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3.3 ORGANIZATIONAL DYNAMICS AND CONTROL

3.31 Control Options

Control is exercised in an organization, as in any system, in order to monitor performance and to institute corrective measures if it is found that performance improvement is possible. In engineering systems, control action is an important and sometimes even the only determinant of their dynamical behavior. One can expect the same to be true of organizations. This expectation is the reason for the emphasis placed in this project on organizational control. This section is devoted to a discussion of it. Sect. 3.4 in effect deals with it also but from a different point of view. As will be seen in both treatments, the manner in which control is exercised does indeed affect the dynamics of an organization in a rather substantial way.

The problem of how to design the control processes within an organization is more complicated in several respects than that of a system of machines. A number of basic questions must be answered, at least some of which have no parallel in engineering. They are the following.

Who has the responsibility for the internal control of an organization?

In practice, there is no clear-cut answer to this question. The responsibility is not very clearly defined in most instances and divided among several member groups, such as supervisors, accounting departments, quality control

units, etc. There may be good practical reasons for this but, in theory, it is more convenient to assume that an organizational division is unequivocally charged with the job. In our work, we have called it the "Control Branch."

What performance characteristics are monitored and how is the possibility of performance improvement diagnosed?

The answer to this question is not much clearer than to the first one, though perhaps for somewhat better reasons. The trouble is that it is often difficult to assess how well an organizational unit performs and whether improvements are possible. Financial statements or production statistics are the performance measures most often relied upon. Their trustworthiness for control purposes is somewhat questionable, however. The same is true of their usefulness because they frequently do not indicate appropriate remedial action. Organization theory is better off. It leads, first of all, to a unique performance measure, for an individual as well as for a group. This measure was in fact mentioned in a superficial way in Sect. 2.21. Its gradient, with respect to the various operational parameters provides the indications for ways of achieving improvements. The theory, however, does not suggest how much of this information is likely to be available and on what confidence levels. It does suggest alternatives, however. For example, rather than seeking data on the performance gradient with respect to the protocol probabilities, the theory suggests that the Control Branch can collect data on those probabilities or on

the processing times and error probabilities. Organizations in practice often do collect such information. Quality control groups in particular are set up to accumulate data on errors probabilities.

What corrective control measures are available?

This question assumes that performance improvements have been found desirable. It inquires into the changes that can be made in order to realize them. There actually is a bewildering variety of ways available for that but, fundamentally, they are of two kinds.

- (a) Changes in the protocol prescribed for an individual, or a group of organization members, including changes in the degree of autonomy.
- (b) Actions, such as incentives, aimed at increasing productivity by motivation (i.e., manipulations of the primary member characteristics via the secondary ones).

There is also a third alternative, namely, the replacement of one or more organization member by other, more productive, ones. This is, however, an option of little theoretical interest.

How can corrective measures be enforced?

Organization units, very much like machines, do not necessarily respond to "control signals" in the desired way. There are again essentially two ways of enforcing (or, perhaps better, of reinforcing) measures intended to improve performance.

- (a) Feedback is the more obvious and more direct of the two. It provides for continued monitoring of the response to measures already taken and for an updating of the new ones.
- (b) Incentives of a positive or negative kind can be employed to induce conformance with control directives.

The two strategams are not mutually exclusive. This report deals chiefly with the first. The second has been of primary interest to economists (see, e.g. [8], for relatively recent review of work in this direction) and to social psychologists (see e.g. [9]).

Interest among economists in the subject of organizational control was perhaps initiated through a thoughtful article by Arrow [10] in which two or our four questions are raised. Sociologists have been well aware of the topic also, though chiefly from three points of view. One is the management of member motivation, an approach which is reviewed in [11]. A second point of view is represented by a handbook article on control in organizations [12]. It is concerned mostly with what can go wrong with control (the "Dysfunctions" of the system, in sociological terminology). The third is documented by an ocean of literature which goes under the heading of "organizational change" (see e.g. [13]) and whose nub is that the stiffness of a person's resistance to change can be lessened by persuading him (her) that it isn't as bad as

(s)he thinks.

3.32 Optimizing Control of Organizational Procedures

In this section of the report, we develop a scheme for the optimization of organizational procedures or, in our terminology, of the organizational protocol. We disregard the effect of autonomy here, as well as all other secondary member characteristics. Some of these will be considered in the next several sections and in Sect. 3.44.

It will be useful to make some general introductory remarks here regarding organizations in general and the mathematical treatment on which this study is based.

The treatment leads, first of all, to the coarse block diagram or organization chart shown in Fig. 3.1.

Fig. 3.1

Coarse Diagram of Organizational Structure

The inputs to the organization, x_i , are pre-processed by a unit labeled the Staff in the figure into u_j . In this form they are passed to the "Executive" who decides on what

output y_j is to be generated by the organization in response to x_i . The division which actually produces this output is represented in the figure by the block marked Line. The arrangement as shown is called "centralized" in the theory because the Line receives its inputs only from the Executive, and not directly from the Staff or from its own sources outside the organization.

Centralized organizations, while a somewhat unrealistic abstraction, are the easiest to analyze and have the simplest properties. One of these is that the structure of the Line is best hierarchical. This conclusion is arrived at in the theory by assuming that the Line members exhibit, in addition to the primary characteristics, only one secondary one (which is too simple to have been mentioned in Chap. 2). It is that the processing times of a member may increase monotonely with the number of destinations to which they addresses his outputs. In organizational practices this is a supervisory characteristic primarily and is evidence of the fact that the time budget of a Supervisor increases with the number of subordinates he has.

The objective of the Line is to produce the desired output, y_j in the notation introduced above, whenever the Executive directs it to do so. Let it be assumed that the Line members are infallible in the absence of overload. The main objective of the design of the Line, i.e., of its structure and protocol, is then to avoid overload among its members. If this objective can be attained, error incidence

can be avoided altogether, under our infallibility assumption. It develops that the hierarchical structure can attain this provided it is large enough and provided that it uses a protocol under which its work load is suitably distributed among its members.

The Line can operate under two basically different kinds of protocol, one which has come to be called "alternate processing" and the other "parallel processing." In practice alternate processing is relied on in organizations or organizational units which are too small to exploit the benefits of job specialization. The task of producing an organizational output y_j can then be assigned by any Line supervisor equally well to any of his subordinates (in contrast to parallel processing under which it is divided among them in accordance with their competencies and carried out piecemeal). The derivation of suitable protocols is simpler, mathematically and notationally, for alternate processing which will therefore be used as the basis for our discussion of the Line operation.

Our discussion will in fact be based on the especially simple two-level hierarchy shown in the figure.

Fig. 3.2

A Simple Hierarchical Line

Each supervisor has only two subordinates. On having decided that the output y_j is to be produced, the Executive issues an instruction y_j^A to this effect to Alfa with the probability $p(y_j^A|u_j)$ or the instruction y_j^B to Bravo with $p(y_j^B|u_j)$. His protocol is thus mixed in general and, as will be mentioned below, it frequently cannot be pure.

Evidently,

$$p(y_j^A|u_j) + p(y_j^B|u_j) = 1$$

if no omissions are to occur. Suppose that his processing times for the issuing of his instructions are t_{jA}^X and t_{jB}^X , respectively. We include in these his decision-making times, as well as a pro-rated portion of the times he spends with his subordinates on other business, including their own

personal ones. His mean processing time will thus be

$$\tau^X = \sum_j p(u_j) [t_{jA}^X p(y_j^A|u_j) + t_{jB}^X p(y_j^B|u_j)] .$$

We regard the input probabilities $p(u_j)$ as given here.

(Sect. 3.34 deals with the considerations that lead to their determination.)

Consider Alfa's operation next. On having received from the Executive her directive y_j^A , she proceeds to relay it as y_j^C or y_j^D to one of the other of her subordinates, and she does so with the probabilities $p(y_j^C|y_j^A)$ and $p(y_j^D|y_j^A)$ such that

$$(3.3.1) \quad p(y_j^C|y_j^A) + p(y_j^D|y_j^A) = 1$$

She, too, uses a mixed protocol. Her mean processing time, calculated in the same way as the Executive's, is

$$(3.3.2) \quad \tau^A = \sum_j p(u_j) p(y_j^A|u_j) [t_{jC}^A p(y_j^C|y_j^A) + t_{jD}^A p(y_j^D|y_j^A)]$$

Bravo's operation is treated analogously.

It remains to discuss the functioning on the lowest (often called the "operator's") level of the Line. Consider Charlie. By contrast to Alfa, he has no one to direct. In other words, one receiving y_j^A from Alfa, he must produce y_j himself. Suppose it takes the time t_j^C for him to do so.

His mean processing time is thus

$$(3.3.3) \quad \tau^C = \sum_j p(u_j) p(y_j^A|u_j) p(y_j^C|y_j^A) t_j^C$$

The mean processing times for the other Line operators Delta, Echo, and Fox are calculated similarly.

The desired operation of the Line will be achieved if all of its members avoid overload, i.e., if

$$(3.3.4) \quad \tau^A \leq \Delta, \tau^B \leq \Delta, \dots, \tau^F \leq \Delta$$

where Δ is the mean inter-arrival time of the inputs (or more precisely, as the discussion in Sect. 32.31 shows, an interval that is short enough to prevent buffer overflow).

The Executive's mean processing time is excluded from (3.3.4), for reasons unrelated to the present topic. At any rate, given the tasks that need to be performed it may not be possible to satisfy all these overload constraints. The problem therefore is this.

Can a set of "assignment protocol probabilities" $p(y_i^A|u_i)$, $p_y(y_i^B|u_i)$, $p(y_j^C|y_j)$, $\dots$, $p(y_j^F|y_j^B)$ be found which satisfy probability constraints such as (3.3.1), and all overload constraints (3.3.4)?

As an examination of the expressions for the mean processing times, in particular for τ^A and τ^C in (3.3.2) and (3.3.3), shows this is a multilinear (or more specifically, a bilinear) programming problem in which no objective function is given and only feasibility is of interest. Multilinear programming problems were discussed in Sect.'s 3.1 and 3.2 above but not the kind in which any feasible solution, if it exists, is acceptable. Some supplementary comment on such problems is therefore indicated.

Since no objective function is prescribed one can introduce one essentially arbitrarily. In linear programming this is usually done as the "Phase I" portion of

the Simplex algorithm, by introducing artificial variables [2] (p. 101). The objective function then involves only those variables and is so chosen that its minimum is zero if and only if the original problem has a feasible solution. In fact, the minimum is attained at that solution. The same approach can be used also with multilinear programs, especially if the curvilinear simplex algorithm described in Sect. 3.13 is relied on. The game-theoretical procedure of Sect. 3.24 on the other hand seems better served by choices of objective functions that are more closely related to the problem at hand. The objective function

$$(3.3.5) \quad w = \sum_J \tau^J \quad (J = A, B, \dots, F)$$

seems especially appropriate. For a reason to be mentioned presently, it is the second algorithm that is best suited in the context of organizational control.

It is perhaps of interest that, in this particular context, two facts are known from earlier work on the subject, namely these.

There exist output generation tasks that can be accommodated without overload on a given Line only if some or all of the supervisors use mixed (i.e., stochastic) protocols.

The search for the optimal Line protocol can be restricted to those protocols which equalize the mean processing times of the subordinates of any one supervisor. Thus, for the Line of Fig. 3.2, one can require that

$$\tau A = \tau B, \tau C = \tau D, \tau E = \tau F,$$

and if any protocols at all are feasible, the equalizing one is certainly among them.

The first of these two results is the main reason for the insistence on mixed protocols in most of the work reported here. The second one indicates that the "size" of the feasible region of the programming problem can be reduced, though at the price of introducing additional constraints.

The incentive for the solution of the programming problem defined here is the same as for organizational control in general. As mentioned in the Overview, it arises whenever it is found that the performance of an organization, or of one of its units, or of one of its members, is not as good as can be expected under the prevailing internal and external conditions. It is then desirable to re-optimize the protocol under which the unit is operating. This, however, requires the solution of the programming problem that has been formulated here. The solution can be achieved by a variety of algorithms, among them those developed in Sect. 3.13 and 3.24. In some respects the last seems the most appropriate.

The algorithm is of the feedback-type. It can thus be re-initiated whenever there is a change in the conditions. It is stable, as has been shown, if those conditions do not change, that is, it converges to the optimal protocol, and it does so regardless of what protocol prevails at the time

it is (re-) initiated. The convergence proof, however, assumes that there are no lags in the system response, such as resistance to change (see Sect.'s 3.34 and Sect. 3.44 for a discussion of the waiver of this assumption).

A few caveats should be mentioned also, however. For one, it should be pointed out that the control process converges on the optimal control but does not necessarily do so in an optimal fashion. It is an "optimizing" control process, in the terminology of the title of this section, but not an optimal one in general. (An optimal process is developed in Sect. 3.44.) Moreover, it is only conjectured, as of this writing, that the process "signals" infeasibility by the convergence to zero of certain solution components. (See Sect. 3.25 for a discussion of this.) If correct, the control process would indicate the non-existence of any protocol that avoids overload on a given Line.

In order to be more specific regarding the nature of the process it is convenient to begin by introducing an abbreviated notation. In the remainder of this section we will write

$$\begin{aligned}
 &u_j^A \text{ for } p(y_j^A | u_j) \quad , \quad u_j^B \text{ for } p(y_j^B | u_j) \\
 (3.3.6) \quad &y_j^C \text{ for } p(y_j^C | y_j^A), \quad y_j^D \text{ for } p(y_j^D | y_j^A) \\
 &z_i^E \text{ for } p(x_i^E | x_i^B) \quad y_i^F \text{ for } p(y_j^F | y_j^B)
 \end{aligned}$$

and

$$\begin{aligned}
 &a_{jc} \text{ for } p(u_j) t_{jc}^A \quad \quad \quad a_{io} \text{ for } p(y_j) t_{jd}^A \\
 (3.3.7) \quad &b_{jE} \text{ for } p(u_j) t_{jE}^E \quad , \quad b_{jF} \text{ for } p(u_j) t_{jF}^F \\
 &c_j \text{ for } p(u_j) t_{jC}^C, \dots \quad \quad \quad f_j \text{ for } p(u_j) t_{jF}^F
 \end{aligned}$$

The expressions for the mean processing times introduced above, other than that of the Executive, are then

$$\begin{aligned}
 (3.3.8) \quad \tau^A &= \sum_j a_{ij} u_j^A y_j^J & (J = C, D) \\
 \tau^B &= \sum_j b_{ij} u_j^B y_j^J & (J = E, F) \\
 \tau^C &= \sum_j c_j u_j^A y_j^C, \quad \tau^D = \sum_j d_i u_j^A y_j^D, \dots, \\
 \tau^F &= \sum_j f_i u_j^B y_j^F
 \end{aligned}$$

These expressions substituted into the overload conditions (3.3.4) and the objective function (3.3.5), together with probability constraints

$$(3.3.()) \quad u_j^A + u_j^B = y_j^C + y_j^D = y_j^E + y_j^F = 1$$

(which hold for every j) define the primal programming problem that needs to be solved. It is clearly bilinear in the variables $u_j^A, u_j^B, \dots, y_j^F$.

In order to execute its solution by the game-theoretical algorithm of Sect. 3.24, one would now have to set up the duals classwise of this problem. This is a straight forward task described in Sect. 3.11. It will not be repeated here. The transition from the programming problem to the game is effected by introducing a new set of variables in accordance to (3.2.15a), i.e., by

$$\begin{aligned}
 (3.3.10) \quad u_j^A &= \frac{p_s}{p_{N+1}}, \quad u_j^B = \frac{p_s}{p_{N+1}} \\
 y_j^C &= \frac{p_s}{p_{N+2}}, \quad y_j^D = \frac{p_s}{p_{N+2}}, \quad y_j^E = \frac{p_s}{p_{N+2}}, \quad y_j^F = \frac{p_s}{p_{N+2}}
 \end{aligned}$$

in which the index s runs lexicographically from 1 to the number of primal variables. N is the total number of

variables, primal as well as dual. Similar substitutions, in accordance with (3.2.15b) are made for the dual variables.

The variables p_s , $s=1,2,\dots,N+2$ which are introduced in this way are the components of the mixed strategy of a player in a three-person, zero-sum, symmetric game. The payoff to the player, if he plays the strategy s , and his opponents u and v , would be denoted here with $f(s,u,v)$, by analogy to Sect. 3.2. The values of these payoffs are identified according to a fairly complicated scheme with the coefficients and right-hand sides of the objective function and the constraints. The scheme is described in Sect. 3.23.

The solution algorithm proceeds according to the formula (3.2.9a), i.e.,

$$(3.3.11a) \quad p_t' = (p_s + \rho \psi_s) / (1 + \rho \sum_{\sigma} \psi_{\sigma}^t) \quad (0 < \rho < 1)$$

where ψ_s is

$$(3.3.11b) \quad \psi_s = \sum_{uv} f(s,u,v) p_u p_v$$

whenever the right-hand side is positive. It is zero otherwise. For certain value of the index s which correspond to designated dual variables, ψ_s is given by a variant on (3.3.11b), namely (3.2.24b).

This control algorithm can be considered a feedback algorithm. In this view, it is assumed that protocol probabilities on the left-hand sides of (3.3.10) are reported back from the Line to the Control Branch, as those currently in use there. The Control Branch converts them to the variables p_s as in (3.3.10), adjusts them to p'_s

according to (3.3.11), and directs the Line unit to change to protocol probabilities that correspond to the p_s' .

It is tacitly assumed that the reports from the Line are truthful to begin with, and that the directives for change are promptly and accurately adopted. Under these assumptions, the feedback link is actually unnecessary. Control can be equally well exercised as an open-loop procedure. The benefit of closed-loop operation arises when the Line members do not follow the control directives promptly and accurately. Lack of promptness may be due to resistance to change among the members, a subject taken up in the next section. Inaccuracy may be due to members fallibility, a subject which is not discussed in this report. The reason is that a realistic treatment of the problem would have to include situations in which the probabilities of error incidence are partly or wholly unknown. Such a treatment was initiated but not completed under this project.

The determination of the directives for change involves the calculation of the quantities ψ_s in (3.3.11b) which require that the protocol probabilities of all Line members be known and available to the agent that does the calculating. The control process, in well established terminology, is then "centralized." It is conceivable that the job of computing the protocol adjustments under this arrangement is time-consuming enough to create overload in the Control Branch. If it does, the overload problem of the

organization has not really been solved, it has merely been shifted from the Line to the Control Branch. In such cases it is desirable to distribute the load among several agents by "decentralizing" the control process.

Decentralization is possible in the control process for the Line under a special assumption, namely that the supervisors' processing times are "destination-independent." In the notation used in (3.3.2) this would mean for Alfa that

$$t_{jC}^A = t_j^A, \quad t_{jD}^A = t_j^A,$$

and similarly for Bravo. Their mean processing times τ^A and τ^B in (3.3.8) then become linear in u_j^A and u_j^B . They do not involve the variables y_j^J , $J=C,D,E,F$ at all. The problem of overload avoidance for Alfa and Bravo by themselves would then be a linear feasibility problem which could be solved separately from its counterparts for their subordinates. Unfortunately such a solution might lead to values for the variables u_j^A and u_j^B which might make the problems for Charlie, etc., infeasible. The coupling between the two problems thus cannot be ignored. It is rather weak, however, a fact which can be exploited towards solution algorithms in a number of ways. Several of these have been investigated. All require that solution algorithms similar to (3.3.11) be run on different time scales, slower for the supervisory levels, and the slower the higher the level. These algorithms will not be reported here, however, for sake of brevity. (No decentralization

has been discovered for Lines whose supervisory processing times are "destination-dependent," and it is doubted that they exist.)

Equally unreported will go similar, and quite analogous, studies of control algorithms for the protocols used by the Staff and the Executive. The main differences between the control of Line and Staff procedures arises when the desire for autonomy is considered. This is done in Sect. 3.34.

3.33 Allowance for Resistance to Change

The conclusions in the preceding section were arrived at under several assumptions, some explicit and some implicit. One of these was mentioned somewhat in passing, namely that the directives of the Control Branch for changes in the Line protocol are adopted instantaneously. However, as was explained in Sect. 2.13, this will not usually be the case because of the phenomenon of resistance to change.

The phenomenon, in brief, is the following. If U_t^A denotes the matrix of the protocol probabilities u_j^A , defined in (3.3.6), which are used by Alfa at the time t then

$$(3.3.12) \quad U_{t+1}^A = U_t^A B_t^A + \Omega^A (I - B_t^A)$$

B_t^A is a diagonal matrix whose elements have certain properties specified in Axioms A.2.1.2 and A.2.1.4. Ω^A is a fix point of the transformation. In fact, if one accepts Axiom A.2.1.3 as well, Ω^A the protocol that Alfa is intended to adopt. In the present instance it is an assignment protocol for Alfa which, together with the protocols of the other Line members, avoids overload among all of them.

The problem is finally whether the Control Branch can issue to Alfa a sequence of directives which will change the protocol U_t^A gradually into the intended Ω^A ; and secondly whether the control process (3.3.11) derived in the preceding section is perhaps capable of achieving this, even though it was designed on the assumption that Alfa will offer no resistance to changes. It will now be shown that the answer is "yes" to both of this question.

The first answer is summarized in the following proposition (which omits the superscript A, to simplify notation).

Proposition 3.2.1. If Ω is the protocol to which Alfa is to change, there is a sequence of directives U_t which ensure that the protocols U_t which Alfa realizes converge to Ω .

Proof. From (3.3.12), we have

$$U_{t+1} - \Omega = (U_t - \Omega)B_t + (\bar{U}_t - \Omega)(I - B_t).$$

Therefore

$$(3.3.13) \quad \|U_{t+1} - \Omega\| \leq \|U_t - \Omega\| \|B_t\| + \|\bar{U}_t - \Omega\| \|I - B_t\|.$$

B_t .

By Lemma 2.1.1, the elements of B_t are bounded away from 1, irrespective of $\bar{U}_t$. There is thus an α , $0 \leq \alpha < 1$, such that $\|B_t\| \leq \alpha$. Also, $\|I - B_t\|$ is bounded by some $d < \infty$. The inequality (3.3.13) can thus be written

$$x_{t+1} \leq \alpha x_t + d u_t$$

where

$$x_t = \|U_t - \Omega\|, \quad u_t = \|\bar{U}_t - \Omega\|.$$

Since α is non-negative, it is clear that, if y_t satisfies

$$(3.3.14) \quad y_{t+1} = \alpha y_t + d u_t, \quad y_0 = x_0,$$

then

$$x_t \leq y_t \quad \text{for all } t \geq 0.$$

We will now show that the sequence $\{\bar{U}_t\}$ can be chosen to make y_t go to zero. Note that (3.3.14) represents a linear discrete-time system with a single input and single output. Since $|\alpha| < 1$ the system is asymptotically stable (its poles

are inside the unit circle). AS is well known from system theory, the output $y_t \rightarrow 0$ as $t \rightarrow \infty$ if the output $u_t \rightarrow 0$. Thus, to guarantee the desired convergence $\{\bar{U}_t\}$ must be such that

$$(3.3.15) \quad \|\bar{U}_t - \Omega\| \rightarrow 0. \quad \parallel$$

This proof shows incidentally that one could set $\bar{U}_t = \Omega$ and achieve the desired convergence but in order to utilize that, one would have to know the desired protocol Ω . That is not known in general, however. In fact the algorithm (3.3.11) is designed to determine it. We will now show that this algorithm can also be used to generate a sequence $\{\bar{U}_t\}$.

Corollary 3.2.1 Every sequence of protocols that corresponds to the vector p_t , $t=0, 1, \dots$, generated by the repeated application of the algorithm (3.3.11) contains a subsequence that converges (in the uniform norm) to an optimal protocol, provided only the last two components p_{N+1}^t and p_{N+2}^t remain bounded away from zero from some t onward.

Proof. Suppose that the n^2 elements of the protocol matrix $\bar{U}_t$ listed in, say, lexicographic order correspond to the first n^2 components of p^t . Thus, if $\bar{u}_s^t$ is one of those elements

$$\bar{u}_s^t = p_s^t / p_{N+1}^t.$$

where the transition from p_s^t to p_s^{t+1} , and from p_{N+1}^t to p_{N+1}^{t+1} is given by

$$p_s^{t+1} = (p_s^t + \rho \psi_s^t) / (1 + \rho \sum \sigma \psi \sigma^t).$$

Convergence of this sequence of transformations is in the sense that the Lyapunov function L^t

$$\lim L^t = \emptyset, \quad L^t = \sum_S (\psi_S^t)^2$$

for $t \rightarrow \infty$. Hence, there is a t_0 for every $\varepsilon > 0$ such that

$$\max_S \psi_S^t \leq \varepsilon \quad \text{for } t > t_0.$$

It follows that, for $t > t_0$,

$$\begin{aligned} |p_S^{t+1} - p_S^t| &= \rho |\psi_S^t| = \rho p_S^t \sum_{\sigma} \psi_{\sigma}^t / (1 + \rho \sum_{\sigma} \psi_{\sigma}^t) \\ &\leq \rho (\psi_S^t + \rho p_S^t \sum_{\sigma} \psi_{\sigma}^t) \leq \rho(N+3)\varepsilon \end{aligned}$$

because p_S^t is a probability, hence no greater than 1, and because the sum $\sum_{\sigma}$ contains $(N+2)$ terms. The quantity ε can be chosen as small as desired. Consequently

$$\max_S |p_S^{t+1} - p_S^t| \rightarrow 0.$$

Suppose now that t_0 is chosen large enough so that not only holds but also

$$p_{N+1}^t \geq \beta, \quad p_{N+2}^t \geq \beta$$

for some $\beta > 0$. This is possible, by assumption. It follows that

$$\begin{aligned} \max_S |\bar{u}_S^{t+1} - \bar{u}_S^t| &= \max_S |p_S^{t+1} - p_S^t| / p_{N+1}^t \\ &\leq \beta \max_S |p_S^{t+1} - p_S^t| \rightarrow 0. \end{aligned}$$

Here we have made use of the fact the elements $\bar{u}_S^t$ are all variables of the same class, in the terminology of Sect. 3.12, e.g., of the first class, hence are all associated with p_{N+1}^t .

The conclusion thus is that the $\bar{u}_S^t$ form an infinite sequence. Since they are defined on a compact space, namely that of the stochastic matrices of dimension $n \times n$, every such sequence has a convergent subsequence. Let its limit of the latter be ω_S . Then

$$\lim_t \max_S |\bar{u}_S^t - \omega_S| = 0,$$

as was to be shown.

Applied to the problem of controlling the protocol of Line member who is resistant to change this means that, in the sequence of directives $\bar{U}_t$ that is generated by the algorithm (3.3.11), some matrices $\bar{U}_t$ may have to be skipped in order to cull a subsequence that has the desired convergence property (3.3.15). It may be noted that the proof of the corollary does not involve the matrices B_t . The control algorithm in other words is ultimately effective regardless of how stiff a member's resistance is to change. The Control Branch thus need not know B_t at all but if it does not, it must implement the algorithm as a feedback procedure.

3.34 Allowance for Desire for Autonomy

In the preceding two sections, we discussed a procedure for adjustments in the protocol that is followed by an organizational Line unit. In our reasoning we disregarded all secondary member characteristics (and in fact a primary one, namely member fallibility, as well).

In this section, we consider one secondary characteristic, namely a member's desire for autonomy. Making allowance for this desire in the design of an organization can have a profound effect on the design techniques. One, relatively simple, example will be given in this section and the other, more complicated ones will be briefly sketched. Others yet are the topic of Sect. 3.4.

The example to be treated here addresses a question regarding the autonomy granted to the Executive. It stands to reason that he is more likely to be granted autonomy (by, say, the Board of Directors) and more generously so, than any other member of the organization. Such a grant will be seen to involve a number of considerations and conclusions which differ fundamentally from those developed in the preceding section. Among the latter is the realization that the responsibility for choosing an appropriate protocol for the Executive and the Staff is best vested in the Executive himself, and not in some separate agency. Fig. 3.1, in other words, is somewhat misleading. The Executive should have a double function, one within and the other without the Control Branch.

The figure also shows the Executive as deriving his inputs only from the Staff. This, too, is often not true in practice. Executives typically draw their inputs also from sources to which only they have access, their personal experience and expertise included. In fact, in many cases, it is such access that qualifies a person for the position of an Executive in the first place. We will call such sources "private." Fig. 3.1 should therefore be amended to show additional detail on the input side, as indicated in Fig. 3.3. The private input is denoted there

Fig. 3.3

Internal and Private Inputs to the Executive with x_p' , and shown along with u_j which reaches the Executive "through channels" from the Staff. The Executive

block overlaps with the one representing the Control Branch.

We assume that the inputs x_i and x_r' occur with a joint probability $p(x_i, x_r')$ and that the Line manages to avoid overload. Thus, if the Executive directs it to produce y_k , it does do so. Our assumption concerning the functioning of the staff and the Executive is that they follow mixed protocols, the Staff with the protocol probabilities $p(u_j|x_i)$ and the Executive $p(y_k|u_j, x_r')$. Our assumption concerning the latter is thus more general here than in the preceding section. The main reason for introducing mixed protocols in the Staff/Executive interaction is to prevent overload on the Executive. Avoiding it among the Staff is, of course, a consideration also, but it will not be taken into account here.

The overload constraint for the Executive then takes the form

$$(3.3.16) \quad \tau \bar{X} = \sum_t \bar{X}(j, r; k) p(x_i, x_r') p(u_j|x_i) p(y_k|u_j, x_r') \leq \Delta,$$

with the same interpretation for Δ as in the preceding section, and the probability constraints

$$(3.3.17) \quad \sum_j p(u_j|x_i) = 1, \quad \sum_k p(y_k|u_j, x_r') = 1.$$

The performance of the organization as a whole under these conditions is

$$(3.3.18)$$

$$E\{\psi\} = \sum \psi(i, r; k) p(x_i, x_r') p(u_j|x_i) p(y_k|u_j, x_r')$$

The sums in (3.3.16) and (3.3.18), according to a convention made earlier in this chapter, range over all indices unless

indicated otherwise.

One can now ask what the best protocols are which the Staff and the Executive can use. This question can be, and has been, answered. For the present discussion, however, this answer would be unhelpful because, as explained at the end of Sect. 2.22, the prescription of any specific protocol would deprive the Executive of all autonomy and hence would depress his productivity. It is, therefore, often better not to make such a prescription, even though this implies that the performance of the organization cannot be uniquely determined. The concept of a definite performance, optimality included, thus becomes meaningless.

The first question that needs to be answered under the circumstances is what kind of operational goal the organization should seek. In this project the answer was borrowed from one that appears to be fairly common in practice (and on occasion also the result of theoretical arguments [14]). According to it, a "target interval" $[L, U]$ is designated within which the actual performance is to lie, if at all possible. Thus, it is desired that

$$(3.3.19) \quad \min E\{\Psi\} \geq L, \quad \max E\{\Psi\} \leq U.$$

if that can be arranged. (We will discuss briefly later on in this section what to do if it cannot.) The extrema above are subject to the constraints (3.3.16) and (3.3.17), and to whatever additional limitations are imposed on the Executive's freedom of choice among his protocols.

A second point in need of clarification is concerned

with the variables with respect to which the maximization and minimization in (3.3.19) is to be carried out. The Executive protocol probabilities $p(y_k|u_j, x_r')$ are certainly among them, but so are presumably the Staff protocol probabilities $p(u_j|x_i)$. The latter cannot be fixed in any rational way without knowing the former. But since only the Executive himself has that knowledge, one is led to the conclusion that he must be the one to attune the Staff operation to the one he chooses for himself. This particular aspect of organizational control is therefore almost inevitably a part of the Executive's job, and it is reflected in our drawing of Fig. 3.3. In view of this we assume that the extrema in (3.3.19) are with respect to both the Executive and the Staff protocols.

A third point that needs to be clarified is the definition of the set Π of protocol probabilities $p(y_k|u_j, x_r')$ from within which the Executive is free to choose them for himself. (We assume that there are no restrictions on his choice among Staff protocols $p(u_j|x_i)$; in particular we assume that he need not make allowances for autonomy among the Staff.) One possibility that comes to mind as a definition for Π is by way of upper and lower limits for the $p(y_k|u_j, x_r')$, i.e., by

(3.3.20)

$$p(y_k|u_j, x_r') \leq p(y_k|u_j, x_r') \leq \bar{p}(y_k|u_j, x_r').$$

This can be done. It leads to a meaningfully formulated problem, however one whose solution seems quite difficult.

A simpler, and (we feel) more realistic, formulation is to specify the Executive protocol set Π indirectly, by way of "budget" constraints. The chief Executive of a utility firm will presumably be instructed by the Board of Directors to be sure to keep the financial of the organization performance within certain bounds but will be given complete autonomy regarding the organizational procedures he institutes to achieve this. Such an instruction, however, is merely a re-statement of the target interval (3.3.19). We will assume here that this is in fact the manner in which the Executive autonomy is delineated. In this way we avoid having to introduce new concepts and new problem complications.

With this assumption, the problem of interest (to the Board of Directors, in our example) is whether or not the Executive can find any protocols at all that satisfy the budget constraints (3.3.19), along with the overload and probability constraints (3.3.16) and (3.3.17). This would be a simple (bilinear) feasibility problem very similar in form, and in solution, to the one which emerged in Sect. 3.32 for finding a feasible assignment protocol for the Line. The only difficulty here is that the mean processing time τ^X in (3.3.16) is a function of the degree of autonomy α^X which the Executive derives from the latitude given to him in his choice of protocol.

As shown in Sect. 2.2, α^X depends on the expectation of the personal utilities ψ^X which the Executive attaches to

the various protocols, his own as well as the Staff's, among which he can choose. This is the quantity

$$E\{\psi^X\} = \sum \psi^X(i, r; k) p(u_j, x_r) p(y_k | u_j, x_r)$$

In fact, the degree α^X of his autonomy was shown as the difference between the smallest and largest values of $E\{\psi^X\}$ which he can attain by varying those protocols, under some very plausible axioms.

The utility matrix ψ^X is presumably unknown to the Board of Directors, hence so are α^X and τ^X . Offhand, therefore, the Board has no way of assessing the effect of the overload constraint (3.3.16). It may, however, be willing to accept two cases as being the worst and best under the circumstances. The worst case in this view would be that the Executive is totally indifferent to the options made available to him by Board. In such a case $\psi^X_{ij}(i, r, k)$ would be a constant and, according to Proposition 2.2.4, α^X is minimal. Nevertheless, this would determine the values of the processing times $t^X(j, r; k)$ in (3.3.16) and, with them, would uniquely define the feasibility problem that needs to be solved.

The best case, in the Board's view, might be the case in which the Executive identifies his preferences with those of the organization, so that

$$\psi^X(i, r, k) = \psi(i, r; k) .$$

If he actually does so then, by (2.2.4), his degree of autonomy is

$$\alpha^X = \max E\{\psi^X\} - \min E\{\psi^X\} =$$

$$= \max E\{\Psi\} - \min E\{\Psi\} = U - L.$$

This would again determine the processing times $t^X(j,r;k)$, at values that are presumably much lower than those in the worst case, and it would uniquely define a second feasibility problem which would have to be solved for the best case.

In either case, a solution procedure along the lines described in the preceding section is clearly applicable also here.

What we have called the worst case above is unarguably the worst, from the standpoint of someone wishing to grant Executive autonomy. On the other hand, there may be cases that are better than our "best." In those, the Executive would value some of his protocol options even higher than the organization.

It may be worth adding some remarks here on what to do if it is found that one or both of the feasibility problem defined above have no solution. A number of alternatives exist under such circumstances. One that is conceptually the closest to the topic of Sect. 3.4 is the following. To fix ideas, suppose that the best-case problem has been found infeasible. One can then ask how "near" to the target interval it is possible to bring the actual performance interval. This question requires first of all a clarification of what is meant by the "nearness" between two intervals, in the present case between the interval $I = [L,U]$ of desired performances and the interval $I_a = [L_a, U_a]$

of actual ones, where

$$L_a = \min E\{\Psi\}, \quad U_a = \max E\{\Psi\}.$$

This is a quantity, sometimes called a "semi-Hausdorff distance," defined by

$$\delta(I, I_a) = \max (r(L - L_a), r(U_a - U))$$

where r is the ramp function

$$x \quad \text{for } x \geq 0,$$

$$r(x) =$$

$$0 \quad \text{for } x < 0.$$

It is known ([15] or [16], p. 170) that the minimization of a quantity such as $\delta(I, I_a)$ is equivalent to that on an artificial nonnegative variable z , subject to the constraints

$$L - L_a \geq z, \quad U_a - U \geq z,$$

in addition to the already established constraints (3.3.16) and (3.3.17), as well as those limiting the Executive's autonomy. If those are again of the budgetary type, they would now be changed from (3.3.19)

$$(3.3.21) \quad E\{\Psi\} \geq L^X, \quad E\{\Psi\} \leq U^X$$

where L^X and U^X would be variables replacing the given constants L and U which had been found infeasible. The problem now would be to minimize z with respect to L^X and U^X , subject to the constraints just mentioned. However, since the mean processing time in the present (best) case would be a largely unspecified function of $(U^X - L^X)$, any solution procedure would have to rely on more or less empirical search algorithms. These are not of sufficient

interest to be reported here.

We also omit a report here on the problem formulation and solution if the set Π defining the Executive's autonomy is given by (3.3.20), instead of by (3.3.19) or (3.3.21).

3.35 Comments

In this section we have presented three applications to organizational control problems of a control algorithm developed in Sect. 3.2. They represent a sample from among those that have been studied under this project. We chose these three as being typical of the rest. To say that they are "typical" is perhaps understating the matter. Actually, each of those other problems has certain features which differentiate it in a significant way from the sample discussed here. With one possible exception, however, they all have one thing in common. They can be implemented by the algorithm we derived in Sect. 3.2, or by some variations on it. (The exception may be the problem mentioned at the end of Sect. 3.34 but even that shows signs of yielding.)

Under the circumstances one can ask whether organizations in practice actually use internal control schemes that resemble the ones we derived in the preceding sections. We rather doubt that they do. In the Overview to this chapter we suggested one possible reason for this. It is that people do not seem to be as conscious of the importance of organizational control now and that, historically, there probably was no good reason for them to be so. However, we suspect that even a recognition of the importance of the subject would not have led to the adoption of the kind of control schemes we have derived. They are not obvious enough intuitively for that to happen. It is thus possible that our theoretical work points to some very

useful practical control innovations.

This work would have to undergo considerable adaptation before it became practical, a process that would no doubt be best attempted in collaboration with an experienced management consulting firm. It would, however, also require additional theoretical work. Ours has so far ignored several important issues. Thus, we have assumed that the reports to the (fictional) Control Branch are accurate and truthful. In practice they need not be. The inputs to that Branch will thus be of uncertain reliability. Such uncertainties would presumably be modeled by intervals in our work, in the present case by confidence intervals on the data supplied to the Control Branch. The theory described so far has nothing to say on how the Branch should operate under those circumstances.

The theory also has little to say on how the Control Branch should proceed when it cannot be sure that its directives will be implemented among the organization as given. The only failure of such an implementation which we have analyzed here is due to resistance to change. There are other possible causes of failure, however. Simple member fallibility is one. Willful disregard is another. The economist's solution to the latter problem is the bestowal of suitable incentives. These should be included in the theory. They are then "loyalty-inducing" incentives, rather than the motivational ones we discussed in Sect. 2.22. The latter, however, should probably be considered as

well. If they are, a theory similar to the one developed in Sect. 3.33 would have to be developed for them.

3.4 PLANNING UNDER UNCERTAINTY

3.41 Uncertainty Modelling and the Planning Problem

The uncertainty structure in many real-world systems is quite often not amenable to probabilistic modelling. This is especially true for systems with economic, social or human elements where information regarding some quantities may be too unstructured to allow setting up of a reasonable probabilistic model. In these cases it seems that a set-valued model of uncertainty is more appropriate. This type of model is then also able to accommodate uncertainties which arise in these systems due to factors like behavioral freedom. Moreover in some systems the state variables themselves are probabilities, and uncertainties in these are in the form of confidence intervals leading to the set-valued uncertainty model (see subsection 3.44 for an example).

In this section we treat the problem of planning for a general discrete-time system with the above-mentioned uncertainty structure. We look at a "satisficing" problem but interpret it in the framework of an optimization problem. The goal is to reach certain target sets over the period of the plan and we interpret this as trying to be "as close as possible" to the targets sets.

Treating the uncertainty as set-valued, i.e., considering knowledge about an uncertain variable x to be of the form $x \in X$ (X known), is not really new. This has been considered by several researchers [17-22]. In the context

of control or planning, however, work has concentrated on reachability problems. Also, the use of observations on the system have not been considered except for the linear case. In recent times the treatment of control problems with this uncertainty structure has been advocated by Drenick [23].

We shall show that the problem can be treated as an optimal control problem for a (set-valued) dynamic system evolving on a metric space formed from the compact subsets of a Euclidean space. Set-valued dynamic systems have been considered by earlier researchers (see, e.g., [24]), but they have concentrated on attainability problems and open loop procedures without observations.

We consider the system

$$(3.4.1) \quad x_{n+1} = f(x_n, w_n, u_n)$$

where x_n is the current ℓ -dimensional control or command vector and w_n is a D -vector of disturbances or choice-variables internal to the system. In some cases w_n can be considered as external uncertain inputs. The a priori knowledge concerning the system is summarized in the following relations:

$$(3.4.2) \quad x_0 \in X_0$$

$$(3.4.3) \quad w_n \in W_n(x_n, u_n)$$

where X_0 is a known compact set and W_n are known set valued mappings or correspondences from $R^\ell \times R^q$ to R^D . Controls are chosen from fixed compact sets U_n , i.e., $u_n \in U_n$.

Thus all knowledge concerning the initial state and subsequent internal choice-variables or disturbances is

set-valued. Observations on the system are also affected by uncertainty and we know only that the p -dimensional observation at time n : y_n satisfies

$$(3.4.4) \quad y_n \in H(x_n)$$

where H is a known set-valued mapping or correspondence from R^l to R^p

We assume for all n

$f_n: R^l \times R^d \times R^q \rightarrow R^l$ is continuous

$(x, u) \mapsto W_n(x, u)$ is a continuous correspondence

$x \mapsto H(x)$ is a continuous correspondence

W_n and H are compact set-valued

(For continuity of correspondences see [26].)

Compact target sets $T_1, T_2, \dots, T_N$ are given in R^l over N periods. The objective is to keep the state as close as possible to the T_n by a suitable choice of controls, which depend causally on the observations.

The "closeness" to the target sets will be measured via a metric or distance function d defined on R^l . The choice of the metric is free and will depend on the particular problem at hand, the nature of the target sets, etc.

Uncertainty concerning the knowledge of f in the form of parameter uncertainty may also be included. This leads to a structure similar to that obtained from the above, but is accompanied by notational and technical complications which we wish to avoid for the present. Some remarks concerning this are presented in Sect. 3.4.

Let $\hat{X}_0$ denote the set of possible state vectors at time

1, given the information at time zero. The set of states at time 1 compatible with the observation at time 1, is denoted by Y_1 . Then we have from (3.4.1) to (3.4.4):

$$\hat{X}_0 = \bigcup_{x \in X_0} \bigcup_{w \in W_0(x, u_0)} \{f_0(x, w, u_0)\}$$

$$Y_1 = \{x \in R^l \mid y_1 \in H(x)\}.$$

Using the above, if X_1 denotes the set of all possible states at time 1, based on all the information at time 1, clearly

$$X_1 = \hat{X}_0 \cap Y_1$$

This process can be continued to generate the set of possible states at each time as follows:

$$(3.4.6) \quad \left\{ \begin{array}{l} \hat{X}_n = \bigcup_{x \in X_n} \bigcup_{w \in W(x, u_n)} \{f_n(x, w, u_n)\} \\ Y_{n+1} = \{x \in R^l \mid y_{n+1} \in H(x)\} \\ X_{n+1} = \hat{X}_n \cap Y_{n+1} \end{array} \right.$$

for $n = 0, 1, \dots, N-1$ where

$$(3.4.7) \quad \left\{ \begin{array}{l} \hat{X}_n = \text{set of all possible states at time } n+1, \\ \quad \text{based on information at time } n. \\ Y_{n+1} = \text{set of state vectors compatible with} \\ \quad \text{the observation at time } n+1 \\ X_{n+1} = \text{set of all possible state vectors, based} \\ \quad \text{on information at time } n+1. \end{array} \right.$$

Let us fix a metric d on R^k . This will be used to measure distances of the state vector from the target sets. Since all we know about the state vector x_n , at time n is : $x_n \in X_n$, it is clear that "making x_n close to T_n " should be interpreted as "making $\sup_{x \in X_n} d(x, T_n)$ small." Here $d(x, A)$ is the usual distance between a point and a set as induced by the metric d .

We have to combine the "costs" for different periods in order to obtain a single objective function to be minimized. Several choices are possible, two of which are exhibited below:

$$\max_{1 \leq n \leq N} \sup_{x \in X_n} d(x, T_n)$$

or

$$1/N \sum_{n=1}^N \sup_{x \in X_n} d(x, T_n)$$

We will work with only one of these (the maximum), though similar results can be obtained for any of the other alternatives. The choice of the actual form will reflect the sociopsychological characteristics of the planner or the organization. For example, the choice of the "max" form would imply a conservative or risk-averse choice.

As it stands, the above objective function depends on future observations. This dependence must be removed for the plan to be feasible, since the control choices have to be causal. This can be done by taking the supremum over all the possible observations which can occur in the future, subject to present knowledge. Since our knowledge concerning the observations is set-valued, this is the only unbiased way of removing the observation dependence.

The problem can now be formally stated as:

$$(P_0) \left\{ \begin{array}{l} \text{Find the infimum of } J \text{ over control choices} \\ u_0, \dots, u_{N-1}, \text{ with the control } u_n \text{ depending} \\ \text{only on } y_1, \dots, y_n \text{ and } x_0 \end{array} \right.$$

where

$$(3.4.8) \quad J(x_0, u_0, u_1, \dots, u_{N-1}) = \sup_{y_1 \in \hat{H}_0} \sup_{y_2 \in \hat{H}_1} \dots \sup_{y_N \in \hat{H}_{N-1}}$$

$$\max_{1 \leq n \leq N} \sup_{x \in X_n} d(x, T_n)$$

with

$$(3.4.9) \quad \hat{H}_n = \bigcup_{x \in X_n} H(x)$$

and $X_n, \hat{X}_n$ given by (3.4.6).

Note that as used in (3.4.8), u_n denotes a function

rather than the value of the control.

3.42 Solution by Generalized Dynamic Programming

We now show that the problem can be treated as one of controlling a dynamic system evolving on an abstract state space. The appropriate state space is the set $\mathcal{X}$ (compact subsets of R^l) endowed with the Hausdorff metric h . By extending Theorem 4.8.1 of [27], it can easily be seen that $(\mathcal{X}, h)$ is a complete metric space.

Consider the functions $F_n : \mathcal{X} \times R^q \rightarrow \mathcal{X}$ and $G : \mathcal{X} \times R^p \rightarrow \mathcal{X}$ defined as

$$(3.4.10) \quad F_n(X, u) = \bigcup_{x \in X} \bigcup_{w \in W_n(x, u)} \{f_n(x, w, u)\}$$

$$(3.4.11) \quad G(X, y) = X \cap H^-(y)$$

where

$$(3.4.12) \quad H^-(y) = \{x \in R^l \mid y \in H(x)\}$$

For F_n and G to be well-defined functions as indicated, we must show that $F_n(X, u)$ and $G(X, y)$ are compact subsets of R^l for every $X \in \mathcal{X}$, $u \in R^q$, $y \in R^p$. This is shown in Appendix 3.4A. Then on using (3.46) we see that the evolution of information can be considered as a dynamic system evolving on $(\mathcal{X}, h)$ described by:

$$(3.4.13) \quad \begin{aligned} \hat{X}_n &= F_n(X_n, u_n) \\ X_{n+1} &= G(\hat{X}_n, y_{n+1}) \end{aligned}$$

and we may call X_n the (information) state at time n , and $\hat{X}_n$ the one-step predicted (information) state at time n .

To complete our representation of the problem, we have to define the objective function J on $\mathcal{X}$. This is easily

done from (3.4.8), if we use a set function related to the Hausdorff distance--the Hausdorff "semi-distance" defined, for all $A, B \in \mathcal{H}$ by

$$(3.4.14) \quad h_-(A, B) = \sup_{x \in A} d(x, B)$$

(see [27], p. 163).

Using this on (3.4.8)

(3.4.15)

$$J(x_0, u_0, \dots, u_{N-1}) = \sup_{y_1 \in \hat{H}_0} \dots \sup_{y_N \in \hat{H}_{N-1}} \max_{1 \leq n \leq N} h_-(X_n, T_n)$$

Note that by (3.4.9), the H_n are also defined on $\mathcal{X}$. The problem now is

Find the infimum of J (of 3.4.15) over controls
(P) $u_0, \dots, u_{N-1}$ which are causal functions of
the observations $y_1, \dots, y_N$ for the dynamical
system (3.4.13) on the metric space $(\mathcal{X}, h)$.

[If a probabilistic view of uncertainty were taken, the problem could be viewed as a control problem for a dynamical system defined on a function space--the space of probability density functions on R^l . The conditional density function (given the observations), would correspond to the state in that framework. We have simpler and smaller state space here.]

The ideas of dynamic programming can be applied to the present problem, though this is an infinite dimensional

system and the state space is not a linear space.* The solution obtained is given below. The necessary steps are outlined in Appendix 3.4B.

Let $J_N(X, u)$ be the "cost-to-go" at stage n , if the control u is used and X is the state, at time n . Let $J_n^*(X)$ be the optimal cost-to-go for the state X . Then the following recursion relations are obtained:

$$(3.4.16) \quad J_{N-1}(X, u) = h_{N-1}(F(X, u), T_N)$$

$$\text{for } (X, u) \in \mathcal{X} \times U_{N-1}$$

$$(3.4.17) \quad J_{N-1}^*(X) = \inf_{u \in U_{N-1}} J_{N-1}(X, u)$$

For $n = N-2, N-3, \dots, 0$:

$$(3.4.18) \quad J_n(X, u) = \max_n \{h_n(F(X, u), T_{n+1}), \sup_{y \in H_n(X, u)} J_{n+1}^*(G(F_n(X, u), y))\}$$

for $(X, u) \in \mathcal{X} \times U_n$, where

$$(3.4.19) \quad \hat{H}_n(X, u) = \{x \in F_n(X, u) \mid H(x)\}$$

$$(3.4.20) \quad J_n^*(X) = \inf_{u \in U_n} J_n(X, u)$$

*Addition of sets in $\mathcal{X}$ can be defined as usual, but the elements of $\mathcal{X}$ have no additive inverse, so that $\mathcal{X}$ is not a group relative to the addition operation [28].

Finally

$J_0^*(X_0)$ is the solution of (P), i.e.,

(3.4.21)

$$J_0^*(X_0) = \inf J$$

If the infima in these relations are actually minima, and $u_n^*(X)$ denotes the minimizing control for $J_n^*(X)$, the actual sequence of optimal controls will be:

$$u_0^*(X_0), u_1^*(X_1), \dots, u_{N-1}^*(X_{N-1}) .$$

Thus the solution is feedback, and the control depends on the current state only. Even if the minima do not exist, we can find a set of controls $\{u_n^\varepsilon(X)\}$ which bring the "costs-to-go" to within ε of the infimum. The sequence of feedback controls: $\{u_n^\varepsilon(X_n)\}$ would then keep the objective function to within ε of the infimum.

In general, continuity of the J_n cannot be shown. It is possible, however, to show that the objective function is upper semi-continuous. The next section contains a discussion of this together with its impact on the robustness of the optimization problem and on the stability of implemented algorithms.

3.43 Stability and Robustness of Solution Algorithms

The continuity problem mentioned at the end of the previous sub-section seems to be inherent in the knowledge structure and does not depend on our formulation.

Since all information is set-valued, the intersection $\hat{X}_n \cap Y_{n+1}$ in (3.4.6) is the most appropriate way to combine. This is precisely the source of the problem since intersections do not preserve continuity. Also, the correspondence $Y_{n+1} \mapsto Y_{n+1}$ is not continuous in general. This propagates through to the cost-to-go functions at every stage. In fact, for the present situation, the cost-to-go functions J_n cannot even be shown to be lower semi-continuous. Hence the minimizing controls in (3.4.16-21) may not exist.

However, it is shown in Appendix 3.4C that $J_n(X, u)$ is upper semi-continuous in X and u . Since U_n is a fixed compact set, by Theorem 4.8.3 of [27], $J_n^*(X) = \inf_{u_n \in U_n} J_n(X, u_n)$ is also upper semi-continuous.

Thus the minimizing problem at any stage is u.s.c. in the previous state and present control, although lower semi-continuity, which would guarantee the existence of the minimizing controls at each stage, is absent. Our discussion below will show that the u.s.c. property is perhaps more advantageous than l.s.c. when we consider robustness of approximate minimizations and the feasibility of minimizing procedures.

Let U be a compact subset of a Euclidean space and

$g : U \rightarrow \mathbb{R}$ a real-valued function which is to be minimized ("infimized") to within an approximation of ε of the true infimum value $= \inf_{u \in U} g(u) = g^*$. A general practical scheme for doing this will involve a sequence of uniform finite grids on U , $\mathcal{G}_n$, such that the grid size δ_n of $\mathcal{G}_n$ approaches zero as $n \rightarrow \infty$. The algorithm will then attempt to choose an N large enough with the hope that there will be a point on the grid $\mathcal{G}_N$ whose g value is within ε of g^* . Can we guarantee this? We shall show that if g is upper semi-continuous, this can be guaranteed. A simple example shows that this is not possible if g is lower semi-continuous (even though a minimum exists!).

Let us consider the example, before we prove our claim. Take $U = [0, 1]$, $g(u) = 1$, if $u \neq \pi/4$ and $g(\pi/4) = 0$. It is clear that g is lower semi-continuous. Now consider the sequence of grids $\mathcal{G}_n = \{0, 1/n, 2/n, \dots, (n-1)/n, 1\}$. Then $\inf_{u \in \mathcal{G}_n} g(u) = 1$ for each n since $\pi/4 \notin \bigcup_n \mathcal{G}_n$. This means that searches over any of the grids, however small the grid size will produce points which are at a distance of one from the true infimum.

Now assume g to be upper semi-continuous. Let $x_0 \in U$ be such that $g^* \leq g(x_0) < g^* + \varepsilon$ (possible by definition of g^*). The set $V = \{x \mid g(x) < g^* + \varepsilon\}$ is open, by upper semi-continuity of g . This means that there is a $\delta > 0$ such that $x \in U$ and $d(x, x_0) < \delta$ implies $x \in V$ or $g(x) < g^* + \varepsilon$ (and of course $g(x) \geq g^*$). Thus all points within δ of x_0 have g values within ε of g^* . Since the grid size δ_n of $\mathcal{G}_n \rightarrow$

0 as $n \rightarrow \infty$, clearly there is an N and a point $x_N \in \mathcal{G}_n$ with $d(x_N, x_0) < \delta$. So by the above $g^* \leq g(x_N) < g^* + \varepsilon$, with $x_N \in \mathcal{G}_N$.

Hence the minimizing problem at each stage in the dynamic programming procedure can be solved to within any approximation of the optimal value, by searching over a fine enough grid. This proves the feasibility of approximate minimizing procedures via quantization.

Consider now the related question of robustness. Suppose the approximate minimizing controls have been generated by some such quantization procedure. These controls are used but the actual initial state x_0 is slightly different from the assumed one. Can we guarantee that by using the calculated controls the actual performance can deteriorate only slightly from the calculated one? For a lower semi-continuous function the answer is negative, as this example shows. Take

$h : [0,1] \times [0,1] \rightarrow \mathbb{R}$ to be

$$h(1/2, 1/2) = 0$$

$$h(u,v) = 1 \text{ otherwise.}$$

Then if v is assumed to be $1/2$, and the control $u = 1/2$ is used, we have

$$h(1/2, 1/2) = 0, \text{ but } h(1/2, 1/2 \pm \varepsilon) = 1$$

for any $\varepsilon > 0$, however small.

With upper semi-continuity, however, the picture is quite different, and the question can be answered

positively. Let $h : U \times V \rightarrow R$ be jointly upper semi-continuous and u_0 be the control being used with v assumed to be v_0 . For every $\epsilon > 0$, there is a $\delta > 0$ such that if $d(v, v_0) < \delta$ then $h(u_0, v) < h(u_0, v_0) + \epsilon$, by upper semi-continuity. Thus, though h could decrease drastically, it cannot increase by much if v is perturbed slightly. Since our problem is a minimization one, we have demonstrated our claim.

Thus the problem is robust in the sense described above. If initial data are close to actual ones, the actual performance can be better than the calculated one, and if it does deteriorate, can deteriorate only slightly.

3.44 Application: Control of Resistance to Change

In this sub-section we apply our method to a concrete problem from organization control. To simplify the presentation we have isolated a sub-system of the organization.

The problem is one of resistance to change in an organization which has been treated in Sect. 2.1.3. In the model presented there, the protocol matrix of the individual may be treated as the original "state." Reminding ourselves that P_n is this probability matrix used by the member at time n , and P_n^* the directive from the organization, we have from eqn. (2.1.11)

$$(3.4.22) \quad P_{n+1} = P_n B_n(P_n^*) + P_n^*(I - B_n(P_n^*)).$$

The problem then is to choose the control directives $P_0^*, \dots, P_{N-1}^*$ over N periods so that the matrices $P_1, \dots, P_N$ lie close to some target sets. The target sets, of course, are decided by the organization.

The uncertainty in the problem arises from the knowledge concerning the B_n , which is interval valued and the measurements of P_n , which also give rise to interval uncertainty. Thus, it is known that $P_0 \in \mathcal{P}_0$ and that $B(P^*) \in \mathcal{B}_n(P^*)$, where $\mathcal{P}_0$ and $\mathcal{B}_n$ have interval components.

The problem then is evidently one of the type presented in sub-section 3.41 and developed in sub-section 3.42. Taking X_n to be identified with the set of possible P_n and u_n with P_n^* , the system (3.4.13) emerges as does the problem

(P). Instead of the objective function (3.4.15), one of the other alternatives discussed after eqn. (3.4.7) can be used, depending on whether the operating protocols for all N periods, or only that of the final period, is important to the organization.

The functions F_n of (3.4.10) are given by

$$(3.4.23) \quad F_n(X, u) = \bigcup_{P \in X} \bigcup_{B \in \mathcal{B}_n(u)} \{PB + u(I-B)\}$$

This system has a special structure in that the states X_n are always convex polytopes. This is because of the interval valued knowledge concerning B and P_0 , and the special form of (3.4.22). The state space for this problem can therefore be considered to be the reduced space $\mathcal{Y}$, where $\mathcal{Y}$ is the class of convex polytopes in R^S . Also at any stage n , the number of possible extreme points for the state polytope is bounded so that the state space is actually a finite-dimensional space for a given N .

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3.45 Computational Aspects and Sub-Optimal Procedures

The computational difficulties associated with the general planning problem under consideration become quite transparent if we concentrate on the example of the previous sub-section.

Consider the special case of two dimensions for the original state space (before uncertainty). In other words if the organization member receives one of two inputs and has to produce one of two outputs, the transition probability matrix can be specified by two independent variables. In this case the original state space dimension is 2, as is the control space dimension. Let each original state space variable be quantized via a grid of n_x points and each control variable by a grid of n_u points.

Then a direct attack on the problem of the previous section via a search over the N controls required for optimality leads to

$$2^2 \sum_{i=1}^N (n_u^2)^i$$

calculations which are of the function evaluation type.

(Note that the initial state set is specified by 4 points.)

The dynamic programming type solution presented in sub-section 3.42, however, results in

$$N n_u^2 \sum_{M=2}^{2n_x} \left(\sum_{n=(n_x-M) \vee 0}^{(2n_x-M)/2} \frac{n_x!}{n!} \right)^2$$

calculations. This bound is obtained by finding a tight bound on the number of possible state sets which can arise after the quantization, using the fact that the state set is a convex polytope.

In $N = 10$, $n_x = n_u = 5$ and a calculation time of 1 s is assumed, the direct optimization leads to a calculation time of 12.6 years vs 93 seconds for the abstract dynamic programming approach. The considerable reduction achieved via our approach is obvious.

If the original state space dimension or the parameters N , n_x , n_u are increased, however, the calculation time shoots up even for the dynamic programming approach (the "curse of dimensionality"). And, for the general problem, the true state space is infinite dimensional exhibiting the impossibility of direct implementation of the solution. This is an inherent limitation of the problem itself, since for most cases its computational complexity is orders of magnitude larger than our computational and storage capabilities as "boundedly rational" human beings.

The way out is via sub-optimality. The main thing is to replace the state X_n (which is an element of an infinite-dimensional space) via another "state" X_n which resides in a finite dimensional space. The state space $(\mathcal{X}, h)$ is not a linear space and hence does not have a nice structure like that of a Hilbert space. So approximation theory does not seem to be of much help. The theory of approximations in Hilbert Space could perhaps be used via

some embedding operations [28], but this is an open area.

An alternative idea is to bound the sets X_n via simpler sets and enlarge the transitions, so as to keep the bounding set within this class. For example in (3.4.13)

$$\hat{X}_n = F_n(X_n, u_n)$$

can be replaced by

$$\hat{\bar{X}}_n = \bar{F}_n(\bar{X}_n, u_n)$$

where

$$\bar{F}_n(X, u) = \text{smallest convex polytope with } n_0 \text{ vertices containing } F_n(X, u)$$

with n_0 fixed. $\bar{X}_0 \supset X_0$ is chosen similarly as is a function $\bar{G}$. Then the new state $\bar{X}_n$ and predicted state $\hat{\bar{X}}_n$ are polytopes with n_0 vertices resulting in a finite-dimensional state space since n_0 is fixed.

Note also that since $X_n \subset \bar{X}_n$, $h_-(X_n, T_n) \leq h_-(\bar{X}_n, T_n)$, so that the actual J_0 value obtained is less than the one calculated ($\bar{J}_0$), using this new system, for any given set of controls.

Hence the actual performance will always be at least as good as the calculated one. This is a feature which is not present in most sub-optimal procedures in stochastic control.

It is perhaps interesting to interpret the sub-optimal procedure we have described from the point of view of information structure. Bounding the set X_n by $\bar{X}_n$ is equivalent to ignoring some information or coarsening the information structure: we know that the state vector

$x_m \in X_n$, but we ignore this and pretend that we know only $x_n \in X_n$, where $X_n \subset \bar{X}_n$. The situation seems analogous to conditioning with respect to coarser σ -algebras in a stochastic framework.

3.46 Some Special Cases

The complexity of the solution for the problem prompts us to look for conditions under which simpler procedures exist. This search was fueled by the fact that in certain situations, it is observed that a one-step plan is usually solved at every period by planners in reality, instead of solving the complete plan for the future. An example of this is the class of farming models considered by Day [29].

In our terminology, we wish to see when the dynamic programming solution procedure (3.4.16)-(3.4.21) of sub-section 3.42 can be replaced by the following at each instant n :

$$\text{Solve for } \inf_{u_n} h_-(\hat{X}_n, T_{n+1})$$

We present one set of conditions, though there are others, as well as possible generalizations.

Our result also has interesting connections with behavioral patterns, and can be considered as an example of making an intuitive socio-psychological statement precise. What we have in mind is the well known association of cautious behavior or short-term planning with risk-averse, pessimistic or conservative attitudes of the planner.

If we interpret these ideas in our framework, we can associate the risk-averseness or conservativeness of the planner with the choice of the conservative loss function

$$\max_{1 \leq n \leq N} h_-(X_n, T_n)$$

among the several alternatives discussed earlier.

The concretization of the behavioral statement above is then:

Proposition 3.4.1. If the objective function is $\max h_-(X_n, T_n)$, then planning for one-step ahead at each state, i.e., solving $\inf_{u_n} h_-(\hat{X}_n, T_{n+1})$, is optimal, under the following conditions:

- (i) $T_n = T$, fixed,
- (ii) for every X , there is a $u_n^*(X)$ such that $(F_n(X, u_n^*(X))) \subset X$. $\square$

Before we prove the proposition, let us consider an example. Condition (i) is a matter of choice, so (ii) is the only one to be checked. This is satisfied in the problem of resistance to change treated earlier. This can be shown using the form (3.4.23) for F_N . Given X , choose any $P_0 \in X$. Let $u_n^* = P_0$. Then for every $P \in X$ and $B \in \mathcal{B}_n(u^*)$, $PB + u^*(I-B) = PB + P_0(I-B)$ is a convex combination of points in X (remember B is diagonal). Since X is a convex polytope, this is in X . Thus $PB + u^*(I-B) \in X$ for $B \in \mathcal{B}_n(u^*)$ and every $P \in X$. Hence $F_n(X, u^*) = \bigcup_{P \in X} \bigcup_{B \in \mathcal{B}_n(u^*)} \{PB + u(I-B)\} \subset X$ and (ii) is proved.

Proof of Proposition 3.4.1. Now to show the sufficiency of the conditions set out above. Let the control strategy at each $n \geq 1$ be u^* , i.e., $u_n = u_n^*(X_n)$. Since X_n depends only on the past and present, these are causal controls. Then at any $n \geq 1$, by condition (ii), (3.4.11) and (3.4.13)

$$X_{n+1} \subset \hat{X}_n = F_n(X_n, u_n) = F_n(X_n, u^*(X_n)) \subset X_n$$

Hence all the $X_n \subset X_1$ for $n \geq 2$ and so by the definition

(3.4.14) of $h_- : h_-(X_n, T) \leq h_-(X_1, T)$, which means that

$\max_{1 \leq n \leq N} h_-(X_n, T) = h_-(X_1, T)$. Since condition (i) implies $T_n = T$, we can use this in (3.4.15) to obtain

$$\begin{aligned} J(X_0, u_0, \underbrace{u^*, \dots, u^*}_{N-1 \text{ places}}) &= \sup_{y_1 \in \hat{H}_0} \dots \sup_{y_N \in \hat{H}_{N-1}} h_-(X_1, T) \\ &= \sup_{y_1 \in \hat{H}_0} h_-(X_1, T) \quad (\text{since the} \\ &\quad \text{dependence on } y_2, \dots, y_N \text{ has} \\ &\quad \text{disappeared}). \end{aligned}$$

Thus

$$\inf_{u_0} \inf_{u_1, \dots, u_{N-1}} J(X_0, u_0, u_1, \dots, u_{N-1}) \leq \quad (3.4.24)$$

$$\inf_{u_0} J(X_0, u_0, u^*, \dots, u^*) = \inf_{u_0} \sup_{y_1 \in \hat{H}_0} h_-(X_1, T).$$

Also $\max_{1 \leq n \leq N} h_-(X_n, T) \geq h_-(X_1, T)$, so from (3.4.15) :

(3.4.25)

$$\inf_{u_0} \inf_{u_1, \dots, u_{N-1}} J(X_0, u_0, \dots, u_{N-1}) \geq \inf_{u_0} \sup_{y_1 \in \hat{H}_0} h_-(X_1, T)$$

From (3.4.24) and (3.4.25)

$$\begin{aligned}
 J^*(X_0) &= \inf_{u_0} \sup_{y_1 \in H_0} h_-(x_1, T) \\
 &= \inf_{u_0} h_-(\hat{X}_0, T) \quad \text{(this follows as in (B.8) of Appendix B).}
 \end{aligned}$$

Thus the problem (P) at time 0 reduces to a one-step problem as desired. The above considerations can be applied at any n to yield a similar conclusion, and our proposition is proved under conditions (i) and (ii). $\square$

We note that other alternative sets of conditions as well as more general ones are possible.

Another special case which leads to a lessening of the computational burden is one where the original state is observed completely without any uncertainty. It turns out that this is the situation in certain formulations of corporate planning for a utility company, and we shall discuss this application in the next sub-section. Here we shall just demonstrate the simplification for this special case.

In this case, the observation correspondence of (3.4.4) is actually a function:

$$H(x) = \{x\}$$

The development of Sect. 3.41 can be followed through to obtain the present state set X_n as

$$X_n = \{Y_n\}$$

and hence the predicted state set $\hat{X}_n$ as

$$\hat{X}_n = \bigcup_{w \in W(x, u_n)} \{f_n(x, w, u_n)\} \text{ with } x = Y_n.$$

These can be used in (3.4.10)-(3.4.13) of Sect. 3.42 to yield the dynamic system as

$$\hat{x}_n = F_n(x_n, u_n)$$

(3.4.26)

$$x_{n+1} = y_{n+1}$$

where

$$(3.4.27) \quad F_n(x, u) = \bigcup_{w \in W_n(x, u)} \{f_n(x, w, u)\}$$

This shows that for the complete certain state observation, the state of the "abstract" dynamic system reduces to a finite-dimensional vector of fixed dimension (the dimension of the original state space). Hence the problem of "growing" state dimension and inherent infinite dimensionality of the problem discussed in Sect. 3.45 vanishes, and we need not consider sub-optimality. Instead we can look at approximations to the optimizations involved, and the discussion in Sect. 3.43 shows the feasibility of such approximations. These approximations being in fixed finite-dimensional spaces should be fairly standard and we shall not consider them any further.

3.47 Application: Utility Planning

We shall now apply our approach, specifically the special case discussed at the end of the previous sub-section, to the problem of financial planning and power generation planning for an electric utility corporation. The plan will be considered at a fairly high-level or as an aggregated plan. We shall spell out our assumptions below in order to indicate the level and nature of our "aggregations" as well as any other simplifications we make.

Our assumptions are

- (i) No new shares or bonds are issued during the period of the plan. (The issuance of new shares, etc., could be accommodated directly in our framework with a concomittant rise in eqn. complexity.)
- (ii) All loans are to be repaid by the end of the plan period.
- (iii) The tax structure is assumed to remain the same over the period of the plan, and the tax rate is taken to be a piecewise linear increasing function of taxable income. (This is compatible with standard tax structures.)
- (iv) The power generation costs are aggregated and treated as a quadratic function of the power generation levels of various generating groups in the utility. (We use the cost structure assumed in [30].) Note that this can be

considered as an external/aggregated model for a detailed production sub-model, which uses production functions of the Cobb-Douglas type or other alternatives and an optimization formulation to generate an optimal cost for each set of generation levels.) However, the specific nature of the non-linearity of the cost structure does not really affect our formulation, and can be changed if necessary.

- (v) The company is assumed to pay interest on all its loans at a uniform aggregated rate. This rate is the basic "prime" rate (given externally via projections) plus an additional incremental rate depending on the risk involved in loaning to the company. The incremental rate is taken to be proportional to the ratio of outstanding debt to total worth (capital + investments + initial worth) in any period.
- (vi) The yields available to the company from its investments is also assumed to accrue at a uniform aggregated rate, which is decided by the economy as a whole.
- (vii) We do not consider separate corporate or other office costs and assume that they have been incorporated into the generation costs.
- (viii) We do not consider different power demands and different power rates for different

groups/sectors of consumers. Instead we consider total demand and an aggregated consumer rate.

- (ix) Projections for various quantities which are decided essentially by the economy as a whole are assumed to be available throughout the period of the plan with interval uncertainties. We assume that the corporate plan can affect these quantities only negligibly, and hence they can be presumed given. Thus over the complete period of the plan, we have bounds for the following quantities:

"prime" interest rate, incremental interest rate ratio, coefficients in the power generation cost functions, range of possible production levels.

- (x) Constraints will be presumed to exist on the possible production levels, on the total consumer budget for power costs, on the amount of loan the utility can obtain and on its loan worthiness.

- (xi) The goal of the corporation will be to attain certain levels of capital, dividends and investment by the end of the plan period, while maintaining some minimum dividend level during the plan, and meeting the demand for power throughout the plan period.

The equations governing the system are described below. Individual equations with brief explanations will be provided first, before converting them to the form of the general problem considered in earlier sections.

The first equation describes the evolution of the outstanding debt incurred by the company:

$$(3.4.28) \quad U_{n+1} = U_n - \frac{1}{N-n} U_n + L_n$$

where U_n is the total unpaid debt at the beginning of the n^{th} period, L_n is the new loan obtained during the n^{th} period and N is the number of periods in the plan. The term $1/(N-n) U_n$ corresponds to the amount of capital in the n^{th} period (under our assumption of full repayment by the N^{th} period).

The total investment at the beginning of the n^{th} period, M_n evolves as

$$(3.4.29) \quad M_{n+1} = M_n + I_n$$

where I_n is the amount invested during the n^{th} period.

According to assumption (v) above, the interest rate on the company's loans is given by

$$(3.4.30) \quad r_{n+1} = r_{0,n} + \delta_n \frac{U_n}{C_n + M_n + P_0}$$

where $r_{0,n}$ is the prime rate in the n^{th} period, P_0 is the initial worth of the utility and C_n is the total capital available at the beginning of the n^{th} period. The coefficient δ_n governing the incremental rate and the prime rate $r_{0,n}$ are decided by the economy as a whole and are presumed known to within intervals. (These should be the

outputs of models for the economy and are only negligibly affected by the utility's decision.)

The net taxable income for the n^{th} period, N_n , is the sum of the investment yields and the revenue from customers less the loan interest payments

$$(3.4.31) \quad N_n = y_n M_n + \sum_{i=1}^K k_{i,n} x_{i,n} - r_n U_n$$

The coefficient y_n is the investment yield rate (see assumption (vi)), $x_{i,n}$ is the power production level for the utilities i^{th} generating sector and $k_{i,n}$ is the consumer payment rate associated with this sector. There are K generation sectors. The taxes for the n^{th} period are

$$(3.4.32) \quad G_n = \nu(N_n) N_n$$

where $\nu(\cdot)$ is the tax-rate function which is piecewise linear and fixed.

The total production cost for operating the K generating sectors at levels $x_{i,n}$ is (see assumption (iv))

$$(3.4.33) \quad K_n = \sum_{i=1}^K (a_{i,n} x_{i,n}^2 + b_{i,n} x_{i,n} + d_{i,n}) .$$

The new capital generation then evolves as

$$(3.4.34) \quad C_{n+1} = C_n + N_n - \frac{1}{N-n} U_n - G_n - K_n - I_n \\ - D_n + L_n$$

where D_n is the total dividends paid out by the utility in the n^{th} period.

Equations (3.4.28) to (3.4.34) provide a model for the system under consideration. The quantities $r_{0,n}$, δ_n , y_n , $a_{i,n}$, $b_{i,n}$, $d_{i,n}$ are assumed known to within intervals via

projections from models of the external economy.

To complete the description we need to describe the constraints:

Production limits:

$$(3.4.35) \quad L_{i,n} \leq x_{i,n} \leq H_{i,n}$$

Consumer budget (aggregate);

$$(3.4.36) \quad \sum_{i=1}^K k_{i,n} x_{i,n} \leq b$$

Non-negativity of investment, loans and capital:

$$(3.4.37) \quad I_n \geq 0, L_n \geq 0, C_n \geq 0$$

Limit on new loans available to the company:

$$(3.4.38) \quad L_n \leq \beta_n (C_n + M_n + P_0)$$

(β_n is an externally available ratio, uncertain to within an interval),

Maximum outstanding debt which debtors allow:

$$(3.4.39) \quad U_n \leq \alpha(C_n + M_n + P_0)$$

All loans paid off by end of plan:

$$(3.4.40) \quad U_N = 0 \text{ (equivably } L_N = 0)$$

Lower bound on dividends issued by utility:

$$(3.4.41) \quad D_n \geq E_n$$

(E_n is a known sequence)

Total power demand to be met by utility:

$$(3.4.42) \quad \sum_{i=1}^K x_{i,n} \geq \mathcal{D}_n$$

($\mathcal{D}_n$ = power demand)

The constraints above involve both state and control variables. The state variables are clearly U_n , M_n , r_n and C_n , while the control variables are L_n , I_n , $\{k_{i,n}, x_{i,n}, i=1$

to $K\}$ and D_n .

The goals of the utility, as described in assumption (xi), is trying to reach the following target set:

$$(3.4.43) \quad T_N = \{(U, M, r, C) \mid C \geq C_d, C + M \geq P_d\}$$

where C_d and P_d are (known) desired levels of capital and total worth of the corporation. WE shall not consider any intermediate target sets.

In the context of the general problem treated in earlier sub-sections, we can identify $x_n = (u_n, M_n, r_n, C_n)^T$, $u_n = (L_n, I_n, x_1, x_2, n, \dots, x_{K,n}, k_{1,n}, \dots, k_{K,n}, D_n)$ as the state and control vectors. The quantities $r_{0,n}, \delta_n, y_n, a_{i,n}, b_{i,n}, d_{i,r}$, $i=1$ to K comprise what we have termed the disturbance/choice variables in Sect. 3.4.1. Of course w_n here is not a disturbance or choice variables vector but an external uncertain input vector. Since these are given external to the planning model we are considering, with interval uncertainties, clearly we are given the sets W_n of eqn. (3.4.3). In the current situation these sets are independent of x_n and u_n . The sets W_n will be Cartesian products of intervals.

The function f_n of (3.4.1) is easily obtained from equations (3.4.28)-(3.4.34) with the above identification, and we shall omit the explicit expression here.

Also, in the present situation, all the state variables--outstanding debt, total investment, interest rate, capital available--can be measured exactly at the end of each period. Hence the system falls into the second

special case discussed in Sect. 3.4.6, and obeys equations (3.4.26) and (3.4.27). The problem of growing state dimension disappears, therefore, and the utility planning problem appears to be computationally tractable. To actually use the approach, of course, the approximations mentioned at the end of Sect. 3.4.6 will have to be considered.

There are, however, a couple of differences with the general problem of section 3.4.1 and hence with the general solution of section 3.4.2. The first is the different objective function because of the single target set. The objective function (3.4.15) has to be replaced by

$$\sup_{Y_1 \in \hat{H}_0} \dots \sup_{Y_N \in \hat{H}_{N-1}} h_-(X_N, T_N) .$$

Due to this the solution (3.4.16)-(3.4.21) will change in its detail but the modifications are easy. For example, (3.4.16), (3.4.17) will remain the same but in (3.4.18) the recursion will be replaced by

$$J_n(X, u) = \sup_{y \in \hat{H}_n(x, u)} J_{n+1}^*(G(F_n(x, u), y))$$

Of course, due to (3.4.26) which the present problem obeys, the state set X will be replaced by a vector x and $G(F_n(X, u), y) = y$.

The second difference is the presence of state and control constraints (3.4.35) to (3.4.42), which we have not considered in the general problem. These constraints do not change the nature of the problem or solution at all. All that occurs is that the cost-to-go functions are defined on

subsets of the state-space at each instant: the parts which satisfy the state constraints. Thus we need evaluate the function only for these states. The control constraints just limit the allowable controls so that all the infinizations in (3.4.16)-(3.4.21) are to be carried out over the controls which satisfy the constraints. Since the dynamic programming solution will be carried out by minimizing via grid searches at each step, these constraints make things easier by restricting the search regions.

3.48 Comments

We have considered a general planning problem with set-valued uncertainty and solved it via an abstract dynamic programming approach. Though a particular form for combining the costs at various periods was used for the most part, the results hold for other reasonable combinations as mentioned in the discussion after (3.47). An example of this was considered in Sect. 3.47 where only nearness to the final target set were important, and the cost function was just $\sup_{x \in X_N} d(x, T_N)$.

Though, in general, the problem is infinite dimensional, special cases (see sub-sections 3.44-3.47) and computational aspects have been discussed which indicate the feasibility of the procedure. We have also demonstrated the robustness and feasibility of approximate procedures. It is hoped that this will lead the way to practical approximate and sub-optimal schemes.

Several generalizations of this paper can be treated in a similar way. Among them is the existence of fixed unknown parameters in the system.

The case where fixed parameters are present can be easily accommodated into the present framework by adjoining the parameter vector to the state vector. This defines the proper state vector for the system. Thus consider the system

$$x_{n+1} = f(\alpha, x_n, w_n, u_n)$$

instead of (3.4.1) where α is the fixed parameter vector.

Knowledge regarding α is that α belongs to a set A_0 . To cast this in our framework, define the state as

$$\begin{pmatrix} x_n \\ \alpha_n \end{pmatrix}$$

and we have

$$\begin{pmatrix} x_{n+1} \\ \alpha_{n+1} \end{pmatrix} = \begin{pmatrix} f(\alpha_n, x_n, w_n, u_n) \\ \alpha_n \end{pmatrix}$$

as the counterpart of (3.4.1). The subsequent development is identical to what we have presented. Note that the function F of (3.4.10) looks like

$$F(X \times A) = \bigcup_{(x, \alpha) \in X \times A} \bigcup_{w \in W(x, u)} \begin{pmatrix} f(\alpha, x, w, u) \\ \alpha \end{pmatrix}$$

and the initial information state is $X_0 \times A_0$.

Appendix 3.4A

In this appendix we prove the compactness of the images of F_n and G of (3.4.10) and (3.4.11), which is mentioned in sub-section 3.42. We also prove some related continuity properties which will be used in Appendix 3.4C to prove upper semi-continuity of the "cost-to-go" functions.

Let $\mathcal{I}_n$ be the correspondence from $R^l \times R^q$ to $R^l \times R^n \times R^q$ defined by:

$$(A.1) \quad \mathcal{I}_n(x, u) = \{x\} \times W_n(x, u) \times \{u\}$$

The function f_n treated as a correspondence is denoted by $\tilde{f}_n$, i.e., $\tilde{f}_n$ is a correspondence from $R^l \times R^D \times R^q$ to R^l defined as

$$(A.2) \quad \tilde{f}_n(x, w, u) = \{f_n(x, w, u)\}$$

Let $\mathcal{F}_n$ be the correspondence defined from $R^l \times R^q$ to R^l via

$$(A.3) \quad \mathcal{F}_n = \tilde{f}_n \circ \mathcal{I}_n$$

Prop. 3.4A1. $\mathcal{F}_n$ is a continuous correspondence.

Proof. Since W_n is a continuous correspondence by assumption, from (A.1) $\mathcal{I}_n$ is clearly continuous. The continuity of $\tilde{f}_n$ is apparent so that (A.3) and the fact that the composition of two continuous correspondences is continuous imply the derived result. The continuity of composition for correspondences follows for example from (iii) of p. 190, (iii) of p. 197 and Def. A. III-5 of [26].

From (A.1) - (A.3) we have

$$(A.4) \quad \mathcal{F}_n(x, u) = \bigcup_{w \in W_n(x, u)} f_n(x, w, u)$$

The definition (3.4.10) of F then gives

$$(A.5) \quad F_n(X, u) = \bigcup_{(x, u) \in X \times \{u\}} \mathcal{F}_n(x, u)$$

Prop. 3.4A2. $F_n(x, u)$ is compact for each $X \in \mathcal{X}$ and $u \in \mathbb{R}^q$.

Proof. Since X is compact, $X \times \{u\}$ is compact. Then by (A.5), $F_n(X, u)$ is the image of a compact set via the continuous correspondence $\mathcal{F}_n$. Theorem A.III.1 of [26] then yields the results.

Prop. 3.4A3. $G(X, y)$ is compact for every $X \in \mathcal{X}$ and $y \in \mathbb{R}^p$.

Proof. Since $G(X, y) = X \cap H^-(y)$ by (3.411), and X is compact, all we have to show is that $H^-(y)$ is closed. Now $H^-(y) = \{x \in \mathbb{R}^l \mid H(x) \cap \{y\} \neq \emptyset\}$, and H is continuous. Since $\{y\}$ is a closed subset, this means that $H^-(y)$ is closed from exercise 29(c) of Chapter 4 of [27]. $\square$

Let (M, d_M) be a metric space and $\mathcal{M}: M \rightarrow \mathcal{X}$ a upper hemi-continuous map. (In other words $\mathcal{M}$ is a compact-valued u.h.c. correspondence from M into $\mathbb{R}^l$.) Define the correspondences $\mathcal{G}$, Δ and ψ as follows

$$(A.6) \quad \begin{aligned} \mathcal{G} & \text{ from } \mathbb{R}^l \times \mathbb{R}^p \rightarrow \mathbb{R}^l, \quad \mathcal{G}(x, y) = \{x\} \cap H^-(y) \\ \Delta & \text{ from } M \times \mathbb{R}^q \rightarrow \mathbb{R}^l \times \mathbb{R}^q, \quad \Delta(m, u) = \mathcal{M}(m) \times \{u\} \\ \psi & \text{ from } M \times \mathbb{R}^p \rightarrow \mathbb{R}^l \times \mathbb{R}^p, \quad \psi(m, y) = \mathcal{M}(m) \times \{y\} \end{aligned}$$

Prop. 3.4A4. The correspondence $(m, u) \mapsto F_n(X(m), u)$ and $(m, y) \mapsto G(X(m), y)$ are upper hemi-continuous (from $M \times \mathbb{R}^q$ and

$M \times \mathbb{R}^p$ respectively to $\mathbb{R}^l$).

Proof. Since $\mathcal{M}$ is u.h.c., the correspondences Δ and ψ are u.h.c. (Prop. A.III.2 of [26]). From (A.5), (A.6), (3.411) we have $F(\mathcal{M}(m), u) = \mathcal{F}_n^{\Delta}(m, u)$ and $G(\mathcal{M}(m), y) = (\mathcal{G} \circ \psi)(m, u)$. Prop. 3.4A shows $\mathcal{F}_n$ to be continuous. If we show $\mathcal{G}$ is u.h.c., we are through on using the u.h.c. of Δ, ψ and the result on compositions of u.h.c. correspondences (p. 190 (iii) of [26]).

To show $\mathcal{G}$ to be u.h.c., note that the correspondence $(x, y) \mapsto \{x\}$ is clearly compact-valued and continuous hence upper hemi-continuous. The result will then follow from Prop. A.III.5 of [26] on intersections, provided we show that the correspondence $y \mapsto H^-(y)$ is closed (i.e., has closed graph). For this, let $(y_n, z_n) \in \text{graph of } H^-$ and $(y_n, z_n) \rightarrow (y, z)$. We have to show $(y, z) \in \text{graph of } H^-$. Now, we know $z_n \in H^-(y_n)$ or $\{y_n\} \cap H(z_n) \neq \emptyset$, i.e., $y_n \in H(z_n)$. Since $z_n \rightarrow z$ and H is continuous (hence u.h.c.), $h_-(H(z_n), H(z)) \rightarrow \emptyset$ as $n \rightarrow \infty$. Also $y \rightarrow y_n$ as $n \rightarrow \infty$. But

$$\begin{aligned} d(y, H(z)) &\leq d(y, y_n) + d(y_n, H(z)) \\ &\leq d(y, y_n) + d(y_n, H(z_n)) + h_-(H(z_n), H(z)) \end{aligned}$$

(The 2nd inequality can be shown directly from the definition of h_-)

and since $y_n \in H(z_n)$, the above convergences show $d(y, H(z)) = 0$, i.e., $y \in H(z)$, since $H(z)$ is closed. Hence $\{y\} \cap H(z) \neq \emptyset$ or $z \in H(y)$ or $(y, z) \in \text{graph of } H^-$, which was to be

shown.

Appendix 3.4B

In this appendix we give a quick outline of the steps leading from the basic problem (P) of sub-section 3.42 to the dynamic programming solution (3.4.16) - (3.4.21). The problem (P) can be written as

$$(B.1) \quad \inf_{u \in U, g_1, g_2, \dots, g_{N-2}} \inf_{g_{N-1}} \sup_{Y_1, \dots, Y_N} \max_{1 \leq n \leq N} h_-(X_n, T_n)$$

were we to have omitted the explicit domains of the various variables to reduce clutter. The variable g_n is the control strategy at time n (≥ 1), i.e., it is a U_n -valued map:

$(Y_1, Y_2, \dots, Y_n) \mapsto g_n(Y_1, \dots, Y_n) \in U_n$. The infima in (B.1) are over all such maps.

A trivial modification of Prop. 5.6.1 of [27] can be used in (B.1) to obtain

$$\inf_{g_{N-1}} \sup_{Y_1, \dots, Y_N} \max_{1 \leq n \leq N} h_-(X_n, T_n) = \sup_{Y_1, \dots, Y_N} \inf_{u_{N-1} \in U_{N-1}} \max_{1 \leq n \leq N} h_-(X_n, T_n)$$

$$\sup_{Y_N} \max_{1 \leq n \leq N} h_-(X_n, T_n)$$

This is possible because g_{N-1} is a function of $Y_1, \dots, Y_{N-1}$ and the infimum relative to g_{N-1} is over all possible functions. This result can be used recursively in (B.1), with respect to each of the infima to obtain (P) as

$$(B.2) \left\{ \begin{array}{l} \inf_{u_0 \in U_0} \sup_{y_1 \in \hat{H}_0} \inf_{u_1 \in U_1} \sup_{y_2 \in \hat{H}_1} \dots \inf_{u_{N-1} \in U_{N-1}} \sup_{y_N \in \hat{H}_{N-1}} \\ \max_{1 \leq n \leq N} h_-(X_n, T_n) \end{array} \right.$$

Now for $X_n = X$ and $u_n = u$, define the "cost-to-go" at the n th stage (for $n \leq N-1$) as

$$(B.3) \left\{ \begin{array}{l} J_n(X, u) = \sup_{y_{n+1} \in \hat{H}_n} \inf_{u_{n+1} \in U_{n+1}} \dots \inf_{u_{N-1} \in U_{N-1}} \\ \sup_{y_N \in \hat{H}_{N-1}} \max_{n+1 \leq i \leq N} h_-(X_i, T_i) \end{array} \right.$$

The max in (B.3) can be written for ($n \leq N-2$), as

(B.4)

$$\max_{n+1 \leq i \leq N} h_-(X_i, T_i) = \max \{ h_-(X_{n+1}, T_{n+1}), \max_{n+2 \leq i \leq N} h_-(X_i, T_i) \}$$

and X_{n+1} does not depend on $u_{n+1}, u_{n+2}, \dots, u_{N-1}$ as well as $y_{n+2}, y_{n+3}, \dots, y_N$ (see (3.4.15)). Using these facts, the inf-sup operations with respect to $u_{n+1}, y_{n+2}, \dots, u_{N-1}, y_N$ can be interchanged with the outer max in (B.4), so that (B.3) becomes

$$(B.5) \quad \left\{ \begin{array}{l} J_n(X, u) = \sup_{y_{n+1} \in \hat{H}_n} \max\{h_-(X_{n+1}, T_{n+1})\}, \\ \\ \inf_{u_{n+1} \in U_{n+1}} \sup_{y_{n+2} \in \hat{H}_{n+1}} \dots \inf_{u_{N-1} \in U_{N-1}} \sup_{y_N \in \hat{H}_{N-1}} \\ \\ \max_{n+2 \leq i \leq N} h_-(X_i, T_i) \end{array} \right.$$

Using (B.3) with n replaced by $n+1$ in (B.5) we have

$$(B.6) \quad \left\{ \begin{array}{l} J_n(X, u) = \sup_{y_{n+1} \in \hat{H}_n} \max\{h_-(X_{n+1}, T_{n+1})\}, \\ \\ \inf_{u_{n+1} \in U_{n+1}} J_{n+1}(X_{n+1}, u_{n+1}) \end{array} \right.$$

for $0 \leq n \leq N-2$.

We can now use (3.4.13) to reduce X_{n+1} to X (remember we have taken $X_n = X$ and $u_n = u$, the definition (3.4.9) of $\hat{H}_n$, and define $\hat{H}_n(X, u)$ & $J_n^*(X)$ as in (3.4.19), (3.4.20)).

$$H_n(X, u) = \bigcup_{x \in F_n(X, u)} H(x), \quad J_n^*(X) = \inf_{u \in U_n} J_n(X, u)$$

so that (B.6) becomes

$$J_n(X, u) = \sup_{y \in \hat{H}_n(X, u)} \max\{h_-(G(F_n(X, u), y), T_{n+1}),$$

(B.7)

$$J_{n+1}^*(G(F_n(X,u), y))$$

The sup and max can be interchanged in (B.7), and the recursion (3.4.18) is obtained provided we show that

$$(B.8) \quad \sup_{y \in \hat{H}_n(X,u)} h_-(G(F_n(X,u), y), T_{n+1}) \\ = h_-(F_n(X,u), T_{n+1})$$

This can be shown once we note that the left hand side is actually (use (3.4.11), (3.4.19))

$$\sup_{y \in \bigcup_{x \in F_n(X,u)} H(x)} \sup_{z \in F_n(X,u) \cap H^-(y)} d(z, T_{n+1})$$

and that (use (3.4.12))

$$\bigcup_{y \in \bigcup_{x \in F_n(X,u)} H(x)} F_n(X,u) \cap H^-(y) = F_n(X,u).$$

We omit the direct proofs.

Thus the recursion (3.4.18) has been shown for $0 \leq n \leq N-2$. For $n = N-1$, steps similar to the last step above can be carried out to obtain (3.4.16). Since $J_0^*(X_0)$ is clearly the solution of (P), we have proved the validity of the solution procedure.

Appendix 3.4C

The upper semi-continuity of $J_n(X, u)$, used in sub-section 3.43, will be proved in this appendix. Most of the work has already been done in Appendix 3.4A. First let us prove a result we shall need concerning the function $h_-(X, T)$. Let (M, d_M) be a metric space.

Proposition 3.4C1. If $\mathcal{M}: M \rightarrow \mathcal{X}$ is an upper hemi-continuous map and T is a fixed set in $\mathcal{X}$, $h_-(\mathcal{M}(m), T)$ is an upper semi-continuous function of m .

Proof. Using the definition (3.4.14) of h_- it can be shown that for any $m \in M$

$$h_-(\mathcal{M}(m), T) \leq h_-(\mathcal{M}(m), \mathcal{M}(m_0)) + h_-(\mathcal{M}(m_0), T)$$

where m_0 is a fixed point in M . Now let $m \rightarrow m_0$, by upper hemi-continuity of $\mathcal{M}$ we have the first term on the right converging to zero so that

$$\limsup_{m \rightarrow m_0} h_-(\mathcal{M}(m), T) \leq h_-(\mathcal{M}(m_0), T)$$

which proves the upper semicontinuity of the map $m \mapsto h_-(\mathcal{M}(m), T)$ at any arbitrary point m_0 in M (Propositions 3.6.1 and 3.7.1 of [27]). $\square$

Fix an n and let $X_n = X$, $u_n = u$. Then using Prop. 3.4A4 of Appendix 3.4A and (3.4.13) recursively we have the fact that X_i is an upper hemi-continuous function in X , controls

$u, u_{n+1}, \dots, u_{i-1}$ and observations $y_{n+1}, \dots, y_i$ -- for any $i > n$. These properties, together with Prop. 3.4C1 and the fact that the maximum of a finite number of upper semicontinuous functions is upper semi-continuous yield

$$(C.1) \quad \max_{n+1 \leq i \leq N} h_-(X_i, T_i) \text{ is upper semi-continuous}$$

in X , controls $u, u_{n+1}, \dots, u_{N-1}$ and observations $y_{n+1}, \dots, y_N$.

The set $\hat{H}_{N-1}$ is from (3.49) and (3.413)

$$\hat{H}_{N-1} = \bigcup_{x \in F_{N-1}(X_{N-1}, u_{N-1})} H(x)$$

Since H is a continuous correspondence, using Prop. 3.4A4 and the preservation of upper hemi-continuity by compositions [26] (p. 190), we have $\hat{H}_{N-1}$ is an upper hemi-continuous function of $X, u, u_{n+1}, \dots, u_{N-1}$ and $y_{n+1}, \dots, y_{N-1}$. This, Theorem 4.8.3 of [26] (modified to the corresponding sup version), and (C.1) then yield

$$\sup_{y_N \in \hat{H}_{N-1}} \max_{n+1 \leq i \leq N} h_-(X_i, T_i) \text{ is}$$

upper semi-continuous in the appropriate variables.

Using Theorem 4.8.3 of [26] directly on this (since U_{N-1} is a fixed set) we have

$$\inf_{u_{N-1} \in U_{N-1}} \sup_{Y_N \in \hat{H}_{N-1}} \max_{n+1 \leq i \leq N} h_-(X_i, T_i)$$

is upper semi-continuous in the appropriate variables.

This process can be continued backwards to obtain $J_n(X, u)$ to be upper semi-continuous in X and u (we use the form of (B.3) for J_n). Since n has been fixed arbitrarily we are through. $\square$

3.5 References

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