DEEP-INELASTIC LEPTON SCATTERING BY NUCLEI THE PION-EXCHANGE MODEL

CONF-8503129--2

F. Coester and E. L. Berger
Argonne National Laboratory, Argonne IL 60439

Quark constituents of nucleons were discovered in deep-inelastic electron scattering.¹ It is not surprising therefore that the nuclear effects in deep-inelastic lepton scattering first observed by the European Muon Collaboration (EMC)²⁻⁴ have spawned many suggestions of revolutionary changes in our views of nuclear structure.⁵ Perhaps quarks play a far more active role in nuclear dynamics than the conventional theory provides. Before following the natural appeal of radical conclusions we believe it is important to examine in quantitative detail the consequences of the conventional nuclear model, in which the nucleon-nucleon potential features a one-pion exchange tail at large distances, an intermediate range attraction dominated by two-pion exchange and a strong short-range phenomenological repulsion. For this purpose let us assume that quarks are strictly confined to single hadrons, and that the quark structure of the hadrons in a nucleus is not affected by their environment. These assumptions allow a consistent description of deep-inelastic lepton-nucleus scattering,⁶ and the quantitative consequences agree well with available data.⁷

Fig. 1. Schematic sketch of the deep-inelastic scattering process.

DEEP-INELASTIC LEPTON SCATTERING BY COMPOSITE TARGETS

The inclusive cross section for the process sketched in Fig. 1 is proportional to the tensor

$$F^{\mu\nu} = \sum_f \langle f | j^\mu(0) | a \rangle^* \langle f | j^\nu(0) | a \rangle \delta^4(p' - p - Q), \quad (1)$$

where $|a\rangle$ is the state of the target nucleus moving with momentum p and the states $|f\rangle$ are the final states of the target material. The operator j^ν is the electromagnetic current density operator of the target material. The tensor $F^{\mu\nu}$ is related linearly by simple kinematic factors to the invariant structure functions F_1 and F_2 , which are functions of Q^2 and $p \cdot k$. Individual hadronic constituents of the target have structure functions which are related to matrix elements of the current in the same way. It is possible to express the structure functions of the nucleus as a linear functional of the structure functions of the nucleons and other hadronic constituents if the following assumptions are adopted:

MASTER

1. The current operator of the target is additive in the currents of the constituents, $j^\mu = \sum_i j_i^\mu$.
2. The constituents contribute incoherently to the cross section, i.e.

$$F^{\mu\nu} = \sum_i \sum_f \langle f | j_i^\mu(0) | a \rangle^* \langle f | j_i^\nu(0) | a \rangle \delta^4(p' - p - Q) ,$$

and the final-state interactions between the residue of the struck hadron and the spectators have a negligible effect on the sum over final states in $F^{\mu\nu}$.

3. The momentum transfer is large, $Q^2 \gg m_N^2$.

Under these assumptions it can be shown⁶ that the structure functions of the nucleus, per nucleon, $F_2^A(x, Q^2)$, and those of the constituents, $F_2^{(i)}(x, Q^2)$, are related by

$$F_2^A(x, Q^2) = \sum_i \int dz F_2^{(i)}\left(\frac{x}{z}, Q^2\right) f_i(z) , \quad (2)$$

where x is the Bjorken scaling variable, $x \equiv Q^2/(-2p \cdot Q)$, and z is the "light-front momentum fraction" of a constituent,

$$z_i \equiv \frac{A p_i^+}{\sum_j p_j^+} . \quad (3)$$

The $+$ component of any four vector is the sum of its time and longitudinal components. The functions $f_i(z)$ are probabilities obtained from the wave function of the target state $|a\rangle$. In the derivation of Eq. (2) it is essential to use wave functions with light-front symmetry, for which the sum of the $+$ components of the hadron momenta is equal to the $+$ component of the momentum of the target, $\sum_i p_i^+ = p^+$. It follows from this symmetry constraint that the probabilities $f_i(z)$ must satisfy the momentum sum rule

$$\sum_i \int dz z f_i(z) = 1 . \quad (4)$$

Similarly, baryon number conservation implies the sum rule

$$\sum_i b_i \int dz f_i(z) = 1 , \quad (5)$$

where b_i is the baryon number of the constituent i .

THE PION EXCHANGE MODEL

The pion exchange model which underlies conventional nucleon-nucleon potentials is specified by a Hamiltonian of the form

$$H = H_N + \int d^3k c_\pi^\dagger(\vec{k}) c_\pi(\vec{k}) \omega_\pi(\vec{k}) + H' , \quad (6)$$

where H_N includes the nucleon kinetic energy and a short-range phenomenological potential and H' is the nucleon-pion vertex interaction,

$$H' = (2\pi)^{-3/2} \int d^3k \int d^3p' \int d^3p \delta(\vec{p}' + \vec{k} - \vec{p}) [(\Lambda^2 - m_\pi^2)/(\vec{k}^2 + \Lambda^2)]^2 / m_\pi \times \int_\rho c_N^\dagger(\vec{p}') i \vec{\sigma} \cdot \vec{k} \tau_\rho c_N(\vec{p}) [c_{\pi\rho}^\dagger(\vec{k}) + c_{\pi\rho}(-\vec{k})] / (2\omega_\pi)^{1/2} \quad (7)$$

Since the pion number density is given by the expectation value of the number operator $c^\dagger(\vec{k})c(\vec{k})$, it can be obtained by calculating the linear response of the ground-state energy to an infinitesimal change $\omega(\vec{k}) \rightarrow \omega(\vec{k}) + \eta(\vec{k})$, in the pion kinetic energy term of (6). After the nucleon self energy is eliminated from the calculation of pion numbers, just as it must be eliminated from the calculation of ground-state energies, we obtain the pion exchange densities per nucleon from the expression

$$\rho_\pi(\vec{k}) = - \left(\frac{1}{A} \frac{\delta E_{\text{pot}}}{\delta \eta(\vec{k})} \right)_{\eta=0} \quad (8)$$

Two points need to be stressed: Any method which allows the calculation of ground-state energies, and hence pion exchange energies, will also yield corresponding pion exchange densities. To obtain consistent results, the parameters and approximations used in obtaining pion-exchange potentials must also be used in the calculation of the pion exchange densities.

COMPARISON WITH DATA

The ratio of the structure functions,

$$R_{\text{EMC}}^A(x) \equiv F_2^A(x, Q^2) / F_2^D(x, Q^2) \quad (9)$$

is related to the ratio of cross sections by

$$R_{\text{EMC}}^A(x) = \frac{\sigma^A(x, Q^2)}{\sigma^D(x, Q^2)} \frac{1-y + 1/2 y^2 / [1+R(D)]}{1-y + 1/2 y^2 / [1+R(A)]} \quad (10)$$

where $y = Q^2 / (2ME_x)$ and $R = \sigma_L / \sigma_T = (F_2 / 2xF_1) - 1$ is the ratio of the cross sections for the absorption of longitudinally and transversely polarized virtual photons.

Ratios of the deep inelastic cross sections have been obtained for muon scattering by iron and deuterium^{2,4} and for electron scattering by many nuclei.³ In the case of the muon data R is measured to be negligible. In the case of the electron data at lower Q^2 , R is not negligible and may exhibit a dependence on A .³ The data for aluminum, iron, and gold are shown in Fig. 2. The apparent discrepancy between the EMC data at small x and the SLAC data³ may be a nonasymptotic dependence on Q^2 and/or might be removed if an A dependence of σ_L / σ_T is confirmed by future experiments.⁸ On the other hand, a possible error of up to 7% in the relative

Fig. 2. Computed values and data for $R_{EMC}(x, Q^2)$ versus x for three nuclei: a) Al, b) Fe, and c) Au.

Fig. 3. Computed values of $R_{EMC}(x, Q^2)$ versus x for Fe compared with data where EMC points are divided by 1.05.

Fig. 4. Theoretical anti-quark enhancement $R_q(x)$ compared with the data of Ref. 10.

normalization of the EMC iron and deuterium data⁹ is within the uncertainty stated by the experimenters.² Fig. 3 shows the same data as Fig. 2b, except that the EMC data have been divided by 1.05. After this adjustment the EMC data are still in agreement with the more recent BCDMS data,⁴ and for $x > 0.1$ they also agree with the SLAC data³, if no A dependence of R is assumed.

Neutrino scattering by iron and hydrogen¹⁰ has yielded data for the ratios of structure functions which correspond to ratios of antiquark densities,

$$R_{\frac{q}{q}}(x) \equiv \frac{(F_1^{\bar{v}} - F_3^{\bar{v}})^{\text{Fe}}}{(F_1^{\bar{v}} - F_3^{\bar{v}})^{\text{H}}} = \frac{(\bar{u} + \bar{d} + 2\bar{s})^{\text{Fe}}}{2(\bar{d} + \bar{s})^{\text{H}}}, \quad (11)$$

and

$$K_{\frac{q}{q}} \equiv \int dx \, x(\bar{u} + \bar{d} + 2\bar{s})^{\text{Fe}} / \int dx \, 2x(\bar{d} + \bar{s})^{\text{H}}. \quad (12)$$

The experimental value for $K_{\frac{q}{q}}$ is $1.11 \pm 0.11 \pm 0.07$;¹⁰ the data for $R_{\frac{q}{q}}(x)$ ¹⁰ are shown in Fig. 4.

The pion exchange model sketched above yields a prediction for the structure function of the nucleus in the form

$$F_2^A(x, Q^2) = \int dz F_2^N\left(\frac{x}{z}\right) f_N(z) + \int dy F_2^{\pi}\left(\frac{x}{y}\right) f_{\pi}(y). \quad (13)$$

The same convolution formula also holds for xF_1 and xF_3 . For the nucleon's quark and antiquark distributions we used the parameterization derived by the CERN-Dortmund-Heidelberg-Saclay (CDHS) collaboration from fits to deep inelastic νH_2 and $\bar{\nu} H_2$ data.¹¹ Our pion structure function was obtained from an analysis of massive lepton pair production.¹²

Fig. 5. The light-front momentum-fraction distribution $f_N(z)$ of the nucleons in deuterium, aluminum, iron and gold.

Fig. 6. The light-front momentum-fraction distribution $f_{\pi}(y)$ of exchange pions in aluminum, iron and gold.

The nucleon and pion distributions $f_N(z)$ and $f_\pi(y)$ must satisfy the integral relations

$$\int dz f_N(z) = 1, \quad (14)$$

$$\int dz z f_N(z) + \int dy y f_\pi(y) = 1. \quad (15)$$

The nucleon distribution $f_N(z)$ for deuterium, shown in Fig. 5, was obtained from the deuteron wave function. For aluminum, iron, and gold we used the Thomas-Fermi approximation. The pion distributions shown in Fig. 6 were obtained from the potential V28Q,¹³ which produces a very good fit to two-nucleon scattering data. Models in which the target is the nucleon with its pion cloud, rather than the nucleus, yield a similar convolution formula with an $f_\pi(y)$ of much shorter range.^{6,14}

The solid curves in Figs. 2, 3 and 4 were obtained with Eq.

(13); our computed value of K_- is 1.11. The enhancement of $R_{EMC}(x)$ above unity for $x < 0.2$ is produced by pion exchange, whereas the depression below unity for $0.2 < x < 0.7$ is principally a reflection of momentum conservation, Eq. (15). The quality of the fit to the data for $R_{EMC}(x)$ is controlled by the mean number of pions per nucleon, $\langle n_\pi \rangle = \int dy f_\pi(y)$, and by $\langle y \rangle = \int dy y f_\pi(y)$. For the V28Q potential $\langle n_\pi \rangle = 0.10$ and $\langle y \rangle = 0.05$.

There are no adjustable parameters in our calculations, but the results are, of course, to some extent model dependent. The choice of the nucleon-nucleon potential fixes the results. The dashed curves in Figs. 2 and 4 are obtained with the potential V28.¹⁵

Our model agrees well with all data, except the magnitude of the original EMC data at small x . Even these data are consistent with our computation if they are reduced in normalization by 5%, consistent with experimental uncertainties. On the other hand, if their magnitude is confirmed, the data would require modification of our assumptions. The magnitude of the small x enhancement of the structure functions

Fig. 7. The ratio $R_{EMC}(x)$ of the structure functions obtained from the pion exchange model with an additional ad hoc alteration of the nucleon's quark structure.

could be accounted for by relatively minor medium dependent changes in the nucleon's quark structure. An ad hoc additional enhancement of the nucleon's ocean by 10% would produce the solid curve shown in Fig. 7a. The effect of a 10% to 15% additional enhancement of the nucleon's ocean on R_g is shown as a shaded region in Fig. 4. Such an enhancement is not ruled out by present data. The magnitude of the small x enhancement could also be produced by a minor medium dependent modification of the nucleon's valence quark distribution, without additional ocean enhancement. An example⁷ is shown in Fig. 7b.

CONCLUSION

Conventional nuclear theory can account quantitatively for the available deep-inelastic lepton scattering data. The data do not require any revision of nuclear structure theories.

REFERENCES

1. J. I. Friedman and H. W. Kendall, *Ann. Rev. Nucl. Sci.* 22, 203 (1972).
2. European Muon Collaboration, J. J. Aubert *et al.*, *Phys. Lett.* 123B, 275 (1983).
3. SLAC experiments: A. Bodek *et al.*, *Phys. Rev. Lett.* 50, 1431 (1983) and 51, 534 (1983); R. G. Arnold *et al.*, *Phys. Rev.* 52, 727 (1984).
4. BCDMS Collaboration (CERN Experiment NA4), as quoted by I. A. Savin, Rapporteur report, Int. Conf. on High Energy Physics, Leipzig, July, 1984.
5. See references quoted in ref. 7.
6. E. L. Berger, F. Coester, and R. B. Wiringa, *Phys. Rev.* D29, 398 (1984).
7. Edmond L. Berger and F. Coester, ANL preprint ANL-HEP-PR-84-97.
8. Y. Sacquin in *New Particle Production*, Proc. XIX Rencontre de Moriond, ed. by J. Tran Thanh Van (Editions Frontieres, France, 1984) Vol. 2, p. 659; I. A. Savin and G. Smirnov, *Phys. Lett.* 145B, 438 (1984).
9. J. J. Aubert, *Phys. Lett.* 105B, 332 (1981), and 123, 123 (1983).
10. CDHS Collaboration, H. Abramowicz *et al.*, *Z. für Phys. C* 25, 29 (1984).
11. CERN-Dortmund-Heidelberg-Saclay collaboration, F. Eisele, in *Proceedings of the 21st International Conference on High Energy Physics, Paris, 1982*, edited by P. Petiau and M. Porneuf [*J. Phys. (Paris) Colloq.* 43, C3-337 (1982)], and private communications, Dec. 1984.
12. CERN-Naples-Ecole Polytechnique-Strasbourg-ETH Collaboration (CERN-NA10), B. Betev *et al.*, CERN reports CERN-EP/85-03 and CERN-EP/85-04, (Jan. 1985), submitted to *Z. für Phys.*
13. R. B. Wiringa, *Recent Progress in Many Body Theories*, Proc. Third Int. Conf., Odenthal-Altenberg, Germany, ed. by H. Kimmel and M. L. Ristig (Lecture Notes in Physics, Springer-Verlag,

1984), Vol. 198, p. 44.

14. C. H. Llewellyn Smith, Phys. Lett. 128B, 107 (1983); M. Ericson and A. W. Thomas, ibid. 128B, 112 (1983).
15. R. B. Wiringa, R. A. Smith, and T. L. Ainsworth, Phys. Rev. C29, 1207 (1984).

DISCLAIMER

This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency thereof, nor any of their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.