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TITLE: FAN TO PARALLEL BEAM CONVERSION IN CAT
BY RUBBER SHEET TRANSFORMATION

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the filter effectively produces the phaseless derivative p_{eff} of the original p.p. c. then using these filtered projections, the cross section σ_{eff} at a given θ may be estimated directly using

$$\sigma_{\text{eff}} = \int \sigma_{\text{eff}}(\theta) d\theta$$

where $\sigma_{\text{eff}}(\theta)$ is the cross section at θ calculated using the filtered projections. The cross section σ_{eff} may be estimated directly using

$$\sigma_{\text{eff}} = \sum \sigma_{\text{eff}}(\theta)$$

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Fig. 1. Fan-beam geometry.

Let $t = t(\theta)$.

Continuing the above calculations, we find that in general:

$$x' = t \left(\frac{1}{\sqrt{1 + t^2}}, \sin^{-1} t \right).$$

The function $p(t, \theta)$ is often called a sinogram since, as a function of the variable t , it can be displayed as a picture in which point objects appear as sinusoidal curves. If we call $h(x', \theta')$ the fan-beam sinogram, then the coordinate transformation of Eq.

$$\begin{cases} x' = \frac{t}{\sqrt{1 + t^2}} & -k \leq t \leq k \\ \theta' = \theta + \sin^{-1} t & 0 \leq \theta \leq \pi \end{cases}$$

defines a rather simple one-to-one and continuous transformation from the parallel-beam sinogram to the fan-beam sinogram. This transformation has the interesting property that parallel-beam projection data $p(t, \theta)$ taken at a fixed rotation angle θ depend only on the beam data for rotation angles θ' with

In the parallel-beam case we take 180 evenly spaced projections over a total rotation of 180° . Each projection has 256 equally spaced sample points over the diameter of the cylinder. In the fan-beam case, we choose the source distance L so that the fan angle α subtended by the cylinder is 70° . Since we take 256 sample points per projection, the choice of 0.357° produces a sinogram array of 256 by 256 fan-beam data points. The sample points of a fan-beam projection are chosen equally spaced in x' across the fan-beam angle α .

Figure 3 shows a noiseless reconstruction of the platinum cylinder. Figures 4a and 4b show respectively the sinograms of the parallel and fan-beam sinograms. Each contains essentially the same noise content. The rotation angle coordinate is taken vertically in both sinograms. Figure 4c shows the converted fan-to-parallel-beam sinogram. Figures 4a and 4c appear identical except for a just noticeable noise reduction in fig. 4c. Finally, figs. 5a and 5b show the respective reconstructions from the noisy parallel sinogram and the converted noisy fan sinogram. As predicted, the reconstruction of fig. 5b shows a perceptible decrease in noise over fig. 5a. One can speculate that the perceptibility detection threshold has been reduced.

Fig. 3. Noiseless reconstruction of the platinum cylinder.

Fig. 4a. Parallel-beam sinogram.

Fig. 4b. Fan-beam sinogram.

Fig. 4c. Converted fan-beam sinogram.

Fig. 5a. Reconstruction from parallel beam.

Fig. 5b. Reconstruction from converted fan beam.

Quantitative results from the simulation are as follows. The RMS errors σ_p for the parallel and fan sinograms are 0.096 and 0.091, respectively. The RMS error in the converted fan sinogram is 0.065, a reduction of approximately 2/3. The discrepancy in the 0.096 and 0.091 RMS errors are probably due to sampling error since the fan data were collected over 256² independent points as compared to the 180 by 256 points of the parallel data. The 2/3 reduction is reasonable, since it lies between 1/2 (all interpolated points equally weighted by interpolation rectangle vertex values) and 1 (all interpolated points fall on lattice positions so no noise reduction).

The reconstruction error σ_R due to photon noise is proportional to σ_p , the sinogram RMS error. This would imply a similar reduction of 2/3 in the σ_R of the reconstruction from a converted fan over that from parallel data. The measured σ_R 's are 0.0060 mm⁻¹ and 0.0035 mm⁻¹, respectively, for the parallel and fan cases, a reduction of 0.53. This unexpected result -- to our advantage -- is presently unexplained.

Summary

In support of our industrial tomographic research and development program at Los Alamos, we have developed a technique to convert fan-beam tomographic projection data into an equivalent set of parallel-beam projection data using a rubber sheet transformation. This approach appears to be satisfactory when operating on densely sampled fan-beam data and permits reconstruction using a fast Fourier implementation of the back-projection algorithm. Indeed, large reconstructions (in sections) of 2048 by 2048 pixels are possible at LANS using appropriately sampled collections of either fan-beam or parallel beam data. The conversion technique also has the additional advantages of reducing noise and of being implemented with software developed for picture registration.

References

1. R. P. Kruger and T. M. Cannon, "The Application of Computed Tomography, Boundary Detection and Shaded Graphics Reconstruction to Industrial Inspection," *Materials Evaluation*, Vol. 36, No. 5, April 1978, 75-80.
2. R. P. Kruger, T. M. Cannon, A. S. Lundy, R. A. Morris, "Simulation and Preliminary Performance Characteristics of a Computed Tomography Device for Industrial Applications," SPIE Vol. 155, *Image Understanding Systems and Industrial Application*, August 1978, 178-183.
3. R. P. Kruger, G. W. Weeksung, R. A. Morris, "Industrial Applications of Computed Tomography at Los Alamos Scientific Laboratory," submitted to *Optical Engineering*.
4. J. W. Beattie, "Tomographic Reconstruction from Fan Beam Geometry Using Radon's Integration Method," *IEEE Trans. Nuc. Sci.*, NS-22 (February 1975) 359-363.
5. P. Dreike and D. P. Boyd, "Convolution Reconstruction of Fan Beam Projections," *Comp. Graphics and Image Proc.* 5 (1976) 459-469.
6. L. Wang, "Cross-Section Reconstruction with a Fan-Beam Scanning Geometry," *IEEE Trans. Comp.* (March 1977) 264-268.
7. G. I. Herman and A. Naparstek, "Fast Image Reconstruction Based on a Radon Inversion Formula Appropriate for Rapid Collected Data," *Siam J. Appl. Math.* 33 (November 1977) 511-533.
8. B.K.P. Horn, "Density Reconstruction Using Arbitrary Ray-Sampling Schemes," *Proc. of IEEE* Vol. 66, No. 5, May 1978, 551-562.
9. J. Radon, "U'er die Bestimmung von Funktion durch ihre Integralwerte large gewisser Mannigfaltigkeiten," *Berichte Saechsische Akademie der Wissenschaften*, 69 (1917) 261.
10. E. L. Hall, *Computer-Image Processing and Recognition* (Academic Press, 1979) Chap. 5, 302.