AMBIPOlar ELECTRIC FIELDS AND TURBULENCE STUDIES
IN THE WISCONSIN LEVITATED TOROIDAL OCTUPOLE

BY

CHARLES JESSE ARMENTROUT

A thesis submitted in partial fulfillment of the
requirements for the degree of

DOCTOR OF PHILOSOPHY

(Physics)

at the

UNIVERSITY OF WISCONSIN-MADISON

1977

DISCLAIMER

This document contains information of work performed by an agency of the United States Government. Neither the United States Government nor any agency thereof nor any of their employees makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.

DISTRIBUTION OF THIS DOCUMENT IS UNLIMITED

DISCLAIMER

This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency Thereof, nor any of their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.

DISCLAIMER

Portions of this document may be illegible in electronic image products. Images are produced from the best available original document.

ABSTRACT

AMBIPOLAR ELECTRIC FIELDS AND TURBULENCE STUDIES IN THE WISCONSIN LEVITATED TOROIDAL OCTUPOLE

Charles Jesse Armentrout

Under the Direction of Professor D. W. Kerst

Detailed studies of hot ion plasmas ($T_i > T_e$) in the poloidal field octupole show that the ambipolar electric field which is perpendicular to the flux surfaces is well explained by the observed properties of the microturbulence structures in the plasma.

The radial electric fields decay at the rate which is the sum of the decay rates of the ion temperature and the inverse density scale length. The potential in the plasma is related to the density by the ambipolar relationship with the diffusion and mobility coefficients treated as unrelated phenomenological parameters. This relationship between the electric field, the ion temperature, and the density is seen to hold for a variety of operating conditions. It is shown that ambipolarity need not require vector equality between the ion and electron fluxes at every point in space and a particle flux imbalance may well exist. This imbalance has important implications for the experimental measurement of various quantities.

The turbulence structure has been measured by correlation techniques which are carefully described. In these experiments, signals were studied which are aperiodic in time and space, short lived compared to the decay times of the bulk plasma parameters, short ranged

compared to the machine size, and are therefore classified as micro-turbulence structures. The resulting spatial and temporal correlation functions (CFs) are well fitted to a Gaussian function and the associated correlation lengths or times are the half width at half maximum of the CFs. The correlation length is measured to be the ion gyro radius for the hot hydrogen plasma and somewhat less for the helium plasma. The measured correlation time is comparable to the disassembly time of such a cell from the influence of the magnetic gradient and curvature drifts.

The observations suggest a model in which the cell-like structures are composed from the electron distribution. Electron diffusion results from the vortex motion of an electron in the cell field until the cell decorrelates. The resulting electron mobility coefficient is zero. The ion-cell interaction results in both diffusion and mobility coefficients. This model explains the observed ambipolar electric field and gives an estimate of the diffusion coefficient which is within a factor of four of the previously measured value.

The amplitude of the cells is larger than predicted for spontaneous random thermal fluctuations. Suggestions have been made that such cells could be driven in amplitude by long wavelength coherent oscillations. Such waves are always seen to coexist with the turbulence structures and are identified as hot ion flute modes.

Donald M. Kerst

ACKNOWLEDGEMENTS

The author would like to express his appreciation to the many people who have helped in this work and in particular to Dr. D. W. Kerst for his patient support of the research project reported here.

I would like to especially thank Dr. K. R. Symon for the many useful discussions which guided my work and my analysis on the proper path. I would also like to thank Dr. G. A. Emmert for his interest, especially in the basic problems involved in the analysis of plasma turbulence.

The many helpful discussions with Dr. J. R. Greenwood on the observed diffusion processes in the octupole are greatly appreciated. His development of the computer interface system made this work feasible. I would also like to express my appreciation to Mike Zarnstorff for the development of the REBEL/BASIC software used to acquire and analyze the majority of the data presented below. The many discussions with Jim and Mike about the care and feeding of an acquisition system have been quite enlightening.

I would in particular like to thank Paul Nonn for his invaluable aid in the design and initial construction of the special probes and circuits used. I also wish to give special thanks to John Laufenberg for advice and aid in the daily effort to keep the machine and the computer running, and for aid in the construction and development of most of the electronic circuits employed.

TABLE OF CONTENTS

CHAPTER I: THE WISCONSIN LEVITATED TOROIDAL OCTUPOLE

A: Description of the Octupole	1
(i) Mechanical and Electrical Construction	1
(ii) The Magnetic Field	4
(iii) Flux Coordinates	11
B: Plasma	15

CHAPTER II: DATA ACQUISITION AND ANALYSIS TECHNIQUES

A: Probes	27
(i) Physical Construction	27
(ii) Simple Probe Theory	31
(iii) Effects of Sheath Expansion	34
(iv) Double Probes	37
(v) Surface Contamination	44
1. Saturation Current Measurements	44
2. Floating Potential Measurements	45
(vi) Vacuum Cleanable and Expandable Probes	51
B: Acquisition Techniques	54
C: Profile Acquisition	61
(i) Internal Averaging	64
(ii) Reference Scaling	65
(iii) Pulse Averaging	67
D: Fluctuation Acquisition and Analysis	68
(i) Normalization of Fluctuation Data	68
(ii) Analysis of Fluctuation Data	76
1. The Correlation Function	78
(a) Length of data train	79
(b) Pulse averaging of C	79
2. Coherent Signals with Slow Frequency Change	80
3. Coherent Signals with Random Frequency Variations	83
(a) Shared turbulence	83
(b) Unshared turbulence	83
4. Coherent Signals with Randomness in Amplitude	84
5. Rapid Random Turbulence	87
6. Random Turbulence of Arbitrary Lifetime	87
(iii) Correlation vs. Fourier Transform	87

CHAPTER III: BULK PROPERTIES OF THE PLASMA

A: Temperature Measurements	95
(i) The Electron Temperature	95
(ii) The Ion Temperature	108
B: Plasma Profiles	108
C: The Ambipolar Density: Potential Relationship	129
D: Analysis of the Profile Data	132
(i) Ambipolar Relation for V_f and I_s	132
(ii) Parameterized (I,V) Plots	133
(iii) $T^*(x,t)$	151
(iv) T^* and T_1	169
E: Errors in Experimentally Estimated Parameters	179
(i) T^* Errors	179
(ii) D Errors	180
(iii) D_a Measurement Errors from an Improper D_a, Γ Relationship	182
(iv) Implications for Striped Particle Collector Measurements	182

CHAPTER IV: TURBULENCE STUDIES

A: Motivation for the Microturbulence Search	187
(i) The Turbulence Explanation for the Observed D_1	187
(ii) Zero Frequency Turbulence: Two Interpretations	188
(iii) Particle Transport in the Two Pictures: Convective Cells and Diffusive Cells	190
(iv) Preliminary Study for Short Range Turbulence	193
B: Experimental Methods for Turbulence Measurements	196
(i) Measurement Techniques	196
(ii) Analysis Techniques	199
C: Observations of the Turbulence Structure	206
(i) Correlation Length	206
1. Hydrogen, $\psi = 6.75$	206
2. Helium, $\psi = 6.75$	218
(a) Apparatus	218
(b) Temperature	219
(c) Results	219
3. Hydrogen, $\psi = 3.4$	226
4. Summary, λ_c	249
(ii) Correlation Time	249
(iii) Amplitude of the Cell	256
D: Kinetic Description of the Empirical Model	257
(i) Approximate Scalings	257
(ii) Electron Transport	258
(iii) Ion Transport	263
1. The kinetic equation	263
2. The ion-cell collision frequency	265
3. The ion particle flux	269
(iv) Calculations from the Model	270
1. Correlation time	270
(a) τ_c from the model	271
(b) Analysis of the data	272
2. Electron-cell encounter rate	276
3. Estimate for D_1	277

CHAPTER IV: TURBULENCE STUDIES (cont.)

E:	Implications	281
(i)	Evaluation of D_1	282
(ii)	T^*	283
(iii)	D_a	284
(iv)	T_e	285
(v)	T_i	287
F:	Turbulence Observations from Other Devices	288
(i)	Berezin, Buligihsky, Vildjuna: PIG Discharge	288
(ii)	Mazzucato: A.T.C.	289
(iii)	Hamberger, Sharp, Lister, Mrowka: TORSO	290
(iv)	Surko, Slusher: A.T.C.	291
(v)	Okabayashi, Arunasalam: FM-1	292
G:	Possible Mechanisms for Cell Formation	292
(i)	Thermal Level Fluctuations	293
(ii)	Mode Coupling as a Turbulence Driving Mechanism	295
	1. Mima, Hasagawa	295
	2. Okuda, Cheng	297
H:	Summary of Chapter IV	300

CHAPTER V: COHERENT FLUCTUATIONS IN THE HOT ION PLASMA

A: Introduction	305
(i) Preliminaries	305
(ii) Observation Conditions	306
(iii) Normalization of the Signals	307
B: Rising Field Data (10 msec)	308
(i) Acquisition Methods	308
(ii) Fluctuations Near ψ_c	311
1. Amplitude	311
2. Frequency	311
3. CF measurements	322
(iii) Toroidal Phase Velocities	323
1. Measurements	323
2. Interpretation	330
3. T* Measurements	333
(iv) Radial Phase Velocity	334
C: Peak Field Data (24 msec)	337
(i) Acquisition Methods	337
(ii) Fluctuation Observations	337
1. Correlation map	337
2. Toroidal phase velocity	346
(iii) Analysis	357
1. Toroidal phase velocity	357
2. Mode identification	364
3. Test of the flute mode	367

CHAPTER I: THE WISCONSIN LEVITATED TOROIDAL OCTUPOLE

A: Description Of The Octupole

(i) Mechanical And Electrical Construction

The Wisconsin levitated toroidal octupole is shown in the diagram of Figure 1. The machine is composed of four current carrying rings inside a 9 cubic meter toroidal vacuum chamber of noncircular cross section and is constructed of high conductivity aluminum. It measures 2.5 meters between the top lid of the chamber and the laboratory floor and 4 meters in outside diameter. The four internal rings shape the magnetic field into an octupole pattern which is modified by the presence of the conductive walls. The current in the rings is inductively driven by the iron core transformer, the rings being four single turn secondary windings linking the core in parallel.

The device is the second generation successor of the smaller octupole located in the Wisconsin physics department. It was designed partly to discover what portion of the plasma loss is due to the supports which hold the rings in position. The machine was built to do this using two major modifications of the design of the first generation device. The outer and inner walls are indented to enclose the rings partially. This allows the rings to be positioned so that the net vertical and horizontal forces on them due to the wall currents are zero. Because of this free-force positioning, the rings merely rest on their supports which need hold them only against gravity. During the field pulse, the supports may be removed from ring contact for a total of about sixty milliseconds to keep the rings from falling more than a

Figure I 1

The Wisconsin Levitated Toroidal Octupole

few millimeters. It requires 20 msec to move the supports approximately 10 cm below the rings (this is outside the confinement region), and 20 msec to reinsert them. The total time they are held out, then, is also 20 msec. During the support free period, the rings receive a gravitational impulse which the supports must absorb after reconnection. The ring-support system then undergoes a series of damped oscillations between the final resting position and a point very near the wall. This process is called levitation because at the largest possible magnetic fields, the wall currents help support the rings in position against the gravitational field and the final cushioning oscillations are small. Levitation may be programmed to occur at any time in the field pulse or not at all and in the levitated experiments reported in the later chapters, all supports are fully removed at the time plasma is injected. Table 1 summarizes the major electrical and mechanical features of the device. More complete descriptions are available elsewhere.^{1,2,3}

(ii) The Magnetic Field

The machine was designed to have the capacitor bank discharge into the transformer to produce a half sine magnetic field pulse 43 msec long. It then was modified to allow the transmission line to be "crowbarred" (CBR) shortly after peak field, that is, shorted at the capacitor bank. The resulting magnetic field decay rate varies according to position but $B/\dot{B}$ is everywhere longer than 70 msec.⁴ It is well known^{5,6} that sometime after peak field in the half sine case the Poynting vector at the surface of the rings reverses direction and flux

TABLE I 1

OPERATING PARAMETERS OF THE LEVITATED OCTUPOLE

Capacitance	0.048 F
Maximum Voltage on Capacitors	5 kV
Energy of Pulse	0.6×10^6 J
Maximum Core Flux	0.72 Wb
Inductance L : $L = N^2 L_o$, $N = 90$	$L_o = 0.6 \mu\text{H}$ (calculated)
Volume of Vacuum Region	8.6 m^3
Volume inside ψ -critical at 20 msec	7.7 m^3
Ring Minor Radius	8.9 cm
Inner Ring Major Radius	0.99 m
Outer Ring Major Radius	1.79 m
Wall Thickness	5 cm
Half Sine Period (Poloidal Field Only)	43 msec

VALUES AT MAXIMUM CORE FLUX

	<u>Wall</u>	<u>Inner Ring</u>	<u>Outer Ring</u>
Current*	1.4×10^6 A	0.5×10^6 A	0.25×10^6 A
B max (at surface)*	6. kG	12. kG	5.5 kG
B min (at surface)*	1. kG	5.3 kG	3.4 kG

* Approximate values

begins to reemerge from the surface. This occurs between 31 and 32 msec in the field pulse. This effect has a major influence on the bulk motion of the plasma as will be shown in chapter III. Plasma profiles of density and potential for the sine field prior to about 30 msec are similar to those seen for the CBR case. The plasma evolution in the sine field becomes dominated after 32 msec by the motion of the magnetic field.

Figures 2 and 3 show the magnetic field lines in the octupole and the lines of constant magnetic field strength at 25 msec, shortly after the CBR. These diagrams are cross sections of the torus and the field lines close in the plane of the cross section. When rotated about the toroidal axis, the closed field lines generate closed toroidal flux shells which are symmetric about the central axis. In some regions of the torus, all flux shells enclose a single ring, in others, they all enclose two and in yet others all four rings. The magnetic lines which generate the boundaries between these regions are called the separatrices and are the dotted lines of Figure 2. An interesting and important observation is that each separatrix passes through the zero magnetic field strength region. These field nulls act as non-adiabatic scattering centers for any charged particle whose center of gyration about a magnetic field line is within its gyro radius of a separatrix. When the gyration center moves along the field line and enters the field null region, it loses all information about its previous magnetic moment and can scatter into any of the magnetic flux regions of the machine. The final destination depends only on the phase of its orbit about the magnetic field as it enters the field null region. This

7

Figure 1 *P*

Cross section of the octupole showing the magnetic field lines at 25 msec into the field pulse. The field lines are numbered with the value of ψ assuming a peak core flux of 10 units.

25 msec

139 cm

Figure 1 3

Cross section of the octupole showing $|B|$ at 25 msec.

101 kG
25 msec

process has been discussed by others^{7,8} and is partially responsible for maintaining continuity of the plasma parameters (density and temperature) across the various regions.

(iii) Flux Coordinates

It is convenient for the purposes of the following discussion and for the work of the later chapters to employ flux coordinates which are non-linear in the usual Cartesian " x, y, z " spatial coordinates but which provide a geometry-independent description of the processes occurring in a magnetic device. Flux coordinates are fully described elsewhere.⁹ For the purpose of orientation, envision two axes for a torus: a linear "toroidal" axis which is the symmetry line of rotation used in Figure 2 to produce the torus from a cross section, and a circular "mid circle" axis which runs through the center of the "donut" and encircles the toroidal axis. In Figure 2, the mid circle would be the locus of the central field null under rotation about the toroidal axis. The "mid cylinder" is the figure generated by moving the mid circle parallel to the toroidal axis. The toroidal position is provided by the azimuthal angle ζ measured about the toroidal axis along the mid circle. The poloidal position (the short way around the torus, in the plane of the cross section of Figure 2) is specified by the poloidal magnetic scalar potential χ which is measured along a field line in the direction of $\underline{B}$. The third coordinate is the total magnetic flux ψ between the toroidal axis and point in space being identified (including the core flux). It may be shown⁹ that ψ is a constant along a field line and is the coordinate used to label the flux shells described above. It serves as a

radial coordinate for the set. The coordinate directions are oriented by the fundamental relationship $\underline{B} = \nabla\chi = \nabla\psi \times \nabla\zeta$. ψ is zero inside the rings (if superconducting) and increases for flux shells successively further from any ring. Since $\hat{\chi}$ is parallel to $\underline{B}$, $\hat{\zeta}$ must be opposite the direction of the ring current.

The field lines of Figure 2 are labeled with the calculated value of ψ . The convention used for the calculation is the assumption that the core contains 10 units of flux at peak field. The units of ψ are named for Dory, who originated the convention. The conversion from the tabulated value ψ_t to the actual value is $\psi = A\psi_t$ where A is .1 times the actual core flux ϕ_c . ϕ_c is easily calculated to be $(LC)^{1/2} V_c/N$ where N is the number of turns on the transformer. L , C , and N are obtained from Table 1 and A is $1.52 \times 10^{-2} V_c$ Wb/d where V_c is the initial capacitor bank voltage in kV. Drake¹⁰ has measured ϕ_c and gives the experimental A to be $(1.45 \pm .04) \times 10^{-2} V_c$. Figure 4 plots $\psi_t(x)$ v.s. position in centimeters across the outer bridge which is the region between an outer ring and the outer wall. ψ_t is shown for various times in the half sine pulse. The $\psi_t(x)$ for the CBR field is assumed to be given by the 25 msec curve. At 25 msec, the separatrices are near 5 cm, the flux lines ψ_g that just graze the walls are near 10 cm, and the flux line ψ_c which separates the region of stably confined plasma from the outer region near the wall of unstable plasma is near 9 cm. The arrow heads indicate the direction these surfaces move in time.

There will be occasions in later chapters when a representative value for the magnetic field is desired for a given flux surface.

Figure I 4

The flux function $\psi_{(t)}$ v.s. position in the outer bridge region, between the ring and the wall. $\psi_{(t)}$ is calculated assuming that the peak magnetic flux in the core is 10 units.

Figure 5 plots the actual magnitude of the magnetic field in kG v.s. ψ at 25 msec for an initial capacitor bank voltage of 2.5 kV. Several average values are given, the maximum and minimum values along the line, as well as the values for the outer bridge region (between an outer ring and the wall) and the mid cylinder. The average of B around a field line is $\langle B \rangle_l = \oint B \cdot d\mathbf{l} / \oint dl$ where the integration is along the field line and $d\mathbf{l} = \hat{\chi} d\chi / B$. The average of B along a tube of constant flux is $\langle B \rangle_{\psi} = \oint B \cdot d\mathbf{l} / \oint dl$. It has been shown elsewhere¹¹ that the flux tube average of a quantity A is $\langle A \rangle_{\psi} = (\oint A \cdot d\mathbf{l} / B) / (\oint dl / B)$ so $\langle B \rangle_{\psi} = \oint dl / \oint dl / B$. The values for the integrals are from computed tabulations.¹²

B: Plasma

There are a number of different mechanisms available for generating plasma in the octupole but in general they fall into one of two classes: (a) hot ion, relatively low density plasma with low neutral pressure, and (b) cold ion, higher density plasma with high neutral background pressure. The principle source for the hot plasma is the small coaxial gun of the type described by Marshal¹³ and others,^{14,15} mounted on the end of the drift tank outside the toroidal chamber (Figure 1). The gun fires its plasma into the torus through a 10 cm aperture and its neutrals are initially deposited in the drift tank. A nude ion gauge in the torus indicated¹⁶ that the neutral gas enters with about a 10 msec rise time. The ambient background pressure in the torus is typically 1 to 2×10^{-7} Torr, just prior to injection of the hydrogen plasma and the initial electron-neutral collision time is

Figure I 5

Representative values of the magnetic field B , in kG, v.s. the ψ of Figure I 4 assuming that the main capacitor bank was charged to 2.5 kV. $\langle B \rangle_v$, is the average of B over the flux tube, $\langle B \rangle_l$ is the average around the field line.

about 2 msec. A helium cooled cryo panel was installed for the majority of the work reported here so the principal initial background constituent is H_2 , as seen by the residual gas analyser installed on the torus. The ion temperature is about 30 eV one millisecond after injection and the electron temperature is 8 eV. Table 2 presents the important plasma parameters for this plasma. The details of their measurement are presented in later chapters.

The cold, dense plasma is produced by either one of the microwave sources or by a somewhat larger gun which discharges directly into the torus.¹⁷ Both plasmas are associated with neutral pressures of at least the upper 10^{-6} Torr region with electron-neutral collision times of less than 100 μ sec. The initially hot ions from the large gun are known to drop in temperature below measurability within the first few milliseconds. The principal advantage of these plasmas over the hot plasma from the small gun is the high particle density, 10^{10} to 10^{12} cm^{-3} , and the high degree of reproducibility. Microwave plasma typically exhibits shot-to-shot variation in its parameters of less than 5% while the drift tank gun user must accept 10 to 20% variation. The high density readily allows probe measurements of most parameters such as potential and density, while the low density plasma measurements have problems because of large debye length and high sheath impedance (see Chapter II). The principal disadvantages are the ion temperature and high collisionality. Thermonuclear plasmas will be nearly collisionless due to their extremely high temperature. In addition, finite ion gyro radius effects should be expected in these plasmas. Neither of these operating regimes are available to cold, high density plasma

TABLE I-2

WISCONSIN LEVITATED TOROIDAL OCTUPOLE

OPERATING PARAMETERS (RINGS LEVITATED AT INJECTION)

	PEAK VALUE IN MACHINE ONE msec AFTER INJECTION	E-FOLDING TIME OF PEAK VALUE
B	5 kG (POLOIDAL ONLY)	-70 msec
NEUTRAL PRESSURE	1×10^{-7} TORR PRINCIPALLY H ₂	+10 msec
HYDROGEN PLASMA DENSITY	8×10^9 cm ⁻³	-18 msec
ELECTRON TEMPERATURE	8 eV (SPATIALLY UNIFORM)	-30 msec
ION TEMPERATURE	30 eV (SPATIALLY UNIFORM)	-11 msec

Table I-2

and it is for this reason that hot ion plasma was chosen for the investigations presented in the later chapters.

Effects of collisionality may be investigated in a controlled manner by puffing neutral gas into the torus just prior to injection. The reproducibility of the gun is another problem. Figure 6 presents three series of oscilloscope records of plasma shots taken to show the gun behaving in different manners. These records were taken weeks apart. The traces are of the plasma current drawn to a single tipped probe biased at -45 V to repel electrons and located on the separatrix in the lower outer bridge region. It is shown in chapter II that such current is proportional to ion density times the square root of the temperature. All three sets are triggered at the beginning of the field pulse and sweep at 5 nsec per division across the photographs. The machine operating parameters are the same in all three cases: B_p only, supported rings, V_c of 2.5 kV, and the torus pressure of about 1×10^{-7} Torr. Figure 6(a) shows the optimal behavior of the gun. The records of ten shots are seen to overlay one another within a 5% deviation. In addition, one levitated trace is shown and demonstrates the fact that levitation causes a factor of 2 increase in the drawn current. An acceptable plasma variation is shown in (b) which is the overlay of 5 shots. This is a typical variation in the gun parameters and appears to be due to variations in the plasma density. Figure 6(c) shows an unacceptable case where the behavior of the gun was pathologically bad. Such behavior is presumably caused by wide independent variation of density and temperature. The gun must be removed from the system at this time and cleaned. Visual inspection of the gun elec-

Figure 1 6

Ion saturation current drawn by the same probe biased to -45 V at ψ_B , 225° lower outer bridge region. Triggered on field pulse, horizontal sweep at 5 msec/division. Gun injection of hydrogen at 25 msec into CBR field. $V_c = 2.5$ kV, $P_T \approx 1 \times 10^{-7}$ Torr.

(A) 10 supported ring traces, 1 levitated trace, 100 μ A/D. This is optimal plasma from gun with newly polished electrodes.

(B) 5 acceptable traces, 50 μ A/div, rings supported.

(C) 7 unacceptable shots, 50 μ A/div, rings supported. After removal of gun, electrodes showed discoloration and severe arcking patterns.

A

B

C

trodes shows surface scarring due to improper arcking of the plasma in the gun. Inspection of the gun surfaces after operation in mode (a) shows no such surface problems. The operation modes of (b) and (c) may generally be restored to that of (a) by careful polishing of the electrodes before reinstallation. Mode (c) has also been caused by external arcking of the plasma between the gun's outer electrode and the conductive drift tank wall. The distribution of plasma inside the octupole for this case is documented in Chapter III. An extender was provided for the outer electrode which moved the muzzle of the gun further from the drift tank lid and appears to solve the external arcking problem. For reproducibly smooth behavior of the gun, it was found necessary to remove and repolish the electrodes every several weeks, after about a thousand shots. This record compares well with what the investigators who develop gun design call reproducible.¹⁸ Such guns are specially assembled, tested for 50 to 200 shots, then disassembled and modified (and cleaned!). Plasma from such guns are reputed "always" to look like Figure 6(a).

Figure 7 is included here for later reference. It plots the gyro radius ρ of a 20 eV proton as a function of the tabulated ψ_c for the octupole field when V_c is 2.5 kV. Shown are the flux tube and line average of ρ which are equivalent to the averages of B^{-1} and the specific values for ρ in the bridge and along the mid cylinder.

Figure I 7

Representative values of the ion gyro radius $\rho_1(\psi)$, in mm,
v.s. ψ for $T_1 = 20$ eV hydrogen, $V_c = 2.5$ kV.

REFERENCES FOR CHAPTER I

1. J. Steben. J. Appl. Phys. 43, 1211 (1972)
2. H. K. Forsen, D. W. Kerst, R. A. Breun, A. J. Cavalo, J. R. Drake, J. C. Sprott. Proceedings of the Fourth European Conference on Controlled Fusion and Plasma Physics (Comitato Nazionale Energia Nucleare, Rome, 1970), p. 24
3. I. N. Sviatoslavsky, N. C. Lien. Hydraulics and Pneumatics (Sept. 1971) p. 86 and PLP-275 (1969)
4. G. A. Navratil. University of Wisconsin Plasma Physics PLP-629
5. K. Evans. University of Wisconsin Plasma Physics PLP-464
6. J. Schmidt. University of Wisconsin Plasma Physics PLP-181
7. M. G. Rusbridge. Plasma Physics 13, 977 (1971)
8. B. Lehnert. Plasma Physics 17, 501 (1975)
9. C. J. Armentrout. University of Wisconsin Plasma Physics PLP-667
10. J. R. Drake. University of Wisconsin Plasma Physics PLP-512
11. C. J. Armentrout. University of Wisconsin Plasma Physics PLP-470
12. A. Butcher Ehrhardt. University of Wisconsin Plasma Physics PLP-703
13. J. Marshal, Phys. Fluids 3, 134 (1960)
14. C. E. Bush. Ph.D. Thesis. University of Wisconsin (1972)
15. C. E. Bush, H. K. Forsen, H. de la Fuente, R. Harder, G. Kuswa. University of Wisconsin Plasma Physics PLP-146
16. D. A. Brouchous. University of Wisconsin Plasma Physics PLP-648
17. G. A. Navratil. Ph.D. Thesis. University of Wisconsin (1976)
18. D. Y. Cheng. Bul. Am. Phys. Soc. 21, 1175 (1976)

CHAPTER II: DATA ACQUISITION AND ANALYSIS TECHNIQUES

A: Probes

(i) Physical Construction

The basic tools for acquiring the data presented in later chapters are high and low impedance probes. These have been discussed in detail many places.^{1,2} Figure 1 shows the diagrams of the three probe constructions used in this work. The tips of all three probes are smaller than an ion gyro diameter (>6 mm) so the ion flux to the tips may be assumed to be isotropic. The high Z probe consists of a tungsten cylindrical tip sealed with epoxy in a glass tube. A 20 M Ω resistance was placed just behind the tip and shielded by a copper foil electrically connected to the tip. Grounded shield coaxial cable was connected to the resistor and led to a "probe box" which provided a frequency-compensated voltage divider and an amplifier. Full description of the circuits used is found elsewhere.² This probe was used to acquire the early injection data of Chapter V.

The type 1 low Z probe consists of a cylindrical copper tip epoxied to a hypodermic needle which is in a teflon sleeve. The needle is electrically connected to ground through the stainless steel main body of the probe. The tip is connected via a wire to grounded shield coaxial cable which leads to the outside world. For frequencies below 500 kHz, it was found that the same response could be obtained for this probe attached to a Tektronix 10x, 10 M Ω attenuating probe as for the 20 M Ω high Z probe with the balance box and amplifier just described. Since this probe has the additional advantage of presenting a smaller

Figure II 1

Diagram of three basic probe types used in the experiments.

HIGH Z

LOW Z 1

LOW Z 2

profile to the plasma, it and the type 2 probe are used in the majority of the experiments.

The type 2 probe is similar but the tip is a small sphere on the end of a thin ceramic stalk. Both platinum and gold were used for the tips but platinum has proven to be most useful because of its high melting temperature and ease of handling. In addition, pure platinum may be purchased while gold is always alloyed, usually with copper. The gold-tipped probe has proven to suffer from problems with surface contamination (see below). The ceramic stalk leads to the hypodermic needle of the type 1 probe.

It is important to bring the coaxial shielding as close to the tip as possible to avoid capacitative coupling to plasma fluctuations which are distant from the tip. R. Richards has reported³ that for the same D.C. plasma, a major difference was observed between the measured potential signals from two probes which differed by the presence of coaxial shielding along the stalk of one. The shielded stalk allowed him to obtain a consistent phase relationship between potential and density fluctuations, which was not obtainable from the unshielded probe.

For all probes of type 2, the ceramic stalk is at least 1 cm long, however. Large obstacles generate turbulence which affects the measurements of a second probe. It is for this reason that the high Z probe was abandoned and the low profile type 2 probe is preferred for all fluctuation measurements.

(ii) Simple Probe Theory

A number of problems arose when these probes were used. As indicated below, they are the result of the expansion of the plasma sheath as a function of probe tip bias, and of surface contamination of the probe tip itself. These problems may be understood using "simple probe theory" which ignores the presence of a magnetic field and the expansion of the sheath. Full details may be found elsewhere.⁴

Probe theory treats the ion or electron collection by an object embedded in the plasma. If the object is at any potential other than the plasma's potential, V_p , a space charge layer called the sheath forms which shields the plasma from the object's potential V . "Simple" probe theory assumes that the ions and electrons have a Maxwellian velocity distribution and that the sheath's thickness is independent of $\Delta V = V - V_p$. This discussion will use the thin sheath approximation where the sheath area is the same as the probe tip area.

Figure 2(a) shows the equivalent circuit for such a probe with impedance R_m to the laboratory ground. For $\Delta V < 0$, all ions incident on the sheath are collected and accelerated⁴ to the ion sound speed $c_i = (eT'/2\pi m_i)^{1/2}$. Here, m_i = ion mass, $T' = T_i$ for $T_i > T_e$ or is T_e otherwise. All temperatures are expressed in volts. The sheath retards the electrons so $R_s \equiv R_e$ is the electron current impedance which grows as $\exp(|\Delta V|/T_e)$. For a tip which is deeply enough biased, $|\Delta V| \approx 3 \dots 5 \times T_e$, the current drawn has a saturated value given by the isotropic ion flux

$$I_{io} = \frac{1}{4} n_i A \left(\frac{T'}{2\pi m_i} \right)^{1/2}. \quad [\text{II-A-1}]$$

Figure II 2

(A) Equivalent circuit for a single tipped probe.

(B) Characteristic plot of the current $I(v)$ drawn to a probe tip v.s. the tip's bias voltage.

(A)

(B)

For $\Delta V > 0$, the situation is reversed and $R_s = R_i$ is the ion current impedance which grows as $\exp(\Delta V/T_i)$. Figure 2(b) shows the characteristic graph of probe bias versus current drawn. When no net current is drawn by the probe, the tip bias is called the floating potential, V_f . An isolated object will charge to V_f and draw no further net current. It is easy to use $I_{i0} = I_{e0} \exp(V_f - V_p)/T_e$ to show that (v_e = the electron thermal speed)

$$V_f = V_p - T_e \ln(v_e/c_i) \quad [\text{II-A-2}]$$

$$R_s = \left(\frac{T_e}{neA} \right) \left(\frac{2\pi m_i}{eT_e} \right)^{1/2} \quad [\text{II-A-3}]$$

The relation for the sheath impedance, R_s , evaluated at $V = V_f$, is given by Spratt.⁵

(iii) Effects of Sheath Expansion

More realistic probe theories^{6,7,8} show that the expansion of the sheath modifies the saturated (constant) value of I_{i0} in [1] into a nearly linear function of $(V_f - V)$. Rudmin has verified⁹ that the predicted shape of the characteristic graph for a probe is very nearly that which is actually observed.

One interesting feature is that the floating potential can vary by as much as 3 or 4 T_e across flux space at any instant. Such plasma profiles are presented in Chapter III. This will cause misinterpretations if simple probe theory is used to construct density profiles from the current drawn by a probe biased to repel electrons. Figure 3 illustrates this point. Consider the exaggerated case of a constant

Figure II 3

Characteristic plots for a probe with a fixed bias V_b at two positions in a plasma with a potential gradient but no density gradient. The operating points of the probe are marked by an x.

density profile (i.e., one with no density gradients), but with a large potential gradient. Two identically shaped characteristic probe curves are drawn for the two different positions in the plasma. A fixed probe bias is assumed and the two currents drawn by the probe are marked by the x's. An observer who uses simple probe theory to obtain a density profile as is commonly done^{10,11} would obtain a density gradient proportional to the potential gradient. One method of circumventing this problem is to bias the probe more deeply into the ion collection branch. A deep bias of $5 \times V_f$ for the octupole would be about -150 V. Before the data from this biased probe could be used, however, one would need to show that the flux tubes were not depleted of ions, especially in the private flux region where the field lines are short.

(iv) Double Probes

A second method, and the one adopted here, would be to use a low impedance double probe¹² as illustrated in Figure 4. Suppose initially $V = 0$. Then both tips 1 and 2 would be at V_f . As V increases, tip 2 moves up the probe characteristic curve drawing ions. Tip 1 rides down the exponential curve collecting electrons until the electron current to 1 equals the ion current to 2. These final points are controlled by T_e , T_i , and n , but are independent of V_f . Figure 4 shows the circuit used in all double probe measurements for the isolated microammeter schematically shown in Figure 5. There is $10^{13} \Omega$ isolation between the diode ammeter circuit and the optically driven differential amplifier circuit. For the circuit shown, the transconductance $I_{\text{through}}/V_{\text{out}}$ is an essentially constant value from 5 μA to 10 mA. Figure 6 shows

Figure II 4

(A) Circuit schematic for double probe operation.

(B) Diagram illustrating the principle of operation.

(A)

(B)

Figure II 5

Circuit diagram of an optically isolated microammeter for the circuit of Figure 4 A.

$P, Q = \text{MCT-2 Opto-isolator}$

Figure II 6

(A), (B) Single tipped probe characteristic for a hot ion gun injected plasma 6 and 11 msec after injection (labeled .031 and .036 on graph). Each point is the average of three to five points. Vertical units: $76.3 \mu\text{A}/\text{division}$. Horizontal units: -45 V to +45 V.

(C), (D) Double probe characteristic for the same shots which were used to generate A. Vertical units: $23.2 \mu\text{A}/\text{division}$. Horizontal units: -45 V to +45 V.

B

D

A

C

double and single probe characteristics taken for the same set of pulses taken at ψ_s in the B_p supported octupole. For the density profiles of Chapter III, the floating double probe has a relative bias of 45 V which places the ion collecting tip well into the repelled electron branch of the curve.

(v) Surface Contamination

The existence of contaminants on the surface of a probe tip generates a wide range of problems. Many of the difficulties have been studied previously.¹³ Only those aspects of the problem will be discussed here which have a direct bearing on the studies presented in the later chapters. The faulty measurement of both density and floating potential profiles is probably the most obvious error caused by surface contamination.

1. Saturation Current Measurements: It is clear, even from the simple probe theory of [1], that a partial coating of the probe tip by an insulating material such as oxide will reduce the effective collection area A and therefore I_{i0} . Figure III-29 presents data taken with three separate gold-tipped probes of type 2. When those data were being acquired, the 45° probe initially measured an I_{i0} which was 1/3 the value of the other two. All three probes were removed, cleaned with alcohol, very lightly abraded, and then the spark of a hand-held Tesla coil was lightly played over the surface to break up any surviving large areas of oxide. The probes were reinserted and all readings were within ten to twenty percent of each other. If a double probe is also to yield a reasonable estimate of n , both tips must have nominally

the same collection area. It is essential that the tips be clean.

2. Floating Potential Measurements: V_f may be measured by making the impedance, R_m , in Figure 2(a) much larger than the sheath impedance. Equation [3] shows that R_s can be very large for a low density hot plasma² and the criterion that $R_m \gg R_s$ may be difficult to meet. If $R_m \approx R_s$, the laboratory ground will be an electron source or sink (depending on the sign of V_f) for the sheath with an "output" impedance comparable to that of the plasma. The operating point for the tip will drift to the region where fewer (or more) plasma electrons are collected and the measuring instruments will no longer read V_f .

A contamination coating on the tip can only worsen the situation. Figure 7 shows the equivalent circuit for a probe tip with a surface coating, such as an oxide, whose impedance is R_c . The tip voltage measured by the experimenter is $V_1 = [R_m / (R_m + R_c)] V_f$, assuming that $(R_m + R_c) \gg R_s$. V_1 may be significantly reduced from V_f if the high resistivity contaminate is very thick. A factor of 2 increase in the magnitude of V_f has been observed after cleaning a badly oxidized (i.e., dark, cloudy surfaced) gold ball tip of a type 2 probe. It is pertinent here to re-emphasize that pure gold is very difficult to obtain because it is usually alloyed with copper which has severe oxidation problems. The best material for a probe tip is platinum, which is easily obtained pure, has a high melting point, and is quite non-reactive.

This contamination problem has important implications both for measurements of floating potential profiles, which are done with a single tip, and for direct measurements of the local electric field E

Figure II 7

Equivalent circuit for a probe tip with a surface contamination layer.

by subtracting V_f from two neighboring probes. Electric field measurements are especially suspect because R_c will certainly have different values for different tips. In the course of these studies no electric field has ever been measured which unambiguously retained its value when the tips were interchanged. Figure 8 shows the best subtracted measurements from the "self cleaning" platinum tipped expandable double probe described below. Two V_f signals A and B are connected to the (+) and (-) input terminals of a 1A7A differential amplifier. The amplifier--probe set-up had a 100:1 common mode rejection ratio, which is well below any error visible in the photographs. Figure 8(a) shows three shots. The top trace of each shot is A-B and the bottom is A. The tips A and B are aligned along the same field line. Figure 8(b) shows another three shots of the same electrical connections (A-B and A) but the tips have been rotated 180° with respect to the field line. The subtracted amplitudes are not the same in both cases for both orientations nor are the shapes the same, but they do at least reverse in sign. Gold and tungsten tips, cleaned as well as possible prior to insertion, when used in this manner have never been observed to reverse. It could easily be true that with a much higher plasma density, the tips might be plasma cleaned and such a measurement might well be meaningful. It should be stressed that this is the best electric field data taken during these studies. The cleaned surfaces of probes inserted through ports exposed to getter shine undergo continuous coating and E measurements must not be considered trustworthy. This is also true of the shot-to-shot reproducibility of a probe in the gettering region which is used to trace out a profile of either I_{10} or V_f .

Figure II 8

Test of V_f subtraction. A and B are oscilloscope traces from the experimental arrangement shown in C. A and B show the data from three successive machine pulses and have two traces for each pulse. Two high Z probes (10 M Ω , 100:1 CMRR) are separated by 1 cm along a field line ($\psi = 3.4$, lower outer bridge, 330°). The tips were made of platinum and were flame cleaned prior to insertion. The upper trace shows the difference signal between tips 1 and 2, and the lower trace shows the signal to tip 1. In A, the tips were arranged as shown in C, and in B they were rotated 180° (interchanged). The scope was triggered 5 msec after injection, the horizontal scale is 5 msec/division, and the vertical scale is .5 V/division.

A

B

C

(vi) Vacuum Cleanable and Expandable Probes

A special design was developed for probes meant to acquire quantitatively trustworthy data. From the above observations, this requires that the tip be perfectly clean when the data are taken. Due to previous usage, however, the quarter inch probe ports typically have their channels coated with vacuum grease which collects dust and grit. No probe may be guaranteed clean after insertion. Electron bombardment of tips prior to insertion into other devices has proven¹⁴ to be an effective cleaning technique since the ports could be maintained clean. Figure 9 illustrates how this technique was adapted to clean a probe tip inside the octupole vacuum chamber. Construction details are available elsewhere¹⁵ but in general the probe has a filament mounted at the end of a sliding narrow tube and inserted into a stainless probe body next to the platinum tip. The vacuum seal is made at the air end of the probe body. After the probe is inserted into the chamber, the filament is moved near the tip and heated with a 6 V power supply. The tip is biased to +400 V and draws several hundred milliamperes of electron current. When used in a glass test chamber, the tips glowed white hot. A 10 sec heating pulse has proven to be sufficient to allow the pressure gauge of the small test chamber to return to normal. The filament is pulled back from the tip into the end of the probe body and the data is taken. When used in the gettering region, this should be done every hour or so.

The debye length, λ_d , in the low density, hot plasma varies from half to one millimeter. A double probe should have its tips 5 to 10 λ_d apart to properly isolate the tips. A one centimeter separation was

Figure II 9

Diagram of a self cleaning probe.

chosen as a safe value and proved to be useful for fluctuation measurements but posed a problem since the ports are only $1/4$ " in diameter. Figure 10 is a photograph of a typical probe which solves the problem. Each platinum tip is part of a type 2 probe stem. The small tube has a double bend in it at the main stainless probe body and extends through spacers in the body, through the compressed viton pad at the air end and out into the laboratory. The small tubes may be rotated nearly 180° from the deployed position shown to the closed position with the small tips less than $1/4$ " apart. When the cleaning filament (not shown) is included in the unit, the entire self-cleaning expandable double probe will fit through the air lock. Figure 11 is a photograph of the main probe used to acquire the levitated data in Chapter III.

B: Acquisition Techniques

The majority of the results presented in this thesis require the systematic processing of a great many data records. Figure 12 shows a block diagram of the basic elements needed for computer management of the acquisition and analysis of data. The computer used is a Digital Equipment Corporation PDP-11/20 with 28 K, 16 bit words. The differential input analog-to-digital (A/D) signal converters used are three Biomation 610-B 6 bit, 256 word transient recorders and one Biomation 610 6 Bit, 128 word unit. These units are capable of digitizing at a 10 MHz rate and are synchronized to a common clock. The signal comes from the probe and is processed by the "probe box" which represents the electronic processing performed on the signal before it reaches the oscilloscope. The oscilloscope amplifier displays the transient signal

Figure II 10

Photographs of an expandable probe designed to acquire data for $\psi > \psi_g$ in the bridge. (A) Tip design, (B) The vacuum seal is made using a Cajon fitting at the air end of the probe.

Figure II 11

Photograph of an expandable self cleaning probe.

A

B

C

D

Figure II 12

Block diagram of the computer acquisition system.

Figure II 13

Comparison of data from an oscilloscope with its digitized image.

(A) Oscilloscope record: 1 V/div, 2 msec full scale horizontal sweep.

(B) Digitized image of A: 1.42 V/div, 2.56 msec full scale.

(C) Oscilloscope record: 1 V/div, 2 msec full scale horizontal sweep.

(D) Digitized image of B: 1.10 V/div, 2.56 msec full scale.

on its screen and drives it down the RG-62/U cable to the A/D station. The A/D is triggered once per machine pulse and the digitized data are read by the computer¹⁶ after each pulse.

The primary purpose of the computer is to acquire the data records and save them on paper tape for subsequent analysis. The time between octupole pulses is adequate, however, for most analytical purposes. The software I have written to acquire and analyze the data is called DATANAL and is described elsewhere.¹⁷ At present, DATANAL is capable of acquiring, saving, and displaying the data, and, within a few additional seconds, finding (i) the least squares best fit to the data, (ii) the correlation function¹⁸ between various data records, (iii) the Fourier transform of the data, and (iv) various other matrix operations useful in acquiring and analyzing data. Figure 13 shows a signal taken directly from the oscilloscope and its digitized image as acquired and displayed by the computer.

C: Profile Acquisition

As used in this thesis, a "profile" is any plot of data versus some parameter fixed throughout the pulse. Examples of such profiles are the graphs of floating potential from a probe versus position in the machine as seen in Figures III 8-13 and graphs of current drawn to a probe tip versus the bias on that tip. This latter type of profile has been used with DATANAL to get I-V plots for a probe in order to obtain computer measurements of T_e .

Figures I 6 and III 7 show a number of shots in the levitated octupole to demonstrate the problems involved in constructing a reason-

able profile. Two major problems are evident: (i) fluctuations or noise in the data signal, and (ii) shot-to-shot irreproducibility in the signal for the same profile parameter. The problems caused by (i) arise, for example, when one wishes to discover the bulk, "zeroth order," distribution and temporal evolution of some property, such as density or potential. This fluctuational uncertainty may be relieved by averaging the signal over a time interval which includes fluctuation periods. The inverse problem arises when the object of the study is the fluctuations themselves, as in subsection D. Problem (ii) may sometimes be helped by using a monitor probe and standardizing the data magnitude with it.

To get a better idea of what is happening, let the data be represented as $d_k(\underline{x}, t)$ for the k -th machine pulse at position $\underline{x}$ and time t . Suppose d_k may be written as

$$d_k(\underline{x}, t) = D_k P(\underline{x}, t) + \delta_k(\underline{x}, T_0, t) \quad [\text{II-C-1}]$$

where $P(\underline{x}, t)$ is the physical profile shape factor, D_k is the data amplitude which depends on such pulse parameters as the filling density or temperature. The fluctuating part of the data is $\delta_k(\underline{x}, T_0, t)$ where T_0 is the longest characteristic time of the oscillations. In the following paragraphs, three separate methods are discussed for making the profile independent of the per shot variability of the plasma.

(1) Interval Averaging

We might define δ as being the portion left over after d_k has been averaged over a time $T_d \gg T_0$:

$$\delta_k(\underline{x}, T_o, t) = d_k(\underline{x}, t) - \frac{1}{T_d} \int_{t-T_d/2}^{t+T_d/2} ds d_k(\underline{x}, s) \quad \text{[II-C-2]}$$

If $P(\underline{x}, t)$ changes appreciably in the time T_d (as it might) this is not satisfactory. A better method would be to curve fit a low order polynomial, $\bar{d}_k$, to the data in the interval and subtract that from the data. For a given position $\underline{x}$,

$$\bar{d}_k(\underline{x}, t) = \sum_{j=0}^M C_j(\underline{x}) t^j \quad \text{[II-C-3]}$$

$$\delta_k(\underline{x}, T_o, t) = d_k(\underline{x}, t) - \bar{d}_k(\underline{x}, t). \quad \text{[II-C-4]}$$

The $\{C_j\}$ are the coefficients of the least squares best fit polynomial of order M defined in the interval $(t - T_d/2, t + T_d/2)$. The l.s.b.f. polynomial should be used because the δ_k defined by [4] has a zero average in the T_d interval. Notice that if $M = 0$, [2] and [4] are identical. DATANAL uses the third order l.s.b.f. polynomial from subroutine CURVE for the values of $\{C_j\}$. In other words, the fluctuations may be practically removed by using $\bar{d}_k$ instead of d_k . Then [1] will be replaced by

$$\bar{d}_k(\underline{x}, t) = D_k P(\underline{x}, t) \quad \text{[II-C-5]}$$

with $\bar{d}_k$ found by [3].

(ii) Reference Scaling

The shot-to-shot variations in D_k may frequently be removed by

standardizing the signal of [1] or [5] with the signal from a data monitor whose profile parameters (such as position, impedance, bias, etc.) are not changed on any shot. This reference monitor is assumed to generate data of the same type as the data which are being profiled. On a given shot, the reference monitor data will also obey [5]:

$$\bar{d}_k^{(r)} = D_k P^{(r)}(\underline{x}, t) \quad [\text{II-C-6}]$$

The idea is to have previously averaged $\bar{d}_k^{(r)}$ over a number of shots and to have obtained $\langle d^{(r)} \rangle$:

$$\langle d^{(r)}(\underline{x}, t) \rangle = \sum_{k=1}^N \bar{d}_k^{(r)} / N = \langle D \rangle P^{(r)}(\underline{x}, t). \quad [\text{II-C-7}]$$

Now, on the k-th pulse, we can multiply [5] by $\langle d^{(r)} \rangle$ and divide by $\bar{d}_k^{(r)}$ and get, using [5, 6 and 7]

$$\langle d(\underline{x}, t) \rangle = \langle D \rangle P(\underline{x}, t). \quad [\text{II-C-8}]$$

This is the pulse independent zeroth order profile of the data d.

It is easy to misuse reference scaling. For example, the average amplitude for the floating potential, $\langle V_f \rangle$, and for the ion saturation current, $\langle I_s \rangle$, can not be the same quantities. I_s data must not be scaled by a V_f monitor because [8] would give a meaningless answer. The same applies to scaling I_s fluctuations with an I_s profile monitor, and so forth. In fact, fluctuation data should be reference scaled only by a fluctuation monitor of the same type. All of these will destabilize the data profiles being constructed. In fact, it is questionable whether V_f may be reference scaled at all, since V_f is the sum

of the plasma space potential and a term proportional to the electron temperature. If there is shot-to-shot variation in T_e , temperature gradients, or if T_e decays at a different rate than V_p then reference scaling must not be used. The rule to use is that data may only be standardized by a like-type reference monitor. Even then, if the k-th pulse's monitor varies too much from its average value, the physics might change and change the data profile factor $P(\underline{x}, t)$. It is a good second rule to always print out $(d_k^{(r)} - \langle d^{(r)} \rangle) / \langle d^{(r)} \rangle$ so that the pulse may be ignored if it is too far out of range. When applicable, however, reference scaling can be invaluable in smoothing profile data. It has significantly smoothed density profiles, I-V probe characteristics, and r.m.s. fluctuation amplitudes of both potential and density versus position. In the data reported in the later chapters, a $\pm 15\%$ acceptance interval was used whenever possible.

(iii) Pulse Averaging

Both of the averaging processes represented by equations [3] and [8] may be done quickly by the computer between shots and involve no additional effort by the experimenter. (The experimenter must initially find $\langle d \rangle$ by [7] but the number of shots need not be very large.) However, it is frequently impossible to completely average out the δ_k because it is impractical to set $T_d \gg T_o$. In that case both [5] and [8] have a small added fluctuating term which will affect the profile analysis. (Note that no assumption of smallness is made in [1].) If the result of [8] is averaged over several (K) pulses, the fluctuating component will (usually) phase mix to zero. The final profile, then,

is

$$d(\underline{x}, t) = \underbrace{\frac{1}{K} \sum_{k=1}^K}_{\text{pulse avg.}} \underbrace{\frac{\langle d(r) \rangle}{\bar{d}_k(r)}}_{\text{reference scaling}} \underbrace{\left[\sum_{j=0}^M C_j(\underline{x}; k) t^j \right]}_{\text{interval average}} \quad \text{[II-C-9]}$$

D: Fluctuation Acquisition and Analysis

The problem divides into two parts: (i) identification of the fluctuation component from the slowly varying "zeroth order" bulk portion of the data, and (ii) analysis of the data using correlation and Fourier transform techniques.

(i) Normalization of Fluctuation Data

As indicated in subsection C, any data signal is composed of a part which varies rapidly in time, and an offset which has a much slower variation, see [II-C-1]. The process of removing the slowly varying component from the fluctuating signal is consistently called "normalization" throughout this thesis. Although it is somewhat arbitrary how the two components should be separated, the choices available to the experimenter will make major differences in fluctuational quantities such as r.m.s amplitudes. It is important that the normalization method, once chosen, be applied consistently to all the data analyzed. For all the data presented here, the fluctuation is what is left after the time average of the signal is removed. This zero average property is the proper choice for the quantities used in the particle transport studies of Chapter V. The turbulence studies of Chapter IV will show that an amplitude offset is generated which might cause

interpretation problems. However, there is no clearly defined prescription for uniquely identifying a random turbulence signal so the above definition is still applied.

Using the notation of subsection C, let $d_k(\underline{x}, t)$ be the input data signal. This signal might well have been processed by an electronic filter with a "D.C." offset added by the filters, line drivers, etc. The fluctuation component of d_k will be written (following [II-C-4]) as $\delta_k(\underline{x}, T_o, t)$ where

$$\delta_k(\underline{x}, T_o, t) = d_k(\underline{x}, t) - \bar{d}_k(\underline{x}, t) \quad [\text{II-D-1}]$$

where $\bar{d}_k$ has the property of being the time average in some manner of d_k over an interval T_d which is much greater than T_o , the longest time variation of δ_k . Following the argument after [II-C-2], $\bar{d}_k$ is not just the simple average of d_k . δ_k ultimately is to be time correlated and Fourier transformed in order to obtain all of the information available. As described in the following paragraphs, the correlation function retains all of the Fourier components of the original data and any slow time variation not removed from d_k will appear in the Fourier spectrum and quite possibly dominate it. Figures 14-16 show the effect of a "D.C." ramp and offset on the analysis of a fluctuation data record. If the ramp and offset are too large they will obscure the fluctuation analysis. For this reason, DATANAL (ref. Section II-B), uses the least squares best fit of an order M polynomial for $\bar{d}_k$ in equation [1]. If $M=0$, $\bar{d}_k$ is the simple average of d_k and any zero frequency offset will be removed. An $M=1$ fit would in addition remove a linear ramp, and an $M=2$ fit would remove a constant curvature in

Figure II 14

(A) Data trace as acquired by the computer.

(B) The same trace after the best fit linear function has been subtracted. Both data are .96 msec long and the vertical units are 59.6 mV per inch.

Figure II 15

(A) Auto correlation function of 14 A.

(B) Auto correlation function of 14 B.

Figure II 16

(A) Real part of the Fourier transform of 15 A.

(B) Same for 15 B. The low f component implies that a second or third order function should have been used to normalize 14 A.

that ramp. Since $M=3$ essentially removes a single oscillation period from the data, it is obvious that high order M must be used only with great care if the fluctuation to be analyzed are not themselves to be damaged. For example, suppose the input data is 256 μsec long and has had all frequencies below 10 kHz electronically filtered from it. An $M=3$ curve fit would numerically remove all frequencies present below 5 kHz and is therefore not an unreasonable normalization. As explained below, good numerical calculation of a correlation function requires that eight to ten cycles of the fluctuation be present in the data record. For this case, the $M=3$ normalization should offer no problem in data destruction.

(ii) Analysis of Fluctuation Data

The principal information desired from any fluctuation data are the average amplitude of the signal, its characteristic times and lengths, and its relative motion with respect to the observer. The data are said to be coherent if the Fourier spectrum is sharply peaked in time and space and to be turbulent if it is not.

For coherent data with well-defined oscillations, this information set is the period and wavelength of the oscillations averaged over time and distance intervals which are long with respect to their averaged values. In addition, the coherence (correlation) time and length of the signal is needed as a measure of the rate at which the period and wavelength change. The phase velocity of the signal is the ratio of the wavelength to the period.

For turbulence data, the relevant information is the lifetime

(correlation time) and velocity of each characteristic spatial length in the signal. The observer also needs the average rate at which the probe encounters each turbulence length in the signal. This indicates whether the turbulence occurs in well separated bursts and is spatially and temporally isolated, or whether it is dense, with the occurrence of the turbulence signals overlapping in time and space. In practice, it is very difficult to obtain this entire set of information.

If the data are oscillations at a single pure frequency, then direct inspection of the data record itself will yield all the necessary information without further analytic effort. Unfortunately, most plasma data usually indicate the presence of multiple coherent frequencies, of random variations (for even a single frequency component) of the frequency, phase and amplitude, or of a dominant non-coherent turbulence component. In these cases, the required information may be extracted from the data by first finding the auto or cross correlations of the data record with itself or similar records, and then Fourier transforming that function in time or space. The correlation function will be defined below. Its purpose is to ensemble average random quantities in the manner to be described. The amplitude, frequency and phase of the signal may then be found from the Fourier transform. It should be noted that this is not the order in which such calculations are occasionally performed.^{19,20} The reason for the present method is that the Fourier transform is a stochastic function and is susceptible to noise, whereas the correlation function is not. This point will be further discussed at the end of this section.

The following six paragraphs define the correlation function and

its response to various sorts of randomness in the input data. Since the first five points to be discussed have been considered in detail in PLP-480, the pertinent problems and principal results will be merely sketched here.

1. The Correlation Function: Let $F_1(t, z_0)$ and $F_2(t, z_0 + z)$ be two data records which have been obtained between the times t_1 and t_2 and at the positions z_0 and $z_0 + z$. The correlation function in time and space $C(F_1, F_2; T; z)$ is defined as^{21,22,23}

$$C(F_1, F_2; T; z) = \frac{1}{T_0} \int_{\bar{t} - T_0/2}^{\bar{t} + T_0/2} F_1(t, z_0) F_2(t + \tau, z_0 + z) dt \quad [\text{II-D-2}]$$

where $\bar{t} = (t_1 + t_2)/2$ and T_0 is the integration time interval. The relative time shift τ between F_1 and F_2 ranges between τ_m where, if the data time interval $T_D = t_2 - t_1$, $\tau_m = (T_D - T_0)/2 > 0$. The theoretical correlation function is the limit of C as T_0 approaches infinity. The convolution, $\bar{C}$, is defined as $\bar{C} = T_0 C$ in [2], or, to be precise, as the limit as T_0 grows without bounds. The correlation C of coherent oscillations is a finite number, whereas $\bar{C}$ is infinite. However, C is zero for a single non-repeated "blip," whereas $\bar{C}$ has a finite value. Notice that [2] does not perform a direct spatial averaging of the data. For steady state ("stationary") fluctuations, the best definition of the correlation function is the ensemble average of the shifted product of two data records. For this type of data, the time average of [2] is equivalent to an ensemble average and the above definition of C is correct. However, C is actually independent of $\bar{t}$ and z_0 only for

stationary data. For nonstationary data, even the concept of frequency runs into problems²⁴ unless the time variations are "slow." See paragraph (2), below.

If F_1 and F_2 may be Fourier decomposed as $F_1(t) = \sum F_{1k} \cos(\omega_k t)$, $F_2(t) = \sum F_{2k} \cos(\omega_k t + \phi_k)$, then

$$C(F_1, F_2; \tau) = \sum (F_{1k} F_{2k} / 2) (\cos[\omega_k \tau + \phi_k] + er_k). \quad [\text{II-D-3}]$$

If the error term er_k is negligible, each component reaches its maximum value at $\tau_k = -\phi_k / \omega_k$. An important theorem is that

$$C(F_1, F_2; \tau) = C(F_2, F_1; -\tau). \quad [\text{II-D-4}]$$

(a) The error term may be shown to be $\leq 1/(\omega_k T_0)$. To keep this term relatively small, T_0 should be eight to ten times larger than the longest period in the data train. This, of course, reduces the significance of the error only for the extrema of the k -th component; the zero of that component is poorly calculated.

(b) Fortunately, there is a second method of reducing the error. It may be shown that er_k is proportional to sine functions of the phase of F_1 and F_2 with respect to t_1 , the beginning of the data record. If C is calculated for N different data records and averaged together to form $\langle C \rangle = \sum C_j / N$, er_k will phase average to zero no matter what $\omega_k T_0$ is. Since this averaging procedure is equivalent to ensemble averaging, $\langle C \rangle$ is always the quantity that should be used whenever possible. This is not always experimentally feasible, however. Small shot-to-shot changes in the plasma parameters may place the fluctuation on significantly different points of its dispersion curve with each

machine pulse, and a sequence of shots will trace out that region of the dispersion relation. $\langle C \rangle$ can not be formed and accuracy may be obtained only through the use of long data records. The BIOMATION 610-B A/D transient recorder described in section B is barely adequate for the requirements if accuracy in both phase and frequency is desired.

2. Coherent Signals With Slow Frequency Change: Consider a single component of [3] only. Suppose that the frequency for F_1 is $\omega(1;t) = \omega_0 + \omega_1(t - \bar{t})$ where ω_0 is the instantaneous frequency at the middle of the data record. F_2 is the same as F_1 but shifted in time by τ_0 . That is, $F_2(t)$ has for its frequency $\omega(2;t) = \omega_0 + (t - \bar{t} - \tau_0)$. It is straightforward to show that the correlation function for this component is

$$C(F_1, F_2; \tau) = \left[\frac{F_1 F_2}{2} \cos(\omega_0 \tau) \right] \frac{\sin(\omega_1 T_0 \Delta \tau)}{\omega_1 T_0 \Delta \tau} \quad [\text{II-D-5}]$$

with $\Delta \tau = \tau - \tau_0$ and $\omega = \omega_0 + \omega_1(\tau - \tau_0)$. The term in square brackets is the expected C for a signal with a constant frequency ω_0 since $\phi = -\omega_0 \tau_0$ is the phase given by [3]. If the integration time T_0 is very large, C approaches a delta function at $\tau = \tau_0$. Figure 17 shows an example of this for data from the octupole. The plasma was injected 5 msec after the start of the multipole field and the data is taken 5 msec later (43° into the sine pulse). Notice that the diffraction envelope of [5] changes signs at $\tau = \tau_0 \pm \pi/\omega_1 T_0$. This well accounts for the otherwise puzzling effect that the amplitude of C first decreased

Figure II 17

(A) Two data records, 871 and 872, which show the effect of a slowly increasing frequency.

(B) The cross correlation between the two records. Notice the phase inversion at $\tau = -3.5 \times 10^{-4}$ and $+2.5 \times 10^{-4}$ sec.

A

C(871, 872) BATCH(12, 12)

B

to a minimum then increases again, and that the phase of C apparently changes at the minimum.

3. Coherent Signals With Random Frequency Variations: On an oscillation-to-oscillation basis, the period of a fluctuation usually varies randomly about some average value $2\pi/\omega_0$. This turbulence in frequency is averaged by C if the data interval is large enough so that the integration samples the ensemble of all probable frequencies, or if C is averaged over many separate pulses to achieve the same ensemble averaging. If the sample length is too small then C for a single shot is quite sensitive to the turbulence. In order to calculate this effect, assume that the probability that at time t the frequency is in the interval $(\omega, \omega + d\omega)$ is $N \exp[-(\omega - \omega_0)^2 / 2\sigma^2] d\omega$. The average frequency is ω_0 , N is the unity normalization constant and σ is the frequency variance (r.m.s. deviation of ω from ω_0). The problem naturally divides into two subparts depending on whether the turbulent signal is the same for F_2 as for F_1 but shifted by τ_0 , or whether the turbulence in F_2 is independent of F_1 .

(a) Shared turbulence: Auto correlations fall under this category. If $F_2(t)$ is just $F_1(t)$ shifted by τ_0 , then each component of [3] becomes

$$\langle C(F_1, F_2; \tau) \rangle = \left[\frac{F_1 F_2}{2} \cos(\omega_0 \Delta \tau) \right] \exp \left[\frac{-\sigma^2}{2} \Delta \tau^2 \right] \quad [\text{II-D-6}]$$

where, again, $\tau_0 = -\phi/\omega_0$.

(b) Unshared turbulence: F_1 and F_2 are averaged over the ensemble separately and, for $\sigma T_0 \gg 1$, $\Delta \tau \ll T_0$:

$$\langle C(F_1, F_2; \tau) \rangle = \left[\frac{F_1 F_2}{2} \cos(\omega_0 \Delta \tau) \right] \frac{\sqrt{\pi}}{\sigma T_0} \exp \left[\frac{-\sigma^2}{\tau} \Delta \tau^2 \right] \quad [\text{II-D-7}]$$

It is easy to mistake the exponential envelope of [6] or [7] with the "diffraction" envelope of [5]. C must extend far enough in τ to assure that the cosine amplitude does not increase again.

4. Coherent Signals With Randomness In Amplitude: Suppose that the Fourier amplitudes of F_1 and F_2 are random functions of time but that the frequency is constant. Then [3] becomes

$$C(F_1, F_2; \tau) = \sum_k \left[\frac{\int F_{1k}(t) F_{2k}(t+\tau) dt}{T_0} \right] \frac{\cos(\omega_k \tau + \phi_k)}{2} \quad [\text{II-D-8}]$$

The amplitude of C is just the average of the product of the correlation time shifted amplitudes. If the average is performed over too small a sample, the non-regular amplitudes can exaggerate or depress the net amplitude of C . Figure 18 shows an example of this accidental enhancement of the correlation peaks. The integration time for C is half the data record and for F_1 is centered in the middle of the record as shown by the dotted box. The integration window for F_2 is time shifted through the entire record. When the window is shifted as shown by the dotted box, the indicated data peaks overlap and enhance the marked correlation peak. There are at least three ways to avoid this problem. The input records may be very large, sometimes a difficult problem for a pulsed device.²⁵ The correlation function may be calculated twice and [4] used to form the net C :

Figure II 18

Two data records are shown in (A) which lead to the heightened peak in (B) which is marked by an arrow. The integration intervals in the records which are used to form the correlation function are shown for $\tau = 0$ by dotted lines. When $\tau = 15 \times 10^{-5}$ sec, the peak indicated by the arrow in 131 overlaps the indicated peak in 132. The data is actually well correlated near $\tau = 0$ as may be visually confirmed by viewing the data from the bottom of the page, at a shallow angle.

A

C(131, 132) BATCH(12, 12)

$$C(F_1, F_2; T) = \frac{1}{2}[C(F_1, F_2; T) + C(F_2, F_1; -T)]. \quad [\text{II-D-9}]$$

In addition, the net C used may be the average of the C calculations for many successive shots.

5. Rapid Random Turbulence: Suppose incoherent spikes appear in the data with no apparent time width. This is the appearance of turbulence with lifetimes shorter than the data sampling period. Suppose that the probability that the amplitude is between $(a, a+da)$ is $N \exp(-a^2/2A^2)$. This assumes that the average amplitude of the noise is zero. The noise correlation is a spike whose amplitude is A^2 which is added to the coherent C at $\tau = \tau_0$.

6. Random Turbulence Of Arbitrary Lifetime: This is a much more serious problem than that posed by the preceding paragraph because the entire central peak of C will be modified. Discussion of this, however, will be delayed until Chapter IV where the analysis techniques for turbulence are presented along with the results of the investigation.

(iii) Correlation v.s. Fourier Transform

It is occasionally stated^{21,26} that the correlation function (CF) and Fourier transform (FT) are completely equivalent functions: the CF defined over the time-space domain, and the FT defined over the frequency: wave-vector domain. This is due to a misuse of the Wiener-Khinchine theorem²⁷ for the case of turbulent signals as has been discussed elsewhere.²⁸ Physically, however, the differences in stability to randomness in data may be readily understood. The CF is a sequence of the time averages of the product of two data records. Each

value in the sequence is the average of a different time shift between these records. For stationary data, this is equivalent to ensemble averaging. The FT, however, is a curve fitting routine. It has been shown²⁹ that the Fourier integral is the amplitude which provides the least squares best fit of a sinusoid to a data train. Thus, if the signal is sufficiently turbulent and lasts for a finite length of time, the algorithm will fit a frequency component with non-zero amplitude to the noise. This situation is a turbulence generalization of the usual²⁶ process of aliasing, which means curve fitting a non-existent high (finite) frequency component to a low (zero) frequency component which has not been sampled sufficiently often.

Interpretation of an FT is also complicated by the phenomenon of "leakage." Even for a pure frequency in a finite data record, the structure of the FT amplitude in frequency is the same as the diffraction structure in space of a pure frequency passing through a single slit. There exist large side lobes to the pattern which make resolution of different frequencies quite difficult. People who apply the FT to data²⁰ usually multiply the data by an artificial amplitude function which suppresses the side lobes. However, when the FT is applied to the CF of data with turbulence, the turbulence itself has modified the amplitude of the coherent component (see [5], [6], and [7]) in a manner similar to the Hanning amplitude popularly used²⁶ to suppress leakage. In principle, one could measure the strength of the turbulence by comparing the size of the side lobes with the diffraction amplitude expected, although in practice this is usually very difficult. Figure 19 A, B, C, and D show a data record, its auto CF and the comparison

Figure II 19 A,B

A data trace is shown in (A) and its auto CF in (B).

Figure II 19 C,D

The FT of B is shown in C and the two frequencies are well isolated. The FT of the data in A is shown in D, which is the real component of the complex FT.

19 A

19 B

19 C

19 D

between the FT of the raw data and the FT of the CF. The CF-FT sequence is much easier to interpret than the direct FT of data.

In summary, then, the Data-CF-FT sequence is the one which should be used since the ensemble averaging of the CF will stabilize the resulting FT. In addition, the FT will retain the phase information contained in [3]. The FT for a given component may be written as $A \exp(i \phi_k)$ and A^2 may be found from the power spectrum which is the sum of the squares of the real (Re) and imaginary (Im) components. The phase angle is $\phi_k = \tan^{-1}(\text{Im}/\text{Re})$ at the peaks in the power spectrum. The information from the FT will be the best fit A , ϕ , and ω from the entire CF, whereas these can be extracted from the CF alone only at the central peak and therefore with a high degree of error. The value normally²¹ called the "power spectrum" is the real component of the FT of the auto CF of data. In general, this value is $(\text{Re}^2 + \text{Im}^2)^{1/2}$ at the peak in the usual power spectrum.

REFERENCES FOR CHAPTER II

1. F. F. Chen. Plasma Diagnostic Techniques, R. H. Ruddlestone, S. L. Leonard, ed. (Academic Press, N. Y., 1965) p. 113
2. J. C. Sprott, University of Wisconsin Plasma Physics PLP-88
3. R. K. Richards. Ph.D. Thesis, University of Wisconsin (1976)
4. W. Lochte-Holtgreven, ed. Plasma Diagnostics (Wiley Interscience, N. Y., 1968)
5. J. C. Sprott. Review of Scientific Instruments 37, 897 (1966)
6. J. G. LaFramboise. UTIAS No. 100 (University of Toronto, Toronto, 1966)
7. J. R. Sanmartin. Phys. Fluids 13, 103 (1970)
8. J. S. Chang, J. G. LaFramboise. Phys. Fluids 19, 25 (1976)
9. J. W. Rudmin. J. Appl. Phys. 47, 4833 (1976)
10. J. R. Drake, J. R. Greenwood, G. A. Navratil, R. S. Post. Phys. Fluids 20, 148 (1977)
11. J. R. Greenwood. Ph.D. Thesis, University of Wisconsin (1975)
12. E. O. Johnson, L. Malter. Phys. Rev. 80, 58 (1950)
13. The following papers have treated the problems of V_f or I_s measurements.
 V_f measurement problems are presented in the following two papers:
 J. C. Sprott, D. E. Lencioni, J. A. Schmidt. PLP-135 (1967)
 J. C. Sprott. PLP-175 (1968)
 Explanations attempted in the following problems:
 G. L. Schmidt. PLP-292 (1969) Problem solved by heating tips.
 G. W. Kuswa. Ph.D. Thesis (1970) Explanation of problem due to work function of tip material.
 I_s problems due to surface contaminants were seen by
 J. Barter. PLP-521 (1973)
 J. W. Rudmin. J. Appl. Phys. 47, 4833 (1976)
14. W. C. Guss. Ph.D. Thesis, University of Wisconsin (1976)
15. C. J. Armentrout. University of Wisconsin Plasma Physics PLP-727
16. M. C. Zarnstorff, J. R. Greenwood. Bul. Am. Phys. Soc. 21, 1080 (1976)

17. C. J. Armentrout. University of Wisconsin Plasma Physics PLP-481
18. C. J. Armentrout. University of Wisconsin Plasma Physics PLP-480
19. D. E. Smith, E. J. Powers, G. S. Caldwell. IEEE Transactions on Plasma Science PS-2, 261 (1974)
20. Y. Amagishi. Institute of Plasma Physics Technical Report IPPJ-T-26 (1976)
21. G. Bekefi. Radiation Processes In Plasmas (John Wiley & Sons, Inc., N. Y., 1966)
22. I. P. Shkarofsky. Proceedings on the Symposium on Turbulence of Fluids and Plasmas. XVIII (Polytechnique Press, Polytechnique Institute of Brooklyn, N. Y., 1968) p. 289
23. E. B. Hooper. Plasma Physics 13, 1 (1971)
24. L. Mandel. Am. J. Phys. 42, 840 (1974)
25. J. A. Schmidt. Ph.D. Thesis. University of Wisconsin (1975)
26. E. O. Brigham. The Fast Fourier Transform (Prentice-Hall, Inc., Englewood Cliffs, N. Y. 1974)
27. N. Wiener. Acta Math. 55, 117 (1930)
28. C. J. Armentrout, K. R. Symon. University of Wisconsin Plasma Physics PLP-726
29. G. Birkhoff, G.-C Rota. Ordinary Differential Equations (Blaisdell Publishing Company, Waltham, Mass., 1962) Ch. XI

CHAPTER III: BULK PROPERTIES OF THE PLASMA

A: Temperature Measurements

(i) The Electron Temperature

The electron temperature, T_e , has been measured using the technique¹ of sweeping the bias of a Langmuir probe (single tipped, low impedance ... see II A) from a negative value of $3 \dots 5 T_e$ below V_f to a positive voltage of about the same amount. If the bias sweeps through its full range much faster than the plasma properties change, then the operating point of the probe will move along the probe's characteristic curve of Figure II 2(b). By sweeping the probe linearly in time, the probe characteristic may be displayed on an oscilloscope screen or digitized by a transient recorder. However, there are two principal difficulties with this scheme.

Firstly, one can not usually place the entire characteristic on a screen or in a 6 bit digitizer and retain the accuracy in the ion saturation branch needed for the processing described below. This is because the ratio of electron to ion saturation currents is the ratio of electron thermal speed to ion sound speed² which for $T_i \leq T_e$ is $(m_i/m_e)^{1/2} \approx 42$. One must usually be content to display the curve from deep into ion "saturation" to somewhere near V_p . Figure III 1(a) shows this data and its ramp for plasma at ψ_s .

Secondly, one may not be allowed to sweep the probe fast enough. Fluctuations occur at the edges of the plasma (near the rings and walls) where the density is quite low. These are waves with between

10 and 30 kHz frequencies and an r.m.s. amplitude between 20 and 50% the time averaged density. On the other hand, at these low densities, the plasma frequency is on the order of a megahertz. Symon has shown³ that as a signal changes on times approaching $1/f_p$, the debye length expands without bounds and a signal at $1/f_p$ would drive all the plasma on the flux shell (assuming density is constant on the shell). The ramp, then, should be quicker than 10 μsec to be "much faster" than the changing plasma density, but should last longer than 10 μsec to be "much slower" than $1/f_p$. Figure 1(b) shows one of the worst cases where the 100 μsec ramp is in a roughly 30 kHz wave near ψ_c . Nevertheless, the main body of the plasma is much higher in density and quieter and the swept probe measurements yield reasonable values.

Figure 2 shows the circuit used to generate the ramp. The ammeter is the same as in Figure II 5. Most advanced probe theories which account for sheath expansion with probe bias (section II A) predict that for potentials below V_p , the current drawn may be written as approximately

$$I(V) = I_{i0}(V) - I_{e0} \exp[(V - V_p)/T_e] \quad [\text{III-A-1}]$$

$$I_{i0}(V) = I_o + I_1(V_f - V) \quad [\text{III-A-2}]$$

where V is the (negative) tip bias and ion current is assumed to be positive. I_o is the ion current drawn at $V = V_f$ when the net current drawn is zero. The proper technique is to fit $I_{i0}(V)$ to the ion saturation branch, subtract it from the total current everywhere below V_p , and plot $\ln[I(V) - I_{i0}(V)]$ versus V to get T_e from the inverse of the

Figure III 1

Swept probe traces for T_e measurement. The period of the sweep is 100 μ sec and the bias voltage (ramp trace) changes from -45 V to +45 V.

(A) Three successive shots of ψ_s , 30 msec into field pulse, and 5 msec after injection.

(B) Three shots at 8 cm in the lower outer bridge region.

A

B

Figure III 2

Circuit used to generate the ramp of 1.

slope. E. J. Strait is currently developing a program for the PDP-11 system to curve fit a line and an exponential from entire characteristic simultaneously in order to obtain unambiguously the coefficients I_0 , I_1 , I_{e0} , and T_e^{-1} . In the future, the best way to measure T_e from probe characteristics will be to abandon the sweeping circuitry and acquire the current : bias curve on a point by point basis as in Figure II 6 and apply the curve fitting routine. Using the considerations of Chapter II C, rapid time variations in the data need not disturb the accuracy of the resulting temperatures. Another advantage to this is that T_e as a function of time is obtained automatically.

The results of swept probe measurements are shown in Figures 3 and 4 for gun plasmas in the poloidal field octupole with the normal operating pressure. The results were summarized in Table I 2. There is no significant difference between the levitated and supported ring cases for either T_e or $\tau_{Te} = -T_e/\dot{T}_e$. Figure 5 shows T_e as a function of position along the mid cylinder from the wall to a point inside ψ_c . The points were obtained by overlaying a number of curves similar to Figure 1(b) and visually tracing the most reasonable characteristic curve. This was done a number of times for each position and the scatter is fairly large. However, the mean value for T_e joins well with the values of Figures 3 and 4 which supports the contention that $\nabla T_e = 0$ in the machine. This spatial uniformity is attributable to the turbulent injection and capture process that accompanies gun generated plasmas.⁴ These data, along with the spot checked observations elsewhere in the device by the author and others, suggest that the behavior of T_e is essentially independent of a wide range of machine parameters.

Figure III 3

T_e for the hydrogen plasma as a function of position in lower outer bridge and of time. Rings levitated and the background pressure is 2×10^{-7} Torr. $V_c = 2.5$ kV, crowbarred field.

Figure III 4

T_e for same operating conditions as 3, but with rings supported.

Figure III 5

T_e 5 msec after injection along the mid cylinder.

The cooling mechanism is still experimentally not determined, although a hypothesis is presented in Chapter IV which might partially explain it.

It should be mentioned in passing that Navratil⁵ has measured a different T_e/T_e using the admittance probe technique.⁶ T_e was seen to drop with about a 20 msec decay time until it reached 1 eV. It stayed at that value for the remainder of his observation time. The transition time would occur after the observation time of Figures 3 and 4, which was chosen to be the time interval for levitation. The final resolution of this data conflict must await the acquisition of point for point data and its analysis using Strait's non-biased computer technique to avoid systematic error.

(ii) The Ion Temperature

Figure 6 of the ion temperature is included for completeness. This is a compilation of T_i measurements made by a number of observers using different techniques.^{5,7,8} The results are summarized in Table I 2 for hydrogen. All T_i measurements were made in the $B = 0$ region of the octupole. However, $VT_i = 0$ may be assumed due to the same processes used to explain the observed flat T_e profile.

B: Plasma Profiles

Figure 7 shows the ion saturation current and floating potential traces as a function of time for four positions across the lower outer bridge region at 330°. See Figure I 5 for the conversion from centimeters from the ring in the bridge to the corresponding ψ value. The data were obtained using the cleaned double platinum probe described in

Figure III 6

$T_1(t)$ from measurements by Breun $\odot$, Navratil $\square$, and Brouchous Δ . All data is acquired at the field null.

Figure III 7

(A-D) $I_s(t)$ data traces at four positions in the bridge for the operating case of Figure 3. A, 1.75 cm; B, 4.5 cm; C, 7.0 cm; D, 8.0 cm.

(E-H) $V_f(t)$ traces for four positions in the lower outer bridge for the same plasma as in (A-D). E, 1.75 cm; F, 5.0 cm; G, 7.0 cm; H, 9.5 cm. Units per vertical division are marked on left of each base line as $ABC \text{ YZ} = ABC \times 10^{\text{YZ}}$ amperes for I_s or volts for V .

Chapter II. In these traces, as well as in all subsequent data, the ion saturation current are plotted in units of current density, $\mu\text{Amperes per square centimeter}$. Profiles for these data and others similar are presented in Figures 8 through 13. In each case, the profiles were acquired according to the prescription of II C. Four profiles, each at a different time, are presented for each of the six different machine configurations. The earliest time given for each set shows the plasma 1 msec after injection. Each time frame pairs the (upper) V_f profile, marked in volts of the peak value, with the (lower) ion current density, in $\mu\text{Amperes/cm}^2$ of its peak value. The particle densities corresponding to the ion current density may be estimated using [II-A-1]. All data were acquired with the clean double probe except the SINE case profiles, which were from single tipped gold probes cleaned as well as possible prior to insertion. The accuracy of these single tipped profiles is discussed in III E where the problems mentioned in II A are again examined.

Figure 14 is included to document the existence of what must be called an atypical profile. It is for the same machine parameters as Figure 9 but shows the profiles peaking on ψ_s with a relatively high I_s , a VI_s near the ring which points toward the ring, and a double humped V_f profile. This sort of profile has been seen on rare occasions by several other observers⁹ since the gun was first installed. It is unknown what is responsible for this distribution, but it has usually been a transient state of the gun's operation which disappears after a day or so. These particular data, however, were observed after the extender on the gun (section I B) was removed and disappeared when the

Figure III.8

Poloidal field only, crowbarred case, supported rings.

Profiles of floating potential (upper) in volts, and ion saturation current density (lower) in $\mu\text{A}/\text{cm}^2$ versus centimeters from the lower outer ring in the bridge at 330° . $V_c = 2.5 \text{ kV}$,

$P_0 = 2 \times 10^{-7} \text{ Torr}$.

- A. 23 msec, 1 msec after injection
- B. 35 msec
- C. 47 msec
- D. 65 msec

A

B

C

D

Figure III 9

Poloidal field only, crowbarred case, levitated rings.

Profiles of floating potential (upper) in volts, and ion saturation current density (lower) in $\mu\text{A}/\text{cm}^2$ versus centimeters from the lower outer ring in the bridge at 330° . $V_c = 2.5$ kV, $P_o = 2 \times 10^{-7}$ Torr.

- A. 23 msec, 1 msec after injection (rings fully out)
- B. 35 msec
- C. 47 msec (rings start to return)
- D. 65 msec (rings reconnected)

A

B

C

D

Figure III 10

Poloidal field only, half sine field case, supported rings. Profiles of floating potential (upper) in volts, and ion saturation current density (lower) in $\mu\text{A}/\text{cm}^2$ versus centimeters from the lower outer ring in the bridge at 330° . $V_c = 2.5 \text{ kV}$, $P_0 = 2 \times 10^{-7} \text{ Torr}$.

- A. 16 msec, 1 msec after injection
- B. 24 msec, peak field
- C. 31 msec
- D. 38 msec

A

B

C

D

Figure III 11

Poioidal field only, half sine field case, levitated rings. Profiles of floating potential (upper) in volts, and ion saturation current density (lower) in $\mu\text{A}/\text{cm}^2$ versus centimeters from the lower outer ring in the bridge at 330° . $V_c = 2.5 \text{ kV}$, $P_o = 2 \times 10^{-7} \text{ Torr}$.

- A. 19 msec, 1 msec after injection (rings fully out)
- B. 24 msec, peak field
- C. 31 msec
- D. 38 msec

A

B

C

D

Figure III 12

Weak toroidal field added, crowbarred, supported rings.

Profiles of floating potential (upper) in volts, and ion saturation current density (lower) in $\mu\text{A}/\text{cm}^2$ versus centimeters from the lower outer ring in the bridge at 330° . $V_c = 2.5 \text{ kV}$, $P_o = 2 \times 10^{-7} \text{ Torr}$.

- A. 23 msec, 1 msec after injection
- B. 35 msec
- C. 47 msec
- D. 65 msec

A

B

C

D

Figure III 13

Weak toroidal field added, crowbarred, levitated rings.

Profiles of floating potential (upper) in volts, and ion saturation current density (lower) in $\mu\text{A}/\text{cm}^2$ versus centimeters from the lower outer ring in the bridge at 330° . $V_c = 2.5 \text{ kV}$,

$P_o = 2 \times 10^{-7} \text{ Torr}$.

- A. 23 msec, 1 msec after injection
- B. 35 msec
- C. 47 msec
- D. 65 msec

A

B

C

D

Figure III 14

Anomalous profiles. Operating conditions same as in Figure 8. Injection at 25 msec. The potential V is plotted in volts, the saturation current in μA .

extender was replaced.

It should also be mentioned that the V_f profiles for the supported ring cases occasionally display an apparently arbitrary D.C. offset from the values shown here. The peak V_f value has actually been seen to be negative, but the potential gradients remain the same except at the walls and very close to the rings where the profile rapidly returns to ground potential. The offset does not appear to affect the density or any other parameter and, in fact, has been restorable to the "typical" value shown by levitating the rings.

Most of the "normal" profiles presented have two striking features in common. Firstly, the sign of ∇V_f implies that the electrons more readily leave the plasma than the ions. Secondly, V_f and I_s peak together in space. Since there are no gradients in temperature, [II-A-1 and 2] imply that the space potential and particle density do also. In addition, a linear relationship is observed between $\ln[I(x)]$ and $V_f(x)$. This is the topic of section D and was first suggested from the fluctuation phase velocity measurements as discussed in Chapter V.

A great deal has previously been discovered about the density profiles in the octupole with the poloidal field only. The magnitude and scaling of the ambipolar diffusion coefficient, D_A , (defined below) has been measured^{10,11,12} and it has been shown that the density profile decays in a stable normal mode due to the diffusion driven particle flux.^{13,14} It seems reasonable in this case to look for a diffusion explanation for the relation between $\underline{E} = -\nabla V_f$ and n .

Rudmin¹⁵ has shown that late in the sine field case, the density distribution moves with the field lines in response to the behavior of

the machine's electric field. Hence a diffusion relation between n and V_f might be expected to fail late in the sine field cycle. In the case of the added toroidal field, the plasma has been seen to subdivide into regions where different processes dominate.¹⁶ This should also confuse any overall pattern to the relationship between V and I .

C: The Ambipolar Density: Potential Relationship

Section III B shows that the probe saturation current and floating potential are related. This section gives a phenomenological explanation for this observed relationship. A kinetic theory model which quantitatively accounts for the data is presented in Chapter IV. The ambipolar assumption is that in every region in space the ion and electron densities increase or decrease at the same rate.¹⁷ The continuity equation relates the particle fluxes, $\underline{\Gamma} = n\underline{v}$, for both species by

$$\nabla \cdot \underline{\Gamma}_i = \nabla \cdot \underline{\Gamma}_e \quad \text{[III-C-1]}$$

The general solution of [1] is

$$\underline{\Gamma}_i = \underline{\Gamma}_e + \underline{\gamma} \quad \text{[II-C-2]}$$

where the particle flow imbalance $\underline{\gamma}$ is the curl of some vector, $\underline{\gamma} = \nabla \times \underline{\beta}$. The vector $\underline{\beta}$ is related to the magnetic field caused by the net charge flow and must be determined by the boundary conditions of the problem. The flux coordinates of Chapter I will be used but the components of a vector parallel to $\underline{B}$ will be labeled with $||$. Since the octupole is azimuthally symmetric, $\Gamma_{i\zeta} = \Gamma_{e\zeta}$. It has been shown¹⁸ that the average of $\underline{\gamma}$ around the closed field line is zero but local paral-

lateral current imbalances may exist even with $E_{||} = 0$. The parallel and perpendicular components then must be left as

$$\Gamma_{i||} = \Gamma_{e||} + \gamma_{||} \quad \text{[III-C-3a]}$$

$$\Gamma_{i\psi} = \Gamma_{e\psi} + \gamma_{\psi} \quad \text{[III-C-3b]}$$

Frequently the vector $\underline{\gamma}$ is arbitrarily set to zero. Although it has been shown that the average current ($e \langle \gamma \rangle$), crossing a closed field line is zero, the perpendicular current at any point on the field line may be a large value. The fact that $\underline{\gamma} = 0$ is not a justifiable relationship causes important problems with the interpretation of many types of data. This point is again discussed in section E.

For small displacements from thermodynamic equilibrium, the thermodynamic fluxes may be related linearly to the thermodynamic forces by the usual¹⁹ Onsager expressions. The most general linear transport equation for the particle flux may be written as ($n_i = n_e = n$)

$$\underline{\Gamma}_e = -\nabla \cdot (\underline{D}_e n) - n \underline{\mu}_e \cdot \underline{E} \quad \text{[III-C-4]}$$

$$\underline{\Gamma}_i = -\nabla \cdot (\underline{D}_i n) + n \underline{\mu}_i \cdot \underline{E} \quad \text{[III-C-5]}$$

The subscripts are for ions and electrons and the equations are written so that the general coefficients $\underline{D}$ and $\underline{\mu}$ are identifiable as diffusion and mobility. $\underline{D}$ and $\underline{\mu}$ are written as second ranked tensors and the equations are structured as shown because these are the forms prescribed for a model dependent case by the fluid equations²⁰ or kinetic theory.²¹ For the purpose of this paper, $\underline{D}$ and $\underline{\mu}$ will be considered

unrelated diagonal phenomenological transport parameters.

Rudmin²² has used simple probe theory and has shown that, to within experimental accuracy, the observed plasma density in the octupole is constant along a field line, as expected by M.H.D. theory. As assumptions, let $E_{||} = 0$ and all quantities be independent of ζ . Let the plasma potential be U so that $E = -\nabla U$, then [3], [4], and [5] yield (see Reference 18)

$$\frac{\partial U}{\partial \psi} = T^* \frac{1}{n} \frac{\partial n}{\partial \psi} + \frac{[\nabla \cdot (\underline{D}_e - \underline{D}_i)]_{\psi} - \gamma/n}{[\underline{\mu}_e + \underline{\mu}_i]_{\psi\psi}} \quad \text{[III-C-6]}$$

where

$$T^* = \frac{[\underline{D}_e - \underline{D}_i]_{\psi\psi}}{[\underline{\mu}_e + \underline{\mu}_i]_{\psi\psi}} \quad \text{[III-C-7]}$$

With [5] and [6] the electron flux becomes

$$\underline{\Gamma}_e = -D_a \nabla n + \underline{E}_e. \quad \text{[III-C-8]}$$

The ambipolar diffusion coefficient D_a is

$$D_a = \frac{\mu_{e\psi\psi} D_{i\psi\psi} + \mu_{i\psi\psi} D_{e\psi\psi}}{\mu_{e\psi\psi} + \mu_{i\psi\psi}} \quad \text{[III-C-9]}$$

and the flux imbalance is

$$\underline{E}_e = \frac{-\hat{\psi}}{\mu_{e\psi\psi} \mu_{i\psi\psi}} [\mu_{e\psi\psi} \gamma + n(\mu_{e\psi\psi} \nabla \cdot \underline{D}_{i\psi} + \mu_{i\psi\psi} \nabla \cdot \underline{D}_{e\psi})]. \quad \text{[III-C-10]}$$

If T^* is a constant in ψ , [6] may be integrated to yield

$$U(\psi) = U_0 - T^* \ln(n_0/n) + W(\psi) \quad \text{[III-C-11]}$$

The subscript zero indicates the maximum value of U or n . As seen in Figures 8-13, these occur at the same place in space which indicates that the function $W(\psi)$ in [11] is a constant. W is the ψ integral of the term in [6] involving γ , which must be zero. This sets

$$\gamma = n \nabla \cdot (\underline{D}_e - \underline{D}_i) \Big|_{\psi} \quad [\text{III-C-12}]$$

and also [10] will become

$$\underline{E}_e = -n(\nabla \cdot \underline{D}_e)_{\psi} \hat{\psi}. \quad [\text{III-C-13}]$$

Note that [11] need be true only in a piecewise fashion, between each region where the gradients go to zero. T^* must be constant in each interval for [11] to be valid, but may change its value at the peaks.

D: Analysis Of The Profile Data

(i) Ambipolar Relation for V_f and I_s

The absence of temperature gradients for the profiles of III B imply that $V_f(\underline{x}) = V_p(\underline{x}) + a$ and $I_s(\underline{x}) = n(\underline{x})b$, where a and b are independent of position. This permits the conversion of [III-C-6] or [11] into a form with measurable parameters. As indicated, $W(\psi) = 0$ and ambipolar diffusion effects predict that

$$V_f(\psi) = V_o + T^* \ln[I_s(\psi)/I_o] \quad [\text{III-D-1}]$$

where T^* is still prescribed by [III-C-7] and V_o , I_o are the maximum value in ψ of V_f , and I_s . T^* may be found from the slope of the plot of $\ln(I_s)$ v.s. V_f using the spatial coordinate as the common parameter to generate the $(\ln I, V)$ graph coordinates.

(ii) Parameterized (I,V) Plots

Figures 15-20 show the parameterized (I,V) plots of the profiles of Figures 8-13, respectively. There are four distinct regions visible in the plots for the B_p only cases. Regions I and II are in the closed field line portion of the octupole. Region I extends from near the rings to the mutual peak in density and potential. Region II extends from the peak to near the wall. As explained below [III-C-1], these represent good piecewise solutions of [III-C-6] in the form of [1]. Region III extends inward from the wall to a point where $VV = 0$, where it becomes II. n does not go to zero at this place, but the $VV = 0$ field line is the one which first terminates on the wall. (See Figure I 2.) Plasma in this region is mirror confined and $T_{i||} = T_{e||}$ is no longer necessarily true. III is external to the confinement zone and is characterized by a high level of fluctuations. Figure 21 shows the transition between regions II and III clearly for data taken along the mid cylinder where $d\psi/dx$ is small.

An example from a completely different plasma device is shown in Figure 22, which is the parameterized (I,V) graph of data from the D.C. spherator ring at Wisconsin taken with R. Richards using his marginally stable mirror configuration.²³ The plasma is mirror confined for this case and is shown for several values of bias on the single tipped probe used to acquire the data. T_i for this plasma is ten to a hundred times lower than T_e .

Region IV in the octupole plots begins at the ring and extends about an ion gyro diameter into the plasma, where it joins I. IV therefore corresponds to the ring scrape-off region.

Figure III 15

(I,V) graph of Figure 8. Heavy dots are data, light dots are the fitted curve to these dots. Abscissa marked in 0.5 V steps.

- A. 23 msec
- B. 35 msec
- C. 47 msec
- D. 65 msec. Short upper line is the ring-peak region I.

A

B

C

D

Figure III 16

(I,V) graph of Figure 9. Abscissa marked in 0.5 V steps.

A. 23 msec. Line connecting smallest J values is peak-wall region II, other line is wall-peak region I, points on left end of I are the near-ring region IV.

B. 35 msec. Line for lowest J is R II, shorter line is R I. Other dots are R IV.

C. 47 msec. Long line is R II, short line is R I.

D. 65 msec. Long line is R II, short line is R I.

A

B

C

D

Figure III 17

(I,V) graph of Figure 10. Abscissa marked in 0.5 V steps.

A. 16 msec. Upper line is ring-peak region I, lower line is peak-wall region II. One point of near-wall region III visible.

B. 24 msec. Long line is R II, short line is R I. Points of R III visible.

C. 31 msec. Long line is R I, short line is R II. T* identification is difficult.

D. 38 msec. Positive slope line is R I and IV, negative slope line is R II and III, other points are near peak.

A

B

C

D

Figure III 18

(I,V) graph of Figure 11. Abscissa marked in 0.5 V steps.

A. 19 msec. Upper line is the ring-peak region I, other line is peak-wall region II.

B. 24 msec. Short line is R I, long line is R II.

C. 31 msec. Lower (right) line is R I, upper (left) line is R II.

D. 38 msec. Negative slope line is R II.

A

B

C

D

Figure III 19

(I,V) graph of Figure 12. Abscissa marked in 0.5 V steps.

A. 23 msec. Negative slope line is ring-peak region I,
positive slope line is peak to wall region II.

B. 35 msec. Short line is R I, other is R II.

C. 47 msec. Lower line is R II, upper is R I.

D. 65 msec. Steepest line is R I, other is R II.

A

B

C

D

Figure III 20

(I,V) graph of Figure 13. Abscissa marked in 0.5 V steps.

A. 23 msec. Negative slope line is near-ring region IV. Positive slope is peak-wall region II. The ring-peak region I is poorly defined.

B. 35 msec. Positive slope line is R II. Negative line joins peak data to data of R I and IV.

C. 47 msec. Positive slope line is R II, negative is R I.

D. 65 msec. Positive slope is R II.

A

B

C

D

Figure III 21 A

Poloidal field only, early rising field, supported rings.
Profiles of $V_f(\psi)$ in volts (upper), and $I_s(\psi)$ in μA (lower) taken
along the lower mid cylinder at 45° . Data at 10 msec into the
pulse, 5 msec after injection.

Figure III 21 B

(I,V) graph of profile data of Figure 21 A. Transition
between regions II and III clearly observable.

Figure III 22

(I,V) graph of data taken on the D.C. spherator ring, obtained with R. Richards. Two different curves are shown corresponding to different $I_{\square}$ biases. T^* is the same for both biases.

(iii) $T^*(x,t)$

Figures 23 through 28 give the plots of T^* v.s. time for the main regions discovered in the parameterized (I,V) plots. The following abbreviations are used: B-P means that the magnetic field is in the poloidal direction only. B-PT means that a weak (≈ 300 G on the mid cylinder) toroidal magnetic field has been added to the usual poloidal field. LEV means that the rings were levitated during the first 20 to 30 msec following injection. SUP means the rings were resting on their supports during the entire pulse. SINE means that the magnetic field was run in the 43 msec half sine mode (see section I A). CBR means that the transmission line was crowbarred at peak field (see section I A).

1. B-P CBR SUP: (Figures 8, 15, 23) Plasma was injected at 22 msec into the magnetic field pulse. T^* regions I and II are essentially identical and decay with an 11 ± 1 msec time constant $\tau^* = -T^*/\dot{T}^*$ until about 53 msec when T^* changes to about 22 ± 2 msec. Region III is clearly visible in the later time frames.

2. B-P CBR LEV: (Figures 9, 16, 24) Plasma is injected 22 msec into the field pulse. Regions I, II, and IV are visible in the early time frames. III is not evident because the density data does not extend to ψ_g (Figure I 4). T^* in region I in the private flux is larger than in region II but both T^* values decay with $\tau^* = 11 \pm 1$ msec. Again as in the supported case T^* changes to roughly 22 msec after about 53 msec.

3. B-P SINE SUP: (Figures 10, 17, 25) Injection was at 15 msec in the half sine pulse. These profile data display the more

Figure III 23

$T^*(t)$ for the data of Figures 8 and 15.

Circles are for data in ring-peak region I.

Triangles are for data in peak-wall region II.

B-P CBR SUP

Figure III 24

$T^*(t)$ for the data of Figures 9 and 16.

Circles are for data in ring-peak region I.

Triangles are for data in peak-wall region II.

Figure III 25

$T^*(t)$ for the data of Figures 10 and 17.

Circles are for data in ring-peak region I.

Triangles are for data in peak-wall region II.

Figure III 26

$T^*(t)$ for the data of Figures 11 and 18.

Circles are for data in ring-peak region I.

Triangles are for data in peak-wall region II.

B-P SINE LEV

Figure III 27

$T^*(t)$ for the data of Figures 12 and 19.

Circles are for data in ring-peak region I.

Triangles are for data in peak-wall region II.

B-PT CBR SUP

Figure III 28

$T^*(t)$ for the data of Figures 13 and 20.

Triangles are for data in peak-wall region II.

B-PT CBR LEV

variation in their values than the other cases presented. This is because no reference scaling was performed and very little pulse averaging done (see Chapter II C). Between the two regions, region I has the largest error in the value of T^* because of its range of the uncertain V_f is smallest. Compare ΔV_f values in Figure 17. T^* is larger in region I than in II and the time behavior is different for the two regions. In I, T^* is constant until about 18 msec. It then decays with an apparent τ^* of 6 msec. Due to the uncertainty in the profile measurements and the reflected errors in T^* of about ± 1 V, τ^* could be different from this by a factor of two in either direction. T^* abruptly rises at 27 msec when the density peak is expelled from private flux region, presumably due to the motion of the flux surfaces. It reaches its maximum value between 30 and 32 msec and decays slowly again. T^* at these late times drops less than an e factor so τ^* may not be reliably estimated. In region II near the wall, T^* is better determined and is seen to fall rapidly for the several msec, the drop much more slowly for the next 8 or 9 msec. A 12 msec τ^* may be fit to this interval but since T^* changes by less than e, no good τ^* estimate may be made. At 27 msec T^* rises, reaches its maximum at about 32 msec then becomes negative. ΔV_f is too small for accurate determination in this region.

The only reason to attempt to fit an exponential decay to the T^* of this case is the very obvious exponential T^* decay seen in the other three B-P cases presented.

4. B-P SINE LEV: (Figures 11, 18, 26) Injection occurred at 18 msec. T^* is very similar to the B-P CBR LEV case until 27-29 msec

when the plasma is expelled from the private flux region by the moving flux surfaces. τ^* is roughly 10 msec for both regions. After about 34 msec, T^* is very clearly negative in region II and positive but difficult to estimate (short ΔV_f) in region I. The negative T^* decays with a 5 msec time constant after its peak value.

It can be stated in summary that hot ion plasma in the half sine poloidal field case is quite similar in the several milliseconds about peak field (at 22 msec) to the hot ion plasma in the CBR field for virtually the entire time that $T_i > T_e$. (T_i is calculated to be $\approx T_e$ at about 50 msec using the measured decay rates.)

5. B-PT CBR SUP: (Figures 12, 19, 27) Plasma injection occurred at 22 msec. In region II, between the peak and the wall, T^* falls with a 10 msec decay rate until 50 msec when it abruptly rises and then falls at about a 20 msec rate. This is similar to the observations of paragraphs 1 and 2. Initially, T^* is negative quite near the ring, in region IV, but after 2 msec regions I and IV merge and T^* is positive. By the time the plasma is 8 msec old, T^* has started to decay with an unchanging 22 msec τ^* .

6. B-PT CBR LEV: (Figures 13, 20, 28) Injection was at 22 msec again. This is the most complicated case of the six presented here. The density between the peak and the wall decays faster than the density between the peak and the ring and the peak itself has an apparent motion toward the ring. This is similar to what has previously been reported for this case. The behavior of T^* between the peak and the ring has several subdivisions and will not be discussed here except to note that T^* is negative very close to the ring during the first 5

msec. T^* between the peak and the wall initially decays from above $2 T_e$ with a 3 msec value then slows to a 13 msec decay time between 28 and 39 msec in the field cycle. T^* then abruptly rises with a -6 msec τ^* and peaks at 55 msec. It might be meaningful to restate that the supports are fully withdrawn at injection and begin reinsertion at about 40 msec. The exact times involved are different for each of the 16 supports. The supports, however, reconnect with the rings about 15 to 20 msec after they begin to reenter the confinement region. The relationship, if any, between the details of the levitation process and the observed time behavior of T^* is unclear and should be further studied. After T^* reaches its peak at 55 msec it decays at a steady 4 msec τ^* for the remainder of the observation period (until 70 msec). The behavior of plasma in the octupole with an added toroidal field is the subject of active investigation at the time of this writing. This configuration will not be further examined by the work presented in later chapters.

7. Azimuthal symmetry of T^* : The dependence of T^* on the toroidal position was investigated for the case of paragraph 3, above. Three single tipped gold ball probes simultaneously acquired data at separate toroidal azimuths around the machine. This represents the worst case test of symmetry because the 45° probe is on a field line that intersects an inner support, the 135° port is midway between outer supports and the 330° port is 10° away from the nearest support. The poloidal gap is at 0 degrees. Figure 29 shows the results. Notice that the slope of the graphs are the same to within experimental error. The region II-III transition is also shown in this figure.

Figure III 29

(I,V) graphs from three separate stations to show the azimuthal symmetry of T*. $45^\circ +$, $135^\circ \circ$, and $330^\circ \Delta$.

(iv) T^* and T_i

An important observation for the poloidal field data of subsection (iii) is that

$$\tau^* = \tau_i \quad \text{[III-D-2a]}$$

where

$$\tau^* = -T^*/(dT^*/dt) \quad \text{[III-D-2b]}$$

and

$$\tau_i = -T_i/(dT_i/dt) \quad \text{[III-D-2c]}$$

with T_i used for the ion temperature. T^* is a positive quantity which indicates that $D_e > D_i$. This can also be seen from the fact that the potential is most positive where the density is greatest. Therefore, the diffusive transport is dominated by electrons. However, T^* itself decays at the ion temperature rate so that the electric field appears to be related to the ion distribution. Without an explicit model for the diffusive transport, it is difficult to see why [III-C-7] should favor ions over electrons.

For a Maxwellian plasma, however, T^* is a temperature-like variable as may be readily seen. The Einstein relation $D = \mu T$ should hold for such plasmas (reference 20, equation [7.56]) if there is a well defined collision frequency. Then D/μ may be factored from [III-C-7] for either T_i or T_e . In this case, T^* equals the temperature times a factor involving the ratio of diffusion coefficients. This would account for the integrability of T^* as indicated in [III-C-10] since

D_e/D_i might be independent of space although D_e itself might not. These arguments, however, would imply that the electrons not only diffuse faster than the ions but are also more mobile. Unfortunately, $D_e > D_i$ and $\mu_e > \mu_i$ indicates that T^* should decay with the electron distribution, which is not observed. This question is treated in the next chapter which satisfactorily explains this problem.

The existence of the relationship between T^* and T_i has been tested in several ways. Figures 30 and 31 show low T_i (< 1 eV) data with essentially the same neutral collisionality as the previous data. The plasma was prepared by placing a brass screen between the gun and the 10 cm entrance port into the torus. The purpose of this screen was to remove the embedded currents in the plasmoid as it drifts from the gun to the containment region. This resulted in a low density and a T_i which was at the threshold of the measuring equipment.⁵ Although quantitative comparisons for T^* are difficult, the resulting T^* is the same on both sides of the peak, reduced in value, but drops at the same rate as before.

Figures 32 and 33 show data for which T_i has been reduced by the high charge exchange probability due to the increased neutral background pressure. The neutral collision frequency is about 10 kHz. The ion temperature drops⁵ essentially to room temperature within 5 msec and, as is seen, E_ψ goes to zero on the same time scale. The floating potential drops to the wall potential in 10 msec. The electron temperature for this case is uniform in ψ and remains above 1 eV. This is a strong argument against any suspicion that these electric fields are induced by magnetic field errors. Such errors would cause the lines to

Figure III 30

I_s and V_f profiles for a low density hydrogen plasma. The peak density is about $5 \times 10^8 \text{ cm}^{-3}$. Data acquired 5 msec after injection and I_s taken with a single tipped probe.

Figure III 31

(A) (I,V) graph of the data of Figure 30. T^* has the same value on both sides of the peak.

(B) $T^*(t)$ for the low density plasma.

A

B

Figure III 32

I_s and V_f profiles for hydrogen plasma 1 nsec after injection.
Background pressure was about 8×10^{-6} Torr.

Figure III 33

Profiles for the plasma of Figure 32 but 5 msec after injection.

spiral into the rings or walls and the resulting parallel electron flow could be responsible²⁴ for the apparent E_{ψ} . However, when T_i is reduced as in this case, the parallel ion flow should be decreased and the electric field would be enhanced, not suppressed as seen.

E: Errors in Experimentally Estimated Parameters

The use of a single tipped probe biased to draw "ion saturation current" will generate errors in measurements if simple probe theory is used to analyze the data. This has been discussed in II A and III A and this section discusses the specific errors caused to the T^* and $D_{\perp}$ estimates. As described in II A, these problems are eliminated if a double probe is used. For completeness, a discussion is included of the error involved in $D_{\perp}$ evaluations caused by the usage of $\Gamma = -D_{\perp} \nabla n$ rather than $-D_{\perp} \nabla n + g$ as in [III-C-8].

(i) T^* Errors

The current drawn to a biased probe is given by [III-A-2] and may be rewritten as

$$I_s(V) = n[I_0 + I_1(V_f - V)]. \quad [\text{III-E-1}]$$

Frequently, the simple probe theory of [II-A-1] is used to estimate the density.^{10,11,22} This value, which may be called the simple density $N(V)$ as opposed to the actual density, n , is

$$N(V) = I_s(V)/eAc. \quad [\text{III-E-2}]$$

From [1], the inverse density scale length is

$$\frac{\nabla n}{n} = \frac{\nabla I_1}{I_s} - \frac{n I_1}{I_s} \nabla V_f. \quad \text{[III-E-3]}$$

Recall that from [III-C-6], $\nabla V_f = T^* (\nabla n)/n$ (this is what $W(\psi) = 0$ in [III-C-10] meant). If I_s rather than n is used, the result is

$$\nabla V_f = \frac{T^*}{\left(1 + \frac{n I_1}{I_s}\right)} \frac{1}{I_s} \nabla I_s$$

which defines an effective T^* as

$$T_{\text{eff}}^* = \frac{T^*}{\left(1 + \frac{n I_1}{I_s}\right)} < T^*. \quad \text{[III-E-4]}$$

Double probe measurements give an $I_s(V)$ which, while still dependent on the bias V , is a function independent of V_f . So a clean double probe will give the actual value for T^* . This explains why the T^* values from single tip data in III D are all smaller than the values from double probe data.

(ii) $D_{\perp}$ Errors

Simple probe theory again causes an underestimate of $D_{\perp}$. Define the effective perpendicular diffusion coefficient D_{eff} as

$$D_{\text{eff}} = \frac{|\Gamma|}{|\nabla N|}. \quad \text{[III-E-5]}$$

Then, using the first two equations of this section, a little algebra shows that

$$D_{\text{eff}} = D_{\perp} \left[\frac{n e A c}{I_s + n T^* I_1} \right]. \quad \text{[III-E-6]}$$

For a large enough T^* or bias, D_{eff} is clearly smaller than the actual D_1 . The use of a double probe to measure VI_s which is proportional to Vn will alleviate part of the problem because the I_1 term in the denominator will be absent. One must still compare I_s from simple probe theory with the actual $I_s(V)$. The attenuation factor in the above equation may be written as $eAc/[I_o + I_1(V_f + T^* - V)]$.

(iii) D_a Measurement Errors From An Improper D_a, Γ Relationship

A different class of measurement errors arises when [5] is used as the operational definition of D_1 , as is often done.

[III-C-8] shows that the particle flux is offset from $-D_a Vn$ by the term g . This is caused by the particle flux imbalance, γ , and by the geometrical properties of D_e and D_1 . In general, γ has no simple prescription, but for the B_p octupole [III-C-12] shows that γ is the geometrical response to any non-zero divergence of the difference between D_e and D_1 . This produces the very simple result [III-C-13],

$$g_e = -n(\nabla \cdot \underline{D}_e) .$$

An effective perpendicular diffusion coefficient, D_1 , has been defined in III E(11) as [5] with n used instead of N . This is clearly related to the true physical diffusion coefficient D_a by

$$D_1(e) = - \frac{\Gamma_e}{|\nabla n|} = D_a - \frac{g_e}{|\nabla n|} . \quad \text{[III-E-7]}$$

For the B_p octupole, this is

$$D_1(e) = D_a + a_n (\nabla \cdot \underline{D}_e) \psi \quad \text{[III-E-8]}$$

where a_n is the density scale length. $D_1(i)$ may be analogously defined

using $\Gamma_i = \Gamma_e + \gamma$:

$$D_i(i) = - \frac{\Gamma_i}{|\nabla n|} \quad \text{[III-E-9]}$$

$$D_i(i) = D_a + a_n (\nabla \cdot \underline{D}_i) \psi. \quad \text{[III-E-10]}$$

This problem will be discussed again in Chapter IV where turbulence measurements generate a kinetic theory model which provides a relationship between D_a and D_e . In principle, then, one could obtain D_i by measuring Γ_i , Γ_e , and ∇n .

(iv) Implications For Striped Particle Collector Measurements

The purpose of these remaining paragraphs is to raise some questions and offer some suggestions about the use of striped particle collectors. Collectors have been described in detail elsewhere.²⁵ Physically, the collector looks like two metallic combs placed on an insulating surface. The combs are pressed into each other ("interdigitated") so that as one moves along the surface, one encounters the teeth of the two combs alternately. One comb is biased positive with respect to a reference potential chosen by the experimenter and the other negative. The electric field of this arrangement decreases exponentially with distance from the surface, the e-folding distance being the stripe separation length. If this distance is smaller than a debye length, then it is hoped that the collector is nonperturbing to the plasma and that the current drawn by the stripes is the total particle flux onto its surface.

The standard connection circuit is similar to that of a double

probe (Figure II 4). The positive terminal of a battery is connected to one set of stripes and the negative to the other. The current drawn is found by measuring the differential voltage across a resistor in series with the battery. This requires that the current to the negative stripe, I^+ , be the same as that to the positive stripe, I^- . The purpose of this subsection is to point out that unless the current excess γ ($= en V \cdot (\underline{D}_e - \underline{D}_1)$ from [III-C-12]) is accidentally zero, an uncollected space charge will be generated over the collector and its insulating substrate which must surely perturb the measurements.

It would appear from the considerations of this chapter that the better operation method would be to center tap the measuring resistor to the wall or ring on which it lies, and measure I^+ and I^- separately. This would allow the flux excess γ to be measured, should help suppress the space charge, and provide a second experimental equation for D_1 and D_e . This can not yet yield self consistent values for D_1 and D_e because theoretical or experimental relationships for the mobilities are needed. The observations of Chapter IV provide such a relationship and in principle D_1 and D_e should be measurable.

Cavallo²⁵ reports that I^+ and I^- are sometimes different when collectors are operated in the center tapped configuration. He, however, used the a priori postulate that γ must be zero (our notation) and so changed the bias on the positive and negative legs or let the collector float until equality was achieved. He observed no differences between the results of the center tapped, differential bias mode and the floating double probe mode. Cavallo made a point of acquiring serious data only for those configurations in which the floating center tap

remained naturally near the potential of the boundary surface. This corresponds to investigating regions where $\underline{y} = 0$.

REFERENCES FOR CHAPTER III

1. J. R. Drake. Ph.D. Thesis. University of Wisconsin (1975)
2. F. F. Chen. Plasma Diagnostic Techniques, R. H. Huddelstone, S. L. Leonard, ed. (Academic Press, N. Y., 1965) p. 113
3. K. R. Symon. University of Wisconsin Plasma Physics PLP-515
4. D. M. Meade. Ph.D. Thesis. University of Wisconsin (1965)
5. G. A. Navratil. Ph.D. Thesis. University of Wisconsin (1976)
6. J. C. Sprott. Rev. Sci. Inst. 39, 1569 (1968)
7. R. A. Breun. Ph.D. Thesis. University of Wisconsin (1975)
8. D. A. Brouchous. University of Wisconsin Plasma Physics PLP-728
9. J. R. Drake, A. J. Cavallo. Private Communication
10. A. J. Cavallo. Phys. Fluids 16, 408 (1973)
11. J. R. Drake, J. R. Greenwood, G. A. Navratil, R. S. Post. Phys. Fluids 20, 148 (1977)
12. G. A. Navratil, R. S. Post, A. Butcher Ehrhardt. Phys. Fluids 20, 156 (1977)
13. J. R. Drake, J. C. Berryman. Phys. Fluids 20, 851 (1977)
14. J. C. Berryman. University of Wisconsin M.R.D. report #1635
15. J. W. Rudmin. Ph.D. Thesis. University of Wisconsin (1975)
16. J. R. Drake. Phys. Fluids 20, 1013 (1977)
17. F. F. Chen. Plasma Physics (Plenum Press, N. Y., 1974)
18. C. J. Armentrout. University of Wisconsin Plasma Physics PLP-667
19. S. R. DeGroot, P. Mazur. Non Equilibrium Thermodynamics (North-Holland Publishing Company, Amsterdam, 1962)
20. D. J. Rose, M. Clark. Plasmas and Controlled Fusion (M. I. T. Press, Cambridge, Mass., 1961)
21. I. P. Shkarofsky, T. W. Johnston, M. P. Bachynski. The Particle Kinetics of Plasmas (Addison-Wesley Publishing Company, Reading, Mass., 1966)

22. J. W. Rudmin. J. Appl. Phys. 47, 4833 (1976)
23. R. K. Richards. Ph.D. Thesis. University of Wisconsin (1976)
24. T. Jernigan, D. Meade, J. Rudmin. Phys. Rev. Lett. 26, 1298 (1971)
25. A. J. Cavallo. Ph.D. Thesis. University of Wisconsin (1975)

CHAPTER IV: TURBULENCE STUDIES

This chapter is grouped into four major parts. Sections A and B give the motivation for the study and the techniques that were needed to perform it. Section C presents the data and the analysis that may be performed without use of a model for the observations. Sections D and E develop a microturbulence model from kinetic theory based on the observations of C. Finally, sections F and G contain short reviews of the relevant turbulence work by other research groups and aid in the understanding and interpretation of the phenomena seen in the levitated octupole.

A: Motivation For The Microturbulence Search

The turbulence study presented in this chapter was conducted for hot ion plasmas in the B_p octupole. These experiments were initiated for several reasons.

(i) The Turbulence Explanation For The Observed $D_\perp$

Firstly, measurements^{1,2,3} show that the diffusion coefficient, $D_\perp$, is large ($\approx 700 \text{ cm}^2/\text{sec}$) and, at the machine parameters of this work, is independent of B and inversely proportional to the square root of the density. This is the scaling predicted by the vortex diffusion model proposed⁴ by Okuda and Dawson (called the OD theory here). This theory, which is a refinement of a previous⁵ model by Taylor and McNamara, calculates the cross field particle transport due to zero frequency random thermal turbulence. To add weight to the turbulence nature of these predictions, it should be mentioned also that the OD

formula differs by a factor of $\sqrt{2}$ (ref. 4, p. 411) from the results of the kinetic theory of strong turbulence by Dupree,^{6,7} Weinstock,⁸ and their coworkers. The OD formula arises from the application of the Fluctuation Dissipation Theorem⁹ to a fluid model plasma, while the strong turbulence model of Dupree, for example, uses the kinetic equations to follow the statistically exact trajectory of a particle in the presence of random electric fields.

Secondly, it was hoped that an explicit understanding of the turbulence contribution to particle transport might help explain the parametric dependence of T^* on T_1 . Recall from Chapter III that $E_1 = -T^*/a_n$ where a_n is the density scale length and T^* is a function of the transport parameters. It was shown that the plasma's potential was functionally dependent on its density. However, other studies^{10,11} have shown that for the B_p only configuration, the density distribution is determined through the diffusion equation by the observed parametric dependence of D_1 on n . Hence it did not seem unreasonable to believe that a turbulence explanation underlies both D_1 and T^* .

(ii) Zero Frequency Turbulence ... Two Interpretations

There are two quite different possible meanings that may be assigned to the phenomenon of a zero frequency turbulence field of random structures. One is to interpret zero frequency to mean infinite duration to each structure, i.e., the structure is fixed in some reference frame and neither grows nor damps away in time. "Random turbulence" means that such structures are spatially aperiodic and generated by no fixed features of the apparatus such as obstacles.

The other interpretation is of the turbulence as a dynamic assembly of random structures which are aperiodic in time so that no frequency component other than zero can match the observed temporal occurrences. The Fourier transform (FT) of a finite segment of such data would peak at $f = 0$ and display decreasing amplitude with increasing f as the aliased fit of the components to the random data structures become progressively more difficult, as discussed in section II D(iii). Each of the turbulence structures might be quite dynamic, lasting only a short time, compared to the duration of the data segment. This, in fact, leads to a requirement for the identification of such turbulence ... the sample interval must be much longer than any characteristic time of the data. If not, the FT of only two or three random structures will display a broad peak at a finite, non-zero frequency.

Fourier transform theory, in fact, provides the wrong language to discuss the turbulence in either picture. Aperiodicity in time or space will cause the FT to peak at frequency f and wave number k , even though each turbulence structure has a well defined average lifetime and spatial extent. In fact, an observer at a fixed point in his reference frame might encounter the structures at random times but with a definite non-zero, non-infinite average time between encounters. ($\tau = T_0/N$, T_0 = observation time and N = number of encounters, assumed large.) The inverse of this average has units of frequency, although the FT of these data would peak at $f \approx 0$. The correlation function (CF), however, which may be defined [section II D(ii) 1] as the ensemble average of time or space shifted products of data, does provide

the proper analysis tool. All average, correlated structure will appear in the CF except the encounter time τ , above. This must still be obtained directly from the data.

(iii) Particle Transport In The Two Pictures: Convective Cells and Diffusive Cells

Figure 1 shows the interaction of a charge with a potential cell. The magnetic field is perpendicular to the page and the size of the cell, L_c , is assumed to be much larger than the particle's gyroradius. The guiding center of the particle drifts from left to right due to the magnetic inhomogeneities. As the particle enters the cell field E_c , it experiences an additional $E_c \times B/B^2$ vortex drift about the cell. In the stationary cell picture, the particle will leave the cell along the same path it would have followed had the cell not been present. No net transport occurs unless an obstacle, such as a machine wall, intercepts the vortex trajectory and removes the particle before it completes its convection about the cell.

Suppose, however, the cell field is extinguished by the time the particle reaches point 1. It now resumes its unperturbed trajectory but displaced Δx from its initial path. A number of such encounters cause a diffusive transport of the particle across space.

The two cases of turbulence may be classified as convective turbulence or diffusive turbulence and are not mutually exclusive processes for plasma transport. Convective cells must clearly be spatially large to transport particles effectively from the confinement region of the machine. Diffusive cells must have very short lifetimes to do the same

Figure IV 1

Physical explanation for convective diffusion.

PHYSICAL PICTURE OF VORTEX DIFFUSION

NO DIFFUSION RESULTS IF $\frac{\partial E_c}{\partial t} = 0$

IF E_c IS EXTINGUISHED WHEN PARTICLE IS AT PT. 1, THEN THE G.C. STEP SIZE IS Δx .

τ_c = CELL CORRELATION TIME (LIFETIME)

T_T = PARTICLE TRANSIT TIME ($\sim L_c/v_M$)

NEEDED FOR DIFFUSION: $\tau_c \leq T_T$

$$D_{\perp} = \frac{(\Delta x)^2}{T_{e-cell}} = \frac{(\vec{v}_c \tau_c)^2}{T_{e-c}} = \frac{E_c^2}{B^2} \tau_c^2 \nu_{e-c}$$

and this probably requires that they be spatially small. Convective cells have been studied previously in other machines^{12,13} and are currently being investigated by A. Butcher Ehrhardt in the Levitated Octupole. To my knowledge, diffusive cells have not previously been studied experimentally and are the subject of the investigations of this chapter.

(iv) Preliminary Study For Short Range Turbulence: Feasibility Test

Before any of the investigations reported in later sections were undertaken, data such as shown in Figure 2 were observed. The upper trace of both records is from a 5 cm $\times$ 5 cm collector mounted on the edge of the outer wall which is indented inwards toward the machine center. The lower trace is from a probe with a 3 mm diameter tip which was placed on the same field line as the collector. The field line is just outside of ψ_c and grazes the wall. See Figure 1 4. The amplitudes of the oscillations are unimportant, as is the original purpose of the experiment ... to look for a $k_{||}$ for the low f long λ waves (≈ 10 kHz, 20 cm) that exist in that region. It is interesting, however, to observe the relative degree of turbulence on the two signals. The collector, which integrates the signal over its 5 cm toroidal extent, is fairly quiet. The 3 mm probe, which is smaller than the ion gyro orbit as explained in section II A, shows a rather high degree of noise. This could be interpreted to indicate that the turbulence structures are larger than 3 mm but shorter than 5 cm and live for times short compared to the wave period. This indicated that the turbulence signal might successfully be separated from the wave signal

Figure IV 2

Preliminary evidence for short range turbulence. In all four photographs, the upper trace shows I_g fluctuations observed by a 5 cm $\times$ 5 cm collector located on the wall and the lower trace shows I_g oscillations seen by a 3 mm diameter probe tip on the same field line. A is the digitized image of the signals. B, C, and D are oscilloscope photographs with a 100 μ sec/division horizontal sweep speed (1 msec full scale) and 2 μ A/division for the probe signal, 2.5 μ A/division for the collector signal.

A **B** **C** **D**

experimentally and that the spatial structure might be resolvable by the sort of expandable multiple probes [section II A(vi)] which may be inserted through the available 1/4" probe ports.

B: Experimental Methods For Turbulence Measurements

(1) Measurement Techniques

The required information is obtained from two probe tips measuring fluctuations in either floating potential or electron saturation current. The probes used were of the expandable double probe type [section II A(iv)] for bridge measurements through the 1/4" air lock, or were constructed with a fixed 3.5 mm separation and inserted through the single available 2" air lock located at 45°, mid cylinder. The platinum tips were flame cleaned prior to insertion but not subsequently cleaned. Figure 3 diagrams the method used to obtain spatial information from the signals. The configuration is shown for data near the ring but the technique is also used to acquire common flux data on the mid cylinder. The probe is inserted perpendicular to the field line and the tips are then expanded to (for example) a one centimeter separation, which is held fixed for the entire experiment. Initially, the tips are placed on the same B line and the relative phase shift is measured. In all the experiments attempted, tips on the same line have always been completely in phase, to within the accuracy of the equipment (below). The tips may be varied in their ζ (toroidal) separation by rotating the probe about its axis. That is, the tips may be rotated with respect to B from an orientation with no toroidal separation (1 cm apart on the same B line) to one with 1 cm toroidal separation (differ-

Figure IV 3

Diagram of the orientation of the two probe tips with respect to a field line. The tips are typically 1 cm apart. The displacement is controlled by rotating the probe on its axis.

ent B lines but on the same flux shell).

The signals are passed through a high pass filter to remove the offset due to the zeroth order "profile" signal [section II D(i)] and its slow time variation. For one set of data, below, it was also necessary to filter out the 10 kHz coherent signal. The signal is then amplified by a Tektronix 1A7A preamplifier and driven down coaxial cable to the Biomation 610 B digitizers. Although a relative time shift between the two data records might be expected, no shift was in fact observed at even the fastest digitizing rate. This places the upper limit on any undetected relative time shift at .05 μ sec, half the smallest sample interval of the digitizers.

(ii) Analysis Techniques

As discussed in IV A(ii), the Fourier transform (FT) is not the proper analytic tool for turbulence studies. The information desired was described in section II D(ii) as being the characteristic lifetimes and spatial lengths averaged over the ensemble of possible values. Define a "stationary" property as one whose average values do not change in time. Then, for stationary turbulence, the correlation function (CF) of [II-D-2] provides the ensemble average of the data. The resulting correlation times and lengths are the desired characteristic values.

Although the FT is unstable when used as the primary analysis tool, important information may be obtained when applied to the CF. As discussed in II D(iii) for coherent oscillations, the Data-CF-FT sequence provides frequency and phase information that is difficult to

obtain from the CF alone. Figures 4 and 5 illustrate this point for turbulence. Data from five successive pulses of the octupole are analyzed with only the first and last being shown. These data are from the same shots which make up the data for the 10 mm cross CF of Figure 8 and are, in fact, the records from one of the two tips used. Four distinct analytic methods are used to process these records. The FT of each data record is displayed, as is the CF and the CF's FT. (Actually, the power spectrum (PS) is shown rather than the complex FT because the PS is independent of the phase of the data with respect to $t = 0$ and is more stable than either of the FT components [see II D (iii)]). It is also the self or "auto" CF which is displayed for the data record. Next, the CF is averaged over successive pulses to form $\langle CF \rangle$. This average and its FT are also shown. In addition, the FT is also averaged over successive shots and its value, $\langle FT \rangle$, is displayed. As is evident from these figures, the $\langle CF \rangle$ has stabilized sufficiently to provide information on the lifetime and amplitude of the average turbulence as seen by the fixed probe. In addition, amplitude and frequency information about the coexisting coherent wave may be found from the FT of the $\langle CF \rangle$. None of these parameters are readily available from the $\langle FT \rangle$. In fact, one would need perhaps an order of magnitude more shots to obtain the same degree of ensemble averaging for the $\langle FT \rangle$ as was done for the $\langle CF \rangle$ with these five shots. Between five and fifteen shots have been averaged together for each point plotted in the following sections. The actual number used depended on the length of the data sample (shorter records = more pulses) and on the reproducibility of the plasma.

Figure IV 4

First of 5 shots to demonstrate difference between the FT of data and the FT of a CF. A is the data record of δI_e to a single tip and is 256 μsec long, 1 $\mu\text{sec}/\text{point}$. The vertical units/division for all photographs is located on the left of the baseline as $ABC \text{ YZ} = ABC. \times 10^{\text{YZ}}$. The data are in units of microamperes. B presents the auto CF of the data $C(\delta I)$, C is the PS of A $P(\delta I)$, and D is the PS of B $P(C(\delta I))$. The PS displays are marked at 7.81 kHz per horizontal step along the axis.

A

B

C

D

A

B

C

D

E

F

Figure IV 5

Fifth of the five shots shown to demonstrate the difference between the FT and the CF of random data. All horizontal units are the same as in Figure 4. A is the fifth data record. B upper is the CF of A, $C(\delta I)$ and B lower is the five shot average $\overline{C(\delta I)}$. C is $P(\delta I)$ and D is $P(C(\delta I))$. E is the five shot average of C, $\overline{P(\delta I)}$, and F is the PS of the averaged CF, $P(\overline{C(\delta I)})$.

The single peak with flat wings is the characteristic correlation of random turbulence. Notice that the central peak extends below the baseline. This is an artifact of the way the data is normalized (see section II D(1)). The profile data of Chapter III was taken as the average of the signal about a given time so the fluctuations in the signal must be what is left over after this average is removed. This is the desired definition for the transport arguments of Chapter V. However, the time average of a CF is zero if the data itself has a zero average and the offset observed is what is necessary to make the total area under the CF curve zero. If the interval is long enough, the turbulence signal will extend only a short way below the axis. This is an intrinsic problem with the Biomation 610 B (section II B) which will not take sufficiently long records with satisfactory time resolution.

The averages of cross CFs between different probes are used below to provide spatial correlations and phase velocities of the signals. The spatial CF is found using [II-D-2] setting $\tau = 0$ as discussed in the paragraphs accompanying that definition. In order to smooth further the correlations against the per-shot variations in signal amplitudes, the spatial coefficient of correlation, rather than the CF itself, is in all cases plotted against the toroidal tip separation, z . The spatial correlation coefficient is the ratio $C(z)/C_{\text{peak}}$, where for each z , the CF $C(\tau, z)$ is calculated, C_{peak} is its maximum value, and $C(z)$ is its value at $\tau = 0$.

Suppose the data, D , are Gaussian. Then $D(t) = D_0 \exp(-t^2/T^2)$. If there are N such Gaussians observed in the time T , the CF of D is $C(D, D; \tau)$

$$C(D,D;\tau) = (\pi/2)^{\frac{1}{2}} \nu T \exp(-\tau^2/2 T^2)$$

[IV-B-1]

where $\nu = N/T$ is the average rate at which the Gaussian peaks are observed. This demonstrates the important feature that the CF of a Gaussian is a Gaussian. The correlation time of the data is T and may be found as the value of τ which reduces C to half of its maximum value. This fact provides a convenient method of obtaining the correlation times and distances directly, without further calculations. The problem is dealt with more thoroughly in D(iv)1.

C: Observations Of The Turbulence Structure

This section presents the basic turbulence data obtained from the octupole. The model which may be derived from it and the subsequent conclusions are presented in the later sections.

(i) Correlation Length

1. Hydrogen, $\psi = 6.75$: Figure 6 shows electron saturation current signals, I_e , from three tips biased to +45 V. The three tips are labeled 45, 41, and 42 and are all on the $\psi = 6.75$ surface. Probe 45 has a 1 mm diameter tip located in the bridge region 7 cm from the lower outer ring at 45°. 41 and 42 are 2 mm diameter tips mounted on a probe 1 cm apart in the lower mid cylinder at 45° essentially on the same field line as 45. 41 and 42 are separated by different toroidal distances, with 41 closest to the gap, by rotating the probe body about its axis. (a) shows the long time behavior of I_e from probe 45. The scope was triggered on the octupole field pulse and the hydrogen plasma was gun injected at 25 msec. (b) and (c) show the fluctuation signals

Figure IV 6

I_e data for a hydrogen plasma at $\psi = 6.75$.

(A) 5 msec/division, triggered on the field pulse, data from the bridge region.

Upper: 10 $\mu\text{A}/\text{div}$, signal from 1 kHz high pass filter

Lower: 100 $\mu\text{A}/\text{div}$, full signal

(B) 256 μsec full scale, 1 $\mu\text{sec}/\text{point}$. Trace starts 5 msec after gun injection, ζ tip separation is 10 mm.

Upper: signal from probe 45 in the bridge

Center: signal from probe 41 in the mid cylinder

Lower: signal from probe 42 in the mid cylinder

(C) Same as (B) but with ζ tip separation of 8.7 mm

A

B

C

from the three probes during the 256 μsec interval 5 msec after injection. In both cases the upper trace is 45, the center trace is 41 and 42 is the lower trace. The microamperes per division on the picture is printed above the left end of the three baselines as ABC YX which means there are $ABC \times 10^{YZ}$ μA per division for that trace. (b) is a shot where the toroidal separation z is 10 mm and (c) has $z = 8.7$ mm.

Figure 7 shows 4 auto CFs for these data for the different z values labeled. Two are from 41 and 42 for the same 10 shots which made up the averages. The points are plotted in 1 μsec steps. This is included to show the similarity between the auto CFs. Figure 8 presents four cross CFs for different values. Figure 9 shows that the turbulence signal is well correlated along a field line. The correlation coefficient typically is .85 which indicates that about 90% of the signals to the probe are shared. The slight shift of the peak from $\tau = 0$ to $\tau < -1$ μsec is due primarily to the misalignment of the probe ports and corresponds to a spatial displacement of less than 3 mm.

A low amplitude signal of about 40 kHz is visible in nearly all the CFs. The possible relationship between these waves and the turbulence signal is discussed in section G. The toroidal velocity of the turbulence is $v = \langle z/\tau_{pk} \rangle$ which is $(2.6 \pm .3) \times 10^5$ cm/sec. The temporal half width of the turbulence signal at half maximum amplitude is the same for the cross CFs as for the auto CFs. As discussed in subsection (ii) below, this indicates that the signals are not decorrelating as they pass the probes. This is true for nearly all of the data to be presented in this chapter.

Figure 10 is a plot of the spatial correlation obtained from the

Figure IV 7

Auto CFs for the case of Figure 6. 1 $\mu\text{sec}/\text{step}$. Peak value in μA^2 labeled on the axis at $\tau = 0$. The stations used for each CF are marked in the upper left of each photograph and the z value is also indicated. For example, A is C(42,42; $z = 8.7$ mm).

A

B

C

D

Figure IV 8

$C(F_1, F_2; z; \tau)$ for hydrogen plasma δI_e signals in the mid cylinder at $\psi = 6.75$.

F_1 , F_2 , and z are labeled in the upper left corner of each photograph.

A

B

C

D

Figure IV 9

Cross and auto CFe for δI_e signals in the bridge (45) and in the mid cylinder (41).

A

B

C

D

Figure IV 10

Spatial correlation $C(z)$ vs. probe tip separation z for I_e fluctuations in hydrogen plasma at $\psi = 6.75$ in the mid cylinder. The solid circles are direct $C(z)$ measurements and the open symbols are the spatial extent calculated from the temporal correlation functions.

$$\psi = 6.75 \quad H'$$

cross CFs as explained in IV B(ii). There are two kinds of data shown. Direct $C(z)/C_p$ values are plotted with solid circles. If there is no decorrelation of the turbulence cell-like structure as it passes the probes, the spatial profile may be obtained using $z = vt$ where v is the measured cell velocity. This calculated structure is shown by open symbols for two cross CFs and an auto CF, and overlay the direct spatial correlation. This matching of actual spatial structure to the calculated structure is indeed the primary evidence that the cells are not decorrelating. This is discussed again in IV C(ii) and again in IV D(iv).

The capacitor bank was charged to 2.5 kV and values of B , ρ_1 and their various spatial averages may be found from Figures I 4 and 5. The mid-cylinder value of ρ_1 is marked on the graph. Notice that $z = \rho_1$ corresponds to the half width of the peak at half maximum. This confirms the preliminary estimate of IV A(iv) of the size of the turbulence structures.

2. Helium, $\psi = 6.75$: The hydrogen data of the last paragraphs indicate that the turbulence cells are limited in spatial extent by the average orbit size of the hot ions. An effort was made to test the mass dependence using a different plasma. Helium was used because hot ion plasmas are much harder to form and maintain with heavier masses. Also, vortex scaling of the diffusion coefficient is known to hold for helium.³

(a) Apparatus. The drift tank gun, DTG, (see section I B) was used rather than the torus gun used in the past. This was done because the DTG injects its neutrals into the drift tank, rather than directly

into the torus as the torus gun does. The neutral background pressure resulting from continuous 1.5 min cycle operation was 2×10^{-6} Torr, 4x to 10x lower than the torus gun produces. When running on the three minute cycle, the ambient pressure is 3×10^{-7} Torr. This is because the drift tank is closer to the turbo pump which is the only pump for helium on the system, and because neutral helium should enter the torus with a greater than 20 msec rise time. (The hydrogen pressure rise time was reported in Chapter I to be greater than 10 msec.)

(b) Temperature. Temperatures were measured¹⁴ for this plasma using the gridded analyser probe used previously. For the slow cycle, low background pressure case, $T_i = (4 \pm .5)$ ev was seen. The rapid cycle higher pressure case yields T_i of $(1 \pm .5)$ ev.

(c) Results. Figure 11 (a) shows the electron saturation current to a tip located on $\psi = 6.75$ on the lower mid cylinder at 45° . This is the same position as for the data of section IV C(1)1 and the machine parameters are also the same. However, the torus pressure is 2×10^{-6} Torr just prior to the shot. (b) shows the I_e fluctuations to two probe tips separated by 3.5 cm but on the same flux surface. The fluctuation data were taken 5 msec after the gun injection and the low frequency cut off is set for 10 kHz.

Figure 12 shows four auto CFs for this plasma. The tips are labeled 44 and 45 and an auto CF is shown for several different toroidal separations. Figure 13 presents several cross CFs with different values of probe separation z . Positive tip separations mean that 44 is closer to the $0^\circ B_p$ gap. Negative values mean that the 45 tip is closer. As seen in these pictures, the turbulence moves without

Figure IV 11

I_e signals for helium plasma at $\psi = 6.75$ in the mid cylinder.

A. Four records of the long time behavior, 10 msec per division with the sweep triggered on the field pulse. In each case the vertical scale is 500 $\mu A/div$.

B and C are both δI_e records taken at 1 $\mu sec/step$, 256 μsec full scale. B is for $z = 1.2$ mm, C is for $z = -2.0$ mm, where z is toroidal separation between tips.

A

B

C

Figure IV 12

Auto CFs for the helium plasma. The stations used are labeled in the upper left corner of the photographs. Plots are made at 1 μ sec per point along the abscissa.

A

B

C

D

Figure IV 13

Cross CFs for the helium plasma, $C(F_1, F_2; z; \tau)$. F_1 , F_2 , and z are labeled in the upper left corner: A is $C(44, 45; -2; \tau)$, plotted at 1 $\mu\text{sec}/\text{point}$.

A

B

C

D

significant decay counterclockwise toroidally, as viewed from above. This is the same direction as for the hydrogen data but the phase velocity is $(3.3 \pm 1.) \times 10^4$ cm/sec, 4x slower than for the hydrogen case.

Figure 14 shows the spatial correlation for the helium plasma in the 2×10^{-6} Torr background, using these probes. Direct $C(z)/C_p$ values are plotted with the solid points. The spatial structure is again calculated from the temporal correlations using the measured turbulence velocity. The open symbols plot the calculated averaged size from two cross CFs and one auto CF. The spatial correlations agree so the cells are again passing the tips without decorrelating. The large uncertainty in T_1 makes identification of ρ_1 difficult but it appears likely that the half width at half maximum is somewhat less than ρ_1 , about $\rho_1/2$.

3. Hydrogen, $\psi = 3.4$: The turbulence study was also conducted on a different ψ surface and at different B values for hydrogen in order to provide λ_c comparisons for these different parameters. The probes were moved near the ring in order to investigate the turbulence contribution to particle transport, the question which initiated this study. Vortex motion is caused by the electric field of a cell so floating potential measurements were taken in the region near a ring where D_1 has been extensively studied. Figure 15 shows hydrogen plasma fluctuations in V_f for two successive shots at $\psi = 3.4$, 1.5 cm from the lower outer ring in the bridge at 330° . This measurement point was chosen because it is inside ψ_s but outside the region IV of section III D(11), the ion scrape-off region. The direct measurements of D_1 have all been

Figure IV 14

Spatial correlation for the helium plasma. The ion gyro radius in the mid cylinder is labeled for three possible temperatures.

 $\psi = 6.75$

He

Figure IV 15

Two δV_f traces in a hydrogen plasma at $\psi = 3.4$, 1.5 cm from the lower outer ring. 10 kHz signal is visible, as well as the higher frequency turbulence signal. The high pass filter is set at 900 Hz. The local magnetic field is 2 kG ($V_c = 2.5$ kV).

$B = 2 \text{ KG}$ $t_0 = 5 \text{ msec}$

performed in this region. The filter is again set at 900 Hz, the B_p capacitor bank set to 2.5 kV, and the rings are fully levitated by injection time, 25 msec into the field pulse. The magnetic field strength at the observation point is 2 kG. The signal shown is composed of two components, a 10 kHz coherent oscillation, and the turbulence signal which rises to full amplitude in 5 to 15 μ sec and decays away in about the same time. The slowly varying ambipolar potential discussed in Chapter III has been removed by the filter. The r.m.s. magnitude of the 10 kHz signal is approximately equal to that of the turbulence, but its spatial coherence is very much longer. These fluctuations are $k_{||} = 0$ modes moving at the $\underline{E} \times \underline{B}$ and the ion diamagnetic velocity (see Chapter V) with a wavelength λ of about 10^8 in ζ . At the observation point, λ is about 18 cm. It should be noted that the maximum 10 kHz wave-particle diffusion coefficient is estimated to be about .01 times smaller than the observed value of about 700 cm^2/sec .

Figure 16 shows typical turbulence data obtained with the low frequency limit of the filter set to 10 kHz. The r.m.s. potential amplitude averaged over 20 shots is 30 mv and shows about a 5 mv deviation. Typically, $1\frac{1}{2}$ to 3 peaks occur in the 51 μ sec data interval with a rough average of two. Therefore 30 to 50 kHz is a reasonable estimate of the probe-turbulence encounter rate.

Figure 17(a) shows the single shot auto CF of the signal with the 10 kHz cutoff on the filter, (b) shows the cross CF for a probe separation of 8 mm with the high pass filter set to 900 Hz. The high f CF shows the typical signature of a turbulence structure. The low f CF

Figure IV 16

δv_f signal as in Figure 15 but with filter set at 10 kHz.

Time interval of the trace is 51.2 msec, .2 μ sec/point.

A. $z = 1.7$ mm, .045 V/div (upper) and .025 V/div (lower)

B. $z = 2.6$ mm, .043 V/div (upper) and .042 V/div (lower)

A

B

Figure IV 17

- A. Single shot auto CF for data of Figure 16, .2 $\mu\text{sec}/\text{point}$.
- B. Average CF of 7 pulses of data in 15. 1 $\mu\text{sec}/\text{point}$ sweep speed, $z = 8 \text{ mm}$.

A

AUTO CORRELATION OF DATA, SINGLE SHOT
 $.2 \mu\text{sec/pt}$ $f_{\min} = 10 \text{ KHz}$ $\psi = 3.3$

B

CROSS CORRELATION OF DATA, AVERAGE OF 7 SHOTS
 $1 \mu\text{sec/pt}$ $f_{\min} = 900 \text{ Hz}$ $\psi = 3.3$

shows the correlation for 7 kHz waves with a turbulence-enhanced central peak. The storage capacity of the transient digitizer (see II B) is too limited to show both many oscillations of the wave and an accurate time shift for the signal. Acquisition of accurate simultaneous information must await future investigators using more powerful equipment.

Figure 18 shows four auto CFs for this plasma. Two are shown for the same z value, but for both probe tips (330 and 331) and the other two are for two different probe separations. The horizontal time axis is .2 μsec per plotted point for these CFs and the vertical axis crosses at $\tau = 0$. Figure 19 presents three cross CFs for different z values. The probe separation of $z = 2.6$ mm gives a time shift of 2.8 microseconds which yield a phase velocity of $.9 \times 10^5$ cm/sec. The best estimate for v , averaged over all the z values is $(1 \pm .2) \times 10^5$ cm/sec for the turbulence signal. The 10 kHz wave moves at 2×10^5 cm/sec in the same direction. Figure 20 shows hydrogen data at $\psi = 3.4$ but with the magnetic field reduced to 1 kG. The frequency cutoff is set at 10 kHz. Figure 21 shows four auto CFs for these data and Figure 22 shows four cross CFs at different z values. These data are plotted at .5 μsec per point. Figure 23 plots the spatial correlation function for these two cases. It is shown in subsection (ii) that the 1 kG turbulence must be decorrelating as it passes the probe and that this effect will artificially reduce the size of the peak. For this reason, the 1 kG correlation coefficient $C(0)/C_p$ may not be plotted. Instead, $C(0)$ is divided by the product of the r.m.s. amplitudes of the data records.

A

B

C

D

Figure IV 18

Four average auto CFs for this data to show comparison between the signals to the two tips. .2 μ sec per point.

A

B

C

Figure IV 19

Three cross CFs of δV_f for hydrogen plasma of $\psi = 3.4$, 2 kG
local field.

A

B

C

D

Figure IV 20

Four data records for hydrogen plasma at $\psi = 3.4$, 1.5 cm from the lower outer ring. This is similar to Figure 16 but with the local $|B|$ reduced to 1 kG. Data taken at 1 μ sec/point.

A and B are for tips on the same field line, $z = 0$. The average correlation coefficient is .84.

C and D are for $z = 6.5$ mm.

A

B

C

D

Figure IV 21

Four auto CFs for the case of Figure 20. These are presented to allow comparisons of the average signals seen by the probe. The CFs are plotted at 1 μ sec/point.

A

B

C

D

Figure IV 22

Four cross CFs for the plasma of Figure 20. The functions are plotted at 1 μ sec/point.

Figure IV 23

Spatial correlation of the turbulence signals for the 2 kG and 1 kG case. The ion gyro radius is marked at $z = 3$ mm for the 2 kG case.

$$\psi = 3.4 \quad H^+$$

(The r.m.s. value is the square root of the corresponding auto CF peak value.) Using this definition, the data are 80% correlated when $z = 0$, and less than that for non-zero z . To make reasonable comparisons, these ratios are all then normalized (divided) by the $z = 0$ values so that the graph is 1 at $z = 0$. The same procedure applied to the 2 kG data does not alter the curve shown. The 2 kG data show a half width at half maximum z_H of 3 mm and the 1 kG data has its $z_H = 6$ mm. The values of ρ_1 for the 2 kG case and its various averages (see Figure I 7) are marked on the z axis. The 1 kG ρ_1 values are twice as large, of course. Both turbulence cells have $\lambda_c = z_H = \rho_1$.

An attempt was made to see the turbulence decorrelation in the full fluctuation spectrum of the 2 kG case shown in Figure 15. Figure 24 shows the spatial correlation for this case. This is the composite of data taken at different times, separated by a month. The drop at 3 mm can be interpreted from the preceding figure as being caused by the decorrelation of the turbulence component.

4. Summary, λ_c : The correlation length λ_c is the e-folding length of the data and is the half width of the CF at half maximum, as shown in [IV-B-1]. For hydrogen, $\lambda_c = \rho_1$ is the appropriate value. Helium data have a larger experimental error, primarily due to uncertainty in T_1 , but roughly have $\lambda_c = .5 \rho_1$. At present, there is no well-formulated theory that will explain these results. This question is again taken up in subsection G.

(ii) Correlation Time

This is a very difficult parameter to measure from the available

Figure IV 24

Spatial correlation of the total δV_f signal with $f_{\min} = 900$ Hz, as in Figure 15. The drop in C at 3 mm can be interpreted from Figure 23 as being due to the turbulence decorrelation. The local $|B|$ is 2 kG.

$$\psi = 3.4 \quad B = 2 \text{ k G} \quad F_{\min} = 900 \text{ Hz}$$

data because in most instances the cells show no indication of decorrelating as they pass the probes. A large array of closely spaced probes would be needed to measure correlation times directly by performing mutual cross-correlations and watching a single cell decorrelate and disappear. Such an array is not available on the octupole and these data would be suspect in any case because of the (unknown) effect of many closely spaced probes on turbulence cells. One method that may be employed is to use the spatial information available from the τ_c measurements to construct a model for the spatial structure of the average cell. Calculations of the decorrelation time τ_c may be made from this model using a data case that displays obvious decorrelation.

The only configuration that allows such a check is the 1 kG case at $\psi = 3.4$. It is fortunate that it is available here because the geometry is sufficiently simple that spatial averages may be readily performed. The τ_c obtained from the data using the spatial structure model may be immediately compared with τ_c estimates using dynamical, "physical" models. It is also convenient because τ_c is needed for the D_1 estimates (chapter IV E(i)1) desired for this location. Details of these calculations are reserved for the analysis section (IV D). All that will be shown in this data section is that the turbulence structures clearly must be decorrelating as they pass the probes for the 1 kG case but not for the 2 kG case. The test is to use the temporal cross correlations to define a velocity $v' = z/\tau_{pk}$ where τ_{pk} is the time coordinate of the CF's peak value. If τ_H is the half width of the peak at half its maximum value, then a cell which does not decorrelate as it passes the probe will obey $L = v'\tau_H = \lambda_c$, where λ_c is the half

width value of the spatial correlation. If $L > \lambda_c$ to within the experimental accuracy, then the data are decorrelating.

It will be shown analytically in D that when there is no significant decorrelation, v' is the signal velocity v . However, when the cell passes the probe in a time comparable to the time required for it to disassemble, v' is much larger than v . This is because τ_{pk} is less than the actual transit time between probes. To see this physically, consider a cell which reaches its temporal maximum when the spatial maximum passes probe 1. Probe 2 measures the temporal maximum value of the tail. As the main body of the cell approaches 2, its overall amplitude decreases due to its decorrelation. By the time the spatial maximum reaches 2, its amplitude could be severely reduced or essentially gone if the cell is moving slowly enough in the lab frame. The maximum amplitude 2 sees will occur in a time interval less than the transit time and this is a time somewhat longer yet than τ_{pk} which is the peak value of the product of 2's signal times the monotonically decreasing signal of 1.

The data presented in Figures 18-22 provide concrete examples of this. The τ_H of the $z = 3.4$ auto CF for $B = 2$ kG is $3.2 \mu\text{sec}$ (for all examples of this case, τ_H is $3 \pm .2 \mu\text{sec}$). It was shown that $v' = (1 \pm .2) \times 10^5$ cm/sec so $L = 3$ mm with an experimental error of about .5 mm. λ_c certainly lies within this estimate of L so these cells do not decorrelate. Figure 25 shows this more graphically. The auto CF data of $z = 3.4$ was plotted using $v = .9 \times 10^5$ cm/sec (the value for the 3.4 mm case's v') and the direct spatial CF is overlapped in solid circles. This is similar to the plots of Figures 10 and 14.

Figure IV 25

Spatial correlation of the 2 kG data of Figure 23 to demonstrate the fact that the signals remain correlated as they pass the probe tips. Solid symbols are $C(z; \tau = 0)/C_{ph}$ taken directly from the cross CFs. The open symbols are the calculated spatial structure from the temporal CFs assuming $v = 1 \pm 1 \times 10^5$ cm/sec. $\square$ is from $z = 3.4$ mm $C(330, 330)$, $\triangle$ is from $z = 3.4$ mm $C(330, 331)$, and $\circ$ is from $z = 2.6$ mm $C(330, 330)$.

$$\psi = 3.4 \quad B = 2 \text{ kG}$$

The 6.4 mm example for the 1 kG case has $\tau_{pk} \approx 3 \mu\text{sec}$ and $\tau_H = 5.5 \mu\text{sec}$. This makes $v' = 2.1 \times 10^5 \text{ cm/sec}$ and $L = v'\tau_H$ of 1.2 cm. This is about twice λ_c and since the errors involved are $\pm 0.25 \mu\text{sec}$ for the τ values, L_c is clearly $> \lambda_c$.

(iii) Amplitude Of The Cell

The r.m.s. amplitude of the turbulence data, V_{rms} , has been discussed at the beginning of this subsection. This is not the amplitude V_c of the cell itself, however. The shapes of the spatial correlations and the temporal correlations indicate that the signal as seen by the probe may on the average be represented by

$$V(t) = V_c \exp\left[-(t - t_0)^2/T_t^2\right] \quad [\text{IV-C-1}]$$

where T_t is the transit time of a cell past the probe and V_c is the cell amplitude. The time convolution of this function with no relative shift is

$$\int V(t) V(t) dt = (\pi/2)^{1/2} T_t V_c^2. \quad [\text{IV-C-2}]$$

The correlation function replaces ∞ by $T_0/2$ and divides the result by T_0 . In the time T_0 , the probe will encounter N such cells and if $v_p = N/T_0$ is the probe-cell encounter rate, the unshifted correlation function, which is the time average of the squared data, becomes

$$V_{rms}^2 = (\pi/2)^{1/2} v_p T_t V_c^2$$

or

$$V_c = V_{rms} / (v_p T_t)^{1/2} \quad [\text{IV-C-3}]$$

since the quantity $(\pi/2)^{1/4}$ is essentially unity. From the data of the preceding subsections, $v_p T_t$ is .24. Since V_{rms} is 30 ± 5 mv, V_c is 60 ± 10 mv. This value will be used in section E to obtain an estimate for D_t .

If the cells are decorrelating as they pass the probe, T_c is replaced by an effective time parameter, T_{ef} . This is further discussed in the paragraphs surrounding [IV-D-21], below.

D: Kinetic Description Of The Empirical Model

(i) Approximate Scalings

The characteristic scale lengths provided by the data of the last section are

$$\rho_e \ll \frac{L}{2} < \rho_i \ll L_o \quad [IV-D-1]$$

where ρ is the gyro radius, L is the characteristic size of the average cell, $2\lambda_c$, and L_o is a macroscopic scale length of the plasma. This is the principal difference between the OD model and the modification required by the data. These scale relationships provide the time scales

$$\frac{2\pi}{\omega_e} \ll \frac{L}{v_i} \ll \frac{2\pi}{\omega_i} \quad [IV-D-2]$$

where ω is the gyro frequency and v_i is the ion thermal velocity. The time variation of all macroscopic parameters is assumed to be much larger than any of these characteristic times.

These scale relationships are appropriate for the hot ion plasma in the octupole, since the hydrogen plasma has $\rho_e = .01 \rho_i$, for exam-

ple, shortly after injection. It follows immediately from [2] that drift ordering is applicable for the interaction of the electrons with a cell but not for the ion interactions. Full description of the ion interaction requires the explicit solution of the kinetic equation. An electron, however, never leaves the guiding center reference frame during its interaction. This allows the following heuristic derivation of $D_{e\perp}$ which, although not rigorous, is physically clear.

Consider Figure 1. If the lifetime of the cell, τ_c , is short compared to the period of the vortex orbit, the displacement experienced by an electron is $\Delta x = v_{Ec} \tau_c = E_c \tau_c / B$. The diffusion coefficient $D_{e\perp}$ is $(\Delta x^2) / T_{ec}$ where T_{ec} is the mean time between electron-cell encounters. Writing $v_{ec} = 1/T_{ec}$, D_e becomes

$$D_e = \frac{E_c^2}{B^2} \tau_c^2 v_{ec}. \quad [\text{IV-D-3}]$$

All ensemble averages have been ignored, but surprisingly enough, [3] is the same as the more carefully argued approximation given by equation [IV-D-30].

(ii) Electron Transport

The drift kinetic equation derived by Hazeltine¹⁵ provides the proper kinetic equation for the electron distribution since the required drift ordering is satisfied by [2]. For the case studied, this is

$$\frac{\partial f}{\partial t} + \underline{v}_d \cdot \nabla f = 0 \quad [\text{IV-D-4}]$$

$$\underline{v}_d = \underline{v}_E + \underline{v}_M + \underline{v}_p + \underline{v}_{||} \quad [\text{IV-D-5}]$$

where f is the gyro phase averaged electron distribution function, $\underline{v}_E$ is the electric drift velocity, $\underline{v}_H$ is the magnetic gradient and curvature drift velocity, $\underline{v}_p$ is the polarization drift velocity, and $\underline{v}$ is the electron's velocity in the direction of $\underline{B}$. Since the cells are formed out of the electron distribution which moves at the ensemble averaged drift velocity, this drift frame is the proper reference frame in which to consider [4]. No attempt will be made to solve the problem self-consistently with Maxwell's equations because both the ambipolar field and the cell field may be approximated from external considerations. The formal solution of [4] is

$$f(t) - f(t - T_0) = - \int_{t-T_0}^t dt_1 \underline{v}_d(t_1) \cdot \nabla f(t_1). \quad [\text{IV-D-6}]$$

Now expand $f(t) = f_0(t) + f_1(t)$ where $f_0(t)$ is the part of f which varies on the macroscopic time scale so that during the interval $(t - T_0, t)$, $f_0(t - T_0) = f_0(t)$. This sets T_0 to be an interval which is long with respect to the lifetime of any given turbulence cell (its correlation time, τ_c) but short compared to the characteristic times of the bulk plasma. For the octupole, the macroscopic time variations are on the order of tens of milliseconds and the cell lifetime is on the order of ten microseconds. This allows $f_1(t)$ to be defined by [6] as

$$f_1(t) = - \int_{t-T_0}^t dt_1 \underline{v}_d(t_1) \cdot \nabla f_0(t). \quad [\text{IV-D-7}]$$

The particle flux in the k -th direction is

$$\Gamma_k(t) = n\langle v_k \rangle = \int d^3v v_{dk}(t) f_1(t).$$

Using [7], this becomes

$$\Gamma_k(t) = - \int d^3v v_{dk}(t) \int_{t-T_0}^t dt_1 v_d(t_1) \cdot \nabla f_0(t). \quad [\text{IV-D-8}]$$

This particle flux function depends on the instantaneous phase of the particle in the cell because of the term $\underline{v} \cdot \nabla f$. The macroscopically meaningful quantity is the time average of $\underline{\Gamma}$ over the previous time interval T_0 , and [8] must be replaced by its average

$(1/T_0) \int_{t-T_0}^t \Gamma_k(t_2) dt_2$. The time averaged particle flux will hence-

forth be called $\bar{\Gamma}$ and is

$$\bar{\Gamma}_k(t) = - \int d^3v \frac{1}{T_0} \int_{t-T_0}^t dt_2 \int_{t-T_0}^t dt_1 v_{dk}(t_2) v_d(t_1) \cdot \nabla f_0(t). \quad [\text{IV-D-9}]$$

The cell fields which generate the transport portion of the velocity $\underline{v}_d$ are turbulence fields. They are associated with a correlation time τ_c and a correlation length L_c which are the lifetime and spatial length of a representative cell. The time ordering for the turbulence-particle interactions is (a) $T_0 \gg \tau_c$, and (b) $f_0(t-\tau_c) = f_0(t)$. In addition, the turbulence is assumed to be a stationary process during the preceding interval, T_0 . This implies that the r.m.s. amplitudes and correlation times and lengths of the cells are slow macroscopic variables of time. The assumption of stationary turbulence leads to two additional properties. (c) The time convolu-

tion of velocities is invariant to small displacements in time ($|a| \ll T_0$): $\int_0^\infty \underline{v}(t+a+\tau) \underline{v}(t+a) dt = \int_0^\infty \underline{v}(t+\tau) \underline{v}(t) dt$ but is a function of the relative shift τ and of τ_c . (d) The time average of the convolution is a function of the correlation time τ_c and the particle-cell encounter rate, v_{ec} . For the purposes of the convolution integral of (c), the upper bound T_0 has been replaced by the essentially equivalent ∞ . Property (d) is further discussed below. The two time integrals in [9] may be rewritten as

$$\frac{1}{T_0} \int_0^{T_0} d\tau \int_0^T ds v_{dk}(s+\tau) v_d(s). \quad [\text{IV-D-9a}]$$

The convolution integral over s is a function of τ and τ_c . The τ integral of [9a] is $N \langle v_{dk}, v_d; \tau_c \rangle$, a function in time of τ_c only. N is the number of separate cells encountered by the test particle in the interval T_0 and $\langle v_{dk}, v_d; \tau_c \rangle$ is the average velocity convolution due to a single representative cell interaction. The encounter rate v_{ec} is the limit of (N/T_0) for very large T_0 . The averaging process over a long time span and many cell encounters is equivalent to the spatial average over the entire cell of the electron's initial position as it enters the cell. That is, this initial position is assumed to be a stochastic variable. This point will again be treated in detail under paragraph D(iv)3 where the diffusion coefficient is evaluated. With these assumptions and definitions, [9] may be rewritten as

$$\Gamma_k(t) = -\int d^3v v_{ec} \langle v_{dk}, v_d; \tau_c \rangle \cdot \nabla f_0(t). \quad [\text{IV-D-10}]$$

The term $v_{ec} \langle v_{dk}, v_d; \tau_c \rangle$ can be seen from [9a] to be merely the long time integral of the velocity correlation of the electron guiding center.

[10] is the fundamental expression for the average particle flux in the vortex picture. It is a generalization of the previous OD result in that the transport is caused by the ensemble average of the interaction between the tensor velocity convolution and the gradient of the distribution function. The OD results may be partially recovered for the closed field line case by averaging [4] around a closed flux and assuming that the lowest order gradient of f_0 is in the ψ direction. Then [10] becomes

$$\Gamma_{e\psi} = - \nabla_{\psi} (D_{e\psi\psi} n) \quad (\text{IV-D-11})$$

$$D_{e\psi\psi} = \frac{1}{n} \int d^3v v_{ec} \langle v_{ed\psi}, v_{ed\psi}; \tau_c \rangle f_{eo}(t) \quad (\text{IV-D-12})$$

$D_{e\psi\psi}$ is the vortex diffusion tensor. The major difference between [12] and the OD results lies in the electron-cell interaction rate, v_{ec} .

Evaluation of [12] is aided by the proper choice of reference frames. The cell is composed from the electron distribution and therefore moves with the distribution's drift due to the ambipolar electric field $\underline{E}_a$ and the magnetic field inhomogeneities. The important remaining drifts are due to the cell field $\underline{E}_c$ and due to a given electron's deviation from the mean magnetic drift. If $\underline{E}_c$ changes sufficiently slowly along the electron's trajectory through the cell, the polarization drift may be ignored completely. There is no contribution to v_d due to the deviation from the average magnetic drift because this is

completely in the ζ direction, after flux tube averaging. The term $v_{d\psi}$ in [12] is entirely from $\underline{E}_c$. The interaction rate v_c , however, must be Λ_c/v_{em} where Λ_c is the averaging spacing between cells and v_{em} is the particle's deviation from the distribution's magnetic drift velocity. [12] is evaluated later using these considerations and the measurements of section IV C.

An important result of the model is that, since there are no sudden adiabaticity breaking interactions, the electron remains continuously in its guiding center drift frame. This explains why [11] shows no mobility response to the ambipolar electric field. [11] may be written in the form of [III-C-4] if $\mu_{e\psi\psi} = 0$ is used.

[10] may be derived in a slightly different manner. The left hand side of [7] is actually $f_1(t) \sim f_1(t-T_0)$ and [8] has added to the right hand side $\int d^3v v_{dk}(t) f_1(t-T_0)$. The assumption of turbulence is such that $f_1(t-T_0)$ is assumed to be completely uncorrelated with the state of the system at time t . The time average of this added term is zero and [9] is unchanged. Turbulence assumption (b) is all that is necessary to make [7] valid.

(iii) Ion Transport

1. The kinetic equation: The dynamics of the ion distribution may be described for a collisionless plasma by the ion Vlasov equation

$$\frac{\partial f}{\partial t} + \underline{v} \cdot \nabla f + (\underline{\varepsilon}_a + \underline{\varepsilon}_c + \underline{v} \times \underline{\omega}) \cdot \nabla_v f = 0 \quad [\text{IV-D-13}]$$

where $\underline{\varepsilon} = e\underline{E}/m$, $\underline{\omega} = e\underline{B}/m$, and m is the ion mass. The term $\underline{\varepsilon}_a$ is the ambipolar electric field which is perpendicular to the flux shells,

slowly varying in time and space, and generated by the unequal ion and electron diffusion rates. The term $\underline{\epsilon}_c$ is the electric field of the turbulence cell and might easily be a strong function of time and space during its interaction with an ion. [13] describes the reduced single particle distribution for an ion and $\underline{\epsilon}_c$ contains the interaction information between the ion and all of the other particles which make up the cell field. However, the electric fields will again be approximated from outside considerations, as in the case of the electrons.

The kinetic equation of [13] contains information about the micro-scale behavior of the ion distribution. It is not very useful mathematically because it is virtually intractable, and it is not very interesting experimentally because the parameters available for comparison are macro-scale averages of all quantities. Therefore, [13] should be averaged over a time, T_0 , which is longer than either the particle-cell interaction time or the cell correlation time. The result is

$$\frac{\partial f}{\partial t} + \underline{v} \cdot \nabla f + (\underline{\epsilon}_a + \langle \underline{v} \times \underline{\omega} \rangle) \cdot \nabla_v f = -\langle \underline{\epsilon}_c \cdot \nabla_v f \rangle. \quad [\text{IV-D-14}]$$

The ion-cell interaction term has been placed on the right-hand side of the equation and plays the role of a collision operator. Specifically, it is

$$\langle \underline{\epsilon}_c \cdot \nabla_v f \rangle = \frac{1}{T_0} \int_{t-T_0}^t (e/m) \underline{E}_c \cdot \nabla_v f \, dt_1. \quad [\text{IV-D-15}]$$

This includes not only the effect on an ion for a single pass through a given cell, but also the non-random effect of multiple particle tran-

sits through the cell as the ion guiding center drifts past. Since [15] follows the ion trajectory, the integral is the number of such cell interactions occurring in T_0 times the average integral for a single encounter with a representative cell. This time average is the equivalent to an average over all possible initial approaches of the ion with the cell. The right-hand side of [15] is clearly proportional to v_{ic} , the ion-cell encounter rate, times a function of the average time for a single transit of an ion through the cell, T_{it} , the average number of successive passes through the same cell, and the cell correlation time, τ_c . The cells are at rest in the electron drift frame so v_{ic} is not the same as v_c in [12]. This is a complicated problem because unless $T_{it} \ll \tau_c$, E_c will change in overall magnitude even during a single pass of the ion through the cell.

2. The ion-cell collision frequency: Although an exact solution of [15] and [14] has not been accomplished, a qualitative understanding of the interaction is not difficult. Assume for simplicity that the cells do not appreciably disassemble during an encounter with an ion. Let N_{ic} be the average number of encounters during T_0 and N_p the average number of passes per encounter. v_{ic} is N_{ic}/T_0 and the effect of multiple transits may roughly be considered as an amplification of the effect of a single pass, i.e., N_p times the effect of a single pass. [15] may be approximated as

$$\langle \frac{E}{c} \cdot \nabla_v f \rangle \approx v_{ic} N_p \int_{t-T_{it}}^t (e/m) \frac{E_c}{c} \cdot \nabla_v f dt, \quad [IV-D-16]$$

Let v_{ic} be the relative velocity between an ion and the cells which are

assumed fixed in the electron drift frame, and Λ_c the mean separation between cells. N_p is the ratio of the total time required for the ion guiding center to drift past the cell to the period of gyration.

Figure IV 26(a) illustrates the ion-cell encounter. If the cell is L wide, the interaction drift path of the guiding center is $L + 2\rho_i$ long.

The relationships are

$$v_{ic} = v_{ic}/\Lambda_c$$

$$N_p = \frac{B e}{2 \pi m} \left[\frac{L + 2\rho_i}{v_{ic}} \right]$$

$$v_{ic} = \langle v_{eM} \rangle + v_{iM}$$

where $\langle \rangle$ indicates a distribution average, v_M the magnetic drift velocity. If an ion-cell collision frequency $\nu' = v_{ic} N_p$ is defined, then the collision operator of [16] becomes

$$\langle \underline{E}_c \cdot \nabla_v f \rangle = \nu' \int_{t-T_{it}}^t (e/m) \underline{E}_c \cdot \nabla_v f dt_1 \quad \text{[IV-D-17a]}$$

$$\nu_i' = \frac{B e}{2 \pi m} \frac{1}{\Lambda_c} (L + 2\rho_i) \quad \text{[IV-D-17b]}$$

The data of section C shows that L is roughly twice the thermal average gyro radius. [17] might be used with [14], the result averaged over a flux tube, and the final equation solved for the ion flux Γ_i . Since ν_i' is a function of v_A , this is still a difficult problem. The qualitative behavior of the interaction indicated by [17b] is sufficient, however, for the purposes of this present discussion. The collision frequency ν_i' is shown in Figure 26(b) as a function of v . For cold

Figure IV 26

A. Diagram of the interaction of an ion whose gyro radius is ρ with a potential cell whose spatial extent is L .

B. v_1' v.s. the ion kinetic velocity.

(A)

(B)

$$u'_{i0} = \frac{BeL}{2\pi m} \frac{1}{\Lambda_c}$$

ions, v_i approaches the constant value of $(e T_i / m)^{1/2} / \pi \Lambda_c$. For ions traveling even at the thermal speed, v_i has only doubled. High speed ions, however, experience a collision frequency proportional to the particle speed v . Hence the ion-cell interaction should effectively depopulate the ion distribution of its large v , "hot" component. This is an important consequence of the turbulence observations and will be discussed again in section IV E.

3. The ion particle flux: For this section, the analysis of [14] will be indicated from the viewpoint of an even more greatly simplified model. The path length through a cell, $L = |\int \underline{v}_\perp dt|$, will be sufficiently smaller than the gyro radius $\rho = v_\perp / \omega$ so that $\langle \underline{v} \times \underline{\omega} \rangle$ may be replaced by $\underline{v} \times \underline{\omega}$. The cell field of [15] causes stochastic scattering only in directions perpendicular to $\underline{B}$, so [14] may be averaged over the (closed) flux tube and the parallel variation in f removed. This reduces the interaction to an essentially two-dimensional problem. The picture of this simplified interaction is similar to the well-understood problem of ion transport due to scattering from small centers such as charged particles or neutrals. The particle flux $\underline{\Gamma} = \int d^3v \underline{v} f$ may be found in the usual¹⁶ manner and the result is

$$\underline{\Gamma}_i = -\nabla \cdot (\underline{D}_i n) + n \underline{\mu}_i \cdot \underline{E}_a. \quad [\text{IV-D-18}]$$

The simplified picture was adopted to demonstrate the important feature that the ion distribution displays a mobility response to the ambipolar electric field, $\underline{E}_a$. This is quite different from the electron case where $\underline{\mu}_e$ has been shown to be zero. If f_i is essentially a Maxwellian, then the standard¹⁷ Fokker-Einstein relation $\underline{D}_i = T_i \underline{\mu}_i$ is

obtained for the simplified model.¹⁸

(iv) Calculations From The Model

The cell observations reported in IV C are sharply limited in space by the ion gyro diameter and live for about 10 μ sec in the 2 kG field. Since ρ_i is much larger than ρ_e and the ion gyro period is about .3 μ sec, these long-lived structures must be composed from the electron distribution. These are the fundamental observations which underlie the model and upon which all the estimates of this section depend. The physical mechanism responsible for these cells must be the random fluctuations in the charge density of the electron distribution. The electron thermal speed is very high and a 7 eV electron will cycle around the 70 cm long B line at $\psi = 3.4$ in about one μ sec. In that time it will have made about 5×10^3 gyrations about the B line. On the time scale of the cell, the electrons will have averaged out the parallel variation effects of the field lines. The fluctuations mentioned above must refer to random variations in the density of guiding centers rather than the electrons themselves. After the parallel averaging has been done, the motion of the guiding center distribution on the flux surface is one-dimensional, in the toroidal direction.

1. Correlation times: There are two mechanisms which might damp out a cell. One is viscosity or the equivalent diffusion damping. These are related effects and will be discussed more fully in IV E. Essentially, it is well known¹⁹ that the diffusion causes viscosity which damps fluid structure on time scales of $L^2/D_\perp$. Using the observed values for $2\lambda_c$ and $D_\perp$ of 6 mm and 700 cm²/sec, the damping

time would be 500 μ sec. Clearly the observed cells are not being damped by viscosity.

(a) τ_c from the model. The other mechanism is the differential motion of the electrons in their average drift frame caused by the presence of gradients and curvature in the magnetic field. If v_M is the average magnetic drift speed of the guiding center distribution, then the r.m.s. velocity, v_m , of the electrons in their own average drift frame is easily shown to be $v_m = (3/4)^{1/2} v_M$, assuming an isotropic Maxwellian distribution. If a potential caused by a charge imbalance exists with a spatial extent of L , it will be disassembled in the transit time of an electron through the cell, L/v_m . This is also the characteristic assembly time for such a cell and the identification

$$\tau_c = L/v_m \quad \text{[IV-D-19]}$$

may reasonably be made. The flux tube averaged v_M is given²⁰ as $2\pi R V''/V'$ where R is the distance from the measurement point to the major axis, V'' and V' are ψ derivatives of flux tube volumes and are given in tables.²¹ For the high field case at $\psi = 3.4$, v_M for a 7 ev electron is $.9 \times 10^5$ cm/sec directed opposite the drift caused by the ambipolar E_a . At this ψ value, a 7 ev electron will cycle the field line roughly 3 times for each 2 mm it drifts toroidally. The observations indicate that the proper cell size to use is some parallel average of twice the observed correlation length, $L = 2\langle\lambda_{\parallel}\rangle$. The flux tube average for L is 9.0 mm, corresponding to τ_c of 11.5 μ sec. The field line average for L is 8.4 mm, corresponding to τ_c of 10.8 μ sec. If the local L value of 6 mm is used (no averaging done), τ_c becomes 7.6 μ sec.

These lie within acceptable limits of the observations of IV C(ii) and $\tau_c = 10 \mu\text{sec}$ will be used in the subsequent analysis.

(b) Analysis of the data. A turbulence cell constructed from a Maxwellian might be expected to obey a Gaussian self correlation time dependence in its own frame of reference. Weinstock has pointed out²² that this is the most probable time dependence expected from strong turbulence theories, and, indeed the one seen by the computer simulation "experiments," such as reported in the OD paper. It should be mentioned, however, that to date all theories and simulations have been for $\rho_i = \rho_e$ plasmas. Extrapolations from these results to the hot ion case of this current work might be risky.

Whatever the theoretical justification, the correlations presented in section C may all be well represented with Gaussian functions. Since the CF of a Gaussian is a Gaussian (see [IV-B-1]), the time and space dependence of a cell in its own reference may be written as

$$V_c(x,t) = V_c \exp[-(x-x_c)^2/\lambda_c^2] \exp[-(t-t_0)^2/\tau_c^2].$$

If the cell is seen from the laboratory frame to move at the speed v and reach its maximum at $x = x_0$ when $t = t_0$, the observed signal is

$$V(x,t) = V_c \exp\{-[(x-x_0) - v(t-t_0)]^2/\lambda_c^2\} \exp[-(t-t_0)^2/\tau_c^2].$$

[IV-D-20]

The CF for the signals from two probes, one at x and the other at $x+y$, may be found with considerable algebraic effort. The result is independent of t_0 and after C has been averaged over the probability of finding the peak at x_0 , the answer is

$$C(\tau, z) = \left(\frac{\pi}{2}\right)^{\frac{1}{2}} v T_{ef} v_c^2 \exp\left[-\frac{\tau^2}{2\tau_c^2} - \frac{(z - v\tau)^2}{2\lambda_c^2}\right] \quad \text{[IV-D-21a]}$$

$$\frac{1}{T_{ef}^2} = \frac{1}{\tau_c^2} + \frac{1}{T_t^2} \quad \text{[IV-D-21b]}$$

$$T_t = \lambda_c / v \quad \text{[IV-D-21c]}$$

where v is the average frequency at which such cells are seen. Compare this with the result of [IV-C-2]. The effective time T_{ef} is the observed correlation time for an autocorrelation function. Notice that [21a] reaches its maximum value at

$$\tau_{pk} = \frac{zv}{\lambda_c^2} T_{ef}^2 \quad \text{[IV-D-21d]}$$

If the transit time of a cell past the probe is much shorter than the correlation time, $T_t \ll \tau_c$, then $T_{ef} = T_t$ and [21d] is $\tau_{pk} = z/v$. This is the usual method of measuring phase velocities with a cross CF.

However, if $T_t \geq \tau_c$, τ_{pk} will be significantly reduced from z/v and an effective velocity $v' = z/\tau_{pk}$ will be much higher than the actual v .

This is a physically plausible result as can be seen from the cell description of [20]. The decorrelation of a cell as it reaches the second probe reduces the amplitude of the spatial peak. The peak of the CF should occur at an intermediate value between the time ($\tau = 0$) it passes the upstream probe and the time ($\tau = z/v$) when the spatial peak passes the downstream probe. This was previously discussed in C(ii). [21a] reduces to [IV-C-2] or [IV-B-1] for the case that $\tau_c \gg T_t$.

The hydrogen data of C(ii) at $\psi = 3.4$ may now be examined in more

detail. The only point that could be made when this work was first discussed was the simple observation that as the signal passed the probes, the 2 kG data did not decorrelate but the 1 kG did. This was the result of using the HWHM value of the auto or cross CF peak, multiplying it by the effective velocity $v' = v/\tau_{pk}$ and comparing this with the observed λ_c . T_{ef} is the HWHM of the CF of [21a] for $z = 0$ and is easily obtained. Since z , τ_{pk} and λ_c are also known or measurable, v may be obtained from [21d].

The decorrelation time τ_c may be estimated for the 1 kG data of $\psi = 3.4$ using [21b] since the measurable quantities are τ_{pk} , z , λ_c , and $T_{ef} = \tau_H$, the half width of the half maximum value. [21d] will give v and [21c] will provide T_c . It is convenient to rewrite [21b-d] in terms of a dimensionless parameter p and $v' = z/\tau_{pk}$

$$p = \frac{\tau_{pk} \lambda_c}{T_{ef} z} \quad [\text{IV-D-21e}]$$

$$v = p^2 v' \quad [\text{IV-D-21f}]$$

$$\tau_c = \frac{T_{ef}}{(1 - p^2)^{1/2}} \quad [\text{IV-D-21g}]$$

T_{ef} is measured from the 8 auto CFs and 4 CFs of the data to range between 5 and 6.5 microseconds with most of the values between 5.5 and 6.0 μsec (the CFs are calculated in .5 μsec steps). Table 1 gives the values of the parameters for a $T_{ef} = \tau_H$ of 5.5 μsec and $\lambda_c = 6 \text{ mm}$. The values in parentheses are for T_{ef} of 6 μsec . The $z = 3.4$ data contain the most error since τ_{pk} is poorly determined so its values are ignored in the error estimates of the summary presented there. In conclusion,

TABLE IV 1

z	τ_{ph}	p	$v'(10^5 \text{ cm/sec})$	$v(10^4 \text{ cm/sec})$	$T_t (\mu s)$	$\tau_c (\mu s)$
6.4	3.25	.55(.50)	2.0	6.3(5.2)	10(12)	6.6(6.9)
5.0	2.75	.6(.55)	1.8	7.2(6.0)	8.3(10)	6.9(7.2)

3.4	1.25	.4(.37)	2.72	4.4(3.7)	14(16)	6.0(6.5)

$$v' = (1.9 \pm .1) \times 10^5 \text{ cm/sec}$$

$$v = (6. \pm 1.) \times 10^4 \text{ cm/sec}$$

$$T_t = (10 \pm 2) \mu\text{sec}$$

$$\tau_c = (6.9 \pm .3) \mu\text{sec}$$

the estimated value of τ_c is about 7 μ sec and is most sensitive to errors in T_{ef} .

2. Electron-cell encounter rate, v_{ec} : This is the average rate that an electron moving in its own average drift frame will meet successive cells. As shown in [12], this number is required to calculate D_L . If the average cell spacing Λ_c were known, say from an array of many small probes, v_{ec} could be estimated as v_{em}/Λ_c . Even if such a large array did exist, however, the data might not be trustworthy due to the (unknown) effect of the many obstacles on the turbulence cells.

An alternate method would be to measure the probe-cell interaction rate, v_p , transform to the turbulence frame and calculate either v_{ec} if $v_p^{-1} \ll \tau_c$ or the cell generation rate in its own frame if $v_p^{-1} \gg \tau_c$. The data presented in IV C(1)2 shows that v_p^{-1} is between 20 and 30 μ sec, $2 \times \dots 3 \times \tau_c$. A more accurate determination would require the average of an unfeasibly large number of machine pulses, or of a fast digitizer with more storage capacity than available.

In the estimates of subsection E, v_{ec} will be assumed to lie within the same range as v_p , 30 to 50 kHz. This is not an unreasonable spontaneous generation rate for a cell in the electron frame which requires τ_c to develop, τ_c to decay and immediately to be replaced by the generation of a new cell. A thermal electron in that frame would be stationary and v_{ec} would indeed be the spontaneous generation rate. From another viewpoint yet, the r.m.s. deviation velocity, v_{em} , is only slightly smaller in the turbulence drift frame than that frame's velocity in the laboratory system (.8:1). An electron moving at v_{em} would encounter the cells at only a slightly lower frequency than does the

probe. For ν_p of 50 kHz, ν_{ec} would be 40 kHz.

3. Estimate for D_L : The formula for D_L is presented in [12], which is the ensemble average of [9a] for the case where $\hat{\psi} \times \nabla n = 0$ and all parallel motion has been averaged away. Properly speaking, the entire problem should be solved from first principles using the kinetic equation and Maxwell's equations. τ_c and λ_c should be natural consequences of the solution, as well as the other parameters. To develop a formula for D_L , however, [9a] will be estimated on the basis of the vortex model shown in Figure 1. This work will be done for a thermal electron since the decorrelation effects have been added to the model on an ad hoc basis. For this case, $D_L = D_{e|\psi}$ is given by [9a] which may be rewritten for convenience as

$$\frac{1}{T_0} \int_{-T_0/2}^{T_0/2} dt \int_{-T_0/2}^{T_0/2} ds v_{d\psi}(s) v_{d\psi}(s+\tau). \quad [\text{IV-D-22}]$$

The cell field will be modeled from the data to be

$$V_c(r,t) = V_c \exp(-r^2/\lambda_c^2) \exp(-t^2/\tau_c^2) \quad [\text{IV-D-23}]$$

$$\underline{E}_c(r,t) = \hat{r} \frac{2r}{\lambda_c^2} V_c(r,t). \quad [\text{IV-D-24}]$$

E_c reaches its maximum value of $.86 V_c/\lambda_c$ at $r = .71 \lambda_c$. At $r = \lambda_c$, E_c has decreased to $.86 E_{\max}$. The cell is quite sharply limited in spatial extent from the viewpoint of particle transport. Using the heuristic formula of [3], D_L is proportional to E_c^2 , so for all else constant, D_L is reduced to 1% of its maximum value where E_c is $.1 E_{\max}$.

This defines the effective cell size and occurs at $r = 1.95 \lambda_c$ and also at $r = .043 \lambda_c$. Since the smaller value is roughly the electron orbit size, essentially the entire cell participates in the diffusion, from $r = 0$ to $r = 2 \lambda_c$.

The first question to resolve is the identification of v_d . The two candidates are the electric drift $v_{E\psi} = (E_c/B) \sin \alpha$ and the polarization drift $v_{p\psi} = -2(t/\tau_c) v_E \cos \alpha / (\omega_{ci} \tau_c)$ where $\alpha = \cos^{-1}(\underline{E}_c \cdot \hat{\psi})$. Since $\omega_{ci} \tau_c$ is on the order of 10^4 , the maximum transport caused by v_p is much smaller than that caused by v_E and so will be neglected.

Both the s and τ integrations of [22] follow the electron trajectory and so for $|\tau|$ large enough, the s integral will convolve velocities from different cells. The cells which have been observed are well isolated in space and do not overlap so that the electron sees one cell at a time. (If the cells overlapped, the experimental correlations would have been zero because the continuous overlapping of many cells would simply add an undetectable "D.C." offset.) The model employed here makes the reasonable assumption that the velocities from different cells are uncorrelated. The s integral therefore becomes the convolution of a single cell. The s integral of [22] is

$$\int_{-\infty}^{+\infty} ds \sin^2 \alpha(r, t) \frac{E_o^2(r)}{B^2} \exp\left[-\frac{s^2}{\tau_c^2}\right] \exp\left[-\frac{(s+\tau)^2}{\tau_c^2}\right] \quad [\text{IV-D-25}]$$

The integrand is non-zero only in the narrow cone of values around $\alpha = \pm \pi/2$. A thermal particle will move in a circle around the cell. If α_0 is the value of the angle at the beginning of the interaction, at any time t , $\alpha = \alpha_0 + v_E t/r$. In order to estimate the total change in

α , assume that the cell maintains its full magnitude of V_o/α_c during the correlation time τ_c . Then $\alpha - \alpha_o = (V_c/\lambda_c B) \tau_c/\lambda_c$ which, for $\lambda_c = \rho_i = 3$ mm, $B = 2$ kG, $\tau_c = 10$ μ sec, and $V_c = 60$ mV, is about .3 radians. α changes by a small amount during the interaction, less than 15 to 20 degrees. The approximation to be used is that if $\sin^2 \alpha_o \approx 1$, then the sine function remains essentially unity throughout. Only the α_o values of $\pm \pi/2$ will be used for this estimate of D. [25] may be integrated to give

$$\frac{E_o^2(r)}{B^2} \left(\frac{\pi}{2}\right)^{\frac{1}{2}} \tau_c \exp(-\tau^2/2\tau_c^2) \quad \text{[IV-D-26]}$$

for the convolution in [23] where

$$E_o(r) = \frac{2r}{\lambda_c^2} V_c \exp(-r^2/\lambda_c^2). \quad \text{[IV-D-27]}$$

τ integrates along the electron trajectory for many cell encounters and is the sum of the direct τ integral of the single cell result of [26] for each. The distance r from the center of a cell is different for each encounter and is a random variable. The τ integral of [22] may be replaced by the number of electron-cell encounters N occurring during the time T_o times the single cell τ integral of [27], using for $E_o^2(r)$ the value for an average encounter distance.

The spatial average to be used is not clearly indicated. The simple average over all space is zero because there is much more volume outside the sphere of interaction than inside. This is a bad estimate because the average should be performed for the interaction.

One could integrate E_c^2 from 0 to some cutoff length such as $2\lambda_c$.

then divide by $2\lambda_c$. The answer should then be doubled to account for the two values of α_0 , $\pm \pi/2$. The value obtained is

$$\langle E_c^2 \rangle = 2 \int_0^{2L_c} dr E_c^2 / 2\lambda_c = .5(\pi/2)^{\frac{1}{2}} V_c^2 / \lambda_c^2 . \quad [\text{IV-D-28}]$$

This is not a satisfactory estimate because the magnitude of the answer depends strongly on the value actually used for the cutoff.

Another method is to use a weighted average. The transport is caused by the vortex drift velocity v_E and the average could be weighted most heavily where this is large. The average so obtained is

$$\langle E_c^2 \rangle = 2 \int_0^\infty dr E_c^3 / \int_0^\infty dr E_c = (8/9) V_c^2 / \lambda_c^2 . \quad [\text{IV-D-29}]$$

The factor of 2 in [29] comes from the two values of α_0 .

Another weighting factor might be the diffusion coefficient itself, or the particle flux. Both of these are proportional to E_c^2 so that the resulting average is

$$\langle E_c^2 \rangle = 2 \int_0^\infty dr E_c^4 / \int_0^\infty dr E_c = (9/8)^{\frac{1}{2}} V_c^2 / \lambda_c^2 . \quad [\text{IV-D-30}]$$

Equations [29] and [30] both yield a value of nearly V_c^2 / λ_c^2 , as would [28] if a cutoff of $1.15 \lambda_c$ were used. (This is where $E_c^2 = .5 E_{\text{max}}^2$.) This will be the value used for the estimate of section E.

Using all these values, [22] may be evaluated. Since $D_\perp$ is [22],

$$D_\perp = \frac{\langle E_c^2 \rangle}{B^2} - \frac{1}{2} \frac{\pi}{2} \tau_c^2 v_{ec}$$

where $v_{ec} = N/T_0$ is the average electron-cell encounter rate. Considering the inexactness of the derivation, the numerical factors in the D_1 estimate will be dropped and the value to be used will be

$$D = \frac{\langle E^2 \rangle}{B^2} \tau_c^2 v_{ec} = \frac{V_c^2}{\lambda_c^2 B^2} \tau_c^2 v_{ec} \quad \text{[IV-D-31]}$$

[31] must be considered to be a lower bound on D_1 because of the various ignored effects which might add significant contributions when summed up over the entire cell. The estimate was performed for thermal electrons which are assumed to move only in response to the cell field. Any other electron will move across the cell with the magnetic drift speed. A particle moving with v_{em} will drift the distance L_c in a correlation time. Much colder or hotter electrons will move farther. Since these electrons will experience a considerable change of E_c along their drift path, the convective term of the polarization drift, $(\mathbf{v}_m \cdot \nabla) E_{c\psi} / \omega_{ci} B$, may be significant. Initial estimates, using the $\langle v_m \rangle$ reported in D(iv)1 and the various scale values, indicate that $v_{p\psi}$ is comparable to $v_{E\psi}$. One would need to include this effect in the solution to [9a], then average over the velocity distribution to get [12]. These geometrical corrections to D_1 can not be obtained from the formalism of the OD theory, or any other, in fact, which does not allow for curvature and gradients in the magnetic field.

E: Implications

The data presented in part C and the resulting kinetic model developed in part D provide the framework in which to view the micro-

turbulence processes in the levitated octupole. The fundamental question remains, however: Are the microturbulence structures important enough to account for the observations? A numerical estimate for $D_{\perp}$ is developed first and shows that $D_{\perp}$ is within an order of magnitude of the reported value. Then, assuming that the turbulence is responsible for the particle losses, several interesting implications on the bulk plasma properties are investigated.

(i) Evaluation of $D_{\perp}$

Equation [IV-D-31] is the estimate for the lower bound on D to be used for this analysis. The numerical evaluation is made for the hydrogen data taken in the bridge region of the lower outer ring at $\psi = 3.4$. This location was chosen because the geometry is simple there and the necessary average values are easily performed. This is also the region where the diffusion coefficient has best been measured.¹ V_c is given in IV C(iii) as 60 ± 10 mv, v_{ec} is given in IV D(iv)2 as 40 ± 10 kHz, λ_c is shown in IV C(i)2 to be 3 mm, τ_c is estimated in IV D(iv)1 to be 10 μ sec, and the magnetic field B is 2 kG. These values yield a lower bound for the local $D_{\perp}$ value of

$$D_{\perp} = 400 \text{ cm}^2/\text{sec}. \quad [\text{IV-E-1}]$$

If the measured value of $\tau_c = 7 \mu$ sec is used (Table 1), then $D_{\perp}$ is 200 cm^2/sec . The maximum value $D_{\perp}$ could have from these data is 680 cm^2/sec (350 cm^2/sec using the upper bound on the measured τ_c). In addition to the pessimistic nature of [IV-D-31], the value measured for V_c could be low for the two reasons discussed in Chapter II A(v)2: If

the sheath impedance were sufficiently high because of the low density, the operating point of the tips would drift on the characteristic curve to a position with V_{tip} below the actual value of V_f . Isothermal oscillations at this point would not necessarily cause probe potential oscillations equal to plasma potential oscillations. The tips were flame cleaned prior to insertion into the vacuum chamber but they could have been contaminated during insertion by vacuum grease or dirt in the air lock channel. (The octupole is a multiuser machine.) A resistive coating on the tips acts as a voltage divider and the measured potentials will again be reduced from V_f . A 15% increase in V_c from the average value given would change D_L of [1] to 500 cm²/sec. In any event, [1] is within a factor of 2 of the reported value of 700, although D_L using the measured r_c is a factor of 4 low.

(11) T^*

Chapter III showed that the electric field E which is perpendicular to the magnetic flux shells is given by $E = -T^*/a_n$, where $a_n = n/|\nabla n|$ is the density scale length. It was further shown in [III-D-2] that $T^*/(dT^*/dt) = T_i/(dT_i/dt)$. By [III-C-7],

$$T^* = (D_{e\perp} - D_{i\perp})/(\mu_{e\perp} + \mu_{i\perp}).$$

In the paragraphs following [IV-D-11], it was shown that $\mu_e = 0$ so T^* may be rewritten as

$$T^* = \frac{D_{i\perp}}{\mu_{i\perp}} \left(\frac{D_{e\perp}}{D_{i\perp}} - 1 \right). \quad \text{[IV-E-2]}$$

If the ion distribution were Maxwellian then $D_{i\perp} = T_i \mu_{i\perp}$ would be true. However, as discussed following [IV-D-17b], the ion distribution function has its hot tail reduced in size or simply missing. The bulk of f_i should be nearly Maxwellian, though, because v_i' is essentially constant for most of the particles. It should be expected that $D_{i\perp} = \alpha T_i \mu_{i\perp}$ for some α which is on the order of 1. [2] then may be rewritten as

$$T^* = T_i \alpha \left[\frac{D_{e\perp}}{D_{i\perp}} - 1 \right] \quad [\text{IV-E-3}]$$

[3] explains the relationship of [III-D-2] if $D_{e\perp}/D_{i\perp}$ vary much more slowly than T_i .

(iii) D_a

The ambipolar diffusion coefficient was given by [III-C-9] as

$$D_a = \frac{\mu_{e\perp} D_{i\perp} + \mu_{i\perp} D_{e\perp}}{\mu_{i\perp} + \mu_{e\perp}}$$

Again applying $\mu_{e\perp} = 0$, D_a becomes

$$D_a = D_{e\perp} \quad [\text{IV-E-4}]$$

The measurements of $D_a = \Gamma/|\nabla n|$ therefore measure $D_{e\perp}$ and should exhibit vortex diffusion scaling because $D_{e\perp}$ does, even though $D_{i\perp}$ does not.

(iv) T_e

The electrons experience adiabatic interactions with the turbulence cells since $\lambda_c \gg \rho_e$ and $\tau_c \gg \omega_{ce}^{-1}$. Okawa²³ has pointed out that this transport mechanism will conserve the magnetic moment of the electrons. Such a process should put energy into the electron distribution as the particles diffuse out of the minimum B well. It is fairly easy to estimate the effect of this process on T_e , as will be done in the following paragraphs. It should be noted that this effect should be true for vortex diffusion in general, whether caused by microturbulence or by large convective cells.

For a density of $3 \times 10^{13} \text{ cm}^{-3}$ and $T_e = 7 \text{ eV}$ the electron Maxwellization time (see Rose and Clarke, equation 8.25) is about 300 μsec . This is much faster than any of the diffusion time scales of the plasma so for the calculation, the electron distribution is assumed to be isotropic and Maxwellian. The ensemble average of the magnetic moment μ_m is (see PLP 470 and PLP 654)

$$\langle \mu_m \rangle = k T_e / B$$

and its time derivative is

$$\frac{d\langle \mu_m \rangle}{dt} = 0 = \frac{k}{B} \frac{\dot{T}}{T} - \frac{k T}{B^2} \dot{B}$$

or

$$\dot{T}/T = \dot{B}/B. \quad [\text{IV-E-5}]$$

$\dot{B}$ is caused by the motion of the particle through the field so

$\dot{B} = \underline{v} \cdot \nabla B$. Parallel motion is to be ignored since flux tube averaging

will be employed. [5], therefore is

$$\dot{T}/T = (\underline{v} \cdot \nabla B)/B.$$

The right-hand side is > 0 since at $\psi = 3.4$, v_ψ is toward the ring and B increases in that direction. The velocity is obtained from $\underline{\Gamma} = n \underline{v} = -D \nabla n$ and, as is well known,²⁴ $\nabla_\psi = 2\pi R B \partial/\partial\psi$. Putting these together, [5] becomes

$$\frac{\dot{T}}{T} = -D \frac{1}{n} \frac{\partial n}{\partial \psi} (2\pi)^2 R^2 B \frac{\partial B}{\partial \psi}. \quad [\text{IV-E-6}]$$

Now perform the flux tube averaging with n , T and D all functions of ψ only. [6] becomes

$$\begin{aligned} \frac{\dot{T}}{T} &= -\frac{1}{V'} \oint \frac{D}{n} \frac{\partial n}{\partial \psi} (2\pi)^2 R^2 B \frac{\partial B}{\partial \psi} \frac{d\chi}{B^2} \\ \frac{\dot{T}}{T} &= -(2\pi)^2 D \frac{1}{n} \frac{\partial n}{\partial \psi} \frac{1}{V'} \oint R^2 \frac{\partial B}{\partial \psi} d\ell. \end{aligned} \quad [\text{IV-E-7}]$$

The integral in [7] must be done numerically.

This has not been done for the octupole geometry but an estimate may be made using the simpler geometry of a linear current carrying conductor. The field is $B(r) = B_0 r_0/r$ where r is the distance from the center of the conductor. [6] becomes $\dot{T}/T = D/a_n r$ where $a_n^{-1} = |\nabla n|/n$. The rings are 8.9 cm in radius and the measuring position is 1.5 cm further out. If $r = 11$ cm, $a_n = 2$ cm, and $D = 700$ cm²/sec, $\dot{T}/T$ is +32 Hz, $\tau_T = T/\dot{T} = +30$ msec. Since this is time scale of the temperature decay, the simple geometry is not sufficient and the problem probably merits more detailed attention.

(v) T_1

The time decay of the ion temperature is the fastest bulk time scale of the plasma. Breun²⁵ postulated that the 10 to 15 msec hydrogen ion cooling time was due to collisions with methane in the background gas. Subsequent to his work, however, a helium cryo panel was installed in the torus and the residual gas analyser on the system showed that the methane had been removed. Hydrogen ion temperature measurements in the clean low background pressure plasma still displayed the 12 ± 5 msec T_1 decay.

The observations of this chapter provide another mechanism for this cooling rate. It was reported in the last subsection that the electron Maxwellization time τ_{em} is typically 300 μ sec. It is well known²⁶ that the ion Maxwellization time, τ_{im} , defined as the time required to scatter an ion 90° off of other ions is $\tau_{em} (m_1/m_e)^{1/2} / Z^2$. (Rose and Clarke's equation 8.23 could also be used.) For hydrogen, this is 13 msec. The close agreement between the measurement and the prediction probably should not be taken too seriously. With all due consideration for the roughness of the estimate, however, one should become interested in possible cooling explanations from some relaxation process of a perturbed ion distribution.

Such a perturbation is required by the model developed in section D(iii). The collision frequency ν_1' of [IV-D-17] was shown to be proportional to v for large v but nearly constant for thermal ions, or colder. The cooling mechanism is then the following. The ion-cathode interaction depopulates the distribution of its high v particles. Ion-ion collisions then refill the tail (or attempt to). These ions come

from the thermal or hotter portion of the distribution, shifting the average kinetic energy of the distribution to a lower value. It is this continuous loss of the hottest ions and the Maxwellizing rearrangement of the distribution that causes the net temperature to decay away. The quantitative decay rate for T_i has not been solved for a truncated distribution, but the Maxwellization time is a qualitatively proper estimate.

As a check, the T_i decay time for the hot drift tank gun helium plasma could be measured. This has not been done, but if the hypothesis is accurate, it should be on this order of magnitude. The estimate is quite sensitive to T_e . For singly ionized helium, if n is $3 \times 10^9 \text{ cm}^{-3}$ and $T_e = 5 \text{ ev}$, τ_{em} is 140 μsec and $\tau_{im} = 11.5 \text{ msec}$. If T_e is 2 ev, τ_{em} is 25 μsec , $\tau_{im} = 2 \text{ msec}$. In all estimates, the coulomb logarithm is taken to be 15.

F: Turbulence Observations From Other Devices

Turbulence studies in several other devices display interesting correspondences to the results reported above for the octupole. Such measurements are relatively recent because they require high speed digital devices for acquiring and computers for processing the data. Earlier techniques using analog methods of acquisition and processing were difficult to adjust and maintain, were limited in their time of frequency resolution, did not offer the same flexibility of control and analysis as digital techniques, and did not gain very wide usage.

(1) Berezin, Buligihsky, Vildjuna: PIG discharge

In this 1970 Soviet study,²⁷ information was acquired by photo-

graphing oscilloscope traces, digitizing them, then using a computer to calculate correlation functions. This is an expensive, tedious, time-consuming method and usually results in a minimal number of data records being evaluated. Using this technique, a helper and I required six months to evaluate the oscilloscope data taken in two weeks on the octupole.

The hydrogen data presented here was correlated in time and space and the CFs were pulse averaged. The CFs shown in the paper are too limited in range for subsequent FT analysis. No information is given on any analog or digital frequency filters employed, nor on the basic plasma parameters. A low level coherent oscillation is visible in the CFs but not discussed. A turbulence correlation time $\tau_c = 10 \mu\text{sec}$ was measured by moving probes far enough apart that the spatial decorrelation of a single cell was observed. This is also much longer than the gyro period of the ions in their 2 kG field. The spatial correlation visible from their data is about 5 mm, but the resolution is poor. Unfortunately, no further information on T_i , T_e or drift speed are available.

(ii) Mazzucato: A.T.C

This study²⁸ in the uncompressed A.T.C. discharge measured k directly from the scattering of 20 W, 70 GHz microwaves by small scale density oscillations in the plasma column. The tokamak was operated with a toroidal field of 20 kG and a discharge current of 70 kA. The electron density was $2 \times 10^{13} \text{ cm}^{-3}$, $T_i = 150 \text{ eV}$, and T_e was $2.6 T_i$.

Six receiving horns were set up in the chamber to allow determination of six different values of k . These values were obtained using a direct FT on the raw data and show the high noise level in the results that should be expected from this technique. Let $\rho_s = c_s/\omega_{ci}$ and $\rho_i = v_{th}/\omega_{ci}$ where c_s is the ion sound speed and v_{th} is the ion thermal speed. ρ_i is the ion gyro radius and ρ_s is a related scale length. Note that for these data $\rho_s = 1.6 \rho_i$. The graph of electron density oscillations amplitude v.s. $|k_A|$ is presented and Mazzucato identifies the single broad peak as occurring at ρ_s^{-1} . However, the peak is relatively flat between $\rho_s^{-1} = 7 \text{ cm}^{-1}$ and $\rho_i^{-1} = 11 \text{ cm}^{-1}$ with rapid fall off for $k_A > \rho_i^{-1}$ and $k_A < \rho_s^{-1}$. These results may be summarized by $k_A \rho = 1$, leaving unanswered the question of whether ρ is ρ_s or ρ_i .

(iii) Hamberger, Sharp, Lister, Mrowka: TORISO

This experiment²⁹ on the Culham torsatron was conducted during the ohmic heating discharge (1 ... 10 kA) at moderate fields (4 ... 10 kG). The density was about 10^{13} cm^{-3} and T_e was less than 40 eV in the data shown. No information was given for T_i . The fluctuation data were taken by four microwave interferometer beams. Fluctuations were important to diffusion only for sharply peaked density distributions when the electron drift velocity was above 2% of the thermal velocity. The information reported in these paragraphs is only for the case of 'wide band, apparently uncorrelated "noise" signals.'

They report that $k_A \rho_s \approx 1$ where ρ_s is defined in (ii). Since no estimates of T_i are provided, ρ_s can not be related to ρ_i . The corresponding turbulence scale length is reported to be 0.5 ... 1.0 cm, the

variation presumably due to changes in B , T_e , $(T_i?)$, I , and n which are not detailed.

(iv) Surko, Slusher: A.T.C.

The turbulent k spectrum of the A.T.C. tokamak was again examined,³⁰ this time for a 10.6 micron CO_2 laser scattering experiment. The data are from nearly forward scattering of the 200 W beam, resolving spatial fluctuation lengths less than 10 cm. The data were acquired in narrow frequency bands at successive steps in f . The filter was set to operate for $\Delta f/f$ at about .1. The resulting frequency information was obtained using the FT of the raw data. The machine parameters of the experiment were $B_t = 16$ kG and $I = 70$ kA. The plasma density was $1 \times 10^{13} \text{ cm}^{-3}$ and T_e was reported to be 800 eV. The ion temperature was nowhere discussed. B_t is lower and T_e higher than in Mazzucato's report, discussed in (ii). Using the symbols defined in (ii), ρ_s is 2.6 mm and ρ_s^{-1} is 3.9 cm^{-1} .

To within the measurements' accuracy, $\underline{k} \cdot \underline{B}_t = 0$ and $k_r = k_\theta$. Plots of $n(k, f)$ v.s. f for k held constant show that f peaks at zero, or at least at very low values, for the different k_r . There is a not very marked tendency for f to peak somewhat below 100 kHz for $k_\theta^{-1} > 4$ mm. Plots of $n(k, f)$ v.s. k for fixed f show a definite tendency to peak at finite $k_\perp > 0$. The error bars in k are wide (5 cm^{-1}) but the same broad peak around $k = 10 \text{ cm}^{-1}$ is observed for $f = 500$ kHz. The peak possibly shifts to smaller k values as f decreases and at the lowest value of $f = 100$ kHz, the peak is nearly ρ_s^{-1} . The uncertainty in the k values makes comparison with Mazzucato's results difficult.

At all frequencies shown, the k error bars of the peak overlap for values of k between about 5 and 8 cm^{-1} . Since T_i is unknown, there is no way of comparing this with ρ_i^{-1} .

(v) Okabayashi, Arunasalam: FM-1

This turbulence study conducted with the FM-1 spherator³¹ was made using microwave scattering to obtain an estimate for the k spectrum. Five antennae were installed inside the vacuum chamber to receive the signals scattered by the plasma. Three different values of the toroidal component of k (k_ϕ) could be measured ($k_\phi \rho_i = .25, .5$, and $.70$) and one value each for the poloidal k (k_χ) and the component perpendicular to the flux surface (k_ψ). Thus the k space resolution of the fluctuations is much poorer than for the A.T.C. results, above.

The helium plasma in a microwave discharge had $n = 2 \times 10^{11} \text{ cm}^{-3}$, $T_e = 3 \text{ eV}$, and $T_i = 2 \text{ eV}$. The turbulence results are quite dependent on the shear length. For very high shear, $n(k, f)$ is strongly peaked in k . The only scattered signal seen was for $k_\phi \rho_i = .5$ (k_χ was not reported). As the shear length increased, the data showed that $n(k_\phi)$ broadened but remained more or less peaked at k_ϕ of $.5/\rho_i$. In addition, signals were seen in the ψ and χ antennae. At low shear lengths, all stations had essentially the same sort of signals. This was interpreted to mean that the turbulence was isotropic. Except for the high shear case, no estimate of $k \rho_i$ is possible from these experiments.

G: Possible Mechanisms For Cell Formation

This turbulence study concludes with a discussion of the theoretical problems that arise from the existence of turbulence cells of the

magnitude that have been observed. The same difficulty is seen in the work on the diffusion coefficient measurements² that showed that the vortex scaling of the OD theory applied to plasma in the poloidal field octupole. The OD theory⁴ is based on the idea of thermal level fluctuations in the plasma, as was the TM theory⁵ before it. In the first paragraphs below, the observations are shown to be much larger than may be accounted for by random thermal fluctuations. The following paragraphs outline some current ideas on how such cells might be generated. At present, there is no really satisfactory explanation available.

(1) Thermal Level Fluctuations

The amplitude of turbulence due to random thermal fluctuations in any medium is predicted by the fluctuation dissipation theorem⁹ (FDT). The OD theory used the FDT to predict the form of the electric fields from which they developed the form of the diffusion coefficient. The FDT most amenable to the present data relates the r.m.s. potential amplitude of the turbulence to the temperature of the plasma. This is the form of the well known^{9,32} Johnson Noise fluctuations in a dielectric medium. The FDT in this case is

$$\frac{1}{2} \epsilon \epsilon_0 V_{th}^2 = \frac{1}{2} k_B T_e . \quad [\text{IV-G-1a}]$$

$$\epsilon = 1 + \frac{\omega_{pi}^2}{\omega_{ci}^2} + \frac{\omega_{pe}^2}{\omega_{ce}^2} \quad [\text{IV-G-1b}]$$

$$\epsilon_0 = 8.85 \times 10^{-12} \text{ Fd/m} .$$

ϵ is the low frequency dielectric constant, V_{th} is the thermal level r.m.s. potential of the turbulent "noise," k_B is the Boltzmann con-

stant, ω_p is the plasma frequency and ω_c is the cyclotron frequency. In order to write ϵ in this form, it must be assumed that the ions participate in the generation and suppression of the turbulence. More on this assumption is in the paragraphs below. For the hydrogen data near the ring, $V_{rms} = 30$ mV, $T_e = 10$ eV, and $\epsilon = 20$ will be used. These parameters violate the estimate of [1] because either V_{rms} is much too big or T_e is exceptionally small. Post and his coworkers have defined an effective temperature which will be called T_p^* here. Since $D_\perp$ in the OD formulation is proportional to $T_e^{1/2}$, they define T_p^* as $T_e \times D_{meas}^2 / D_{OD}^2$ and obtain a value 10^5 to 10^6 larger than T_e . It must be emphasized that T_p^* has no direct relationship to the ambipolar parameter T^* defined in Chapter III. Although a possibility of confusion exists, it was decided to maintain continuity of notation with my much earlier preliminary report.

In order for the data to be used in the FDT, [1] requires one of two ad hoc modifications. A temperature amplification factor K_T or a potential attenuation factor K_V may be defined as

$$\frac{1}{2} \epsilon \epsilon_0 (K_V V_{rms})^2 = \frac{1}{2} k_B T_e \quad [IV-G-2]$$

$$\frac{1}{2} \epsilon \epsilon_0 V_{rms}^2 = \frac{1}{2} k_B K_R T_e \quad [IV-G-3]$$

Using the above data, $K_T \approx 10^5$ or $K_V \approx 1/300$.

While it is obvious that one factor is the inverse square of the other, the use of [2] or [3] implies differing viewpoints of the generation process. The use of [2], which is preferred here, implies that a turbulence cell has its origin in thermal fluctuations of the particle

density but has its amplitude driven to larger values by some natural plasma process. This is the viewpoint upon which Hasagawa's work is based. (See below.) [3] follows from the view of Post, et al.^{2,33,34,35} that the turbulence enhancement of the temperature is due to the violent plasma injection/entrapment process. The injection turbulence temperature is somehow frozen into the plasma and must persist through the assembly and subsequent destruction of many individual cells. As yet, there is no theoretical model offered for this process, nor, according to these workers (reference , page) any possible experimental way of varying the enhancement.

(ii) Mode Coupling As A Turbulence Driving Mechanism

1. Mima, Hasagawa: In a series of papers^{36,37,38} Hasagawa and several different coworkers have established that the plasma can be affected significantly by the transmission of energy from long wavelength oscillations to short wavelength modes via the convective term in the polarization drift. In a recent paper,³⁹ Mima and Hasagawa have applied this mechanism to explain the A.T.C turbulence data. Their results are device independent and independent even of the specific long wavelength modes involved. They treat the problem of turbulence in a slab model plasma using the strong turbulence renormalization techniques of Dupree^{6,7} and Weinstock.⁸ For simplicity, they assume that $T = T_e \gg T_i$, $n/n_0 = e\phi/T$, and use the ion continuity equation with the polarization included:

$$\frac{\partial n}{\partial t} + \nabla_1 [n_0 (\frac{v}{E} + \frac{v}{p})] = 0 \quad \text{[IV-G-4a]}$$

$$\underline{v}_E = \underline{B} \times \nabla \phi / B^2 \quad [\text{IV-G-4b}]$$

$$\underline{v}_p = - \left[\frac{\partial \nabla \phi}{\partial t} + (\underline{v}_E \cdot \nabla) \nabla \phi \right] / B \omega_{ci} \quad [\text{IV-G-4c}]$$

The potential is expanded in a Fourier series in space and the wave vector $\underline{k}$ is normalized (multiplied) by $\rho_s = c_s / \omega_{ci}$, where c_s is the ion sound speed $(T/n_1)^{1/2}$. With $\phi_{\underline{k}}$ normalized by T/e , [4] may be rewritten as

$$(1 + k^2) \frac{\partial \phi_{\underline{k}}}{\partial t} + i \omega_{\underline{k}}^* \phi_{\underline{k}} = \frac{1}{2} \sum_{\underline{k} = \underline{k}' + \underline{k}''} \Lambda_{\underline{k}', \underline{k}''} \phi_{\underline{k}'} \phi_{\underline{k}''} \quad [\text{IV-G-5a}]$$

$$\Lambda_{\underline{k}', \underline{k}''} = (\underline{k}' \cdot \underline{k}'') \cdot \hat{z} [(k'')^2 - (k')^2] \quad [\text{IV-G-5b}]$$

$$\omega_{\underline{k}}^* = - \frac{k_y T}{e B \omega_{ci}} \frac{1}{n} \frac{\partial n}{\partial x} \quad [\text{IV-G-5c}]$$

It is assumed that $\hat{z} = \underline{B}/B$, and $n = n(x)$. The mode coupling coefficient Λ arises from the non-linear convection term in [4c]. [5] is then solved for the case of a long wavelength toroidal mode ($k_z \ll 1$), and a short wavelength radial mode ($k_y \approx 1$). The solution for the spectrum near $k = 1$ is $|\phi_{\underline{k}}|^2 \approx k^{1.8} / (1 + k^2)^{2.2}$, which peaks reasonably close to the peak of $|n(k)/n_0|^2$ that is observed in Mazzucato's data. As discussed in section F, this broad peak in the unnormalized $|k_{\perp}|$ is somewhere near ρ_i^{-1} or ρ_s^{-1} . The theory is actually a surprisingly good fit to the data because no finite ion gyro radius corrections were made to the guiding center drift equations.

Mima and Hasegawa state that, because their result is independent of any specific linear mode of the system, their mechanism of energy transference from small $\underline{k}$ waves to nearly unity orthogonal $\underline{k}$ turbulence

modes should be device independent. These results should be true for any magnetized non-uniform collisionless plasma. They speculate that the energy source for the turbulence on the A.T.C could be (1) long wavelength drift waves, (2) M.H.D. or tearing modes, (and) or (3) long wavelength convective cells.

In the light of this analysis, it should be considered significant that long toroidal wavelength, low frequency flute modes are always observed coexisting with the turbulence cells in the octupole. These waves have saturated at a relatively low, "harmless" amplitude (they produce negligible particle transport) presumably because of the octupole geometry in the M.H.D stable region. The turbulence amplitude may well be driven above the thermal level by Hasagawa's mode coupling mechanism.

The model has not yet been solved for $T_i \geq T_e$ nor with finite gyro radius effects and fluctuations of the electron distribution included. The turbulence enhancement process can not be considered well understood until the full problem has been treated and the mechanism capable of predicting the observed amplitude and correlation lengths and times.

2. Okuda, Cheng: This work,⁴⁰ to be called the OC paper, is a direct outgrowth of the work³⁹ by Mima and Hasagawa (to be called MH), which they cite. The ion continuity equation provides the expression for the fluctuating ion density and the force balance equation yields the electron density perturbation. Poisson's equation is then applied to yield the potential fluctuation $k^2 \phi_k^C = 4\pi e(n_i^C - n_e^C)$ where the C superscript refers to quantities in the convective cell. The result is an expression similar to [5] but with a much stronger coupling coeffi-

cient. This work must be considered more general than the MH treatment because no assumptions were made of quasi neutrality $\delta n_i = \delta n_e$ or thermal equilibrium $\delta n_e/n = \delta\phi/T_e$. Consequently, large electric cell fields are allowed to develop. No attempt will be made in this summary to repeat the equations they derive, because this work is readily available currently. Convective cells are purely damped modes ($\gamma = -k_i^2 D_i$) but the resulting non-linear resonant growth rate is much larger than the linear damping rate and non-linear excitation is very important even for low amplitude waves.

The emphasis of this study is the reverse of that of the MH work. The mechanism studied is the "beating together of short wavelength drift waves" to produce long wavelength convective cells. They use the dispersion relation for drift waves in the case of $v_{th,i} \ll v_{ph} \leq v_{th,e}$ which are unstable modes. The superscript D will be used to label drift modes. The graphs of frequency ω^D and growth rate γ^D v.s. $k_i^D \rho_i$ peaks near $k_i^D \rho_i \approx 1/4$ for ω^D and near .8 for γ^D . The product $\omega^D \gamma^D$ peaks at about $k_i^D \rho_i \approx .5$. By their mechanism, they require $\underline{k}_i^C = \underline{k}_{i1}^D + \underline{k}_{i2}^D$. They pick two terms on either side of the ω^D maximum and show from the drift dispersion curve that long wavelength ($\underline{k}_{i1}^D + \underline{k}_{i2}^D \approx 0$) convective modes grow most quickly. It is unclear from their graphical argument why the same should not be true for $\rho_i (\underline{k}_{i1}^D + \underline{k}_{i2}^D) \approx 1$. They point out that with the rapid growth, the time behavior of the convective mode is non-linear and a full non-linear strong turbulence analysis is required, which they do not provide.

The two studies (MH and OC) strongly differ in the regimes for the modes which they find interesting. The MH study examines excitation of

short wavelength convective turbulence ($k_{\parallel}^C = 0$, $\rho_i k_{\perp}^C \approx 1$) modes by long wavelength coherent modes ($k_{\perp}^D \approx 0$). The OC work shows that long wavelength convective turbulence ($k_{\parallel}^C \approx 0$, $k^C \approx 0$) may be excited by short wavelength drift modes ($k_{\perp}^D \rho_i \approx 1$). The results of the MH paper are compared favorably with data from the A.T.C tokamak. The results of the OC paper are compared qualitatively to computer simulations which they present. The simulations are for $m_i/m_e = 100$ and $T_e/T_i = 4$ and for various shear lengths L_s , presented in units of density scale length L_n . For L_s/L_n values as low as 50 (shearless to weak shear) convective modes were seen to grow twice as fast as the drift modes. With strong shear ($L_s/L_n = 10$) no convective cells were observed. No comment was included on either the presence or absence of short wavelength convective modes. However, the conditions of their simulations (and all such simulations reported to date) $\rho_i/\rho_e \leq 5$. The results of the turbulence studies which report short range modes all have ρ_i/ρ_e on the order of 100. It is this large ratio, in fact, which prescribes a fluid model electron-cell interaction but a single particle ion-cell interaction and leads to the interesting plasma properties discussed in section E. When the ions are sufficiently cold that this ratio is 5, such cells can not be generated and live for the observed times of $10000 f_{ce}^{-1}$.

The mode coupling equations are developed in the OC study in a much more general way than is done in the MH study. The only additional effects not included are the finite gyro radius corrections needed for a proper discussion of $k_{\perp}$ modes when $k_{\perp} \rho_i \approx 1$.

H: Summary of Chapter IV

It is probably useful to summarize the important points of this rather long development. The spatial correlation of random turbulence in hot ion plasmas ($T_i > T_e$) was obtained by plotting the CF at $\tau = 0$ v.s. probe separation z . The CF was normalized in various ways to smooth the shot-to-shot variations in amplitude. The resulting averaged CFs in time and space were Gaussian-like appearance rather than exponential with a linear argument. With the assumption that the CFs are in fact Gaussian and since a Gaussian CF comes from Gaussian data, correlation lengths of the average data were readily obtained. If the correlation length of an $\exp(-r^2/\lambda^2)$ structure is defined as the e-folding length λ_c , then this is the half width at half maximum of the spatial CF. λ_c was shown to be ρ_i for hydrogen plasmas and about $\rho_i/2$ for helium. λ_c obeys ρ_i scaling for different masses, for different magnetic fields at the same location, and for different locations. Further, the cells were observed to be well correlated along a field line. The cells were seen to decorrelate for only one case, and using the apparent Gaussian time dependence of the CFs, a correlation time τ_c of 7 μsec was measured. Again, τ_c is the e-folding time of the amplitude of the average structure.

The structures which are used to form the CFs are sharply limited in spatial extent and well separated in time. Indeed, if there had been significant overlapping of the structures, the CF techniques employed could not have resolved them. It is for this reason that the word "cell" has been applied to the average appearance of the turbulence signal.

The observations suggest the following model. Since $\rho_i/\rho_e = 100$ for hydrogen, the electron-cell interaction does not break the electron adiabaticity. The electrons never deviate from their guiding center drift frame so that there is no mobility response to external $\underline{E}$ fields in the cross field transport. The ions, however, interact with structures of sizes comparable to their own gyro orbit. The interaction must break adiabaticity and so ion transport has both diffusion and mobility components.

Due to their size, the long-lived cells ($\tau_c \gg 2\pi M_i/eB$) must be constructed from the electron distribution. The large drifts due to magnetic gradients and curvature must play a major role in the dynamics of the cell. A cell should be stationary in the average electron magnetic drift frame. The cells may be either diffusion damped on time scales of 500 μsec or magnetically damped in the 10 μsec time scale required for an average electron to drift across the cell. The measured 7 μsec τ_c indicates that magnetic damping is the dominant process.

The consequences of these turbulence cells are (a) the estimated electron diffusion coefficient is within a factor of two to four of the measured value, (b) because of the zero electron mobility μ_e , the measured ambipolar diffusion coefficient must be the electron coefficient, (c) again, because $\mu_e = 0$, the ambipolar electric fields must be controlled by the ion diffusion and T^* ($= E_a a_n$) must be proportional to T_i . This last point presents an interesting picture. Since $D_e > D_i$, the plasma charges up to some positive value. The resulting $\underline{E}_a$ has no retarding effect on the electrons ($\mu_e = 0$) and so only oper-

ates to expel the ions at the same rate that the electrons leave.

Two other effects are proposed and should operate. (1) The adiabatic diffusion of the electrons out of a minimum B well should add energy to the electron distribution, albeit slowly. T_e should therefore decay more slowly than predicted by collisions. If diffusion were to occur for large scale convective cells, this effect should work for both the electron and the ion temperatures. (2) The short range turbulence cells were shown to effectively depopulate the high energy tail of the ion distribution. The subsequent attempted relaxation of the distribution back to a full Maxwellian should cause the measured temperature to decay with a time constant on the order of the 10 μ sec Maxwellian time.

Finally, the cells were observed to have an amplitude enhanced above the thermal levels. It is possible that these cells are receiving energy from long wavelength coherent modes, as proposed by Mima and Sasagawa, and therefore have their amplitudes enlarged. The theory behind this speculation is still at an unsatisfactory state of development.

REFERENCES FOR CHAPTER IV

1. A. J. Cavallo. *Phys. Fluids* 16, 408 (1973)
2. J. R. Drake, J. R. Greenwood, G. A. Navratil, R. S. Post. *Phys. Fluids* 20, 148 (1977)
3. G. A. Navratil, R. S. Post, A. Butcher Ehrhardt. *Phys. Fluids* 20, 156 (1977)
4. H. Okuda, J. M. Dawson. *Phys. Fluids* 16, 408 (1973)
5. J. B. Taylor, B. McNamara. *Phys. Fluids* 14, 1492 (1971)
6. T. H. Dupree. *Phys. Fluids* 9, 1773 (1966)
7. T. H. Dupree. *Phys. Fluids* 11, 2680 (1968)
8. J. Weinstock, R. H. Williams. *Phys. Fluids* 14, 1472 (1971)
9. G. Bekefi. Radiation Processes in Plasmas (John Wiley & Sons, Inc., N. Y., 1966)
10. J. R. Drake, J. C. Berryman. *Phys. Fluids* 20, 851 (1977)
11. J. C. Berryman. University of Wisconsin Plasma Physics PLP-680
12. J. A. Schmidt. *Phys. Rev. Lett.* 24, 721 (1970)
13. A. W. Molvik, Ph.D. Thesis. University of Wisconsin (1970)
14. D. A. Brouchous. University of Wisconsin Plasma Physics PLP-728
15. R. D. Hazeltine. *Plasma Physics* 15, 77 (1973)
16. I. P. Shkarofsky, T. W. Johnston, M. P. Bachynski. The Particle Kinetics of Plasma (Addison-Wesley Publishing Company, Reading, Mass., 1966)
17. S. R. DeGroot, P. Mazur. Non Equilibrium Thermodynamics (North-Holland Publishing Company, Amsterdam, 1962)
18. D. J. Rose, M. Clark. Plasmas and Controlled Fusion (M. I. T. Press, Cambridge, Mass., 1961)
19. S. I. Braginskii. Reviews of Plasma Physics 1, M. A. Leontovitch, ed. (Consultants Bureau, N. Y., 1965) pp. 205-311
20. G. J. Armentrout, J. C. Sprott. University of Wisconsin Plasma Physics PLP-470

21. A. Butcher Ehrhardt. University of Wisconsin Plasma Physics PLP-703
22. J. Weinstock. *Phys. Fluids* 19, 531 (1976)
23. T. Okawa. Private Communication
24. C. J. Armentrout. University of Wisconsin Plasma Physics PLP-667 (Appendix)
25. R. A. Breun. Ph.D. Thesis. University of Wisconsin (1975)
26. Lyman Spitzer, Jr. Physics of Fully Ionized Gases (Interscience Publishers, N. Y., 1962)
27. A. B. Berezin, D. G. Buliginsky, M. I. Vildjumas. A. F. Ioffe Physico-Technical Institute, Leningrad. Report 253 (1970)
28. E. Mazzucato. *Phys. Rev. Lett.* 36, 792 (1976)
29. S. M. Hamberger, L. E. Sharp, J. B. Lister, S. Mrowka. *Phys. Rev. Lett.* 37, 1345 (1976)
30. C. M. Surko, R. E. Slusher. *Phys. Rev. Lett.* 37, 1747 (1976)
31. M. Okabayashi, V. Arunasalam. Princeton Plasma Physics Laboratory, PPPL-1310 (Jan, 1977)
32. F. W. Sears. An Introduction to Thermodynamics, The Kinetic Theory of Gases, and Statistical Mechanics (Addison-Wesley Publishing Company, Inc., Reading, Mass., 1953)
33. J. R. Greenwood. Ph.D. Thesis. University of Wisconsin (1975)
34. G. A. Navratil. Ph.D. Thesis. University of Wisconsin (1976)
35. G. A. Navratil, R. S. Post. University of Wisconsin Plasma Physics PLP-671
36. A. Hasagawa. *Phys. Lett.* 57A, 143 (1976)
37. A. Hasagawa, L. Chen. *Phys. Rev. Lett.* 36, 1362 (1976)
38. A. Hasagawa. *Bul. Am. Phys. Soc.* 21, 1180 (1976)
39. K. Mima, A. Hasagawa. Unpublished as yet.
40. H. Okawa, C. Z. Cheng. Princeton Plasma Physics Laboratory, PPPL-1328 (May, 1977)

CHAPTER V: COHERENT FLUCTUATIONS IN THE HOT ION PLASMA

A: Introduction

These studies of low frequency coherent fluctuations are presented for two purposes. Firstly, the waves are identified as flute modes with $\underline{k} \cdot \underline{B} = 0$. The measured phase velocities are shown to be comparable to the sum of the $\underline{E} \times \underline{B}$ velocity due to the ambipolar field and the ion diamagnetic velocity, both of which are estimated from the measured flux space gradients. This velocity decomposition identifies the specific branch of the flute dispersion relation as hot ion flutes driven by the expansion energy of the ion distribution. Secondly, these observations provide an important, independent method of checking the density/potential relationship discussed in Chapter III. The discussion in this chapter is restricted to these two topics, with additional supporting details included.

(i) Preliminaries

Earlier work¹ on coherent fluctuations in the small supported octupole indicated that the important particle losses in an octupole are not due to the particle interactions with the spectrum of coherent low frequency oscillations ($f \ll f_{ci} = eB/2\pi m_i \approx 1$ MHz). The studies presented here were undertaken to investigate the low frequency fluctuation in the levitated octupole, their relationship to the behavior of the plasma, and the correspondence of the results to the earlier work. The studies are restricted to this frequency regime because the facilities do not exist for systematic (correlation) studies of fluctuations

near the plasma frequency (300 MHz) or the electron cyclotron frequency (2 GHz). In principle, one could examine these high frequency regions using the photographic reduction technique described below for the rising field studies, but in practice, the process is too time-consuming and expensive to be practical. Major requirements for a comprehensive study of any plasma parameter range are the acquisition of hundreds of separate shots and the relatively rapid analysis and tabulation of the results. Too few samples will only lead to confusion if on a shot-to-shot basis the fluctuations fall on different portions of the relevant dispersion curve. Too long a time between acquisition and analysis will, in addition, lose for the experimenter the ability to correlate a change in the data with accidental variation in the machine parameters such as obstacles or magnets left in or near the vacuum chamber.

(II) Observation Conditions

The observations presented below were acquired 5 msec after gun injection into a background pressure of 2 to 4×10^{-7} Torr. It was shown in Chapter III that the bulk properties of the plasma in the sine field configuration prior to 27 msec were not significantly different from those of the plasma in the crowbarred field. Data from two types of field configurations are presented. Rising field data were taken at 10 msec (43°) into the rising magnetic field. Peak field data were taken at 24 or 25 msec and are the same for either the half sine or the crowbarred cases. The observed waves were all classified as flutes ($\mathbf{k} \cdot \mathbf{B} = 0$) or "flute-like" because small deviations from the zero $k_{||}$

can not be detected. Molvik² established that rising field waves were in phase between the lower outer bridge region and the upper mid cylinder region about ψ_c . Section B shows that the signals are also correlated along a ψ line between the upper and lower mid cylinder regions. Section C shows that the waves seen at peak field are also classifiable as flutes. These results should not be surprising because flutes were the principal modes observed under normal conditions in the small octupole. (Ref 1, p. 80.)

(iii) Normalization Of The Signals

The stated purpose of initiating these studies was to examine the effect that waves have on cross field plasma transport. As suggested in Chapter II D, this effectively defines the proper normalization of fluctuation data, i.e. the prescription used to "draw the baseline" through the data records. The plasma in the wave can use the electric field of the wave and the magnetic field of the machine to cross flux shells and escape from the confinement region. This familiar result has been discussed elsewhere.^{1,3} The particle flux due to this is $J(t) = n(t) \underline{E}(t) \times \underline{B}/B^2$. If $n(t) = n_0 + n_1$, $E(t) = E_0 + E_1$ where $n_1 = \delta n \cos(\omega t)$, $E_1 = \delta E \cos(\omega t + \phi)$, and the time averages of n_0 and E_0 are both zero, then the time average of $J(t)$ over many oscillations is $J = \delta n \delta E \cos(\phi)/B = C(n_1, E_1, \tau = 0)/B$. C is the correlation function between n_1 and E_1 . The proper normalization of fluctuating data then is to give it a zero time average. Since such signals are usually obtained with a D.C. offset from the line drivers and are in addition processed with a high pass electronic filter to remove the large slowly

varying "profile" component to the plasma signal, no other meaningful normalization is usually possible.

B: Rising Field Data (10 msec)

(i) Acquisition Methods

The data were obtained 5 msec after gun injection, 10 msec into the machine cycle. At this time $\psi_c = 5.53$ and ψ_s is 3.82 and 3.85 (see Figure 1 4). The data were acquired from two type 1 (section II A) high impedance probes with a 20 M Ω resistor just behind the tip. One probe was located in the lower mid cylinder at 45° and the other in the upper mid cylinder in the swivel port which enabled it to move in the $\hat{z}$ direction. The probe signal was frequency compensated to a 1 μ sec rise time by a voltage divider, then preamplified and sent to the oscilloscope. The two probes were balanced to a 100:1 common mode rejection ratio prior to insertion. The signals were fed into two dual beam oscilloscopes carefully calibrated and matched in gain and time sweep rate. The second scope was triggered before the first scope completed its data sweep so that the signals overlapped. The sweeps were photographically recorded, the photographs cut and carefully overlapped, and the completed extended record was hand digitized on a digitizing table such as used in meteorological and cartographical studies of aerial photographs. The digitized traces were stored on computer cards and processed by the large computer at the campus computing center. Figure 1 shows an oscilloscope record and its digitized image. The computer plot was drawn on a 1:1 scale with the photograph so that the digitizing accuracy could be checked.

Figure V 1

Oscilloscope record (A) and digitized image (B) of an early injection pulse. The probe is 6.4 cm from the lower lid. The scales for the scope trace are 50 mV/vertical division, 50 μ sec/horizontal division. The digitized record's scales are 149 mV/vertical inch, .92 msec full time interval or record.

A

B

(11) Fluctuations Near ψ_c

1. Amplitude: Figure 2 is the plot of the r.m.s. amplitude of the floating potential oscillations. These data are from the lower probe which was reference scaled (section II C) using the signal from the upper probe fixed at 10 cm below the upper lid. The corresponding profile values of I_g and V_f for this case, and parameterized (I,V) plot of these data are shown in Figure III 21. δV_f is seen to reach a maximum value of 20 mV at $\psi = 5.9$, outside ψ_c , approximately at ψ_g which is the flux surface which first enters the wall.

2. Frequency: Figure 3 shows the auto and cross correlation functions (CF) and a Fourier transform (FT) of a CF for the data from the two probes located at 10 cm from the lids on the mid cylinder. This is at $\psi = 5.3$, just inside ψ_c . The upper probe is displaced 2 cm in ζ from the azimuth of the lower. Figure 4 shows the same sort of comparisons with the probe 6.6 cm from the lids ($\psi = 5.3$) just outside ψ_c . Figure 5 shows two more 10 cm FTs to indicate reproducibility and the FT at 7.6 cm ($\psi = 5.6$) and at 14 cm ($\psi = 4.9$). Figure 6 presents the plot of frequency $f(\psi)$ as a function of ψ from a range of data such as these. It is important to note that for $\psi > \psi_c$, f is a single valued function, but for $\psi < \psi_c$ f has two values. This may be interpreted in the following manner. Flute waves are interchange instabilities⁴ which are in the class of fluctuations which multipoles were designed to suppress.⁵ Inside the critical surface ψ_c , the linear growth rate ψ_L is negative and outside it is positive. Outside ψ_c these waves which may be excited by a number of different mechanisms will grow to non-linear saturation and would appear to be frozen in

Figure V 2

R.m.s. value of V_f v.s. position from the lid along the mid cylinder.

Figure V 3

(A), (B), (C) are CFs for tips at 10 cm from the lids. Tip 1 is above the lower lid and 2 below the upper lid, displaced 2 cm in $\hat{\xi}$. (C) is the cross CF between 1 and 2 and (D) is the FT of (A).

Figure V 4

A, B, C are CFs for tips 6.4 cm from the lids. Tip 1 is again above the lower lid, tip 2 below the upper lid. C is the cross CF of A and B, and D is the FT of A.

Figure V 5

A and B are two more 10 cm CFs. C is at 7.6 cm and D is at 14 cm.

Figure V 6

Frequency (kHz) of signal v.s. ψ .

amplitude in a frame moving with the wave. Inside ψ_c , γ_L is negative and one would possibly expect to find no waves at all.

The octupole is what Hastie and Taylor⁶ call a "shallow well" device because ψ_c is weakly defined over a region rather than sharply on a surface. There is a region in ψ where γ_L is negative but almost zero and they identify a large number of different types of oscillations which could exist there. The flutes which are seen to exist must have the least negative (largest) γ_L of the set. If the frequency is completely real ($\gamma_L = 0$) then there will be a reference frame in which the waves are seen as non-translating, standing oscillations in amplitude and may be represented as $V_w(t) = V_0 \cos(\omega t) \cos(k\zeta)$. One may view such fluctuations as being composed of two mathematical wave trains whose phase velocities have the same magnitude but opposite directions. Transform to the laboratory frame of reference where the wave frame moves with speed v . The phase velocities of the two phase trains will separate which, since k is the same for both, will result in a Doppler separation of the measured frequencies. If v_c is the angular toroidal velocity of the wave frame and $\Omega = k v_c$ is the $\omega = 0$ frequency of the waves measured in the laboratory, then V is $V_0 \cos(k\zeta - \Omega t) \cos(\omega t) = (V_0/2)[\cos(k\zeta - \Omega_+ t) + \cos(k\zeta - \Omega_- t)]$ where $\Omega_{\pm} = \Omega \pm \omega$ are the Doppler shifted frequencies. The toroidal wave number k is $2\pi R/\lambda$ where R is the distance to the toroidal axis and λ is the toroidal wavelength.

3. CF measurements: The CF of such a signal may be easily integrated to be $C(V_1, V_2; \tau) = (V_0^2/r) \cos(\phi - \Omega \tau) \cos(\omega \tau)$ where $\phi = k\Delta\zeta$ and $\Delta\zeta$ is the angular separation between V_1 and V_2 . If $V_1(t)$ has the amplitude V_1 and $V_2(t)$ has V_2 then the amplitude of the CF is $V_1 V_2/4$ rather

than that given. (Notice that the r.m.s. of V is $V_0/2$ instead of the $(V_0/2)^{1/2}$ for the single frequency correlations of Chapter II and $V_1 V_2/4 = V_{1rms} V_{2rms}$.) The central maximum of the CF is at $\tau = 0$ if $\Delta\zeta = 0$ and one might guess that the maximum would be at $\tau_0 = \phi/\Omega$ so that ϕ could be measured. Notice, however, that $(\partial C/\partial \tau)_{\tau_0} \neq 0$. If the maximum occurs at $\tau_{pk} = \tau_0 + t$ with $t < \tau_0$ and τ_0 much less than Ω^{-1} or ω^{-1} , then $\tau_{pk} \approx [1 + \omega^2/(\Omega^2 - \omega^2)]\tau_0$. For $\omega = \Omega/3$, no more than a 15% error is introduced if $\phi = \Omega\tau_{pk}$ is used rather than $\Omega\tau_0$. Since this is well within the experimental error, $\tau_{pk} = \phi/\Omega = \Delta\zeta/v$ will be used to measure phase velocities.

The data of Figure 5 are replotted in Figure 7 to show ω and Ω in units of kHz. For $\psi > \psi_c$, $\omega = 0$ and $\Omega = f$ but for $\psi < \psi_c$, ω is $.5(\Omega_+ - \Omega_-)$ and Ω is $.5(\Omega_+ + \Omega_-)$. At 10 cm, Ω is 11 kHz, ω is 2 kHz, and v is measured (below) to be 3.4×10^5 cm/sec. These values yield $\lambda = 30$ cm (12° in ζ). Figure 8 shows the frequencies Ω and ω as functions of B at 10 cm from the lid, $\psi = 5.35$. The various averages for B may be approximately scaled from Figure I 5 since B at the mid cylinder at 10 msec is 370 G and B is proportional to V_c .

(iii) Toroidal Phase Velocities

1. Measurements: The toroidal phase velocities were measured by placing the lower probe a certain distance from the lower lid, positioning the upper probe the same distance from the upper lid and moving it through various displacements toroidally, keeping the probe on the same flux surface. Figure 9 shows the value of z vs. τ_{pk} , the toroidal displacement vs. the time value of the maximum of the cross CF between

Figure V 7

Data of Figure 6 replotted to show the average frequency Ω and the half difference frequency ω .

Figure V 8

Ω and ω as functions of magnetic field strength, expressed as V_c .

Probe 10 cm from lid

Figure V 9

Toroidal displacement z between probe tips on $\psi = 5.3$
($x = 10$ cm) plotted as functions of their correlation time shift τ .

the probes located 10 cm inside the lids, on $\psi = 5.3$. Positive z indicates that the probe was displaced counterclockwise as viewed from above and the positive τ for these separations indicate that the phase velocity v moves in that direction. Figure 10 is the superposition of phase velocities at 6.4, 10, and 15 cm for the supported case and 10 cm for the levitated case. The error in the v estimates are roughly 1×10^5 cm/sec and to within this accuracy $v = (3 \pm .5) \times 10^5$ cm/sec for all cases. The fact that τ_{pk} is not zero when z is zero is not a cause for concern. The displacements from the mid-plane of the upper and lower lids are known to be asymmetrical by 0.5 to 1 cm and this is sufficient to displace the graphs from the (τ, z) origin.

2. Interpretation: These numbers may be compared to various calculated velocities. It is immediately clear that the phase velocities of these flutes are not related to the magnetic (gradient and curvature) drift velocities. The flux tube average of the magnetic velocity⁷ goes to zero at ψ_c but the observed v_{ph} does not. The $\underline{E} \times \underline{B}$ drift velocity is quite simple to calculate assuming the potential is constant along a ψ line and azimuthally symmetric, $U = U(\psi)$. $\underline{E} = -\nabla U - \dot{\underline{A}}$ and

$$\nabla U = \hat{\psi} 2\pi R B \frac{\partial U}{\partial \psi}$$

$$\underline{A} = \frac{\psi}{2\pi R} \hat{\zeta}$$

$$\dot{\underline{A}} = \frac{\dot{\psi}}{2\pi R} \hat{\zeta}.$$

Therefore

Figure V 10

(z, r) plots for three ψ surfaces for the supported ring case and one for the levitated ring case. —6.4 cm ($\psi = 5.75$), —10 cm ($\psi = 5.3$), —15 cm ($\psi = 4.8$), —10 cm ($\psi = 5.3$) levitated.

X	MODE	V_p (10^5 cm/sec)
10 cm	SUP	3.4
10 cm	LEV	3.5
6.4 cm	SUP	3.2
15 cm	SUP	2.6

$$\mathbf{v}_E = \hat{\mathbf{z}} \, 2\pi R \frac{\partial U}{\partial \psi} + \hat{\boldsymbol{\phi}} \, \frac{1}{2\pi R B} \psi .$$

The diamagnetic velocity is $\nabla n \times \mathbf{B}/B^2$ and this becomes, for $n = n(\psi)$, $\mathbf{v}_{dia} = 2\pi R (T/n) (\partial n / \partial \psi)$. The associated angular velocities are $\Omega = v/R$, so

$$\Omega_E = 2\pi \frac{\partial U}{\partial \psi}, \quad \Omega_{dia} = 2\pi T \frac{1}{n} \frac{\partial n}{\partial \psi} . \quad [V-B-1]$$

Positive Ω indicates clockwise rotation as viewed from above. The gradients are obtained from Figure III 21 if ∇V is ∇U and ∇I_s is proportional to ∇n . $2\pi \partial/\partial\psi$ may be converted from the tabulated ψ_t as $(413/V_c) \partial/\partial\psi_t$ with V_c expressed in kV. From the data at 10 cm, $\partial V/\partial\psi_t$ is -4 to -5 V/d, $\partial I_s/\partial\psi_t$ is -12 to -15 A/d, I_s is 19 μ A, and $R = 140$ cm. Ω_E is $-(.82 \dots 1.0) \times 10^3 \text{ sec}^{-1}$ and Ω_{dia} is $-(1.3 \dots 1.6) \times 10^{-3} \text{ sec}^{-1}$ for $T = +10$ eV. Using the ion temperature, Ω_{net} is $-(2.1 \dots 2.6) \times 10^3 \text{ sec}^{-1}$ counterclockwise. The electron temperature gives about $.5 \times 10^3 \text{ sec}^{-1}$ clockwise. This is to be compared to the measured counterclockwise velocity of $\Omega = v/R = 2.5 \times 10^3 \text{ sec}^{-1}$. Clearly, the flutes are moving under the combined influence of the electric drift due to the ambipolar E field and the ion diamagnetic drifts due to the density gradient.

3. T* measurements: Equation [III-D-1] may be combined with the angular velocity estimates to give a T* estimate of $T^*/T = \Omega_E/\Omega_{dia} = .6$ for $T = 10$ eV. T* was obtained directly from the same profiles and is shown in Figure III 21 to be 4.5 eV. The latter is probably a more reliable estimate because the single measured slope of a curve on a semilog graph is more accurately obtained than the ratio of slopes from

curves from linear graphs.

(iv) Radial Phase Velocity

Figure 11 presents data which are difficult to interpret and are included primarily for reference purposes. The upper probe was held fixed at 10 cm below the lid and correlated with the lower lid probe which was moved along the mid cylinder. The figure is the plot of τ_{pk} v.s. position x above the lower lid. Since $v = x/\tau_{pk}$, a possible interpretation of these data is of waves originating around ψ_c and propagating away in both directions across the ψ surfaces. Another interpretation is to recognize the fact that v is not the group velocity of the waves and this result might be due to fluctuations moving toroidally with a slight shift in phase between the waves on adjacent surfaces. The surfaces of constant phase in this case would be cocked at an angle to the planes at constant azimuth which contain the field lines. The toroidal motion of such planes will generate both toroidal and radial phase velocities in the same manner that the helical stripes on a rotating barber pole will display a phase velocity parallel to the rotation axis. A third interpretation is based on the fact that the correlation function of two signals will always return an answer. Figures 5 and 6 show that f is roughly an increasing function of ψ until it reaches ψ_g where it decreases again. As the probes separate in ψ space the f components also separate and C will yield a correlation not of the peak frequencies but of the overlapping portion of the peaks in Fourier space. The larger the separation the lower $C(\tau_{pk})$ should become, which roughly corresponds to the observed behavior. The diffi-

Figure V 11

CF time shift τ between two probes v.s. position of probe along lower mid cylinder. Other probe held stationary 10 cm below upper lid.

culty with this interpretation is that f peaks at about 5 cm but τ_{pk} is most negative about 8 cm, near ψ_c . This problem will be mentioned again in the next section.

C: Peak Field Data (24 msec)

(i) Acquisition Methods

The hydrogen plasma was injected at 18 msec into the field pulse and the data were obtained 4 to 6 msec later. Five msec after the injection, T_i is 10 ... 20 eV, T_e is 6 ... 7 eV, and the peak density is approximately 1 to $3 \times 10^9 \text{ cm}^{-3}$ for $V_c = 1.5 \text{ kV}$ with the rings levitated. Peak density for the supported rings case is 2 ... 3 $\times$ smaller. The correlation studies employed single-tipped low-impedance probes biased to -45 V for saturation current studies or attached to a 10 M Ω frequency compensated impedance to ground for V_f studies. Phase velocity measurements were made using a tungsten expandable double probe with the tips separated by 1 cm (see section II A).

(ii) Fluctuation Observations

1. Correlation map: Figures 12-15 show a correlation map for different field strengths between a collector (#65) located at ψ_g on the upper edge of the nose and an I_g probe at the same azimuth in the upper outer bridge at 330°. The collector was run in the floating mode and measured δI_g . A high pass filter was used with the lower cutoff set at 1 kHz. The upper half of each graph is a plot of the correlation coefficient (CC) between the probe and the collector versus probe position. The temporal CC in this case is defined as the maximum of the cross CF divided by the product of the r.m.s. amplitudes of the two

Figure V 12

Fluctuation map for the supported ring case with $V_c = 1.0$ kV.

Figure V 13

Fluctuation map for the supported ring case, $V_c = 1.5$ kV.

1.5 kV

Figure V 14

Fluctuation map for the supported ring case, $V_c = 2.0$ kV.

2.0 KV

Figure V 15

Fluctuation map for the supported ring case, $V_c = 2.5$ kV.

2.5 KV

signals. In each case the fluctuations are well correlated ($CC = .8$) until some distance inside ψ_c where an abrupt drop is noticed and the CC drops essentially to zero at ψ_g (as does the r.m.s. amplitude of δI_g). The frequency range for these fluctuations is 10 ± 2 kHz. The lower half of each graph is the plot of the correlation time shift between the probe and the collector as a function of the probe's position. The shift goes to zero when the probe is on the collector's ψ line. The data for study have already been shown in Figure IV 2. As stated in section IV A, probe signals are significantly more noisy than collector signals, an effect which is explained by the spatial averaging property of the collector. This is primarily why the CC is reduced to .8 when the signals along a common ψ line follow each other as well as they do. A radial phase velocity may be measured from these data but interpretation of v_r has the same uncertainties as in V B(iv), above. Figure 16 shows the structure of the fluctuations at 1.5 kV in more detail. The ion saturation current, I_g , its fluctuating value δI_g , and the ratio $\delta I_g / I_g$ are plotted as a function of ψ . The surface area of the probe tip was about $.2 \text{ cm}^2$ and the density may be estimated from simple probe theory for 10 eV ions as $n = k I_g$ where k is about $10^7 \text{ cm}^{-3}/\mu\text{A}$.

2. Toroidal phase velocity: The toroidal phase velocities were measured for the $V_c = 1.5$ kV field for the levitated ring case by measuring the time shift between probe tips separated toroidally by z . Figure 17 shows that the correlation time shift between tips on the same $\underline{B}$ line ($z = 0$) is zero. This establishes the lower bound for $v_{||}$ of $4 \times 10^6 \text{ cm/sec}$ and for a 10 kHz wave, λ_{min} is about 4 m. Since this

Figure V 16

I_s (●), $\delta I_{s,rms}$ (○), $\delta I/I$ (Δ), v.s. position in the bridge
for $V_c = 1.5$ kV, levitated.

Figure V 17

Correlation time shifts from tips separated by 1 cm along the same B line provide estimates of parallel phase velocities. The CF integrals are performed over intervals which are 75% of the data interval. The CFs themselves range over $\pm 1/8$ the data interval. The tips are 7.6 cm from the upper outer ring in the bridge region at 330° . Records begin at 24 msec, 5 msec after injection.

(A) The data record is 1.28 msec long, 5 μ sec per point, with the vertical scale of .85 μ A/division. The r.m.s. value of the upper (lower) trace is .6(.5) μ A. (B) CF of the signals of (A). The peak value is $C_p = .236 \mu A^2$ and the CC is .94. The time step is 5 μ sec/point. C_p occurs at $\tau = 0$. (C) Same as (A) but with a time step of .5 μ sec and with .21 μ A per vertical division. The r.m.s. of the upper (lower) trace is .22(.17) μ A. (D) CF for the data of (C). The time sweep is .5 μ sec. C_p is .032 μA^2 , occurs at $\tau = 0$, and the CC is .88.

A

B

C

D

is the length of a common flux field line, $k_{||}$ is essentially 0. The errors involved in measuring the toroidal velocity v are a combination of the small range of z and the inaccuracy in the τ_{pk} measurements. The tip separation z is controllable to within a half mm so if z is assumed well determined, $|\delta v/v| = |\delta \tau/\tau|$. The time step for the measurements is 1 μsec so $\delta \tau = .5 \mu\text{sec}$ and τ is measured to be between 1 and 2 μsec . Figure 18 shows a case where $\tau_{pk} \approx 2 \mu\text{sec}$ for tips on the same ψ surface but separated by 1 cm in $\hat{\zeta}$. The error is between 20 and 50% with the largest error corresponding to the largest measured velocities (smallest τ_{pk}). This rough time step was required so that the CF integration interval contained enough oscillation periods to calculate a meaningful τ_{pk} . (See section II E.) These measurements are intrinsically more error prone than those of the early injection measurements where large z values were possible and $|\delta \tau/\tau|$ was about 5% for the largest z . On the other hand, bridge measurements of $v_{\perp}$ could only be performed with an expandable double probe of this type with a limited range in z . Accuracy could only be increased if an A/D converter were used with a much expanded storage capacity (section II B). Table 1 gives the time shift measurements of δI signals and the corresponding velocity. Figure 19 shows the plot of v vs. x in the bridge for the 10 kHz fluctuations at 24 msec, 6 msec after injection. The background pressure for these data was 2 to 4×10^{-7} Torr and the capacitor bank was charged to 1.5 kV. All velocity measurements are within the error interval 6.5×10^5 cm/sec.

Figure V 18

Perpendicular velocities are measured by separating the tips 1 cm in $\hat{z}$ and measuring the CF time shift. Probe at the same position as in Figure 17. (A) Data from these tips. Records are 256 μsec long, 1 μsec per point. The vertical scale is .85 $\mu\text{A}/\text{division}$. The r.m.s. of the upper(lower) trace is .59(.66) μA . The lower trace is from the tip closest to the poloidal field gap. (B) $C(\text{upper trace, lower trace}, \tau)$. C_p is .29 μA^2 , occurs at -2 μsec , and the CC = .9. The signal travels counterclockwise as viewed from above.

A

B

TABLE V 1

TOROIDAL PHASE VELOCITY MEASUREMENTS FROM TIPS SEPARATED 1 cm IN ζ

Position (cm)	τ_{pk} (μ sec)	Error (%)	v_{ph} (10^5 cm/sec)
10.2	-2.0	25.	5.0 ± 1.2
9.5	-1.5	33.	6.7 ± 2.2
8.9	-1.5	33.	6.7 ± 2.2
8.3	-1.0	50.	10.0 ± 5.0
7.6	-1.0	50.	10.0 ± 5.0
7.0	-1.0	50.	10.0 ± 5.0
6.4	-1.5	33.	6.7 ± 2.2

Figure V 19

Toroidal phase velocity v v.s. position x in the bridge region of the upper outer ring at 330° . Data acquired at 24 msec, 6 msec after injection.

(iii) Analysis

1. Toroidal phase velocity: Figures 20 and 21 show hydrogen plasma profiles at 25 msec for injection into the levitated octupole at 18 msec. The background pressure was 2 to 4×10^{-7} Torr and the profiles were formed while the v measurements above were being conducted. The profiles are from single tipped probes with the I_g probe biased to -45 V and the V_f probe connected with 10 M Ω between the tip and ground. I_g and V_f are plotted vs. the tabulated ψ of Figure 14. The most striking observation is that V_f is quite linear in ψ between the peak and ψ_g but I_g is not. The (I, V) graph of these profiles is shown in Figure 22 and displays the linear relation between $\ln(I)$ and V_f with T^* of 2.7 eV. This implies that the density scale length in ψ space is also a constant between the peaks and ψ_g .

Using simple probe theory, an ion saturation current of 10 μ A to a $\frac{1}{8}$ " $\times$ $\frac{1}{8}$ " cylindrical tip corresponds to a density of 10^6 cm^{-3} . The corresponding debye length of 2 mm is comparable to the 3 mm linear dimensions of the tip. The sheath impedance at the floating potential is about 1 M Ω . The considerations of section III E hold and this T^* value must be considered a lower estimate.

$\partial V / \partial \psi_c$ is 3.75 V/d from the graph which corresponds to 1.65×10^5 V/Wb for $V_c = 1.5$ kV. The symbol "d" stands for the unit "Dory" which is the standard unit for ψ for all tabulated values. See section I A for details. The angular velocity is found from [V-B-1] to be 1×10^3 sec^{-1} . The inverse density scale length $(1/n) \partial n / \partial \psi = (1/T^*) \partial V / \partial \psi = 61$ Wb $^{-1}$. At 7 cm from the outer ring ($\psi = 6.75$) $R = 195$ cm and $v_E = 1.9 \times 10^5$ cm/sec. The data is taken 6 msec after injection and for

Figure V 20

$I_g(\psi)$ for $V_c = 1.5$ kV, rings levitated. Peak density is approximately $2 \dots 3 \times 10^9 \text{ cm}^{-3}$.

Figure V 21

$V_f(\phi)$ for $V_c = 1.5$ kV, rings levitated. Same conditions as in Figure 20.

Figure V 22

Parameterized (I,V) plot of the data from Figures 20 and 21.

$T^* = 2.7$ eV.

$T_1 = 10$ eV, $v_{dia} = 7.2 \times 10^5$ cm/sec. The net velocity is then $v_{net} = 9.1 \times 10^5$ cm/sec.

If the value used for T^* were 5 eV taken from Figure III 26 instead of the measured 2.7 eV, v_{dia} is 3.9×10^5 cm/sec and v_{net} is 5.8×10^5 cm/sec. The data in Chapter III are for $V_c = 2.5$ kV and the observed density is higher (corresponding to smaller λ_d and R_g) so the measurements provide a more reliable T^* estimate. Since T^* has been interpreted to be T_1 times a factor involving the diffusion coefficients which are independent of V_c (for $V_c > 800$ V), T^* itself is probably independent of V_c . The use of either T^* values, however, provides a v_{net} estimate which is within the accepted error range of the measured values. These velocities and v_{net} are also shown in Figure 19. It may be pointed out in passing that the v_A studies of this chapter led to the identification of T^* and the work of Chapter III. Since $\Omega_E + \Omega_{dia} = \Omega_{net}$ and the plasma undergoes rigid body rotation between the peak density and ψ_g , it was tempting to write $\Omega_E = \Omega_{dia}$, or (from [V-B-1])

$$\frac{\partial V}{\partial \psi} = \alpha T_1 \frac{1}{n} \frac{\partial n}{\partial \psi}.$$

This is indeed true, as shown in Chapter III.

2. Mode identification: To summarize the data presented in sections B and C, the coherent oscillations are $k_{||} = 0$ flute waves with a laboratory toroidal phase velocity equal to the sum of the $\underline{E} \times \underline{B}$ drift (from the ambipolar field) and the ion density diamagnetic drift. In the $\underline{E} \times \underline{B}$ frame moving with the plasma, these waves are ion flutes moving with the diamagnetic velocity.

The fact that these are not drift modes with a finite $k_{\parallel}$ has been established by the observation that the waves move in the ion diamagnetic direction. A fundamental result from elementary theory^{8,9} is that a drift mode moves in the electron diamagnetic direction. More sophisticated theories^{6,10} support this conclusion, at least for low frequency (ω less than an electron bounce time in the magnetic wells) positive energy waves. Yoshikawa¹¹ has observed that it is the closed field lines in a multipole which provide the stability against the drift modes. Physically, the argument is that there must be an integral number of parallel wavelengths along the field lines and the collisionless parallel streaming of the electrons can easily short out the required $E_{\parallel}$.

These flute waves appear to be in conflict with the results of the elementary developments found in many texts.^{8,9} The magnitude of the phase velocity does not agree with the predictions. These treatments assume that the energy source for these waves are the magnetic inhomogeneities of the magnetic field. The resulting phase velocity is the magnetic drift velocity for waves which obey $k^2 > 4\omega_{ci}/(v_n a_n)$ and are therefore unstable with a linear growth rate of $\gamma_L = (v_n \omega_{ci}/a_n)^{1/2}$. The ion drift speed due to gradients and curvature in the magnetic field is v_n , ω_{ci} is the ion cyclotron frequency eB/m , and a_n is the density scale length. When the magnetic drift is zero, the critical wavelength of the instability, the growth rate, and the frequency ($= k v_n$) are all zero. The development leading to this result is readily available and will not be repeated here. For the closed field line octupole, these results are modified by flux tube averaging^{1,12,13} but in any case the

phase velocity of these $k_{||} = 0$ modes should be zero or nearly so at ψ_c where $\langle v_n \rangle = 0$. The mode actually observed certainly shows no tendency to approach zero in this region. The proper analysis of this problem is to use the fact that the expansion energy of the plasma can act as a driving source for these waves. This was done first by Rosenbluth et al.¹⁴ for slab model geometry and has been reviewed extensively elsewhere. They showed from kinetic equations that when $v_{1,n} \ll v_{1,dia}$ the frequency is $\omega = \omega^* = k v_{1,dia}$. Hastie and Taylor^{15,6} did the problem independently for the case of arbitrary geometry. They solved Poisson's equation for the fluctuating charge density and obtained general dispersion relations for the specific case of multipoles. Their treatment includes all drifts, trapped particle effects and geometrical complications. For the case of flute waves, they showed that $\omega = k v_{1,dia}$ is an exact solution to all flute dispersion relations at all frequencies. For the case of uniform temperatures, their treatment showed that the flutes are fluid unstable when $\langle K \rangle = 0$ where

$$K = 2 \frac{n}{B} \left(\frac{\partial B}{\partial \psi} \right) / \left(\frac{\partial n}{\partial \psi} \right) \quad [V-C-1]$$

and $\langle \rangle$ indicates flux tube averaging. If s_n is the ψ space scale length $[(n^{-1} \partial n / \partial \psi)^{-1}]$, then $\langle K \rangle = s_n V''(\psi) / V'(\psi)$. This is < 0 for $\psi < \psi_c$ and > 0 for $\psi > \psi_c$. It therefore corresponds to the usual⁴ criterion for stabilization against interchange modes. The fraction of the ions whose magnetic drift is the same as the wave phase velocity can pass energy to the wave even in the fluid stable region and cause a weakly unstable condition. In this case, the waves are said to be residually unstable. Hastie and Taylor showed in a semi-quantitative

manner that this condition occurs when $\tau \langle \psi \rangle + \langle K \rangle > 0$ where $\tau = T_e/T_i$, and $b = k^2 \rho_i^2$ with ρ_i written for the ion gyro radius. Flute waves are residually unstable, then, for $\psi > \psi_r$ where ψ_r is determined by

$$\frac{T_e}{T_i} \langle k^2 \rho_i^2(\psi_r) \rangle + s_n \frac{V''(\psi_r)}{V'(\psi_r)} = 0. \quad [V-C-2]$$

ψ_r has been found by calculating the averages for a number of different ψ values until [2] has been met. Notice that [2] is actually independent of T_i . For the case where $T_e = 7$ eV, $s_n = 1.33$, and $k = 12 \text{ m}^{-1}$ at 7 cm ($\psi = 6.75$) in the outer bridge. The value obtained for ψ_r is 7.6. Since ψ_c is 7.8, this helps to explain the existence of waves inside the fluid stable region. These waves are being driven by the resonating ions and grow until they saturate non-linearly inside the fluid stable region. The quantitative agreement is poor between the predicted estimate for the residual instability and the presence of the waves which are observed to extend to $\psi = 6.5$, 6 cm from the ring in the outer bridge. The resonance effect should operate, however, much deeper than ψ_r to drive waves which are strongly damped by the fluid stabilizing process. It is clear from the preceding chapters that such driven long wavelength oscillations could themselves be the energy source needed to drive the turbulence cells to the observed amplitudes which are larger than predicted from thermal equilibrium theory.

3. Test of the flute mode: Figure 23 shows a plot of $\delta I/I$ (proportional to $\delta n/n$) v.s. δV_f for the waves in the $V_c = 1.5$ kV field. The data were taken simultaneously from the two tips separated by 0.5 cm toroidally. The coordinates of each point use the spatial location of

Figure V 23

$\delta I/I$ v.s. δV_f for $V_c = 1.5$ kV, levitated ring data.

the probe tips as the common parameter. The tips themselves were tungsten cylinders 3 mm long and 1 mm in diameter. Again, the I_s measurements employed a -45 V bias on the tip and V_f , a 10 M Ω impedance to ground. δI and δV were from a high pass filter with a 900 Hz cutoff. The line drawn is the best line passing through the origin which fits the points. The fluctuating amplitudes are merely the r.m.s. value of the total signal recorded and are the averages of several shots each. The inverse slope of the graph is 4.1 V.

If the fluctuations are $k_{||} = 0$ flutes, then no equilibrium is established across the phase surfaces and both elementary theory and slab model theory (Chen, [6-49], Kadomtsev, [IV.63]) show that the amplitudes obey

$$\frac{\delta n}{n} = \frac{\delta V}{(B a_n \omega/k)} = \frac{\delta V}{T_i} \frac{v^*}{v_{ph}} \quad [V-C-3]$$

for $v_m \ll v^*$. No such relationship was provided in the paper by Hastie and Taylor. Notice that the elementary treatments require that δV goes to zero in the region around ψ_c because v_{ph} would be zero there. If the data are used to select the hot ion branch of the flute wave dispersion relation where $v_{ph} = v^*$, then δV is multiplied by only T_i^{-1} .

The inverse slope of Figure 23 is about a factor of 10 below T_i and nearly 5 to 7 times less than T_e which would have been expected for drift waves (Kadomtsev, [IV.36]). The problem probably lies with the probe. The tungsten tips have never satisfactorily given a reliable V_f measurement (see section II A(v)) and δV_f could well be low. In addition, the large sheath impedance of about 1 M Ω ($R_s / [R_s + R_m] = 0.1$)

means that the operating point of the probe could have drifted closer to $V = 0$ than expected. The low density plasma has a debye length greater than the 1 mm probe diameter so the biased probe operates in the sheath dominated regime and, since δI is comparable to I , the ratio of $\delta I/I$ for a bias of $-5 T_e$ may not have been sufficient to yield $\delta n/n$. As a result, no conclusions may confidently be drawn about the relationship between the amplitudes of δn and δV .

REFERENCES FOR CHAPTER V

1. J. A. Schmidt. Ph.D. Thesis. University of Wisconsin (1969)
2. A. W. Molvik. University of Wisconsin Plasma Physics PLP-453
3. K. Chung, D. J. Rose. Proceedings of the Symposium on Turbulence of Fluids and Plasmas. XVIII (Polytechnique Press, Polytechnique Institute of Brooklyn, Brooklyn, N. Y., 1968) pp. 311-332
- 4(a) I. B. Bernstein, E. A. Frieman, M. D. Kruskal, R. M. Kulsrud. Proc. Roy. Soc. A 244, 17 (1958)
- 4(b) J. L. Johnson, C. R. Oberman, R. M. Kulsrud, E. A. Frieman. Phys. Fluids 1, 281 (1958)
5. T. Okawa, D. W. Kerst. Il Nuovo Cimento 22, 784 (1961)
6. R. J. Hastie, J. B. Taylor. Plasma Physics 13, 265 (1971)
eqn 30-34, 56
7. C. J. Armentrout, University of Wisconsin Plasma Physics PLP-470
8. F. F. Chen. Plasma Physics (Plenum Press, N. Y., 1974)
9. G. Schmidt. Physics of High Temperature Plasmas (Academic Press, N. Y., 1966)
10. B. B. Kadomtsev. Plasma Turbulence (Academic Press, N. Y., 1965)
11. S. Yoshikawa. Methods of Experimental Physics, 9 A, 305 (1971)
sections 8.1.2 and 8.3.2
12. J. A. Schmidt. University of Wisconsin Plasma Physics PLP-144
13. J. A. Schmidt. University of Wisconsin Plasma Physics PLP-209
14. M. N. Rosenbluth, N. Rostoker, N. A. Krall. Nucl. Fusion, Suppl.
Pt 1, 143 (1962)
15. J. B. Taylor, R. J. Hastie. Plasma Physics 10, 479 (1968)