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Introduction

The focusing system that is to be discussed here has long been used in an adiabatically tapered solenoid... The positrons ... distribution in energy of the ... and that after ... the positrons lies within the ... M.V ... the debunching effect is ... the drift space following the target, and in the first few cm of the acceleration, leading to a broadening of the ... energy ...

Description

Focusing system

The focusing system has been described.[1] It consists of a 2-meter long solenoid (up to an energy of 55 MeV) and of quadrupole triplets and doublets. The emittance matching between the plane of the target and the solenoid uses an adiabatically tapered solenoid. Recent computations made by M.B. James et al.[2] showed that the transverse momentum accepted by the system remains high ($p_t \leq 0.6$ MeV/c) over a very broad energy band (2.5 MeV $\leq E \leq 30$ MeV). In addition they computed the distribution in energy of the accepted positrons from which one can estimate that approximately one-half of the total number of accepted positrons lies within the energy band from 2 – 6 MeV. To maximize the current of positrons it is then necessary to accelerate the broadest energy band possible, starting from the lowest energy possible. The limitation in that process is the debunching effect in the drift space following the target, and in the first few cm of the acceleration, leading to a broadening of the energy spectrum.

RF System

The distance between the target and the first accelerating section had been set rather large (93 cm) mainly to provide space for collimators. It was decided last year to insert in that space a special short section, Fig. 1. This section is a 5 λ long (52.5 cm) constant im-
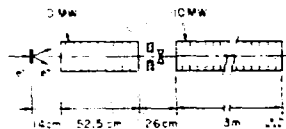
Fig. 1.
Positron source accelerator structure.

pedance structure, the group velocity is vg/c = 0.0204, the accelerating field is related to the input power by E (MeV/m) = 3.35 $\sqrt{P}$ (MW).

The distance between the target and the first section is then reduced to 14.2 cm. The section is followed

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... in the drift space from the target ... the beam is debunched because of different ... different velocities ... of positrons with different ...

... of the time length of the drift ... slippage, with respect to a wave at the ... of light, is given by

$$\delta = \frac{1}{\beta} - 1$$

where β is the particle velocity ... in ... of wavelengths. The initial momentum is also

$$p_z^2 = p^2 - p_t^2 ...$$

where p_t is the transverse momentum ... $p = E/c^2$ ... The axial velocity in ... tapered field region the transverse momentum ... cording to:[1]

$$p_t(z) = p_{t_0} \left(\frac{B(z)}{B_0} \right)^{\frac{1}{2}}$$

where p_{t_0} and B_0 are the transverse momentum ... netic field on the target. The phase slippage ...

$$\delta(\gamma_{o1}, p_{to}) = \int_0^D \frac{z}{\lambda} \left(1 - ... \right)$$

with $p_t(z)$ given by Eq. (2), which can be approximated ...

$$\delta = -\frac{1}{2} p_{to}^2 \frac{1}{B_0} \int_0^D B(z)dz - ...$$

The total longitudinal phase space can be calculated for a distance D and a range of energy and transverse momentum, for a given law of magnetic field $B_{(z)}$. Fig. ...
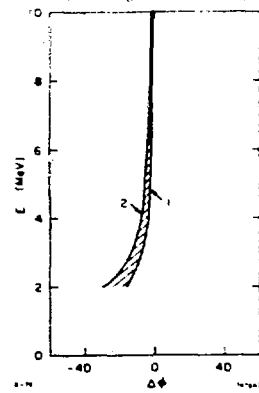
Fig. 2. Longitudinal phase space after a 14.2 cm drift. Curve 1 for p_{to} = 0; Curve 2 for p_{to} = 0.6 MeV/c.

when the positron ... and if the ... accelerates to ... MeV ... maximum transverse momentum in the target ... MeV ... The total ... drift ... MeV ...

(several lines illegible)

... solution for the real field

$$ \cdots \cdots \cdots \cdots \cdots \cdots \frac{1}{r^2} \qquad (4) $$

where ... ... the total gain per wavelength in ... units ... ... the asymptotic phase. The ... is ... that the electric field is ...

... if ... that there exists a relation between phase and ... as the beam enters the accelerator, which can be written, using Eq. (3):

$$ \phi_a = \phi_0 + \cdots \left(\phi - \frac{\cdots}{\cdots^2} - 1 \right) \qquad (5) $$

ϕ_0 being a parameter which represents the input phase of the field for a 1 positron.

By using Eqs. (4) and (5) one can compute optimum values for the parameters ϕ, D and ϕ_0 in order to obtain a minimum dispersion in asymptotic phase for a given range of ... The asymptotic phase ϕ_a must be chosen near $-90°$ in order to obtain the maximum acceleration. The orbits ϕ_a, ϕ resulting from Eq. (4) are symmetric with respect to $\phi = 0$. The positive values of ϕ_0 (decelerating field) give a positive slope $d\psi_a/d\psi_0$. Eq. (5) gives a positive slope $d\psi/d\psi_0$. The best fitting between relations (4) and (5) will then be obtained when the beam enters the accelerator in a decelerating field. (The optimum value of ϕ_0 will be near $90°$ for $\phi_a = -90°$). This fact can be easily understood by looking at Fig. 2 and comparing it with the general shape of the phase space orbits in Fig. 3.

As an illustration of this bunching process, Fig. 4 shows the phase space resulting from the drift space of 14.2 cm for energies from 2 MeV to 10 MeV, (solid curve) together with the orbits $\phi_\infty = -85°$, $-90°$ and $-95°$ for an accelerating field of 12 MeV/m (dotted curves) and $\phi_0 = 86°$ (decelerating field). It is seen that the
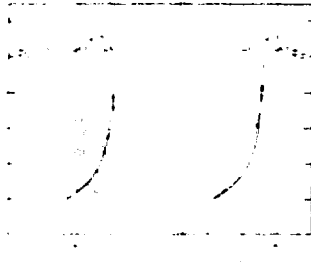
Fig. ... distribution phase space of a frame of ... accelerator section.

... ... that a ... effect is achieved within less than ... and ... to ... a dispersion of ... in the phase ... ... and that is less than ... on the other ... has ... as the same phase space for the beam ... with the orbits $-75°$, $-80°$, $-90°$ and $-100°$ in ... ... accelerating fields. It is evident that the dispersion in asymptotic phase will be greater ... ... By integrating the equations of motion over a distance of ... gives a total bunch length of $30°$, the total energy varies from 47 MeV to 49 MeV.

It appears that this bunching process is very efficient for an initially bunched beam, which contains particles within a large energy band extending down to a very low energy, and which has been debunched under the action of different velocities over a drift space.

2. **Positrons With Transverse Momentum** – If we assume that there exists no other transverse force than the focusing effect of the dc magnetic field, then the transverse momentum remains constant in a constant focusing field, or varies according to Eq. (2) if the magnetic field varies slowly. The equations of longitudinal motion can be written in that case.

$$ \begin{cases} d\psi = a \sin\phi \, dz \\ d\phi = 2\pi \left(1 - \frac{\cdots}{(\gamma^2 - 1 + p_t^2(z))^{\frac{1}{2}}} \right) dz \end{cases} \quad z \text{ in units of } \lambda $$

For constant p_t these equations can be combined and integrated, giving

$$ \cos\phi_0 - \cos\phi_\infty = \frac{2\pi}{\lambda} [\gamma_0 - (\gamma_0^2 - 1 - p_t^2)^{\frac{1}{2}}] \qquad (6) $$

which defines new orbits in the phase space E, ϕ. Let us consider two positrons of the same energy γ_0, one moving on the axis and one leaving the target with a transverse momentum p_{t0}. Let ϕ_{01} be the entrance phase for the first one and $\phi_1 = \phi_{01} - \delta\phi$ the phase for the second one, where

$$ \delta\phi = \Delta\phi(\gamma_0, 0) - \Delta\phi(\gamma_0, p_{t0}), $$

where $\Delta\phi$ is given by Eq. (3). In the approximation of small angle of emission one can write

$$ \delta\phi = \frac{\pi}{\lambda} \frac{p_{t0}^2}{\gamma_0^2} \frac{1}{B_0} \int_0^D B \, dz $$

which can also be written

$$ \delta\phi = \frac{\pi}{\lambda} \frac{p_t^2}{\gamma_0^2} \cdot K $$

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where p_t is the transverse momentum in the section and

$$K = \frac{1}{B_2} \int_0^D B_{(z)} \, dz,$$

B_2 being the magnetic field in the section, which is sup-
posed to be constant. Eq. (4) can be approximated by:

$$\cos z_o - \cos z_w = \frac{2\pi}{\lambda} \left[z_o - (v_o^2 - 1)^{\frac{1}{2}} + \frac{1}{2} \frac{v_o^2}{(v_o^2-1)} z_o \right] \quad (6b)$$

Using Eqs. (4) and (6b) together with the expression for
z_o the dispersion in asymptotic phase is given by:

$$z_o = \sin z_{i,o} = \frac{1}{2} \left[k \sin z_{i,o} - \frac{1}{(v_o^2-1)} z_o \right] \quad (7)$$

It is seen that one can choose the accelerating
field and input phase, for a given energy, to make the
asymptotic phase independent of the transverse momentum.

It is evident from Eq. (7) that z_o must be posi-
tive, i.e., the beam must be decelerated. The accelerat-
ing field E' and input phase are simply related to the
energy E by:

$$E' \sin z_{i,o} \approx \frac{E}{K}$$

In the case of SLAC, K is of the order of 0.45 m.
For $E_o = 2$ MeV, we must choose $E' \sin z_{i,o} \approx 4.4$ MeV/m.
The equations of motion have been integrated over a dis-
tance of 3 m for $E_o = 2$ MeV, $E' = 10$ MeV/m, $p_{to} = 0$ to
0.6 MeV/c, and $z_{o,i} = 25°(z_o$ in Eq. (5) $= 40°)$. The re-
sulting phase dispersion is less than 1°, for an input
dispersion of 15°.

This bunching process could be very useful in the
case of a high transverse acceptance over a narrow energy
band centered on a low energy (use of a quarter wave
transformer).

We can conclude that, by first decelerating the
positron beam, one can take advantage of using very low
energy and high acceptance (in our case the angular ac-
ceptance at 2 MeV is 18°) together with large energy
band, leading to a maximum current of positrons within
a narrow energy spectrum. The limitation may be the
initial energy dispersion in the case of a low final
energy.

Results of Calculations in the Case of SLAC

The first section is a very short one, followed by
a drift space (Fig. 1). With an accelerating field of
the order of 10 MeV/m it is evident that one cannot take
full advantage of the bunching process described above,
mainly because the minimum energy at the end of the first
section must be high enough to avoid a debunching effect
over the second drift space. We have computed the longi-
tudinal motion for several values of the parameter z_o
(see Eq. (5)) and of the accelerating field. Fig. 5
shows the resulting phase space at the entrance of the
second section in the cases $\phi_o = 70°$ and $30°$ (beam ini-
tially decelerated) and $\phi_o = -70°$ (beam accelerated),
and a field of 10 MeV/m. (For each case, the two solid
curves with cross hatching between them represent the
phase space for positrons with initial transverse momenta
between 0 and 0.6 MeV/c.) The effect of the second drift
space is clearly seen in the case $\phi_o = 70°$: the minimum
energy is about 1 MeV and the phase space is completely
distorted. In the case $\phi_o = 30°$ the minimum energy is
2.5 MeV and the distortion of the phase space is much
less important. The orbits for $\phi_o = -40°$ and $\phi_o = -50°$
are shown on the same scale for an accelerating field of
10 MeV/m (the second section is long enough to consider
at a first approximation the asymptotic behavior). The
fitting between these orbits and the phase space of the

Fig. 5. Longitudinal phase space at the entrance of
the second section; $z_o = 2$ MeV ... positrons
entering with $\phi_o = -70°$ and ...
$\phi_o = +70°$ and two ... and $\phi_o = +30°$ ...

beam is reasonably good and one can expect that the bunch
length of less than 10°. In addition it can be seen that
the bunching of low energy positrons with transverse
momentum from 0 to 0.6 MeV/c has been achieved in the
short section.

In the case $\phi_o = -70°$ (beam accelerated) no bunch-
ing effect has occured, the total bunch length is 48°
and no further bunching can occur in the second section.
The phase space at the end of the second section has been
computed for various values of the accelerating field,
input phase and different phase space configurations re-
sulting from the first section. Fig. 6 shows on the

Fig. 6. Phase space at output of second section:
Left: New section in use.
Right: Without new section.

left the result in the case of the phase space shown in
Fig. 5 for $\phi_o = 30°$. The input phase for the second
section is $\phi_o' = 50°$ (phase of the field for a $z = 1$
particle). The average output phase is -45°, giving a
total phase slippage for the beam of about 95°. The
bunch length is 6° and the energy dispersion 0.5 MeV.
Shorter bunches can be obtained at the expense of final
energy and energy dispersion. The transition between the
solenoid and the quadrupole must take place at an energy
of at least 55 MeV in order to keep an admittance of
0.15 π MeV/c.cm[1]. With the phase space of Fig. 4, add-
ing a 3 m-long section with the same field gives an
energy from 44 MeV to 54 MeV. We have investigated two

ways of increasing the energy without perturbing the
bunching process, for the same accelerating field:

- Retuning of three quarters of the long section
  at half-emfromits gradient, giving a phase
  shift of 270° ± 45° in places the output
  energy is about 5 MeV, within a bunch length
  of 5°.

- Inverting the temperature in the section by
  45°, it increases the output energy, given
  that azimuth the energy shift.

These two approaches are in very good agreement
and give a total slippage over the output meter of the
beam ... with the calculated values.

... (illegible lines) ...

- Influence of the B_ρ and B_ϕ components of the
  field.

When positrons begin leaving their motion
along the axis of the wave, in a region where $zB_\rho \neq 0$,
they will experience a defocusing force due
to the axial and radial components of the field. The radial
equation of motion can be written [6]:

$$\frac{dr}{dz} = r\left[\frac{E}{2E} - r\frac{B_{ext}^{2}}{4E_0}\right]\cos\tau - \frac{B_{ext}^{2}}{4E_0}\right] \quad (8)$$

where r = distance from the axis, E = accelerating field
in units of 10 MeV/m, and B_{ext} is the dc magnetic
field in units of $m c^2/e \cdot cm$ (= 1704 gauss). The first
term, which is due to the rf fields, can have a magni-
tude about 1/4 the magnitude of the second term for
distances of a few centimeters when a low energy posi-
tron passes through zero phase.

Longitudinal Motion - When we studied
the bunching of positrons of the same energy and dif-
ferent transverse momentum, the effect of the axial
force resulting from the interaction of radial velocity
and azimuthal magnetic field was neglected. The com-
plete equation of motion can be written in the case of
the fundamental above [7]:

$$\Delta\gamma = -q\left[\sin\tau - \frac{2\pi}{\lambda}\frac{r r_z}{v_z}\cos\tau\right] dz \quad (9)$$

This equation has been integrated in the case mentioned
before (E_0 = 2 MeV, E' = 10 MeV/m, τ_0 = 25°, P_{t0} = 0 to
0.6 MeV/c) for various values of the initial azimuth
angle. It was found that the maximum output phase
variation is about 2°.

Comparison with the Present Mode of Operation

In the present mode of operation, the short sec-
tion is not used. The rf phases of the linac are not
changed from electron to positron acceleration. The
positron beam is then decelerated in the 3 m long sec-
tion. The phase space at the end of the section was
computed for different values of the input phase and
field which give a total phase slippage for the beam
of about 180°. Fig. 6 shows the resulting phase space
for a field of 10 MeV/m and an input phase of ϕ_0 = 80°.

Positrons with energies from 4.5 MeV to 10 MeV are bunched
within approximately 10°. Positrons from 2 to 4 MeV are
far away in phase and will not contribute to the useful
beam (within 12 energy bins).

Results of Experiment

The short section was used in June 1979 for an ex-
periment with positron beams of final energy 1.6 GeV and
20 GeV. The analyzed current within 1% was increased
in both cases by at least a factor of two with respect
to the usual current. The optimum shift for the rf
downstream of the target, versus electron operation was
found to be about 60°, giving a phase slippage for the
beam of about 190°. These results are in good agree-
ment with the calculated values. The short section will
be put into normal operation next October with 360° pulsed
phase shifters for the klystrons upstream the converter.

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