SINGLE PASS COLLIDER MEMO

SLAC-CN--326

DE86 010092

AUTHOR: M. Sands

DATE: Apr. 14, 1986

TITLE: CENTRIFUGAL SPACE-CHARGE FORCES IN SLC*

1. Introduction

The purpose of this note is to make a preliminary estimate of the effect of the centrifugal space charge forces on SLC - assuming that the results of Piwinski¹ are correct.

2. Estimate of the chromaticity

According to Piwinski the new space-charge effect is a highly nonlinear radial force whose important term is the second derivative at the beam center. He evaluates this term numerically but gives his results in terms of the horizontal chromaticity for various storage rings. I use here a scaling law to relate his results to the arcs of SLC.

The horizontal chromaticity ξ is given by

$$\xi = -\frac{R}{2E} \left\langle \beta \eta \left(\frac{d^2 F_r}{dx^2} \right)_0 \right\rangle \quad (1)$$

where E is the energy, R the radius of the ring, β and η are the radial betatron and off-energy functions, and F_r is the radial space-charge force. From Piwinski's calculations we can expect the second derivative of F_r to be proportional to $I/\sigma_x^2 R$, where I is the local, instantaneous current density in a bunch and σ_x is the r.m.s. bunch width. The worst case is at the center of the bunch where $I = \bar{I}$, the peak bunch current, so I will use this value. This scaling assumption presumes that the transverse aspect ratio of the bunch σ_z/σ_x is a constant. I ignore this problem for now.

*Work supported by the Department of Energy, contract #DE-AC03-76SF00515

MASTER

JSD

We may then expect the chromaticity to scale in proportion to a scaling parameter S given by

$$S = \frac{\beta\eta\hat{I}}{\sigma_x^2 E} \quad (2)$$

Actually, since β , η , and σ_x^2 are correlated, I should take $\langle\beta\eta/\sigma_x^2\rangle$, but I will not make this refinement here.

Piwinski gives average values of the factors in Eq. (2) for three rings, CESR, PETRA, and LEP, together with a calculated chromaticity. In the following table I list these values together with a calculated S and the ratio ξ/S . You see that, for these rings at least, S is a proper scaling parameter.

TABLE 1

Parameter	CESR	PETRA	LEP	SLC
$\hat{I}$ (amp)	152	460	1350	1000
β (m)	13	18	80	4
η (m)	1.0	1.1	1.64	0.035
E (GeV)	5	7	20	50
σ_x (mm)	1.0	0.9	0.87	0.035
S (10^8 A/GeV)	4.0	16.8	117	23
ξ	8.6	39	300	{58 }
ξ/S	2.2	2.3	2.5	{2.5 }

The table gives also the value of S obtained for SLC. Using this S together with the factor $\xi/S = 2.5$ obtained from the LEP column, we get the value of 58 for the space-charge chromaticity expected for SLC. In the table I have taken for σ_x only the betatron oscillation width ($=\sqrt{\epsilon\beta_x}$). More about this later.

The biggest extrapolation is in the value of σ_x - which enters as the square. One might worry because all of the rings considered have about the same σ_x while SLC is twenty to thirty times smaller. I believe that ξ must vary as σ_x^{-2} from quite general arguments; and the results of Piwinski (his Fig. 10), which gives ξ for various σ_x agree very well with a σ_x^{-2} dependence.

On the other hand Piwinski considers only flat bunches with $\sigma_z/\sigma_x \approx 0.2$ or so, while SLC has a more nearly round beam. This should decrease ξ somewhat but probably by not as much as one-half.

Two important points to remember: First, the space-charge field does not correspond to a magnetic sextupole field. The radial dependence of the force on the central, horizontal plane ($z = 0$) is indeed like a sextupole field, but the variation of the radial force with z is different, and also there is no significant, off-axis z -component of the force as there would be for a real sextupole. Second, the space-charge field is centered at the bunch and not at the central orbit as is the field of the built-in sextupoles of the arcs.

3. Effect on the Beam

Let's say that the space-charge chromaticity is 58 for the arcs of SLC. What would that do? It is quite difficult to say, because the effects depend quite sensitively on the random imperfection in the guide field. I will make here only some semi-quantitative comments. A chromaticity in the arcs will mean that there is a phase slip $\Delta\phi(\Delta E)$ of the betatron oscillations in traversing the arcs between electrons of different energies. And I believe that this number gives some measure of the sensitivity to random errors. Indeed, I believe that if the maximum $\Delta\phi$ (for the expected energy spread) is much less than 1, there will be no significant effect, whereas if $\Delta\phi$ is much greater than 1, the effect will probably be serious. If $\Delta\phi$ is near 1 the situation will require further study.

Since the arcs are only the fraction $f = 0.6$ of a whole ring, $\Delta\phi(\Delta E)$ is related to ξ by

$$\Delta\phi(\Delta E) = 2\pi f \xi \Delta E/E_0. \quad (3)$$

Or for $\xi = 58$,

$$\Delta\phi(\Delta E) = 220 \Delta E/E_0. \quad (4)$$

Let me use this result in two ways. First, suppose we take $\Delta E/E_0$ to be $\pm 2 \times 10^{-3}$, the presently expected energy spread from the linac. Then $\Delta\phi = 0.44$, which is less than one, but not by a lot. But wait, we should also include the additional beam width due to the energy spread. Then

$$\sigma_x^2 \approx \sigma_{x\beta}^2 + (\eta \Delta E/E)^2$$

$$\approx (35^2 + 70^2)(\mu m)^2,$$

or

$$\sigma_x \approx 78 \mu m.$$

And since the chromaticity goes as σ_x^{-2} , it is reduced by the factor $(0.35/78)^2 = 0.2$. The appropriate $\Delta\phi$ is now 0.09 which would seem to be quite benign.

Notice that any larger ΔE would give an even lower $\Delta\phi$, because then ξ would be proportional to ΔE^{-2} and $\Delta\phi$ would vary as ΔE^{-1} .

Now let me show that the above estimates are not quite correct, because I have assumed that ΔE represent the energy spread at the bunch center. In reality we expect that there will be a strong correlation between the mean energy deviation ΔE at each longitudinal location within the bunch and the longitudinal coordinate, say τ . It is this ΔE which is expected to vary between $\pm 2 \times 10^{-3}$ of the mean energy. In addition, there will be an energy spread $\pm \delta E$ at each longitudinal position. This spread is related to the energy spread injected into the linac from the damping ring and compressor. I am told that the expected spread gives $\delta E/E = \pm 4 \times 10^{-4}$ at the arcs. In other words as the bunch enters an arc the energy varies from one τ to another within the bunch (with a total range $\pm \Delta E/E \approx 2 \times 10^{-3}$) which at each τ there is a local energy spread of $\pm \delta E/E \approx 4 \times 10^{-4}$.

In the arcs then the bunch will have a snake-like shape because the axis of the bunch is displaced laterally (because of the η -function) proportional to ΔE , which is varying with τ . At each τ , however, the bunch has an energy spread $\pm \delta E$, and a width σ_x which is given almost completely by the beta-tron emittance - because η times the "inner" energy spread δE is quite small. Now since the space-charge nonlinearity is centered at the displaced bunch axis, it will produce a phase displacement $\Delta\phi$ given by the chromaticity times the inner energy spread δE .

The phase spread $\Delta\phi$ produced by an energy spread δE of 2×10^{-4} is only 0.04 which is even smaller than the $\Delta\phi$ estimated above.

There is an additional complication in the arcs. As the bunch goes along it gets shorter and its energy spread increases. Both of these effects will increase

the space-charge effect as the bunch moves along. I have made a rough estimate that the actual $\Delta\phi$ in the arcs might be increased by a factor of 2 or 3.

Both estimates still give us that the effects of the centrifugal space-charge effects in the arcs should be comfortably small. Some questions remain, however.

Questions

The results found above follow from (a) the space-charge calculations of Piwinski, (b) the scaling law derived here, and (c) my assumption that it is the "inner" (or local) energy spread which determines the chromatic tune shift.

I have some serious reservations about the Piwinski calculation. (For one, there are no radiation fields that I can see. What happened to them?) In particular, our results depend strongly on his conclusion that the space charge force depends only on the local bunch density. Can that be strictly true? If there is any long longitudinal spreading (which would be reasonable physically) it could make matters worse.

I believe that the scaling law for the chromaticity is correct. In particular, I think that d^2F_r/d^2x should scale as σ_x^{-2} from quite general arguments.

On the last point (c), we may reasonably question whether the space-charge forces for a "snake-shaped" bunch are the same as for a nice sausage, which was considered by Piwinski. Conceivably, the local "tilt" of the bunch axis could change the fields significantly.

We should try to resolve these questions before concluding that the centrifugal space-charge effects will do no harm.

1. A. Piwinski. CERN/LEP-TH/85-43.

DISCLAIMER

This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency thereof, nor any of their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.