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THE STRUCTURE OF ^{13}C STUDIED BY
PION SCATTERING NEAR THE [3,3] RESONANCE

by

Susan Joyce Seestrom-Morris

ABSTRACT

Good resolution ($\pi^+, \pi^{+ \prime}$) and ($\pi^-, \pi^{- \prime}$) data were obtained for many states in ^{13}C using the Energetic Pion Channel and Spectrometer at the Los Alamos Meson Physics Facility. Differential cross sections were measured for angles between 20° and 105° at an incident pion energy of 162 MeV for the elastic scattering as well as for states at excitation energies of 3.09, 3.68, 3.85, 7.55, 8.86, 9.5, 11.82, 16.05, 17.92, 21.37, and 21.6 MeV. Excitation functions were measured at momentum transfers of $q = 1.1 \text{ h fm}^{-1}$ and $q = 1.4 \text{ h fm}^{-1}$, for energies between 100 and 300 MeV. A $\sigma(\pi^-)/\sigma(\pi^+)$ ratio of 9:1 was observed for the first time, indicating a pure neutron particle-hole excitation of a high spin state ($J^\pi = 9/2^+$). Strikingly different energy dependences were found for $\Delta S = 0$ and $\Delta S = 1$ transitions. Data for all states were compared with the microscopic model calculations of Lee and Kurath. Very good agreement was found between experiment and theory for the strongly excited states and the $9/2^+$ state at 9.5 MeV, in sharp contrast to the disagreement for weak transitions. The excitation function data and comparisons with the microscopic model calculations were used to identify the states at 16.05, 17.92, 21.37, and 21.6 MeV as either $7/2^+$ or $9/2^+$ states. The elastic scattering data were analyzed with an optical model and the results were found to be consistent with neutron and proton distributions having equal rms radii. The optical potentials generated were used in a collective model analysis of the $3/2^-$ (3.68 MeV) and $5/2^-$ (7.55 MeV) states. The reduced transition probabilities ($B(E2)$) derived from the proton parts of the transition strength are in agreement with those determined from electromagnetic measurements.

Chapter I

Introduction

A longstanding problem in studies of the structure of the nucleus has been the separation of the contributions of neutrons and protons to the ground state densities and to the transition densities between the ground state and excited states of the nucleus. Elastic scattering of elementary particle probes can be used to derive the distributions of protons and neutrons in the ground states of nuclei. Neutron and proton wave functions in nuclear excited states are studied by inelastic scattering measurements. The analysis of the scattering data to extract nuclear structure information contains model-dependent uncertainties, and these uncertainties vary with the probe being used. When investigating some aspect of nuclear structure, it is desirable to use the probe, or probes, for which the model-dependent errors are smallest.

Elastic electron scattering is well suited to the determination of ground-state charge distributions. The interaction is the well known electromagnetic interaction, which is weak compared to the nuclear force and thus does not appreciably alter the nuclear distribution. A point proton distribution of the nucleus can be extracted from the charge distribution by unfolding the finite charge distribution of the proton from the charge distribution of the nucleus.

Forward-angle inelastic electron scattering can be used to obtain the proton parts of the transition density. Inelastic electron scattering data at 180° can be used to extract information on the transition densities involving protons and neutrons. From the 180° data, the transverse form factors and hence the matrix elements of magnetic multipole operators between excited states and the ground state are obtained. This "magnetic" scattering is due to the interaction between the electron current and the magnetic moments of both the protons and neutrons as well as the magnetic moment caused by the orbital motion of the protons. This complicated interaction makes it difficult to extract information about the relative contributions of neutrons and protons from back-angle electron scattering.

There is a possibility of extracting information about the differences between neutron and proton contributions to ground-state [Ra-79] or transition densities [Ad-80] by a comparison of proton and electron scattering. The proton-proton and proton-neutron interactions are thought to be the same for the same isospin state (neglecting the Coulomb interaction) and thus proton scattering is sensitive to both protons and neutrons in the nucleus. The fact that protons are strongly interacting is a disadvantage since they are more likely to distort the nuclear distribution than are weak or electromagnetic probes. This effect is smaller at "high" energies where the projectile essentially interacts with only one nucleon and is not affected very much by any nuclear rearrangement it might have induced. Another advantage of choosing a medium or high energy probe is that then the nuclear binding energy is a small fraction of the

total energy, making it possible to develop a microscopic reaction theory relating the proton-nucleus scattering amplitude to the proton-nucleon amplitudes (using either a multiple or single scattering approximation). Intermediate-energy proton scattering has been used to determine neutron distributions for ground states of nuclei. In those works, the parameters of the proton distribution are fixed at the values determined by electron scattering and the parameters of the neutron distributions are varied to fit the data [Ra-79]. This technique seems not to be applicable to inelastic scattering to weak, single-particle type states. Often the theory does not correctly reproduce the data, possibly due to two-step contributions to these transitions which make it very difficult if not impossible to extract information on the relative contributions of neutrons and protons to the transition density.

Inelastic pion scattering provides perhaps the most sensitive method to extract the relative contributions of neutrons and protons to inelastic scattering. At pion energies around 180 MeV, the pion-nucleus scattering process is dominated by the $[3,3]$ resonance (the $J=3/2, T=3/2$ term in the pion-nucleon interaction). For $[3,3]$ resonance dominance, the isospin properties of the pion-nucleon system result in the cross section for $\pi^+ p$ ($\pi^- n$) elastic scattering being nine times that for $\pi^- p$ ($\pi^+ n$) elastic scattering (see App. A for derivation). If an optical potential for pion-nucleus scattering can be described in terms of the amplitudes for pion scattering from free nucleons by an impulse approximation, this 9:1 enhancement should also be observed in pion-nucleus inelastic scattering. Thus transitions

which proceed purely through proton excitation should be nine times stronger in π^+ scattering than in π^- scattering. Approximate values for the ratios A_n/A_p of the neutron and proton transition density amplitudes can be obtained from the simple ratio $R = \sigma(\pi^+)/\sigma(\pi^-)$ using the plane wave impulse approximation result $R = \left| \frac{3A_p + A_n}{A_p + 3A_n} \right|^2$.

For dominance of the [3,3] resonance and the assumption of a one-step-direct reaction mechanism it has been shown [Si-80] that pion inelastic scattering can be used to separate transitions that proceed with or without a spin-transfer, $\Delta S = 1$ or $\Delta S = 0$, respectively. Since the spin-dependent and spin-independent parts of the pion-nucleus interaction have quite different energy dependences, the relative contributions of $\Delta S = 1$ and $\Delta S = 0$ to a transition may be extracted.

Pion elastic scattering has not met early expectations of determining the neutron and proton distributions in nuclear ground states. The difficulty lies in the interpretation of the elastic scattering which is complicated by some properties of the [3,3] resonance that make pion inelastic scattering so useful. The pion-nucleon phase shifts in the region of the resonance are strongly energy dependent. Thus it is difficult to make nuclear-matter corrections to the pion-nucleon amplitudes. This leads to large ambiguities in the pion-nucleus optical potential.

If the simple impulse approximation is valid, pions have a distinct advantage over conventional probes in separating neutron and proton contributions to nuclear excitations. An examination of the pion inelastic-scattering experiments performed up to now gives some

idea of the validity of this claim. None of the early studies found the large ratio $\sigma(\pi^+)/\sigma(\pi^-)$ $\{\sigma(\pi^-)/\sigma(\pi^+)\}$ of 9:1 predicted for a pure proton {neutron} transition. The emphasis in those experiments was on the strong collective transitions in which one does not expect large asymmetries. Such states, even in nuclei with one or two particles outside a closed shell, are reached by many single-particle excitations and thus have about equal proton and neutron components. For a single state, the largest ratio found has been $\sigma(\pi^-)/\sigma(\pi^+) = 1.86 \pm 0.16$, for the first 2^+ state of ^{18}O (1.982 MeV) [Iv-79]. Large asymmetries have been observed in ^{12}C [Mo-79], at an excitation energy of about 19 MeV. In the ^{12}C case one state was seen only in π^+ scattering (19.25 MeV) and another only in π^- scattering (19.65 MeV). Such asymmetries are not expected in a self-conjugate nucleus for transitions between states of pure isospin. The asymmetries for ^{12}C have been interpreted as due to strong isospin mixing of two neighboring 4^- states.

The interpretation of the $^{12}\text{C}(\pi, \pi')$ data in terms of isospin mixing between states near 19 MeV relies heavily on the validity of the impulse approximation and on the predicted 9:1 ratio for pure neutron or proton excitation. Although the distorted-wave impulse approximation (DWIA) has been reasonably successful in explaining the existing body of pion inelastic-scattering data, [Le-80] and [Le-80a], at the time of this experiment there had as yet been no observation of a 9:1 or 1:9 ratio for $\sigma(\pi^+)/\sigma(\pi^-)$ for the excitation of a state expected to be reached either by a pure proton or pure neutron excitation. It seems reasonable to look for such a large ratio in

transitions to "single-particle" states in a nucleus with one particle outside a closed-shell core. The nucleus ^{13}C has a number of fairly good single-neutron states known from $^{12}\text{C}(\text{d},\text{p})$ [Da-73]. A simplified picture of the structure of these states can be obtained from the diagram of shell model levels in Fig.1. In the extreme shell model the ^{12}C ground state consists of filled $1s_{1/2}$ and $1p_{3/2}$ proton and neutron shells. The ^{13}C ground state is then a $1p_{1/2}$ neutron outside this 0^+ core, yielding spin, parity $J^\pi = 1/2^-$. Single-particle states can be reached by promoting this neutron into the $2s_{1/2}$ ($1/2^+$), $1d_{5/2}$ ($5/2^+$), and $1d_{3/2}$ ($3/2^+$) levels. States with these spins and parities, at 3.09, 3.85, and 8.25 MeV, respectively, have been observed to be strongly excited in $^{12}\text{C}(\text{d},\text{p})$ [Da-73] and have been identified with these single-particle configurations. The ratio of the cross section for exciting these states with π^- to that for exciting them with π^+ would be 9:1 if this simple picture were valid. It is known, however, that the $1p_{3/2}$ and $1p_{1/2}$ orbitals are strongly mixed in the ^{13}C ground state [Co-65] and this could decrease the value of $\sigma(\pi^-)/\sigma(\pi^+)$ from the extreme shell model value of 9. However, as discussed in some detail below, some π^+/π^- asymmetry is expected even in the collective $3/2^-$ and $5/2^-$ states at 3.68 and 7.55 MeV with a principal $[^{12}\text{C}(2^+) \otimes p_{1/2}]$ configuration

FIGURE I-1

because of Pauli blocking of the quadrupole transition strength in ^{12}C by the extra neutron.

This dissertation describes a study of the nucleus ^{13}C using π^+ and π^- inelastic scattering. The single-particle states, $1/2^+$ (3.09) and $5/2^+$ (3.85), were investigated, as were the collective states, $3/2^-$ (3.68) and $5/2^-$ (7.55). Data were obtained for many other states, some of which had not been observed before. The EPICS channel and spectrometer at the Clinton P. Anderson Meson Physics Facility was used to measure differential cross sections for angles between 25° and 105° at an incident pion energy of 162 MeV. In addition, excitation functions were measured at energies between 100 and 300 MeV for two momentum transfers, 1.1 k fm^{-1} and 1.4 k fm^{-1} .

The most unexpected result of this experiment is that the largest value of $\sigma(\pi^-)/\sigma(\pi^+)$ was seen for a state at 9.5 MeV about which relatively little was known. This state has been cataloged as a $3/2^-$ state [Aj-76], in contradiction to the results of this experiment which indicate that it is a high-spin state excited by a simple particle-hole transition. The π^-/π^+ ratios observed for the "single-neutron" states were only about two. A strongly π^+ enhanced state was found at 16.05 MeV. At about 21 MeV a group of states was excited in which the centroid shifted between π^- and π^+ , similar to the π^-/π^+ differences seen at about 19 MeV in ^{12}C [Mo-79]. These three states, or groups of states, were previously unknown, but two of them (16.05 and 21.37) have since been seen in 180° electron scattering [Hi-79]. The excitation functions for the 9.5 MeV state and the states around 16 and 21 MeV have energy dependences that are

dramatically different from those for the other states in ^{13}C and contain unique spectroscopic information.

A description of the LAMPF accelerator and the EPICS facility is given in Section II followed by a presentation of the data in Section III. Theoretical interpretations of the data are discussed in Section IV.

Chapter II

Experimental Methods

A. Accelerator

The positive and negative pions were produced as secondary beams using the 800 MeV protons of the LAMPF linear accelerator as the primary beam. This accelerator has three major parts. The first of these is the injector, with three separate ion sources. A Von Ardenne duoplasmatron source is used to produce the primary H^+ (proton) beam. An H^- beam, used for proton scattering experiments, is produced by a charge-transfer type source developed at LASL. The third source is a polarized H^- (P^-) source. Each of these sources has a separate 750-keV Cockcroft-Walton high-voltage generator to provide a pre-acceleration voltage. The pulsed beams from two of the ion sources (H^+ and either H^- or P^-) are timed to enter the accelerator during selected parts of the rf cycle. This allows the simultaneous acceleration of H^+ and H^- (P^-) beams by timing the H^- (P^-) beam to be accelerated during the reverse phase of the rf cycle from the H^+ .

The second section of the accelerator is an Alvarez type drift tube linear accelerator that is operated at 201.25 Mhz. There are four tanks making up this section, producing terminal voltages of 5, 41, 73, and 100 MeV respectively. The Alvarez section is followed by a resonantly-side-coupled standing-wave linear accelerator, which is operated at 805 Mhz. In this section the protons are accelerated to

their final energy of 800 MeV. The production beam consists of about 120 500- μ sec-long pulses per second, giving a duty factor of 6%. The average beam current is about 550 μ A.

After leaving the accelerator, the H^+ and H^- beams enter the switchyard where the H^- beam is split off and sent to Experimental Area C (High Resolution Spectrometer) and Experimental Area B. The H^+ beam then passes through a thin target before reaching the A-1 production target. The A-1 target consists of a graphite wheel that rotates in the beam to facilitate cooling. The areal density of this target is approximately 5 gm/cm². The Energetic Pion Channel and Spectrometer (EPICS) comes off the main beam line at an angle of 35° (see Fig. II-1).

B. Energetic Pion Channel and Spectrometer

The 35° angle between the EPICS channel and the main beam line represents a compromise between high intensity (maximal at 0°) and space requirements for movement of the spectrometer. Momentum selection in the channel is done by four dipole magnets (BM01-BM04 in Fig. II-2) that bend the particle trajectories in a vertical plane. The channel contains three multipole focusing magnets (FM01-FM03) and four adjustable collimating jaws (FJ01-FJ04). The channel provides for point to point focusing in the vertical plane and point to parallel focusing in the horizontal plane.

FIGURE II-1

FIGURE II-2

The pion beam incident on the scattering target is about 20cm high by 8cm wide. Such a dispersed beam, in which the particle momentum is correlated with its' vertical position in the beam, has a much larger phase space than a monochromatic beam of the same solid angle and yielding the same resolution in the focal plane of the spectrometer. This allows approximately 200 times greater intensity than would be available from a monochromatic beam. The momentum spread of the beam across the scattering target is $\delta P/P = 2\%$.

The EPICS spectrometer is not dispersion matched to the channel and therefore the momentum of the incident particle must be measured in order to determine the momentum loss in the scattering process. For this reason there are three quadrupole magnets and a set of wire chambers between the scattering target and the spectrometer (Fig. II-3). The focusing of this triplet is point to point in the vertical plane, with a magnification of -1, and parallel to point in the horizontal plane. The set of multiwire chambers placed near the focal point of the triplet is used to measure the x , x' and y , y' coordinates of each particle (the coordinate system is defined such that z is the particle trajectory direction and positive x is toward large bending radius in the magnets). This information is used to calculate the momentum of the incident particle.

Momentum analysis of the scattered particles is done in a vertical plane. Two 45° bending magnets (Fig. II-3) are used. In the focal plane of the spectrometer there are sets of wire chambers, scintillators, and a Čerenkov counter. When the major parts of the current experiment were performed, the spectrometer had not been

FIGURE 11-3

vacuum coupled to the scattering chamber. There was a 0.015 in. thick Mylar window on the scattering chamber and another one at the entrance to the vacuum jacket of the spectrometer. The space between these two windows was filled with a helium bag to minimize the degradation in resolution due to multiple scattering in the air. Inside the spectrometer vacuum jacket a metal flange extended into the beam. Scintillators were placed on either side of the flange and the resulting signals were used to veto events in which particles had passed through the flange.

Beam monitoring at EPICS is done by three ion chambers. One of these is located near the A-1 target and measures the secondary emission due to the proton beam striking the target (BOT). The other two ion chambers (IC1 and IC2) are located in the scattering chamber, on the same axis as the incoming beam. These ion chambers are used to monitor the pion intensity incident on the scattering target. A mixture of 90% Argon and 10% CO₂ gas, by volume, is used in the chambers and they are operated at atmospheric pressure (650 mm of Hg in Los Alamos). A graphite absorber (3 cm thick) is placed before the first ion chamber to stop protons before they enter the chamber.

The scattered particle detection system at EPICS is composed of eight sets of position-sensitive detectors, a Čerenkov counter, and a variable number of scintillators. The position-sensitive detectors used are delay-line readout drift chambers, designed at LAMPF. In these chambers, the anode wires run perpendicular to the direction in which position is being measured, and they are all connected to a delay line. The time difference between the signals at the two ends

of the delay line are used to determine which of the anode wires fired. Additional position information is obtained by summing the drift times from each end of the delay line and comparing this to some "prompt" time to get drift time information. To apply the drift time correction it is necessary to know on which side of a wire the particle passed. This left-right ambiguity is eliminated by having two planes of anode wires, offset from each other by half a wire spacing. The details of this correction are described in [At-80]. The x', y' angle measurements require two sets of two wire planes, separated along the particle trajectory axis. Eight planes located close to the focus of the quadrupole triplet measure two x and two y positions and the quantities $X_f, Y_f, \theta_f (dX_f/dz)$, and $\varphi_f (dY_f/dz)$ (X, Y, θ , and ϕ at the front of the spectrometer) are calculated from these four measurements. There are eight more wire planes located in the focal plane and the position signals from these chambers are used to calculate the quantities X_r, Y_r, θ_r , and φ_r .

The standard set of scintillators used at EPICS consists of two scintillators in the focal plane (S2 and S3). These scintillators are located after the last set of wire chambers. An event is required to generate pulses in both S2 and S3 for it to be considered a valid event. The pulse heights in S2 and S3 and the time of flight between S2 and S3 are used for particle identification (see Sec. III-A).

A third scintillator was used during the current experiment. A layer of graphite was placed after S3 and a scintillator, S4, was placed after the graphite. The graphite was intended to remove pions through nuclear interactions. Muons, which are weakly interacting

particles, would pass through. This was the initial muon rejection system at EPICS, the signal from S4 being used to veto events associated with muons instead of pions. The performance of this device is described in Sec. III-A.

The other detector in the EPICS focal plane is a Čerenkov counter, located between S2 and S3. The signals from this counter can be used to identify electrons. This is most useful for very low count rate experiments in which a majority of events might be due to electrons. Such was not the case during this experiment and, thus, the Čerenkov counter was not used.

C. Data Acquisition

The raw signals from the chambers pass through amplifiers and fast discriminators in the experimental area. Cables carry the signals from the chambers and scintillators into the counting house where they are shaped, amplified, and delayed as necessary. The signals then go to CAMAC modules, Analog to Digital Converters (ADC's) for the scintillator signals and Time to Digital Converters (TDC's) for the chamber pulses. The chamber signals are used as the stop for the TDC's and the output (S2^{*}S3) of a logical "and" unit (with signals from S2 and S3 as inputs) is used as the start. The gate signal for the CAMAC modules, known as the trigger, is generated only if there are pulses from S2 and S3 and at least two pulses each from the four x planes and the four y planes in the front chambers.

The CAMAC system is interfaced to a PDP 11-45 computer through a LAMPF designed Microprogrammable Branch Driver (MBD). The process of reading in the approximately 50 data words associated with each event and subsequently writing them to magnetic tape is controlled by the LAMPF standard data acquisition system Q.2 [Ke-76]. In addition to reading the CAMAC modules when a trigger signal is present, the program also reads out various scalers at fixed time intervals. The trigger signals (called Event in the scaler file) are scaled. The computer provides a logical signal, Busy, when it cannot accept any events. The logical "and" of Events and not Busy ($\text{Events} \cdot \overline{\text{Busy}}$) is another one of the scaled quantities and the ratio of $\text{Events} / \text{Events} \cdot \overline{\text{Busy}}$ is used to calculate the computer live time.

During this experiment a system was developed to veto elastic scattering events before they are read into the computer, so that inelastic scattering may be measured at forward angles where the computer would otherwise be kept busy by the high rate of elastic events. A hardware signal was made that is roughly proportional to the momentum loss in the scattering process, as will be discussed below.

The quantity δ_{sp} can be defined as:

$$\delta_{sp} = \frac{P_{\pi} - P_{sp}}{P_{sp}} \quad \text{II-1}$$

where P_{π} is the momentum of the outgoing pion and P_{sp} is the central

particles, would pass through. This was the initial muon rejection system at EPICS, the signal from S4 being used to veto events associated with muons instead of pions. The performance of this device is described in Sec. III-A.

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The quantity δ_{sp} can be defined as:

$$\delta_{sp} = \frac{P_{\pi} - P_{sp}}{P_{sp}} \quad \text{II-1}$$

where P_{π} is the momentum of the outgoing pion and P_{sp} is the central

momentum of the spectrometer. The x position in the focal plane, X_r , to first order is:

$$X_r \approx M_{sp} \cdot X_f + D_{sp} \cdot \delta_{sp} \quad \text{II-2}$$

The magnification of the spectrometer in the x direction, M_{sp} , is -1, hence the term $M_{sp} \cdot X_f$ (X_f is x at the front chambers). D_{sp} is the dispersion of the spectrometer in units of cm/(% difference of P_π from P_{sp}). Equation II-2 can be solved for δ_{sp} , yielding

$$\delta_{sp} \approx \frac{X_f + X_r}{D_{sp}} \quad \text{II-3}$$

The vertical position of the incident particle at the scattering target, X_{tgt} , is given by:

$$X_{tgt} = D_c \cdot \delta_c \quad \text{II-4}$$

where D_c is the dispersion of the channel and $\delta_c = P_\pi - P_c / P_c$ is the percent difference between the incident particle momentum P_π and the central momentum of the channel P_c . The x position at the front chambers is related to the position at the target, to first order, by

$$X_f = -1 \cdot X_{tgt} \quad \text{II-5}$$

Therefore,

$$\delta_c \approx \frac{-X_f}{D_c}$$

11-6

The quantity δ_{loss} , the percent momentum lost in the scattering process is defined as:

$$\delta_{loss} = \delta_{sp} - \delta_c = \frac{X_r}{D_{sp}} + X_f \left(\frac{1}{D_{sp}} + \frac{1}{D_c} \right) \quad 11-7$$

A quantity proportional to δ_{loss} is approximated in hardware by adding $X_r + C \cdot X_f$. The constant C is adjusted to optimize the resolution in δ_{loss} . A single channel analyzer (SCA) is used to put a window on the elastic peak in the δ_{loss} spectrum. The resulting output pulse is called the fast clear signal. The fast clear signal is fed into a fan-out unit, and one of the outputs of the fan-out unit is fed into a decade scaler. The decade scaler provides one output pulse for each ten fast clear signals. The fast clear and the output of the decade scaler go into an anti-coincidence unit whose output is used to veto the trigger signal before an event is read into the computer. This results in nine of ten elastic events being rejected in the hardware. The one in ten elastic events that is not rejected has a data word associated with it so that it may be identified and its energy loss calculated. This enables a histogram to be set up which contains one in ten of the events that are rejected as elastic events. This histogram can be used to monitor the fast clear system to assure that events are not being rejected from higher-lying states.

After the raw data have been read into the computer, the Q program transfers control to various subroutines to perform calculations that transform the raw data to physical quantities of interest if time is available. A first-order position measurement can be obtained from the difference in the times of the signals from the two ends of the delay line for a given wire plane. The position resolution that can be obtained with such a first-order calculation is limited to a distance of the order of a wire spacing, about 8 mm. The resolution can be significantly improved by using the drift-time information to correct the first-order positions. Because this requires a knowledge of which side of a wire the particle trajectory was on, there must be two planes of wires, offset by half a wire spacing, for each position measurement. The details of the drift time correction have been presented in detail in [At-80].

The quantity that we are ultimately interested in is the "missing mass", or equivalently, the excitation energy of the residual nucleus. The subroutine CALKIN calculates the missing mass, Q , from the following equation:

$$(Q + M_D)^2 + P_D^2 = (E_{\text{miss}} + M_D)^2 \quad \text{II-8}$$

Here M_D is the mass of the residual nucleus and P_D is the momentum transfer. The missing energy, E_{miss} , is calculated from the following equation:

$$E_{\text{miss}} = (T_A + M_A) + M_B - \sqrt{P_C^2 + M_C^2} - M_D \quad \text{II-9}$$

T_A and M_A are the kinetic energy and rest mass energy of the projectile, respectively. M_B is the rest mass energy of the target nucleus, and P_C and M_C are the momentum and rest mass energy of the detected particle respectively. All the masses are looked up in a table using the reaction information that is typed into the computer at the start of a run. The input parameters to CALKIN are P_C, P_A (the projectile momentum), and θ_{sc} (the scattering angle), and from these parameters CALKIN calculates T_A and P_D .

The subroutine PROCO6 calculates P_C , P_A , and θ_{sc} from the eight measured quantities X_f , θ_f , Y_f , ϕ_f , X_r , Y_r , θ_r , ϕ_r that are the result of the analysis of the chamber signals. P_A can be written:

$$P_A = (\delta_c + 1) \cdot P_C \quad \text{II-10}$$

where,

$$\delta_c = \frac{X_{tgt}}{D_c} \quad \text{II-11}$$

and P_C is the central momentum of the channel beam (determined by the magnetic field of the channel bending magnets). The first order expression for X_{tgt} in terms of X_f (Eqn. II-5) is modified by aberrations in the fields of the quadrupole magnets and by any misalignment of the system. X_{tgt} is therefore calculated as a polynomial of up to fourth order in the quantities X_f , Y_f , θ_f , ϕ_f , and X_r . The technique for determining the coefficients in the polynomial

is outlined in [Bo-79]. Similarly, y at the target (Y_{tgt}), ψ at the target (ψ_{tgt}), and ϕ at the target (ϕ_{tgt}) are calculated as polynomials, i.e.

$$X_{tgt} = P(X_f, \theta_f, Y_f, \psi_f, X_r)$$

$$Y_{tgt} = P(X_f, \theta_f, Y_f, \psi_f, X_r)$$

$$\theta_{tgt} = P(X_f, \theta_f, Y_f, \psi_f, X_r)$$

$$\phi_{tgt} = P(X_f, \theta_f, Y_f, \psi_f, X_r)$$

The coefficients for all the polynomials are stored in a file called POL.DAT which is read by the analyzer when it is initialized.

To calculate P_C , the outgoing particle momentum, the quantity δ_{sp} must be evaluated. δ_{sp} is calculated as a polynomial in $X_f, \theta_f, Y_f, \psi_f$, and X_r . A typical set of coefficients for the δ_{sp} polynomial are shown in Table II-1. The coefficients of X_f and X_r reflect the first order expression for δ_{sp} given by Eqn. II-3. P_C is then given by,

$$P_C = (\delta_{sp} + 1) \cdot i_{sp} \quad \text{II-12}$$

P_{sp} is determined from the magnetic fields in the spectrometer bending magnets.

The final input parameter to CALKIN, θ_{sc} , is given by the following expression:

$$\theta_{sc} = \theta_{spec} + \phi_{tgt} \quad \text{II-13}$$

where θ_{spec} is the angle of the spectrometer with respect to the incident beam direction.

The data acquisition system is set up so that once an event has been processed, i.e. the various relevant quantities calculated, any data word may be tested. The results of the tests are used for the classification of the event. The program ALLTST [Am-79] controls this testing in a way that allows a large amount of flexibility. There are tests on raw or calculated data words, microtests, and tests on the results of previous tests, macrotests. Microtests can be either gate tests, which are true if the data word being tested is between some user defined limits, or box tests, which are true if each of two data words fall within their respective limits. Macrotests are of three types, corresponding to the logical functions "and", "inclusive or", and "exclusive or". The tests to be performed on the data are specified in a test file which is made up to suit the particular needs of a given experiment.

TABLE II-1: Typical Set of Coefficients in δ_{sp} Polynomial

Term	Coefficient
1.0	-0.416
X_f	0.236
θ_f	-2.24E-5
X_r	0.230
Y_f	1.98E-3
ψ_f	-1.76E-4
$X_f \cdot X_f$	8.35E-4
$X_f \cdot \theta_f$	2.35E-5
$X_f \cdot X_r$	-4.56E-5
$\theta_f \cdot \theta_f$	6.29E-5
$\theta_f \cdot X_r$	6.20E-5
$Y_f \cdot Y_f$	4.21E-4
$Y_f \cdot \psi_f$	8.97E-5
$\psi_f \cdot \psi_f$	1.08E-5
$X_f \cdot X_f \cdot X_f$	9.11E-7
$X_f \cdot X_f \cdot \theta_f$	4.76E-6
$X_f \cdot X_f \cdot X_r$	-5.81E-6
$X_f \cdot \theta_f \cdot \theta_f$	1.03E-6
$X_f \cdot Y_f \cdot Y_f$	5.41E-5
$X_f \cdot Y_f \cdot \psi_f$	-5.93E-7
$X_f \cdot \theta_f \cdot \theta_f$	-1.67E-6
$\theta_f \cdot \theta_f \cdot \theta_f$	1.07E-7
$\theta_f \cdot \theta_f \cdot X_r$	7.41E-6
$\theta_f \cdot Y_f \cdot Y_f$	5.33E-6
$\theta_f \cdot Y_f \cdot \psi_f$	3.40E-6
$\theta_f \cdot \psi_f \cdot \psi_f$	-4.05E-7
$\theta_f \cdot X_r \cdot X_r$	1.60E-6
$Y_f \cdot Y_f \cdot X_r$	2.84E-5
$Y_f \cdot \psi_f \cdot X_r$	-6.74E-6
$\psi_f \cdot \psi_f \cdot X_r$	-3.22E-7
$X_r \cdot X_r \cdot X_r$	1.72E-6
$X_r \cdot X_r$	9.94E-4

The set of tests defined in the test file are used to gate various histograms and to calculate quantities used for monitoring and normalization of the data. The program DSP controls the definition and display of the data in the form of histograms and two-dimensional dot plots. Histograms are defined by the user by specifying the data word to be histogrammed, a lower and an upper limit on the channel number, a rebinning factor, and the number of the test that had to have been passed for the event to be recorded in the histogram. Any two data words may be displayed on-line as dots on a plane resulting in a "scatter plot". These plots are used to diagnose problems in the system and to set up box cuts. For example, the box cut for particle identification is made by setting upper and lower limits on the mean energy loss in scintillators S2 and S3 versus the time of flight between S2 and S3.

During this experiment between 9 and 15 histograms were used. There were histograms of the various quantities which are important for background rejection; X_{tgt} , Y_{tgt} , θ_{tgt} , ϕ_{tgt} , θ_{chk} , and ψ_{chk} (see Sec. III-A), and up to five missing-mass histograms. The missing-mass histograms used channels that were 50 KeV wide. One histogram contained all counts from the full angular acceptance of the spectrometer, another only those in the central three degrees, and three others those in one-degree bins, centered at $+1^\circ$, 0° , and -1° away from the central angle. When the muon rejection was used there was also a histogram of the rejected counts in addition to the histograms with the unrejected counts.

The counting rate at forward angles was very high and therefore sometimes as few as 30% of the taped events could be analyzed on-line because of computer limitations. The data tapes for these high-count-rate runs were replayed "off-line" to analyze all of the events.

Chapter III

Data Reduction and Results

A. The Missing-Mass Histogram

The reaction yields were extracted from the missing-mass histograms. The data recorded in the missing-mass histogram have passed a number of the tests defined in the test file (see Sec. II-C). A sample ALLTST output is shown in Appendix B. The good event test, #93, was used for all the missing-mass histograms. This test is combined with various angular acceptance cuts, i.e. for the full angular acceptance, the central 3° of the full angular acceptance, and three cuts of 1° acceptance each, centered at $+1^\circ$, and 0° , and -1° . The good event test consists of the particle identification test, the good chambers tests, and background rejection tests.

The particle identification (PID) test is a box cut on the mean energy loss in S2 and S3 versus the time of flight between S2 and S3 (Fig. III-1). The main purpose of this box cut is to eliminate protons, which for the same momentum as pions have a smaller velocity and hence a longer time of flight. The PID box used for most of this experiment was the smaller box in Figure III-1. This particular box eliminates approximately 15% of the good events (pions). The number of pions lost depends on the energy of the scattered pion because the energy lost in the scintillators increases as the pion energy decreases. Thus the danger existed that the fraction of pions lost by

use of the small PID box depended on scattering angle. Therefore, a number of the 162 MeV runs were replayed with both the original and a larger PID box and it was found that the fraction of pions lost in the small PID cut was independent of scattering angle. The replays also served to renormalize the runs taken using the small PID box. Furthermore, all the excitation function data and the Hydrogen normalization runs were replayed with the larger PID box shown in Figure III-1 because over the wide range of energies covered in these runs the fraction of pions lost by use of the small box was clearly not constant.

The "good-chamber" tests are of two types, drift-time tests and drift-difference tests. The drift-time tests are gates on the drift times from each of the sixteen chamber planes. These tests serve to eliminate events which did not generate signals in one or more planes. The drift-difference tests eliminate events for which the drift-time correction has not been correctly made. Types of events for which the drift-time correction fails are events where a pion has decayed between two chambers ($\approx .5 - 1\%$ of all events) and those in which two particles passed through the chambers at the same time. The results of these tests are used to calculate the chamber and drift efficiencies (See Sec. III-B).

There are six background rejection tests. Four of these are tests on the quantities calculated at the target; X_{tgt} , Y_{tgt} , θ_{tgt} , and ϕ_{tgt} (see Fig. III-2 for X_{tgt} histogram). The histograms for each of these quantities have a fairly sharp edge. Events outside these sharp edges are assumed to be due to background. The two other

FIGURE III-1

background rejection tests are gates on θ_{chk} and ϕ_{chk} . These quantities are calculated from the difference between the angles measured at the front of the spectrometer and those calculated from information measured at the rear of the spectrometer. A typical ϕ_{chk} histogram is shown in Figure III-3. The θ_{chk} and ϕ_{chk} gates are especially useful in reducing the background due to muons from pions that decay in the spectrometer. When a pion decays in flight the muons are emitted in a forward cone about the pion direction of motion. The angles calculated for these muons will be different from those that would have been calculated for the pions, and therefore different from the angles measured at the front of the spectrometer.

One other source of background is muons that scatter from the target. The θ_{chk} and ϕ_{chk} gates will not eliminate these muons. An additional muon rejection system was tried out during this experiment. This consisted of a graphite absorber (23 cm) after S3 to stop pions through nuclear interactions. Muons however, being weakly interacting particles, would usually pass through the graphite. Another scintillator, S4, was placed after the graphite absorber and events that generated pulses in S4 should be mainly muons. It was found that along with muons a significant fraction of pions had passed through the absorber and thus were also rejected. This fraction was found to depend on the pion energy so that the efficiency of the system was different for different excited states. Therefore, the resulting muon-rejected histograms were not used to extract the cross sections presented here.

FIGURE III-2

FIGURE III-3

A typical missing mass histogram obtained during the first part of this experiment is shown in Figure III-4. This spectrum was taken at 62° with 162 MeV π^+ incident on the 209 mg/cm² ^{13}C target. The energy resolution width is approximately 250 keV (FWHM). For that part of the excitation function data that was taken after the spectrometer and scattering chamber had been vacuum coupled, the energy resolution width was about 200 keV (FWHM). The main effect of this improved resolution was that the very weak states, $1/2^+$ (3.09) and $1/2^-$ (8.86), were more easily seen above the background.

B. Peak Fitting

The peak areas used to calculate cross sections were obtained using the lineshape oriented fitting program LOAF, written at LAMPF by L. E. Smith. This program uses as a line shape a reference peak that can be specified from the data. Up to ten peaks can be fitted simultaneously. Peaks can be handled as "free peaks", whose centroids are allowed to vary freely, or as a group of "delta peaks". In the latter case (the delta mode), the separations between the peaks are kept fixed as the entire group is allowed to move to minimize the chi-squared. The background can be fitted as a polynomial of up to 10th order, or it can be specified by a number of background points which are fitted with a spline curve.

FIGURE III-4

The same reference peak shape was used for all the states below 11 MeV in the 3^0 histograms at 162 MeV. This shape was derived from the elastic peak in a π^+ spectrum taken at 62 0 . This reference peak had a tail that extended to almost 4 MeV. The 1^0 histograms were fitted with a narrower reference shape. The excitation-function spectra were analyzed using a different reference shape for each incident energy because the resolution changed as a function of energy.

The missing-mass spectrum was divided into groups of four or less peaks. This made it possible to fit the background with a lower order polynomial, which, in most cases, yielded more consistent results. The first group contained the elastic-scattering peak and the inelastic peaks due to the 3.09-MeV $1/2^+$, 3.68-MeV $3/2^-$, and 3.85-MeV $5/2^+$ states. The long tail on the reference peak shape was important for extracting reproducible peak areas for the weak $1/2^+$ state.

The states at 3.68 and 3.85 MeV were not resolved experimentally, as can be seen from Fig. III-4, but definitely showed a larger width than other, single-state, peaks. In an attempt to separate the doublet by peak fitting, all four peaks in the first group were fitted together in the delta mode. A typical result for this group is shown in Fig. III-5. This attempt to extract separate areas for the two states seemed to be successful, but the area obtained for the weaker of the two states, the $5/2^+$ state at 3.85 MeV, was very sensitive to the width of the reference peak shape. Consistent areas could be obtained as long as the reference peak shape gave a good fit to the elastic peak. However, the errors given by the code LOAF and

tabulated in App. D do not correctly reflect the uncertainty in the area of the 3.85 MeV state. The error due to uncertainties in the peak fitting is estimated to be at least 20% for this relatively weak state.

The states at 7.55, 8.86, 9.50, and 9.90 MeV were analyzed together in the delta mode. The large peak at 11.82 MeV had a shape quite different from those of the lower lying states. A reference shape was made up from this peak in a good statistics run and was used for the spectra at all other angles. This peak was fitted as a free peak, in a region that included the weakly-excited states at 10.78 and 11.08 MeV. The higher-lying states at 16.05, 17.92, 21.38, and 21.60 MeV were also analyzed with the 11.82 MeV shape. A broad group of states at 15.1 MeV, had to be included with a special peak shape in order to be able to extract areas for narrower groups in this region. The 15.1-MeV peak areas were not used to calculate cross sections. All of these higher lying states were treated as free peaks, and the mean value of the centroid from all the runs was used to determine the energy of the state.

In almost every case a linear background was used. Exceptions were made when the peak due to the Hydrogen contamination in the target was near the edge of the fitted region. Then it was necessary to use a second or third order background. The results with an error greater than 80% were not used unless a peak could be seen very clearly in the spectrum.

FIGURE III-5

C. The Differential Cross Section and Errors

The differential cross section in the center-of-mass system is given by the following expression:

$$\frac{d\sigma}{d\Omega} = \frac{G \cdot \text{YIELD}}{\Delta\Omega \cdot N_{\text{tgt}} \cdot N_{\text{beam}}} \quad \text{III-1}$$

Here G is the Jacobian of the transformation from the laboratory to the center-of-mass system, $\Delta\Omega$ is the solid angle subtended by the experimental apparatus, N_{tgt} is the number of target nucleons per unit area, and N_{beam} is the number of particles in the pion beam incident on the target. YIELD is the peak area multiplied by a correction factor, CORR:

$$\text{YIELD} = \text{AREA} \cdot \text{CORR} \quad \text{where,}$$

$$\text{CORR} = \frac{1}{\text{LT} \cdot \text{CHEFF} \cdot \text{DREFF} \cdot \text{SF} \cdot \text{LEV}} \quad \text{III-2}$$

LT is the computer live time, which is calculated from the scaler ratio $\text{EVENTS} \cdot \overline{\text{BUSY}} / \text{EVENTS}$. This ratio gives the fraction of the total number of events for which the computer was not busy. The factor LEV, the loops per event, is the ratio of the number of times the analyzer was called to the total number of events. LEV corrects the measured number of counts by the fraction of total events that were analyzed;

LEV is 1.0 for replayed runs and >.80 for all runs that were not replayed. CHEFF is the chamber efficiency which is calculated as a product of the efficiencies of each of the sixteen chambers. The efficiency of each chamber is given by:

$$EFF1 = \frac{\text{All Chmbrs OK}}{\text{All Chmbrs but \#1 OK}} = \frac{TST(46)}{TST(49)} \quad III-3$$

EFF1 measures the fraction of events for which only chamber #1 did not fire. The quantities (ALL Chmbrs but #1 OK), etc. are calculated by ALLTST. DREFF is the drift efficiency. It is given by the fraction of events for which all the chambers fired which however did not pass the drift-difference tests. The factor SF is the survival fraction, the fraction of pions that do not decay between the target and the last scintillator used in the trigger, S3.

$$SF = \exp \left(- \frac{t}{\tau'} \right) \quad III-4$$

Here τ' is the moving pion's mean lifetime in the laboratory frame and t is the time it takes the pion to travel from the target to S3. The ratio of the path length, L , over the velocity, v , gives t . In terms of the pion lifetime in the rest frame, $\tau = \tau'/\gamma$, the pion's momentum p and the rest mass m , SF can be written as:

$$SF = \exp \left(- \frac{L \cdot m}{p \cdot \tau} \right) \quad III-5$$

where,

$$p = \gamma \cdot M \cdot v$$

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Here p is the pion's momentum and M is the pion's rest mass. The path length depends on δ_{sp} and is given by:

$$L = (12.28 - .035 \cdot \delta_{sp}) \text{ meters} \quad \text{III-6}$$

At EPICS, neither of the factors $\Delta\Omega$ nor N_{beam} is known absolutely. The design solid angle of the spectrometer is 10 msr. The solid angle varies as a function of δ_{sp} and therefore a "delta scan" must be measured to determine the acceptance of the spectrometer as a function of δ_{sp} . This was done by measuring the yield for scattering to some state near a flat portion of its angular distribution and varying the spectrometer field to vary δ_{sp} . The results of such a delta scan for the full angular acceptance of the spectrometer measured at the second maximum in the angular distribution of $\pi^+ + {}^{12}\text{C}$ elastic scattering is shown in Figure III-6. The delta scan determines only the relative spectrometer solid angle for different values of δ_{sp} . The shape of this acceptance curve is somewhat different for the full acceptance than for the 3° bin or the 1° bins. Therefore a delta scan is separately extracted for each angular bin used. Most of the data presented here were taken with the

FIGURE III-6

elastic peak at $\delta_{sp} = 5\%$, which puts an excitation energy of 9.5 MeV at about $\delta_{sp} \approx +1\%$ for 162 MeV incident energy.

A quantity proportional to the factor N_{beam} can be determined from the currents measured in the ion chambers. The current in the ion chamber is due to the energy lost not only by the pions but also by the muons and electrons in the pion beam. The relative numbers of pions, muons, and electrons in the beam has been measured at a number of energies for both π^+ and π^- . This was done by using a low intensity H^+ beam, chopped at 80 nsec. The spectrometer was placed at 0° and time of flight from the A-1 target to the focal plane was measured relative to a start signal derived from the chopped H^+ beam. A resulting time-of-flight spectrum, in which peaks due to pions, muons, and electrons can clearly be seen, is shown in Figure III-7. The measured fractions of pions, muons, and electrons as a function of energy is shown in Table III-1. The numbers in Table III-1 can be used to correct the ion chamber currents to get a number, N_π , that is proportional to the number of pions in the beam, N_{beam} , i.e.

$$N_\pi = \frac{F_\pi \cdot ICI}{\Delta E_\pi \cdot F_\pi + \Delta E_\mu \cdot F_\mu + \Delta E_e \cdot F_e} = ICI \cdot FPI \quad \text{III-7}$$

Here F_π , F_μ , and F_e are the fractions of pions, muons, and electrons, respectively, and ΔE_π , ΔE_μ , and ΔE_e are the energy losses of the pions, muons, and electrons, respectively. The value of FPI is calculated as a function of energy and pion charge and is used to correct ICI.

FIGURE III-7

TABLE III-1: Beam Composition (in per cent) as a Function of Energy

T_{π} (MeV)	F_{π}^{+}	F_{μ}^{+}	F_e^{+}	F_{π}^{-}	F_{μ}^{-}	F_e^{-}
310	97.6	1.30	1.10	97.7	1.20	1.10
290	97.2	1.40	1.30	96.9	1.30	1.80
260	97.1	2.10	0.80	95.2	1.50	3.30
230	96.6	2.40	1.00	93.4	1.70	5.00
170	94.2	3.70	2.10	86.3	3.00	10.7
140	91.7	4.30	4.00	78.1	4.50	17.40
110	84.6	7.50	7.90	66.3	5.40	28.2
90	77.1	10.2	12.7	54.8	6.40	38.8
70	67.9	13.4	18.6	44.1	7.90	48.2

Because N_{beam} and $\Delta\Omega$ are not known absolutely, the yield for a reaction for which the cross section is known must be measured to determine the absolute normalization of the other cross sections. In this experiment this was done in two ways; by measuring elastic scattering (1) from hydrogen (in a CH_2 target) and (2) from ^{12}C (in a graphite target). The expression for the differential cross section can be written to include the normalization factor, N ,

$$\frac{d\sigma}{d\Omega} = \frac{G \cdot \text{YIELD}}{\Delta\Omega_{\text{rel}} \cdot N_{\text{tgt}} \cdot \text{FPI} \cdot \text{ICI}} \cdot N \quad \text{III-8}$$

Where $\Delta\Omega_{\text{rel}}$ is the relative solid angle, which varies with δ_{sp} . The normalization factor, N , is determined by setting $d\sigma/d\Omega$ equal to the known (^{12}C or H) cross section. Once N has been determined in this way for one reaction it can be used to obtain absolute cross sections for other reactions.

The data in this dissertation were taken in two runs, approximately one year apart, and the two sets of data were normalized in slightly different fashions. The first set of data, consisting of the 162-MeV angular distributions and the large-momentum-transfer excitation-function points at 116, 162, 180, 200, and 240 MeV, was originally normalized to hydrogen cross sections measured using a CH_2 target that was thought to have an areal density of $\rho\delta x = 68.14 \text{ mg/cm}^2$. N_{tgt} , in nucleons per millibarn, is obtained from the following formula:

$$N_{tgt} = 68.14 \times 10^{-3} \text{ g/cm}^2 \cdot N_A \cdot \frac{2}{14g} \cdot 10^{-27} \text{ cm}^2/\text{mb}$$

III-9

Here N_A is Avogadro's number and the factor $\frac{2}{14}$ reflects the fact that there are two moles of hydrogen atoms for each 14 gm of CH_2 . The factor N was measured at each energy for π^+ and π^- , by measuring the scattering from hydrogen at 40° as a function of energy and comparing with the calculated cross sections of Dodder [Do-79]. These calculated cross sections are based on an energy dependent phase shift analysis of much of the available $\pi^+ + p$ data. The π^+ normalizations at the different energies all agreed to within 2% and the π^- normalizations also agree to 2% except for the 240 MeV run which is 7% different from the mean value determined from all the others. The overall π^- normalization was found to be 5% lower than that for π^+ .

When these normalizations were used to calculate the cross section for elastic scattering from ^{12}C , measured at 65° , the results disagreed with the previously measured EPICS [Bo-80] and SIN[Pi-79] data. To agree with the previous data the π^- cross sections had to be multiplied by .763 and the π^+ by .845. The reason for the discrepancy between the average of these numbers, .80, and the old normalization is thought to be an error in the CH_2 target thickness. The CH_2 target that was used is no longer available and so the thickness can not be checked. The ^{12}C normalization is the one used for the data presented here, and this normalization results in agreement with the new set of ^{13}C data at every overlap point.

During a development run before the data set 2 was obtained, hydrogen yields were measured for π^+ and π^- at energies from 290 MeV down to 70 MeV. An attempt was made to calculate the number of pions absolutely from the ion chamber response (see App. C). If N_{beam} is known, then the only undetermined quantity in Eqn. III-1 for the differential cross section is $\Delta\Omega$. After N_{beam} was related to ICI as described in Appendix C, Eqn. III-1 was solved for $\Delta\Omega$ at each energy for π^+ and π^- using the measured hydrogen yields and comparing with the known hydrogen cross sections. Although the factors $\Delta\Omega$ obtained were near 10 msr, they varied with energy and were different for π^+ and π^- . Since the solid angle should be constant and the same for π^+ and π^- , the variation in $\Delta\Omega$ is probably due to an error in the way N_{beam} was related to ICI. It is useful to know N_{beam} in terms of the ion chamber charge and therefore $\Delta\Omega$ was assumed to be 10 msr (at $\delta_{\text{sp}} = 0\%$) and Eqn. III-1 was solved again for N_{beam} . Factors $CF^\pm$ were determined to relate N_{beam} to ICI, $N_{\text{beam}} = CF^\pm \cdot \text{ICI}$. Although the factors $CF^\pm$ and $\Delta\Omega$ contain a normalization factor, including the normalization factor in $CF^\pm$ has the advantage that the quantity $CF^\pm \cdot \text{ICI}$ would be the number of pions if the solid angle were 10 msr. Absolute cross sections for the second set of data were obtained using Eqn. III-1, with $N_{\text{beam}} = CF^\pm \cdot \text{ICI}$ and an acceptance scan with $\Delta\Omega = 10 \text{ msr}$ at $\delta_{\text{sp}} = 0\%$. Yields for elastic scattering from hydrogen were measured at 240, 162, and 140 MeV for π^+ and π^- , and at 100 MeV for π^- only, and the absolute cross sections obtained using this procedure agree with the calculated cross sections at all energies to $\pm 3\%$.

The errors quoted for the tabulated data in this dissertation reflect only the statistical and the fitting errors estimated by the code LOAF. The fitting errors are based on the χ^2 of the fit, but there are cases in which a bad fit yields a small χ^2 because of an unreasonable background. The possibility of such bad fits being taken as good data points was minimized by making hard copies of each fit and carefully examining them a number of times. It was found that, with the exception of the unresolved doublet mentioned in Sec.III-A, the peak areas of the fits were reasonably insensitive to the peak shape used. Fits made with reference shapes that were quite noticeably too wide (or too narrow) gave the same areas within 3% as fits made with reference shapes that had just the right width. However, uncertainties in the definition of the background must also be considered, and therefore we estimate an overall error of $\pm 5\%$ from peak fitting in addition to the statistical errors.

Another source of error is in the focal plane acceptance scan. The acceptance scans were measured with 1-2% statistics in the measured peak and therefore we include a contribution of $\pm 2\%$ statistical error from this source. The survival fraction correction introduces the possibility of an additional systematic error. Uncertainties in the path length through the spectrometer will cause uncertainties of about 3% in the survival fraction that can be angle dependent. The possible errors in measuring chamber efficiencies and in beam monitoring are estimated to be $\pm 3\%$ each.

In normalizing to ^{12}C , the statistical error in the number of counts was better than 3%. In normalizing to hydrogen, there is an additional source of error in that the peak shapes for scattering from hydrogen and ^{13}C are quite different. The hydrogen peak areas were extracted by subtracting a background and including all counts above the background in the peak area. In the ^{13}C spectra, peak areas were obtained by peak fitting using some lineshape (see Sec. III-B). Because peak fitting generally underestimates the number of counts in a peak, this could introduce a difference between hydrogen and ^{13}C yields for the same cross sections. However, the hydrogen yields were measured with better statistics than the ^{12}C yields and thus the normalization error is estimated to be 5% for both the ^{12}C and the Hydrogen normalizations.

The total sources of error in addition to the statistical error are therefore $\pm 5\%$ in normalization, $\pm 5\%$ from peak fitting, $\pm 2\%$ from focal plane variation of solid angle, $\pm 3\%$ in the survival fraction, $\pm 3\%$ in the chamber efficiency, and $\pm 3\%$ from beam monitoring. Adding these in quadrature yields a total error of 9% in the absolute normalization. Some of the above items must be the same for π^+ and π^- , namely the errors in peak fitting, focal plane acceptance, and survival fraction. The uncertainty in the relative normalization between π^- and π^+ is therefore estimated to be 6%.

These errors do not include one additional source of an angle-dependent error. The evidence for this additional error results from the fact that for the first set of data there were some efficiency problems associated with high count rates in the

scintillators located in the focal plane of the spectrometer. During a development run after the first part of this experiment, it was found that the yield for $\pi^+ + p$ elastic scattering dropped by 15% compared to calculated values as the scattering angle was changed from 20° to 120° . At back angles there was a high count rate in S2 and S3 because of high background radiation levels near the shielding wall around the A-1 target. This high count rate probably caused the inefficiency. Since data set 1 was normalized at 65° , the forward-angle data could be too high by as much as 5% while at the most backward angle (105°) it could be as much as 5% low. Additional shielding installed before the second part of the experiment appeared to cure this problem.

D. The Data

In this section the main features of the data are discussed and a few brief comments regarding their theoretical interpretations are made. These comments are not to preclude the detailed comparisons with theory in Chapter IV. The spectra presented in Figure III-8 are the result of summing normalized spectra taken at angles of 62° , 68° , 74° , 80° , and 86° . Part a shows the π^- spectrum Y_{π^-} , part b the π^+ spectrum Y_{π^+} , and part c gives the difference, $Y_{\pi^-} - Y_{\pi^+}$. The most striking feature seen in the spectra is a state at 9.5 MeV that is excited very strongly by π^- but only very weakly excited by π^+ . Another state (or group of states) at about 16 MeV is found to be much more strongly excited by π^+ than π^- . There is also a significant

π^+/π^- difference at 21 MeV, as indicated by a shift of the centroid of a group of states excited by π^+ with respect to the same group of states excited by π^- . Many other transitions exhibit differences between π^+ and π^- cross sections, but not as dramatic as for the above three cases. The numerical results for all states that were strong enough to warrant extraction of angular distributions will be presented below.

The angular distributions for π^+ and π^- elastic scattering are shown in Figure III-9. The angular distributions have the diffractive oscillatory pattern characteristic of strong absorption. At forward angles the π^+ and π^- cross sections are essentially the same, while in the region of the first minimum and second maximum the π^- cross sections are larger than the π^+ cross sections. The depth of the first minimum relative to the second maximum is slightly deeper for π^+ than for π^- . The minimum in the π^- angular distribution occurs at a smaller angle than the minimum in the π^+ angular distribution. This latter effect is mostly due to the fact that for π^- the Coulomb potential is attractive whereas for π^+ it is repulsive. Thus the "black disc" size of the nucleus is effectively larger for π^- than for π^+ . No need was found for invoking differences in neutron and proton distributions to fit the observed shift in the minimum (see Sec. IV-A).

Figures III-10 and III-11 contain the angular distributions for the transitions to the collectively enhanced $3/2^-$ (3.68 MeV) and $5/2^-$ (7.55 MeV) states. The latter state was not resolved from the $7/2^+$ state at 7.49 MeV, which is believed to be only weakly excited. The

FIGURE III-8

FIGURE III-9

effect of the $7/2^+$ state should be small, except perhaps in the region of the minimum of the $5/2^-$ angular distribution. The similarity of the $3/2^-$ and $5/2^-$ angular distributions supports this assumption. As expected for a collective transition, the π^+ and π^- cross sections for the $3/2^-$ state are approximately equal at forward angles but the minimum in the π^- angular distribution occurs at a smaller angle for π^- than for π^+ , similar to the shift seen in the elastic scattering angular distributions. At the forward peak of the $5/2^-$ angular distribution the π^+ cross section is larger than the π^- cross section by a factor of 1.7. This enhancement can be qualitatively understood in terms of partial blocking of the neutron part of the collective quadrupole strength by the $1p_{1/2}$ neutron, as will be discussed in Sec. IV-C.

The large cross section for a strong state at 11.82 MeV (Fig. III-12) (≈ 5 nb/sr at the peak) suggests that the transition to this state is also collectively enhanced. At the peak of the angular distributions $\sigma(\pi^+)$ is larger than $\sigma(\pi^-)$ by a factor of 1.2. A state at this energy has $J^\pi = 3/2^-$ [Aj-80] but the angular distributions that we see are more characteristic of an orbital angular-momentum transfer of three than of two (see Sec. IV-B). Further, the $3/2^-$ state observed in $^{14}\text{N}(d, ^3\text{He})$ is expected to be only very weakly excited in inelastic scattering. This leads us to believe that pion inelastic scattering excites another state, or states, with $J^\pi = 5/2^+$ and/or $7/2^+$.

FIGURE III-10

FIGURE III-11

FIGURE III-12

The transition to the $1/2^+$ state at 3.09 MeV is π^- enhanced, with a ratio $\sigma(\pi^-)/\sigma(\pi^+)$ of 1.7 ± 3 (FIG. III-13). Although the results of the $^{12}\text{C}(d,p)^{13}\text{C}$ reaction indicated [Ho-74] that the transition to this state contains a large fraction of the $2s_{1/2}$ single-particle strength, the small experimental π^-/π^+ ratio indicates a large deviation from a simple $1p_{1/2} \rightarrow 2s_{1/2}$ single-neutron excitation.

The same discrepancy exists for another "single-neutron" state known from stripping reactions. Figure III-14 shows the angular distributions for the $5/2^+$ state at 3.85 MeV. Here again, the $\sigma(\pi^-)/\sigma(\pi^+)$ ratio is only about two.

As already mentioned briefly, a $\sigma(\pi^-)/\sigma(\pi^+)$ ratio consistent with the value of 9:1 expected for a pure neutron transition was found for a state at 9.5 MeV (Fig. III-15). We have identified this state as a $9/2^+$ state based on the excellent agreement of the cross section ratios and the shapes of the angular distributions with the predictions of [Le-80] for which the shell-model calculations of [Mi-75] were used. A straightforward explanation of the large π^- enhancement can be made in a simple weak-coupling model in terms of a $1d_{5/2}$ neutron coupled to the first excited $2^+ T = 0$ state of ^{12}C , as will be discussed in Sec IV-C.

The π^+ cross section for the $1/2^-$ state at 8.86 MeV is about a factor of two larger than the cross section for π^- (Fig. III-16). These cross sections are very small, and hence the error bars are large. As is apparent from Figure III-16, most of the π^- data points are only upper limits.

FIGURE III-13

FIGURE III-14

FIGURE III-15

Figures III-17 and III-18 show the angular distributions obtained for four states, or groups of states, in the region from 15 to 22 MeV. All of these angular distributions have shapes characteristic of a large angular momentum transfer, similar to that for the $9/2^+$ state at 9.5 MeV. The state at 16.05 MeV is strongly π^+ enhanced. The π^- cross section is so weak that an angular distribution could not be extracted. The π^- cross section obtained at one point ($\theta_{lab} = 62^\circ$) gives a $\sigma(\pi^+)/\sigma(\pi^-)$ ratio of $R > 4$. The state at 17.92 MeV is excited about equally by π^+ and π^- . The group centered at 21.5 MeV appears as if π^+ and π^- excite different states. The centroid of the group seen with π^+ is at 21.37 MeV and the centroid of the group seen with π^- is at 21.6 MeV. This structure is similar to that observed at about 19.5 MeV in $^{12}\text{C}(\pi, \pi')$ by Morris, et al. [Mo-79] and interpreted by them as due to an isospin-mixed doublet of 4^- states.

The excitation-function data show two strikingly different types of behavior (Fig. III-19, III-20, III-21). The excitation functions for $(\pi^+, \pi^{+'})$ to the $3/2^-$ and $5/2^-$ states were taken at a momentum transfer q near the maximum in the angular distribution for $\Delta L = 2$ and $\Delta L = 3$ transitions, $q = 1.1 \text{ fm}^{-1}$. For both of these transitions the cross sections increase uniformly with energy (Fig. III-19). In sharp contrast is the gradual decrease of the cross sections with increasing energy in the excitation functions taken for the states at 9.5 and 21.6 in $(\pi^-, \pi^{-'})$ and 16.0, 17.9, and 21.4 MeV in $(\pi^+, \pi^{+'})$ at a momentum transfer of 1.4 fm^{-1} (Figure III-20) ($q = 1.4 \text{ fm}^{-1}$ is close to the momentum transfer at which the angular distributions for these transitions have a maximum). Excitation-function data for the

FIGURE III-16

FIGURE III-17

FIGURE III-18

$1/2^+$ (3.09 MeV), $5/2^+$ (3.85 MeV), and $1/2^-$ (8.86 MeV) states were taken at a momentum transfer of $1.1 \text{ } \hbar \text{ fm}^{-1}$ with π^+ . They show the same basic trend as the data for the $3/2^-$ and $5/2^-$ states, although they do not fall off as quickly toward low energies. Finally, the cross sections for the $3/2^-$ and $5/2^-$ states obtained at $q = 1.4 \text{ } \hbar \text{ fm}^{-1}$, i.e. off the $L = 2$ maximum in the angular distributions, first decrease sharply with decreasing energy until about 140 MeV where they start to increase. These very different excitation functions at the maxima of the angular distributions can be explained in terms of the spin transfer ($\Delta S = 1$ or 0) to the nucleus (see Sec. IV-D). The excitation functions taken off the maxima in the angular distributions are in addition affected very strongly by the energy dependence of the pion-nucleus optical potential.

Tabulations of the angular distributions and excitation-function data are presented in Appendix D.

FIGURE III-19

FIGURE III-20

FIGURE III-21

FIGURE III-22

Chapter IV

Theoretical Interpretations of the Data

A. Elastic Scattering

The elastic-scattering data has been analyzed using an optical model that describes the scattering of the pion by the nucleus in terms of an interaction potential that is independent of the coordinates of the individual nucleons and depends only on the coordinates of the pion with respect to the nucleus as a whole. This optical potential, V_N , can be related to the pion-nucleus scattering amplitude, T_N , by the following expression [Fr-53]:

$$T_N = V_N + V_N a^{-1} T_N \quad \text{IV-1}$$

$$a \equiv E_a + i\eta + H_0$$

and where $H_0 = H_N + h$; H_N is the nuclear Hamiltonian, h is the kinetic energy operator for the pion and E_a is the total energy. η is the standard parameter inserted to keep a from being equal to zero when H_0 operates on an eigenfunction. Then V_N can be related to the amplitudes for scattering of a pion from a bound nucleon, t'_i :

$$V_N = \sum t'_i \quad \text{IV-2}$$

Here the summation is over all the nucleons in the nucleus.

In the impulse approximation the t'_i are replaced by the amplitudes for scattering from free nucleons, t_i . In coordinate space the optical potential can be written in the Kisslinger form [Ki-53],

$$V_N^\pm(r) = (k_\pi^2 b_0^\pm \rho(r) - b_1^\pm \vec{\nabla} \rho(r) \cdot \vec{\nabla}) \quad \text{IV-3}$$

Here $\rho(r)$ is the radial dependence of the nuclear matter density and k_π is the pion momentum. The $(\pm)$ indicates that the potential is different for π^+ and π^- . The complex constants $b_0^\pm$ and $b_1^\pm$ are related by kinematic factors [Au-67] to the $L = 0$ and $L = 1$ pion-nucleon phase shifts, $\delta_{2T,2J}^L$. $\rho(r)$ in expression IV-3 is the radial dependence of the matter density, assumed to be the same for neutrons and protons, the b 's must be isospin averaged and are written:

$$b_0^- = F(E_\pi, E_N) \left\{ Z \frac{\alpha_{33}^0 + 2\alpha_{11}^0}{3} + (A - Z) \alpha_{31}^0 \right\} A^{-1} \quad \text{IV-4}$$

$$b_1^- = F(E_\pi, E_N) \left\{ Z \frac{2\alpha_{33}^1 + \alpha_{31}^1 + 4\alpha_{13}^1 + 2\alpha_{11}^1}{3} + (A - Z)(2\alpha_{33}^1 + \alpha_{31}^1) \right\} A^{-1}$$

where,

$$\alpha_{2T,2J}^L = \exp(i\delta_{2T,2J}^L) \sin \delta_{2T,2J}^L$$

and where $F(E_\pi, E_N)$ is the kinematic factor defined by [Au-67]. For b_0^+ and b_1^+ the factors Z and $(A - Z)$ are interchanged.

If the optical potential is to be calculated with different density distributions for neutrons and protons, equation IV-3 has to be rewritten in terms of four other coefficients, $T_{p0}^{\pm}$, $T_{n0}^{\pm}$, $T_{p1}^{\pm}$, $T_{n1}^{\pm}$:

$$V_N^{\pm} = \frac{k_{\pi}^2}{A} \{ (Z T_{p0}^{\pm} \rho_p(r) + N T_{n0}^{\pm} \rho_n(r)) - (Z T_{p1}^{\pm} \vec{\nabla} \rho_p(r) \cdot \vec{\nabla} + N T_{n1}^{\pm} \vec{\nabla} \rho_n(r) \cdot \vec{\nabla}) \} \quad \text{IV-5}$$

Where ρ_n and ρ_p are the radial dependences of the neutron and proton density distributions. The complex factors $T^{\pm}$ are defined by:

$$\begin{aligned} T_{p0}^{-} &= F(E_{\pi}, E_N) \frac{\alpha_{31}^0 + 2\alpha_{11}^0}{3} = T_{n0}^{+} \\ T_{p0}^{+} &= F(E_{\pi}, E_N) \alpha_{31}^0 = T_{n0}^{-} \\ T_{p1}^{-} &= F(E_{\pi}, E_N) \frac{2\alpha_{33}^1 + \alpha_{31}^1 + 4\alpha_{13}^1 + 2\alpha_{11}^1}{3} = T_{n1}^{+} \\ T_{p1}^{+} &= F(E_{\pi}, E_N) (2\alpha_{33}^1 + \alpha_{31}^1) = T_{n1}^{-} \end{aligned} \quad \text{IV-6}$$

The elastic scattering is calculated by solving a Klein-Gordon equation, which in the absence of potentials is given by:

$$E^2 - p^2 c^2 = m_0^2 c^4 \quad \text{IV-7}$$

where E is the total energy, m_0 is the pion rest mass and p the momentum operator, given by $\vec{p} = -i\hbar \vec{\nabla}$. The optical potential, $V_N(r)$, and the Coulomb potential, $V_C(r)$, are included by replacing E by $E' = (E - V_C - V_N)$. The numerical calculation is done using the code

NDWPI [Ei-76], which approximates E'^2 by $(E - V_c)^2 - 2EV_N$, i.e. neglecting the second order terms $2V_c V_N$ and V_N^2 [Ei-74]. This program integrates the Klein-Gordon equation in coordinate space for each partial wave out to a match point, where the internal logarithmic derivatives are compared to the external logarithmic derivatives to obtain the phase shifts used to calculate the differential cross sections. In the exterior, non-relativistic Coulomb wave functions are used.

The factors $T_{p0}^{\pm}$ and $T_{p1}^{\pm}$ are calculated from the pion-nucleon phase shifts of Rowe, Saloman, and Landau [Ro-78]. Two sets of T 's were used in this work, and are denoted as potential Set I and potential Set II. Set I was calculated by the code NDWPI, using the approximation of Auerbach, et al. [Au-67] for the transformation from the pion-nucleon center-of-mass system to the pion-nucleus center-of-mass system. The second set was generated using the method of Cottingham, et al., in which the pion-nucleon to pion-nucleus transformation is done exactly and where the pion-nucleon phase shifts are evaluated at an energy 30 MeV less than the incident pion energy [Co-80]. Table IV-1 contains the two sets of T coefficients.

TABLE IV-1: Proton and Neutron Potential Parameters

Set	$TP0^+(TN0^-)$	$TP1^+(TN1^-)$	$TN0^+(TP0^-)$	$TN1^+(TP1^-)$
I	$-1.807 + i0.4079$	$5.590 + i14.155$	$0.4075 + i0.3241$	$1.494 + i4.731$
II	$-1.887 + i0.3512$	$9.817 + i8.915$	$0.5812 + i0.332$	$2.811 + i2.982$

The only other variable inputs to the calculation are the neutron and proton density distributions. Two density distributions were used, a p-shell Gaussian and a three-parameter Fermi function. The expression for a two-parameter Gaussian is given by:

$$\rho(r) = \rho_0 \left(1 + z \frac{r^2}{c^2}\right) \exp\left(-\frac{r^2}{c^2}\right) \quad \text{IV-8}$$

Where ρ_0 , z , and c can be different for neutrons and protons. A p-shell Gaussian is obtained by constraining z to be $(Z-2)/3$ for protons (z_p) and $(N-2)/3$ for neutrons (z_n). The parameter c for protons and neutrons (c_p and c_n) has been adjusted to give the same rms radius for both the protons and neutrons as had been determined for protons from electron-scattering measurements [He-70]. The elastic electron scattering determines the mean square radius only of the charge distribution $\langle r^2 \rangle_{\text{chg}}$, and the radius of the point proton distribution is obtained from $\langle r^2 \rangle_{\text{chg}}$ by subtracting the mean square charge radius of the proton, $\langle r^2 \rangle_{\text{proton}}$, i.e.

$$\langle r^2 \rangle = \langle r^2 \rangle_{\text{chg}} - \langle r^2 \rangle_{\text{proton}}$$

The parameters of the p-shell Gaussian distribution, ρ_1 , are presented in Table IV-2. The results of the elastic-scattering calculations with density 1 (ρ_1) are shown in Figure IV-1. For the dashed lines potential Set I was used and for the solid lines potential Set II. The agreement with the data is improved considerably with Set II over the results with Set I. The agreement

TABLE IV-2: Neutron and Proton Density Distribution Parameters

Density	c_p (fm)	c_n (fm)	$\langle r^2 \rangle_p^{1/2}$ (fm)	$\langle r^2 \rangle_n^{1/2}$ (fm)
ρ_1^a	1.565	1.549	2.305	2.305
ρ_2^b	2.40	2.40	2.28	2.28
ρ_3^a	1.548	1.532	2.28	2.28

a) Two-parameter gaussian distribution

$$\rho(r) = \rho_0 \left(1 + z \frac{r^2}{c^2}\right) \exp\left(-\frac{r^2}{c^2}\right)$$

with $z = z_p = (Z-2)/3$ for protons and $z = z_n = (N-2)/3$ for neutrons.

b) Three parameter fermi distribution (wine bottle)

$$\rho(r) = \rho_0 \left(1 + \exp\left(\frac{r-c}{a}\right)\right)^{-1} \left(1 + \frac{wr^2}{c^2}\right)$$

with $a = 0.41$ fm and $w = -0.16$.

FIGURE IV-1

is still not very good in the region of the second minimum and third maximum, however.

This disagreement was the motivation to determine if the use of a different density distribution, i.e. the three-parameter Fermi function, would generate better fits than obtained with the p-shell Gaussian. The form of this distribution is given by:

$$\rho(r) = \rho_0 \frac{1}{1 + \exp(\frac{r-c}{z})} \left(1 + \frac{wr^2}{c^2}\right) \quad \text{IV-9}$$

The same parameters were used for neutrons and protons. The proton distribution parameters were derived from (e,e') data for use in the analysis of 800 MeV $^{13}\text{C}(p,p')$ data by Blanpied [Bl-78]. Blanpied had used a Fermi-like density distribution, slightly modified by the addition of a Gaussian correction term. This additional term was omitted in the present analysis, however, and to account for this omission the parameter c was adjusted to give the "correct" rms radius. It was found that the agreement with the data for both the three-parameter Fermi and the p-shell Gaussian distributions was slightly improved by reducing the rms radius from 2.31 to 2.28 fm for both neutrons and protons. The p-shell Gaussian with this radius is labeled ρ_2 and the three-parameter Fermi is labeled ρ_3 . The parameters for these distributions are found in Table IV-2.

Figure IV-2 shows the calculations done with densities ρ_2 (solid line) and ρ_3 (dashed line), both using potential Set II. The two densities produce results that agree equally well with the data out to the second maximum. The agreement is much better near the second minimum and third maximum for ρ_2 than it is for ρ_3 . Since both distributions have the same rms radius, the calculations show some sensitivity to the details of the density distribution. This is probably due mostly to the difference between the radial fall-off at large radii of the two distributions. Since pions are strongly absorbed they are particularly sensitive to small differences in the surface of the nucleus.

The quality of the agreement of the calculations with both the π^+ and π^- data shows no indication that a neutron distribution rms radius larger than the proton distribution rms radius is needed. If the neutron radius is increased from 2.28 to 2.35 fm the position of the first minimum in the angular distribution changes by $\approx 1^\circ$. However, the rms radius needed to reproduce the position of the first minimum depends slightly on the energy shift used in calculating the optical potential. Thus, it seems clear that a systematic study of elastic scattering at many energies and from many nuclei should be done before reliable values can be extracted for the radii of neutron and proton distributions. The calculations are, however, sensitive to the difference between the neutron and proton radii.

FIGURE IV-2

B. Collective Model of Inelastic Scattering

In descriptions of inelastic scattering, the transition matrix element between the initial and final nuclear states can be described in terms of either a microscopic or a macroscopic model. In the microscopic model (to be discussed in Sec.IV-C) the initial and final states are described by shell-model wave functions and are connected by particle creation and annihilation operators. In the macroscopic model, the nuclear excitations are treated in an analogy with the collective motions of a liquid drop, as collective oscillations of the nuclear surface. When the neutron and proton strengths in a transition are very different, the physical basis of the model in nuclear deformations is no longer valid, but it does provide a simple model to provide form factors to determine the relative neutron and proton strengths. The two approaches are connected in that the collective modes of the nucleus can be described by a coherent superposition of a large number of single-particle excitations. However, the calculations involved in reproducing the collective behavior of nuclei from a microscopic approach are very large and unwieldy.

In the model used here the radius parameter c of the nuclear density distribution $\rho(r) \equiv \rho(r,c)$ is written with an explicit angular dependence [Ei-76] :

$$c = c_0 \left(1 + \sum_{\lambda\mu} \alpha_{\lambda\mu} Y_{\lambda\mu}(r) \right) \quad \text{IV-10}$$

For rotational states the $\alpha_{\lambda\mu}$ are the static deformation parameters β_λ of the ground state. For vibrational excitations, the factors $\alpha_{\lambda\mu}$ are linear combinations of phonon creation and annihilation operators, given by:

$$\alpha_{\lambda\mu} = \frac{\beta_\lambda}{(2\lambda+1)^{1/2}} \{ b_{\lambda\mu} + (-1)^\mu b_{\lambda, -\mu}^\dagger \} \quad \text{IV-11}$$

The explicit angular dependence of c is substituted into the expression for $\rho(r, c)$ and ρ is expanded about c_0 to first order in $(c - c_0)$:

$$\begin{aligned} \rho(r, c) &= \rho(r, c_0) + \delta\rho \\ &= \rho(r, c_0) + \sum_{\lambda\mu} \alpha_{\lambda\mu} F_\lambda(r) Y_{\lambda\mu}(\hat{r}) \end{aligned} \quad \text{IV-13}$$

$$F_\lambda(r) = c \left. \frac{\partial \rho(r, c)}{\partial c} \right|_{c=c_0}$$

Where $F_\lambda(r)$ is the transition density. The elastic-scattering optical potential was related in Sec. IV-A to the ground-state density. The potential causing the nuclear excitation is generated by substituting the deformed part of the density $\rho(r, c)$ of Eqn. IV-13 for $\rho_p(r)$ and $\rho_n(r)$ in Eqn. IV-5.

Calculations were performed with these optical potentials and transition densities with a version of the code DWPI, called NDWPI, which had been modified by R. Eisenstein to use separate deformation parameters, β_n and β_p , for the neutron and proton density

distributions. Calculations were done with $\beta_p = \beta_n = 1$ and then β_p and β_n were adjusted to reproduce the experimental peak cross sections by solving the following equations:

$$R^+ = (3Z\beta_p + N\beta_n)^2 / (3Z + N)^2 \quad \text{IV-14}$$

$$R^- = (Z\beta_p + 3N\beta_n)^2 / (Z + 3N)^2$$

Where R^+ and R^- are defined by:

$$R^\pm = \frac{\sigma^\pm(\text{exp})}{\sigma^\pm(\text{DW})}$$

Here $\sigma^\pm(\text{DW})$ are the cross sections calculated with $\beta_p = \beta_n = 1$. After β_p and β_n had been determined from Eqn. IV-14 calculations were done with these values of $\beta_{p,n}$ and those calculations are presented in this section.

The model outlined above may be applicable to the collectively enhanced $3/2^-$ and $5/2^-$ states at 3.68 MeV and 7.55 MeV because these states appear to be a weak-coupling doublet obtained by coupling a $1p_{1/2}$ neutron to the lowest 2^+ , $T = 0$ state in ^{12}C . We therefore did the calculations with an angular momentum transfer $\Delta L = 2$ and parameters μ_2^n and β_2^p . The angular distributions for the strongly excited 11.82-MeV state are suggestive of $\Delta L = 3$ and its energy is close to the energy of the collectively enhanced 3^- state at 9.63 MeV in ^{12}C . Thus a calculation was done with $\Delta L = 3$ and assuming a final state spin of $5/2$. All calculations were done with potential set II,

but both ρ_2 and ρ_3 were used. The results are shown in Figure IV-3. The solid curves correspond to ρ_2 and the dashed curves to ρ_3 . The quality of the agreement between the calculations and the data is better for ρ_2 than ρ_3 for all three states. For the $3/2^-$ and $5/2^-$ states, the shape of the calculations reproduces the experimental shapes quite well. For the 11.82-MeV state the agreement is not as good, especially at angles larger than 75° .

In a collective model analysis of inelastic scattering it is customary to quote the deformation length, βR , obtained from the analysis. The parameter R is the undeformed value of the radius parameter that is given the explicit angular dependence of Eqn. IV-10. In the present case, since the deformation parameters have been separately obtained for neutrons and protons, we quote values of $\beta_{\lambda p} R$ and $\beta_{\lambda n} R$. Table IV-3 contains the values determined in this work, using both ρ_2 and ρ_3 , and the values obtained by Blaupied [Bl-78] from the analysis of 0.8-GeV proton scattering. The two density distributions give quite different values of $\beta_{\lambda p} R$ and $\beta_{\lambda n} R$. The values extracted with ρ_2 are much closer to the values measured with protons. The reason for this discrepancy is that parameter c in the p -shell Gaussian distribution does not correspond to the half-density radius for that density distribution, as it does for the three-parameter Fermi function. Therefore, for the p -shell gaussian we are not deforming the half-density radius and the values of βR should not be directly comparable to those obtained for the other distributions. It should be noted, however, that the value of the ratio $(\beta_{\lambda p} R)/(\beta_{\lambda n} R)$ is about the same for both density distributions.

FIGURE IV-3

TABLE IV-3: Deformation Lengths Obtained for ^{13}C

State	λ	$(\beta_{\lambda p} R)_2$	$(\beta_{\lambda n} R)_2$	$(\beta_{\lambda p} R)_3$	$(\beta_{\lambda n} R)_3$	$(\nu R)_\pi$	$(\nu R)_p$
$3/2^-$	2	1.40(10)	1.08(08)	0.72(05)	0.56(04)	1.22(12)	1.404(07)
$5/2^-$	2	1.48(09)	0.77(07)	0.80(05)	0.39(03)	1.09(12)	1.23(06)
$5/2^+$	3	1.9(13)	1.31(10)	0.99(07)	0.72(06)	1.60(16)	1.32(07)

The values of (βR) in the last column of Table IV-3 come from proton scattering data [Bl-78], and thus are an average over proton and neutron excitations. As a quantity for pion scattering that is comparable to these values we define $(\beta_\lambda R)_\pi$:

$$(\beta_\lambda R)_\pi = (N\beta_{\lambda n} R + Z\beta_{\lambda p} R) A^{-1} \quad \text{IV-15}$$

The values of this quantity, for ρ_2 , are shown in column 7 of Table IV-3. The agreement between $(\beta_\lambda R)_\pi$ and $(\beta_\lambda R)_p$ is not spectacular. Only for the $5/2^-$ state are the two numbers within errors. The reduction in the strength of the $5/2^-$ state compared to the $3/2^-$ is seen in both the pion and the proton work. The ratio $(\beta_\lambda R)_{3/2^-}/(\beta_\lambda R)_{5/2^-}$ is the same for the pion and the proton results within errors.

The interpretation of the "deformation lengths" obtained in terms of static or dynamic deformations of the nuclear shape is misleading. In cases, such as the $5/2^-$ state, where the proton and neutron deformation lengths are very different this would imply dramatically different shapes for the neutron and proton distributions in the nucleus. This is not at all reasonable. The justification for the use of this model is that it provides a very simple procedure to generate a semi-realistic, surface-peaked transition density that can be used to extract reasonable estimates of the relative strengths of the contributions of neutrons and protons to the transition.

In the next few paragraphs it will be pointed out that the strengths of the proton parts of the transition densities obtained in this procedure are in good agreement with electromagnetic transition strengths. The results of the analysis of the (π, π') data will be compared to the reduced transition probability, $B(T_\lambda; I_1 \rightarrow I_2)$, from other measurements:

$$B(T_\lambda; I_1 \rightarrow I_2) = \frac{1}{(2I_1 + 1)} |\langle I_2 || T_\lambda || I_1 \rangle|^2 \quad \text{IV-16}$$

for a transition from an initial state with angular momentum I_1 to a final state with angular momentum I_2 . Here $\langle I_2 || T_\lambda || I_1 \rangle$ is the reduced matrix element of the transition operator, T_λ , defined according to the convention of Bohr and Mottelson [Bo-69]. The transition probability for the inverse transition, $I_2 \rightarrow I_1$, is related to $B(T_\lambda; I_1 \rightarrow I_2)$ by :

$$B(T_\lambda; I_2 \rightarrow I_1) = \frac{(2I_1 + 1)}{(2I_2 + 1)} B(T_\lambda; I_1 \rightarrow I_2) \quad \text{IV-17}$$

Within the framework of the collective model the reduced matrix element for an electric transition is related to the "deformation parameter" β and the radial part of the transition density $F_\lambda(r)$ by:

$$M(E\lambda) = Ze\beta \int F_\lambda(r) r^{\lambda+2} dr$$

In a nuclear transition the neutron and proton parts may be different and we define:

$$M(E\lambda) = Ze\beta_p \int F_{\lambda p}(r) r^{\lambda+2} dr$$

$$M(N\lambda) = Ne\beta_n \int F_{\lambda n}(r) r^{\lambda+2} dr \quad \text{IV-18}$$

Where $F_{\lambda p}$ ($F_{\lambda n}$) is the proton (neutron) transition density from Eqn. IV-13 (the factor e has been included to facilitate comparison with electromagnetic measurements). The program NDMPI was modified to do the radial integrals of the transition densities in Eqn. IV-18. The value of $B(E\lambda) + (B(N\lambda) +)$ is used here, because it is directly comparable to the $B(E\lambda) +$ value from γ -ray transitions. Table IV-4 presents the values extracted for $B(E\lambda) +$ and $B(N\lambda) +$ in Weisskopf single-particle units, w.u.. The Weisskopf unit, $B_w(E\lambda)$, is defined as [Bo-69] :

$$B_w(E\lambda) = \frac{(1.2)^{2\lambda}}{4\pi} \frac{3}{\lambda + 3} A^{2\lambda/3} e^2 (\text{fm})^{2\lambda} \quad \text{IV-19}$$

This table also presents experimental $B(E\lambda) +$ from γ -ray transition strengths [En-79]. In addition, the $B(E\lambda) +$ for the mirror nucleus is presented, if available. This may be compared with $B(N\lambda) +$ for ^{13}C .

The $B(E2)$'s obtained for the $3/2^-$ and $5/2^-$ states are in agreement with the γ -ray measurements. For the $3/2^-$ state, $B(N2)$ agrees with the $B(E2)$ for the mirror nucleus, but the errors for the mirror nucleus value are very large. There are no measured $B(E3)$ values for the 11.82 MeV state. However, it is useful to compare the

strength obtained herein with the $B(E3)$ measured for the $^{14}\text{C } 3^-$ state, $B(E3) = 14 \pm 1 \text{ w.u.}$. The strength that we measure is only about 70% of the $^{12}\text{C } 3^-$ strength assuming that we measure only a single $5/2^+$ state at 11.82 MeV. There is no evidence for an equally strong $7/2^+$ state, which should be present if the $5/2^+$ is part of a weak-coupling doublet. This might indicate that both the $5/2^+$ and $7/2^+$ states are unresolved in the 11.82 MeV group. If the cross section seen for the 11.82 MeV group is due to both the $5/2^+$ and $7/2^+$ states, it corresponds to only 30% of the $^{14}\text{C } 3^-$ strength.

TABLE IV-4: Neutron and Proton Reduced Transition Probabilities for ^{13}C

J^π	λ	$B(E\lambda) +$ (w.u.)		$B(N\lambda) +$ (w.u.)		$B(E\lambda) +$ (w.u.)	$B(N\lambda) +$ (w.u.)
		ρ_2	ρ_3	ρ_2	ρ_3	$\gamma\text{-ray}$	Mirror
$3/2^-$	2	4.89(67)	4.62(64)	3.90(57)	3.80(56)	4.10(90)	6(2)
$5/2^-$	2	3.60(46)	3.80(47)	1.30(23)	1.20(21)	3.10(20)	----
$5/2^+$	3	9.90(135)	11.0(145)	6.60(99)	6.80(108)		

C. Microscopic Model of Inelastic Scattering

The microscopic model calculations presented in this section were done by T.-S. H. Lee and D. Kurath at Argonne National Laboratory. Their method, described in detail in reference [Le-80], uses a momentum space formulation of the distorted-wave impulse approximation and microscopic form factors derived from shell-model calculations [Co-65] and [Mi-75]. The pion-nucleus interaction potential is factored into two terms, one of which contains the nuclear transition form factors and the other the pion-nucleon interaction matrix elements. The pion-nucleon interaction matrix elements and the distorted waves are obtained from the momentum-space elastic scattering code, PIPIT [Ei-76a].

The shell-model states in ^{13}C may be divided into two groups, those of "normal" parity and those of "non-normal" parity. In the Lee and Kurath calculations, the transition densities for the normal-parity states (whose parity is the same as the ground state) are obtained from the lp-shell wave functions of Cohen and Kurath [Co-65]. As the name implies, these wave functions involve only the excitation of particles within the lp shell. This p-shell model does not reproduce the collective quadrupole strength seen in ^{12}C and therefore the $\Delta J = 2$, $\Delta L = 2$, and $\Delta S = 0$ transition density amplitudes are multiplied by enhancement factors like those needed to reproduce the measured $B(E2)$ values. Enhancement factors of 2.1 for neutrons and 1.4 for protons were used for ^{13}C [Le-80]. The transition densities for the non-normal parity states are obtained using a

modification of the Millener-Kurath wave functions [Mi-75]. These wave functions are generated from a basis consisting of three particles in the 1s shell and ten particles in the 1p shell $[(1s)^3(1p)^{10}]$ and of four particles in the 1s shell, eight particles in the 1p shell, and one particle in the 2s1d shell $[(1s)^4(1p)^8(2s1d)^1]$. Such states are connected to the ^{13}C ground state by operators of the form $b^\dagger(1p)b(1s)$ and $b^\dagger(2s1d)b(1p)$. In the calculations presented in [Le-80b] and used here the transition densities for the 1s to 1p excitations were omitted and the Millener-Kurath effective interaction had been modified to reproduce more exactly the low lying energy levels of ^{13}C . In this model some of the low-lying states of ^{13}C have large overlaps with states formed by coupling a (2s1d)-shell neutron to the low-lying $T = 0$ states of ^{12}C . Such states are expected to show large π^- enhancements.

The large π^- enhancement observed for the state at 9.5 MeV can be explained in a simple weak-coupling model in terms of a $1d_{5/2}$ neutron coupled to the first-excited 2^+ , $T = 0$ state of ^{12}C . When a $1d_{5/2}$ neutron is added to the 2^+ state, the diagram given in Figure IV-4, a represents one important component in the wave function. The term on the right can be reached from the ^{13}C ground state (Fig. IV-4, b) by a one-step, single-neutron excitation from the $1p_{3/2}$ to the $1d_{5/2}$ shell. The excitation of the other term from the main component in the ^{13}C ground state requires a two-step process; a proton must be promoted from the $1p_{3/2}$ to the $1p_{1/2}$ shell and a neutron from the $1p_{1/2}$ to the $1d_{5/2}$ shell. Therefore, under the assumption of a one-step direct

reaction mechanism, such a state can be reached only by a neutron particle-hole excitation.

In this model, states of $J^\pi = 1/2^+, 3/2^+, 5/2^+, 7/2^+, \text{ and } 9/2^+$ may be formed. However, an identification of these states with physical states is not simple. States of spin-parity $1/2^+, 3/2^+, 5/2^+, \text{ and } 7/2^+$ can also be obtained from many other configurations, and there will be strong mixing between the different configurations, many of which could be reached by simple particle-hole excitations. However, the "stretched" $9/2^+$ state can be excited only by moving a $1p_{3/2}$ neutron to the $1d_{5/2}$ shell. The transition to the $9/2^+$ member of the multiplet should therefore be purer than those to the other members of the multiplet. This is in agreement with the ratio of 9:1 observed for the $9/2^+$ state.

a)

b)

FIGURE IV-4

This $9/2^+$ state can also be identified with the first $9/2^+$ state predicted by Millener and Kurath. The identification of the theoretical $9/2^+$ state with a state at 9.5 MeV was first suggested by Millener and Kurath [Mi-75] on the basis of three-particle-transfer data [Ho-74]. Indeed, the overlap of the weak-coupling wave functions with the microscopic model wave function is very large [Ku-79]. The DWIA calculations with Millener and Kurath wave functions for this state reproduce the shape of the angular distributions very well (Fig. IV-5). The very large $\sigma(\pi^-)/\sigma(\pi^+)$ ratio of ≈ 9 is also very well reproduced although the absolute magnitude of the calculations is larger than the data by about 20%. [This new absolute cross section is different from the preliminary results presented in references [De-79] and [Le-80b]. A compelling reason was recently found for a new overall normalization of all the data. See Chapter III-C.] The weak-coupling model is also supported by the $^{12}\text{C}(n,n'\gamma)$ data [Ha-59] which show preferential decay of the 9.5-MeV state to the 2^+ state in ^{12}C . Furthermore, the widths of the 9.5-MeV state and its analog in ^{13}N were found to be very different [Ho-74]. This result is also consistent with a preferential decay through the first 2^+ state in ^{12}C .

The transition density predicted by Lee and Kurath for this state is purely $\Delta J = 4$, $\Delta L = 3$, and $\Delta S = 1$, as outlined in the following considerations. A value of $J = 9/2$ is the largest angular momentum that can be obtained by a particle-hole ($1p \rightarrow 2s1d$) excitation with respect to the ^{13}C ground state, i.e. the $9/2^+$ state is a "stretched" state. When a particle is promoted from a ($1p$) to a ($1d$) orbital, the

FIGURE IV-5

orbital angular momentum transfers allowed are 1, 2, or 3. Since this transition is from a $1/2^-$ to a $9/2^+$ state, a total angular momentum transfer of at least 4 is required. This can be achieved only by combining $\Delta L = 3$ and $\Delta S = 1$ to give $\Delta J = 4$. Thus the transition to the $9/2^+$ state is a pure "unnatural-parity" transition (with $\Delta S = 1$ only) unlike most transitions in odd-mass nuclei to lower values of J which usually proceed with a combination of $\Delta S = 0$ and $\Delta S = 1$.

A number of $7/2^+$ and $9/2^+$ states between 15 and 22 MeV are predicted to be dominated by the $\Delta J = 4$, $\Delta L = 3$, $\Delta S = 1$ amplitude [Le-80b]. Experimentally, we find several transitions that appear to contain large $\Delta J = 4$, $\Delta L = 3$, $\Delta S = 1$ components, since the angular distributions resemble the predicted curves for these angular momentum transfers. However, for only one case is it possible to make a fairly unambiguous identification with one of the predicted states, namely the state seen experimentally at 16.05 MeV. The calculations for the third predicted $9/2^+$ state are very close to the absolute magnitude of the π^+ data for the 16.05-MeV state (Fig. IV-6). The observed $\sigma(\pi^+)/\sigma(\pi^-)$ ratio of 4 ± 2 is also successfully predicted; however the experimental errors in the ratio are quite large.

None of the calculations for individual states around 21 MeV reproduce the magnitude of the cross sections or the $\sigma(\pi^+)/\sigma(\pi^-)$ ratio seen in the 21-MeV complex. However, the calculations do predict about equal π^+ and π^- strength and the total experimental π^- strength is about equal to the total experimental π^+ strength. Furthermore, the summed theoretical cross sections for states in this region are in qualitative agreement with the summed experimental cross sections.

FIGURE IV-6

However, the calculations do not predict the large $\sigma(\pi^+)/\sigma(\pi^-)$ asymmetries as observed experimentally. This is possibly due to the fact that the calculations did not include the effect of isospin mixing. Evidence for strong isospin mixing has been deduced for two 4^- states at ≈ 19 MeV in ^{12}C [Mo-79] from $^{12}\text{C}(\pi, \pi')$ data. The great similarity in the ^{13}C and $^{12}\text{C}(\pi, \pi')$ spectra in this energy region suggests that the states seen in ^{13}C have a large overlap with states formed by coupling a $1p_{1/2}$ neutron to the isospin-mixed 4^- states in ^{12}C .

In Figure IV-7 three sets of angular distributions ($\pi^+, \pi^{+'}$) and ($\pi^-, \pi^{-'}$) are presented, one each for the states at 3.09 MeV ($1/2^+$), 3.85 ($5/2^+$), and 8.86 MeV ($1/2^-$). In contrast to the results for the first $9/2^+$ state at 9.5 MeV, the agreement between the calculations and the data is not very good for the $1/2^+$ state at 3.09 MeV (first set of angular distributions in Fig. IV-7). In the extreme shell model the wave function for this state is that of a $2s_{1/2}$ neutron coupled to the ^{12}C ground state. It would be reached by a $1p_{1/2}$ to $2s_{1/2}$ pure neutron transition with $\sigma(\pi^-)/\sigma(\pi^+) \approx 9$. Experimentally we find a ratio of $1.7 \pm .3$. According to Lee and Kurath the π^- transition is dominated by the ($L = 1, S = 0$) transition density amplitudes, while for π^+ the ($L = 1, S = 0$) and ($L = 1, S = 1$) amplitudes are of comparable magnitude. The excitation of the ($L = 1, S = 0$) strength is inhibited for both π^+ and π^- by the destructive interference between the amplitudes for $(1p) \rightarrow (1d)$ and $(1p) \rightarrow (2s)$ excitation. Most of the $(1p) \rightarrow (2s)$ amplitude is $1p_{1/2}$ to $2s_{1/2}$ which is almost pure neutron. The difference in the effect of

the interference on the π^+ and π^- scattering results in quite different angular distribution shapes for π^+ and π^- . However, the quality of the data is not good enough to determine if the π^+ and π^- angular distributions have different shapes. Nevertheless, it is apparent that the calculated π^+ cross sections are a factor of 3 to 5 smaller than the experimental cross sections. The magnitude of the π^- cross section, however, is consistent with the data. A plausibility argument for the gross underestimation of the π^+ cross section can be made by assuming that the ground state of ^{13}C as described by the Cohen and Kurath wave functions has too few protons in the $1p_{1/2}$ orbit. An increase in the ground-state components that involve $1p_{1/2}$ protons would increase the proton $1p_{1/2}$ to $2s_{1/2}$ amplitude, decrease the $1p_{3/1}$ to $1d_{5/2}$ amplitude, and result in less destructive interference for π^+ scattering so that the π^+ cross section would increase relative to the π^- cross section. This would result in the ($L = 1$, $S = 0$) amplitudes becoming stronger relative to the ($L = 1$, $S = 1$) amplitudes. Components in the ^{13}C ground state with two particles in the (2s1d) shell would also increase the cross section for exciting this type of non-normal parity state, and make the neutron and proton components of the transition density more equal.

The second set of angular distributions in Figure IV-7 shows the comparison of the calculations with the π^+ and π^- data for the $5/2^+$ state at 3.85 MeV. The calculations successfully predict the positions of the maxima and minima in the angular distributions but the absolute magnitude is too low for both π^+ and for π^- . In addition, the calculated ratio $\sigma(\pi^-)/\sigma(\pi^+)$ is 4.2, compared to the

FIGURE IV-7

experimental value of ≈ 2 . As for the $1/2^+$ state, the transition to this state is not as neutron dominated as the shell-model predictions indicate.

The extra strength that we see for this $5/2^+$ state may be explained in terms of mixing with the $5/2^+$ state of the [^{12}C $3^- \times p_{1/2}$] $5/2^+, 7/2^+$ weak-coupling doublet. As was discussed in Section IV-B, the $B(E3)$ measured for the state (or states) at 11.82 MeV (calculated assuming a single $5/2^+$ state) contains only about 2/3 of the E3 strength that is seen in the ^{12}C 3^- state at 9.63 MeV. The enhanced cross section for the $5/2^+$ state at 3.85 MeV could be due to splitting of the octupole strength between the $5/2^+$ state in the 11.82 MeV complex and the $5/2^+$ state at 3.85. Such mixing would also reduce the ratio $\sigma(\pi^-)/\sigma(\pi^+)$ since the collective E3 excitations would involve neutrons and protons approximately equally.

The last set of angular distributions in Figure IV-7 are for the $1/2^-$ state at 8.86 MeV. This transition is very weak and the error bars are quite large. The calculations fail to reproduce both the absolute cross sections and the π^+/π^- ratio. The predicted π^+/π^- ratio is about ten, whereas the experimental ratio is between one and two. For π^+ the calculated angular distribution is shifted out of phase with respect to the data. The p -shell model predicts [Le-80] proton amplitudes much larger than the neutron amplitudes only for the ($L=0, S=1$) amplitude. For the other amplitudes the proton to neutron ratio is about two, indicating that the relative importance of the ($L=0, S=1$) amplitudes has been overestimated.

Figure IV-8 shows the shell model calculations and the data for those states that have previously been discussed within the framework of the collective model. In agreement with the data, the calculations (in which enhancement factors are included) give about equal cross sections for π^+ and π^- scattering to the $3/2^-$ state. However, the calculations are about 20% higher than the data for this state at the peak of its angular distribution, indicating the need for a slightly smaller (10%) enhancement factor. The π^+ cross section to the $5/2^-$ state is predicted to be larger than the π^- cross section. For π^- scattering there is a non-negligible contribution from the experimentally unresolved $7/2^+$ state at 7.49 MeV, which contains a large component of the wavefunction of the $7/2^+$ member of the $[2^+ \times d_{5/2}]$ weak coupling multiplet. In Figure IV-8 the dot-dash lines give the results for the $7/2^+$ calculation, the dashed line for the $5/2^-$ and the solid line for the sum of the two. The contribution from the $7/2^+$ state is quite large near the minimum in the π^- angular distribution but is barely visible for π^+ . Even when the effects of the $7/2^+$ state are included, the calculations predict a π^+ enhancement slightly larger than is observed experimentally.

The π^+ enhancement of the $5/2^-$ state can be understood qualitatively as follows. The $3/2^-$ and $5/2^-$ state are often interpreted as states formed by coupling a neutron in the $1p_{1/2}$ shell to the collective 2^+ state in ^{12}C . A large component in the 2^+ wave function consists of equal parts $(1p_{3/2})^{-1}(1p_{1/2})$ neutron particle-hole and proton particle-hole excitations relative to the ^{12}C ground state. When a $1p_{1/2}$ neutron is added to this part of the 2^+

state wave function, the term with the proton in the $1p_{1/2}$ shell can couple to either $J^\pi = 3/2^-$ or $5/2^-$ but the term with the neutron in the $1p_{1/2}$ shell can couple only to $3/2^-$. This is because the two neutrons in the $1p_{1/2}$ shell must couple to 0^+ so that the total angular momentum of this term can only be $3/2^-$ due to the $1p_{3/2}$ hole. Thus the $5/2^-$ member of the $\{ [(1p_{3/2})^{-1} \times 1p_{1/2}]2^+ \times p_{1/2} \}_{3/2^-, 5/2^-}$ doublet could be reached only by proton particle-hole excitation from the ^{13}C ground state; or in other words, the neutron particle-hole excitation is blocked by the $1p_{1/2}$ neutron. Of course, neither the ground state nor the $5/2^-$ state can be described successfully by the extreme shell model. However, the experimental ratio $\sigma(\pi^+)/\sigma(\pi^-) \approx 1.7$ instead of 1.0 for a purely collective transition or 9.0 for an extreme shell model transition is a measure of the relative importance of the collective and single-particle aspects of the transition. The difference in the $\sigma(\pi^+)/\sigma(\pi^-)$ ratio for the $3/2^-$ and $5/2^-$ states constitutes a nice example of a state-dependent blocking effect for the two states of a weak-coupling doublet.

As mentioned before, the $\Delta J = 2$, $\Delta L = 2$, $\Delta S = 0$ transition density amplitudes have been multiplied [Le-80] by enhancement factors that are also needed to account for the fact that the collective E2 transitions are not adequately described within the $1p$ -shell model space. Use of the same enhancement factors as for the $3/2^-$ state reproduces the magnitude of the cross section for the $5/2^-$ state very well. Possibly different enhancement factors are needed for the $3/2^-$ and $5/2^-$ states to reproduce all the experimental cross sections.

The last set of curves in Figure IV-8 presents the theoretical angular distributions for the second predicted $7/2^+$ state (dot-dash line) and the fourth predicted $5/2^+$ state (dashed line), both calculated to lie near 12 MeV and to be the strongest states in the region. The summed curves (solid lines) reproduce the shape and magnitude of the data quite well, although they are slightly below the π^- data and slightly above the π^+ at their peaks. The calculations contain no enhancement factors for E3 type transitions and the wave functions for these states are not dominated by terms corresponding to a $1p_{1/2}$ neutron coupled to the 3^- state in ^{12}C [Le-80b]. The agreement with the data indicates that no enhancement is necessary. However, because of the underprediction of the $5/2^+$ state at 3.85 MeV it is plausible that there is mixing between that state and the $\approx 12\text{-MeV}$ $5/2^+$ state, with the $5/2^+$ collective octupole strength of ^{12}C being split between them. In this case, the total experimentally observed E3 cross section is not quite reproduced in the calculations, indicating the need for a small enhancement of the $\Delta J = 3$, $\Delta L = 3$, $\Delta S = 0$ amplitudes.

FIGURE IV-8

D. Excitation Functions

The use of excitation functions to distinguish between $\Delta S = 1$ and $\Delta S = 0$ transitions is a new application of inelastic pion scattering to the study of nuclear structure. This technique is based on the fact that at energies near the $[3,3]$ resonance the spin-dependent and spin-independent parts of the pion-nucleus scattering amplitude have quite different energy dependences for a given momentum transfer [Si-80].

The pion-nucleon scattering amplitude can be written in the following form if the interaction is dominated by the $[3,3]$ resonance [Ko-69]:

$$f(k,k') = \alpha(k) [2 \cos(\theta) + i \vec{\sigma} \cdot \hat{n} \sin(\theta)] \quad \text{IV-20}$$

where k (k') is the pion momentum before (after) the collision, θ is the laboratory scattering angle, $\vec{\sigma}$ is the spin of the nucleon, $\hat{n}$ is the normal to the scattering plane, and the coefficient $\alpha(k)$ is energy dependent. The operator $\vec{\sigma} \cdot \hat{n}$ is the only term that can lead to a spin transfer ($\Delta S=1$). Siciliano and Walker have shown that a similar expression holds for pion-nucleus scattering [Si-80]. In their formalism, the differential cross section for pion-nucleus scattering can be approximated by:

$$\sigma(\theta) = \Gamma(E) [4 M^2(q,E) \cos^2(\theta) + S^2(q,E) \sin^2(\theta)] \quad \text{IV-21}$$

assuming dominance of the $[3,3]$ resonance and a one-step reaction mechanism. Here E is the pion energy, q is the momentum transfer, θ is the center-of-mass scattering angle, and $M(q,E)$ and $S(q,E)$ are the spin-independent and spin-dependent form factors, respectively. The energy dependences of the form factors $M(q,E)$ and $S(q,E)$ are very similar for values of q,E near the maximum of the form factors. The energy dependence of the pion-nucleon amplitudes as well as distorted-wave effects are contained in the factor $F(E)$. Both $M(q)$ and $S(q)$ can contribute to $\Delta S = 0$ transitions, although $M(q)$ usually dominates the $\Delta S = 0$ transitions. Only the $S(q)$ form factor can contribute to $\Delta S = 1$ transitions (if Fermi motion corrections are neglected). Thus the difference in the energy dependences of $\Delta S = 1$ transitions and $\Delta S = 0$ transitions measured at constant momentum transfer is due to the difference between the $\sin^2(\theta)$ and $\cos^2(\theta)$ angular dependences. This simple energy dependence can be observed experimentally by decreasing the scattering angle as the pion energy is increased to keep the momentum transfer $[q = 2 k \sin(\theta/2)]$ constant. Consequently, for the data presented in this work the scattering angle was varied from 90° at the lowest energy to 31.5° at the highest energy. Therefore the cross sections for $\Delta S = 1$ transitions, with pure $\sin^2(\theta)$ dependence, should decrease with increasing energy. In contrast, the cross sections for $\Delta S = 0$ transitions, which are usually dominated by $M(q)$ and the associated $\cos^2(\theta)$ dependence, should increase with increasing energy. Indeed, the transitions to known natural- and unnatural-parity states in the

^{12}C data of Moore, et al. [Mo-78] agree with these simple predictions, and indicate the presence of previously-unknown unnatural-parity states [Co-79]. It should be noted that the energy dependences at constant q associated with $\sin^2(\theta)$ and $\cos^2(\theta)$ dominate even the energy dependence of $\Gamma(E)$ in the region of the $[3,3]$ resonance. The energy dependence $\Gamma(E)$ is expected to be close to the energy dependence of the pion-nucleus total reaction cross section.

The $(\pi^+, \pi^{+\prime})$ cross sections measured at $q = 1.1 \text{ K fm}^{-1}$ for the transitions to the strongly excited low-lying states, at 3.68 MeV ($3/2^-$), and the unresolved $5/2^-, 7/2^+$ doublet at 7.5 MeV, increase uniformly with energy (Fig. III-20). The collectively enhanced $3/2^-$ and $5/2^-$ states are predicted to be dominated by the $\Delta L = 2, \Delta S = 0$ transition density amplitude [Le-80]. At $q = 1.1 \text{ K fm}^{-1}$ the differential cross section is close to the peak for an $L = 2$ transition and the doublet at 7.5 MeV is probably dominated by the transition to the $5/2^-$ state. Thus the excitation function for this group should show the features of a $\Delta L = 2, \Delta S = 0$ transition, as do the excitation functions for the transition to the $3/2^-$ state. For both states the simple $\cos^2(\theta)$ relation predicts the trend of the data very well. The difference between the data and the $\cos^2(\theta)$ curve reflects the importance of distorted-wave effects and the energy dependence of the pion-nucleus total cross section.

The excitation function for the transition to the 9.5-MeV state decreases uniformly with energy (Fig. III-21), in sharp contrast to the data for the $\Delta S = 0$ transitions discussed above. The energy dependence is very well accounted for by $\sin^2(\theta)$. The excitation of

this state was interpreted (Sec. IV-C) as due to the promotion of a neutron from the $p_{3/2}$ to the $d_{5/2}$ shell. As pointed out before, to make a $9/2^+$ state in such a transition, a spin transfer $\Delta S = 1$ is required. The experimental excitation function, which has the energy dependence characteristic of a $\Delta S = 1$ transition, strongly supports this interpretation.

Also shown in Fig. III-21 are the excitation functions for the states seen at 16.05, 17.92, 21.37, and 21.6 MeV, along with curves of $\sin^2(\theta)$, plotted on the appropriate energy scale. In all four cases the data are very well accounted for by $\sin^2(\theta)$, indicating that the transitions to these states involve a spin transfer $\Delta S = 1$. This supports an identification of these states as either $7/2^+$ or $9/2^+$ states, reached with an angular momentum transfer $\Delta J = 4$. Recent electron scattering measurements are in agreement with the $\Delta S = 1$ assignment for two of these states [Hi-80]. In that work the form factors measured for the states at 9.5, 16.1, and 21.4 MeV were well described by an M_4 shape.

The transition to the $1/2^-$ state at 8.86 MeV is predicted to be a pure $\Delta S = 1$ transition [Le-80]. The excitation-function data, however, indicates that this is not the case. The data in Fig. III-22 for this state do not fall off towards low energies as rapidly as do the data for the $3/2^-$ and $5/2^-$ states, but they are falling off enough to be distinguished from the pure $\Delta S = 1$ transitions in Fig. III-20. The difference in slope between the energy dependence of the $3/2^-$, $5/2^-$ and the $1/2^-$ transitions may be due to a significant $\Delta S = 1$ amplitude for the $1/2^-$ state, but the $\Delta S = 1$ amplitudes are clearly

not the only ones contributing in contrast to the expectations based on the shell model.

The excitation function for the $1/2^+$ state at 3.09 MeV is similar to that of the $1/2^-$ state. This transition appears not to be dominated by $\Delta S = 1$ amplitudes. The shell-model transition densities predict, for π^+ , that the $\Delta S = 0$ and $\Delta S = 1$ amplitudes should be of comparable magnitude. This cannot be ruled out from the experimental excitation function. Similarly, the $5/2^+$ state (3.85 MeV) excitation function does not fall off towards lower energies as rapidly as the $3/2^-$ and $5/2^-$ state excitation functions do. Here again we conclude that the transition is not purely $\Delta S = 1$. Future microscopic DWIA calculations may allow a quantitative separation of the $\Delta S = 1$ and $\Delta S = 0$ contributions to the transitions. Here again it is only possible to state that the transition is not purely $\Delta S = 1$.

CHAPTER V

Summary

This work comprises a detailed study of the nucleus ^{13}C with pion inelastic scattering. High resolution ($\pi^+, \pi^{+'}$) and ($\pi^-, \pi^{-'}$) data have been taken for many states in ^{13}C using the EPICS facility at the Clinton P. Anderson Meson Physics Facility. Differential cross sections were measured for angles between 20° and 105° at an incident pion energy of 162 MeV. Excitation functions were measured at momentum transfers of $q = 1.1 \text{ K fm}^{-1}$ and 1.4 K fm^{-1} for energies between 100 and 300 MeV. The excellent agreement between theoretical predictions and the data for a few simple transitions provides a convincing confirmation of the impulse approximation. First of all, the observation for the first time of a 9:1 ratio of $\sigma(\pi^-)/\sigma(\pi^+)$ was reproduced in a microscopic calculation [Le-80a] in terms of a pure neutron transition. This ratio is one of the important predictions of the impulse approximation and all claims for the sensitivity of pion inelastic scattering to neutron-proton differences are based on it.

Secondly, strikingly different energy dependences were observed for $\Delta S = 0$ and $\Delta S = 1$ transitions. A major success of the impulse approximation is the reproduction of the different energy dependences for a few well understood transitions. This data, along with that of Cottingame, et al. [Co-80], beautifully verify the simple $\sin^2(\theta)$ and $\cos^2(\theta)$ energy dependences at a constant momentum transfer predicted

by Siciliano and Walker [Si-80] based on the plane-wave impulse approximation. For a number of states not previously known the measured energy dependences were used to determine whether a spin transfer occurs or not.

Elastic-scattering data have also been taken and analyzed by an optical model. Reasonable fits were obtained by using the empirically-determined energy-shifted optical potential of Cottingham and Holtkamp [Co-80]. No need was found to use different density distributions for neutrons and protons. The best fit r.m.s. radius was slightly smaller than that determined from electron scattering. However, the radius parameter necessary to reproduce the position of the first minimum in the angular distributions is sensitive to the value of the energy shift. Therefore, no claim is made at present regarding the significance of this difference between the r.m.s. radius from electron scattering and from this work.

These optical potentials were used in calculations of the inelastic scattering to the $3/2^-$, $5/2^-$, and $(5/2^+, 7/2^+)$ states in a collective model. The reduced transition probabilities obtained were found to be in agreement with those determined from electromagnetic measurements on ^{13}C and its mirror nucleus ^{13}N .

The data have also been compared with microscopic-model predictions of Lee and Kurath [Le-80],[Le-80b]. To reproduce the data for the collective states by the microscopic calculations, enhancement factors had to be included for the $\Delta J = 2$, $\Delta L = 2$, $\Delta = 0$ amplitudes. Calculations with enhancement factors determined from electromagnetic measurements were successful in reproducing the magnitudes of the

cross sections. The same enhancement factors were used for the $3/2^-$ and $5/2^-$ states. This may not be justified since the blocking effect of the $1p_{1/2}$ neutron is larger for the transition to the $5/2^-$ state than the transition to the $3/2^-$ state.

Large discrepancies were found between theory [Le-80],[Le-80b] and experiment for the weak transitions. In a few cases large $\sigma(\pi^-)/\sigma(\pi^+)$ or $\sigma(\pi^+)/\sigma(\pi^-)$ ratios were predicted but only small ratios (≈ 2) were found. In such cases the weaker transition (either π^+ or π^-) is usually considerably stronger than the predictions, indicative of neglected terms in the wave functions or in the reaction mechanism.

One case in which the agreement between the shell model calculations and the data is very good is the transition to the $9/2^+$ state at 9.5 MeV. The experimental ratio $\sigma(\pi^-)/\sigma(\pi^+)$ is reproduced exactly within errors while the magnitude of the predicted cross sections are higher than the data by about 20%. Within the p-sd shell-model space there is only one particle-hole excitation ($1p_{3/2} \rightarrow 1d_{5/2}$) that can contribute to the transition to such a high-spin state. Therefore calculations for such a state would be expected to fit the data if the impulse approximation is valid. The good agreement between theory and experiment for the $9/2^+$ state is in contrast to (p,p') and (e,e') results for such "stretched" states where the calculations are generally too large by a factor of 2.

Experimental excitation functions and predictions of the shell model [Le-80b] have been used to identify four previously unknown states between 15 and 22 MeV as high-spin states. The transitions to these states all proceed with a spin transfer $\Delta S = 1$, similar to the

transition to the $9/2^+$ state at 9.5 MeV. The π^+/π^- asymmetry in the group at 21.5 MeV suggests that there may be isospin mixing between $T = 1/2$ and $T = 3/2$ states of $J^\pi = 7/2^+$ and/or $9/2^+$.

The sensitivity of pion inelastic scattering to neutron-proton differences in inelastic transitions has proved to be especially useful in testing nuclear wave functions. Such tests carry additional weight in view of the fact that all comparisons of theory and data for well understood transitions indicate that the Distorted-Wave Impulse Approximation is valid for pion scattering at energies near the $[3,3]$ resonance. Furthermore, pion excitation functions have been shown to be a valuable spectroscopic tool for determining the spin transfer in inelastic excitations.

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APPENDIX A

Isospin Properties of the Pion-Nucleon System

The isotopic spin formalism, developed to exploit the hypothesis of charge independence in the nuclear force, can be extended to pions. In this formalism, the proton and the neutron are regarded as two states of the same particle, the nucleon, corresponding to a new degree of freedom, isospin. Because this variable can have just two values, it is reasonable to regard it as a quantum-mechanical vector of length $1/2$. As such it can have two projections on some axis ($\hat{x}_3$) given by $+1/2$ and $-1/2$, in analogy with ordinary spin. Thus the isospin of the nucleon is $\hat{t} = 1/2$ with the two projections $t_3 = 1/2$ corresponding to a proton and $t_3 = -1/2$ to a neutron. Pions exist in three charge states, π^+ ($q = e$), π^- ($q = -e$), and π^0 ($q = 0$), and in view of this the pion is characterized by an isospin of 1 ($\hat{\tau} = 1$) and therefore has three substates; π^+ ($\tau_3 = 1$), π^0 ($\tau_3 = 0$), and π^- ($\tau_3 = -1$).

The hypothesis of charge independence in the strong interaction implies that the interaction is determined by the total isospin ($\hat{T}$) of a system but is independent of the third component (T_3). It is therefore useful to describe the various pion-nucleon systems in terms of states of good total isospin. Using the notation $|T T_3\rangle$ to denote

a state having isospin T and 3-component T_3 , the various states of the nucleon and the pion can be denoted as follows:

$$p \equiv |1/2 \ 1/2\rangle \quad n \equiv |1/2 \ -1/2\rangle$$

$$\pi^+ \equiv |1 \ 1\rangle \quad \pi^0 \equiv |1 \ 0\rangle \quad \pi^- \equiv |1 \ -1\rangle$$

The total isospin of a system composed of a pion and a nucleon is determined by the rules for addition of quantum-mechanical vectors:

$$\vec{T} = \vec{T}_1 + \vec{T}_2 \quad \text{and} \quad |t_2 - t_1| \leq T \leq t_1 + t_2$$

Therefore addition of $\vec{T}_1 = 1/2$ and $\vec{T}_2 = 1$ can result in a total isospin of either $T = 1/2$ or $T = 3/2$. The resulting states of good isospin are given by the following expression (where z now denotes the third component):

$$|T \ T_z\rangle = \sum_{t_1 t_2 t_{1z} t_{2z}} (t_1 t_2 t_{1z} t_{2z} | T T_z) |t_1 t_{1z}\rangle |t_2 t_{2z}\rangle$$

where $(t_1 t_2 t_{1z} t_{2z} | T T_z)$ is the Clebsch-Gordan coefficient and $|t_1 t_{1z}\rangle |t_2 t_{2z}\rangle$ is the wave function of the pion-nucleon system being considered. This expression can be inverted to give $|t_1 t_{1z}\rangle |t_2 t_{2z}\rangle$ in terms of states of good total T :

$$|t_1 t_{1z}\rangle |t_2 t_{2z}\rangle = \sum_T (t_1 t_2 t_{1z} t_{2z} | T \ t_{1z}+t_{2z}) |T \ t_{1z}+t_{2z}\rangle$$

The various pion-nucleon systems are then given by:

$$\pi^+_p = |11\rangle|1/2 \ 1/2\rangle = (1 \ 1/2 \ 1 \ 1/2|3/2 \ 3/2) \ |3/2 \ 3/2\rangle$$

$$\pi^0_p = |10\rangle|1/2 \ 1/2 \rangle = (1 \ 1/2 \ 0 \ 1/2|3/2 \ 1/2) \ |3/2 \ 1/2\rangle +$$

$$(1 \ 1/2 \ 0 \ 1/2|1/2 \ 1/2) \ |1/2 \ 1/2\rangle$$

$$\pi^-_p = |1-1\rangle|1/2 \ 1/2\rangle = (1 \ 1/2 \ -1 \ 1/2|3/2 \ -1/2) \ |3/2 \ -1/2\rangle +$$

$$(1 \ 1/2 \ -1 \ 1/2|1/2 \ -1/2) \ |1/2 \ -1/2\rangle$$

$$\pi^+_n = |11\rangle|1/2 \ -1/2\rangle = (1 \ 1/2 \ 1 \ -1/2|3/2 \ 1/2) \ |3/2 \ 1/2\rangle +$$

$$(1 \ 1/2 \ 1 \ -1/2|1/2 \ 1/2) \ |1/2 \ 1/2\rangle$$

$$\pi^0_n = |10\rangle|1/2 \ -1/2\rangle = (1 \ 1/2 \ 0 \ -1/2|3/2 \ -1/2) \ |3/2 \ -1/2\rangle +$$

$$(1 \ 1/2 \ 0 \ -1/2|1/2 \ -1/2) \ |1/2 \ -1/2\rangle$$

$$\pi^-_n = |1-1\rangle|1/2 \ 1/2\rangle = (1 \ 1/2 \ -1 \ -1/2|3/2 \ -3/2) \ |3/2 \ -3/2\rangle$$

Thus only the systems π^+_p and π^-_n are states of good total isospin.

We can now calculate the relative cross sections for the following processes:

$$\pi^+ + p \rightarrow \pi^+ + p$$

$$\pi^- + p \rightarrow \pi^- + p$$

$$\pi^- + p \rightarrow \pi^0 + n$$

The cross section is proportional to the square of the matrix element of the operator connecting the initial and final states:

$$\sigma \propto |\langle \Psi_f | H | \Psi_i \rangle|^2$$

Because of charge independence, H may be written in terms of H_1 and H_3 , where H_1 acts between $T = 1/2$ states and H_3 acts between $T = 3/2$ states:

$$H = H_1 + H_3 \quad \text{where}$$

$$\langle \Psi_f(T_1) | H_1 | \Psi_i(T_m) \rangle = M_1 \delta_{1m}$$

$$\langle \Psi_f(T_3) | H_3 | \Psi_i(T_m) \rangle = M_3 \delta_{3m}$$

and M_1 (M_3) contains the matrix element of H_1 (H_3) between the space-spin portions of the wave functions. Therefore,

$$\sigma(\pi^+p \rightarrow \pi^+p) \propto |M_3|^2$$

$$\sigma(\pi^+n \rightarrow \pi^+n) \propto \left| \frac{1}{3}M_3 + \frac{2}{3}M_1 \right|^2$$

$$\sigma(\pi^-p \rightarrow \pi^0n) \propto \left| \frac{\sqrt{2}}{3}M_3 - \frac{\sqrt{2}}{3}M_1 \right|^2$$

Similar relations hold true for $\pi^-n \rightarrow \pi^-n$, $\pi^-p \rightarrow \pi^-p$, and $\pi^+n \rightarrow \pi^0p$. If one of the isospin amplitudes dominates the scattering process, the following limiting ratios are obtained. If $M_3 \gg M_1$,

$$\sigma(\pi^+p \rightarrow \pi^+p) : \sigma(\pi^+n \rightarrow \pi^+n) : \sigma(\pi^-p \rightarrow \pi^0n) = 9:1:2$$

and

$$\sigma(\pi^-n \rightarrow \pi^-n) : \sigma(\pi^-p \rightarrow \pi^-p) : \sigma(\pi^+n \rightarrow \pi^0p) = 9:1:2$$

If $M_1 \gg M_3$,

$$\sigma(\pi^+p \rightarrow \pi^+p) : \sigma(\pi^+n \rightarrow \pi^+n) : \sigma(\pi^-p \rightarrow \pi^0n) = 0:4:2$$

and

$$\sigma(\pi^-n \rightarrow \pi^-n) : \sigma(\pi^-p \rightarrow \pi^-p) : \sigma(\pi^+n \rightarrow \pi^0p) = 0:4:2$$

The $[J=3/2, T=3/2]$ pion-nucleon resonance corresponds to a short lived particle (the Delta) of isospin $T = 3/2$ and spin $J = 3/2$. In the region of the resonance the reaction is dominated by the $T = 3/2$ amplitude, and the expected ratio of 9:1 is seen for the ratio of $\pi^+ + p$ elastic scattering to $\pi^- + p$ elastic scattering.

APPENDIX B

Sample ALLTST Output

*****SUMMED ALL TEST RESULTS FROM [135,146]EORDAT.DAT

RUNS 126 127
17-AUG-80

21:15:34 EVENT 6 FILE [135,146]EPX.TST

9048. = GAT.259.10000.12000..	:1	:S1 DE/DX	:EPX.TST
9048. = GAT.251.10000.12000.	:2	:S2 DE/DX PRODUCT	:LARGE JAWS
9048. = GAT.252.10000.12000.	:3	:S3 DE/DX PRODUCT	
9048. = GAT.253.10000.12000.	:4	:S2*S3 DE/DX	
40. = GAT.250.10301.32000.	:5	:CERENKOV PH SUM	
0. = B1.2.5.	:6	:HOOP P	
0. = B1.2.6.	:7	:HOOP M	
0. = B1.2.7.	:8	:HOOP R	
0. = B1.2.8.	:9	:HOOP L	
0. = GAT.3.1.32000.	:10	:FAST CLEAR (1/10)	
8767. = BOX.1.	:11	:BOX 1 ==> PID	
9048. = AND.2.3.4.	:12	:GOOD DE/DX	
0. = IOR.6.7.8.9.	:13	:HOOP COUNTER	
34. = AND.11.5.	:14	:PID=CERENKOV	
8733. = AND.11.-5.	:15	:PID=NOT(CERENKOV)	
9048. = IOR.1.-1.	:16	:LOOP1 COUNTER	
8890. = GAT.141.9990.10140.	:17	:1A DRFTMA	:CHAMBER LOOP 2
8893. = GAT.142.9990.10140.	:18	:2A DRFTMA	
8876. = GAT.143.9990.10140.	:19	:3A DRFTMA	
8685. = GAT.144.9990.10140.	:20	:4A DRFTMA	
8907. = GAT.145.9990.10140.	:21	:5A DRFTMA	
8890. = GAT.146.9990.10140.	:22	:6A DRFTMA	
8870. = GAT.149.9990.10140.	:23	:9A DRFTMA	
8910. = GAT.150.9990.10140.	:24	:10A DRFTMA	
8929. = GAT.201.9990.10140.	:25	:1C DRFTMC	
8951. = GAT.202.9990.10140.	:26	:2C DRFTMC	
8947. = GAT.203.9990.10140.	:27	:3C DRFTMC	
8945. = GAT.204.9990.10140.	:28	:4C DRFTMC	
8930. = GAT.205.9990.10140.	:29	:5C DRFTMC	
8929. = GAT.206.9990.10140.	:30	:6C DRFTMC	
8923. = GAT.209.9990.10140.	:31	:9C DRFTMC	
8906. = GAT.210.9990.10140.	:32	:10C DRFTMC	
0. = GAT.147.9926.9982.	:33	:7 CHECKSUM (QUAD)	
0. = GAT.148.9949.10002.	:34	:8 CHECKSUM (QUAD)	
8620. = GAT.272.9990.10100.	:35	:FRONT DRF DIF PLANE A	
8683. = GAT.273.9990.10100.	:36	:FRONT DRF DIF PLANE C	
8251. = GAT.274.9990.10100.	:37	:REAR DRF DIF PLANE A	
8314. = GAT.275.9990.10100.	:38	:REAR DRF DIF PLANE C	
7220. = AND.11.35.36.37.38.	:39	:DRF DIFF TST*PID ==> DRF	
8558. = AND.11.25.26.27.28.	:40	:FRONT C CHAMBERS OK*PID	
8541. = AND.11.17.18.19.20.	:41	:FRONT A CHAMBERS OK*PID	
8501. = AND.11.29.30.31.32.	:42	:REAR C CHAMBERS OK*PID	
8451. = AND.11.21.22.23.24.	:43	:REAR A CHAMBERS OK*PID	
8415. = AND.40.41.	:44	:FRONT OK	
8233. = AND.42.43.	:45	:REAR OK	
7907. = AND.44.45.	:46	:ALL EXCEPT QUAD OK*PID ==> CHMBS	
0. = IOR.33.34.	:47	:QUAD OK	
0. = AND.46.47.	:48	:ALL OK	
7927. = AND.18.19.20.40.45.	:49	:ALLBUT 1A CHAMBER EFFICIENCY TEST	
7932. = AND.17.19.20.40.45.	:50	:ALLBUT 2A	
7935. = AND.17.18.20.40.45.	:51	:ALLBUT 3A	
7929. = AND.17.18.19.40.45.	:52	:ALLBUT 4A	
7941. = AND.22.23.24.42.44.	:53	:ALLBUT 5A	
7954. = AND.21.23.24.42.44.	:54	:ALLBUT 6A	
7979. = AND.21.22.24.42.44.	:55	:ALLBUT 9A	
7952. = AND.21.22.23.42.44.	:56	:ALLBUT 10A	
7933. = AND.26.27.28.41.45.	:57	:ALLBUT 1C	
7936. = AND.25.27.28.41.45.	:58	:ALLBUT 2C	
7941. = AND.25.26.28.41.45.	:59	:ALLBUT 3C	
7935. = AND.25.26.27.41.45.	:60	:ALLBUT 4C	
7951. = AND.30.31.32.43.44.	:61	:ALLBUT 5C	
7943. = AND.29.31.32.43.44.	:62	:ALLBUT 6C	
7960. = AND.29.30.32.43.44.	:63	:ALLBUT 9C	
7969. = AND.29.30.31.43.44.	:64	:ALLBUT 10C	
8639. = GAT.233.9000.11000.	:65	:XTGT	:BACKGROUND REJECTION LOOP 3
8582. = GAT.234.8000.11000.	:66	:THTTGT	
8489. = GAT.235.9500.10500.	:67	:YTG	
8391. = GAT.236.9496.10385.	:68	:PHITGT	
6717. = GAT.237.9900.10100.	:69	:THTCHK	
7055. = GAT.238.9900.10100.	:70	:PHICLK	

1493.	=	GAT.233.9075.10139.	:71	:XTARG COPPER ROD
0.	=	GAT.266.9961.10069.	:72	:PION CTOF
8472.	=	GAT.221.9095.10740.	:73	:XFRONT PHASE SPACE CUTS
8500.	=	GAT.222.9137.10951.	:74	:THETA FRONT
7292.	=	GAT.223.9649.10501.	:75	:Y FRONT
8509.	=	GAT.224.9668.10401.	:76	:PHI FRONT
20.	=	BOX.2.	:77	:BOX 2
0.	=	BOX.3.	:78	:BOX 3
0.	=	BOX.4.	:79	:BOX 4
3877.	=	IGAT.6.	:80	:GATE 6 ==> ELASTIC PEAK
0.	=	IGAT.1.	:81	:ROD 1 OR ELASTIC PEAK
4788.	=	IGAT.2.	:82	:CTOF PIONS
4020.	=	IGAT.3.	:83	:CTOF MUONS
0.	=	IGAT.4.	:84	:ROD 4
0.	=	IGAT.5.	:85	:ROD 5
5080.	=	AND.39.46.73.74.75.76.	:86	:PHASE SPACE CUTS-FRONT
2213.	=	AND.39.46.69.70.80.	:87	:THETACHECK*PHICHECK*DRF*BOXS
0.	=	AND.39.46.47.	:88	:GOOD CHMBS. QUAD DK
0.	=	AND.-1.1.	:89	:SPARE
6618.	=	AND.39.46.65.66.67.68.	:90	:TARGET TESTS*GOOD CHMBS ==> TGT
5620.	=	AND.39.46.69.70.	:91	:THYCHK AND PHICLK*GOOD CHMBS ==> ANG CHK
6979.	=	AND.39.46.	:92	:DRF*CHKSUM ==> GOOD CHMBS
5393.	=	AND.90.91.	:93	:TGT*ANG CHK ==> GOOD EVENT (INCLUDES PID)
3025.	=	AND.80.92.	:94	:ELASTIC. GOOD CHMBS
2109.	=	AND.80.93.	:95	:ELASTIC. GOOD EVENT
0.	=	AND.47.94.	:96	:ELASTIC. GOOD EVENT. QUAD
2276.	=	GAT.236.9739.9913.	:97	: -1.0 DEG
2280.	=	GAT.236.9913.10087.	:98	: 0.0 DEG
1679.	=	GAT.236.10087.10261.	:99	: 1.0 DEG
1117.	=	GAT.236.10080.10087.	:100	: 0.25 DEG
930.	=	GAT.236.10087.10174.	:101	: 0.75 DEG
749.	=	GAT.236.10174.10261.	:102	: 1.25 DEG
6243.	=	GAT.236.9739.10261.	:103	: 3DEG
8801.	=	GAT.236.9000.11000.	:104	: FULL ACCEP
9040.	=	GAT.1.-1.0.	:105	: SPARE
1500.	=	AND.93.97.	:106	: -1.00 DEG. GOOD EVENT
1555.	=	AND.93.98.	:107	: 0.00
1048.	=	AND.93.99.	:108	: 1.00
762.	=	AND.93.100.	:109	: 0.25
502.	=	AND.93.101.	:110	: 0.75
466.	=	AND.93.102.	:111	: 1.25
4103.	=	AND.93.103.	:112	: 3DEG
5393.	=	AND.93.104.	:113	: FULL
0.	=	AND.-1.1.	:114	: SPARE
0.	=	AND.-1.1.	:115	: SPARE
0.	=	AND.-1.1.	:116	: SPARE
0.	=	AND.-1.1.	:117	: SPARE
0.	=	AND.-1.1.	:118	: SPARE
4103.	=	AND.112.-10.	:119	: 3DEG BIN WITH -1/10 VETO
0.	=	AND.93.10.	:120	: GOOD EVENT. 1/10
5393.	=	AND.93.-10.	:121	: GOOD EVENT. -1/10
0.	=	AND.82.120.	:122	: PION. 1/10
3041.	=	AND.82.121.	:123	: PION. -1/10
0.	=	AND.83.120.	:124	: MUON. 1/10
2195.	=	AND.83.121.	:125	: MUON. -1/10
0.	=	AND.-1.1.	:126	: SPARE
9040.	=	IOR.1.-1.	:127	: LOOP 4 COUNTER (WHOOPEE WERE DONE)

BOX CUTS FOR EVENT 6

BOX 1.	X VAR=253.	Y VAR=260	9886<X<10686	10470<Y<11190
BOX 2.	X VAR=246.	Y VAR=236	9724<X< 9801	9690<Y<10393
BOX 3.	X VAR=246.	Y VAR=222	9990<X< 9990	8640<Y<11706
BOX 4.	X VAR= 0.	Y VAR= 0	0<X< 0	0<Y< 0

INDIRECT GATES FOR EVENT 6

GATE 1.	INDEX= 0:	0-	0
GATE 2.	INDEX= 23:	1941-	2616
GATE 3.	INDEX= 23:	88-	904
GATE 4.	INDEX= 0:	0-	0
GATE 5.	INDEX= 0:	0-	0
GATE 6.	INDEX= 23:	144-	454

APPENDIX C

Ionization Chamber Response

The ionization chambers in the EPICS scattering chamber are each 6.96 cm thick. They are operated at a pressure of 650 mm Hg (one atmosphere in Los Alamos). The average temperature of the experimental area is 293 K. The gas mixture used in the chambers is 90% Argon, 10% CO₂ by volume. The density of the gas, at STP, can be calculated from the following:

$$\rho_{\text{Ar}} = \frac{M(\text{Ar}) \text{ gm/mole}}{22.4 \cdot 10^3 \text{ cm}^3/\text{mole}} = 1.78 \cdot 10^{-3} \text{ gm/cm}^3$$

$$\rho_{\text{CO}_2} = \frac{(2M(\text{O}) + M(\text{C})) \text{ gm/mole}}{22.4 \cdot 10^3 \text{ cm}^3/\text{mole}} = 1.97 \cdot 10^{-3} \text{ gm/cm}^3$$

where M stands for the Atomic weight. The areal density is obtained by multiplying ρ by the thickness and correcting for the fact that the gas is not at standard temperature and pressure, as follows:

$$t_{\text{Ar}} = 6.96 \cdot \rho_{\text{Ar}} \frac{P_{\text{Ar}}}{760}$$

$$t_{\text{CO}_2} = 6.96 \cdot \rho_{\text{CO}_2} \frac{P_{\text{CO}_2}}{760}$$

and where

$$P_{Ar} = .9 \cdot 650 \text{ mm Hg} \cdot \frac{273}{293} = 545.1 \text{ mm Hg}$$

$$P_{CO_2} = 1 \cdot 650 \text{ mm Hg} \cdot \frac{273}{293} = 60.6 \text{ mm Hg}$$

This yields:

$$t_{CO_2} = 1.09 \cdot 10^{-3} \text{ gm/cm}^2$$

$$t_{Ar} = 8.91 \cdot 10^{-3} \text{ gm/cm}^2$$

$$t = t_{CO_2} + t_{Ar} = 10.0 \cdot 10^{-3} \text{ gm/cm}^2$$

To calculate the number of pions per ampere of ion chamber current one needs to know the ionization potential of the chamber gas and the relative fractions of all particles making up the "pion" beam. The ionization potential of Argon gas is 26.4 eV and that of CO_2 is 32.9 eV. The average ionization potential, $\langle IP \rangle$, is given by:

$$\langle IP \rangle = \frac{t_{Ar} \cdot IP_{Ar} + t_{CO_2} \cdot IP_{CO_2}}{t} = 27.0 \text{ eV}$$

In this case, in which the beam is made up of pions, muons, and electrons, the ionization chamber charge is given by:

$$I = \frac{(\Delta E_{\pi} N_{\pi} + \Delta E_{\mu} N_{\mu} + \Delta E_e N_e)}{\langle IP \rangle} 1.6 \cdot 10^{-19} \text{ coul/e} \quad C-1$$

where ΔE_{π} , ΔE_{μ} , and ΔE_e are the energies lost in the gas by pions,

muons, and electrons, respectively. N_π , N_μ , and N_e are the number of pions, muons, and electrons respectively. Equation C-1 may be solved to yield an expression for N_π in terms of the ionization chamber charge:

$$N_\pi = \frac{I \cdot \langle IP \rangle}{\Delta E_\pi + \Delta E_\mu N_\mu / N_\pi + \Delta E_e N_e / N_\pi} \frac{1}{1.6 \cdot 10^{-19}}$$

$$= I \cdot \text{FAC}^\pm(E) \quad \text{C-2}$$

$\text{FAC}^\pm(E)$ is a function of both the pion charge and energy because the relative fraction of pions, muons, and electrons are different for π^+ and π^- and are also a function of energy, as are the energy losses.

An absolute cross section can be related to the yield through the factors $\text{FAC}^\pm(E)$:

$$\frac{d\sigma^\pm}{d\Omega} = \frac{G \cdot \text{YIELD}}{\text{FAC}^\pm \cdot I \cdot N_{\text{tgt}} \cdot \Delta\Omega} \quad \text{C-3}$$

where N_{tgt} is the number of target nuclei per millibarn and $\Delta\Omega$ is the solid angle. If a known cross section is measured, then Eqn. C-3 can be solved to give the solid angle, $\Delta\Omega$. The cross section for scattering from hydrogen were measured at energies between 50 and 310 MeV for both π^+ and π^- . Equation C-3 was used to determine from this data values of $\Delta\Omega$ for π^+ and π^- . Figure C-1 shows the resulting values as a function of energy. The solid angle obtained in this way is different for π^+ and π^- and it is not constant as a function of

FIGURE C-1

energy. Part of the fall off at low energy could be explained if the path length through the spectrometer was assumed to be 14m instead of 12.28 m, which would differentially increase the yields at low energies and require a larger $\Delta\omega$ to give the same cross section. The difference between π^+ and π^- , however, cannot be explained in this way. The difference must reflect an incomplete understanding of the ion chamber response.

The fact that $\Delta\omega$ was not constant was empirically corrected by setting $\Delta\omega$ equal to its nominal value of 10 msr and correcting $FAC^\pm(E)$ to give the correct hydrogen cross sections at each energy. The resulting numbers, $CF^\pm(E)$, are shown in Table C-1. An absolute cross section is obtained by multiplying the ion chamber integrated charge (in units of 10^{-10} coul) by $CF^\pm$ and using the following:

$$\frac{d\sigma^\pm}{d\omega}(\text{mb/sr}) = \frac{Q \cdot YIELD^\pm}{CF^\pm \cdot I \cdot \Delta\omega \cdot N_{tgt}}$$

where N_{tgt} is the number of target nuclei/mb and $\Delta\omega$ is the solid angle normalized to be 10 msr at $\delta_s = 0\%$.

TABLE C-1: Ion Chamber Correction Factors as a Function of Energy

T_{π}	$CF^{-}(10^5)$	$CF^{+}(10^5)$
300	12.00	9.40
280	11.75	9.30
260	11.50	9.20
240	11.25	9.15
220	10.90	9.00
200	10.50	8.85
180	9.90	8.60
160	9.20	8.30
150	8.80	8.05
140	8.25	7.75
130	7.70	7.35
120	7.00	6.90
110	6.25	6.40
100	5.45	5.85
80	3.90	4.75
70	3.10	4.30

APPENDIX D

Tabulation of Differential Cross Sections

The errors quoted in the tables are the statistical errors only. The possible systematic errors are enumerated in Sec. III-C. The overall additional error in the absolute normalization is 9%, while the error in the relative π^+ to π^- normalization is 6%.

TABLE D-1

 $^{13}\text{C}(\pi^+, \pi^+)^{13}\text{C}$ Elastic, 162 MeV

θ_{lab}	θ_{cm}	$d\sigma/d\Omega$	$\Delta d\sigma/d\Omega$
(Deg)	(Deg)	(mb/sr)	(mb/sr)
22.00	22.53	1.70 E+02	1.97 E+00
23.00	23.55	1.50 E+02	1.71 E+00
24.00	24.57	1.51 E+02	1.86 E+00
25.00	25.60	1.28 E+02	1.03 E+00
26.00	26.62	1.14 E+02	8.97 E-01
27.00	27.64	1.13 E+02	9.64 E-01
28.00	28.67	9.87 E+01	1.13 E+00
29.00	29.68	8.41 E+01	9.56 E-01
30.00	30.70	8.20 E+01	1.03 E+00
31.00	31.73	6.73 E+01	5.37 E-01
32.00	32.75	5.58 E+01	4.49 E-01
33.00	33.77	5.13 E+01	4.68 E-01
34.00	34.79	4.42 E+01	5.36 E-01
35.00	35.81	3.43 E+01	4.32 E-01
36.00	36.83	3.10 E+01	4.45 E-01
37.00	37.85	2.44 E+01	2.20 E-01
38.00	38.87	1.89 E+01	1.79 E-01
39.00	39.88	1.68 E+01	1.82 E-01
40.00	40.91	1.25 E+01	2.34 E-01
41.00	41.92	9.53 E+00	1.88 E-01
42.00	42.94	7.99 E+00	1.87 E-01
43.00	43.96	6.07 E+00	6.26 E-02
44.00	44.98	4.49 E+00	4.98 E-02
45.00	45.99	3.62 E+00	4.87 E-02
46.00	47.02	2.04 E+00	6.20 E-02
47.00	48.03	1.43 E+00	4.84 E-02
48.00	49.04	9.86 E-01	4.35 E-02
49.00	50.07	4.54 E-01	1.23 E-02
50.00	51.08	2.45 E-01	8.57 E-03
51.00	52.09	1.56 E-01	7.50 E-03
52.00	53.11	1.29 E-01	1.10 E-02
53.00	54.12	1.41 E-01	1.05 E-02
54.00	55.14	2.35 E-01	1.44 E-02
55.00	56.16	3.74 E-01	9.04 E-03
56.00	57.17	4.75 E-01	9.38 E-03
57.00	58.18	6.11 E-01	1.14 E-02
58.00	59.20	8.24 E-01	2.61 E-02
59.00	60.21	9.24 E-01	2.52 E-02
60.00	61.21	1.01 E+00	2.87 E-02
61.00	62.23	1.16 E+00	1.11 E-02
62.00	63.24	1.18 E+00	1.03 E-02

TABLE D-1

θ_{lab}	θ_{cm}	$d\sigma/d\Omega$	$\Delta d\sigma/d\Omega$
(Deg)	(Deg)	(mb/sr)	(mb/sr)
63.00	64.25	1.27 E+00	1.17 E-02
64.00	65.27	1.41 E+00	3.69 E-02
65.00	66.27	1.30 E+00	3.26 E-02
66.00	67.28	1.27 E+00	3.50 E-02
67.00	68.30	1.39 E+00	1.93 E-02
68.00	69.30	1.21 E+00	1.66 E-02
69.00	70.31	1.16 E+00	1.77 E-02
70.00	71.32	1.25 E+00	3.39 E-02
71.00	72.33	1.11 E+00	2.95 E-02
72.00	73.33	1.05 E+00	3.10 E-02
73.00	74.35	1.00 E+00	1.43 E-02
74.00	75.35	8.47 E-01	1.21 E-02
75.00	76.35	7.58 E-01	1.24 E-02
76.00	77.36	7.18 E-01	2.60 E-02
77.00	78.37	5.94 E-01	2.19 E-02
78.00	79.37	5.39 E-01	2.25 E-02
79.00	80.38	4.14 E-01	1.30 E-02
80.00	81.38	3.78 E-01	1.14 E-02
81.00	82.38	2.98 E-01	1.11 E-02
82.00	83.39	3.15 E-01	1.34 E-02
83.00	84.39	2.11 E-01	1.01 E-02
84.00	85.39	1.70 E-01	1.00 E-02
85.00	86.40	1.43 E-01	6.16 E-03
86.00	87.40	1.10 E-01	5.08 E-03
87.00	88.40	9.29 E-02	5.04 E-03
88.00	89.40	6.54 E-02	3.62 E-03
89.00	90.40	4.44 E-02	2.76 E-03
90.00	91.40	4.10 E-02	2.92 E-03
91.00	92.40	4.19 E-02	2.95 E-03
92.00	93.40	3.47 E-02	2.44 E-03
93.00	94.39	3.52 E-02	2.77 E-03
94.00	95.40	4.92 E-02	3.19 E-03
95.00	96.39	5.75 E-02	3.14 E-03
96.00	97.39	6.27 E-02	3.58 E-03
97.00	98.39	8.29 E-02	4.39 E-03
98.00	99.38	8.93 E-02	4.31 E-03
99.00	100.38	1.06 E-01	4.98 E-03
100.00	101.38	1.21 E-01	7.92 E-03
101.00	102.37	1.32 E-01	7.73 E-03
102.00	103.36	1.54 E-01	8.83 E-03
103.00	104.36	1.76 E-01	7.32 E-03
104.00	105.35	1.77 E-01	6.75 E-03
105.00	106.34	1.79 E-01	7.46 E-03

TABLE D-2

 $^{13}\text{C}(\pi^-, \pi^-)^{13}\text{C}$ Elastic, 162 MeV

θ_{lab}	θ_{cm}	$d\sigma/d\Omega$	$\Delta d\sigma/d\Omega$
(Deg)	(Deg)	(mb/sr)	(mb/sr)
24.00	24.58	1.59 E+02	1.92 E+00
25.00	25.60	1.40 E+02	1.39 E+00
26.00	26.62	1.31 E+02	1.39 E+00
27.00	27.64	1.15 E+02	2.32 E+00
28.00	28.67	9.81 E+01	1.56 E+00
29.00	29.68	8.35 E+01	1.34 E+00
30.00	30.70	7.68 E+01	1.38 E+00
31.00	31.73	6.17 E+01	1.71 E+00
32.00	32.75	5.45 E+01	1.49 E+00
33.00	33.77	4.51 E+01	1.49 E+00
34.00	34.79	3.89 E+01	1.01 E+00
35.00	35.81	3.19 E+01	8.51 E-01
36.00	36.83	2.82 E+01	8.59 E-01
37.00	37.85	2.19 E+01	1.10 E-01
38.00	38.87	1.67 E+01	8.94 E-02
39.00	39.88	1.43 E+01	8.97 E-02
40.00	40.91	1.03 E+01	3.26 E-01
41.00	41.92	7.51 E+00	2.55 E-01
42.00	42.94	5.90 E+00	2.44 E-01
44.00	44.98	2.76 E+00	3.66 E-02
45.00	45.99	2.19 E+00	3.55 E-02
46.00	47.02	1.24 E+00	7.31 E-02
47.00	48.03	7.00 E-01	5.19 E-02
48.00	49.04	4.87 E-01	4.76 E-02
49.00	50.07	2.81 E-01	1.20 E-02
50.00	51.08	2.29 E-01	1.02 E-02
51.00	52.09	2.69 E-01	1.19 E-02
52.00	53.11	4.52 E-01	3.39 E-02
53.00	54.12	4.81 E-01	3.27 E-02
54.00	55.14	6.82 E-01	4.22 E-02
55.00	56.16	9.27 E-01	3.44 E-02
56.00	57.17	1.09 E+00	3.39 E-02
57.00	58.18	1.21 E+00	3.90 E-02

TABLE D-2

θ_{lab}	θ_{cm}	$d\sigma/d\Omega$	$d\sigma/d\Omega$
(Deg)	(Deg)	(mb/sr)	(mb/sr)
58.00	59.20	1.41 E+00	6.43 E-02
59.00	60.21	1.48 E+00	6.00 E-02
60.00	61.21	1.63 E+00	6.88 E-02
61.00	62.23	1.83 E+00	2.84 E-02
62.00	63.24	1.75 E+00	2.56 E-02
63.00	64.25	1.91 E+00	2.90 E-02
64.00	65.27	1.90 E+00	7.46 E-02
65.00	66.27	1.70 E+00	6.51 E-02
66.00	67.28	1.75 E+00	7.14 E-02
67.00	68.30	1.70 E+00	2.79 E-02
68.00	69.30	1.48 E+00	2.42 E-02
69.00	70.31	1.36 E+00	2.52 E-02
70.00	71.32	1.53 E+00	6.78 E-02
71.00	72.33	1.27 E+00	5.72 E-02
72.00	73.33	1.18 E+00	6.06 E-02
73.00	74.35	1.07 E+00	2.30 E-02
74.00	75.35	8.68 E-01	1.91 E-02
75.00	76.35	7.99 E-01	2.00 E-02
76.00	77.36	8.37 E-01	3.66 E-02
77.00	78.37	6.69 E-01	3.02 E-02
78.00	79.37	5.43 E-01	3.01 E-02
79.00	80.38	4.46 E-01	1.36 E-02
80.00	81.38	3.62 E-01	1.14 E-02
81.00	82.38	3.01 E-01	1.13 E-02
82.00	83.39	2.56 E-01	2.02 E-02
83.00	84.39	1.81 E-01	1.59 E-02
84.00	85.39	1.54 E-01	1.61 E-02
85.00	86.40	1.53 E-01	1.53 E-02
86.00	87.40	7.42 E-02	9.58 E-03
87.00	88.40	6.28 E-02	1.00 E-02
91.00	92.40	6.11 E-02	1.02 E-02
92.00	93.40	4.58 E-02	8.30 E-03
93.00	94.39	5.18 E-02	9.61 E-03

TABLE D-3

 $^{13}\text{C}(\pi^+, \pi^+)^{13}\text{C } 1/2^+ (3.09), \quad 162 \text{ MeV}$

θ_{lab}	θ_{cm}	$d\sigma/d\Omega$	$\Delta d\sigma/d\Omega$
(Deg)	(Deg)	(mb/sr)	(mb/sr)
38.00	38.87	1.73 E-02	1.10 E-02
41.00	41.93	2.04 E-02	1.38 E-02
44.00	44.98	2.48 E-02	4.29 E-03
47.00	48.03	1.39 E-02	4.64 E-03
50.00	51.08	1.21 E-02	1.60 E-03
53.00	54.13	7.58 E-03	2.07 E-03
56.00	57.17	5.89 E-03	1.01 E-03
62.00	63.24	4.91 E-04	1.15 E-03
68.00	69.31	2.49 E-04	1.07 E-03
74.00	75.35	5.19 E-03	1.00 E-03
77.00	78.37	4.85 E-03	1.99 E-03
80.00	81.38	4.78 E-03	1.22 E-03
83.00	84.39	3.23 E-03	1.23 E-03
86.00	87.40	5.47 E-03	9.45 E-04
89.00	90.40	4.48 E-03	6.68 E-04
92.00	93.40	4.70 E-03	7.18 E-04
95.00	96.40	2.01 E-03	5.74 E-04
98.00	99.39	1.70 E-03	5.90 E-04
101.00	102.37	3.78 E-04	8.86 E-04

TABLE D-4

 $^{13}\text{C}(\pi^-, \pi^-)^{13}\text{C } 1/2^+ (3.09), \quad 162 \text{ MeV}$

θ_{lab}	θ_{cm}	$d\sigma/d\Omega$	$\Delta d\sigma/d\Omega$
<u>(Deg)</u>	<u>(Deg)</u>	<u>(mb/sr)</u>	<u>(mb/sr)</u>
32.00	32.75	7.54 E-02	9.95 E-02
44.00	44.98	2.39 E-02	3.52 E-03
47.00	48.03	3.12 E-02	9.34 E-03
50.00	51.08	2.08 E-02	2.35 E-03
53.00	54.13	1.82 E-02	5.12 E-03
56.00	57.17	1.29 E-02	2.14 E-03
59.00	60.21	9.69 E-03	5.58 E-03
62.00	63.25	6.28 E-03	1.93 E-03
65.00	66.28	3.24 E-03	5.15 E-03
68.00	69.31	5.31 E-03	1.81 E-03
71.00	72.33	1.09 E-02	5.65 E-03
74.00	75.35	7.26 E-03	1.74 E-03
77.00	78.37	4.87 E-03	3.09 E-03
80.00	81.38	9.13 E-03	1.49 E-03
83.00	84.39	5.93 E-03	2.50 E-03
86.00	87.40	3.64 E-03	2.22 E-03
92.00	93.40	8.23 E-03	2.53 E-03

TABLE D-5

 $^{13}\text{C}(\pi^+, \pi^+)^{13}\text{C } 3/2^- (3.68), \quad 162 \text{ MeV}$

θ_{lab}	θ_{cm}	$d\sigma/d\Omega$	$\Delta d\sigma/d\Omega$
(Deg)	(Deg)	(mb/sr)	(mb/sr)
23.00	23.55	6.92 E-01	9.16 E-02
26.00	26.62	6.26 E-01	2.79 E-02
29.00	29.69	4.73 E-01	6.55 E-02
32.00	32.75	7.40 E-01	2.05 E-02
35.00	35.81	7.81 E-01	5.54 E-02
38.00	38.87	7.58 E-01	3.28 E-02
41.00	41.93	6.35 E-01	4.17 E-02
44.00	44.98	6.95 E-01	1.72 E-02
47.00	48.03	4.67 E-01	2.44 E-02
50.00	51.08	4.09 E-01	9.32 E-03
53.00	54.13	2.82 E-01	1.26 E-02
56.00	57.17	1.83 E-01	5.15 E-03
59.00	60.21	8.80 E-02	7.46 E-03
62.00	63.25	5.05 E-02	2.15 E-03
65.00	66.28	2.77 E-02	5.10 E-03
68.00	69.31	1.59 E-02	2.17 E-03
71.00	72.33	1.12 E-02	3.76 E-03
74.00	75.35	2.14 E-02	1.82 E-03
77.00	78.37	2.99 E-02	4.46 E-03
80.00	81.39	4.25 E-02	3.44 E-03
83.00	84.40	5.47 E-02	4.64 E-03
86.00	87.40	4.68 E-02	3.08 E-03
89.00	90.40	5.04 E-02	2.67 E-03
92.00	93.40	5.28 E-02	2.69 E-03
95.00	96.40	3.81 E-02	2.39 E-03
98.00	99.39	3.28 E-02	2.43 E-03
101.00	102.38	2.40 E-02	3.25 E-03
104.00	105.36	2.08 E-02	2.31 E-03

TABLE D-6

 $^{13}\text{C}(\pi^-, \pi^-)^{13}\text{C } 3/2^- (3.68), \quad 162 \text{ MeV}$

θ_{lab}	θ_{cm}	$d\sigma/d\Omega$		$\Delta d\sigma/d\Omega$	
(Deg)	(Deg)	(mb/sr)		(mb/sr)	
25.00	25.60	5.03	E-01	1.01	E-01
26.00	26.62	6.61	E-01	2.51	E-02
29.00	29.69	7.01	E-01	1.01	E-01
32.00	32.75	7.68	E-01	3.17	E-02
35.00	35.81	5.24	E-01	1.16	E-01
38.00	38.87	6.27	E-01	2.45	E-02
41.00	41.93	5.12	E-01	6.54	E-02
44.00	44.98	5.16	E-01	1.46	E-02
47.00	48.03	4.30	E-01	3.64	E-02
50.00	51.08	3.30	E-01	1.07	E-02
53.00	54.13	1.99	E-01	2.02	E-02
56.00	57.17	1.61	E-01	7.62	E-03
59.00	60.21	9.22	E-02	1.54	E-02
62.00	63.25	5.95	E-02	4.99	E-03
65.00	66.28	2.11	E-02	9.67	E-03
68.00	69.31	2.40	E-02	3.44	E-03
71.00	72.33	2.59	E-02	9.13	E-03
74.00	75.35	3.87	E-02	3.97	E-03
77.00	78.37	4.21	E-02	7.39	E-03
80.00	81.39	5.14	E-02	4.02	E-03
83.00	84.40	6.73	E-02	8.52	E-03
86.00	87.40	5.60	E-02	7.45	E-03
92.00	93.40	4.45	E-02	7.08	E-03

TABLE D-7

$^{13}\text{C}(\pi^+, \pi^+)^{13}\text{C } 5/2^+ (3.85), \quad 162 \text{ MeV}$
 (Additional Fitting Errors 20%)

Θ_{lab}	Θ_{cm}	$d\sigma/d\Omega$	$\Delta d\sigma/d\Omega$
(Deg)	(Deg)	(mb/sr)	(mb/sr)
32.00	32.75	2.04 E-02	1.82 E-02
35.00	35.81	1.25 E-02	5.94 E-02
38.00	38.87	7.14 E-02	2.54 E-02
41.00	41.93	1.88 E-01	3.35 E-02
44.00	44.98	9.64 E-02	1.31 E-02
47.00	48.03	1.64 E-01	2.04 E-02
50.00	51.08	1.26 E-01	7.61 E-03
53.00	54.13	1.06 E-01	1.05 E-02
56.00	57.17	8.22 E-02	4.44 E-03
59.00	60.21	6.08 E-02	6.98 E-03
62.00	63.25	4.23 E-02	2.10 E-03
65.00	66.28	2.03 E-02	4.87 E-03
68.00	69.31	1.35 E-02	2.14 E-03
71.00	72.33	1.18 E-02	3.79 E-03
74.00	75.35	9.28 E-03	1.58 E-03
77.00	78.37	1.05 E-02	3.72 E-03
80.00	81.39	1.29 E-02	2.73 E-03
83.00	84.40	1.43 E-02	3.77 E-03
86.00	87.40	3.78 E-02	2.95 E-03
89.00	90.40	2.76 E-02	2.39 E-03
92.00	93.40	2.51 E-02	2.34 E-03
95.00	96.40	2.94 E-02	2.29 E-03
98.00	99.39	2.64 E-02	2.33 E-03
101.00	102.38	3.21 E-02	3.49 E-03
104.00	105.36	2.03 E-02	2.31 E-03

TABLE D-8

$^{13}\text{C}(\pi^-, \pi^-)^{13}\text{C } 5/2^+ (3.85), \quad 162 \text{ MeV}$
 (Additional Fitting Errors 20%)

θ_{lab}	θ_{cm}	$d\sigma/d\Omega$	$\Delta d\sigma/d\Omega$
(Deg)	(Deg)	(mb/sr)	(mb/sr)
32.00	32.75	5.01 E-02	2.50 E-02
35.00	35.81	1.92 E-01	1.02 E-01
38.00	38.87	2.40 E-01	2.13 E-02
41.00	41.93	3.45 E-01	5.98 E-02
44.00	44.98	2.69 E-01	1.30 E-02
47.00	48.03	2.38 E-01	3.27 E-02
50.00	51.08	2.40 E-01	1.01 E-02
53.00	54.13	2.28 E-01	2.09 E-02
56.00	57.17	1.62 E-01	7.67 E-03
59.00	60.21	1.08 E-01	1.61 E-02
62.00	63.25	7.13 E-02	5.18 E-03
65.00	66.28	5.39 E-02	1.13 E-02
68.00	69.31	2.62 E-02	3.50 E-03
71.00	72.33	6.11 E-03	7.23 E-03
74.00	75.35	1.86 E-02	3.52 E-03
77.00	78.37	2.47 E-02	6.90 E-03
80.00	81.39	2.70 E-02	3.57 E-03
83.00	84.40	2.84 E-02	7.34 E-03
86.00	87.40	2.86 E-02	6.71 E-03
92.00	93.40	4.13 E-02	6.55 E-03

TABLE D-9

 $^{13}\text{C}(\pi^+, \pi^+)^{13}\text{C } 5/2^-, 7/2^+ (7.55), 162 \text{ MeV}$

θ_{lab}	θ_{cm}	$d\sigma/d\Omega$	$d^2\sigma/d\Omega d\Omega'$
(Deg)	(Deg)	(mb/sr)	(mb/sr)
23.00	23.56	7.44 E-01	8.74 E-02
32.00	32.75	9.87 E-01	1.77 E-02
35.00	35.81	1.08 E+00	5.21 E-02
38.00	38.87	1.03 E+00	2.76 E-02
41.00	41.93	9.05 E-01	3.80 E-02
44.00	44.99	9.30 E-01	1.46 E-02
47.00	48.04	7.29 E-01	2.18 E-02
50.00	51.09	5.79 E-01	8.10 E-03
53.00	54.13	4.12 E-01	1.10 E-02
56.00	57.18	2.73 E-01	4.50 E-03
59.00	60.21	1.52 E-01	6.73 E-03
62.00	63.25	9.23 E-02	1.92 E-03
65.00	66.28	4.71 E-02	4.35 E-03
68.00	69.31	2.72 E-02	1.76 E-03
71.00	72.34	2.86 E-02	3.29 E-03
74.00	75.36	3.95 E-02	1.80 E-03
77.00	78.38	5.31 E-02	4.33 E-03
80.00	81.39	6.12 E-02	2.98 E-03
83.00	84.40	8.40 E-02	4.20 E-03
86.00	87.41	9.89 E-02	3.06 E-03
89.00	90.41	9.13 E-02	2.55 E-03
92.00	93.41	8.07 E-02	2.41 E-03
95.00	96.40	7.05 E-02	2.24 E-03
98.00	99.39	5.71 E-02	2.22 E-03
101.00	102.38	5.02 E-02	3.14 E-03
104.00	105.36	4.17 E-02	2.22 E-03

TABLE D-10

 $^{13}\text{C}(\pi^-, \pi^-)^{13}\text{C } 5/2^-, 7/2^+ (7.55), \quad 162 \text{ MeV}$

θ_{lab}	θ_{cm}	$d\sigma/d\Omega$	$\Delta d\sigma/d\Omega$
(Deg)	(Deg)	(mb/sr)	(mb/sr)
25.00	26.56	5.05 E-01	2.03 E-02
29.00	29.69	7.13 E-01	9.41 E-02
32.00	32.75	5.61 E-01	1.98 E-02
35.00	35.82	5.97 E-01	8.30 E-02
38.00	38.87	6.27 E-01	1.76 E-02
41.00	41.93	6.00 E-01	4.85 E-02
44.00	44.99	5.54 E-01	1.10 E-02
47.00	48.04	4.19 E-01	2.58 E-02
50.00	51.09	3.69 E-01	8.12 E-03
53.00	54.13	2.64 E-01	1.56 E-02
56.00	57.18	1.95 E-01	5.93 E-03
59.00	60.21	1.19 E-01	1.15 E-02
62.00	63.25	7.71 E-02	3.78 E-03
65.00	66.28	5.16 E-02	8.56 E-03
68.00	69.31	2.98 E-02	2.59 E-03
71.00	72.34	4.83 E-02	8.25 E-03
74.00	75.36	3.82 E-02	2.83 E-03
77.00	78.38	3.67 E-02	5.09 E-03
80.00	81.39	5.54 E-02	3.03 E-03
83.00	84.40	5.90 E-02	5.97 E-03
86.00	87.41	7.19 E-02	6.06 E-03
92.00	93.41	5.39 E-02	5.37 E-03

TABLE D-11

 $^{13}\text{C}(\pi^+, \pi^+)^{13}\text{C } 1/2^- (8.86), \quad 162 \text{ MeV}$

θ_{lab}	θ_{cm}	$d\sigma/d\Omega$	$\Delta d\sigma/d\Omega$
(Deg)	(Deg)	(mb/sr)	(mb/sr)
35.00	35.82	3.49 E-02	2.39 E-02
38.00	38.88	1.24 E-02	8.52 E-03
41.00	41.93	3.45 E-02	1.28 E-02
44.00	44.99	2.93 E-02	4.22 E-03
47.00	48.04	1.60 E-02	6.06 E-03
50.00	51.09	1.80 E-02	2.22 E-03
53.00	54.13	7.17 E-03	2.80 E-03
56.00	57.18	4.28 E-03	1.18 E-03
62.00	63.25	1.30 E-03	8.24 E-04
68.00	69.31	4.09 E-03	1.08 E-03
71.00	72.34	4.30 E-03	2.09 E-03
74.00	75.36	6.73 E-03	9.97 E-04
77.00	78.38	6.02 E-03	2.13 E-03
80.00	81.39	6.87 E-03	1.34 E-03
83.00	84.40	5.59 E-03	1.63 E-03
86.00	87.41	4.20 E-03	1.05 E-03
89.00	90.41	1.51 E-03	7.78 E-04

TABLE D-12

 $^{13}\text{C}(\pi^-, \pi^-)^{13}\text{C } 1/2^- (8.86), \quad 162 \text{ MeV}$

θ_{lab}	θ_{cm}	$d\sigma/d\Omega$	$\Delta d\sigma/d\Omega$
(Deg)	(Deg)	(mb/sr)	(mb/sr)
38.00	38.88	1.24 E-02	7.19 E-03
44.00	44.99	1.70 E-02	4.03 E-03
50.00	51.09	3.79 E-03	2.63 E-03
56.00	57.18	2.59 E-03	3.45 E-03
68.00	69.31	8.98 E-04	1.50 E-03
74.00	75.36	3.71 E-03	1.56 E-03
80.00	81.39	4.10 E-03	1.39 E-03

TABLE D-13

 $^{13}\text{C}(\pi^+, \pi^+)^{13}\text{C } 9/2^+ (9.5), \quad 162 \text{ MeV}$

θ_{lab}	θ_{cm}	$d\sigma/d\Omega$	$\Delta d\sigma/d\Omega$
(Deg)	(Deg)	(mb/sr)	(mb/sr)
47.00	48.04	8.13 E-03	5.49 E-03
50.00	51.09	5.23 E-03	2.17 E-03
53.00	54.13	4.37 E-03	2.70 E-03
56.00	57.18	6.49 E-03	1.45 E-03
59.00	60.22	3.91 E-03	2.32 E-03
62.00	63.25	6.24 E-03	8.39 E-04
65.00	66.29	8.40 E-03	2.67 E-03
68.00	69.31	2.85 E-03	1.08 E-03
74.00	75.36	4.79 E-03	9.30 E-04
80.00	81.39	3.73 E-03	1.20 E-03
83.00	84.40	3.16 E-03	1.49 E-03
86.00	87.41	2.79 E-03	1.03 E-03
89.00	90.41	2.33 E-03	8.21 E-04
92.00	93.41	4.16 E-03	8.56 E-04
95.00	96.41	2.41 E-03	7.79 E-04
98.00	99.40	2.33 E-03	8.81 E-04
104.00	105.37	1.24 E-03	9.48 E-04

TABLE D-14

 $^{13}\text{C}(\pi^-, \pi^-)^{13}\text{C } 9/2^+ (9.5), \quad 162 \text{ MeV}$

θ_{lab}	θ_{cm}	$d\sigma/d\Omega$	$\Delta d\sigma/d\Omega$
<u>(Deg)</u>	<u>(Deg)</u>	<u>(mb/sr)</u>	<u>(mb/sr)</u>
32.00	32.75	2.63 E-02	1.24 E-02
41.00	41.93	3.38 E-02	2.38 E-02
44.00	44.99	2.27 E-02	4.29 E-03
47.00	48.04	4.65 E-02	1.25 E-02
50.00	51.09	4.71 E-02	3.96 E-03
53.00	54.14	5.84 E-02	9.49 E-03
56.00	57.18	7.25 E-02	4.21 E-03
59.00	60.22	6.86 E-02	1.02 E-02
62.00	63.25	6.35 E-02	3.80 E-03
65.00	66.29	7.24 E-02	1.00 E-02
68.00	69.31	5.88 E-02	3.52 E-03
71.00	72.34	5.85 E-02	9.06 E-03
74.00	75.36	4.82 E-02	3.38 E-03
77.00	78.38	5.05 E-02	6.25 E-03
80.00	81.39	3.67 E-02	2.68 E-03
83.00	84.40	3.69 E-02	5.22 E-03
86.00	87.41	2.95 E-02	4.32 E-03
92.00	93.41	1.77 E-02	4.05 E-03

TABLE D-15

 $^{13}\text{C}(\pi^+, \pi^+)^{13}\text{C}$ (11.82), 162 MeV

θ_{lab}	θ_{cm}	$d\sigma/d\Omega$	$d^2\sigma/d\Omega d\Omega$
(Deg)	(Deg)	(mb/sr)	(mb/sr)
26.00	26.63	2.75 E-01	2.20 E-02
29.00	29.69	1.92 E-01	4.83 E-02
32.00	32.76	2.03 E-01	1.42 E-02
38.00	38.88	4.22 E-01	2.21 E-02
41.00	41.94	4.97 E-01	3.26 E-02
44.00	44.99	5.15 E-01	1.26 E-02
47.00	48.04	4.25 E-01	1.93 E-02
50.00	51.09	4.71 E-01	7.95 E-03
53.00	54.14	3.85 E-01	1.13 E-02
56.00	57.18	3.18 E-01	5.14 E-03
59.00	60.22	2.65 E-01	9.09 E-03
62.00	63.26	2.07 E-01	2.93 E-03
65.00	66.29	1.42 E-01	7.46 E-03
68.00	69.32	1.02 E-01	3.43 E-03
71.00	72.34	6.58 E-02	5.37 E-03
74.00	75.37	5.70 E-02	2.43 E-03
77.00	78.38	5.25 E-02	4.90 E-03
80.00	81.40	6.24 E-02	3.49 E-03
83.00	84.41	6.81 E-02	4.27 E-03
86.00	87.41	9.27 E-02	3.30 E-03
89.00	90.42	9.84 E-02	2.79 E-03
92.00	93.41	1.00 E-01	2.88 E-03
95.00	96.41	1.03 E-01	2.95 E-03
98.00	99.40	1.06 E-01	3.26 E-03
101.00	102.39	1.17 E-01	5.08 E-03
104.00	105.37	1.08 E-01	3.79 E-03

TABLE D-16

 $^{13}\text{C}(\pi^-, \pi^-)^{13}\text{C}$ (11.82), 162 MeV

θ_{lab}	θ_{cm}	$d\sigma/d\Omega$	$\Delta d\sigma/d\Omega$
(Deg)	(Deg)	(mb/sr)	(mb/sr)
26.00	26.63	2.32 E-01	2.24 E-02
29.00	29.69	3.02 E-01	9.00 E-02
32.00	32.76	2.64 E-01	2.13 E-02
38.00	38.88	3.79 E-01	1.76 E-02
44.00	44.99	3.87 E-01	1.10 E-02
47.00	48.04	4.24 E-01	2.89 E-02
50.00	51.09	3.78 E-01	9.39 E-03
53.00	54.14	3.59 E-01	2.01 E-02
56.00	57.18	2.52 E-01	7.36 E-03
59.00	60.22	2.18 E-01	1.63 E-02
62.00	63.26	1.68 E-01	5.93 E-03
65.00	66.29	1.14 E-01	1.31 E-02
68.00	69.32	7.37 E-02	4.36 E-03
71.00	72.34	4.12 E-02	1.00 E-02
74.00	75.37	3.28 E-02	3.44 E-03
77.00	78.38	4.44 E-02	6.87 E-03
80.00	81.40	4.88 E-02	3.45 E-03
83.00	84.41	4.93 E-02	6.87 E-03
86.00	87.41	7.50 E-02	6.97 E-03
92.00	93.42	9.33 E-02	7.78 E-03

TABLE D-17

 $^{13}\text{C}(\pi^+, \pi^+)^{13}\text{C } 9/2^+ (16.05), \quad 162 \text{ MeV}$

θ_{lab}	θ_{cm}	$d\sigma/d\Omega$	$\Delta d\sigma/d\Omega$
(Deg)	(Deg)	(mb/sr)	(mb/sr)
53.00	54.14	2.39 E-02	5.39 E-03
56.00	57.19	2.62 E-02	3.08 E-03
59.00	60.23	3.32 E-02	5.26 E-03
62.00	63.26	3.11 E-02	2.00 E-03
65.00	66.29	3.19 E-02	4.90 E-03
68.00	69.32	3.01 E-02	2.71 E-03
74.00	75.37	2.32 E-02	2.41 E-03
77.00	78.39	3.06 E-02	4.84 E-03
80.00	81.40	2.04 E-02	3.05 E-03
83.00	84.41	2.46 E-02	3.56 E-03
86.00	87.42	2.19 E-02	2.51 E-03
89.00	90.42	2.25 E-02	2.06 E-03
92.00	93.42	1.71 E-02	1.86 E-03
95.00	96.42	1.07 E-02	1.81 E-03
98.00	99.41	1.20 E-02	1.93 E-03
101.00	102.39	1.13 E-02	2.86 E-03
104.00	105.38	9.23 E-03	2.09 E-03

TABLE D-18

 $^{13}\text{C}(\pi^-, \pi^-)^{13}\text{C } 9/2^+ (16.05), \quad 162 \text{ MeV}$

θ_{lab}	θ_{cm}	$d\sigma/d\Omega$	$\Delta d\sigma/d\Omega$
<u>(Deg)</u>	<u>(Deg)</u>	<u>(mb/sr)</u>	<u>(mb/sr)</u>
62.00	63.26	1.14 E-02	3.00 E-03

TABLE D-19

 $^{13}\text{C}(\pi^+, \pi^+)^{13}\text{C } 7/2^+, 9/2^+ (17.92), \quad 162 \text{ MeV}$

θ_{lab}	θ_{cm}	$d\sigma/d\Omega$	$\Delta d\sigma/d\Omega$
<u>(Deg)</u>	<u>(Deg)</u>	<u>(mb/sr)</u>	<u>(mb/sr)</u>
53.00	54.15	3.53 E-02	6.40 E-03
56.00	57.19	2.45 E-02	3.41 E-03
59.00	60.23	2.84 E-02	5.58 E-03
62.00	63.26	2.18 E-02	2.00 E-03
65.00	66.30	4.52 E-02	5.64 E-03
68.00	69.33	2.41 E-02	2.81 E-03
74.00	75.37	2.38 E-02	2.43 E-03
77.00	78.39	3.69 E-02	5.60 E-03
80.00	81.41	2.17 E-02	3.35 E-03
83.00	84.42	2.38 E-02	4.06 E-03
86.00	87.42	2.56 E-02	2.88 E-03
89.00	90.43	2.43 E-02	2.32 E-03
92.00	93.42	2.21 E-02	2.27 E-03
95.00	96.42	1.61 E-02	2.14 E-03
98.00	99.41	1.33 E-02	2.31 E-03
101.00	102.40	1.68 E-02	3.49 E-03
104.00	105.38	9.44 E-03	2.46 E-03

TABLE D-20

 $^{13}\text{C}(\pi^-, \pi^-)^{13}\text{C } 7/2^+, 9/2^+ (17.92), \quad 162 \text{ MeV}$

θ_{lab}	θ_{cm}	$d\sigma/d\Omega$	$\Delta d\sigma/d\Omega$
(Deg)	(Deg)	(mb/sr)	(mb/sr)
62.00	63.27	1.67 E-02	3.87 E-03
68.00	69.33	1.30 E-02	3.37 E-03
74.00	75.37	1.92 E-02	3.62 E-03
80.00	81.41	1.90 E-02	3.25 E-03
86.00	87.42	2.05 E-02	5.98 E-03
92.00	93.42	2.11 E-02	5.78 E-03

TABLE D-21

 $^{13}\text{C}(\pi^+, \pi^+)^{13}\text{C } 7/2^+, 9/2^+ (21.37), \quad 162 \text{ MeV}$

θ_{lab}	θ_{cm}	$d\sigma/d\Omega$	$\Delta d\sigma/d\Omega$
(Deg)	(Deg)	(mb/sr)	(mb/sr)
53.00	54.15	3.40 E-02	7.94 E-03
56.00	57.19	6.10 E-02	4.80 E-03
59.00	60.23	6.12 E-02	7.59 E-03
62.00	63.27	5.62 E-02	3.00 E-03
65.00	66.30	9.29 E-02	8.21 E-03
68.00	69.33	5.66 E-02	4.05 E-03
74.00	75.38	5.76 E-02	3.56 E-03
77.00	78.40	6.80 E-02	7.91 E-03
80.00	81.41	4.45 E-02	4.78 E-03
83.00	84.42	3.13 E-02	5.42 E-03
86.00	87.43	5.05 E-02	4.28 E-03
89.00	90.43	3.93 E-02	3.32 E-03
92.00	93.43	3.08 E-02	3.18 E-03
95.00	96.43	2.45 E-02	3.12 E-03
98.00	99.42	1.49 E-02	3.20 E-03
101.00	102.40	2.53 E-02	5.28 E-03
104.00	105.39	1.59 E-02	3.79 E-03

TABLE D-22

 $^{13}\text{C}(\pi^-, \pi^-)^{13}\text{C } 7/2^+, 9/2^+ (21.60), \quad 162 \text{ MeV}$

θ_{lab}	θ_{cm}	$d\sigma/d\Omega$	$\Delta d\sigma/d\Omega$
(Deg)	(Deg)	(mb/sr)	(mb/sr)
62.00	63.27	9.90 E-02	6.56 E-03
65.00	66.30	7.48 E-02	1.53 E-02
68.00	69.33	7.50 E-02	5.90 E-03
71.00	72.36	8.39 E-02	1.65 E-02
74.00	75.38	7.41 E-02	6.10 E-03
77.00	78.40	9.19 E-02	1.20 E-02
80.00	81.41	6.18 E-02	5.50 E-03
83.00	84.42	5.97 E-02	1.10 E-02
86.00	87.43	5.82 E-02	1.04 E-02
92.00	93.43	3.74 E-02	9.15 E-03

TABLE D-23

 $^{13}\text{C}(\pi^+, \pi^+)^{13}\text{C } 1/2^+ (3.09), \quad q = 1.1 \text{ h fm}^{-1}$

E_{lab}	$d\sigma/d\Omega$	$\Delta d\sigma/d\Omega$
(MeV)	(mb/sr)	(mb/sr)
300.0	7.93 E-02	8.41 E-03
260.0	3.03 E-02	5.18 E-03
220.0	3.07 E-02	3.20 E-03
180.0	1.63 E-02	1.61 E-03
162.0	1.20 E-02	1.57 E-03
140.0	1.03 E-02	1.25 E-03

TABLE D-24

$$^{13}\text{C}(\pi^+, \pi^+)^{13}\text{C } 3/2^- (3.68), \quad q = 1.1 \text{ } \hbar \text{ fm}^{-1}$$

E_{lab}	$d\sigma/d\Omega$	$\Delta d\sigma/d\Omega$
(MeV)	(mb/sr)	(mb/sr)
300.0	1.43 E+00	2.44 E-02
260.0	1.21 E+00	2.51 E-02
220.0	8.68 E-01	1.61 E-02
180.0	5.50 E-01	9.51 E-03
162.0	4.31 E-01	9.20 E-03
140.0	2.80 E-01	6.12 E-03
120.0	1.47 E-01	1.13 E-02
100.0	5.12 E-02	5.15 E-03

TABLE D-25

$$^{13}\text{C}(\pi^+, \pi^+)^{13}\text{C } 5/2^+ (3.85), \quad q = 1.1 \text{ } \hbar \text{ fm}^{-1}$$

E_{lab}	$d\sigma/d\Omega$	$\Delta d\sigma/d\Omega$
(MeV)	(mb/sr)	(mb/sr)
260.0	7.07 E-02	1.76 E-02
220.0	8.98 E-02	1.15 E-02
180.0	1.09 E-01	7.02 E-03
162.0	8.59 E-02	7.60 E-03
140.0	4.11 E-02	4.38 E-03
120.0	2.91 E-02	8.65 E-03
100.0	6.29 E-03	3.97 E-03

TABLE D-26

 $^{13}\text{C}(\pi^+, \pi^+)^{13}\text{C } 5/2^-, 7/2^+ (7.55), \quad q = 1.1 \text{ } \hbar \text{ fm}^{-1}$

E_{lab}	$d\sigma/d\Omega$	$\Delta d\sigma/d\Omega$
(MeV)	(mb/sr)	(mb/sr)
300.0	1.73 E+00	2.58 E-02
260.0	1.57 E+00	2.13 E-02
220.0	1.16 E+00	1.43 E-02
180.0	7.31 E-01	8.43 E-03
162.0	5.84 E-01	8.19 E-03
140.0	3.63 E-01	5.45 E-03
120.0	1.76 E-01	9.71 E-03
100.0	5.49 E-02	4.29 E-03

TABLE D-27

 $^{13}\text{C}(\pi^+, \pi^+)^{13}\text{C } 1/2^- (8.86), \quad q = 1.1 \text{ } \hbar \text{ fm}^{-1}$

E_{lab}	$d\sigma/d\Omega$	$\Delta d\sigma/d\Omega$
(MeV)	(mb/sr)	(mb/sr)
300.0	3.04 E-02	6.60 E-03
260.0	3.39 E-02	5.10 E-03
220.0	1.73 E-02	3.15 E-03
180.0	1.43 E-02	1.95 E-03
162.0	1.47 E-02	2.00 E-03
140.0	7.00 E-03	1.38 E-03
100.0	7.19 E-03	2.45 E-03

TABLE D-28

$$^{13}\text{C}(\pi^+, \pi^+) ^{13}\text{C } 3/2^- (3.68), \quad q = 1.4 \text{ } \hbar \text{ fm}^{-1}$$

E_{lab}	$d\sigma/d\Omega$	$\Delta d\sigma/d\Omega$
(MeV)	(mb/sr)	(mb/sr)
260.0	5.87 E-01	8.60 E-03
240.0	4.74 E-01	1.74 E-02
220.0	3.17 E-01	6.91 E-03
200.0	2.00 E-01	8.90 E-03
180.0	1.16 E-01	4.00 E-03
162.0	5.05 E-02	2.15 E-03
140.0	3.79 E-02	2.08 E-03
116.0	7.15 E-02	6.78 E-03
100.0	2.12 E-01	7.01 E-03

TABLE D-29

$$^{13}\text{C}(\pi^-, \pi^-) ^{13}\text{C } 3/2^- (3.68), \quad q = 1.4 \text{ } \hbar \text{ fm}^{-1}$$

E_{lab}	$d\sigma/d\Omega$	$\Delta d\sigma/d\Omega$
(MeV)	(mb/sr)	(mb/sr)
260.0	5.19 E-01	1.09 E-02
240.0	3.52 E-01	1.50 E-02
220.0	2.55 E-01	6.48 E-03
200.0	1.50 E-01	1.20 E-02
180.0	8.97 E-02	9.77 E-03
162.0	6.48 E-02	6.00 E-03
140.0	5.44 E-02	5.85 E-03
116.0	1.04 E-01	1.04 E-02
100.0	2.66 E-01	1.18 E-02

TABLE D-30

$$^{13}\text{C}(\pi^-, \pi^-)^{13}\text{C } 5/2^-, 7/2^+ (7.55), \quad q = 1.4 \text{ h fm}^{-1}$$

E_{lab}	$d\sigma/d\Omega$	$\Delta d\sigma/d\Omega$
(MeV)	(mb/sr)	(mb/sr)
260.0	5.40 E-01	7.92 E-03
240.0	3.88 E-01	1.20 E-02
220.0	3.04 E-01	5.14 E-03
200.0	1.95 E-01	7.84 E-03
180.0	1.14 E-01	6.46 E-03
162.0	7.89 E-02	4.00 E-03
140.0	4.15 E-02	4.15 E-03
116.0	1.01 E-01	7.16 E-03
100.0	2.49 E-01	8.18 E-03

TABLE D-31

$$^{13}\text{C}(\pi^-, \pi^-)^{13}\text{C } 9/2^+ (9.50), \quad q = 1.4 \text{ h fm}^{-1}$$

E_{lab}	$d\sigma/d\Omega$	$\Delta d\sigma/d\Omega$
(MeV)	(mb/sr)	(mb/sr)
260.0	1.91 E-02	2.64 E-03
240.0	3.05 E-02	5.80 E-03
220.0	3.30 E-02	2.30 E-03
200.0	5.30 E-02	5.39 E-03
180.0	6.39 E-02	5.51 E-03
162.0	6.89 E-02	4.00 E-03
140.0	8.59 E-02	5.78 E-03
116.0	8.42 E-02	7.02 E-03
100.0	9.35 E-02	5.89 E-03

TABLE D-32

 $^{13}\text{C}(\pi^+, \pi^+)^{13}\text{C } 9/2^+ (16.05), \quad q = 1.4 \text{ } \hbar \text{ fm}^{-1}$

E_{lab}	$d\sigma/d\Omega$	$\Delta d\sigma/d\Omega$
<u>(MeV)</u>	<u>(mb/sr)</u>	<u>(mb/sr)</u>
240.0	2.39 E-02	3.90 E-03
200.0	2.96 E-02	4.03 E-03
180.0	2.99 E-02	3.26 E-03
162.0	3.11 E-02	2.00 E-03
116.0	5.57 E-02	6.17 E-03

TABLE D-33

 $^{13}\text{C}(\pi^+, \pi^+)^{13}\text{C } 7/2^+, 9/2^+ (17.92), \quad q = 1.4 \text{ } \hbar \text{ fm}^{-1}$

E_{lab}	$d\sigma/d\Omega$	$\Delta d\sigma/d\Omega$
<u>(MeV)</u>	<u>(mb/sr)</u>	<u>(mb/sr)</u>
240.0	1.09 E-02	3.77 E-03
200.0	9.06 E-03	3.85 E-03
180.0	2.19 E-02	3.34 E-03
162.0	2.18 E-02	2.00 E-03
116.0	4.58 E-02	6.93 E-03

TABLE D-34

 $^{13}\text{C}(\pi^+, \pi^+) ^{13}\text{C } 7/2^+, 9/2^+ (21.37), \quad q = 1.4 \text{ } \hbar \text{ fm}^{-1}$

E_{lab}	$d\sigma/d\Omega$	$\Delta d\sigma/d\Omega$
(MeV)	(mb/sr)	(mb/sr)
240.0	2.88 E-02	4.52 E-03
200.0	4.16 E-02	5.05 E-03
180.0	5.17 E-02	4.43 E-03
162.0	5.62 E-02	3.00 E-03
116.0	8.85 E-02	1.36 E-02

TABLE D-35

 $^{13}\text{C}(\pi^-, \pi^-) ^{13}\text{C } 7/2^+, 9/2^+ (21.60), \quad q = 1.4 \text{ } \hbar \text{ fm}^{-1}$

E_{lab}	$d\sigma/d\Omega$	$\Delta d\sigma/d\Omega$
(MeV)	(mb/sr)	(mb/sr)
240.0	6.35 E-02	8.83 E-03
200.0	4.49 E-02	6.97 E-03
180.0	6.37 E-02	7.09 E-03
162.0	9.89 E-02	6.56 E-03
116.0	7.54 E-02	1.86 E-02