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Ion Temperature Drift Instabilities in a Sheared
Magnetic Field

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ABSTRACT

Results from the first particle code simulations of the ion-temperature-gradient-driven instabilities in sheared slab geometry are reported. In the linear stage of the instability, the results are in very good agreement with the theoretical calculations of the mode frequency, growth rate and radial mode structure. Ion energy transport caused by the instability is found to be the process primarily responsible for nonlinear saturation. Enhanced fluctuations associated with marginally stable eigenmodes have been observed.

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The recent neutral beam heating experiments in tokamaks have generated renewed interest in the ion-temperature-gradient-driven instabilities.¹ It is widely believed that the resulting $\gamma_i (= d(n T_i)/d(n))$ driven modes might be detrimental to confinement. The observation of the enhanced density fluctuations and the anomalous thermal conduction in the latest Princeton Large Torus experiments² underscores the point. In this letter, we present the first particle simulation results confirming the existence of unstable γ_i driven eigenmodes and their associated anomalous transport in a sheared slab geometry. The simulation has been carried out using a 2-1/2 dimensional electrostatic particle code,³ in which the adiabatic approximation for the electrons has been invoked.⁴ The measured frequency, growth rate, and mode structure for the linear stage of the instability are found to agree very well with the predictions from a linear theory which takes into account the detailed kinetic response of the ions. During the nonlinear phase, large ion energy transport caused by the unstable modes has been observed. This is apparently the mechanism primarily responsible for the nonlinear saturation of the instability. For a system where only marginally stable modes exist, we have also observed enhanced fluctuations whose frequencies and spatial structures match closely with the theoretical values.

The ion-temperature-gradient-driven instability in the local limit was first discovered by Rudakov and Sagdeev.⁵ It propagates in the ion diamagnetic direction and belongs to one of the two electrostatic drift branches - the other being the well-known branch associated with the electron drift waves. In recent years,

the basic formulation of the problem has been improved by various authors.^{1, 6, 7, 8} In the sheared slab limit, unstable eigenmodes are found to exist with the onset condition for the instability being $\tau_i > 1$.⁸ Such an instability threshold can be easily exceeded in many present day tokamak experiments. This is in contrast to the situation for the electron drift branch, where the eigenmodes are stable,^{9, 10} and the instability is believed to be dominated by transient fluctuations.¹¹

Let us first describe the relevant linear formalism for the mode in a sheared slab. Taking the electron response to be adiabatic, $(\Delta n/n)_e = e\phi/T_e$, and using the quasineutrality condition, $\Delta n_e = \Delta n_i$, the governing equation for the perturbed potential $\phi = \phi(x) \exp(ik_y y - i\omega t)$ can be written as

$$\begin{aligned} & \left(1 + \frac{\tau_i}{2} \frac{T_i}{T_e} + \frac{1}{2} \frac{T_i}{T_e} \frac{\tau_i}{\tau_i} + \frac{\tau_i^*}{2} \frac{\tau_i}{\tau_i} \right) \\ & - \alpha (b + 1/2) (v_{Te}^*/\omega) I_0(b) \exp(-b) \tau_i^{-2} Z(\tau_i) \\ & + \alpha (v_{Te}^*/\omega) I_0(b) \exp(-b) \tau_i^{-2} [1 + \tau_i^{-2} Z(\tau_i)] \phi(x) = 0. \end{aligned} \quad (1)$$

where $T_i = T_i(x)$ is the ion temperature, τ_i is its spatial average, $\tau_i = T_i/T_e$, $\omega_{Ti}^* = k_y v_{Ti} T_i / c / eB$, $\tau_i = -T_i'/T_i$, $\alpha = -n'/n$, $v_{Ti} = v_{Ti}/v$, $b = (-\partial^2/\partial x^2 + k_y^2) (T_i/T_e) v_s^2$, $v_s = (T_e/m_i)^{1/2}/\omega_{ci}$, $\tau_i = \omega L_s / \sqrt{2} k_y x v_{Ti}$, $v_{Ti} = (T_i/m_i)^{1/2}$, L_s is the shear length, I_0 is the Bessel function, Z is the usual plasma dispersion function.¹² The parameter $\alpha [= \kappa_T(t)/\kappa_T(t=0)]$, which will be used later to study nonlinear saturation processes,

is here taken to be unity. It is necessary to include the ion temperature variation in x in Eq. (1), because T_i may vary significantly within the radial extent of the mode under the simulation conditions. In the limit of $k_x \rho_{i1} \ll 1$, the normal mode equation of the form $\nabla^2 + Q(x, y) = 0$ can be obtained by the standard small argument expansion of the Bessel function terms in Eq. (1).⁷⁻¹⁰

The simulation code used for the present study is basically the 2 1/2-D (x, y, v_x, v_y, v_z) model developed by Lee and Okuda.³ However, thermal fluctuations associated with the electron motion may destroy the coherence of the weak ion drift waves, because the frequencies and the growth rates for these modes are extremely low. The problem is alleviated by incorporating into the Poisson equation the Debye shielding effects which are induced by the adiabatic electrons when responding to the ion density fluctuations.⁴ Thus, the Poisson's equation becomes,

$$-\nabla^2 \phi = k_{De}^2 [n_e(x)/n_0] = -4\pi e [n_i(x, y) - n_e(x)] \quad (2)$$

where $k_{De}^2 = 4\pi e^2 n_0 / T_e$ is the Debye wavenumber and n_0 is the average number density. Note that Eq. (2) is valid only when $k \cdot B \neq 0$. In the case of $k \perp B$, the adiabatic approximation breaks down and the shielding term should be ignored. With the introduction of Eq. (2) into the code, the electron dynamics can be completely neglected. However, the fluctuations associated with the ion motion, which is accounted for exactly in the code, remains unchanged. The average electron number density, $n_e(x)$ cannot be determined uniquely in the present scheme, and is

calculated using two basic models: 1) high electron mobility model where $n_e(x) = \langle n_i(x, y) \rangle_y$ or 2) low electron mobility model where $n_e(x) = n_e(x, t = 0)$.

The simulation has been carried out in a simple slab geometry. In the inhomogeneous x direction, the system has a length L_x and is bounded by two conducting plates where the electrostatic potential ϕ goes to zero. The inhomogeneity is represented by an exponential density profile, $n(x) = \langle n \rangle L_x \exp(-\kappa x) / [1 - \exp(-\kappa L_x)]$, and an exponential ion temperature profile, $T_i(x) = \langle T_i \rangle \kappa_T L_x \exp(-\kappa_T x) / [1 - \exp(-\kappa_T L_x)]$. These profiles give a constant $L_n (= 1/\kappa)$ and $L_T (= 1/\kappa_T)$ throughout the system. There is no electron temperature gradient. In the y direction, the system is homogeneous and periodic with a length L_y . The magnetic field is described by $B(x) = B_0(\hat{z} + \hat{y}x/L_S)$. Since the rational surface is at $x = 0$, we are simulating a half space in which only modes with $\phi(0) = 0$ are allowed. Using the grid size Δ and ω_{pi}^{-1} as basic units, the simulation parameters are $L_x \times L_y = 64\Delta \times 32\Delta$, $\langle n \rangle = 16/\Delta^2$, $T_e/\langle T_i \rangle = 1$, $\omega_{ci}/\omega_{pi} = 1$, $\lambda_{De}/\Delta = 1.5$, $\omega_{pi}\Delta t = 0.5$, (particle size)/ $\Delta = 1.5$, $\rho_s/\Delta = 1.5$, $\kappa_T \rho_s = 0.105$, $L_S/L_T = 28, 70, 112$, $n_i = 1.4, 4, \infty$. The system has a set of discrete wavelengths of $k_y \rho_s = 0.3m$ where $m = 0, \pm 1, \pm 2, \dots$, and $L_x/\rho_s = 42.7$. The corresponding drift frequencies are $\omega_T^*/\omega_{pi} = 0.0315m$.

Using the parameters described above, we have solved Eq. (1) numerically with a WKB-shooting code.¹³ The n_i -eigenmodes are found to exist and their frequencies and growth rates for

$n = 1, 3$ (odd) modes are shown in Figs. 1(a) - 1(b). In the calculation, we have used $k_y \rho_s = 0.3$, which corresponds to the first k_y mode ($m = 1$) in the simulation and is usually most unstable. Results qualitatively similar to those of Fig. 1 have also been obtained from Eq. (1) when the spatial variation in T_i is neglected. However, they are less meaningful when compared with the simulation results. Figure 1(a) indicates that $n_{icritical} \approx 1.4$ for a weak shear. This is in good agreement with the recent calculations.^{1,8} It is interesting to note that, for $n_i < 0$, unstable eigenmodes associated with the ion drift branch still exist. As shown in Fig. 1(b), the instability can also be stabilized by shear, where $(L_s/L_T)_{critical} = 28$ for $n_i = 4$. However, the growth rate becomes rather insensitive to shear when the shear is weak. In addition to the ion modes, Eq. (1) also give a set of highly damped eigenmodes in the electron drift direction.¹⁴

Let us first discuss the simulation using the high electron mobility model. Since there is no ambipolar potential build-up due to diffusion in this model, it is most ideal for comparison with the linear theory. The frequency spectrum and the time history of the potential, $e\phi/T_e$, for the case with $n_i = \infty$, $L_s/L_T = 112$ and $k_y \rho_s = 0.3$ are shown in Fig. 2. The data are taken at $x/\rho_s = 16.7$ where the mode amplitude has its maximum. A band pass filter with the width of ω_T^* has been used to obtain the time history. The corresponding theoretical values ($n = 1$) are also shown in Fig. 2. These results indicate that the wave, which indeed propagates in the ion diamagnetic direction, follows

very closely the prescription given by the theory. The instability saturates in about 4 e-folding times with an amplitude of $e\phi/T_e \approx 8\%$. The presence of $n = 3$ mode has also been detected. However, in the linear stage, it is dominant in a region further away from the rational surface and has a smaller amplitude. When a stronger shear is used, its amplitude is further diminished. The comparison between the theoretical values and the measured frequencies and growth rates for all the cases studied are shown in Figs. 1(a) - (b). The agreement between them is quite good. The oscillation levels for higher k_y modes are much lower and will not be discussed here. For cases of marginally stable modes ($L_S/L_T = 112$, $\eta_i = 1.4$, and $L_S/L_T = 28$, $\eta_i = 4$), transient growths, in which there is no clearly defined linear stage, has been observed. They are caused by the re-distribution of the equilibrium fluctuation energy due to the presence of the eigenmodes.¹⁵ In addition, we have detected the oscillations with $\omega/\omega_{Ti}^* \approx 0.3 - 0.7$, presumably, corresponding to the eigenmodes of the electron drift branch. They are usually damped out completely in a few oscillations at the beginning of the simulation. The linear mode structures ($n = 1$) for $L_S/L_i = 112$, 28 and $\eta_i = 4$ obtained from the WKB-shooting code are shown in Fig. 3(a) - (b) along with the simulation results, which are calculated in terms of the correlation length for the characteristic frequency.¹² Note that, for the marginally stable case, total time record has been used for the calculation. Again, the agreement is quite good. The modes have more of a standing wave structure and their widths are proportional to L_S/L_T . The

deviation from the theory as shown in the middle of the system in Fig. 3(a) is the result of the oscillations of $n = 3$ mode.

The saturation levels for these instabilities are shown in Fig. 4(a). For the marginally stable modes, the peak amplitude of $e\phi/T_e \sim 1 - 2\%$ is quite substantial. When the modes are linearly unstable, the saturation amplitude, ranging from $e\phi/T_e \sim 2 - 8\%$, depends more on the linear mode width than the growth rate. The saturation is caused by the quasilinear diffusion of the ion temperature profile. Figure 4(b) gives the results for the two most unstable cases. Using the resultant temperature profiles to calculate the term α in Eq. (1), we have found that they are indeed stable configurations (i.e., $\gamma \rightarrow 0$). The heat diffusion coefficient $D_{\perp}$, which can be calculated by $\partial T_{\perp} / \partial t = D_{\perp} \partial^2 T_{\perp} / \partial x^2$, is found to be $0.016 \omega_{pi} \Delta^2$ for $L_S/L_T = 112$. This measurement is about four times bigger than the value obtained from the existing quasilinear theory based on the local approximation.¹⁵

The results for the low electron mobility model are similar to those given above. However, associated with this model is the ambipolar ($k_y = 0$) mode produced by the temperature diffusion, which induces a large velocity shear in the y direction and, in turn, further stabilizes the instability.

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Fig. 1. Frequencies and growth rates for $n=1$ (solid line) and $n=3$ (dashed line) modes and the simulation results.

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 Fig. 2. Time history and frequency spectrum at $x/s = 16.7$ for $U_s/U_T = 112$ and $n_1 = 4$.

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 Fig. 3. Theoretical (solid line) and simulation (dashed line) mode structure for $n_i = 4$ and (a) $L_S/L_T = 112$, (b) $L_S/L_T = 28$.

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Fig. 4. (a) Saturation levels and (b) the resultant ion temperature profiles.