Ion Production and Trapping in Electron Rings

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I. Introduction

The electron beam in the VUV and X-ray rings of NSLS will ionize residual gas by collisions. Positive ions will be produced with low velocity, and will be attracted by the electron beam to the beam axis. If they are trapped in stable (transverse) orbits, they may accumulate, thereby increasing the $v_{x,z}$ of the individual electrons. Since the accumulated ions are unlikely to be of uniform density, a spread in $v_{x,z}$ will also occur. Should these effects be serious, it may be necessary to introduce clearing electrodes, although this may increase Z/n in the rings, thereby adding to longitudinal instability problems.

The purpose of this note is to estimate the seriousness of the above effect for the VUV and X-ray rings. Our starting point for the analysis is Internal Report DESY H1-71/2 (Dec. 1971), R.D. Kohaupt. We will proceed along the following lines:

Section II Estimation of ion production and transverse spatial distribution.

Section III Estimation of conditions for ion trapping in the transverse direction.

Section IV Calculation of v shift (and spread) from trapped ions.

Section V Estimation of longitudinal motion of the trapped ions and the need for clearing electrodes.

II. Ion Production

Many species of gas are expected to be present in the rings. Although the dominant species will be molecular hydrogen, it is also necessary to consider heavier species like CO which, although they are less plentiful, may be more difficult to clear from the beam.

The rate of production of ions/electron is:

$$\frac{1}{\tau_p} = \sigma_i \rho_A c \quad (1)$$

where ρ_A is the density of ions of molecular number, A, given by:

$$\rho_A = \frac{6 \times 10^{23}}{22.4 \times 10^{-3} \text{ m}^3} \frac{P_A}{760} = 3.5 \times 10^{22} P_A / \text{m}^3 \quad (2)$$

and P_A is the residual pressure of species A in Torr. The cross section for ionization, σ_i , is calculated by Mott and Massey (Eq. (50), p. 182) to be of the form

$$\sigma_i = \frac{2\pi K e^4}{m c^2} \frac{Z}{\langle E_{\text{ion}} \rangle} \quad (3)$$

where $\langle E_{\text{ion}} \rangle$ is some sort of average ionization energy and K is a number depending only slightly on the specific electronic wave functions. We can get reasonable agreement with Kohaupt's numbers for σ_i (source not known) by choosing $K \approx 6$, in which case, with $E_{\text{ion}} \approx 15 \text{ eV}$

$$\sigma_i \approx Z \times 10^{-19} \text{ cm}^2 \approx \begin{cases} 2 \times 10^{-19} \text{ cm}^2 & \text{for } H_2 \\ 1.4 \times 10^{-18} \text{ cm}^2 & \text{for } CO \end{cases} \quad (4)$$

Using (2) and (4), one finds

$$\tau_p = \frac{10^{-8}}{Z P_A^{\text{Torr}}} \text{ sec} \approx \begin{cases} 5 \text{ sec for } H_2 \text{ at } 10^{-9} \text{ torr} \\ 3.5 \text{ sec for } CO \text{ at } 2 \times 10^{-10} \text{ torr} \end{cases} \quad (5)$$

Pressures at the start of acceleration may be 10 times lower, in which case the ion production will be 10 times longer.

The ions will be produced essentially at rest. In fact, their velocities will be thermal, with each component having the rms value, at 20°C of:

$$\begin{aligned} \langle v_x \rangle_{rms} = \langle v_z \rangle_{rms} = \langle v_s \rangle_{rms} &\approx \sqrt{\frac{2.4}{A}} \frac{\text{km}}{\text{sec}} \\ &= \begin{cases} 1100 \text{ m/sec} & \text{for } H_2 \\ 290 \text{ m/sec} & \text{for } CO \end{cases} \end{aligned} \quad (6)$$

III. Ion Trapping

If the electron beam is assumed to consist of relatively long bunches (of the order of a few centimeters) with uniform density in the transverse direction, and with elliptical cross section of semi-major and semi-minor axes σ_x, σ_z , the electric field within the beam is given by:

$$E_{x,z} = - \frac{120 (\text{ohm}) I (\text{amp}) (x,z)}{\sigma_{x,z} (\sigma_x + \sigma_z)} \text{ volt/m} \quad (7)$$

where I is the electron current (averaged over several bunches) in amperes. The ions are, therefore, attracted to the beam axis, and would perform oscillations of frequency given by:

$$\overline{\omega_{x,z}^2} = \frac{120 I e}{M \sigma_{x,z} (\sigma_x + \sigma_z)} \quad (8)$$

unless the size of the periodic electric impulses causes them to be unstable.

If the pulse duration is much shorter than the pulse separation T , the transverse ion motion will be governed in x, x' or z, z' space by the repeated matrix

$$M = \begin{pmatrix} 1 & 0 \\ -\overline{\omega^2} T & 1 \end{pmatrix} \begin{pmatrix} 1 & T \\ 0 & 1 \end{pmatrix} \quad (9)$$

for which the stability condition is $\text{Tr}(M) < 2$, or

$$0 < 2 - 2 \cos \mu = \overline{\omega^2} T^2 < 4 \quad (10)$$

The maximum current for which ion motion would therefore be stable is given by:

$$\frac{120 I^S e T^2}{M \sigma_{xz} (\sigma_x + \sigma_z)} = 4 \quad (11)$$

Or

$$I^S = 3.1 A \times 10^7 \frac{\sigma_z (\sigma_x + \sigma_z)}{L^2} \text{ amp} \quad (12)$$

where L is the separation between bunches and where

$$\frac{Mc^2}{30e} = \frac{938 \text{ MeV}}{30 \text{ ohm} \times e} A = 3.1 \times 10^7 \text{ amp}$$

Since σ_z is rarely larger than σ_x , the stability condition for $\overline{\omega_z^2}$ dominates.

Obviously, the stability limit will vary with position around the ring because of the variation of beam envelope with β_x, β_z . Using

$\sigma_{x,z} = \sqrt{\epsilon_{x,z} \beta_{x,z}}$, one writes for (12)

$$I^S = 3.1 A \times 10^7 \frac{\sqrt{\epsilon_x \epsilon_z}}{L} \frac{\sqrt{\beta_x \beta_z}}{L} \left(1 + \sqrt{\frac{\epsilon_z}{\epsilon_x}} \sqrt{\frac{\beta_z}{\beta_x}} \right) \text{ amp} \quad (13)$$

If the current exceeds the stable value in Eq. (13), the ions will be given transverse impulses which will kick them out of the electron beam. Successive kicks will be reduced in size, but will still continue large transverse velocities which will cause the ions to strike the wall with energies up to

$$W_{ion} \sim \frac{M}{2} \left(\overline{\omega_{x,z}^2} T_{\sigma_{x,z}} \right)^2 \sim \frac{I^2}{A} \times \begin{cases} 1 \text{ KeV, (VUV)} \\ 10 \text{ KeV, (XR)} \end{cases} \quad (14)$$

During the course of build-up of the electron current, the stable current may exceed the actual current. However, by leaving m consecutive rf buckets empty during buildup, the stable current can be reduced by a factor $\frac{1}{(m+1)^2}$, thus decreasing the possibility of trapping at low currents.

Let us now make some numerical estimates:

A. VUV Ring

$$\text{Here } \frac{\sqrt{E_x E_z}}{L} = \frac{\sqrt{8.7 \times 10^{-8} \times 8.8 \times 10^{-10}}}{5.7} = 1.6 \times 10^{-9}$$

$$\frac{\sqrt{\beta_x \beta_z}}{L} = .55 \text{ to } 1.6 \text{ (around the ring)} \quad (15)$$

$$I_{VUV}^s = A \times (30 \text{ to } 80) \text{ mA} = \begin{cases} 60 \text{ to } 160 \text{ mA for } H_2 \\ 850 \text{ to } 2200 \text{ mA for CO} \end{cases}$$

B. X-Ray Ring at 700 MeV

$$\text{Here } \frac{\sqrt{E_x E_z}}{L} = \frac{\sqrt{6.5 \times 10^{-9} \times 6.5 \times 10^{-11}}}{5.7} = 1.15 \times 10^{-10}$$

$$\frac{\sqrt{\beta_x \beta_z}}{L} = .12 \text{ to } 3.0$$

$$I_{XR \text{ at } .7}^s = A (.4 \text{ to } 11) \text{ mA} = \begin{cases} 1 \text{ to } 22 \text{ mA for } H_2 \\ 12 \text{ to } 300 \text{ mA for CO} \end{cases} \quad (16)$$

C. X-Ray Ring at 2 GeV

$$\text{Here } \frac{\sqrt{E_1 E_2}}{L} = \frac{\sqrt{5 \times 10^{-8} \times 5 \times 10^{-10}}}{5.7} = .88 \times 10^{-9}$$

$$\frac{\sqrt{\beta_1 \beta_2}}{L} = .12 \text{ to } 3.0 \quad (17)$$

$$I_{\text{XR at 2.}}^s = A (3 \text{ to } 80) \text{ mA} = \begin{cases} 6 \text{ to } 160 \text{ mA for H}_2 \\ 85 \text{ to } 2200 \text{ mA for CO} \end{cases}$$

Clearly, the smaller the emittances, the less likely the beam is to get trapped. Nevertheless, there is a possibility that the heavier molecules may get trapped in particular locations around both of the rings, although the situation may be correctable by not filling one or two (adjacent) rf buckets.

IV. Transverse Frequency Shift and Spread

If the electrons have a transverse density $\rho_e(\hat{x}, \hat{y})$, ions will be produced at $\hat{x}, \hat{y}$, essentially at rest with probability proportional to

$$\rho_e(\hat{x}, \hat{z}) d\hat{x} d\hat{z}$$

Those ions which are trapped will therefore perform simple harmonic motion in both the x and y directions and will distribute themselves over the rectangle $|x| < \hat{x}$, with density

$$\frac{1}{\pi^2 \sqrt{\hat{x}^2 - x^2} \sqrt{\hat{z}^2 - z^2}}$$

The ion density will then be given by:

$$\rho_{\text{ion}}(x, y) = \eta \frac{1}{\pi^2} \iint \frac{d\hat{x} d\hat{z}}{\sqrt{\hat{x}^2 - x^2} \sqrt{\hat{z}^2 - z^2}} \rho_e(\hat{x}, \hat{z}) \quad (18)$$

where η is the degree of neutralization reached.

Unfortunately, Eq. (18) leads to infinite density at $x=y=0$, unless the electron beam density vanishes along the axis. However, there are also mitigating effects from the pulsed nature of the forces. Since we will be concerned with the ion density distribution only when the electron beam bunches pass over the ion. For these reasons, Kohaupt chooses a model which makes the ion distribution to duplicate the electron distribution.

The formula for the frequency shift is given by:

$$\Delta \nu_{x,z} = \frac{e}{4\pi m \gamma^2 c} \int_C ds \beta_{x,z}(s) \frac{\partial E_{x,z}^{ion}(s)}{\partial x,z} \quad (19)$$

where the contributions to the integral come only from regions where the ions are trapped. From Eq. (7) one can write, for uniform electron density,

$$\frac{\partial E_{x,z}^{ion}}{\partial x,z} = \eta \frac{120 I}{\sigma_{x,z}(\sigma_x + \sigma_z)} \quad (20)$$

where η is the degree of neutralization, which may vary around the ring. This leads to

$$\Delta \nu_{x,z} = \frac{a_{x,z} I}{\gamma} \quad (21)$$

where

$$a_{x,z} = 1.9 \times 10^{-5} \text{ amp}^{-1} C < \frac{\eta \beta_{x,z}}{\sigma_{x,z}(\sigma_x + \sigma_z)} > C \quad (22)$$

and where contributions to the average over the entire circumference of the ring, C , comes only where there is ion trapping. In most circumstances,

a_z will dominate. Numerical estimates can be made as follows:

$$C \left\langle \frac{\eta \beta_z}{\sigma_x \sigma_z} \right\rangle = \frac{C}{\sqrt{E_x E_z}} \left\langle \eta \sqrt{\frac{\beta_z}{\beta_x}} \right\rangle = \frac{NL}{\sqrt{E_x E_z}} \bar{\eta} f \quad (23)$$

where $\bar{\eta} f$ is the average of $\eta(\beta_z/\beta_x)^{1/2}$ around the ring only where ion trapping occurs, and where N is the number of rf bunches (9 for VUV ring, 30 for x-ray ring).

A. VUV Ring

$$a_z = \frac{1.9 \times 10^{-5}}{1.6 \times 10^{-9}} \text{ amp}^{-1} \times 9 \bar{\eta} f = 1.1 \times 10^5 \text{ amp}^{-1} \bar{\eta} f_{\text{VUV}} \quad (24)$$

$$\text{For } I = 1 \text{ amp}, \gamma = 1400, \bar{\eta} = 10^{-3}$$

$$\Delta p_z = 0.08 f \quad (25)$$

B. X-Ray Ring at 700 MeV

$$a_z = 5 \times 10^6 \text{ amp}^{-1} \bar{\eta} f_{\text{XR at } 7} \quad (26)$$

$$\text{For } I = 1 \text{ amp}, \gamma = 1400, \bar{\eta} = 10^{-3}$$

$$\Delta p_z = 3.5 f_{\text{XR at } 7} \quad (27)$$

However, Eq. (16) suggests that there is essentially no ion trapping in this case, so that f will either vanish or be extremely small.

C. X-Ray Ring at 2 GeV

$$a_2 = 6.5 \times 10^5 \text{ amp}^{-1} \bar{\eta} f_{XR \text{ at } 2}. \quad (28)$$

$$\text{For } I = 1 \text{ amp}, \gamma = 4000, \bar{\eta} = 10^{-3}$$

$$\Delta \nu_2 = 0.16 f_{XR \text{ at } 2}. \quad (29)$$

Once again, Eq. (17) suggests that ions will be trapped in only a small portion of the ring.

In the above estimates, we have arbitrarily chosen $\bar{\eta} = 10^{-3}$ as the maximum degree of neutralization that can be tolerated. More accurate calculations of the average around the circumference are necessary to refine this estimate.

With our assumption of uniform electron and ion density distribution, we obtain no spread in transverse frequency. If, however, the ion density is of Gaussian shape, with a width comparable with the electron beam, one expects to have a spread in transverse frequency of the order of the shift calculated in Eqs. (19) and (21), since electrons of different transverse betatron amplitudes will see different average ion gradients.

V. Longitudinal Motion of Trapped Ions

Those ions which are trapped within the electron beam will drift with a thermal longitudinal velocity given by Eq. (6). If we are to keep the degree of neutralization below $\eta = 10^{-3}$, we must remove the ions when they have travelled a distance l in the longitudinal direction given by:

$$l = v_s t$$

where

$$t = \eta \tau_p$$

Using Eqs. (5) and (6), and $\eta = 10^{-3}$,

$$l = \eta v_s \tau_p = \frac{\eta 10^{-8}}{Z \frac{P_A}{A} \tau_{\text{ion}}} \sqrt{\frac{2.4}{A}} \sim \begin{cases} 5.5 \text{ m} \\ 1.0 \text{ m} \end{cases}$$

If larger values for η can be tolerated because most ions will increase accordingly. Of course, l will increase in pressure even further.

Within the bending magnets, the trapped ions will fashion as derived by Kohaupt. His result is basically $\vec{E}_x \times \vec{B}_z / B_z^2$ modified by the ratio of the Larmor frequency ion oscillation frequency ω_x . Specifically,

$$v_D(x) = \frac{E_x(x)}{B_z} \frac{\omega_L^2}{\omega_L^2 + \omega_x^2} = x \frac{\omega_L \omega_x^2}{\omega_L^2 + \omega_x^2}$$

The drift velocity at the edge of the beam, $x = \sigma_x$, is then

$$v_D(\sigma_x) = \frac{\left(\frac{\omega_L \sigma_x}{c}\right) \left(\frac{\omega_x \sigma_x}{c}\right)^2}{\left(\frac{\omega_L \sigma_x}{c}\right)^2 + \left(\frac{\omega_x \sigma_x}{c}\right)^2}$$

From Eq. (8)

$$\left(\frac{\overline{\omega_x} \sigma_x}{c} \right)^2 = \frac{1.3 \times 10^{-7}}{A} I (\text{amp})$$

and for $B_z = 12.2$ kG (in VUV ring and in X-ray ring at 2.5 GeV)

$$\frac{\omega_L \sigma_x}{c} = \frac{3.9 \times 10^{-3} \sigma_x (\text{cm})}{A} = \begin{cases} \times 1 & \text{for VUV} \\ \times 0.3 & \text{for X at .7 GeV} \\ \times 0.8 & \text{for X at 2 GeV} \end{cases}$$

Setting $I = 1$ amp, one eventually finds,

$$V_D(\sigma_x) \approx \begin{cases} \frac{1200 \sigma_x (\text{cm})}{A + 120 \sigma_x^2 (\text{cm}^2)} \frac{\text{km}}{\text{sec}} & \text{for VUV} \\ \frac{37000 \sigma_x}{A + 1300 \sigma_x^2} \frac{\text{km}}{\text{sec}} & \text{for X at .7 GeV (33)} \\ \frac{1500 \sigma_x}{A + 190 \sigma_x^2} \frac{\text{km}}{\text{sec}} & \text{for X at 2.5 GeV} \end{cases}$$

A. VUV Ring

In the dipole magnet

$$\sigma_x = \sqrt{8.7 \times 10^{-8} \times 1.2} \text{ m} \approx 3.2 \times 10^{-2} \text{ cm}$$

so that:

$$V_D(\text{VUV}) \approx \frac{40}{A + .1} \frac{\text{km}}{\text{sec}} \approx \begin{cases} 20 \frac{\text{km}}{\text{sec}} & \text{for H}_2 \\ 1.4 \frac{\text{km}}{\text{sec}} & \text{for CO} \end{cases}$$

B. X-Ray Ring at 700 MeV

In the dipole magnet

$$\sigma_x = \sqrt{6.5 \times 10^{-9} \times 0.8} \text{ m} \approx 7.2 \times 10^{-3} \text{ cm}$$

so that

$$v_D (\text{X at } 7 \text{ GeV}) \approx \frac{27}{A} \frac{\text{km}}{\text{sec}} = \begin{cases} 14 \frac{\text{km}}{\text{sec}} & \text{for H}_2 \\ 1.0 \frac{\text{km}}{\text{sec}} & \text{for CO} \end{cases}$$

C. X-Ray Ring at 2 GeV

In the dipole magnet

$$\sigma_x = \sqrt{5 \times 10^{-8} \times 0.8} \text{ m} = 2 \times 10^{-2} \text{ cm}$$

so that

$$v_D (\text{X at } 2 \text{ GeV}) \approx \frac{30}{A} \frac{\text{km}}{\text{sec}} = \begin{cases} 15 \frac{\text{km}}{\text{sec}} & \text{for H}_2 \\ 1.1 \frac{\text{km}}{\text{sec}} & \text{for CO} \end{cases}$$

These drift velocities at the edge of the electron beam are reduced by x/σ_x within the beam. They are to be compared with the thermal drift velocity in Eq. (6), which is

$$\langle v_s \rangle_{\text{rms}} = \sqrt{\frac{2.4}{A}} \frac{\text{km}}{\text{sec}} = \begin{cases} 1.1 \frac{\text{km}}{\text{sec}} & \text{for H}_2 \\ 0.3 \frac{\text{km}}{\text{sec}} & \text{for CO} \end{cases}$$

As a result

$$\begin{aligned}
 \ell_{\text{dipole}}^{\text{VUV}} &\approx \begin{cases} 100 \text{ m} & \text{for } \text{H}_2 \\ 5 \text{ m} & \text{for } \text{CO} \end{cases} \\
 \ell_{\text{dipole}}^{\text{x.7}} &\approx \begin{cases} 70 \text{ m} & \text{for } \text{H}_2 \\ 3 \text{ m} & \text{for } \text{CO} \end{cases} \\
 \ell_{\text{dipole}}^{\text{x.2}} &\approx \begin{cases} 75 \text{ m} & \text{for } \text{H}_2 \\ 4 \text{ m} & \text{for } \text{CO} \end{cases}
 \end{aligned}$$

The smallest value of ℓ_{dipole} is in the x-ray ring at 700 MeV, but according to the estimate in Eq. (16), none of the ions will be trapped. It is, therefore, clear that no clearing electrodes will be needed within the dipoles.

If clearing electrodes are needed in the portions of the ring, their spacing should be governed approximately by Eq. (30), modified by whatever improvement can be made in the vacuum quality and by whatever $\Delta v_{x,z}$ can be tolerated. The clearing gradients are those associated with Eq. (7) at $x, z = \sigma_{x,z}$; namely ($\sigma_x \gg \sigma_z$)

$$E_{\text{clearing}} \approx \frac{120(\text{ohm}) I(\text{amp})}{\sigma_x (\text{cm})} \quad \frac{\text{volt}}{\text{cm}} \quad (34)$$

For $I = 1$ amp,

$$E_{\text{clearing}} \approx \begin{cases} 12 \text{ kV/cm} & \text{where } \sigma_x = 0.01 \text{ cm} \\ 1.2 \text{ kV/cm} & \text{where } \sigma_x = 0.1 \text{ cm} \end{cases}$$

Therefore, because of the rectangular-shaped vacuum chamber in both rings, it seems that clearing in the vertical plane requires less voltage.

The length of the plates can be estimated with the following arguments. The time τ required to remove (by clearing) one ion from the center of the beam to the plate ($h/2$ cm away) is

$$\tau = \sqrt{\frac{Mh}{e\Delta E}}$$

where M is the mass of the ion and ΔE the effective sweeping field, namely the difference between the field actually applied and the beam field as given by Eq. (34). During the same time the ion will travel a distance $\lambda = v_s \tau$, where v_s is thermal velocity as given by Eq. (6). The requirement is that the clearing electrode plates have a length larger than $\lambda = v_s \tau$.

Combining, we obtain

$$\lambda = v_s \sqrt{\frac{Mh}{e\Delta E}}$$

If we take $\Delta E \sim 10$ V/cm (quite small), then $\lambda = 0.1$ cm, for any ion species. Therefore, a plate of 1 cm length would do.

VI. Conclusions

We believe we have outlined all the basic ingredients to understand the physics of the ions produced in the two storage rings. Yet, because of local properties which depend on the lattice functions, a more detailed calculation should be carried out, may be with the help of a computer code. One should consider one storage ring with well defined lattice parameters and map the behaviour of the ion along the main orbit of the ring. Take one location where the beta values are known, assign the beam emittances and the linear coupling. For a given beam current and bunch spacing is then possible to estimate the ion production and which species would be trapped.

From the ion longitudinal drift and clearing electrode locations, it is also possible to estimate the time to clear the ions trapped. As a consequence, one determines the "local" neutralization factor, the tune shift and spread can then be calculated by integrating around the circumference. Note that a small tune spread could be useful against coherent instabilities.

The calculations we have shown earlier seem to indicate that light ions should escape easily whereas heavier ions, like CO, would resist clearing. We estimate that this should also be the situation in ADONE where, indeed, a tune shift induced by the ions was measured. H_2 ions are safely trapped in DORIS and cause a considerable shift (IX Int. Conf., 1974 Stanford, p. 45). The absence of ions for SPEAR can be explained with the large drift between consecutive bunch crossings; indeed, there is only one bunch per beam.