{M(p - p_m)} \quad ,$$

$$K_{16} = \frac{-K_{14} M}{L_1 (p - p_m)} \quad .$$

This detector has an output equal to the voltage across the normal zone $i_1 r$ when the normal zone is in the lower portion of the tapped coil. When it is in the upper part of the tapped coil the output is

$$v_o = \frac{p}{p - \frac{1 - K^2}{1 - K^2} \frac{p_m}{p}} i_1 r ,$$

Where K is the coupling coefficient, $K = \frac{M}{\sqrt{L_1 L_2}}$

For a small K or if $p_m \approx p$,

$$v_o \approx \frac{p}{p - 1} i_1 r = - \frac{L_b}{L_t} i_1 r .$$

Therefore, if the coil is center-tapped, the detector is equally sensitive to normal zones in the top or the bottom.

When the normal zone is in the untapped coil the output is

$$v_o = K_{14} i_2 r .$$

The gain K_{14} , which is defined above, contains the factor $p - p_m$. In most cases the ratio of the mutual inductances p_m is almost equal to the ratio of the self-inductances p . When this is the case, $p - p_m$ is small, and the detector is relatively insensitive to a normal zone in the untapped coil.

A SUPERCONDUCTING COIL AND A NON-SUPERCONDUCTING COIL

A normal zone can be detected in a superconducting coil that is magnetically coupled to a coil that is not superconducting. Both coils must have a tap. The method and analysis are similar in many respects to the method of the previous section. Two bridges, a mutual inductance bridge and a self-inductance bridge, are associated with the superconducting coil as shown in Figure 3. The tap in the normal coil is used to form a bridge that balances out the resistive component of the normal coil. The output of this bridge, in addition to the two bridge outputs of the superconducting coil, are the inputs to the detector.

The two bridges on the superconducting coil provide expressions for the derivatives of the currents. The superconducting side is the same as in the previous section. Therefore, Equations 11 and 12 apply.

The output of the normal coil bridge v_R is

$$v_R = L_{b2} \frac{di_2}{dt} + M_{b2} \frac{di_1}{dt} + i_2 R_{cb} - \frac{R_{b2}}{R_{t2} + R_{b2}} \left(L_2 \frac{di_2}{dt} + \frac{M di_1}{dt} + i_2 R_C \right).$$

The bridge is balanced to cancel the resistive component.

$$\frac{R_{b2}}{R_{t2} + R_{b2}} = \frac{R_{cb}}{R_{cb} + R_{ct}} = p_r.$$

Then,

$$v_R = (L_{b2} - p_r L_2) \frac{di_2}{dt} + (M_{b2} - p_r M) \frac{di_1}{dt}.$$

This equation is similar to Equation 13 of the previous section except that $L_{b2} - p_r L_2$ replaces L_2 in Equation 13, and $M_{b2} - p_r M$ replaces M in Equation 12. Therefore,

$$v_0 = K_{17}v_2 + K_{18}v_L + K_{19}v_{LM}$$

where

$$\left. \begin{array}{l} K_{17} = K_{14} \\ K_{18} = K_{15} \\ K_{19} = K_{16} \end{array} \right\} \quad \text{with} \quad \begin{array}{l} L_2 = L_{b2} - p_r L_2 \\ M = M_b - p_r M \end{array}$$

In the previous section it was found that the detector is insensitive to normal zones in the untapped coil. In the non-superconducting and superconducting pair, the normal coil replaces the untapped coil. Therefore, the detector is relatively insensitive to changes in the resistance of the non-superconducting coil, a desirable property.

MORE THAN TWO INDUCTIVELY COUPLED COILS

The methods developed in the preceding sections for two coils can be extended to any number of inductively coupled coils. The principles of this extension are illustrated for the three coupled coils shown in Figure 4.

Each of the coils in Figure 4 has a tap and a bridge associated with it. If a coil does not have a tap, this lack can be compensated for by placing an additional bridge on another of the coils. If a coil is not superconducting, a tap is necessary so that a bridge balancing out the resistive component can be constructed.

The bridges provide measures of the rates of change of current. For the left coil and bridge,

$$v_L = L_{b1} \frac{di_1}{dt} + M_{b12} \frac{di_2}{dt} + M_{b13} \frac{di_3}{dt} + i_1 r$$

$$- \frac{R_{b1}}{R_{B1}} \left(L_1 \frac{di_1}{dt} + M_{12} \frac{di_2}{dt} + M_{13} \frac{di_3}{dt} + i_1 r \right) .$$

The bridges can be either all mutual inductance bridges, all self-inductance bridges, or a mix of the two kinds of bridges. For purposes of illustration, it is assumed that all of the bridges are self-inductance bridges. For the left coil,

$$\frac{L_{b1}}{L_1} = \frac{R_{b1}}{R_{B1}} = p_1 .$$

Then,

$$v_L = (M_{b12} - p_1 M_{12}) \frac{di_2}{dt} + (M_{b13} - p_1 M_{13}) \frac{di_3}{dt} + i_1 r(1-p_1) .$$

And similarly,

$$v_{R2} = (M_{b21} - p_2 M_{12}) \frac{di_1}{dt} + (M_{b23} - p_2 M_{23}) \frac{di_3}{dt} ,$$

$$v_{R3} = (M_{b31} - p_3 M_{13}) \frac{di_1}{dt} + (M_{b32} - p_3 M_{23}) \frac{di_2}{dt} .$$

An expression for each of the derivatives in terms of the bridge voltages and the circuit parameters is obtained by solving the three equations simultaneously. If there are n coils, n equations must be solved. This may be an onerous task but the solution of these equations is part of the design procedure and not part of the detector's operation.

The detector can be designed around the measurement of any of a number of voltages in addition to the bridge voltages. If, for example, the object is to detect a normal zone in coil 1 or coil 2, the measurement of $v_C - v_B$ is appropriate.

$$v_C - v_B = (L_{b1} - M_{b21}) \frac{di_1}{dt} + (M_{b12} - L_{b2}) \frac{di_2}{dt} + (M_{b13} - M_{23}) \frac{di_3}{dt} + i_1 r$$

The values of the derivatives are substituted into the equation and it is solved for $i_1 r$. The result is a detector equation of the form

$$v_0 = K_{20}(v_C - v_B) + K_{21}v_L + K_{22}v_{R1} + K_{23}v_{R3}.$$

The output of this detector is equal to the voltage across the normal zone when the normal zone is in the bottom of coil 1.

BALANCING THE BRIDGES AND TUNING THE DETECTOR

A desirable feature of the method described in this paper is that the detector can be tuned under conditions that correspond to the most difficult detection task. A normal zone is most difficult to detect during a change in the coil currents from one level to another. During this time the mutual and self-induced voltages are large compared with the normal zone voltage. The detector must be finely tuned to detect this voltage. If it was necessary to rely on calculated or even measured values, it might be impossible to obtain the degree of accuracy needed. However, as this section describes, the

bridges can be balanced and the detector tuned to eliminate the inductive voltages during a period of changing current.

One method of balancing the bridges is to operate each coil by itself with the others off and open-circuited. The method can only be used to balance the self-inductance bridges. A further disadvantage is that the inductance may change with current because of structural deflections caused by the magnetic forces. Indeed, as is the case for the MFTF magnets, the forces may be such that one coil cannot be operated alone.

A better method is to balance the bridges while the current is being changed near the normal operating current. Balance can be effected during the steady state portion of the change, after the transients have died away. For large coils there is no difficulty obtaining a long steady state period during which the bridges can be balanced.

Steady State Conditions During a Change in Current

During constant current operations a superconducting magnet is usually supplied by a current regulated power supply. Changing the current can be accomplished in one of two ways: by controlling the current output of the power supply or by introducing a voltage into the circuit. Circuit models of the two methods are shown in Figures 5 and 6.

The resistances R in these figures are low resistance resistors connected directly across the coils. Their function is to provide a path for the inductive current should an accident break the lines from the power supply to the coils. They can also be used as dump resistors if the magnet is protected by the external dissipation of energy.

Before the change begins, the currents are at equilibrium. For simplicity it is assumed that $i_1 = i_2 = I$. Since the bridge resistors are relatively large compared with R , the current through the bridge can be neglected.

When a voltage source is introduced as in Figure 5, there is a brief transient due to cable resistance before the entire voltage V appears across the left coil. The current in the left coil then increases linearly,

$$i_1 = I + \frac{V}{L} t .$$

The changing current in the left coil induces a voltage in the right coil. After a transient period, with a length depending on R , the currents in the right coil stabilize and a constant current,

$$i_2 = I - \frac{MV}{LR} ,$$

flows through the coil. Since the current in the right coil is constant, there is no mutually induced voltage reflected back to the left coil. Therefore, the voltages on the left coil are entirely self-inductive, while those in the right are mutually inductive.

A similar analysis can be conducted for the ramp change in the current source shown in Figure 6. After the transients have died away the currents are

$$i_1 = I \left[\left(1 - \frac{L}{R} f \right) + ft \right] ,$$

$$i_2 = I \left(1 - \frac{M}{R} f \right) .$$

The resulting voltages are shown in the figure.

The bridge outputs can be written down from the voltages shown on the diagrams. For the voltage driven change of Figure 5 they are

$$v_L = V \left(\frac{L_{b1}}{L_1} - \frac{R_{b1}}{R_{B1}} \right) ,$$

$$v_R = \frac{VM}{L_1} \left(\frac{M_b}{M} - \frac{R_{b2}}{R_{B2}} \right) .$$

For the current driven change of Figure 6 they are

$$v_L = fIL_1 \left(\frac{L_{b1}}{L_1} - \frac{R_{b1}}{R_{B1}} \right) ,$$

$$v_R = fIM \left(\frac{M_b}{M} - \frac{R_{b2}}{R_{B2}} \right) .$$

Balancing the Bridges

A ramp change in the current of one coil causes self-induced voltages in that coil and mutually induced voltages in the other coil after the transients have died away. During the period of steady state change, a self-inductance bridge across the coil undergoing change can be balanced. A mutual inductance bridge across the other coil can be balanced.

If each coil has a self-inductance or a mutual inductance bridge, the balancing of the bridges must be done in two stages. The current in one coil is changed, and bridges are balanced; then the current in the other coil is changed and the remaining bridges are balanced.

Tuning the Detector

The detector gains are tuned after the bridges are balanced. For the preferred detector, which contains a mutual or a self-inductance bridge across each coil, the output of the detector is of the form

$$v_o = K_a (v_C - v_B) + K_b v_L + K_c v_R.$$

Tuning the three gains is done in two stages, during a steady state change in one coil current, and during a steady state change in the other coil current.

During a steady state change in a coil current, one of the bridge outputs, either v_L or v_R , is zero. If the bridges are both self-inductance bridges, the bridge output associated with the coil having the changing current is zero. If they are mutual inductance bridges, the output of the other bridge is zero.

Since one bridge output is zero, only two gains need be adjusted during the first stage of the tuning. The first step is to establish the level of one of the gains. This level can be set arbitrarily at a suitable sensitivity. After the first gain is fixed, the second gain is adjusted until the output of the detector is zero.

Fixing the first gain establishes the absolute calibration of the detector, that is, the relationship between the voltage output and the voltage across the normal zone. Ordinarily, absolute accuracy is not as important as adequate sensitivity and accurate tuning or zeroing. However, if an accurate absolute calibration is needed, the first gain can be determined from the calculated inductances. Since the calculated inductances are only approximate it may be necessary to measure the inductances. The mutual and self-inductances can be determined from measurements made during a steady state change in the current of the voltages, currents, and rate of change of current in the coils.

During the second stage, the current in the second coil is changed while the current in the first remains constant. The third gain is then adjusted until the output of the detector is zero.

A detector for two similar coils can be tuned during a single ramping since it contains only two gains. However, since no two coils are identical, it is best to treat them as dissimilar and provide three gains until it is demonstrated that two gains are sufficient.

ERRORS

The detector is a composite balance and deflection type instrument. Balancing the bridges and detectors eliminate a host of errors associated with an all deflection type instrument. However, errors can still occur in the deflection phase of the detector's operation. These errors are due to factors which cause the output of the detector to deviate from zero when there is no normal zone present.

The detector output can be affected in two different ways: by an unbalance in the bridge and by a change in the detector gains. Bridge unbalance is caused by either a change in the two bridge resistors or a change in the coil. The main factor affecting the resistors is temperature.

The mutual and self-inductances are relatively constant. However, in some coils, the magnetic forces cause small changes in the physical dimensions of the coil, making the inductances somewhat dependent on current.

The bridges are relatively insensitive to changes in resistance caused by temperature and to changes in the mutual or self-inductances. Bridge balance depends on ratios of the resistances and ratios of the inductance. That is to say, changes that affect adjacent arms tend to cancel. A discussion of errors due to changes in the detector gain requires a closer look at the composition of the gains. Typical of these gains are the two gains, K_6 and K_7 , of a detector associated with two self-inductance bridges, Equation 8. The gains contain ratios of mutual inductance to mutual inductance, self-inductance to self-inductance, and mutual inductance to self-inductance. The ratios of similar kinds of inductance change little compared with the total change in inductance. However, the way in which the ratios of dissimilar kinds of inductance change must be investigated for each pair of coils. If physical distortions caused by the magnetic forces change the mutual and self-inductances in the same direction, the change in gain is small. If the change is in opposite directions, the change in gain is relatively large.

CONCLUSIONS

A normal zone in inductively coupled superconducting coils can be detected using detectors that employ mutual and self-inductance bridges.

It is desirable that the coils be designed with the detector in mind. For a pair of coils, center taps are desirable, but additional taps can be used to increase the sensitivity of the detector and eliminate blind spots.

Although taps in both coils are helpful, a normal zone can be detected if only one of the coils has taps. It is also possible to detect a normal zone in a superconducting coil coupled to a coil that is not superconducting. The method can be extended to a number of coils, with or without taps, and with a mixture of superconducting and non-superconducting coils.

The bridges associated with the detectors are balanced and the detectors tuned during a steady state change of the coil currents. Balancing and tuning, to a large extent, eliminates errors. The remaining errors can be estimated if it is known how the bridge resistors and coil inductances are affected by outside factors.

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TWO INDUCTIVELY COUPLED SUPERCONDUCTING COILS EACH WITH ONE TAP. THE RESISTORS CONNECTED ACROSS THE COILS FORM BRIDGES WITH THE COILS, $R_b + R_t = R_b$, $L_b + L_t = L$ AND $M_b + M_t = M$

FIGURE 1.

TWO INDUCTIVELY COUPLED SUPERCONDUCTING COILS, ONE WITHOUT A TAP.

FIGURE 2

A SUPERCONDUCTING COIL ON THE LEFT COUPLED TO A NORMAL COIL ON THE RIGHT

FIGURE 3

THREE INDUCTIVELY COUPLED SUPERCONDUCTING COILS. FOR CLARITY NOT ALL OF THE INDUCTANCES ARE SHOWN. HOWEVER, THERE IS MUTUAL COUPLING BETWEEN EACH OF THE THREE COILS AND BETWEEN EACH PART OF A COIL AND THE OTHER TWO COILS. $M_{12} = M_{21}$ BUT IN GENERAL $M_{t12} \neq M_{t21}$.

FIGURE 4

VOLTAGES IN A PAIR OF COUPLED COILS DURING A VOLTAGE DRIVEN CHANGE IN THE CURRENT, AFTER THE TRANSIENTS HAVE DIED AWAY.

FIGURE 5

VOLTAGES IN A PAIR OF COUPLED COILS DURING A RAMP INCREASE IN CURRENT, AFTER THE TRANSIENTS HAVE DIED AWAY.

FIGURE 3

DETECTOR EQUATION	COIL QUADRANT			
	LEFT		RIGHT	
	LOWER	UPPER	LOWER	UPPER
EQUATIONS 8 AND 9	$i_1 r$	$\frac{-L_b}{L_t} i_1 r$	$-i_2 r$	$\frac{L_b}{L_t} i_2 r$
EQUATIONS 5 AND 10	$i_1 r$	$\frac{-L_b}{L_t} i_1 r$	$\frac{M}{L} i_2 r$	$\frac{-M_b}{L_t} i_2 r$

THE OUTPUTS OF DETECTORS FOR TWO SIMILAR COILS. THE TOP ROW IS FOR DETECTORS THAT USE $v_c - v_b$ AND HAVE EITHER TWO MUTUAL INDUCTANCE OR TWO SELF-INDUCTANCE BRIDGES. THE BOTTOM ROW IS FOR A DETECTOR WITH A MUTUAL INDUCTANCE BRIDGE IN ONE SIDE AND A SELF-INDUCTANCE BRIDGE IN THE OTHER SIDE AND A DETECTOR WITH TWO SELF-INDUCTANCE BRIDGES THAT USES v_2 .

TABLE 1