TITLE: FAN TO PARALLEL BEAM CONVERSION IN CAT
BY RUBBER SHEET TRANSFORMATION

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Fan-to-parallel-beam conversion in CAT by rubber sheet transform

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Abstract

A technique for converting fan-beam projections to parallel-beam projections via rubber sheet transformation is presented. The problem is approached by use of a rubber sheet transformation. Since the data is discretized, an interpolation step is necessary. The sampled data this approach appears satisfactory and a significant reduction in the number of calculations observable in computer simulations.

Introduction

A research and development program was begun at the Los Alamos Scientific Laboratory (LASL) to study industrial applications of computer tomography. One of the goals of this program was to perform extensive computer simulations to determine the feasibility of tomographic reconstruction with a variety of hardware configurations. It was assumed that the need for reconstruction was that of reconstructing the fan-beam data of a number of different hardware configurations and that of determining the feasibility of the hardware configurations.

The basic approach to this problem takes place. The method is based on the fact that the fan-beam data can be described as a set of parallel-beam projections. After the parallel-beam data of the fan-beam projections are obtained, the fan-beam data are reconstructed by using a computer program which operates directly on the fan-beam data.

The fan-beam data are obtained by first calculating the differentials of the fan-beam projections with respect to the different fan-beam angles. These differentials are then used to calculate the parallel-beam projections.

The differentials of the fan-beam projections are calculated using a numerical differentiation method. The method used is the finite difference method. The fan-beam projections are then transformed to parallel-beam projections using a rubber sheet transformation.

The rubber sheet transformation is performed under the assumption that the data is sampled uniformly. The transformation is performed by first calculating the differentials of the fan-beam projections with respect to the different fan-beam angles. These differentials are then used to calculate the parallel-beam projections.

The parallel-beam projections are then reconstructed using a computer program which operates directly on the parallel-beam data. The program is a modified version of the algorithm developed by C. H. Becking and R. A. Moses.

The parallel-beam data are then used to reconstruct the fan-beam data using the same algorithm. The reconstructed fan-beam data are then compared to the original fan-beam data to determine the accuracy of the reconstruction.

The rubber sheet transformation has been shown to be an effective method for converting fan-beam data to parallel-beam data. The method is computationally efficient and can be applied to a variety of hardware configurations.
be the filter effectively produces the phaseless derivative $p_1$ of the original function. Using these filtered projections, the cross-section $f_2$ at a given $z$ and $y_2$ can be reconstructed exactly as

\[ f_2 = \sum \]
The function \( p(x, \theta) \) is often called a sinogram since, as a function of the variable \( \theta \), it can be displayed as a picture in which point objects appear as sinusoidal curves. To map from the parallel-beam sinogram to the fan-beam sinogram, then the coordinate transformation of the form:

\[
\begin{pmatrix}
  x' \\
  y' \\
  z'
\end{pmatrix} = \begin{pmatrix}
  x \\
  y \\
  z
\end{pmatrix}
\begin{pmatrix}
  \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\
  \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\
  0 & 1 & 0
\end{pmatrix}
\]

defines a rubber sheet one to one and continuous transformation from the parallel-beam sinogram to the fan-beam sinogram. This transformation has the interesting property that parallel-beam projection data \( p(x, \theta) \) taken at a fixed rotation angle \( \theta \) depend only on the fan-beam data for rotation angles \( \theta' \) with
\[ \theta = \sin \theta \Rightarrow \theta = \sin \theta \]

for Fig.: or see that

\[ \theta = \sin \theta \]

as just the angle subtended by the object at the point where \( \theta \) is evaluated and have the well known project.

\[ \theta = \theta \]

Thus project is determined for the few cases to be considered. For instance, this is \( \theta \) is the interior angle of a regular polygon of \( n \) sides.

Thus the angle \( \theta \) is subtended by the object.

As \( i \) the angle of revolution, or rotation, of the observer, is \( \theta \) the angle for the first case. Since the object is 180°.

1. Project

2. Rotation

3. Revolution

Thus the angle of rotation is 180°.

As \( i \) the angle of revolution, or rotation, of the observer, is \( \theta \) the angle for the first case. Since the object is 180°.

for appropriate sample indicate.

be use linear interpolation is.

\[ \theta = \theta \]

where the weights \( \omega_1, \omega_2 \) are selected. That is, \( \omega_1, \omega_2 \) is the angle where the object is 90°. A small interpolation error may limit the use of this technique.
We have noticed in computer simulations that significant decreases in the uncertainty of parameters in the model can be achieved. The primary cause of this is the introduction of a damping factor in the equations of the model, which reduces the variance of the parameters relative to the variance of the initial conditions. This is particularly evident when considering the smoothing of trajectories, where a damping factor can significantly reduce the variance in the model.

\[
\Sigma \rightarrow \Sigma' 
\]

where the weights \( \Sigma \) and \( \Sigma' \) are defined in (3). It can be further assumed that the values of the coefficients are independent. The damping factor creates a new set of \( \Sigma' \) values that are used in the next iteration of the model.

\[
x_i = x_i - \alpha \frac{dx_i}{dt} 
\]

where \( x_i \) is a state variable in the model, \( \alpha \) is the damping factor, and the derivative is taken at each step of the simulation.

In the case of a model with a damping factor, we can calculate the exact time interval \( \tau \) corresponding to a given change in the variable \( x_i \), by using the simulation results and applying the formula:

\[
\tau = \frac{\Delta x_i}{\alpha \Delta x_i / dt} 
\]

where \( \Delta x_i \) is the initial change in the variable and \( \Delta x_i / dt \) is the rate of change at the initial state.
In the parallel beam case, we take the evenly spaced projections over a total rotation of 360°. Each projection has 356 equally spaced sample points over the diameter of the cylinder. In the fan-beam case, we choose the source distance so that the fan angle is 10°. Since we take 216 sample points per projection, the choice of 10° produces a sinogram array of 256 by 256 fan-beam data points. The sample points of a fan-beam projection are uniformly spaced in x across the fan-beam angle.

Figure 5 shows a noiseless reconstruction of the plutonium cylinder. Figures 4a and 4b show respectively the sinograms of the parallel and fan-beam samples. Each sample point yields the same net content. The rotation angle and coordinates are taken vertically and horizontally. Figure 4a shows the converted parallel-beam samples. Figures 4a and 4b appear identical except for a just noticeable noise reduction in Fig. 4b. Finally, Figs. 5a and 5b show the respective reconstructions from the noise parallel samples and the converted fan samples. As predicted, the reconstruction of Fig. 5b shows a little less detail than that of Fig. 5a. One can speculate that the susceptibility detection error in has been reduced.

![Reconstruction from parallel beam](image1)

![Reconstruction from converted fan beam](image2)
Quantitative results from the simulation are as follows. The RMS errors for the parallel and fan sinograms are 0.096 and 0.071, respectively. The RMS error in the converted fan sinogram is 0.065, a reduction of approximately 2/3. The discrepancies in the 0.096 and 0.091 RMS errors are probably due to sampling error since the fan data were collected over 256 independent points as compared to the 180 by 256 points of the parallel data. The 2/3 reduction is reasonable, since it lies between 1/2 (all interpolated points equally weighted by interpolation triangle vertex values) and 1 (all interpolated points fall on lattice positions so no noise reduction).

The reconstruction error due to photon noise is proportional to \( p^2 \), the square of the error. This would imply a similar reduction of 2/3 in the square of the reconstruction value, a converted fan over that from parallel data. The measured values are 0.0144 and 0.0095 \( \text{mm}^2 \), respectively, for the parallel and fan cases, a reduction of 0.35. This unexpected result -- to our advantage -- is presently unexplained.

Summary

In support of our industrial tomographic research and development program at Los Alamos, we have developed a technique to convert fan-beam tomographic projection data into an equivalent set of parallel-beam projection data using a rubber sheet transformation. This approach appears to be satisfactory when operating on densely sampled fan-beam data and permits reconstruction using a fast Fourier implementation of the back-projection algorithm. Indeed, large reconstructions (in sections) of 2048 by 2048 pixels are possible at Los Alamos using appropriately sampled collections of either fan-beam or parallel beam data. The conversion technique also has the advantage of reducing noise and of being implemented with software developed for picture registration.

References


