Perturbative QCD: An Overview*

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Recent progress in the utilization of perturbative quantum chromodynamics to study lepton-hadron and hadron-hadron reactions is reviewed. Interest is focused on the connections between the various results and the implications for the parton model as a phenomenological tool and as a vehicle to test quantum chromodynamics.

The following discussion is intended to serve as an overview of the recent progress made in using perturbative quantum chromodynamics (QCD) as a phenomenological tool for studying lepton-hadron and hadron-hadron reactions. It is also intended as an introduction to and interpolation between the more specialized companion papers.¹⁻⁵

The basic framework for perturbative QCD arises from a purely pre-QCD notion, the parton model.⁶ In this picture the constituents of hadrons, the partons (originally taken to be only quarks), are assumed to suffer confining interactions which are characterized by a (long) time constant of order

$$\tau_L \sim 1/m_h, \quad (1)$$

where $m_h \sim$ a few hundred MeV is characteristic of hadron sizes (and masses). Reactions characterized by a momentum Q larger than a few GeV and a correspondingly short time scale

$$\tau_S \sim 1/Q, \quad (2)$$

($\tau_S \ll \tau_L$) as, for example, in deep inelastic lepton-hadron scattering with Q being the large momentum transfer, may be treated by an impulse approximation scheme. In such hard interactions the partons are considered as essentially free and "on shell" (i.e., with fixed small mass as $|Q^2| \rightarrow \infty$) and the full process is assumed to factorize into a convolution of the probability of finding a parton within a hadron with

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the cross section for the hard process involving the parton. In the "classic" example of lepton-hadron scattering, illustrated in Fig. 1a, the hard process involves the large angle elastic scattering of a lepton and quark. Originally⁶ the distributions of partons within hadrons were assumed to be functions of only the fraction x of the hadron's momentum carried by the parton ($x = p_q/p_h$, the scaling hypothesis) and to be nonzero only for partons collinear with the parent hadron. The parton (quark) distribution $F(x)$ could then be experimentally determined in ep scattering, for example. On the long time scale τ_L , the scattered partons interact and fragment into the observed hadrons. In the naive parton model the resulting hadrons are assumed to be aligned with the parent parton in momentum space and to be described by a fragmentation function $D(z)$, depending only on the ratio z of the hadron's momentum to that of the parent parton ($z = p_h/p_q$). These fragmentation functions can be determined from the analysis of the annihilation of e^+e^- into hadrons as illustrated in Fig. 1b. Again for this process factorization is assumed such that the quark and antiquark fragment independently. Thus a cross section involving such a hard process assumes a general integral form schematically given by

$$d\sigma_{\text{hadron}} = \sum_{i,j} \int \int dx dz F_i(x) d\sigma_{ij}(x,z,Q^2) D_j(z) \quad (3)$$

(quarks)

All of the soft, long-time, confining effects reside in the distributions $F(x)$ and $D(z)$ while $d\sigma_{ij}$ contains all the hard, short-time processes.

While deep inelastic lepton-hadron scattering and the total cross section for e^+e^- annihilation are amenable to more orthodox analyses⁷ in terms of the short distance behavior of an operator product expansion, this is not the case for the distribution of hadrons in the final state and the description of this distribution represents a true application of the underlying simple (naive) physical picture of the parton model. This picture was further extended to describe the hadronic production of massive lepton pairs⁸ via quark-antiquark annihilation, as in Fig. 1c, and to large p_T hadronic production⁹ via quark-quark scattering, as in Fig. 1d. Corrections to the naive picture⁶ were presumed to be characterized by the ratio $\langle k_T^2 \rangle / Q^2$, where $\langle k_T^2 \rangle$ is the average squared transverse momentum of quarks in hadrons or hadrons in quarks (jets), assumed originally to be a (few hundred MeV)². This measure essentially characterizes the "size" of hadrons or the time scale τ_L . Hence the corrections were of approximate magnitude τ_S^2 / τ_L^2 .

Within this parton model framework it is now possible (and necessary) to study the implications of QCD as the theory of the strong interactions. In the context of QCD, hadrons contain not only quarks but also vector gluons which mediate the interactions between the quarks and, unlike QED, between

themselves. Both these new types of partons and the new interactions they imply must be included in the parton model.

This is accomplished by assuming that the naive parton model constitutes a zeroth order approximation, calculating perturbative corrections to it, and then demonstrating that the result is consistent with the initial assumption. Typical corrections are illustrated in Fig. 2. Note that there are not only truly perturbative corrections due to gluon emission and exchange contributions to the original diagrams of Fig. 1 but also totally new terms arising from gluon initiated reactions.

This program is at first very attractive due to the fact that QCD exhibits asymptotic freedom.¹⁰ Simply stated, the effective coupling of the theory decreases with shorter distances and times or larger momenta as

$$\alpha_s(Q^2) \underset{Q^2 \rightarrow \infty}{\approx} \frac{\text{const.}}{\ln(Q^2)} \quad (4)$$

This renormalization of the coupling already represents the summation of a certain subset of perturbative corrections. Furthermore it suggests that the remaining perturbative corrections will be small at large Q^2 . However, the theory also exhibits mass singularities arising from both the collinear fragmentation of, for example, a quark into a quark and a gluon and from the true infrared singularities due to soft gluon emission. This feature of the theory is realized in the perturbation approach as factors of $\ln(Q^2/m_q^2)$ essentially

$\ln(\tau_L/\tau_S)$ which serve to cancel or overwhelm the logarithmic damping inherent in the asymptotically free coupling $\alpha_s(Q^2)$ (which represents the summation of a different set of $\ln(Q^2)$ terms). Thus the factorization [Eq. (3)] into long time scale effects and short time scale effects which is central to the initial parton picture is threatened. However, recent work of many physicists^{1,11} has led to the realization that even in the presence of the logarithms the soft, long-time, quark mass dependent physics still essentially factors from the hard, short-time physics in a universal fashion. Thus the qualitative structure of the parton model remains basically intact but with certain quantitative modifications.

As in the naive model, the asymptotically leading, "scaling," contribution to a given process corresponds to the hard scattering of "on shell" (massless) quarks and gluons. However, in order to actually perform the factorization of the long time scale physics from the hard physics, an arbitrary scale, call it M^2 , must be introduced. (Care must be exercised to perform this factorization in an identical fashion for all processes.) The result is then a convolution of distributions for quarks and gluons, evaluated at the scale M^2 , with a hard scattering cross section which has a perturbative expansion in terms of $\alpha_s(M^2)$ (for convenience the renormalization point is taken to be also M^2). The hard scattering cross section involves logarithms of both the ratio Q^2/M^2 and the ratios of other large kinematic variables. The schematic form is

$$d\sigma_{\text{hadron}} = \sum_{i,j} \int \int dx dz F_i(x, M^2) d\sigma_{ij}(x, z, \alpha_s(M^2), \theta) D_j(z, M^2) \quad (5a)$$

(quarks, gluons)

where ϵ represents the ratios of other invariants. To insure the rapid convergence of this expression it is useful to choose $M^2 \sim Q^2$ (this is equivalent to using the renormalization group) and thus to express the hard scattering cross section as a series in $\alpha_s(Q^2)$. In this case the distributions will be evaluated at scale Q^2 , giving

$$d\sigma_{\text{hadron}} = \sum_{i,j} \int \int dx dz F_i(x, Q^2) d\sigma_{ij}(x, z, \alpha_s(Q^2), \theta) D_j(z, Q^2) \quad (5b)$$

(quarks, gluons)

For large Q^2 ($\alpha_s(Q^2) \ll 1$) the Q^2 dependence (scaling violation) of the distribution and fragmentation functions is predicted by perturbative QCD and the renormalization group to be as a specific inverse power of $\ln(Q^2)$ [the anomalous dimension]. The present lepton-hadron data are suggestive of this behavior but make no decisive statement, as discussed by L. Abbott³ in this volume.

In the above factorization analysis the actual singularities associated with "on shell" partons have been hidden in the fully renormalized distribution functions which are assumed

to be finite due to nonperturbative confinement physics not calculable by present techniques. It is only the large Q^2 behavior which is amenable to perturbative analysis. It now seems clear that this same noncalculable confinement physics must also induce a nonzero transverse momentum distribution in the functions F and D in addition to that which arises perturbatively¹² from the emission processes illustrated in Fig. 2. Phenomenological studies suggest that the mean of this "primordial" distribution is perhaps as large as 700 MeV/c for quarks in hadrons but more like 350 MeV/c for hadrons resulting from quark fragmentation. Furthermore, applications of this notion, e.g., in $\mu\mu$ pair production or large p_T physics, are sensitive to the shape of this noncalculable distribution. The situation is reviewed in the talks of E. L. Berger⁴ and J. Owens.⁵

In addition to the power behaved (nonscaling) corrections of the form $\langle k_T^2 \rangle / Q^2$ present in the naive parton model, perturbative QCD predicts further corrections essentially of the form $\langle n_q^2 \rangle / Q^2$ which are process dependent and do not factor into a sum of terms of the above $\langle k_T^2 \rangle$ form.

Finally, the perturbative QCD improved parton model exhibits infrared finite i.e., genuinely calculable) contributions¹² not present in the naive model which are both process dependent and asymptotically significant in the sense that they do not vanish as a power of $1/Q^2$. There are, for example,

the gluon initiated processes illustrated in Figs. 2c and 2d. For the pair production process (see E. L. Berger⁴) these contributions may be very important for nucleon-nucleon collisions due to their small antiquark content. In the fragmentation process the emission of gluons leads to large k_T within a fragmentation jet, Fig. 3a, which on purely dimensional grounds can be understood to behave as

$$\langle k_T^2 \rangle_{\text{QCD}} \sim \alpha_s(Q^2) \cdot Q^2 \sim Q^2 / \ln(Q^2) \quad (6)$$

In the large angle limit this process presumably leads to two distinct jets as illustrated in Fig. 3b (and 2b). Fig. 3 clearly indicates that the physics is the same in the two cases and one must be careful when using this jet language not to treat "fat jets" and "double jets" as formally distinct objects.

The successes of this perturbative approach suggest the possibility of performing precise tests to establish whether QCD is indeed the correct theory of the strong interactions. Unfortunately, the nonperturbative, noncalculable contributions discussed briefly above guarantee that, in general, the QCD improved parton model will remain largely a phenomenological tool for the near future. At the same time considerable progress¹³ has been made in the direction of avoiding these difficulties by a judicious choice of what experimental measure to study as discussed in the talk by L. S. Brown.²

To constitute a satisfactory test of QCD such a measure should satisfy the following constraints:

- (a) It should exhibit properties characteristic of QCD, e.g., the presence of both quarks and vector gluons.
- (b) It should be reliably calculable, which at present means entirely by the methods of perturbation theory. In order to avoid the necessity of nonperturbative renormalization of infrared singularities, the quantity should be free of explicit infrared logarithms when evaluated perturbatively. It should also be insensitive to corrections due to such nonperturbative confinement effects as the "primordial" k_T distribution.
- (c) Finally, it should be accessible to experiment. This condition argues against small effects on a large background or measures which are sensitive to missing neutrals. An example of the latter is a measure which requires event by event determination of an axis.

The clear suggestion is to study various inclusive measures of the final state of e^+e^- annihilation into hadrons where there are no initial hadrons and no reference is made to specific final hadrons. The study of calorimetric, i.e., energy weighted, measures appears to avoid the mass singularities from both the collinear and true infrared divergences (the energy-momentum tensor is not renormalized).

Another area of recent and rapid development is the study¹⁴ of methods to actually sum the perturbative expansion of the

hard scattering process discussed above. Since the perturbative expansion typically contains logarithms of ratios of various kinematic variables (the Cheshire cat smile of the mass singularities which have been eliminated by factorization or by a careful choice of the quantity), there are kinematic regions where the logarithms are divergent and thus where a low order expansion is not useful. Successful techniques for summation would serve to enlarge the region of validity of the perturbative approach. A specific example of this approach is discussed in the presentation of L. S. Brown.²

Not discussed in any detail in these proceedings, due simply to lack of time, are the perturbative study of spin effects¹⁵ and the interesting comparisons of the structure of gluon initiated jets versus quark initiated jets.¹⁶

Overall there has been considerable progress in understanding and implementing perturbative QCD. This trend will undoubtedly continue, accompanied by increasing emphasis on the consideration of those quantities which are readily accessible to both theory and experiment. This will in turn result in the clear confirmation (or refutation) of the role of QCD.

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FIGURE CAPTIONS

1. Processes discussed in the parton model:
 - (a) Lepton-hadron scattering
 - (b) e^+e^- annihilation into hadrons
 - (c) Lepton pair production by hadrons
 - (d) Large p_T hadron scattering.
2. Typical QCD corrections to the naive parton model description of the processes in Fig. 1. Individual processes are as labeled in Fig. 1.
3. Two descriptions of the result of energetic gluon emission in the fragmentation process.

Figure 1

Figure 2

(a)

(b)

Figure 3