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VERTICAL AXIS WIND TURBINE DEVELOPMENT

Final Report, March 1, 1976—June 30, 1977

By
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FOREWORD

The research described in this report was performed for the United States Energy Research and Development Administration (ERDA), with funding by Contract E(40-1)-5135. This report details progress from March 1, 1976 to June 30, 1977. Dr. Robert Thresher, ERDA Division of Solar Energy, was the Technical Monitor for the project; Mr. R. A. Fahey, Oak Ridge, Tennessee, was the ERDA Contracting Officer.

The research was concerned with investigation of concepts for two innovative wind machines which can convert wind energy to more useful forms: a vortex concentrator device, and a vertical axis wind turbine operating with circulation controlled airfoils. The purpose of the work was to investigate the technical feasibility of these concepts, both theoretically and experimentally, and to evaluate the structural and economic aspects of the designs. This report describes the vertical axis wind turbine development. Results for the vortex concentrator device are given in the following report:

Loth, J. L., Ness, N., and Yen, R.-H., "Vortex-Type Wind Machines; Final Report", ORO/5135-77/6, Department of Aerospace Engineering TR-59, West Virginia University, July, 1977.

A listing of all presentations, reports, and theses which have been accomplished by workers at WVU concerning wind energy is given in Appendix A.

Work was accomplished by several members of the staff and graduate students of the Department of Aerospace Engineering of West Virginia

University. Portions of this work were performed by personnel at the Allegany Ballistics Laboratory (ABL) of Hercules, Incorporated. Dr. P. Ward Hill was in charge of the ABL effort. The efforts of individuals in the preparation of sections of this report is acknowledge in the CONTENTS section; the Project Manager, Richard E. Walters, was the report editor. Mrs. Cheryl Fries typed the manuscript.

Editor's Note: Administration of this continuing project was transferred from D.O.E. to the Solar Energy Research Institute (SERI), Golden, Colorado, in 1978. Dr. Irwin Vas, SERI Program Manager, provided the final approval for this research report.

SUMMARY

This report describes theoretical and experimental research accomplished in evaluating an innovative concept for vertical axis wind turbines (VAWT). The concept is that of using straight blades composed of circulation controlled airfoil sections. The theoretical analysis has been developed to determine the unsteady lift and moment characteristics of multiple-blade cross-flow wind turbines. To determine the drag data needed as input to the theoretical analysis, an outdoor test model VAWT has been constructed; design details, instrumentation, calibration results, and initial test results are reported herein. Initial testing was with fixed pitch blades having cross-sections of conventional symmetrical airfoils. Future tests will include tests of symmetrical-airfoil blades having higher solidity, cyclic changes of blade pitch, and the use of circulation controlled airfoil sections. Costs of building the test model are included, as well as cost estimates for blades constructed with composite materials. These costs are compared with those of other types of wind turbines.

TABLE OF CONTENTS

FOREWORD	iii
SUMMARY	v
TABLE OF CONTENTS	vi
LIST OF FIGURES	vii
LIST OF TABLES	x
1. INTRODUCTION - R. E. Walters	1
2. NUMERICAL SOLUTION FOR THE UNSTEADY LIFTING CHARACTERISTICS OF VARIABLE PITCH CROSS-FLOW WIND TURBINES - P. G. Migliore, J. B. Fanucci, and W. Squire	8
3. BOUNDARY LAYER CONSIDERATIONS FOR A VERTICAL AXIS WIND TURBINE - J. B. Fanucci	40
4. WVU VAWT OUTDOOR TEST MODEL - R. E. Walters and D. G. Elko . . .	52
5. LOW SOLIDITY BLADE TESTS - P. G. Migliore and R. E. Walters . .	130
6. HIGH SOLIDITY BLADE DESIGN - G. M. Palmer and R. E. Walters . .	144
7. COST ANALYSIS OF THE WVU VAWT TEST MODEL - R. E. Walters	167
8. STRUCTURAL PARAMETRIC ANALYSIS OF VAWT BLADES - P. W. Hill . . .	178
9. COST STUDY OF CURRENT WECS - R. E. Walters	215
10. CONCLUSIONS - R. E. Walters	223
11. RECOMMENDATIONS - R. E. Walters	224
APPENDICES	
A. WVU PRESENTATIONS AND PUBLICATIONS REGARDING WIND ENERGY . .	225
B. SYMBOL DEFINITION FOR BLADE STRUCTURAL ANALYSIS, SECTION 4 .	228
C. COMPUTER PROGRAM FOR UNIFORM BLADE FORCE CALCULATIONS, SECTION 4	230
D. COMPUTER PROGRAM FOR BLADE MOMENTS, STRESSES, AND DEFLEC- TIONS, SECTION 4	233

LIST OF FIGURES

<u>Figure Number</u>		<u>Page</u>
1.1	Cross-section of circulation controlled airfoil	5
1.2	Calculated effect of circulation controlled blades on vertical axis wind turbine performance	6
1.3	WVU test model vertical axis wind turbine	7
2.1a	Coordinate systems	23
2.1b	Four-vortex model of the lifting surface and the shed vortex model of the wake	23
2.2	Three bladed turbine. (For numerical convenience it is assumed that each blade advances through $\Delta\theta$ inde- pendently of the other blades.)	24
2.3a	Determination of bound and shed vortex strengths for blade number 1 in the first time step	25
2.3b	Determination of bound and shed vortex strengths for blade number 2 in the first time step. (Blade 2 is influenced by the vortices "previously formed" at blade 1.)	26
2.3c	Determination of bound and shed vortex strengths for blade number 3 in the first time step. (Blades 1 and 2 influence the vortices formed at blade 3.)	27
2.4	Calculated power coefficient vs. number of rotor revolutions	28
2.5	Calculated power coefficient (C_p) vs. tip speed ratio (TSR), for a two bladed VAWT	29
2.6	Calculated VAWT performance, (C_p) vs. tip speed ratio (TSR), straight blade, no pitch, 3 blades, $C/R = 0.08$, $\sigma = 0.12$	30
2.7	Calculated VAWT performance, C_p vs. TSR, straight blades, no pitch; 2 blades, $C/R = 0.12$, $\sigma = 0.12$	31
2.8	Induced drag used for C_p estimates	32
2.9	Calculated VAWT performance, C_p vs. TSR, straight blade, no pitch; various blade numbers, C/R ratios, solidities, and aspect ratios	33
3.1	Geometry of inertial and noninertial coordinate systems	50
3.2	Curvilinear blade surface coordinate system	51
4.1	WVU VAWT test model	93
4.2	Stand with generator, starter motor, and instrumenta- tion controls	94
4.3	Test model airfoil cross-section.	95
4.4	Diagram of forces on the blade	96
4.5	The blades supported by the two struts	97
4.6	Blade with two simple supports and pre-tension cables .	98
4.7	Blade with three simple supports and pre-tension cables	99
4.8	Force body diagram of blade and its two equivalents . .	100

LIST OF FIGURES (Continued)

<u>Figure Number</u>		<u>Page</u>
4.9	Moment and deflection diagrams.	101
4.10	VAWT rotor with strain gage and load cell locations .	102
4.11	Load cells with gage orientations	103
4.12	Shaft gage configuration.	104
4.13	Blade gages and their placement	105
4.14	Calibration curve for load cell #1.	106
4.15	Torque load cell calibration	107
4.16	Rotor static load test results for load cell on ten- sion cable L-2.	108
4.17	Rotor static load test results for load cell on ten- sion cable L-3.	109
4.18	Rotor static load test results for load cell on cen- ter support wire L-5.	110
4.19	Required force for blade pitch change vs. blade loading	111
4.20	Rotor static load tests for blade strain gage B-3 . .	112
4.21	Rotor static load tests for blade strain gage B-2 . .	113
4.22	Rotor static load tests for blade strain gage B-6 . .	114
4.23	Rotor static load tests for blade strain gage B-4 . .	115
4.24	Rotor static load tests for blade strain gage B-5 . .	116
4.25	Rotor static load tests for blade strain gage B-8	117
4.26	Blade stresses at 24 in from the end of the blade (0 lb pre-tension load).	118
4.27	Blade stresses at 62 in from the end of the blade (0 lb pre-tension load).	119
4.28	Blade stresses at 176.4 kg (338 lb) uniform load and 0 kg (0 lb) pre-tension load.	120
4.29	Blade stresses at 168 kg (369.5 lb) uniform load and 227 kg (500 lb) pre-tension load.	121
4.30	Blade stresses at 768.4 kg (370.75 lb) uniform load and 455 kg (1000 lb) pre-tension load	122
4.31	Rotor tests for blade strain gage B-4	123
4.32	Rotor tests for blade strain gage B-8	124
4.33	Cup anemometer calibration.	125
4.34	Hot wire anemometer calibration	126
4.35	Wind speed calculation chart for cup and hot wire anemometers with corrected pressure and temperature .	127
4.36	Sample data showing cup anemometer outputs (1 and 2), hot wire output (3), torque (4), and RPM (5).	128
4.37	WVU VAWT outdoor test site with anemometer installa- tion.	129

LIST OF FIGURES (Continued)

<u>Figure Number</u>		<u>Page</u>
5.1	Typical test results for VAWT support structure counter torque (tare torque)	139
5.2	Plot illustrating the method of obtaining net blade torque from VAWT test data	140
5.3	Experimental VAWT blade power coefficients at various blade rotational speeds	141
5.4	Coordinate system for VAWT blade	142
5.5	Drag coefficients of small chord NACA 0015 airfoil from torque characteristics of VAWT outdoor test model	143
5.6	Minimum drag coefficients for a NACA 0015 airfoil from wind tunnel and WVU VAWT outdoor test model experiments	143
6.1	High solidity blade under construction	159
6.2	Completed high solidity blade with tail and counter-weights	160
6.3	High solidity blades being installed on WVU VAWT test model	161
6.4	High solidity blades installed on WVU VAWT test model.	162
6.5	Test data extrapolation to find inertia, I_{yy}	163
6.6	Response angle vs. angular velocity	164
6.7	Blade angle vs. torque for unbalanced blade	165
6.8	Tail force vs. orbit angle at various tip speed ratios, $\omega = 60$ RPM	166
8.1	Description of fixed blade rotor geometry ($\alpha_0 = 90^\circ$)	197
8.2	Angle of attack vs. orbit angle Darrieus mode operation	198
8.3	Full range section lift coefficients for the NACA 0015 airfoil at Reynolds numbers of 0.36×10^6 , 0.50×10^6 and 0.68×10^6	199
8.4	Full range section drag coefficients for the NACA 0015 airfoil at Reynolds numbers of 0.36×10^6 , 0.50×10^6 and 0.68×10^6	200
8.5	Maximum normalized load coefficients vs. tip speed ratio	201
8.6	Blade structural analysis model	202
8.7	Design criteria	203
8.8	Blade mass/unit length and blade cost/unit length vs. blade length for S2-glass composites (normal operating conditions)	204
8.9	Blade mass/unit length and blade cost/unit length vs. blade length for E-glass composites (normal operating conditions)	205

LIST OF FIGURES (Continued)

<u>Figure Number</u>		<u>Page</u>
8.10	Blade mass/unit length and blade cost/unit length vs. blade length for graphite composites (normal operating conditions).	206
8.11	Blade mass/unit length and blade cost/unit length vs. blade length for Kevlar-49 composites (normal operating conditions).	207
8.12	Blade mass/unit length and blade cost/unit length vs. blade length for 2024-T6 aluminum (normal operating conditions).	208
8.13	Comparison of blade mass/unit length vs. blade length for composites and aluminum (normal operating conditions) with $\omega R/V_m = 4.0$	209
8.14	Blade mass/unit length (gust conditions) vs. blade mass/unit length (normal operating conditions)	210
9.1	Initial cost per unit output vs. turbine capacity of wind turbines.	221
9.2	Initial cost per unit output vs. turbine output efficiency	222

LIST OF TABLES

<u>Table</u>		<u>Page</u>
2-I	History of important kinematic parameters.	34
2-II	Aerodynamic loads for blade number 1 in revolution number 9	35
2-III	Power coefficients for blade number 1 in revolution number 9	36
2-IV	Summary of power coefficients for all blades in revolution number 9.	37
2-V	Key to symbols used in Tables 2-I to 2-IV.	38
7-I	WVU VAWT test model cost summary	173
7-II	WVU VAWT test model cost breakdown	174
7-III	Comparison of some hybrid and composite designs for straight VAWT blades	177
8-I	Nominal static material properties, candidate blade materials.	211
8-II	Degradation to various environments.	212
8-III	Design allowables - Static Criteria (2σ low, 50°C , 10^5 hours.	213
8-IV	Design allowables - Dynamic Criteria (2σ low, 50°C , 10^8 cycles).	214
9-I	Characteristics of some wind turbines	220

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VERTICAL AXIS WIND TURBINE DEVELOPMENT

FINAL REPORT, MARCH 1, 1976 - JUNE 30, 1977

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1. INTRODUCTION

In the continuing effort to evaluate various ways to obtain more energy, wind turbines are under extensive study. The many types of turbines and other machines under consideration are in different stages of development. Some, such as the horizontal-axis "propeller-type" turbines, are relatively well developed from a design standpoint. Other types are much newer in their concepts and development procedure. The wind turbine discussed in this report, the circulation controlled vertical axis wind turbine (CC VAWT), is one of the newer types being investigated.

This concept is that of a vertical axis wind turbine which uses straight blades composed of airfoil shapes having high efficiency, i.e., high lift to drag ratios. This would be attained by using circulation controlled airfoils for the blades; these airfoils contain slots near the rounded trailing edges through which a small amount of compressed air is blown to obtain high lift forces. Straight blades allow cyclic pitch control, and also locate each blade element at a large radius from the shaft so that the maximum rotor torque is produced. The vertical axis design allows extraction of power from winds of varying directions with only minor adjustments of the blade pitching and air blowing cyclic schedules. Fig. 1.1 shows a cross sectional view of a circulation controlled airfoil.

As evaluated by using strip theory, the qualitative effect of

using CC blades is shown in Fig. 1.2. These initial calculations showed that in addition to increasing the aerodynamic efficiency of the blades which in turn increases the power obtained, the turbine with CC blades reaches peak efficiency at a lower rotational speed than one with conventional blades. The decrease in rotational speed is important, since it greatly reduces the critical blade centrifugal forces. However, lower turbine speeds are not generally desired for electrical generation, since greater transmission costs and losses would be anticipated. A more advanced and realistic aerodynamic model has now been developed for calculating the power output of the turbine, and is described in this report.

In the final evaluation of a wind powered energy conversion system (WECS), it is well recognized that the "bottom line" is simply the total costs of installation and operation per unit of power obtained over a certain period of time. Unfortunately, to obtain this information requires several intermediate steps involving knowledge of the WEC's initial cost, operating cost, efficiency, operating lifetime, and the wind intensity duration characteristics at the installation site. The machine size, aerodynamic and structural design, and construction materials used, are all important factors. In attempting to evaluate these factors for the CC VAWT, it was found that several basic "tools" had to be developed.

Specifically, the aerodynamic theory utilizing a non-steady lifting surface theory was required to calculate the real effects of finite blade size, true blade shed-vortex/blade interactions, and blade pitch change. This theory development has been approached in steps starting with a) one conventional blade (symmetrical airfoil)

having no pitch change and subsequently adding, b) multi-blades, c) variable pitch, and finally, d) CC blade calculations. Steps a and b are reported herein, and some parametric calculations are included.

Another factor which is under theoretical consideration is that of boundary layer flow on the turbine blades. Initial formulation of the flow equations gives results which suggest that the high centrifugal forces present on the blades during their circular motion may cause important changes, and perhaps premature separation, of the boundary layer flow.

For use with the theoretical aerodynamic calculations, it was necessary to provide input data for the blade and rotor drag coefficients. This requirement led to the development of a test model VAWT at WVU (Fig. 1.3). The primary purpose of this test model is to provide accurate aerodynamic data for computational use. However, the construction and testing of this turbine allows realistic cost analysis, hardware and detail design experience, and operating experience to be obtained which would not otherwise be possible. It is believed to be an important part of the real-world process necessary to develop a practical CC VAWT. Initial testing is with conventional blades, to be followed with higher solidity and CC blades. Mechanisms to change the blade pitch and to supply blowing air to the blades are also under investigation. Some initial results of tests with low solidity conventional blades (0015 airfoil) are reported herein.

The remainder of this report consists of the following parts:

section 2 details the non-steady lifting surface theory development, while section 3 describes the formulation of the boundary layer equations. The outdoor VAWT test model and initial operational results are described in sections 4 and 5, and section 6 contains the design for the high solidity blades. Section 7 concerns the cost of the WVU test model, while section 8 contains structural parametric analysis of VAWT blades for larger machines. Section 9 includes summaries of the costs of other WECS designs of the past and present. Sections 10 and 11 present the conclusions and recommendations, respectively.

Fig. 1.1. Cross-section of circulation controlled airfoil.

Fig. 1.2. Calculated effect of circulation controlled blades on vertical axis wind turbine performance.

Fig. 1.3. WVU test model vertical axis wind turbine.

2. NUMERICAL SOLUTION FOR THE UNSTEADY LIFTING CHARACTERISTICS OF VARIABLE PITCH CROSS-FLOW WIND TURBINES

Introduction

A numerical procedure has been developed for determining the unsteady lift and moment characteristics of multiple-blade cross-flow wind turbines. The method employs a force-free wake model, accounts for wake/blade interaction and reflects transient aerodynamics. Introduction of experimental drag data will permit calculation of the energy extraction capacity for such devices.

The basic theoretical approach was reported^{2.1} in September, 1975. The mathematical model was conceptually simple, but development of the computer codes was predicted to be demanding and tedious. The first stage of analysis was therefore confined to the single bladed rotor with emphasis on investigating the effect of computational variables and scrutiny of preliminary results.

This single-bladed analysis was completed in February, 1976, and produced some fascinating results. Of particular interest are the lift coefficient (C_L) transients, the effect of wake-blade interaction on shed vortex trajectories, and observation of an expanded zone of influence upstream of the rotating turbine. Reference 2.2 summarizes the preliminary results and presents relevant details of the methodology. Much of this material was presented^{2.3} at the 11th

Intersociety Energy Conversion Engineering Conference, Lake Tahoe, Nevada, on September 14, 1976.

This report outlines the progress of the extension of the analysis to turbines with multiple blades. During this time computer codes have been modified to handle multi-bladed machines with a variety of blade pitch schedule options.

Review of the Mathematical Model

The problem is approached in the spirit of classical potential theory; that is, fluid viscosity is neglected and the flow is assumed to be irrotational. The blades are assumed to be of infinite span, and their speeds are assumed small enough to treat the flow as incompressible. Under these circumstances lifting characteristics are well predicted theoretically by replacing the airfoil with its mean camber line. The problem is then resolved into one of finding a flow pattern having the instantaneous streamline coincident with the mean camber line (the flow tangency boundary condition at the body) and insuring that the flow leaves the trailing edge smoothly (the Kutta condition).

Using the appropriate mathematical relationships for the flow tangency condition, an expression is derived for the normal velocity (resulting from the free stream wind and the blade motion) of a fluid particle on the blade surface. In terms of the moving non-inertial (x,y) coordinate system, the linearized expression for a symmetrical airfoil is

$$U_n \equiv U_y = x \dot{\alpha} + V_R \alpha \quad . \quad (2-1)$$

U_n and U_y are the fluid velocity components normal to the chord line and normal to the relative inflow velocity respectively; α is the blade angle of attack and $\dot{\alpha}$ is its time rate of change. V_R and x are defined in Fig. 2.1.

The loading on the lifting surface is represented by a distribution of bound vorticity, and, for numerical convenience, it is assumed that this distribution can be represented by discrete bound vortices attached to the cambered surface. The physical principle of primary importance in this unsteady problem is the conservation of circulation. This implies that the time rate of change of the bound and wake circulation is zero, and that the net circulation around the airfoil and wake must remain constant. Thus, if the bound circulation changes due to airfoil motion or flow fluctuations, there must be an equal but opposite change in the wake. This is possible when the bound circulation (Γ_b) changes by shedding into the wake vortices of appropriate strength and sense (Γ_w) from the blade trailing edge. The shed vortices convect at their own local velocities and their trajectories describe the wake.

The bound and shed vortices induce velocities everywhere in the flow field. At a "control point" on the blade chord the sum of the normal velocities induced by the bound vortices (u_{n_b}) and the

shed vortices in the wake (u_{n_w}) must be equal in magnitude but opposite in sign to U_n in order that the flow tangency condition be satisfied. For the symmetrical airfoil,

$$u_{n_b} + u_{n_w} + U_n = 0. \quad (2-2)$$

Eq. (2-2) is a generalized form of the governing flow equation. The Biot-Savart law provides an expression for the induced velocities in terms of the unknown discrete bound vortices. The velocity induced at $\bar{\zeta}_c = x_c \bar{x} + y_c \bar{y}$ by a bound vortex of strength γ located at $\bar{\zeta}_b = x_b \bar{x} + y_b \bar{y}$ is given by

$$u_x = - \frac{\gamma}{2\pi} \frac{(y_c - y_b)}{(x_c - x_b)^2 + (y_c - y_b)^2}, \quad (2-3a)$$

and
$$u_y = \frac{\gamma}{2\pi} \frac{(x_c - x_b)}{(x_c - x_b)^2 + (y_c - y_b)^2}. \quad (2-3b)$$

Resolved into components in the (ξ, η) and (s, n) coordinate systems, the equations are

$$u_\xi = - (u_x \sin \beta + u_y \cos \beta), \quad (2-3c)$$

$$u_\eta = - (u_x \cos \beta + u_y \sin \beta), \quad (2-3d)$$

$$u_s = (u_x \cos \alpha + u_y \sin \alpha), \quad (2-3e)$$

and
$$u_n = - (u_x \sin \alpha - u_y \cos \alpha). \quad (2-3f)$$

In complex mathematical notation, Eq. (2-3f) is

$u_n = \gamma \text{Im} \{ C_{ij} \}$ where $C_{ij} = C(\bar{\zeta}_{c_i}, \bar{\zeta}_{b_j}, \alpha)$ is a complex constant. If there are ns bound vortices on the chord line (representing the lifting surface) and m shed vortices representing the wake, Eq. (2-2) applied to the i th control point is

$$\sum_{j=1}^{ns} C_{ij} \gamma_j + \sum_{l=1}^m C_{il} \Gamma_w + \{ x \dot{\alpha} + v_R \alpha \} = 0 \quad (2-4)$$

The equation is written for ns control points and a system of ns equations for the ns unknown γ 's is developed.

The generalized aerodynamic forces on the blade are a local lift normal to the inflow velocity, a local drag parallel to the inflow velocity, and a moment about the mid-chord. The lift and moment are derived from Bernoulli's equation for unsteady, incompressible, inviscid flow written in the (ξ, η) frame. Potential theory provides the mechanism for evaluating these equations in terms of the vortex strengths found by application of Eq. (2-4)

Numerical Solution

For single bladed turbines application of Eq. (2-4) to the various control points gives a system of linear equations for the unknown γ 's. With multiple blades, however, the influence of the bound vortices of the additional blades must be considered. If there are p blades, each represented by ns bound vortices, the number of resulting equations is ns times p . The additional

complexity of this situation is not readily apparent, but in developing the computer codes for the multi-bladed case it was quickly ascertained that the computer time required for solution would be significantly increased, if the mathematical model was adhered to rigidly. A numerical technique was therefore developed which considers each blade individually and assumes that blade motion occurs in discrete time steps and is independent of the motion of other blades. In the limit of infinitesimally small time steps this model corresponds exactly to the actual situation of simultaneous blade motion. To clarify the procedure a three bladed turbine having four segments per blade (four bound vortices and four control points) is considered. Figs. 2.2 and 2.3 illustrate the situation.

Blades 1, 2, and 3 are considered sequentially in a clockwise fashion. At some initial time, $t = 0$, the lift on each blade is zero. It is assumed that blade number 1 is impulsively started while blades number 2 and 3 remain stationary. The strength of the bound vortices are found from the following system of equations:

$$c_{11} \gamma_1 + c_{12} \gamma_2 + c_{13} \gamma_3 + c_{14} \gamma_4 = - \left\{ x_1 \dot{a} + v_R a \right\}, \quad (2-5a)$$

$$c_{21} \gamma_1 + c_{22} \gamma_2 + c_{23} \gamma_3 + c_{24} \gamma_4 = - \left\{ x_2 \dot{a} + v_R a \right\}, \quad (2-5b)$$

$$C_{31} \gamma_1 + C_{32} \gamma_2 + C_{33} \gamma_3 + C_{34} \gamma_4 = - \{ x_3 \dot{a} + v_R a \}, \quad (2-5c)$$

$$\text{and } C_{41} \gamma_1 + C_{42} \gamma_2 + C_{43} \gamma_3 + C_{44} \gamma_4 = - \{ x_4 \dot{a} + v_R a \}. \quad (2-5d)$$

Since the strength of the bound vortices on the other blades is still set at zero, and no vortices have been shed into the wake, the above terms are the only ones that need be considered. Now the bound circulation is given by $\Gamma_{b_1}^1 = \sum_{j=1}^4 \gamma_j^1$. Following the conservation of circulation principle, a vortex is formed at the trailing edge (Fig. 2.3a); its strength is $\Gamma_{w_1}^1 = - \Gamma_{b_1}^1$. In these equations the second subscript on Γ indicates the blade under consideration. The superscript indicates the mth time step, in this case $m=1$. Since all the required information is available, the position that this shed vortex will have at the next time step is calculated and stored for future use. Ref. 2.3 presents more of the details of the procedure.

The second blade is considered next. It is impulsively started while blades 1 and 3 remain stationary. The strength of the bound vortices are found from expressions similar to Eqs. (2-5) except that the influence of the bound and shed vortices of blade number 1 must be considered. Herein lies the advantage of the simplified numerical technique previously mentioned.

The position* and strength of these vortices are known from the previous calculation. It is not necessary to determine them anew by simultaneous consideration in the equations developed for blade number 2. The strength of the vortex formed at the trailing edge of blade number 2 is $\Gamma_{w_2}^1 = -\Gamma_{b_2}^1 = -\sum_{j=1}^4 \gamma_j$. Its position at the next time step is calculated and stored as was done for blade number 1.

Blade number 3 is considered next. In determining the strength of its bound vortices the influence of the bound and shed vortices of blade numbers 1 and 2 must be considered, since their formation has preceeded the calculations for blade number 3. This completes the procedure for the first time step. Since the strength and position of all vortices are known each time a blade is isolated for consideration, the aerodynamic lift and moment coefficients can be determined. From these the torque and power coefficients of the turbine can be derived.

* It should be noted that in considering a particular blade the position of all vortices in the field must be known in the spatial (x,y) coordinates of that particular blade. This requires that after each calculation the (x,y) coordinates of each shed and bound vortex be transformed to the (ξ , η) system. After proceeding to the next blade the coordinate transformation is reversed so that vortex locations are known in the new (x,y) system.

In succeeding time steps the blades are moved individually through an arc corresponding to an angular increment, $\Delta\theta$. Determination of the vortex strengths proceeds as previously described, taking into consideration all bound and shed vortices of known strength and position. In the general case for the m th time step the shed vortex strength is obtained from the relation,

$$\Gamma_{w_p}^m = \Gamma_{b_p}^{m-1} - \Gamma_{b_p}^m. \quad (2-6)$$

Computer Codes

The numerical procedure described above has been programmed on the West Virginia University IBM 360 computer. The complex arithmetic facility of FORTRAN IV has been used extensively to facilitate the analysis. The acronym CROFTAN I (CROss Flow Wind Turbine Analysis) is applied to the computer code which determines the blade lift and moment coefficients. It consists of approximately 500 cards and requires 130 K of central processor storage. Usually five blade revolutions at $\Delta\theta$ steps of 8° are required to reach a periodic solution to the unsteady problem. This requires substantial computer time. In order to avoid lengthy turn around times, CROFTAN I output is stored on a disk file (also printed and punched on cards) for later processing by the data analysis program, CROFTAN 2. Coincidentally, a library of aerodynamic data is created

which may be reprocessed later pursuant to the investigator's particular interests.

Computational Results

Initial results for the theoretical analysis of the VAWT performance have been obtained. The vortex model numerical solution works well at low solidities and moderate tip speed ratios. At high tip speed ratios (approximately 7.0 and greater) and at high solidities (greater than 0.15), the convergence of C_p values to some asymptotic periodic value is questionable. Figure 2.4 illustrates the situation. Deviation from the anticipated smooth curve occurs when the wake/blade interaction becomes so strong that the mathematical model breaks down due to vortex instability. The C_p oscillations may in fact damp out, but limitations in computer time preclude investigation of this possibility at the present time.

Several methods are being investigated to alleviate this vortex instability. These include

- a) application of an artificial damping term to the magnitude of the shed vortices,
- b) combination of proximate shed vortices by adding strengths and relocating at a centroidal position,
- c) variations in bound vortex spacing on the blade chord, and
- d) variations in the step size (orbital divisions) in numerical solution.

Preliminary indications suggest that these techniques will remedy the vortex instability problem.

The expected sequence for completing the parametric analysis is as follows:

- a) Evaluate C_p characteristics at constant blade pitch of 90° and low solidities (0.08 and 0.12).
- b) Evaluate the effect of varying skin friction and induced drag at constant blade pitch of 90° and low solidities.
- c) Evaluate the optimum C_p characteristics ($C_D = 0$) at constant blade pitch of 90° and low solidities.
- d) Repeat (a), (b), and (c) at constant blade pitch other than 90° .
- e) Evaluate C_p characteristics and the effect of varying drag at constant blade angle of attack (variable blade pitch).
- f) Modify the vortex model numerical solution and CROFTAN to eliminate vortex instability. Repeat (a) through (e) for high solidities (0.18, 0.24 and 0.30).

Items (a), (b) and (c) are substantially completed, (d) and (e) are in progress and (f) will be the subject of continuing research.

VAWT C_p characteristics at constant blade pitch, $\alpha_B = 90^\circ$, are shown in Figs. 2.5, 2.6 and 2.7 for a variety of skin friction drag coefficients, numbers of blades, N_B , and chord to radius ratios, C/R . The induced drag coefficients used are as shown in Fig. 2.8 (curve #1). Several important observations should be noted:

- a) The maximum power coefficient, $C_{p_{max}}$, is extremely sensitive to the assumed aerodynamic drag coefficient (in this case, C_{D_0} , the zero lift drag coefficient was varied). This effect was, of course, anticipated, but not to the degree indicated. The tip speed ratio, TSR, at which $C_{p_{max}}$ occurs is lowered as drag increases.
- b) Fig. 2.9 shows the effect of increased drag due to lift, C_{D_i} , on $C_{p_{max}}$. The larger C_{D_i} values of Fig. 2.8 (curve #2) were determined by increasing the basic levels of curve #1 by the inverse ratio of the blade aspect ratios, AR . Going from $C/R = 0.08$ to $C/R = 0.12$ at constant blade span decreases AR by 50%. The effect of increased C_{D_i} is essentially the same as that of increased C_{D_0} .
- c) The effect of increasing solidity, σ , by increasing the number of blades is to actually decrease $C_{p_{max}}$. This phenomenon results from the strong "blockage effect" of multi-bladed turbines. Incorporation of the force-free wake model in the vortex model numerical solution allows the CROFTAN program to reflect this behavior. It appears that other analytical attempts to predict VAWT performance fail in this regard. Continuing research to verify this trend is extremely important. Other investigators of Darrieus turbines seem to have settled on a three bladed configuration as optimum. Other considerations, of course, influence this selection. The calculations shown here

offer some evidence that this may be a poor choice. The result of selecting three blades rather than two blades will certainly be increased cost, and yet may yield poorer performance. The WVU experimental VAWT is a two bladed machine and is apparently the more efficient configuration.

- d) The effect of increasing solidity by increasing the blade chord is to improve $C_{p_{max}}$ and reduce TSR for $C_{p_{max}}$.

It is apparent that the effect of aerodynamic drag on VAWT performance is as important as the proper selection of the number of blades and the solidity. It is also apparent that the prediction of C_p values is practically meaningless unless the drag characteristics are known. At best, only the performance trends can be illustrated with varying turbine parameters. This is the case regardless of what analytical tool is chosen. With this situation in mind, as well as the expense of exercising the CROFTAN program, further in-depth analytical work should be delayed until the experimental results from the WVU VAWT can be analyzed to give a better appreciation of its true drag characteristics.

The zero drag C_p vs. TSR curve of Fig. 2.7 is of considerable interest. It represents the theoretical optimum performance of a fixed pitch ($\alpha_B = 90^\circ$) straight bladed Darrieus turbine, and can be compared to the so-called "Betz limit" for optimum performance of propeller type machines. Analytical attempts to arrive at this type of result have failed thus far; however, the CROFTAN program

gives a numerical solution, if the assumption is made that the unsteady lift coefficients predicted by CROFTAN are substantially correct. A similar curve can be constructed for other solidities, number of blades, and for variable blade pitch/constant angle of attack. The fact that the VAWT $C_D = 0$ curve gives greater C_p values than the Betz limit lends further support to the contention that VAWT performance potential may, in fact, surpass that of propeller type machines.

It should be noted that these C_p curves for $\alpha_B = 90^\circ$ are not really correct for low tip speed ratios where the blade stall angle is exceeded for a substantial portion of an orbit. They are presented here in order to exhibit the important trends and will be modified to reflect variable pitch or blade flip. This would be the actual case for an operational VAWT and represents an advantage over curved bladed Darrieus turbines where the stall cannot be avoided.

An abbreviated copy of CROFTAN output is included in Tables 2-I through 2-IV. These results illustrate the capacity of the CROFTAN code to examine important kinematic parameters, aerodynamic loads, and power coefficients over the entire orbital range for successive blade revolutions. CROFTAN is in the process of continuous update.

Table 2-V gives a key to symbols used in Tables 2-I to 2-IV.

Conclusions

Substantial progress has been made in formulating this complex problem in the detail believed necessary to obtain useful results. Parametric results have been shown for values of power coefficient as influenced by blade number, chord-to-radius ratio, solidity, and tip speed ratio. Some of the results given here are compared to the available experimental data in section 5 of this report, where general agreement is noted.

References

- 2.1 Walters, R. E., et. al., "Innovative Wind Machines." ERDA/NSF/00367-75/T1, Department of Aerospace Engineering TR-47, West Virginia University, September, 1975.
- 2.2 Walters, R. E., et. al., "Innovative Wind Machines." ERDA/NSF/00367-76/2, Department of Aerospace Engineering TR-50, West Virginia University, June, 1976.
- 2.3 Migliore, P. G., Fanucci, J. B. and Squire, W., "Numerical Solution for the Unsteady Lifting Characteristics of Variable Pitch Cross-Flow Wind Turbines," 11th Intersociety Energy Conversion Engineering Conference, State Line, Nevada, September, 1976.

Fig. 2.1a. Coordinate systems

Fig. 2.1b. Four-vortex model of the lifting surface and the shed vortex model of the wake.

Fig. 2.2. Three bladed turbine. (For numerical convenience, it is assumed that each blade advances through $\Delta\theta$ independently of the other blades.)

Fig. 2.3a. Determination of bound and shed vortex strengths for blade number 1 in the first time step.

Fig. 2.3b.

Determination of bound and shed vortex strengths for blade number 2 in the first time step. (Blade 2 is influenced by the vortices "previously formed" at blade 1.)

Fig. 2.3c. Determination of bound and shed vortex strengths for blade number 3 in the first time step. (Blades 1 and 2 influence the vortices formed at blade 3.)

Fig. 2.4. Calculated power coefficient vs. number of rotor revolutions.

Fig. 2.5. Calculated VAWT performance, C_p vs. TSR,
straight blade, no pitch; 2 blades, $C/R = 0.08$, $\sigma = 0.08$.

Fig. 2.6. Calculated VAWT performance, C_p vs. TSR, straight blade, no pitch; 3 blades, $C/R = 0.08$, $\sigma = 0.12$.

Fig. 2.7. Calculated VAWT performance, C_p vs. TSR, straight blades, no pitch; 2 blades, $C/R = 0.12$, $\sigma = 0.12$.

Fig. 2.8. Induced drag used for C_p estimates.

curve number	1	2	3	4
N_b , blade number	2	3	2	2
C/R, chord/radius	0.08	0.08	0.12	0.12
σ , solidity	0.08	0.12	0.12	0.12
AR, aspect ratio	18.7	18.7	18.7	12.5

Fig. 2.9. Calculated VAWT performance, C_p vs. TSR, straight blade, no pitch; various blade numbers, C/R ratios, solidities, and aspect ratios.

TABLE 2-I

AERODYNAMIC CHARACTERISTICS OF A 2 BLADED VERTICAL AXIS WIND TURBINE
(C/R = .12, TSR = 4.5)

HISTORY OF IMPORTANT KINEMATIC PARAMETERS

θ	V_R	α_B	α	$\dot{\alpha}$	β	Ω
THETA	VR	BSETDG	ALPHA	ALPHDT	BETA	OMEGA
0	1.024	90.000	12.529	0.047	-12.529	0.953
15	0.967	90.000	12.330	-0.009	2.170	1.009
30	0.909	90.000	12.216	-0.075	17.784	1.075
45	0.857	90.000	10.560	-0.147	34.440	1.147
60	0.815	90.000	7.334	-0.215	52.166	1.215
75	0.787	90.000	4.189	-0.267	70.811	1.267
90	0.778	90.000	0.000	-0.286	90.000	1.286
105	0.787	90.000	-4.189	-0.267	109.188	1.267
120	0.815	90.000	-7.334	-0.215	127.834	1.215
135	0.857	90.000	-10.560	-0.147	145.560	1.147
150	0.909	90.000	-12.216	-0.075	162.216	1.075
165	0.967	90.000	-12.330	-0.009	177.830	1.009
180	1.024	90.000	-12.529	0.047	-167.472	0.953
195	1.079	90.000	-11.474	0.092	-153.527	0.908
210	1.128	90.000	-9.326	0.126	-140.174	0.874
225	1.168	90.000	-7.733	0.151	-127.267	0.849
240	1.198	90.000	-5.323	0.169	-114.677	0.831
255	1.216	90.000	-2.711	0.179	-102.289	0.821
270	1.222	90.000	0.000	0.182	-90.001	0.818
285	1.216	90.000	2.711	0.179	-77.711	0.821
300	1.198	90.000	5.323	0.169	-65.324	0.831
315	1.168	90.000	7.733	0.151	-52.734	0.849
330	1.128	90.000	9.326	0.126	-39.827	0.874
345	1.079	90.000	11.474	0.092	-26.474	0.908

TABLE 2-II

AERODYNAMIC CHARACTERISTICS OF A 2 BLADED VERTICAL AXIS WIND TURBINE
(C/R = .12, TSR = 4.5)

AERODYNAMIC LOADS FOR BLADE NUMBER 1 IN REVOLUTION NUMBER 9

THETA	CLS	CLUN	CLT	CMS	CMUN	CMT	CDO	CDI	CDT
0	-0.9555	-0.0209	-0.9765	0.0214	-0.0001	0.0213	0.0150	0.0408	0.0558
15	-0.9888	0.0138	-0.9748	0.0222	0.0001	0.0223	0.0150	0.0406	0.0556
30	-0.9190	0.0571	-0.8619	0.0209	0.0004	0.0214	0.0150	0.0293	0.0443
45	-0.7435	0.1071	-0.6364	0.0173	0.0008	0.0181	0.0150	0.0135	0.0285
60	-0.4485	0.1587	-0.2898	0.0110	0.0011	0.0121	0.0150	0.0027	0.0177
75	-0.0477	0.1971	0.1495	0.0024	0.0014	0.0038	0.0150	0.0007	0.0157
90	0.6318	0.3253	0.9571	-0.0189	0.0014	-0.0175	0.0150	0.0387	0.0537
105	0.5717	-0.0364	0.5353	-0.0136	0.0003	-0.0132	0.0150	0.0089	0.0239
120	0.6470	0.0321	0.6791	-0.0139	0.0004	-0.0135	0.0150	0.0159	0.0309
135	0.7508	0.0564	0.8071	-0.0156	0.0005	-0.0151	0.0150	0.0246	0.0396
150	0.5957	-0.0299	0.5658	-0.0130	-0.0003	-0.0133	0.0150	0.0102	0.0252
165	0.5980	0.0281	0.6211	-0.0131	0.0002	-0.0129	0.0150	0.0127	0.0277
180	0.5042	-0.0121	0.4921	-0.0116	-0.0002	-0.0118	0.0150	0.0077	0.0227
195	0.4604	0.0073	0.4677	-0.0111	-0.0000	-0.0111	0.0150	0.0070	0.0220
210	0.4119	-0.0020	0.4099	-0.0097	0.0000	-0.0097	0.0150	0.0054	0.0204
225	0.5108	0.0501	0.5609	-0.0108	0.0005	-0.0103	0.0150	0.0099	0.0249
240	0.2730	-0.0752	0.1984	-0.0066	-0.0006	-0.0073	0.0150	0.0013	0.0163
255	0.2531	-0.0047	0.2484	-0.0064	-0.0001	-0.0065	0.0150	0.0020	0.0170
270	0.0132	-0.0516	0.0516	-0.0049	-0.0006	-0.0055	0.0150	0.0001	0.0151
285	-0.0339	-0.0694	-0.1033	-0.0002	-0.0003	-0.0005	0.0150	0.0003	0.0153
300	-0.1794	-0.0698	-0.2493	0.0037	-0.0004	0.0003	0.0150	0.0020	0.0170
315	-0.3658	-0.0762	-0.4420	0.0079	-0.0005	0.0074	0.0150	0.0062	0.0212
330	-0.5638	-0.0658	-0.6298	0.0124	-0.0004	0.0119	0.0150	0.0132	0.0282
345	-0.7629	-0.0468	-0.8097	0.0169	-0.0003	0.0166	0.0150	0.0249	0.0399

TABLE 2-III

AERODYNAMIC CHARACTERISTICS OF A 2 BLADED VERTICAL AXIS WIND TURBINE
(C.R = .12, TSR = 4.5)

POWER COEFFICIENTS FOR BLADE NUMBER 1 IN REVOLUTION NUMBER 9

THETA	CPCLS	CPCLUN	CPCLT	CFCMS	CPCMUN	CPCMT	CPCDO	CPCDI	CPCDT	CPALL
0	1.1899	0.0261	1.2160	-0.0147	0.0001	-0.0147	-0.0841	-0.2289	-0.3130	0.8884
15	1.1230	-0.0157	1.1673	-0.0136	-0.0001	-0.0137	-0.0748	-0.2026	-0.2774	0.8161
30	0.3814	-0.0547	0.8268	-0.0114	-0.0002	-0.0116	-0.0664	-0.1295	-0.1958	0.6192
45	0.5494	-0.0791	0.4703	-0.0084	-0.0004	-0.0087	-0.0593	-0.0535	-0.1128	0.3487
60	0.2232	-0.0790	0.1442	-0.0048	-0.0005	-0.0053	-0.0540	-0.0096	-0.0637	0.0753
75	0.0119	-0.0493	-0.0374	-0.0010	-0.0006	-0.0015	-0.0507	-0.0024	-0.0532	-0.0921
90	-0.0017	-0.0009	-0.0025	0.0075	-0.0005	0.0069	-0.0496	-0.1279	-0.1775	-0.1731
105	0.1401	-0.0089	0.1311	0.0055	-0.0001	0.0054	-0.0507	-0.0300	-0.0807	0.0558
120	0.3187	0.0158	0.3345	0.0061	-0.0002	0.0059	-0.0539	-0.0571	-0.1111	0.2293
135	0.5511	0.0414	0.5925	0.0075	-0.0002	0.0073	-0.0592	-0.0973	-0.1565	0.4433
150	0.5687	-0.0285	0.5402	0.0070	0.0002	0.0072	-0.0662	-0.0448	-0.1110	0.4364
165	0.6774	0.0262	0.7036	0.0080	-0.0001	0.0079	-0.0746	-0.0634	-0.1380	0.5735
180	0.6271	-0.0151	0.6121	0.0080	0.0001	0.0081	-0.0839	-0.0432	-0.1271	0.4931
195	0.5832	0.0092	0.5924	0.0085	0.0000	0.0085	-0.0935	-0.0435	-0.1370	0.4639
210	0.4393	-0.0023	0.4870	0.0081	-0.0000	0.0081	-0.1027	-0.0367	-0.1393	0.3557
225	0.5138	0.0503	0.5642	0.0096	-0.0004	0.0092	-0.1107	-0.0734	-0.1841	0.3892
240	0.2000	-0.0550	0.1451	0.0062	0.0006	0.0068	-0.1171	-0.0098	-0.1269	0.0250
255	0.0978	-0.0018	0.0960	0.0062	0.0001	0.0063	-0.1211	-0.0159	-0.1370	-0.0347
270	0.0005	-0.0002	0.0002	0.0048	0.0006	0.0054	-0.1225	-0.0007	-0.1232	-0.1176
285	0.0128	0.0262	0.0391	0.0002	0.0003	0.0005	-0.1212	-0.0027	-0.1239	-0.0843
300	0.1299	0.0506	0.1804	-0.0035	0.0003	-0.0031	-0.1172	-0.0155	-0.1327	0.0446
315	0.3550	0.0783	0.4423	-0.0071	0.0005	-0.0066	-0.1109	-0.0461	-0.1570	0.2787
330	0.6580	0.0780	0.7461	-0.0103	0.0004	-0.0100	-0.1029	-0.0903	-0.1932	0.5429
345	0.9659	0.0593	1.0252	-0.0129	0.0002	-0.0127	-0.0937	-0.1552	-0.2489	0.7635

TABLE 2-IV

AERODYNAMIC CHARACTERISTICS OF A 2 BLADED VERTICAL AXIS WIND TURBINE
(C/R = .12, TSR = 4.5)

SUMMARY OF POWER COEFFICIENTS FOR ALL BLADES IN REVOLUTION NUMBER 9

THETA (1)	BLDE 1	BLDE 2	BLDE 3	BLDE 4	BLDE 5	CFAVG
0	0.888	0.498	*****	*****	*****	0.693
15	0.816	0.462	*****	*****	*****	0.639
30	0.619	0.330	*****	*****	*****	0.474
45	0.349	0.308	*****	*****	*****	0.328
60	0.075	0.061	*****	*****	*****	0.068
75	-0.092	-0.022	*****	*****	*****	-0.057
90	-0.173	-0.121	*****	*****	*****	-0.147
105	0.056	-0.092	*****	*****	*****	-0.018
120	0.229	0.058	*****	*****	*****	0.144
135	0.443	0.278	*****	*****	*****	0.361
150	0.436	0.548	*****	*****	*****	0.492
165	0.574	0.768	*****	*****	*****	0.671
180	0.493	0.888	*****	*****	*****	0.691
195	0.464	0.816	*****	*****	*****	0.640
210	0.356	0.620	*****	*****	*****	0.488
225	0.389	0.349	*****	*****	*****	0.369
240	0.025	0.077	*****	*****	*****	0.051
255	-0.035	-0.095	*****	*****	*****	-0.065
270	-0.118	-0.099	*****	*****	*****	-0.108
285	-0.084	0.066	*****	*****	*****	-0.009
300	0.045	0.222	*****	*****	*****	0.133
315	0.279	0.446	*****	*****	*****	0.362
330	0.543	0.436	*****	*****	*****	0.489
345	0.764	0.566	*****	*****	*****	0.665

TABLE 2-V

KEY TO SYMBOLS USED IN TABLES 2-I TO 2-IV
(also see Fig. 2.1a)

Fortran	Math	
<u>Name</u>	<u>Symbol</u>	<u>Description</u>
THETA	θ	blade orbital angle, degrees
VR	V_R	relative inflow velocity; nondimensionalized by uR
BSETDC	α_B	blade setting, degrees
ALPHA	α	blade angle of attack, degrees
ALPIDT	$\dot{\alpha}$	time rate of change of α
BETA	β	angle between axis of fixed and rotating coordinate systems
OMEGA	Ω	time rate of change of β ; nondimensionalized by ω
CLS	-	steady part of the lift coefficient
CLUN	-	unsteady part of the lift coefficient
CLT	C_L	sum of steady and unsteady parts of the lift coefficient
CMS	-	steady part of the moment coefficient
CMUN	-	unsteady part of the moment coefficient
CMT	C_M	sum of steady and unsteady parts of the moment coefficient
CDO	$C_{D_{min}}$	minimum drag coefficient

TABLE 2-V (cont.)

Fortran	Math	
<u>Name</u>	<u>Symbol</u>	<u>Description</u>
CDI	C_{D_i}	induced drag coefficient
CDT	C_D	total drag coefficient
CPCLS	-	contribution of CLS to power coefficient
CPCLUN	-	contribution of CLUN to power coefficient
CPCLT	-	contribution of CLT to power coefficient
CPCMS	-	contribution of CMS to power coefficient
CPCMUN	-	contribution of CMUN to power coefficient
CPCMT	-	contribution of CMT to power coefficient
CPCDO	-	contribution of CDO to power coefficient
CPCDI	-	contribution of CDI to power coefficient
CPCDT	-	contribution of CDT to power coefficient
CPALL	-	total blade power coefficient
THETA (1)	θ_1	blade 1 orbital angle, degrees
BLDE 1	-	blade 1 power coefficient
BLDE 2	-	blade 2 power coefficient
CPAVG	-	average power coefficient for all blades at a particular θ

3. BOUNDARY LAYER CONSIDERATIONS FOR A VERTICAL AXIS WIND TURBINE

Introduction

In the design of a vertical axis wind turbine, the success or failure of the configuration is highly dependent upon the aerodynamic drag associated with the blades, as well as the supporting structure. This can readily be seen from the theoretical calculations (see section 2 of this report) where the power coefficient, C_p , vs. tip-speed-ratio curves are shown for different values of the parasitic drag coefficient. The curves show that the power coefficient, C_p , is highly dependent upon this drag coefficient.

The purpose of this section is to derive the appropriate boundary layer equations for the blade of a vertical axis wind turbine rotating with a constant angular velocity, ω , while the blade is simultaneously executing pitch changes about its pivot axis with an angular velocity, Ω . As previously shown (Ref. 3.1), Ω is in general not constant as a result of the coordinate system that is selected, and is also different in magnitude than the turbine angular velocity, ω . Furthermore, the angular velocity, Ω , may be due to pitch changes of the blade which are programmed in order to maintain a constant blade angle of attack as the blade rotates about the axis of the turbine.

Boundary Layer Equations

For the case where the flow is incompressible, the Navier-Stokes equations written in an inertial frame ξ, η (see Fig. 3.1) are:

$$\nabla' \cdot \bar{V}' = 0 \quad (3-1)$$

and

$$\bar{a}' = \frac{\partial \bar{V}'}{\partial t'} + \bar{V}' \cdot \nabla' \bar{V}' = -\frac{1}{\rho} \nabla' p' + \nu \nabla'^2 \bar{V}', \quad (3-2)$$

where the primes refer to quantities with respect to the ξ, η inertial frame and $\bar{V}'$ is the absolute velocity.

It is convenient to transform the above equations to a frame attached to the blade. For this purpose the equations are first transformed to a frame with the x axis in the direction of the relative wind as shown in Fig. 3.1. This coordinate system executes a rotational motion with respect to the inertial frame. In addition, it translates in a circular path with the origin of the inertial frame at the center.

The relationship between the velocity in the inertial frame $\bar{q}$ and the blade attached frame $\bar{U}$ is given by

$$\bar{q} = \bar{U} + \bar{\Omega} \times \bar{\zeta} + \bar{\omega} \times \bar{R}, \quad (3-3)$$

where $\bar{\zeta}$ is a position vector in the blade frame,

$$\bar{\zeta} = \hat{x} x + \hat{y} y,$$

$\bar{\Omega}$ is the angular velocity of the x, y frame given by $\hat{z} \Omega$.

$$\Omega(t)^* = \frac{d\beta}{dt} = \frac{\omega [1 - \epsilon \sin \theta]}{[1 + \epsilon^2 - 2 \epsilon \sin \theta]} \quad (3-4)$$

and $\beta = \tan^{-1} [(\sin \theta - \epsilon)/\cos \theta]$,

where $\epsilon = V_\infty/\omega R$ is the inverse tip speed ratio.

The relative velocity $\bar{U} = \hat{x} U_x + \hat{y} U_y$ and the absolute velocity $\bar{q} = - (V_\infty \sin \beta - u_x) \hat{x} - (V_\infty \cos \beta - u_y) \hat{y}$ are both written in terms of components in the x, y noninertial frame and u_x and u_y are the absolute perturbation velocities produced by the presence of the blades on the free stream velocity, V_∞ . These components are computed from the potential flow program as described in section 2 of this report.

In the blade fixed frame the velocity is $\bar{U} = \frac{\partial \bar{\zeta}}{\partial t}$, $\bar{\zeta}$ being in the blade fixed frame as mentioned previously.

The absolute acceleration of the fluid is given by

$$\bar{a} = \frac{D\bar{U}}{Dt} + \frac{d\bar{\Omega}}{dt} \times \bar{\zeta} + \bar{\Omega} \times (\bar{\Omega} \times \bar{\zeta}) + 2 \bar{\Omega} \times \bar{U} + \bar{\omega} \times (\bar{\omega} \times \bar{R}). \quad (3-5)$$

Here, $\frac{D\bar{U}}{Dt}$ = the Eulerian acceleration with respect to the blade fixed frame;

*See Ref. 3.1 for details.

- $\frac{d\bar{\Omega}}{dt} \times \bar{\zeta}$ = the tangential acceleration perpendicular to $\bar{\zeta}$ due to $\bar{\Omega} = \bar{\Omega}(t)$;
 $\bar{\Omega} \times (\bar{\Omega} \times \bar{\zeta})$ = the radial acceleration toward the origin of the blade fixed frame;
 $2 \bar{\Omega} \times \bar{U}$ = the Coriolis acceleration due to rotation;
 $\bar{\omega} \times (\bar{\omega} \times \bar{R})$ = the radial acceleration due to translation in circular path where $\bar{R}$ and $\bar{\omega}$ are constant vectors and $\bar{\omega} = \hat{z} \omega$.

Since the primed and unprimed systems are both Cartesian, the operators ∇' and ∇ are of identical form, i.e.:

$$\nabla' = \hat{\xi} \frac{\partial}{\partial \xi} + \hat{\eta} \frac{\partial}{\partial \eta},$$

$$\nabla = \hat{x} \frac{\partial}{\partial x} + \hat{y} \frac{\partial}{\partial y},$$

and $\nabla' = \nabla$.

Using the above equations, the final forms of the conservation equations as given by Eqs. (3-1) and (3-2) are as follows:

x-component of momentum

$$\begin{aligned}
 \frac{\partial U}{\partial t} \bar{x} + U_x \frac{\partial U}{\partial x} \bar{x} + U_y \frac{\partial U}{\partial y} \bar{x} &= y \frac{d\Omega}{dt} + x \Omega^2 + 2 U_y \Omega - \omega^2 R \sin(\theta - \beta) \\
 - \frac{1}{\rho} \frac{\partial p}{\partial x} + \nu \left(\frac{\partial^2 U}{\partial x^2} \bar{x} + \frac{\partial^2 U}{\partial y^2} \bar{x} \right) &.
 \end{aligned} \tag{3-7}$$

y-component of momentum

$$\begin{aligned} \frac{\partial U_y}{\partial t} + U_x \frac{\partial U_y}{\partial y} + U_y \frac{\partial U_x}{\partial y} = -x \frac{d\Omega}{dt} + y \Omega^2 - 2 U_x \Omega \\ + \omega^2 R \cos(\theta - \beta) - \frac{1}{\rho} \frac{\partial p}{\partial y} + \nu \left(\frac{\partial^2 U_y}{\partial x^2} + \frac{\partial^2 U_y}{\partial y^2} \right) . \end{aligned} \quad (3-8)$$

Continuity

$$\frac{\partial U_x}{\partial x} + \frac{\partial U_y}{\partial y} = 0 . \quad (3-9)$$

At this point the equations are referred to a blade fixed system wherein the x axis is oriented in the direction of the relative wind V_R as shown on Fig. 3.1. The next step is to write eqs. (3-7), (3-8), and (3-9) with respect to the chord line which is at an angle α to the x axis. From Fig. 3.1 the relationship between the axes s, n, and x, y are

$$s = x \cos \alpha + y \sin \alpha$$

$$\text{and} \quad n = -x \sin \alpha + y \cos \alpha . \quad (3-10)$$

As a result, the partial derivatives required for the transformation are

$$\frac{\partial}{\partial x} = \cos \alpha \frac{\partial}{\partial s} - \sin \alpha \frac{\partial}{\partial n}$$

$$\text{and} \quad \frac{\partial}{\partial y} = \sin \alpha \frac{\partial}{\partial s} + \cos \alpha \frac{\partial}{\partial n} . \quad (3-11)$$

The relationship between the unit vectors is

$$\hat{x} = \hat{s} \cos \alpha - \hat{n} \sin \alpha$$

and

$$\hat{y} = \hat{s} \sin \alpha + \hat{n} \cos \alpha . \quad (3-12)$$

One can then formally transform eqs. (3-7), (3-8), and (3-9) to the s, n blade fixed system. The final step is to transform the resulting s, n equations to curvilinear surface coordinates X, Y as shown in Fig. 3.2.

The relationships between the s, n axis and X, Y surface coordinates are given by

$$\begin{aligned} s &= \int_0^X \cos \lambda \, dX - Y \sin \lambda \\ \text{and} \quad n &= \int_0^X \sin \lambda \, dX + Y \cos \lambda, \end{aligned} \quad (3-13)$$

where $R_b(X)$ is the radius of curvature of the curved blade surface and is only a function of X.

The resulting substantial operator is

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + v_x \left[\frac{R_b}{R_b + Y} \right] \frac{\partial}{\partial x} + v_y \frac{\partial}{\partial y} .$$

It is noted that except for points on the surface where R_b is the same order as Y, the substantial operator may be approximated by the usual two dimensional cartesian form, i.e., the quantity $[R_b / (R_b + Y)] \approx 1$.

In the interests of brevity, if the above two coordinate transformations were carried out, and if furthermore, it were assumed that the angle of attack α was small and the body radius of curvature R_b was large, the resulting Navier-Stokes equations in a blade fixed frame would essentially be similar in form to equations (3-7), (3-8) and (3-9).

Our primary concern here is to examine the boundary layer form of the Navier-Stokes equations in order to access the effects of rotation of the blade with regard to the validity of the boundary layer approximation.

If we assume, in view of the above assumptions, that eqs. (3-7), (3-8) and (3-9) are appropriate, the boundary layer equations are derived by first nondimensionalizing the dependent and independent variables and then stretching the η_y and y variables by the factor $\sqrt{R_e}$ where R_e is large. Here R_e is defined as $R_e = UL/\nu$, where U and L are any reference speed and length, respectively, and are constant. The barred quantities denote the nondimensionalized forms:

$$\tilde{x} = \frac{x}{L} ; \tilde{U}_x = \frac{U}{U} x ; \tilde{t} = \frac{tU}{L} ; \tilde{\Omega} = \frac{\Omega L}{U} ;$$

$$\tilde{R} = \frac{R}{L} ; \tilde{y} = \frac{y}{L} \sqrt{R_e} ; \tilde{U}_y = \frac{U}{U} y \sqrt{R_e} ;$$

$$\tilde{p} = \frac{p}{\rho U^2} ; \tilde{\omega} = \frac{\omega L}{U} . \quad (3-14)$$

Upon nondimensionalizing the equations and letter $R_e \rightarrow \infty$, the final results are the following equations, written now in terms of dimensional variables:

$$\begin{aligned} \frac{\partial U_x}{\partial t} + U_x \frac{\partial U_x}{\partial x} + U_y \frac{\partial U_y}{\partial y} = x \Omega^2 - \omega^2 R \sin (\theta - \beta) \\ - \frac{1}{\rho} \frac{\partial p}{\partial x} + \nu \frac{\partial^2 U_x}{\partial y^2}, \end{aligned} \quad (3-15)$$

$$x \frac{d\Omega}{dt} + 2 U_x \Omega - \omega^2 R \cos (\theta - \lambda) + \frac{1}{\rho} \frac{\partial p}{\partial y} = 0, \quad (3-16)$$

and

$$\frac{\partial U_x}{\partial x} + \frac{\partial U_y}{\partial y} = 0. \quad (3-17)$$

Equations (3-15), (3-16) and (3-17) are the required boundary layer equations for the rotating blades with respect to a blade fixed coordinate system.

Normal Pressure Gradient

The conservation equation in the normal direction is

$$\frac{1}{\rho} \frac{\partial p}{\partial y} = -x \dot{\Omega} - 2 U_x \Omega + \omega^2 R \cos (\theta - \beta). \quad (3-18)$$

In order to access the order of magnitude of the normal pressure gradient, the case of large tip speed ratio (i.e., small

ϵ) is investigated. Furthermore, it is assumed that $\dot{\Omega}$ is zero.

Then it can be shown that $U_x \approx \omega R$ and $\Omega \approx \omega$, which leads to

$$\frac{\partial p}{\partial y} \approx 0 \left[\frac{1}{\epsilon^2} \frac{\rho V_\infty^2}{R} \right] .$$

The above equations show that for tip speed ratios of 3 or 4, the pressure gradient through the boundary layer is at least one order of magnitude higher than that associated with classical boundary layer flows over either curved surfaces or flows associated with rotating surfaces.

Conclusions

The boundary layer equations for a vertical axis wind turbine blade have been derived. It is shown that the usual classical assumption that the pressure gradient normal to the blade surface is small may be in serious error. In fact, the pressure gradient is proportional to the square of the tip speed ratio ($1/\epsilon^2$). This implies that the usual procedures in boundary layer theory, that the pressure may be calculated from potential theory and assumed constant through the boundary layer, may not be sufficiently accurate. Furthermore, this strong radial pressure gradient may lead to premature laminar boundary layer separation with its attendant increase in drag. This effect will be studied in the future experimental investigation of blade aerodynamics.

It is suggested that further work, both theoretically and experimentally, be performed in order to assess the increases in parasitic drag of a rotating blade over its value determined in the steady state two dimensional configuration.

References

3.1. Walters, R. E., et al, "Innovative Wind Machines; Executive Summary and Final Report", ERDA/NSF/00367-76/2, Department of Aerospace Engineering, WVU, June, 1976.

Fig. 3.1. Geometry of inertial and noninertial coordinate systems.

Fig. 3.2. Curvilinear blade surface coordinate system.

4. WVU VAWT OUTDOOR TEST MODEL

Introduction

The basic design features of the WVU outdoor test model vertical axis wind turbine (VAWT) have been described in reference 4.1. In this section, a more detailed description of the VAWT will be given, with particular emphasis on the rotor design.

The purpose of this section is to describe the design, instrumentation, and calibration of the vertical axis wind turbine test model built by the Department of Aerospace Engineering at West Virginia University (WVU). The turbine was designed for research purposes, in order to confirm theoretical performance calculations developed by the Department. Areas of special interest are both aerodynamic performance and structural analysis.

The vertical axis turbine has a central axis perpendicular to the wind direction, around which the blades rotate (Fig. 4.1). Vertical axis machines with fixed blade pitch have the advantage of operating independently of wind direction, and since they are mechanically somewhat more simple than the more popular horizontal axis machine, construction and maintenance costs are potentially lower. The WVU turbine is similar in operation to a Darrieus rotor, in the respect that both generate their torques from the lifts of the blades, not blade drag, as does the Savonius rotor. The difference between the Darrieus rotor and the WVU Vertical Axis Wind Turbine

(VAWT) is that the Darrieus rotor has curved fixed position blades, while the WVU VAWT has straight blades that can be articulated about their quarter chord to produce higher torques. In general, vertical axis turbines that operate by lift have higher efficiencies but have the disadvantage of performing best at high rotational velocities. This is a drawback because the large angular velocities produce large centrifugal forces on the blades. The large centrifugal forces create the possibility of blade failure due to the high stresses. For this reason care had to be taken when designing the WVU VAWT rotor to make sure the blades were adequately supported for the calculated limits of operable angular velocities and wind speeds. Calculations were performed to determine stresses on the blades and support wires. To verify these calculations, the wind machine was extensively instrumented with strain gages on one of the blades and load cells on various guy wires. Also, the instrumentation could be used to monitor the VAWT while operating to keep the blades and other parts from approaching critical stresses when operating at high angular velocities.

The turbine is a two bladed model with a rotor diameter of 3.05 m (10 ft) and blades 3.25 m (10 ft 8 in) in length (Fig. 4.1). The rotor was built with provisions to allow for easy future addition of blades and change of parameters such as blade length and machine radius. The total height of the machine, measured from the top of the shaft to the bottom of the base pads, is 7.62 m (25 ft). Three equally spaced cables connected to a combination

radial/thrust bearing support the top of the shaft. At the base, a cylindrical sleeve was placed over the shaft, which houses two self-aligning radial/thrust bearings. These bearings support the bottom of the main shaft. On the outside of the sleeve, eight legs complete the base of the machine. The four cables that are each attached from one end of the blade to a collar on the shaft are the pre-load tension cables. They add support to the overall structure and extra rigidity to the blades. Lead-lag force cables are also present on the VAWT; these cables help the blade struts resist torsion loads about the shaft during starting, stopping, and wind gusts. They are the cables that run from one end of the strut, through a small pole extended from the shaft and over the opposite strut end. There are four such cables on the machine. The center of each of the blades is supported by two cables which are attached to the blade cuff which clamps over the blade. The other ends of the cables are attached to a center sleeve on a bearing at the middle of the shaft. Each blade has a separate sleeve, so the blades can change pitch independently of each other. These cables will be used to actuate the blades to different blade pitch angles during operation of the VAWT. For the initial testing described here, the blade angles were fixed at 0° (measured perpendicular to the struts). The cables also provide structural support to the center of the blades.

Under the VAWT and inside the instrumentation building, a stand (Fig. 4.2) supports various instrumentation and control devices. The shaft of the VAWT extends through the roof and into the stand as

shown. On the stand are a generator, a motor used to drive the VAWT for starting and testing purposes, a pulley drive system, and instrumentation. The instrumentation on the stand consists of a torque cell for power measurements, a tachometer generator to measure RPM, and 10 high quality low-noise slip rings which allow the various strain gages and other equipment on the rotor to be monitored while it is turning.

Rotor Design

Structural Forces on the Blades of the VAWT

The blades which were selected for initial operation of the wind turbine were originally used on a helicopter. These blades were used to lower construction time and costs, since they were already available. Thus, operational experience could be gained and data collected for later comparison to data obtained with circulation controlled blades.^{4.1}

The blades used were manufactured with a 15.5% thick symmetrical airfoil and a chord length of 17.27 cm (6.8 in). A cross-sectional view of the airfoil is shown in Fig. 4.3. They were constructed of an aluminum alloy with a mass (weight) of 2.091 kg/m (1.4 lb/ft) of span; after adding structural reinforcement and balancing weights, the mass (weight) of the blades was 2.98 kg/m (2.0 lb/ft) of span. The blades have lengths of 3.25 m (10 ft 8 in).

Before the blades could be installed on the machine it was first necessary to determine what operational forces would be. When the wind machine is in operation the blades are loaded approximately uniformly with a combination of aerodynamic and centrifugal forces.

The uniform load on the blade is given by the following equations derived from Fig. 4.4:*

In the X direction

$$F_x = -L\sin\alpha + D\cos\alpha - m\omega^2 R\cos\alpha_0, \quad (4-1)$$

where $-L\sin\alpha + D\cos\alpha$ is due to the aerodynamic loads and $-m\omega^2 R\cos\alpha_0$ is due to centrifugal loads.

In the Y direction

$$F_y = L\cos\alpha + D\sin\alpha - m\omega^2 R\sin\alpha_0, \quad (4-2)$$

where $L\cos\alpha + D\sin\alpha$ is due to the aerodynamic loads and $-m\omega^2 R\sin\alpha_0$ is due to centrifugal loads.

The aerodynamic loads of both F_x and F_y are then put into non-dimensional coefficient form giving

$$F_x = \frac{1}{2}\rho V_{REL}^2 C \left[-C_L \sin\alpha + C_D \cos\alpha \right] - m\omega^2 R\cos\alpha_0, \quad (4-3)$$

and

$$F_y = \frac{1}{2}\rho V_{REL}^2 C \left[C_L \cos\alpha + C_D \sin\alpha \right] - m\omega^2 R\sin\alpha_0, \quad (4-4)$$

Eqs. 4-3 and 4-4 give the uniform loads on the blade. The equations were then nondimensionalized by dividing by $\frac{1}{2}\rho V_\infty^2 C$,

*Symbols are defined in Appendix B.

giving the forces in coefficient form, giving

$$C_{F_x} = \frac{F_x}{\frac{1}{2}\rho V_\infty^2 C} = \frac{V_{REL}^2}{V_\infty^2} \left[-C_L \sin\alpha + C_D \cos\alpha \right] - \frac{2m}{\rho} \left(\frac{\omega R}{V_\infty} \right)^2 \frac{1}{RC} \cos\alpha_0, \quad (4-5)$$

and

$$C_{F_y} = \frac{F_y}{\frac{1}{2}\rho V_\infty^2 C} = \frac{V_{REL}^2}{V_\infty^2} \left[C_L \cos\alpha + C_D \sin\alpha \right] - \frac{2m}{\rho} \left(\frac{\omega R}{V_\infty} \right)^2 \frac{1}{RC} \sin\alpha_0. \quad (4-6)$$

This was done to make the equations simpler for computer programming. Eqs. 4-5 and 4-6 are now independent of V_∞ (wind speed) and are dependent instead on turbine tip speed ratio, $\omega R/V_\infty$. The value V_{REL}^2/V_∞^2 can be found by trigonometry from Fig. 4.4 to be the following:

$$\frac{V_{REL}^2}{V_\infty^2} = 1 - 2 \frac{\omega R}{V_\infty} \sin\theta - \left(\frac{\omega R}{V_\infty} \right)^2, \quad (4-7)$$

where

$$\alpha = \alpha_1 + \alpha_0, \quad (4-8)$$

$$\alpha_1 = \tan^{-1} \left[\frac{\sin\theta - \omega R/V_\infty}{\cos\theta} \right], \quad (4-9)$$

and α_0 is the pitch angle of the blade. These equations are all derived from Fig. 4.4.

For numerical calculations, the values of $C_L = 0.1\alpha$ and $C_D = 0.02 + 0.026C_L^2$ may be used, as was done with strip-theory calculations.^{4.2} A computer program (Appendix C) was written to calculate these uniform blade forces.

Structural Analysis of the Blades of the VAWT

After the aerodynamic loads and centrifugal forces were determined for the blades at different tip speed ratios and wind velocities, a structural analysis of the blades was performed. The analysis was needed to determine what method should be used to attach the blades to the VAWT. The blade supports had to provide adequate strength for the blades to maintain stresses and deflections within reasonable limits.

Simply Supported Case

The first design configuration studied was the blade supported at each end by struts from the shaft. Self-aligning bearings at each end of the blade provided supports which are equivalent to a simply supported beam. A uniform load on the blade was assumed, consisting of the aerodynamic loads and centrifugal forces. The loads were determined from the computer program of Appendix C. The loads fluctuate throughout the 360° blade revolution; therefore, only the maximum values were used for the stress, moment, and deflection calculations. The formulas for maximum moment and deflection can easily be derived or found in most structural handbooks.^{4.3} Fig. 4.5 illustrates the blade as a simply supported beam with a uniform load. The maximum moment can be calculated from the following equation: $M = \frac{W\ell^2}{2}$, where $\ell = 1.63\text{m}(64\text{in})$, half the blade length, and the uniform load due to the centrifugal forces and aerodynamic loads is W .

To find the maximum stress on the blade the equation for bending stress was used:

$$\sigma_b = \frac{Me}{I_x}, \quad (4-10)$$

where e equals the distance between the extreme fiber and the neutral axis of the blade. I_x is the moment of inertia about the chord of the blade and is equal to 5.62 cm^4 (0.135 in^4).

The equation for stress at the midpoint of the blade can be rewritten as

$$\sigma_b = \frac{Wl^4 e}{2I_x} \quad (4-11)$$

The equation for the deflection at the blade midpoint is

$$y = \frac{5}{24} \frac{Wl^4}{EI_x} \quad (4-12)$$

For the configuration of Fig. 4.5, the only variable in the equations for the maximum stress and deflection is W , the uniform load (lb/in).

Calculations from the previous sections were used to determine a maximum value for W . $\omega R/V_\infty = 6$ and $V_\infty = 6.29 \text{ m/sec}$ (20 ft/sec) are good estimates of actual running conditions. Using these values the maximum W for the blade would be 2.04 kg/cm (11.4 lb/in).

The maximum stress and deflection calculations gave $\sigma_{\max} = 6119 \text{ kg/cm}^2$, ($86,850 \text{ psi}$) and $y_{\max} = 75.2 \text{ cm}$ (29.6 in). These figures were above reasonable limits; therefore, a different method

for attaching the blades had to be devised.

Simply Supported Case with Longitudinal Tension Loads

The second configuration considered used blade pre-tension cables from the ends of the blades to the shaft as shown in Fig. 4.6. These cables reduce the blade stresses and deflection. The maximum stress and deflection occur at the center of the blade as in the simply supported case.

The equation for deflection at any point on the blade can be written as^{4.4}

$$y = \frac{W}{SK^2} \left[\frac{\cosh(K\ell - Kx)}{\cosh K\ell} - 1 \right] + \frac{w}{2S} (2\ell - x)x, \quad (4-13)$$

where $K^2 = S/EI_x$ and $S = P \sin 28.8^\circ$. The second derivative of the deflection equation gives the moment equation:

$$\frac{d^2y}{dx^2} = \frac{W}{S} \left[\frac{\cosh(K\ell - Kx)}{\cosh K\ell} - 1 \right], \quad (4-14)$$

$$M = EI_x \frac{d^2y}{dx^2}, \quad (4-15)$$

and

$$M = \frac{W}{K^2} \left[\frac{\cosh(K\ell - Kx)}{\cosh K\ell} - 1 \right]. \quad (4-16)$$

From the moment equation the stress can be written as

$$\sigma = \frac{Me}{I_x} + \frac{S}{A}. \quad (4-17)$$

If large enough pre-tension loads (P) are applied in the cables, the blade deflections and stresses can be greatly reduced. However, because of limited length of the shaft the maximum cable angles could only be 28.8° ; therefore, for the tension load applied in the cable, only about one half of it acts in the axial direction, S. Tensions of about 2727 kg (6000 lbs) were needed to reduce the stresses to reasonable limits of below 1761 kg/cm^2 (25,000 psi). It was felt that this much tension would be difficult to obtain and maintain because of the possibility of cable failure. For these reasons it was decided that a center wire support should be added between the blades and shaft for extra support, and the loads in the pre-tension cables could be reduced.

Statically Indeterminate Beam With Combined Loading of Uniform Lateral Load and Longitudinal Load

The final blade support configuration considered is shown in Fig. 4.7. With the addition of the center support to the blade, the beam has three simple supports as shown in Fig. 4.8a. The center support cable will also be used to articulate the blade. The equations for moment, stress, and deflection can be solved by using the method of superposition. The blade structural analysis, as shown in Fig. 4.8a, can be separated into two independent problems, as shown in Fig. 4.8b and Fig. 4.8c. The solutions obtained from each can be added together to give the total solution to the original problem.^{4.4}

Using Fig. 5.8b, the solutions for moment, stress, and deflection

were derived. First, summation of moments were taken about the left and right halves; for the left half

$$M = -Sy + \frac{Rx}{2} , \quad (4-18)$$

and from beam deflection theory,

$$M = EI \frac{d^2y}{dx^2} . \quad (4-19)$$

Substituting this into the above equation gives

$$\ddot{y} + K^2 y = \frac{Rx}{2EI} , \quad (4-20)$$

where $K^2 = S/EI$.

The homogeneous solution for Eq. 4-20 can be written in the form

$$y_h = C_1 \sinh Kx + C_2 \cosh Kx. \quad (4-21)$$

For a particular solution

$$y_p = \alpha x^2 + \beta x + \gamma , \quad (4-22)$$

$$\dot{y}_p = 2\alpha x + \beta , \quad (4-23)$$

and

$$\ddot{y}_p = 2\alpha .$$

Substituting Eqs. 4-22 and 4-24 into 4-20 gives (4-24)

$$2\alpha + K^2(\alpha x^2 + \beta x + \gamma) = \frac{Rx}{2EI} .$$

Therefore, $\alpha = 0$, $r = 0$ and $\beta = \frac{R}{2S}$. For the left half, the general solution $y = y_h + y_p$ can be written as

$$y = C_1 \sinh Kx + C_2 \cosh Kx + \frac{R}{2S} x, \quad (4-25)$$

and using the same procedure the solution for the right side can be derived giving

$$y = C_3 \sinh Kx + C_4 \cosh Kx + \frac{R}{2S} (2\ell - x). \quad (4-26)$$

The constants C_1 , C_2 , C_3 , and C_4 can now be found using the boundary condition at the end supports, plus the requirement of continuity of deflection and slope at the center of the blade.

The boundary conditions (B.C.) are

$$1. \quad x = 0, y = 0,$$

$$\text{and } 2. \quad x = 2\ell, y = 0.$$

From B.C. 1 for the left half of the beam,

$$C_2 = 0, \quad (4-27)$$

and from B.C. 2 for the right half,

$$C_4 = -C_3 \frac{\sinh K2\ell}{\cosh K2\ell}. \quad (4-28)$$

From the continuity at $x = \ell$ the deflection equations give

$$C_1 \sinh K\ell = C_3 \left(\sinh K\ell - \frac{\sinh K2\ell}{\cosh K2\ell} \cosh K\ell \right). \quad (4-29)$$

The slope equations for left and right halves give

$$C_1 K \cosh K\ell = C_3 K \left[\cosh K\ell - \frac{\sinh K2\ell}{\cosh K2\ell} \sinh K\ell \right] \quad (4-30)$$

Solving 4-29 and 4-30 as simultaneous equations for C_1 and C_3 gives

$$C_1 = \frac{R \sinh K\ell}{SK \sinh K2\ell} \quad , \quad (4-31)$$

and

$$C_3 = \frac{-R \sinh K\ell \cosh K2\ell}{SK \sinh K2\ell} \quad . \quad (4-32)$$

Only the solution for the left half is needed since the blade loadings are symmetrical about the center support. Because of this, the deflections, moments, and stresses are the same on the left half and the right half of the blade. The solution for the left half of the blade can now be written as follows:

$$y = - \frac{R \sinh K\ell}{SK \sinh K2\ell} \sinh Kx + \frac{Rx}{2S} \quad . \quad (4-33)$$

To obtain the full solution, the beam problem shown in Fig. 4.8c must be solved. The problem is the same as the one discussed before, i.e., a beam simply supported at the ends with longitudinal tension loads. This solution is

$$y = \frac{W}{SK^2} \left[\frac{\cosh(K\ell - Kx)}{\cosh K\ell} - 1 \right] + \frac{W}{2S} (2\ell - x)x \quad . \quad (4-34)$$

The total solution is the sum of both of the deflection equations; however, first the center reaction (R) must be found. To find the center reaction, the sum of the two deflection equations are set

equal to 0 deflection at the point of reaction R, which gives the following:

$$- \frac{R \sinh K\ell}{SK \sinh K2\ell} \sinh K\ell + \frac{R\ell}{2S} + \frac{W}{SK^2} \left[\frac{1}{\cosh K2\ell} - 1 \right] + \frac{W\ell^2}{2S} \quad (4-35)$$

This can be then reduced to

$$R = \frac{2W}{K2 \sinh^2 K\ell - \ell K \sinh K2\ell} \left[\frac{\sinh K2\ell}{\cosh K\ell} - \sinh K2\ell \right] + \frac{W\ell^2 K \sinh K2\ell}{2 \sinh^2 K\ell - \ell K \sinh K2\ell} \quad (4-36)$$

The total deflection for any point on the left half of the blade from superposition can be written as

$$y = - \frac{R \sinh K\ell}{SK \sinh K2\ell} \sinh Kx + \frac{Rx}{2S} + \frac{W}{SK^2} \left[\frac{\cosh(K\ell - Kx)}{\cosh K\ell} - 1 \right] + \frac{W}{2S} x(2\ell - x) \quad (4-37)$$

To obtain the moment equation the second derivative of the deflection equation must be taken. The first derivative gives the slope equation:

$$\frac{dy}{dx} = - \frac{R \sinh K\ell}{S \sinh K2\ell} \cosh Kx - \frac{WK}{SK^2} \frac{\sinh(K\ell - Kx)}{\cosh K\ell} + \frac{W}{S} (\ell - x) + \frac{R}{2S} \quad (4-38)$$

Taking the second derivative gives

$$\frac{d^2 y}{dx^2} = -RK \sinh K\ell \sinh Kx + \frac{W}{S} \frac{\cosh(K\ell - Kx)}{\cosh K\ell} - \frac{W}{S} \quad (4-39)$$

The moment equation for beam deflection problems is

$$M = EI_x \frac{d^2 y}{dx^2} \quad (4-40)$$

To obtain the moment for this particular case Eq. 4-39 is substituted into Eq. 4-40:

$$M = -\frac{R}{K} \frac{\sinh K\ell}{\sinh K2\ell} \sinh Kx + \frac{W}{K^2} \frac{\cosh(K\ell - Kx)}{\cosh K\ell} \quad (4-41)$$

The stress at any point on the blade can then be determined from the bending stress plus the stress due to the longitudinal tension loads:

$$\sigma = \frac{Me}{I_x} + \frac{S}{A} \quad (4-42)$$

The quantity e is the distance between the extreme fiber and the neutral axis of the blade and can be taken on the top side or bottom side of the blade depending on whether tension or compression is present in the blade.

Fig. 4.9 shows the moment and deflection curves for pre-tension load, S , equal to zero. From the curves it can be seen that the

maximum moments occur at the center of the blade and $\frac{3}{8}l$ from the ends of each blade, having values of $\frac{Wl^2}{8}$ and $\frac{9}{128} Wl^2$ respectively. The maximum deflection occurs at $0.42l$.

A computer program was written to calculate the moments, stresses and deflection for different input values of uniform load and longitudinal tension loads. The analysis showed that the pre-tension loads have little effect on the stresses and deflections. Also, the program results showed the maximum allowable uniform load to be about 2.06 kg/cm (11.5 lb/in) if stress is kept under 1761 kg/cm^2 (25,000 psi). This uniform load corresponds to wind speed of 6.1 m/sec (20 ft/sec) at $\omega R/V_\omega = 6$. The program was also used to compare the results from the experimental static blade load tests, which will be discussed later. The program is located in Appendix D.

Instrumentation

General

The VAWT was instrumented with strain gages for the purpose of measuring the loads on various support wires, blades, and the shaft, when the machine was in operation. Also, strain gages were used to construct a torque cell to measure operating torque, so that power output of the VAWT could be obtained. All of the strain gages, except three load cells that were designed for the three guy wires running from the bearing at the top of the shaft, were located on the

rotor part of the VAWT. This meant that slip rings were needed to connect the wiring from the strain gages on the rotor to the readout equipment. Because slip rings are an expensive item, the slip ring used was limited to 10 connectors. For this reason only one strain gage bridge could be read at a time (other than the torque cell output, which was measured continuously) when the VAWT was in operation. A stepper switch was employed to make it possible to switch from one bridge to another. A 22 position stepper switch was available for this purpose. For reasons that will be discussed later, full bridges had to be employed at all test points. The location of all the strain gages on the VAWT are shown in Fig. 4.10.

Design of Load Cells

The load cells were designed to measure the forces on the structural support wires on the rotor itself and also on the guy wires that support the shaft top bearing. Another use of the load cells was to set desired pre-tension loads in certain wires. Seven of the load cells were designed for the structural support guy wires on the rotor itself. Four of those were designed for the pre-tension cables, two for the center blade support wires, and one was placed on one of the lead-lag wires. The other three load cells were designed to be attached to the three guy wires that run from the top bearing on the main shaft to three support poles located about 10.06 m (33 ft) from the building. The wiring from these three gages did not have to pass

through the slip rings since they were located on the stationary cables. These guy wires were used as a safety feature which would be valuable in case of vibration, and also to add more support to the shaft.

Since drag is a major factor in the performance of the VAWT, the load cell shape and placement were important. For this reason, the seven load cells on the rotor were made cylindrical in shape and were kept short in length. Also, they were located as close to the shaft as possible to lower their angular velocities when the VAWT was in operation. The cells were fabricated of 2024 T-4 aluminum rods of various diameters.

It was necessary to determine what diameters and lengths would be needed to give the load cells satisfactory strength and yet give sensitive strain readings. From the design criteria of the VAWT the expected loads on the load cells could be determined and used to calculate the diameters of the load cells. It was determined from the program for the structural analysis of the blades that pre-tension loads of 1818 kg(4000 lb) would be used on the 4 tension cables on the VAWT to add structural strength to the blades. Also from the program, calculations showed that at high tip speed ratios and high wind velocities the forces on the center wire supports would be around 454.5 kg (1000 lb). This gave an average of 227.3 kg (500 lb) on each blade center support wire. The load on the lead-lag wires was estimated to be around 90.9 kg (200 lb) during operation and slightly higher during stopping, starting, and wind gusts. Using

the load figures given above, and choosing different diameters of aluminum rods that were available, strain calculations were performed. These involved using a form of the deflection equation for axial loads, changing $y = FL/AE$ to $y/L = \epsilon = F/AE$, where $A = \frac{1}{4}\pi D^2$, in order to find what diameters would give sensitive enough strain readings. These calculations also took into account whether the load cells would be big enough in diameter and length to be able to drill 2.54 cm (1 in) deep holes with diameters of 0.635 cm (1/4 in), 0.794 cm (5/16 in) and 0.953 cm (3/8 in). These were to be used to attach the load cells in conjunction with the support wires and tension adjusting turnbuckles. The sizes of the holes that were to be drilled in the ends of the load cells were determined by the sizes of the eye bolts that were used in the turnbuckles. This allowed using a straight piece of bolt threaded on both ends to connect one end of the turnbuckle with one end of the load cells, and in the other end of the load cell the old eye bolt from the turnbuckle could be used.

In designing the three load cells for the guide wire supports running from the top bearing on the shaft to the three poles 10.06 m (33 ft) from the building, load cell size and shape was not a factor. It was decided to use flat plates for the design of the load cells in order to facilitate construction. The maximum tension to be applied in the cables would be about 227.3 kg (500 lb). This would be enough to minimize cable sag and would keep the top of the shaft from moving off center while holding the down force (thrust) on the bearings to a minimum. Again, 2024 T-4 aluminum was the material chosen to be

used for the load cells because of its availability. Doing the calculation for strain from the equation $\epsilon = F/A E$, the cross-section of the load cell was chosen to be 0.3175 cm (1/8 in) by 2.54 cm (1 in).

Construction of the Load Cells

After the load cells were machined the strain gages were applied. Since full bridges were chosen to be used at all data points, the gages had to be placed on the load cells (Fig. 4.11) for the measurement of axial loads. In this bridge configuration, gages 1 and 4 are mounted in the axial direction and gages 2 and 3, referred to as Poisson ratio gages, are mounted laterally. When a tension load is applied gages 1 and 4 will react to the tensile strain, while gages 2 and 3 will sense a compressive strain resulting from the lateral contraction of the load cell. The bridge configuration will provide temperature compensation and cancel any effect from torsion or bending. Also, it provides a bridge constant of $2(1 + \nu)$ where ν is Poisson's ratio and would be 0.03.

The gages used in constructing the load cells were foil-type strain gages and were made by Micro-Measurements. The gages had a resistance of $120.0 \pm 15\%$ OHMS, gage factor of $2.11 \pm 0.5\%$, and a temperature range of -73.3°C (-100°F) to 176.6°C (350°F). These gages were chosen because of their size, relatively low cost, and because of the large pads they had which made it easier to solder

wire leads to them. M-Bond 200 epoxy was used to apply the gages to the surface of the load cells. After the wires were soldered on, a protective acrylic coating was applied to the surface of the gage and to a portion of the wire leads to provide partial protection from weather and damage. Then a silicone coating was applied over the acrylic coating for further protection against weather and possible damage.

For purposes of identification all of the load cells were stamped with numbers from one to ten. The four load cells on the four pre-tension load cables were numbered L-1 through L-4. The two load cells on each of the blade center support wires were numbered L-5 and L-7. The lead-lag wire load cell was stamped L-6 and the load cells on the guy support wires were numbered L-8 through L-10.

When mounted, the wires from the seven load cells on the rotor (L-1 through L-7) were all fastened to the shaft and run down to the bottom of the shaft. There they were wired to the stepper switch box which was fastened to the shaft just above the bottom collar on the shaft. The wires running from the three load cells on the guy wires (L-8 through L-10) were run down one of the guy wires and directly into the instrumentation room bypassing the stepper switch and slip rings, since these load cells do not rotate with the shaft.

Design and Construction of Torque Load Cell

A torque cell was designed for the purpose of measuring the torque output of the VAWT when in operation. From the torque output

and the angular velocity of the machine, the power output of the VAWT can be found.

The torque cell design had to be both sensitive enough to torque changes and also strong enough to be an integral part of the shaft. From the equation for shear stress for a hollow pipe,

$$\tau_{MAX} = \frac{16 M_t}{\pi d^3 \left(1 - \frac{d_1^4}{d^4}\right)},$$

and substituting approximate values, an inside and outside diameter was chosen for the torque cell.^{4.3} The torque cell was designed to have an inside diameter of 4.7625 cm (1.875 in) and an outside diameter of 5.08 cm (2.0 in) and was made of type 2024 T3 aluminum. The torque cell is shown in Fig. 4.2, just above the large pulley. The cell strain gage installation is given in Ref. 4.1.

In use, the torque load cell was located above the instrumentation package and above the pulley belt drive system, where the machine was loaded by the generator and/or the motor. The torque cell rotated with the shaft and the wire leads from the cell passed through slip rings which were located inside of the power slip ring assembly. The torque cell had its own set of slip rings, unlike the strain gages on the blades and wires, since the torque was desired to be constantly monitored.

The strain gage bridge on the torque cell was excited by a Brush model 13-4312-00 D.C. bridge amplifier, which also amplified the signal from the bridge. The amplifier output was connected to a voltmeter or strip chart recorder.

Shaft Gages

A full bridge was placed on the shaft to measure bending forces on the shaft during operation. The orientation of the gages is shown in Fig. 4.12. The gages were placed 45.72 cm (18 in) below the center of the shaft. The gages were similar to those on the load cells, and were bonded to the shaft with M-bond 200 epoxy. This strain gage configuration is relatively insensitive to axial and torsion loadings, and provides temperature compensation.

Application of Blade Gages

Strain gages were applied at certain strategic locations on one of the blades to monitor blade stresses during operation of the VAWT. Monitoring the blade stress was important because they were the weakest part in the design, and thus the governing factor in the maximum allowable running speed.

Using the calculations for the deflections, moments, and stresses on the blades developed with the theory previously described, the locations for the strain gages were selected. Sixteen gages were mounted in all; eight on one side of the blade and eight more at the same positions on the other side of the blade. All of the strain gages were placed on only one half of one of the blades, since the stresses, moments, and deflections were assumed the same for each half of the blade. The strain gage positions are shown on Fig. 4.13; gages B-1 through B-8 were mounted on the side of the blade facing the shaft.

Gage B-3 was mounted at the quarter chord 60.96 cm (24 in) from the end of the blade to measure the maximum compressive stress on

that side. Gages B-6 and B-2 were mounted on either side of B-3 at the quarter chord to see how the stresses decreased on both sides of the maximum compressive stress point and gages B-4 and B-5 were mounted behind B-3 to see what effect bending had near the trailing edge. Gage B-8 was located near the middle of the blade where the maximum tensile stress was located. Gages B-7 and B-1 were mounted in the position shown to fill out the rest of the stress curve. Gages B-9 through B-16 were mounted in the same positions on the other side of the blade in the positions shown in Fig. 4.13. They were placed as before to show the maximum stress points, but the tension and compression positions will be reversed. The reason for placing gages on both the top and bottom of the blade was that the stresses on the tension side and compressive side of a point on the blade were not the same. This was because the pre-loaded tension cables added an axial tension load on the blade.

The type of gages used were made by Micro-Measurements, and had a large grid 1.27 cm (1/2 in) by 0.635 cm (1/4 in). The large grid was needed to reduce the possibility of measuring concentrated load points on the skin due to the thinness of the skin on the blades. The gages were 120-ohm gages with a gage factor of $2.125 \pm 0.05\%$.

The gages were bonded to the surface of the blade and then were wired. A flat strip of 16 conductor color-coded 24 gage wire was glued to both sides of the blade near the trailing edge and was used to wire the gages. The strip ran the length of half of the blade and along the strut and to the stepper switch box. The gages were weather

coated with an acrylic coating. A coating of silicone was then applied on the gages and spread over the wires and to the trailing edge for further weather protection and to smooth out the surface of the blade aerodynamically.

Since full bridges had to be used at all test points, three dummy gages had to be employed for each of the 16 gages on the blade. Eight pieces of aluminum bars were used for this purpose. Three dummy gages were placed on each side of the bar and were placed in the stepper switch box. There they were wired to the stepper switches and to the corresponding active gages on the blade. The signals from the blade gages, as for the load cells, pass through the stepper switch box, then through the slip rings, and finally to the bridge amplifier.

Stepper Switch Box

A 22 position stepper switch was used to switch the bridge amplifier signal from one strain gage on the VAWT to another. The stepper switch was needed because only 10 slip rings were available for the instrumentation packet. Four of those slip rings were needed for the torque cell and would be used only for the torque cell, since torque was desired to be constantly monitored. Two more slip rings were used for the two potentiometers which measured pitch angles of the blade. The four remaining slip rings, enough for one full bridge, are used for the 16 blade gages, 10 load cells, and 1 shaft bridge.

This came to a total of 27, which was more than the 20 positions available on the stepper switch after it was wired. The stepper switch was wired to be able to use the 16 blade bridges and 4 other full bridges. It was decided to monitor the 16 blade gages, one of the load cells on the pre-tension wires, one on the lead lag wire, another on the flip wire and, lastly, the bridge on the shaft. This arrangement could be switched around to read other load cells if desired.

Along with the stepper switch mechanism, 48 dummy gages were located in the box. These were installed to be wired to the blade gages to make each of the 16 blade gages a full bridge, providing 3 gages for each gage on the blade. The full bridge arrangement provides complete temperature compensation, which was needed since the stepper switch mechanism and slip rings were being used. The reason for this was that when metallic gages and bridge switching are simultaneously employed, variations in switch resistance can be a problem. There exists the possibility that the change in switch resistance from one stepper switch to the next might be of the same order of magnitude as the strain gage resistance that is of interest, thus resulting in large errors in readings. It was therefore necessary that all the switches be placed completely outside the bridge circuit and complete bridges are switched into and out of the measuring system, which is known as interbridge switching. The bridge balance is not altered using interbridge switching even though the same variations in switch resistance occur as before. The only effect is that bridge sensitivity is altered slightly.

The problems encountered with slip rings are similar to the ones for switching. Factors such as ring and brush wear and changing contact temperature, also cause the need for the use of full bridges at all test points.^{4.5}

The stepper switch box was fastened to the shaft near the bottom, just above the bottom collar to which the pre-tension cables were attached. The leads from the gages were connected to it, connecting with the 4 power and signal leads that run inside the building through the slip rings at the base of the shaft, then to the control box and finally to the strain gage amplifier.

Calibrations

General

Calibrations were performed for the 10 load cells and the torque cell, and static load tests were performed on the assembled rotor. The static load tests were performed in the lab where the VAWT rotor was assembled and supported with the shaft in a horizontal position. The aerodynamic and centrifugal loads on the machine were simulated with weight placed on one of the blades.

Calibration of Load Cells

The load cells L-1 thru L-10 had to be calibrated before they could be installed on the rotor and guy supports of the wind turbine. Load cells L-1 thru L-4 were designed to be placed with the turnbuckles

and the cables of the four pre-tension cables on the rotor and would have tensions of up to 1818 kg (4000 lb). These four load cells were calibrated on a Baldwin Universal testing machine for loads ranging from 0 kg - 1818 kg (0 - 4000 lb) against the corresponding strain in μ in/in using a Budd strain gage indicator.

The load cells were tested with the eye bolts and turnbuckles connected to the load cells to simulate actual operating conditions and to observe whether the turnbuckles and eye bolts could undergo the high tension loads. The calibration curves were all linear. A sample calibration curve for load cell L-1 is shown in Fig. 4.14.

Load cells L-5 thru L-10 had to be calibrated only for maximum loads of 227.3 kg (500 lb). The load cells were calibrated by the method of dead weight testing. The calibration was done for load in pounds verses strain in μ in/in. A Budd Strain Gage Indicator was used to measure the strain. The results of the calibrations were linear curves of load vs. strain.

Torque Cell

The torque load cell was calibrated on a torsional testing machine. Calibration was done for the range of zero to 346.4 (cm-kg) (25 ft-lb) for both increasing and decreasing torque. Both strain and millivolt output were recorded. Both curves were very linear. Only the curve for millivolts vs. in-lb is shown on Fig. 4.15.

Testing

Rotor Static Lab Load Test Procedure

After the load cells were calibrated the VAWT rotor was then assembled in the aerospace lab facility in a horizontal position. The rotor was supported by wooden supports at each end of the shaft. One support was located under the bearing support at the top of the shaft and two other supports were located under the bearing casing near the bottom of the shaft. In this position the static lab load tests were conducted.

The purpose of static tests was to conduct a controlled study of the VAWT under static conditions, and most importantly, to study the effect of different loadings on the blades while varying the pre-tension loads in the tension cables. It was also desired to examine what effect the blade loadings would have on the pre-tension cables, the shaft, the lead-lag wires, and the two center support wires. Another piece of information desired was to determine the forces needed to actuate blade pitch change at different loading conditions. This information was necessary to design a future blade pitch actuating device. Finally, the information from the tests could be used for comparison with the theory from earlier blade analysis.

Sand bags and lead bars were used to apply an incremental uniform load on the lower blades. This was done to simulate the effect of aerodynamic and centrifugal loads that would normally act on the blades

when the machine was in operation. Because just one of the blades was loaded, the shaft was supported at the center using a hydraulic crane. The reason for doing this was that the shaft would otherwise deflect at the center due to the absence of an equal and opposite load on the other blade, as would occur in normal operation.

All of the 24 bridges in the rotor were monitored during these tests. This included the 16 blade gages, 1 shaft bridge, 4 load cells on pre-tension cables, 1 lead-lag wire load cell and 2 load cells on the center cables. The readings from the gages were recorded using strain gage amplifiers. A Vishey-Ellis digital strain gage amplifier with 10 channels was used to record 10 of the blade gage readings, and two Budd strain gage amplifiers, each with 10 channels, were used to record the rest of the data. Strain readings were taken for increasing and decreasing uniform loadings. The tests were performed for three different tensions in the pre-tension wires: 0 kg (0 lb), 227.3 kg (500 lb), and 454.5 kg (1000 lb).

Results of Static Load Tests

Typical data illustrating some of the more important results is given here. Data from each strain gage was compared for the three different pre-tension loads applied during the testing. This was done to study the effectiveness of the pre-tension loads and to study how the strain varied with the addition of blade loading. Also included in the results are comparisons between the static load tests and values obtained from the computerization of the theory for equivalent loading

conditions.

The first two graphs in the results (Fig. 4.16 and 4.17) are data obtained from load cells L-2, and L-3. The location of these gages are shown in Fig. 4.10. Load cells L-2 and L-3 are located on each of the two pre-tension wires that run between the shaft and the ends of the blade on which the uniform load was applied for the tests. It can be observed that the strain readings of the load cells increase with additional uniform load. This trend is especially noticeable for pre-tension loads of 0 kg (0 lb) and 227.3 kg (500 lb). But strain reading for both L-2 and L-3 at 454.5 kg (1000 lb) preload appears to decrease instead.

This tends to indicate that at higher loadings the turn buckles and wires were slacking, thus relieving the load on the wires. This problem was corrected by tightening the connections between the wires. Also from the two graphs it can be seen that increase in the blade loadings does not have any major effect on the tension wires which could cause failure of the cables.

Fig. 4.18 shows test results of load cell L-5, which is located on one of the center blade support wires as shown in Fig. 4.10. The graph shows that strain readings decrease for equivalent distributed loads with increased load in the pretension cables. This trend is in agreement with the theory developed in the program of Appendix C.

A graph for the required force to actuate the blade pitch change is shown in Fig. 4.19. The data is from the tests performed on the two wires with a spring scale and weights described in the test

procedure. Only a few data points were obtained, since the blade load was difficult to distribute evenly fore and aft of the blade quarter chord. Also, once the blade moved slightly, the load center of gravity moved from directly above the quarter chord, often causing a dumping of the load. Although the data has considerable scatter, it is useful to indicate the approximate magnitudes of the forces needed to change blade pitch under load. The differences in leading and trailing edge forces were probably caused by improper placement of the sandbags slightly to the rear of the quarter chord; the two forces would be equal if the load acted at the quarter chord. In practice, the proper design value to be used would probably be an average of that shown by the two curves.

The remaining graphs from the static load tests are based on the blade strain gages results and also blade stress theory. Figs. 4.20 to 4.25 are graphs of test results from the load tests corresponding to blade gages B-3, B-2, B-6, B-4, B-5, and B-8, respectively. All of these are located on the top side of the blade which faces the shaft (Fig. 4.13). The graphs were plotted for strain versus total distributed load for each of the three pre-tension loads tested: 0 kg (0 lb), 227.2 kg (500 lb) and 454.5 kg (1000 lb). Fig. 4.20 is a graph of data from strain gage B-3 located 60.96 cm (24 in) from the end of the blade. This is the location of the maximum compressive stress on that side of the blade. The results of the tests contain some scatter in data, especially at 454.5 kg (1000 lb) preload of tension in the cables, but in general the results are fairly linear. Figs. 4.21 and 4.22 are graphs of strain gage locations B-2 and B-6. These gages are located at 2.62 cm (3 in) on either side of strain gage

B-3 as shown in Fig. 4.13. The values of strain for these two gages should be slightly less than Fig. 4.20 for the same corresponding distributed loads and pretension loads, according to the theory. As it can be seen from the two graphs when compared to Fig. 4.20 this was not always true. This error can probably be contributed to the nonuniformities in the distributed loads represented by the sandbags and lead bars. In general the results are representative of the theory.

Figs. 4.23 and 4.24 are results of test data gathered from strain gages B-4 and B-5 respectively (Fig. 4.13). These gages are located at 60.96 cm (24 in), the same as B-3, but not at the quarter chord, as are the other gages. They were placed behind the quarter chord as shown to study the loading effect at these locations. The strain readings, as can be seen from the two graphs, are mostly less than Figs. 4.20, 4.21 and 4.22, which demonstrates that the major part of the load on the blade acts at the quarter chord. Fig. 4.25 is a graph of readings from blade gage B-8. The gage is located 5.08 cm (2 in) from the middle of the blade which is location of the maximum tensile and overall stress. The graph is fairly linear for tension loads of 0 kg (0 lb) and 227.2 kg (500 lb) but not so for the 454.5 kg (1000 lb) tension load; this could be caused by unevenness in the distributed load. The strain curves decrease with increasing tension loads, as should all of the figures. Deviations away from this trend are probably due to nonuniformities in the distributed load.

The strain gage readings of the bottom side of the blade are not included here, since the same trends occur but of opposite sign. However, the tension load on the bottom of the blade will be slightly larger in magnitude than the corresponding compression load on the top of the blade, due to the added axial pre-tension load on the blade.

The five graphs, Figs. 4.26 to 4.30 are plots of selected static load test results compared with corresponding data of the theory. Fig. 4.26 shows a graph of blades stresses at 60.96 cm (24 in) from the end of the blade on the top surface. The graph is plotted for stress versus total distributed load at 0 kg (0 lb) pre-tension loads. It can be seen that the test matches fairly well with the theory, especially at the lower distributed load values. This graph is for the blade location of maximum compressive stress on the top side of the blade. Fig. 4.27 is a similar curve illustrating the blade stresses at 157.48 cm (62 in) from the end of the blade. This is the location of the gage with the largest tension and overall stress values on the blade. The load test data and theory match extremely well.

The last 3 graphs of this section are Figs. 4.28, 4.29 and 4.30. These graphs are plotted for blade stress versus one half the blade length for a set distributed load and pre-tension. This was chosen to investigate how the overall blade stresses from the tests compare with the theory. Fig. 4.28 is plotted for a total distributed load of 176.4 kg (388 lb) and a pretension of 0 kg (0 lb). The test results match very well with the theory. The bottom of the test

curve is slightly flatter, which is probably due to extra weight added in the middle of the blade from the improper distribution of the sandbags. Fig. 4.29 shows a similar graph for 167.95 kg (369.5 lb) distributed load and 227.2 kg (500 lb) pre-tension load in the cables. The test curve generally has the same shape as the curve from the theory, but is displaced slightly lower. This difference is probably due to improper distribution of the sandbags. In general the test and theory agree well. The last graph of this section is of a total distributed load of 168.5 kg (370.75 lb) and a pre-tension load of 454.5 kg (1000 lb). Fig. 4.30 also agrees well with the theory. Again, the only real difference is at the blade location of 53.3 cm (21 in) to 68.58 cm (27 in) from the end. The error again is probably due to improper test load distribution.

Initial VAWT Outdoor Tests

The VAWT was taken outside to be assembled at the test site after the static load tests were finished. For ease of transportation the machine was disassembled and moved outside in sections. The shaft was placed on the roof of the instrumentation building along with its eight support legs. The stand, with its instrumentation, controls, generator, and starter motor, was then positioned in its place under the shaft inside the building and connected to the rotor shaft by an extension of the rotor shaft. Before the rest of the machine was assembled, initial spin tests were performed, using the motor to rotate the shaft.

Considerable effort was spent to assure the correct operation of the instrumentation and control systems. The guy wire adjustments were made, and set at 68 kg (150 lb) tension; this insured small guy wire vibrations with minimum tension to allow low thrust bearing forces and friction. The struts, cables, and blades of the machine were installed as spin tests progressed. Difficulties which developed included noise pickup from outside RF sources in the strain gage signals, inconsistent torque cell readings, and low torque cell sensitivity. These problems were solved by increasing signal cable shielding, repairing poor cable connections, and reducing the wall thickness of the torque cell from 0.3176 cm (0.125 in) to 0.1587 cm (.0625 in). New strain gages and wiring were installed on the torque cell.

Tests were also performed on the VAWT to evaluate the proper mechanical performance of the machine and its associated instrumentation system. The turbine was run by the electric motor during these initial tests.

A two-trace Brush recorder was used to monitor and record the wind velocity, which was measured with a cup-type anemometer, and also the torque readings from the torque cell. A d.c. tachometer generator, with a voltmeter attached was used to measure the angular velocity of the machine.

Performance measurements, per se, were not made during this initial testing of the VAWT. However, the method for determining the performance of a wind turbine is well known^{4.1} and will not be

repeated here. The measurement requirements to determine power coefficient are torque, rpm, air density, and wind velocity.

Some initial problems with vibrations from the shaft were encountered. It was initially uncertain what the higher limit of machine angular velocity would be. The reason for this was that mild vibrations in the shaft occurred between 120 and 125 rpm. After careful evaluation of vibration levels, it was determined that the machine could be run over 120 rpm. The machine has been successfully run to the motor's rpm limit of 143 rpm with vibrations damping out at values of rpm higher than 120-125.

During the performance testing a few of the strain gages on the blade were connected to the stepper switch and were monitored on a strain gage amplifier. This was done to study the actual loading effects on the blade during increases and decreases of rpm and wind velocity. The results of some of the tests were used for comparison with the static load test results for the same strain gages. Shown on the two graphs for strain gages B-4 and B-8 (Figs. 4.31 and 4.32) are strain vs. total distributed load on the blade. The graphs compare results for the static and dynamic tests at 0 lb load in the pretension wires. From the graphs it can be seen that the data compares very well, and both curves are fairly linear which gives an indication that the strain gages were functioning correctly. The good agreement adds confidence to the validity of the static testing method used on the rotor.

Instrumentation System

Torque Measurement

After the torque cell was modified (see previous section) by boring to achieve a thinner wall section, new strain gages were applied. The increased sensitivity gives a signal-to-noise ratio of about 10 to 20 without additional shielding, filtering, or differential amplification. The transducer output range is 0-490 mv for the torque range of 0-33.9 N-m (0-25 ft-lbs).

RPM Measurement

The turbine rotational speed is measured with a d.c. tachometer generator. Readout is with a digital voltmeter or strip chart recorder. The output range is 0-6800 mv for the RPM range of 0-150.

Wind Speed Measurement

Accurate wind speed measurement is the most important requirement of the instrumentation system. Since C_p is inversely proportional to the cube of the wind speed, measurement errors have a drastic effect on C_p values. The basic instruments used include a hot wire anemometer and two cup-type anemometers:

a) Hot Wire Anemometer - A Thermo-Systems Inc. Model 1366 ruggedized temperature compensated probe is used. Automatic temperature compensation is for relatively slow changes of environmental temperatures. The basic Thermo-Systems (TS) Series 1050 unit consists of a (TS) Monitor and Power System, Model 1052-6, and an Anemometer Module, Model 1050.

b) Cup Anemometer - The two cup anemometers are produced by Weathermeasure Corporation and are Model W121-SD. A 3-cup anemometer for wind speed, and an airfoil vane for wind direction, are both mounted on a common vertical axis. An A.C. generator attached to the shaft produces an output voltage directly proportional to wind speed.

c) Recorder - The anemometer readout device is a Clevite Brush Model Mark 260 6-channel chart recorder.

Prior to outdoor use, the cup anemometers were calibrated in the test section of the W.V.U. 0.81 x 1.14 m (32" x 45") subsonic wind tunnel. Fig. 4.33 shows the characteristics of the two anemometers for a range of speeds from 0-22.3 m/s (0-50 mph). Fig. 4.34 shows the characteristics of the hot wire anemometer which was calibrated in the same wind tunnel and using the same procedures as were used for the cup anemometers. Fig. 4.35 also includes the necessary information for conversion from indicated to true wind speed.

A six channel strip recorder is currently being used to record transducer output for wind speed, torque and RPM. Fig. 4.36 is a sample of recorder data taken during checkout of the wind speed sensors. Data is arranged as follows:

- Channel 1 - cup anemometer #1
- Channel 2 - cup anemometer #2
- Channel 3 - hot wire anemometer
- Channel 4 - torque
- Channel 5 - RPM

The information contained on the strip recorder, plus a knowledge of ambient temperature and barometric pressure, is sufficient to determine the C_p vs. TSR characteristics. Preliminary analysis may be performed immediately since calibration curves (as well as tabulated data) are available in the test instrumentation room. In addition, an off-line data reduction program has been developed and the Hewlett-Packard real time data reduction system^{4.1} will ultimately be linked to the instrumentation system.

The test instrumentation installed at the outdoor test site is shown in Fig. 4.37. The two cup anemometers and the hot wire anemometer are installed at the height of the mid point on the turbine blades. The tower upstream of the turbine rotor contains one cup and the hot wire anemometer, and is located about 3 turbine diameters away.

Conclusions

The WU Vertical Axis Wind Turbine Test Model has been designed and constructed, its instrumentation calibrated, and some initial testing performed. The limited initial testing has shown acceptable system operation. A discussion of design concepts, construction techniques and instrumentation has been given and should be of aid to persons interested in designing similar facilities. Static load tests and some dynamic load tests were performed, and good agreement obtained.

The WVU Vertical Axis Wind Turbine Test Model is a well instrumented test machine which will allow a flexible and useful test program to be accomplished.

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Fig. 4.1. WVU VAWT test model.

Fig. 4.2. Stand with generator, starter motor, and instrumentation/controls.

Chord = 0.174 m (6.85 in)
Maximum Thickness = 3.69 cm (1.06 in)
15.5% Thick Symmetrical Airfoil
Maximum Thickness at 30% Chord

Fig. 4.3. Test model airfoil cross-section.

Fig. 4.5. The blade supported by the two struts.

Fig. 4.6. Blade with two simple supports and pre-tension cables.

Fig. 4.7. Blade with three simple supports and pre-tension cables.

Fig. 4.8. Force body diagram of blade and its two equivalents.

Moment Diagram:

Deflection Curve:

Fig. 4.9. Moment and deflection diagrams.

Fig. 4.10. VAWT rotor with strain gage and load cell locations.

Fig. 4.11. Load cells with gage orientations.

Fig. 4.12. Shaft gage configuration.

B-1 through 8 on Top Half of Blade
 B-9 through 16 on Bottom Half of Blade

Fig. 4.13. Blade gages and their placement.

Fig. 4.14. Calibration curve for load cell #1.

Fig. 4.15. Torque load cell calibration.

Fig. 4.15. Rotor static load test results for load cell on tension cable L-2.

Fig. 4.17. Rotor static load test results for load cell on tension cable L-3.

Fig. 4.18. Rotor static load test results for load cell on center support wire L-5.

Fig. 4.19. Required force for blade pitch change vs. blade loading. (See text, pp. 82-83)

Fig. 4.20. Rotor static load tests for blade strain gage B-3.

Fig. 4.21. Rotor static load tests for blade strain gage B-2.

Fig. 4.22. Rotor static load tests for blade strain gage B-6.

Fig. 4.23. Rotor static load tests for blade strain gage B-4.

Fig. 4.24. Rotor static load tests for blade strain gage B-5.

Fig. 4.25. Rotor static load tests for blade gage B-8.

Fig. 4.26. Blade stresses at 0.6m (2 ft.) from blade end (zero pre-tension).

Fig. 4.27. Blade stresses at 1.53m (5.16 ft.) from blade end (zero pre-tension).

Fig. 4.28. Blade Stresses at 176.4 kg (388 lb) uniform load and 0 kg (0 lb) pre-tension load.

Fig. 4.29. Blade stresses at 168 kg (369.5 lb) uniform load and 227 kg (500 lb) pre-tension load.

Fig. 4.30. Blade stresses at 768.4 kg (370.75 lb) uniform load and 455 kg (1000 lb) pre-tension load.

Fig. 4.31. Rotor tests for blade strain gage B-4.

Fig. 4.32. Rotor tests for blade strain gage B-8

Fig. 4.33. Cup anemometer calibration.

Fig. 4.34. Hot wire anemometer calibration.

1. Calculate K or select from Fig. 1 for ambient P & T .
2. Set K value on index below.
3. Enter Fig. 2 horizontally at readout voltage.
4. Read indicated and true wind speed on upper and lower scales respectively.

Fig. 4.35. Wind speed calculation chart for cup and hot wire anemometers with corrected pressure and temperature.

Fig. 4.36. Sample data showing cup anemometer outputs (1 and 2), hot wire output (3), torque (4), and RPM (5).

Fig. 4.37. WVU VAWT outdoor test site with anemometer installation.

5. LOW SOLIDITY BLADE TESTS

Introduction

Information and technical data on straight bladed VAWTs are extremely limited. Thus, design data and "conventional" airfoil blade data are lacking which are needed for efficient c.c. blade design and meaningful interpretation of theoretical and test results. Results of WVU numerical solutions of a discrete vortex model (section 2 of this report) indicated that the power coefficient is a strong function of blade profile drag coefficient (Fig. 2.7). Unfortunately, this drag coefficient is difficult to predict for the blade flow where Reynolds number and centrifugal effects exist in a curved flow field. For this reason, emphasis has been placed on obtaining experimental results from an outdoor VAWT built at WVU. This turbine, described in detail in section 4 of this report and in refs. 5.1 and 5.2, has a 3.05m (10 ft) diameter, and the blades tested to date have a 3.25m (10 ft 8 in) span. The WVU VAWT has been designed primarily as a test device for various blade configurations, and therefore little attempt was made to obtain an optimum structural/aerodynamic compromise for the blade support structure. Thus far, blades of NACA 0015 airfoil sections and $C/R = 0.114$ have been tested (Fig. 4.1). This section describes test progress to date, and emphasizes the results of aerodynamic study of the blades. Such aerodynamic data must be known accurately to make any system/cost study meaningful.

Test Method

The form of the torque equation for the VAWT support structure can be derived^{5.3} as

$$T = T_S + k(\omega R)^2 \left(\frac{1 + \text{TSR}^2}{\text{TSR}^2} \right) . \quad (5-1)$$

Using $\epsilon = 1/\text{TSR}$, the inverse tip speed ratio, gives

$$T = T_S + k(\omega R)^2 (1 + \epsilon^2) . \quad (5-2)$$

In these equations T_S is the shaft bearing torque, and the remaining term is the torque caused by the aerodynamic drags of the rotor components; k is a constant, ω is the rotor angular velocity, and R is the rotor radius. Thus, the total torque is the sum of the shaft bearing torque plus an aerodynamic torque term related to the tip speed (ωR), and the inverse tip speed ratio ($\epsilon = 1/\text{TSR}$).

Test data (Fig. 5.1) confirms this relationship. The data is shown for the blade support structure, i.e., no blades are installed on the VAWT. The rotor and blade support structure is rotated using the electric motor installed at the base. The solid lines of Fig. 5.1 are least square fits to the data shown, and show excellent agreement with the form of eq. (5-2).

With the tare torques established, torque test data with blades installed can be reduced to yield the net blade torque (Fig. 5.2). Blade torque can then be presented in the form of power coefficient vs. TSR, or, as will be shown later, in the form

of blade drag coefficient. Fig. 5.2 shows a typical collection of data points for gross torque. In order to obtain reduced test data in the form of the power coefficient, C_p , which has the least amount of scatter due to the inaccuracies of wind velocity measurement, curves were faired through the torque data and C_p curves reduced from the faired data. Fig. 5.3 shows such data, where the symbols show calculated points as taken from the faired blade torque curves, and are not individual test points. Note that such curves are for the blade C_p , not the total C_p developed by the VAWT, which will be smaller (or negative) because of the retarding torque of the rotor shaft bearings and blade support structure. Thus, the resulting data allows analysis of the aerodynamics of the VAWT rotor blades, which is of primary interest.

Test Results

Blade Power Coefficient

Fig. 5.3 presents power coefficient data from experiments of the two bladed ($N_b = 2$) WVU VAWT with the low solidity blades (0015 airfoil sections) installed. The blade solidity was 0.114, and the angle, α' , equal to the angle between the blade chord and the tangent to the blade path, was 0° . (This is not necessarily the optimum blade setting angle.) The figure shows results at various values of tip speed ratio and rotor rpm, giving various values of rotor tip speed, ωR . The curves are in general agreement with

those of the theory developed for this case (Fig. 2.7), but are more peaked. The variation of C_p values with ωR suggests that perhaps both blade Reynolds number and centrifugal force effects are present. An increase in blade Reynolds number (increasing ωR) would be expected to reduce the blade profile drag coefficient; however, the data does not show the increase in C_p at the higher ωR values which would result. The possibility exists that the blade boundary layer centrifugal forces are influencing the airflow around the blade in such a way as to cause boundary layer separation at certain Reynolds numbers. This is discussed in more detail in section 3 of this report.

It should be noted that the "correct" value of blade profile drag coefficient, C_{D_o} , is difficult to deduce by comparison of the experimental results (Fig. 5.3) and the theoretical results (Fig. 2.7). Ashley^{5.4} suggests that a C_{D_o} value of about 0.015 to 0.017 would be appropriate to allow a reasonable agreement with his theory and experimental data from the SANDIA 2-meter turbine (curved blade of NACA 0012 section at an average Reynolds number of 2.9×10^5). The data shown in this report suggests a somewhat higher value for the blade setting tested ($\alpha' = 0^\circ$).

Although the theory and experiment do not show exact agreement at this time, consideration should be given to the range of blade C_p values produced by the VAWT test model. These values, approaching blade C_p 's of 0.4 (Fig. 5.3), are not optimum for several reasons:

1. The optimum fixed-pitch blade angle, α' , probably was not used.
2. The optimum turbine solidity probably was not used.
3. Variable cyclic blade pitch, or articulation, should improve performance considerably.
4. The optimum blade cross-section (airfoil shape) probably was not used.

In view of these considerations, the relatively high values of C_p obtained are encouraging.

Blade Drag Coefficient

Further analysis of the data can be made to determine a blade drag coefficient, for the special case of zero wind velocity. Using the coordinate system of Fig. 5.4 gives the following equation with torque as a function of theta:

$$T(\theta) = \frac{1}{2} \rho V_R^2 (2bC) R [C_L \sin(\theta - \beta) - C_D \cos(\theta - \beta) + C_{M_{c/4}} C/R] \quad (5-3)$$

is valid for any operating conditions. For the special case of zero wind velocity, $V_\infty = 0$, it follows that $\beta = \theta$ and $V_R = \omega R$; therefore, the torque becomes independent of θ :

$$C_D = - \left(\frac{T}{\frac{1}{2} \rho (\omega R)^2 2bCR} - C_{M_{c/4}} \frac{C}{R} \right) . \quad (5-4)$$

The blade drag coefficient is seen to be a function of the torque (T), air density (ρ), blade velocity (ωR), blade span (b), blade chord (C), rotor radius (R), and the blade moment coefficient about the quarter chord mounting point ($C_{M_{c/4}}$). If the quarter chord moment coefficient is assumed to be zero, which is valid for a thin symmetrical airfoil at low angles of attack in a uniform flow, the torque data for the blade (e.g., Fig. 5.2) can be converted to blade drag coefficient form. The results of this conversion is shown in Fig. 5.5, for the same data which produced Fig. 5.3. The drag data is shown as a function of the blade setting angle, α' , which is referenced at $\alpha' = 0$ for the blade chord set perpendicular to the blade support arm of the rotor.

The data (Fig. 5.5) show that as blade speed (and Reynolds number) increases, the drag coefficient increases, a perhaps surprising result. Also, the curve shapes are similar to those for a slightly cambered airfoil. Reconsideration of the assumption of zero moment coefficient about the symmetrical airfoil's quarter chord point should be made. The curved-path trajectory of the blade at the end of the rotating blade support arm causes a "virtual" camber effect to occur, even though the blade is physically symmetrical. The finite length of the blade chord results in different points along the blade chord being at larger radii from the axis of rotation than

the radius at the attachment point; thus, the local velocities along the blade, and their different resultant vector angles referred to the blade chord, result. The result is similar to that of a cambered airfoil operating in a uniform flow, as far as the aerodynamic characteristics of the airfoil are concerned. The effect is increased for increasing sizes of blade chord-to-radius ratios and at low TSR values. The effect of the blades having a non-zero moment coefficient would be to reduce the torque output which the operating VAWT could deliver, and to cause the effective value of C_D , as calculated with eq. (5-4), to be increased. This "virtual camber" effect appears to be of considerable importance in determining the VAWT blade aerodynamic characteristics.

Values of NACA 0015 airfoil minimum drag coefficients from several test sources are shown in Fig. 5.6, as a function of Reynolds number. Two dimensional wind tunnel test data obtained at WVU (circles) for a section of the small VAWT blade show the expected decrease in blade drag with increasing Reynolds number. The dashed line extends the data to that obtained by extrapolating NACA test data of Ref. 5.5. The Sandia two dimensional tunnel data^{5.6} is for a blade assumed to be smoother than that used in the WVU tunnel tests. Triangles indicate data reduced from the blade torque of the WVU VAWT. The smaller blades tested on the WVU VAWT show an increasing drag coefficient with Reynolds number, a trend opposite to that of the two dimensional data. The non-zero moment coefficient term, eq. (5-4), is not separated from the "drag" coefficients shown for the VAWT outdoor tests. Yet to be explained are the relatively low drag values obtained for the outdoor VAWT tests with $C/R = 0.114$,

and the increase with Reynolds number. It is hypothesized that the centrifugal force effect within the blade boundary layer flow is a factor.

Conclusions

A complex flow situation exists around the blades of a VAWT, where a combination of Reynolds number, centrifugal force, and virtual camber determine the aerodynamic characteristics of the blade and its "effective" drag. Present test results have not included optimum performance conditions. Blade power coefficients determined by experiment show fair correlation with the WVU theoretical calculation. However, theoretical prediction methods must be improved for design purposes. A more complete knowledge of the characteristics of VAWT blades with conventional airfoils is needed to allow meaningful comparison with c.c VAWT blades.

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Fig. 5.1. Typical test results for VAWT support structure counter torque (tare torque).

Fig. 5.2. Plot illustrating the method of obtaining net blade torque from VAWT test data.

Fig. 5.3. Experimental VAWT blade power coefficients at various blade rotational speeds.

Fig. 5.4. Coordinate system for VAWT blade.

Fig. 5.5. Drag coefficients of small chord NACA 0015 airfoil from torque characteristics of VAWT outdoor test model.

Fig. 5.6. Minimum drag coefficients for a NACA 0015 airfoil from wind tunnel and WVU VAWT outdoor test model experiments.

6. HIGH SOLIDITY BLADE DESIGN

Introduction

In order to better evaluate the aerodynamic performance of the VAWT and the accuracy of the theoretical performance calculations, two blades of higher solidity were designed and fabricated. These blades were designed to produce a solidity of 0.26, while the initial blades (see sections 4 and 5 of this report) gave a solidity of 0.11.

Another feature to be evaluated was the concept of using aerodynamic means, i.e., a tail surface, to achieve blade cyclic pitch changes. The tail surfaces would be actuated by electrical solenoids in order to change the tail surface angle relative to the blade. The solenoids would be actuated to produce two tail pitch positions per cycle. While other actuation methods might be preferable for production models, this actuation method was relatively easy to construct for test purposes. It should be noted that blade pitch control using an aerodynamic surface has the potential advantage of being able to hold the blade near an aerodynamically optimum angle of attack throughout the rotation. This, together with the possible reduction of forces required to change blade pitch, made the tail surface concept an attractive one for study.

This section of the report gives information on the high solidity blade physical characteristics and construction methods. Details

of the measurements of blade and tail moments of inertia are discussed, together with results of the testing of the tail-position actuator system. An analysis of the response times of the actuator system is given, with results of initial rotational tests of the blades performed in order to verify their structural integrity. Calculations giving estimates of the dynamic tail forces required to articulate and produce blade tracking are also included.

Blade Physical Characteristics

The large solidity blades are constructed using a NACA 0015 airfoil section and have a span of 3.25 m (10 ft 8 in) and chord of 0.395 m (15-9/16 in). They are constructed of aluminum with a single spar at the quarter chord and covered with a skin 0.08 cm (0.032 in) thick (see Figs. 6.1 and 6.2). Mounting blocks attached to the spar at either end are tapped for 1.11 cm (7/16 in) bolts. When mounted the blades can rotate about the quarter chord line. Each blade has a weight of about 15.4 kg (34 lb) (without tail and counterweights) and has its center of gravity about 5.84 cm (2.3 in) behind the pivot line. Figs. 6.3 and 6.4 show the blades (without tails) installed on the WVU VAWT.

There are two supports (Fig. 6.2) on each blade that extend 0.24 m (9-1/2 in) behind the trailing edge and along the chord line for mounting the tails. The pivot point of the tails may be positioned at any one of five positions from the pivot point of the blade. Two solenoids are located on the spar of each blade and

are used to rotate the tail, using pull rods, either direction from the chord line of the blade.

The tail has a chord of 13.02 cm (5.125 in) span of 0.81 m (31.75 in), maximum thickness of 1.91 cm (0.75 in) at a point 4.06 cm (1.6 in) from the leading edge, a leading edge radius of 0.20 cm (0.08 in), and a weight of 0.79 kg (1.75 lb). This tail is not balanced, i.e., its center of gravity is not at its quarter chord point.

The blades as stated above are not balanced and must have mass added forward of the quarter chord point in order to be balanced. With the tails in place it was found that a weight of 6.12 kg (13.5 lb) must be added at a distance of 21.84 cm (8.6 in) in front of the pivot. This weight was added in the form of two conical lead weights of 3.06 kg (6.75 lb) each at the ends of the span (Fig. 6.2). Once these counterweights were added to the blades the moment of inertia, I_{yy} , about the line of the quarter chord was measured.

Measurement of the Moment of Inertia, I_{yy}

The moment of inertia of the blade, tail, and counterweights was measured by rotating the blade about its pivot line (quarter chord) with falling weights.

A small rod 3.18 cm (1.25 in) in diameter was attached to the support shaft of the blade. A chord was wrapped around this rod and

connected to a mass, M . When the mass accelerated under the action of gravity the blade rotated. The equation of motion of the blade is

$$I_{yy} \alpha = \Gamma , \quad (6-1)$$

where I_{yy} is the moment of inertia of the blade, α is the angular acceleration of the blade, and Γ is the torque applied to the blade. The equation of motion of the mass, M , is

$$Ma = Mg - T \quad (6-2)$$

where a is the acceleration of the mass, g is the acceleration of gravity, and T is the tension in the cord.

Eqs. (6-1) and (6-2) are connected by

$$RT = \Gamma \quad (6-3)$$

and

$$R \alpha = a , \quad (6-4)$$

where R is the radius of the rod. Using eqs. (6-3) and (6-4) and Newton's third law of motion, one may solve for the acceleration of the system. The acceleration is given by

$$a = \frac{MR^2 g}{I_{yy} + MR^2} \quad (6-5)$$

Since $a = \frac{ds^2}{dt^2}$, where s is the distance the mass moves and t is time, one may find s by integration of eq.(6-5) for the case where

$ds/dt = 0$ and $s = 0$ at $t = 0$. The result of this integration is

$$s = \frac{MR^2 g}{2(I_{yy} + MR^2)} t^2 . \quad (6-6)$$

Solving eq. (6-6) for I_{yy} gives

$$I_{yy} = \frac{MR^2 g t^2}{2s} - MR^2 . \quad (6-7)$$

Eq. (6-7) was used to calculate the moment of inertia. The blade was rotated through one, two, and three complete revolutions with masses (weights) of 1.81, 2.27, 3.18, 3.63, and 4.08 kg (4, 5, 7, 8, and 9 lbs). It was found that the I_{yy} 's for each number of revolutions were about the same but differed from those of another number of revolutions. This difference was due to the drag of the blade as it moved through the air. The actual moment of inertia of the blade less drag was found by plotting I_{yy} as a function of number of revolutions and extrapolating to zero revolutions. The plot of this data is given in Fig. 6.5. The value of I_{yy} from a least-squares fit of this data is 0.68 kg-m^2 (0.50 slug-ft^2).

The moment of inertia of the blade, tail, and counterweights as measured above was found to be 0.68 kg-m^2 (0.50 slug-ft^2). The contributions of the three main components of this system were calculated and are given as follows:

Moments of Inertia, $I_{yy} \sim \text{kg-m}^2 \text{ (slug-ft}^2\text{)}$

Blade	0.22 (0.161)
Tail	0.17 (0.122)
Counterweights	0.29 (0.217)
<hr/>	
Total	0.68 (0.500)

These values are based on the tail in the middle position, i.e., 0.454 m (17-7/8 in) from the pivot point of the blade.

While the moment of inertia of the blade is relatively small compared to the blade, tail, and counterweight system, it is misleading since the blade itself is not balanced, and, as will be shown later, cannot be used without counterweights.

Testing

Tail Actuator Testing

The tail may be rotated out of the plane of the chord of the blade by a two solenoid actuators located in the blade. These actuators are connected to the tail by pull rods with threaded sleeves. The tail angle, α_T , measured as the angle between the chord planes of the blade and tail, may be changed by as much as $\pm 20^\circ$ by the actuators.

The time needed to move the tail through $2 \alpha_T$ ($-\alpha_T$ to $+\alpha_T$) was measured by simulating the signals to the solenoids. The solenoids were driven by symmetric square wave signals. The time relationship of these signals is shown here:

Every t seconds the tail would be moved from one position to another through an angle of $2 \alpha_T$.

The actuator signal and the tail position signal, as derived from a position potentiometer on the tail's rotational axis, were recorded on two channels of a chart recorder. The difference in rise times between the two signals represents the time lag of the tail actuator system. This time lag was found to be independent of the period ($2t$) of the square wave but did vary with applied actuator voltage. The results of this experiment are given here:

<u>Solenoid Voltage ~ RMS</u>	<u>Tail Actuator Response Time ~ ms</u>
60	51
90	44
120	33

The angle through which the VAWT rotor turns during the time the tail is in motion may be calculated and will depend on the rotor angular velocity. A plot of the response angle, θ_r , as a function of angular velocity (rpm) for the three actuator voltages is shown in Fig. 6.6.

Rotational Testing of Blades without Tails

The tails were removed from the blades and the weight of the tails simulated by lead weights attached to the tail support members. In this configuration the blades are balanced and have about the same moment of inertia as with the tails in place. The blades were mounted on the VAWT (Figs. 6.3 and 6.4) and rotational tests were conducted.

The rotor was rotated from about 50 rpm to 100 rpm in 5 rpm steps by the electric motor, and observed for any unusual vibrations or behavior. After running smoothly up to 100 rpm a vibration was noted after a short period of time at 100 rpm, and the rotor was stopped. This vibration seemed not too unusual since past tests with other blades had uncovered vibrations in the 100 to 110 rpm ranges. It was decided to move quickly through this range and continue the testing at 115 rpm and above. Everything was smooth from 115 rpm up to 130 rpm. A very severe vibration abruptly appeared as the speed was being changed from 130 to 135 rpm. Also, it was noticed that parts of the blade were becoming dislodged. The rotor was stopped, and upon inspection of the blades it was found that the bolts holding the counterweights had failed. It was apparent that much stronger bolts would have to be used.

The blades were removed from the VAWT. It was decided that the blades could be run at fixed pitch conditions without counterweights and they were remounted on the VAWT in this configuration.

Rotational tests as described above were performed with a slight vibration noted in the 100-110 rpm range. However, a position potentiometer on one of the blades indicated that the blade angle, α_B (the angle made by the chord line of the blade and the tangent line to the blade's circle of rotation), was changing. Furthermore, when the rotor was stopped the angle did not return to zero, its initial setting. Upon inspection it was found that the center cable mounting fixtures at the main shaft had slipped. These fixtures were repaired and the testing continued.

With no slippage of the center support cables the blade angle still varied with rotor speed but returned to the zero position when the rotor was stopped. The movement of the blade (α_B) with rotor rpm apparently has its origin in the unbalance of the blade. The torque placed on the blade by the unbalanced condition is given by

$$\Gamma(\alpha) = \alpha^2 R \cos \omega_B (\Delta mr), \quad (6-8)$$

where ω is the angular velocity of the rotor, R is the radius of the rotor, and Δmr is the mass moment needed to balance the blade. In this case Δmr is 3.20 kg-m (0.205 slug-ft), R is 1.524 m (5 ft), and $\cos \alpha_B \approx 1$ for the small angles under consideration. Using this data a plot (Fig. 6.7) of $\Gamma(\omega)$ as a function of α_B was made. The data show that the variation of $\Gamma(\omega)$ with α_B is linear and does not pass through the origin.

The non-zero interception of Fig. 6.7 is thought to be due to pre-load forces in the center cables. As the torque increases the center cables change their loadings, one increasing and the other decreasing. At some point the cable attached to the leading edge of the blade is no longer in tension and the blade starts to twist. The twist is not along the pivot line of the blade, but rather along lines from the pivot points to the point where the center support cable is attached near the trailing edge of the blade.

The changes in α_B observed in the tests of the unbalanced blade are too large to be acceptable in any work on the aerodynamics of these blades.

Therefore, the blade counterbalances will be redesigned for increased strength and retested to evaluate the amount of blade twist prior to aerodynamic blade testing. The blade angular position potentiometer, located at one end of the blade, will be used to monitor the blade twist and insure that it is an acceptable amount.

Estimation of Dynamic Tail Forces Required to Articulate and Produce Tracking of the Blades

Estimation of Articulation Tail Forces

The tail will be rotated through an angle of $2\alpha_T$ (say $-\alpha_T$ to $+\alpha_T$) at about 90° through the blade's orbit and then the reverse rotation will occur at about 270° . The tail is designed to produce a force, F_T , that will transmit a torque to the blade and rotate it

through an angle $2\alpha_B$ (say $+\alpha_B$ to $-\alpha_B$). The tail force must be large enough to move the blade through the angle $2\alpha_B$ in a time period that is small compared to the period of rotation of the blade.

The moment of inertia of the blade, tail, and counterweights is 0.68 kg-m^2 (0.5 slug-ft^2) and the equivalent moment of inertia of other hardware that must move with the system is about 0.04 kg-m^2 (0.03 slug-ft^2). Thus the tail must produce torque that will move a system with a moment of inertia of 0.72 kg-m^2 (0.53 slug-ft^2). The equation of motion for the rotational motions and the blade is

$$\Gamma_B = I_{yy} \ddot{\alpha}_B, \quad (6-9)$$

where Γ_B is the torque on the blade, I_{yy} is the total moment of inertia of the system, and $\ddot{\alpha}_B$ is $d^2\alpha_B/dt^2$. If it is assumed that the angular acceleration of the blade is linear and positive for one-half the time period during which the blade is in motion, and linear and negative for the second one-half of the period, the angular displacement of the blade is given by

$$\alpha_B = 1/2 \ddot{\alpha}_B (t/2)^2, \quad (6-10)$$

where t is the total time of the blade's motion. Solving eq. (6-10) for $\ddot{\alpha}_B$ gives

$$\ddot{\alpha}_B = \frac{8 \alpha_B}{t^2}. \quad (6-11)$$

If it is assumed that the blade is to respond, i.e., move from $+\alpha_B$ to $-\alpha_B$, in 1/10 of an orbit (36°), then the time, t , will be given by

$$t = 1/10 \frac{2\pi}{\omega} , \quad (6-12)$$

where ω is the angular velocity of the blade in radians per second. Combining eqs. (6-11) and (6-12) and substituting into eq. (6-9), the torque can be written as

$$\Gamma_B = I_{yy} \frac{200 \alpha_B \omega^2}{\pi^2} . \quad (6-13)$$

The following listing gives values of Γ_B for angular velocities of 30, 60, 90 and 120 rpm for the case where I_{yy} is 0.72 kg-m^2 (0.53 slug-ft^2):

<u>Γ_B, Torque Required ~ Nm (ft-lb)</u>			
<u>$\omega \sim \text{rpm}$</u>	<u>$\alpha_B = 3^\circ$</u>	<u>$\alpha_B = 6^\circ$</u>	<u>$\alpha_B = 9^\circ$</u>
30	15.1 (11.1)	30.1 (22.2)	45.1 (33.3)
60	60.2 (44.4)	120.4 (88.8)	180.6 (133.2)
90	135.4 (99.9)	270.9 (199.8)	406.3 (299.7)
120	240.8 (177.6)	481.6 (355.2)	722.4 (532.8)

The force, F_T , which the tail must produce when located a distance, S_T , from the pivot point of the blade, is given by

$$F_T = \Gamma_B / S_T . \quad (6-14)$$

Using eq. (6-13), the tail force may be written as

$$F_T = \frac{200}{\pi^2} \frac{I_{yy} \alpha_B \omega^2}{S_T} . \quad (6-15)$$

The following gives values of F_T for the corresponding values of Γ_B from the previous listing and using $S_T = 0.14 \text{ m}^2$ (1.5 ft^2):

$\omega \sim \text{rpm}$	F_T , Tail Force Required $\sim \text{N (lb)}$		
	$\alpha_B = 3^\circ$	$\alpha_B = 6^\circ$	$\alpha_B = 9^\circ$
30	32.9 (7.4)	65.8 (14.8)	98.7 (22.2)
60	131.7 (29.6)	263.3 (59.2)	395.0 (88.8)
90	296.2 (66.6)	592.4 (133.2)	888.7 (199.8)
120	526.6 (118.4)	1053.3 (236.8)	1579.9 (355.2)

The values of Γ_B and F_T are based on the assumption of constant acceleration and most likely will not represent the actual values. However, the order of magnitude of these values will be correct and thus will give an indication of the dynamic response of the blade.

Estimation of Tracking Tail Forces

Aside from the tail force required to articulate the blade, the tail must produce forces throughout the orbit of the blade to keep the angle of attack of the blade constant. The size of these forces may be estimated by calculation of the torque the blade must experience. The blade angle for tracking, α_B , is a function of the turbine tip-speed ratio, TSR, and the orbital angular position, ωt . Letting $\epsilon = 1/(\text{TSR})$, where $\text{TSR} = \omega R/V_\infty$, V_∞ is the free stream speed of the air upstream of the turbine, ω is the angular velocity of the rotor and R is the radius of the rotor, the functional relationship for α_B is

$$\alpha_B = \alpha - \omega t + \tan^{-1} \left[\frac{\sin \omega t - \epsilon}{\cos \omega t} \right], \quad (6-16)$$

where α is the angle of attack of the blade.

The angular acceleration, $\ddot{\alpha}_B = d^2 \alpha_B / dt^2$, is found by differentiation of eq. (6-16) and is given by

$$\ddot{\alpha}_B = \epsilon \omega^2 \cos \omega t \left[\frac{(1 - \epsilon^2)}{(1 + \epsilon^2 - 2 \epsilon \sin \omega t)^2} \right]. \quad (6-17)$$

The torque needed to produce tracking, Γ_B^T , is given by

$$\Gamma_B^T = \ddot{\alpha}_B I_{yy} = I_{yy} \omega^2 \cos \omega t \left[\frac{(1 - \epsilon^2)}{(1 + \epsilon^2 - 2 \epsilon \sin \omega t)^2} \right]. \quad (6-18)$$

The corresponding tail force, F_T^T , for tracking may be expressed as

$$F_T^T = \frac{\Gamma_B^T}{S_T} = \frac{I_{yy} \epsilon \omega^2 \cos \omega t}{S_T} \left[\frac{(1 - \epsilon^2)}{(1 + \epsilon^2 - 2 \epsilon \sin \omega t)^2} \right]. \quad (6-19)$$

A plot of tail forces, F_T^T , as a function of orbital angular position, for various values of tip speed ratios is given in Fig. 6.8. The strong effect of increasing dynamic pressure at higher TSR values in reducing the tail force required for blade tracking is easily seen.

A much more thorough analysis of blade tracking using aerodynamic control, i.e., tail surfaces, is underway, and will be completely documented in ref. 6.1 and subsequent project reports.

Conclusions

Two high solidity (0.26) blades with a symmetrical airfoil section have been fabricated and installed on the WVU VAWT. Aerodynamic tests will be conducted to evaluate the blade characteristics for comparison with those obtained for the low solidity blades and the theoretical results. Provisions have also been made to evaluate aerodynamic blade pitch change and blade tracking using a tail surface. The moments of inertia of the blade assemblies have been measured, and an analysis conducted of the aerodynamic (tail) forces required to accomplish blade pitch control and tracking. Effort will continue on the analysis of this concept.

References

6.1 Hoy, D. L., "Theoretical Study of Vertical Axis Wind Turbines Using Aft Tail for Blade Tracking", M.S. Thesis, West Virginia University, Morgantown, WV, December, 1977.

Fig. 6.1. High solidity blade under construction.

Fig. 6.2. Completed high solidity blade with tail and counterweights.

Fig. 6.3. High solidity blades being installed on WVU VAWT test model.

Fig. 6.4. High solidity blades installed on WVU VAWT test model.

Fig. 6.5. Test data extrapolation to find inertia, I_{yy} .

Fig. 6.6. Response angle vs. angular velocity.

Fig. 6.7. Blade angle vs. torque for unbalanced blade.

Fig. 6.8. Tail force vs. orbit angle at various tip speed ratios, $\omega = 60$ RPM.

7. COST ANALYSIS OF VERTICAL AXIS WIND TURBINES

Introduction

At the present time, it is not possible to establish definite costs for various sized vertical axis wind turbines. Optimum turbine configurations have not yet been determined, and the exact power produced with such machines is rather uncertain. Optimization of various design parameters, such as machine size, structural materials, etc., remains to be accomplished. However, the cost of the WVU test model has been determined during its construction. Table 7-I summarizes these costs; Table 7-II gives a detailed breakdown of individual items. These figures do not include the research instrumentation unique to the test model, but do include the generator required to produce electrical power and the control system required to perform blade attitude changes. Several items included in this cost analysis were available without cost, such as the generator and low solidity (~ 0.11) conventional airfoil blades; purchase costs have been estimated and included in the analysis.

Cost of the WVU Test Model

Several items should be considered when reviewing the cost analysis. The test model is a singular unit, and no economy could

be attained by using mass-production techniques. The test model is a research machine, and so in many cases the machine is over-designed in the sense that parts were fabricated to provide versatility and large safety factors so that various future tests may be performed. Also, in order to expedite construction, many items were purchased from vendors where minimum time delays would occur, and, therefore, not necessarily at the lowest possible cost. Some extra costs were incurred due to the site (building roof and surrounding terrain) selected, which could be minimized with other sites.

In order to initiate testing as soon as possible, surplus blades, which were originally designed for helicopter use, were installed on the test model. These blades consisted of a symmetrical airfoil section 15.5% thick with a 0.174 m (6.85 in) chord. Two such blades installed on the test model resulted in a rotor solidity of ~ 0.11 . These straight blades are of simple construction, consisting of a c-shaped extrusion which forms the leading edge of the airfoil shape, and to which is bonded an aluminum skin having the top and bottom surfaces riveted together at the trailing edge. Due to this simplicity of design, the costs were estimated on a unit weight basis as being \$5/lb. Even if this estimate is considered to be low by a factor of two or three,

the cost of these relatively small blades would still only be about 10% of the cost of the complete VAWT.

In order to test the aerodynamic performance of the VAWT and to compare results with theoretical calculations, higher solidity blades have been designed and fabricated. Section 6 of this report describes the design and construction of these blades in detail. They consist of conventional light-aircraft type usage of a main spar, aluminum ribs, and riveted structure. The blade chord is 0.397 m (15.625 in) which gives the two bladed rotor a solidity of 0.26. Construction costs of these blades have not yet been completely determined, but the more extensive labor involved will result in higher costs per unit weight than the smaller blades. This cost will be determined in the near future.

Thus far, it appears that the cost of the actual blades for the straight-bladed rotor, will be much less than that for the curved blade Darrieus VAWT. However, for larger machines the relative percentage of the blade cost might be expected to increase. Also, the cost of the rotor for the straight bladed design must include the costs of blade support arms and other supports; thus final total costs may be comparable.

Circulation Control System

Fabrication of two circulation controlled turbine blades is

also underway. These blades will provide the same rotor solidity (0.26) as the larger symmetrical blades. Construction will be similar, with the sharp blade trailing edge replaced with the round contour and air slots required for the blowing of the circulation control compressed air. This added complexity will increase the blade cost. However, since the blades are straight, mass production techniques would significantly lower the blade cost.

The compressed air system required to provide the blowing air consists of a compressor, ductwork, an air supply "slip-ring" which pressurizes the main rotor shaft, and solenoid valves which control the timing of the air on-and-off cycle to the blade slots. This system is currently under construction, and complete cost breakdowns will be available in the near future.

Cost Estimation of an Operational VAWT Blade Design

It is recognized that for a large operational VAWT, the cost of the blades as a percentage of the total cost will probably be much greater than for the small test models utilized thus far. In order to obtain information which will lead to an evaluation of the economic feasibility of straight-bladed VAWTs, the Allegany Ballistics Laboratory of Hercules Incorporated is participating in the current research effort. The scheduled work includes a) a parametric blade analysis (weight and cost), b) a blade design analysis, c) a

system structural analysis, and d) estimation of facility costs.

The approach selected to accomplish this analysis is summarized in the following list of sequential steps:

1. Find the influence of VAWT operating conditions, material properties, and design features on blade weight and cost.
2. Screen various blade designs for structural efficiency and cost for both conventional and circulation controlled blade types.
3. Select a VAWT operating point and prepare prototype blade designs and analysis.
4. Prepare layouts for the VAWT tower, arms, etc., and design for structural adequacy.
5. Prepare cost estimates of the developed design using the best available performance predictions.

To date, step 1 has been accomplished and detailed results are in section 8 of this report.

Table 7-III shows some results of step 2 studies by ABL. Structural design conditions and production costs are shown for blades of various configurations and materials. Blade costs are seen to vary by about a factor of 2, emphasizing the need for careful optimization of blade design in order to reduce costs. In general, the composite material blades are of significantly less weight per unit length, and would reduce the structural requirements for blade struts and other supporting structure. This would further reduce overall machine costs.

Conclusions

A detailed cost analysis has been performed for the WVU VAWT test model, in order to gain insight as to what approximate costs would be likely for production models of similar configurations. The use of aluminum and composite material blade construction has been studied, and the initial results show promise that the use of the composite materials will be cost-effective, and will also allow machine operation at higher rotational speeds (see section 8 of this report). The overall costs are on the order of those given for other wind turbine designs (see section 9 of this report). Exact performance data must be obtained for the WVU VAWT test model in order to allow a determination of the initial turbine cost per unit of power output. When this is accomplished a meaningful cost comparison may be made with other machines.

TABLE 7-I

WVU VAWT TEST MODEL COST SUMMARY

<u>ITEM</u>	<u>MATERIAL</u>	<u>LABOR</u>	<u>TOTAL</u>	<u>% OF TOTAL</u>
Support Stand	\$ 486	\$ 638	\$1124	23%
Rotor *	744	712	1456	28
Blades *	164	60	224	4
Guy Wire System	492	181	673	13
Power Generating System and Control System	1073	450	1523	29
Assembly and Painting	33	160	193	4
	<hr/>	<hr/>	<hr/>	<hr/>
Totals	\$2992	\$2161	\$5153	100%
	<hr/>	<hr/>	<hr/>	<hr/>
% of Totals	58	42	100%	
	<hr/>	<hr/>	<hr/>	

*Two conventional symmetrical airfoil blades, 0.174 m (6.85 in) chord.

TABLE 7-II

WVU VAWT TEST MODEL COST BREAKDOWN

<u>ITEM</u>	<u>MATERIAL</u>	<u>LABOR</u>	<u>TOTAL</u>
SUPPORT STAND:			
Building pads (4)	\$ 110	\$ 85	\$ 195
Back up plates (4)	12	15	27
Bolts, nuts	8	-	8
Drill concrete walls	12	40	52
Ground wire	2	1	3
Ground rod	15	5	20
Stand leg angles (8)	86	20	106
Bearing casing and assembly	66	200	266
Bearings (2)	124	-	124
Bearing casing caps (2)	8	20	28
Bearing casing retaining rings	5	20	25
Bearing casing grease fittings	1	5	6
Bearing casing tabs (8)	10	10	20
Attachment bolts	10	1	11
Leg bottom angles (4)	10	5	15
Drill bottom angles/pads	-	15	15
Cut roof holes (3)	-	40	40
Roof flashings (3)	7	20	27
Roof sealing	-	136	136
	<hr/>	<hr/>	<hr/>
	\$ 486	\$ 638	\$1124
ROTOR:			
Main shaft	\$ 34	\$ 250	\$ 284
Shaft top extension	30	10	40
Top bearing cup	15	40	55
Grease fitting-top bearing	1	2	3
Arm shaft fittings (2)	40	40	80
Arm inner inserts (4)	8	10	18
Arms (4)	40	10	50
Arm outer inserts (4)	10	30	40
Arm bearing blocks (4)	40	40	80
Blade bearings (4)	86	-	86
Arm bearing block covers (4)	1	5	6
Arm bearing block fairings (4)	19	5	24
Arm fairing cement	4	-	4
Arm block guy fittings (4)	2	5	7
Bolts	4	-	4
Actuator rings (2)	50	20	70
Actuator ring bearings (2)	50	-	50
Actuator ring shaft sleeve	5	10	15

TABLE 7-II

(continued)

<u>ITEM</u>	<u>MATERIAL</u>	<u>LABOR</u>	<u>TOTAL</u>
ROTOR: (cont'd.)			
Pretension cables (4)	\$ 15	\$ 5	\$ 20
Pretension shaft fittings (2)	40	30	70
Blade end fittings, clamps (4)	10	30	40
Pretension turnbuckles (4)	6	10	16
Torsion wires (8)	20	-	20
Torsion wire struts (4)	8	-	8
Torsion wire clamps	2	-	2
Torsion wire turnbuckles	4	-	4
¹ Pitch actuator mechanism	100	100	200
Pitch actuator cables (4)	10	-	10
Actuator clamps, bearings	16	-	16
Actuator turnbuckles	4	-	4
Brake disk	20	20	40
Brake actuator	40	-	40
Brake actuator attachment	10	40	50
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	\$ 744	\$ 712	\$1456
BLADES (2):			
Bearing inserts (4)	\$ 2	\$ 20	\$ 22
End bolts (4)	2	-	2
² Center cuffs (2)	10	40	50
Blades (2)	150	-	150
	<hr/>	<hr/>	<hr/>
	\$ 164	\$ 60	\$ 224
GUY WIRE SYSTEM:			
Rotor top cap	\$ 30	\$ 40	\$ 70
Rotor top bearing	15	-	15
Guy poles (3)	120	10	130
Concrete	71	-	71
Pole installation	-	80	80
Pole guy ground screws (6)	45	10	55
Pole caps	15	-	15
Guy wire (220')	110	30	140
Guy wire clamps, thimbles	40	-	40
Ground rods (2)	30	10	40
Ground wire	1	1	2
Turnbuckles (10)	15	-	15
	<hr/>	<hr/>	<hr/>
	\$ 492	\$ 181	\$ 673

TABLE 7-II

(continued)

<u>ITEM</u>	<u>MATERIAL</u>	<u>LABOR</u>	<u>TOTAL</u>
POWER/CONTROL SYSTEM:			
Shaft extension	\$ 10	\$ 5	\$ 15
Rainshield	2	5	7
Base bearing	45	10	55
Generator stand	170	40	210
Generator speed increaser	60	80	140
Generator belts	15	-	15
³ Generator	200	-	200
Selsyn control/stand	50	100	150
Wind sensor/controls	100	200	300
Tachometer and drive	21	10	31
Electronic speed control	400	-	400
	<hr/>	<hr/>	<hr/>
	\$1073	\$ 450	\$1523
ASSEMBLY AND PAINTING:			
Crane	\$ -	\$ 130	\$ 130
Paint	33	30	63
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	\$ 33	\$ 160	\$ 193

NOTES:

1. Based on completed design.
2. Surplus item - cost based on \$5/#.
3. Surplus item - cost based on replacement.

TABLE 7-III

COMPARISON of SOME HYBRID and COMPOSITE DESIGNS for STRAIGHT VAWT BLADES

BLADE MODEL	DESIGN			ADEQUACY			BLADE COST FOR LOW RATE PRODUCTION (\$)*	TOOLS & EQUIPMENT COST \$
	MATERIAL	DESIGN ALLOW PSI	MODULUS (PSI X 10 ⁶)	MARGIN OF SAFETY	DEFLECTION	NOMINAL WEIGHT (LBS/FT)
2024-T4 ALUMINUM	10,000	10.3	.26	1.6	32	14,200	225,000
2024-T4 ALUMINUM 30° E GLASS	7000 7600	10.3 3.4	.3	1.7	40	15,000	190,000
30° E GLASS	7600	3.4	.6 SKIN NO STRENGTH	4	21	9700	160,000
30° E GLASS 0° E GLASS	7600 10,000	3.4 6.7	.68	2.75	22	13,400	162,000
30° E GLASS	7600	3.4	.6	4	25	16,000	195,000
UNI GRAPHITE 30° E GLASS	44,000 7600	20 3.4	2.0	2	19	18,000	162,000
2024-T4 ALUM. TUBE 30° E GLASS	7000 7600	10.3 3.4	.18	1.9	80	19,000	160,000
SPRUCE	900	1.5	.3	1.4	95	11,400	100,000

BLADE CONFIGURATION

* NO TOOL PRORATED COST. INCLUDES END FITTINGS AND BALANCE WEIGHTS

8. STRUCTURAL PARAMETRIC ANALYSIS OF VAWT BLADES

Introduction

This section of the report details works performed by the Allegany Ballistics Laboratory of Hercules, Inc., a subcontractor to WVU providing structural analysis services. The objective of this task was to find the influence of operating conditions, material properties and design features on blade mass/unit length and cost/unit length. For both functional and economic requirements, blades are the most critical component of wind energy conversion systems (WECS). A principal reason for studying vertical axis wind turbines (VAWT) of the straight blade type is the conceptual simplicity of the blade, expected to provide both lower cost and higher efficiency.

A systematic study of blade mass/unit length and cost/unit length as affected by configuration, material, and process is essential to a rational WECS development program. The work of this task is to provide the initial step in this direction for VAWTs. This is the first work on the subject including advanced materials and processes in the author's knowledge, and has special value in presenting a fabricator's (Hercules, Inc.) position prior to design fixation by systems contractors, thereby tending to avoid problems later in the program. In particular, the notion that the cheapest material provides the least cost blade, or more to the point, the least cost electricity, is challenged.

This report presents results of structural modeling and parametric analyses of blades for VAWTs which aid in the selection of material and

configuration on the basis of cost, performance, and produceability. Initial parametric studies based on a generalized structural model of the blade provide plots of blade mass/unit length and cost/unit length as a function of blade length for various design features and materials. The generalized blade model provides a closed form equation for mass/unit length and cost/unit length given the design conditions, material properties, cost factors, and a parametric description of the blade geometry.

The remainder of this section will discuss the development of the model and results obtained thereby.

Model Development

Loads

Using the aerodynamic theory developed by WVU, blade loadings under all operating conditions were determined. Blade loading as a function of the orbital position of the blades about the axis with reference to the wind direction was obtained for both rated, 8.05 m/sec (26.4 ft/sec), and gust, 26.82 m/sec (88 ft/sec), conditions.

For the purposes of blade design, only the design-controlling loads were sought; i.e., peak operating loads associated with rated or gust wind conditions. A conservative estimate of these loads can be obtained from a streamtube lifting body theory using a fixed blade (Darrieus mode) machine model.

Using the geometry notation in Fig. 8.1, the forces/unit length on the blade, resolved parallel to chord (x) and normal to chord (y) are

$$F_x = -L \sin \alpha + D \cos \alpha \quad ,$$

and
$$F_y = L \cos \alpha + D \sin \alpha - m \omega^2 R$$

Replacing L by $(1/2 \rho_A V_{REL}^2 C) C_L$, D by $(1/2 \rho_A V_{REL}^2 C) C_D$, and non-dimensionalizing by $1/2 \rho_A V_{\infty}^2 C$ yields the following form, where ρ_A is the density of air and m is the blade mass/unit length:

$$C_{F_x} = \left(\frac{V_{REL}}{V_{\infty}} \right)^2 \left[-C_L \sin \alpha + C_D \cos \alpha \right] , \quad (8-1)$$

$$C_{F_y} = \left(\frac{V_{REL}}{V_{\infty}} \right)^2 \left[C_L \cos \alpha + C_D \sin \alpha \right] - \frac{2m}{\rho_A R C} \left(\frac{\omega R}{V_{\infty}} \right)^2 , \quad (8-2)$$

where $(V_{REL}/V_{\infty})^2$ is found from solution of the wind triangle

$$\left(\frac{V_{REL}}{V_{\infty}} \right)^2 = 1 + \left(\frac{\omega R}{V_{\infty}} \right)^2 - 2 \left(\frac{\omega R}{V_{\infty}} \right) \sin \theta , \quad (8-3)$$

and the angle of attack is

$$\alpha = \tan^{-1} \left[\frac{\cos \theta}{\frac{\omega R}{V_{\infty}} - \sin \theta} \right] . \quad (8-4)$$

Note that for high tip speed ratios $\left(\frac{\omega R}{V_{\infty}} > 2 \right)$, the angle of attack remains well behaved and in the range of initial linear approximations for the lift coefficient, C_L (see Fig. 8.2). However, for low tip speed ratios, associated with gusts, the angle of attack takes on all possible values, and lift and drag coefficients must be estimated correspondingly.

In Fig. 8.3 lift coefficient data for the NACA-0015 airfoil are plotted. The bold line represents the curve fit used to model the data for design load computation. Fig. 8.4 presents similar data for the drag coefficient.

The critical loads are those which produce bending about the chord-wise axis. Therefore, the load coefficient calculated from eq. (8-2) is required to calculate the critical bending loads. Eq. (8-2) can be

divided into its two components, aerodynamic and centrifugal:

$$C_{F_{ya}} = \left(\frac{V_{REL}}{V_{\infty}} \right)^2 \left[C_L \cos \alpha + C_D \sin \alpha \right] , \quad (8-5)$$

$$C_{F_{yc}} = \frac{2m}{\rho_A RC} \left(\frac{\omega R}{V_{\infty}} \right)^2 . \quad (8-6)$$

Careful examination of eqs (8-1) to (8-6) yields the fact that

$$C_{F_{ya}} = f \left(\frac{\omega R}{V_{\infty}} \right)$$

and

$$C_{F_{yc}} \propto \frac{1}{RC} \left(\frac{\omega R}{V_{\infty}} \right)^2 , m$$

Redefining the load coefficients so that positive loads are defined radially outward,

$$F_{ay} = -C_{F_{ya}} , \quad (8-7)$$

and

$$F_{cy} = \frac{2}{\rho_A} \left(\frac{\omega R}{V_{\infty}} \right)^2 . \quad (8-8)$$

Then the chordwise blade load/unit length can be written as

$$w = 1/2 \rho_A V_{\infty}^2 C \left[F_{ay} + \frac{m F_{cy}}{RC} \right] . \quad (8-9)$$

The normalized load coefficients defined by eqs. (8-7) and (8-8) were calculated for tip speed ratios between 0 and 7.5 and orbit angles each 10° beginning at 5° (F_{ay}). Using those results, the orbital maximum or peak value of F_{ay} can be determined for each tip speed ratio. These maximum normalized load coefficients are plotted vs. tip speed ratios in Fig. 8.5. Using these normalized load coefficients in conjunction with eq. (8-9) , the maximum bending loads on the VAWT can then be determined.

Blade Model and Analysis

For these studies the blade was idealized as a beam subject to loading in analytical functional form along the span. Using fundamental beam theory, blade mass/unit length (kg/m) and cost/unit length (\$/m), estimates were determined for the VAWT blade. The structural model for the blade is shown in Fig. 8.6.

Using fundamental beam theory, an expression can be developed for the total mass/unit length of the blade as a function of the operating conditions, material properties and design features.

In order to develop a model which can be used to parametrically characterize the "straight-bladed" VAWT blades, it is necessary to make assumptions which result in a tractable model. The assumptions incorporated in the VAWT blade model are listed below:

- *1. The mass of the spar is proportional to the total mass of the blade.
- *2. All of the spar mass is concentrated in the spar caps.
- *3. The mass of the ribs and all internal parts other than the spar is equal to the mass of the skin.
- *4. A fraction of the skin contributes to the bending stiffness of the blade. In this study it is assumed that 25% of the skin contributes to the bending stiffness.

*Note-Assumptions 1 through 4 are based on Hercules, Inc. experience with similar composite material designs. Such assumptions should be checked when detailed designs are developed.

5. The blade is a simply supported beam with uniform loading.
The blade loading consists of the constant centrifugal loads and the variable aerodynamic loads resolved normal to the blade chord.
6. The cross sectional wind blade shape is a 15% symmetrical airfoil (NACA 0015).
7. The failure criteria are:
 - (a) Stress: Controlled by the allowable strength of the unidirectional spar material.
 - (b) Deflection: Controlled by an appropriate deflection limit for a simply supported blade. For this study the maximum deflection is assumed to be 1% of the total blade length.

From Assumption #1:

$$m = k_w m_{\text{spar}} \quad , \quad (8-10)$$

where

m = mass/unit length of the blade, kg/m (slugs/ft) ,

m_{spar} = mass/unit length of the spar, kg/m (slugs/ft) ,

k_w = proportionality constant, dimensionless .

From Assumption #2:

$$m_{\text{spar}} = 2 A_c \rho_c \quad , \quad (8-11)$$

where

A_c = area of one spar cap, m^2 (ft^2) ,

ρ_c = mass density of spar, kg/m^3 (slugs/ ft^3) .

From Assumption #3:

$$m = m_{\text{spar}} + 2 m_{\text{skin}} \quad , \quad (8-12)$$

where

$$m_{\text{skin}} = \text{mass/unit length of the blade skin,} \\ \text{kg/m (slugs/ft) .}$$

From Assumption #4:

$$I_{\text{skin}} = 2 k_B A_{\text{skin}} \left(\frac{T}{2} \right)^2 \quad (\text{For this study, } k_B = 0.25) \quad , \quad (8-13)$$

where

$$I_{\text{skin}} = \text{effective moment of inertia of the blade} \\ \text{skin, } m^4 \text{ (ft}^4\text{)} \quad ,$$

$$A_{\text{skin}} = \text{cross sectional area of one blade skin,} \\ m^2 \text{ (ft}^2\text{)} \quad ,$$

$$k_B = \text{proportionality constant, dimensionless .}$$

Combining the above equations,

$$m = k_w m_{\text{spar}} \\ = m_{\text{spar}} + 2 m_{\text{skin}} \quad .$$

Then

$$(k_w - 1) m_{\text{spar}} = 2 m_{\text{skin}} \quad , \\ (k_w - 1) 2 A_c \rho_c = 2 (2 A_{\text{skin}} \rho_{\text{skin}}) \quad ,$$

where

$$\rho_{\text{skin}} = \text{mass density of the blade skin, kg/m}^3 \\ \text{(slugs/ft}^3\text{)} \quad .$$

Therefore

$$A_{\text{skin}} = \frac{1}{2} \left(\frac{\rho_c}{\rho_{\text{skin}}} \right) (k_w - 1) A_c \quad . \quad (8-14)$$

Calculating the flexural rigidity of the blade,

$$\overline{EI} = 2 A_c \left(\frac{T}{2} \right)^2 E_c + 2 k_B A_{\text{skin}} \left(\frac{T}{2} \right)^2 E_s \quad ,$$

where

E_c, E_s = modulus of elasticity of the spar and skin material, respectively, Pa (lb/ft²) .

Substituting from eq. (8-14),

$$\overline{EI} = \frac{1}{2} A_c T^2 E_c \left(1 + \beta_1 \frac{E_s}{E_c} \right) , \quad (8-15)$$

where

$$\beta_1 = \frac{1}{2} k_B \left(\frac{\rho_c}{\rho_{\text{skin}}} \right) (k_w - 1) .$$

Failure Criterion: Stress

Using classical beam theory, the maximum bending stress can be determined in both the spar and the skin:

$$\sigma_c = \frac{E_c M_{\max} \left(\frac{T}{2} \right)}{\overline{EI}} , \quad (8-16)$$

$$\sigma_s = \frac{E_s M_{\max} \left(\frac{T}{2} \right)}{\overline{EI}} ,$$

where

σ_c, σ_s = maximum bending stress in the spar and skin, respectively, Pa (lb/ft²) ,

$M_{\max}$ = maximum bending moment, Nm (lb-ft) .

For a simply supported beam under a uniform load (Assumption #5)

$$M_{\max} = \frac{wL^2}{8} , \quad (8-18)$$

where

w = uniform blade load/unit length, N/m (lb/ft) ,

L = blade length, m (ft) .

For the vertical axis wind turbine system, the blade loads consist of the centrifugal and aerodynamic loads resolved normal to the blade chord:

$$w = \frac{1}{2} \rho_A V_\infty^2 C \left[F_{ay} + \left(\frac{m}{RC} \right) F_{cy} \right] f_s , \quad (8-19)$$

where

C = blade chord, m (ft) ,

R = orbit radius, m (ft) ,

ρ_A = mass density of air, kg/m³ (slugs/ft³) ,

V_∞ = upstream wind velocity, m/sec (ft/sec) ,

F_{ay} = normalized aerodynamic load coefficient (dimensionless) ,

F_{cy} = normalized centrifugal load coefficient, m³/kg (ft³/slug) ,

f_s = general safety factor (dimensionless) .

For this study, the factor of safety, f_s equals 2.0 .

In order to calculate the total blade mass/unit length, the failure criterion must be applied to both the spar and skin to determine the critical component. Assuming that the blade skin will not fail catastrophically, but rather will craze and/or yield, then the blade spar, which is the major structural element, can be considered to be the critical component.

Substituting eqs. (8-18) and (8-19) into (8-16) yields

$$\sigma_c = \frac{\rho_A V_\infty^2 C L^2 \left[F_{ay} + \left(\frac{m}{RC} \right) F_{cy} \right] f_s}{16 A_c T \left(1 + \beta_1 \frac{E_s}{E_c} \right)} .$$

Substituting eqs. (8-10) and (8-11) into the above equation yields

$$\sigma_c = \frac{\rho_A V_\infty^2 C L^2 k_w \rho_c \left[F_{ay} + \left(\frac{m}{RC} \right) F_{cy} \right] f_s}{8 m T \left(1 + \beta_1 \frac{E_s}{E_c} \right)}$$

Define

$$k_T = \frac{T}{C}$$

Rearranging,

$$m = \frac{1}{8} \left(\frac{k_w}{k_T} \right) \left(\frac{\rho_c}{\sigma_c} \right) \rho_A V_\infty^2 L^2 \frac{\left[F_{ay} + \left(\frac{m}{RC} \right) F_{cy} \right] f_s}{\left(1 + \beta_1 \frac{E_s}{E_c} \right)}$$

Therefore

$$m = \frac{\beta_2 f_s F_{ay}}{(1 - \beta_2 f_s F_{cy}/RC)},$$

where

$$\beta_2 = \frac{1}{8} \left(\frac{k_w}{k_T} \right) \left(\frac{\rho_c}{\sigma_c} \right) \frac{\rho_A V_\infty^2 L^2}{\left(1 + \beta_1 \frac{E_s}{E_c} \right)}$$

Failure Criterion: Deflection

For a simply supported beam, uniformly loaded, the maximum deflection is given as

$$\Delta_{\max} = \frac{5wL^4}{384 EI},$$

where

$$\Delta_{\max} = \text{maximum deflection, m (ft)}$$

Substituting eqs. (8-15) and (8-19) into the above equation yields

$$\Delta_{\max} = \frac{5 \rho_A V_{\infty}^2 C L^4 \left[F_{ay} + \left(\frac{m}{RC} \right) F_{cy} \right] f_s}{384 A_c T^2 E_c \left(1 + \beta_1 \frac{E_s}{E_c} \right)}$$

Substituting eqs. (8-10) and (8-11) yields

$$\Delta_{\max} = \frac{10 \rho_A V_{\infty}^2 C k_w \rho_c L^4 \left[F_{ay} + \left(\frac{m}{RC} \right) F_{cy} \right] f_s}{384 m T^2 E_c \left(1 + \beta_1 \frac{E_s}{E_c} \right)}$$

Therefore

$$m = \frac{\beta_2 f_s F_{ay}}{(1 - \beta_2 f_s F_{cy}/RC)},$$

where

$$\beta_2 = \frac{10}{384} \left(\frac{k_w}{k_T} \right) \left(\frac{\rho_c}{E_c} \right) \left(\frac{L^2}{T \Delta_{\max}} \right) \frac{\rho_A V_{\infty}^2 L^2}{\left(1 + \beta_1 \frac{E_s}{E_c} \right)}$$

Blade Model Analysis Results

The preceding analysis allows the mass/unit length of the blade to be determined using either a stress criterion or a deflection criterion. The following equations summarize the results of the analysis.

The expression for the mass/unit length of the blade can be written as

$$m = \frac{\beta_2 f_s F_{ay}}{1 - \beta_2 f_s F_{cy}/RC},$$

where

$$\beta_2 = \frac{1}{8} \cdot \left(\frac{k_W}{k_T} \right) \left(\frac{\rho_C}{\sigma_C} \right) \frac{\rho_A V_\infty^2 L^2}{\left(1 + \beta_1 \frac{E_S}{E_C} \right)}, \quad \text{Stress Criterion}$$

or

$$\beta_2 = \frac{10}{384} \left(\frac{k_W}{k_T} \right) \left(\frac{\rho_C}{E_C} \right) \left(\frac{L^2}{T \Delta_{\max}} \right) \frac{\rho_A V_\infty^2 L^2}{\left(1 + \beta_1 \frac{E_S}{E_C} \right)}, \quad \text{Deflection Criterion}$$

and

$$\beta_1 = k_B \left(\frac{\rho_C}{\rho_S} \right) (k_W - 1).$$

Also,

- m = mass/unit length of the blade, kg/m (slugs/ft) ,
- k_W = ratio of total blade mass to spar mass ,
- k_T = thickness to chord ratio ($k_T = T/C$) ,
- k_B = fraction of skin contributing to blade bending stiffness ,
- E_S = modulus of skin in span direction, Pa (psi) ,
- E_C = modulus of spar (cap) in span direction, Pa (psi) ,
- ρ_C = specific mass of spar (cap) material, kg/m³ (slugs/ft³) ,
- ρ_S = specific mass of skin material, kg/m³ (slugs/ft³) ,
- σ_C = allowable strength of spar material, Pa (psi) ,
- $\Delta_{\max}$ = maximum deflection of blade, m (ft) ,
- ρ_A = mass density of air, kg/m³ (slugs/ft³) ,
- L = blade length, m (ft) ,
- V_∞ = up-stream air speed, m/sec (ft/sec) ,
- F_{ay} = normalized aerodynamic load perpendicular to the blade chord (dimensionless) ,
- F_{cy} = normalized centrifugal load perpendicular to the chord, m³/kg (ft³/slug) ,
- C = chord, m (ft) ,

R = orbit radius, m (ft) ,

f_s = general factor of safety (dimensionless) .

For this study the factor of safety, f_s , equals 2.0 .

Cost Model

In addition to the above structural model used to determine the blade mass/unit length, an approximate model was also developed to compare the cost/unit length (\$/ft) of the VAWT blades. The cost model can be expressed by the following relationship:

$$P = (R_1 + k_p R_2) m + R_3 C ,$$

where

P = price/unit length of VAWT blade f.o.b. fabricator's plant, \$/m (\$/ft) ,

m = blade mass/unit length, kg/m (slugs/ft) ,

C = blade chord, m (ft) ,

R_1 = cost/unit mass of raw material, including wastage factor, \$/kg (\$/slugs) ,

R_2 = prefabrication manufacturing cost/unit mass, \$/kg (\$/slugs) ,

k_p = fraction of blade which is prefabricated (dimensionless) ,

R_3 = assembly cost/unit area, \$/m² (\$/ft²) .

The fabrication processes and associated cost factors used in this parametric analysis are listed below:

Material	Fabrication Description	R_1 (\$/kg)	R_2 (\$/kg)	R_3 (\$/m ²)	k_p
2024-T6 Aluminum	Extruded Spar, Riveted Skin	4.41	1.10	48.44	0.50
E-Glass Composite	Filament Wound Spar/Skin	1.65	6.61	26.91	0.90
S2-Glass Composite	Filament Wound Spar/Skin	4.41	6.61	26.91	0.90
Kevlar Composite	Filament Wound Spar/Skin	15.43	6.61	26.91	0.90
Graphite Composite	Filament Wound Spar/Skin	52.91	6.61	26.91	0.90

This model was representative of costs in 1976 for quantity produced articles and was checked against known prices for filament wound helicopter spars and wind turbine blades. Comparisons between materials will change as their prices fluxuate.

Blade Materials

In this parametric study, the materials considered for the VAWT blades are composite materials. Composite materials were chosen for consideration for several reasons. First, composite materials have a higher specific strength (α/ρ) and specific modulus (E/ρ) than traditional blade materials (i.e., aluminum). Therefore, the use of composite materials will result in VAWT blades which are lighter and stiffer than traditional metallic blades. Second, the fabrication processes associated with composite materials lend themselves readily toward the manufacture of "straight" VAWT wind blades. In particular, the pultrusion process can be used to form the unidirectional spar, then the blade skins can be formed using filament winding techniques. The cost factors included in this parametric study reflect the costs associated with a pultruded spar/filament wound skin fabrication process.

The composite materials considered in this study are: S2-Glass, E-Glass, Kevlar-49 and Graphite composites. The nominal static material properties for these candidate blade materials are listed in Table 8-I. The design approach criteria used in the parametric study are summarized in Fig. 8.7. In Table 8-II, the degradation factors are tabulated for temperature and static and dynamic fatigue environments. The design allowables are listed for the static and dynamic criteria in Tables 8-III and 8-IV, respectively. The results indicate that for the design criteria associated with VAWT blades, the critical material properties are essentially governed by the dynamic requirements.

In the parametric study, the candidate composite blades are compared with a traditional metallic blade composed of 2024-T6 Aluminum. The material properties for 2024-T6 Aluminum are:

Modulus of Elasticity: 71.0 GPa (10×10^6 psi) .

Tensile Strength: 448 MPa (65,000 psi) .

Dynamic Fatigue Allowable: 96.5 MPa (14,000 psi).
(Allowable strength representative of aluminum skin,
 10^8 cycles, with spanwise rivet holes)

Density: 2768 kg/m³ (5.37 slugs/ft³).

Results and Conclusions

Using the blade model previously described, a parametric study was conducted to characterize the "straight" VAWT blades. The blade mass/unit length was determined for composite (S2-Glass, E-Glass, Graphite, and Kevlar-49) and metallic (2024-T6 aluminum) blades for the conditions of interest. The conditions which were considered in this study were

1. Normal Operating Conditions.

This condition should occur routinely. For this condition, the blade components are designed for fatigue life (stress criterion) and deformation limit (deflection criterion) subject to rated wind speeds, $V_\infty = 8.05$ m/sec (26.4 ft/sec).

2. Gust Conditions.

This condition is not expected to occur frequently. For this condition the blade components are designed not to yield (stress criterion) under the gust conditions, $V_\infty = 26.82$ m/sec (88 ft/sec), which occurs during blade operation. This condition provided a means of checking the sensitivity of the system operating parameters to a strength criterion as well as a fatigue criterion.

Having determined the blade mass/unit length, the blade cost/unit length can then be determined using the cost model previously described. In Figs. 8.8 through 8.11 the blade mass/unit length is shown for the composite blades, and Fig. 8.12 presents the results of the aluminum blade for comparison. In these figures, the blade mass/unit length was plotted as a function of the blade length for various operating tip speed ratios. These results represent blades under normal operating conditions (rated wind conditions). Only the results for the stress criterion (fatigue) are presented for the blades subject to normal operating conditions. In all cases considered, the critical blade mass/unit length was obtained using the stress criterion as opposed to the deflection criterion. The results obtained using the deflection criterion are similar to those from the stress criterion with the exception that the deflection criterion yields consistently "lighter" blade structures.

Also presented in Figs. 8.8-8.12 are the minimum blade mass/unit length and the corresponding minimum blade cost/unit length for each of the composite blades and the metallic blades. The minimum blade mass/unit length and the minimum blade cost/unit length are representative of the "lightest" structural blade which can be fabricated consistent with the assumptions inherent in the blade model and accepted fabrication practice.

It is clear from these results that minimum fabrication practices will dominate mass and cost of rotor blades over a wide range of design conditions. For this reason least cost raw materials and highly mechanized fabrication methods are basic objectives for future development. Designs

will be modified to sacrifice material performance to reduce either or both material cost or fabricating cost.

The general shape of the curves presented in Fig. 8.8-8.12 indicates the critical relationship between the blade mass/unit length and the blade length. In general, for small blade lengths the blade mass/unit length is nearly constant for a given tip speed ratio. As the blade length increases, the blade mass/unit length also increases but at an accelerating rate. As the blade length is increased, a point is reached at which further increase in the blade length will yield infeasible results. The last statement indicates that, consistent with the blade model and its assumptions, for a given tip speed ratio ($\omega R/V_\infty = \text{constant}$) there exists a maximum length blade.

The same general critical relationship exists between the blade mass/unit length and the tip speed ratio ($\omega R/V_\infty$); that is, for the given blade length ($L = \text{constant}$), there exists a maximum safe tip speed ratio.

Based on the results presented in Fig. 8.13 the blade fabricated from Kevlar composite yields the minimum blade mass/unit length. However, the blades which yield the lowest cost/unit length would be those fabricated from E-glass composite. In order to properly evaluate the selection of the blade material based on either minimum mass/unit length or minimum cost/unit length, it is necessary to consider the effect of blade mass/unit length on the cost of the entire VAWT. If the blade mass/unit length has a marked effect on the cost of the entire VAWT system, then it may be necessary to

use the more expensive blade materials to minimize the cost of the entire system. Results of the systems study planned for future work should provide the data to answer this question. In summary it can be seen that those materials which exhibit the highest specific strength (σ_{all}/ρ) will yield the "lightest" blade mass/unit length.

In Fig. 8.14, the results are compared for blades designed for operation under normal operating conditions and gust conditions. The blade mass/unit length for the gust condition is plotted versus the blade mass/unit length for the normal operating conditions (stress criterion). Results in this figure should be evaluated in terms of the "dashed" line which represents equal blade mass/unit length predictions from both the normal operating conditions (rated) and the gust conditions. Results which fall above the "dashed" line represent blade designs which are governed by the gust conditions, whereas data which fall below that line represent blade designs which are governed by the normal operating conditions (stress criterion).

Examination of the results presented in Fig. 8.14 reveals that, in general, VAWT blades fabricated from either S2-glass, E-glass and graphite composites or 2024-T6 aluminum should be designed based on normal operating conditions, whereas those blades fabricated from Kevlar composite should be designed based on gust conditions. The results do indicate that at low tip speed ratios ($\omega R/V_\infty$ rated ≈ 1.0) the blades for all the materials considered, should be designed based on gust conditions; however, under normal operating conditions the tip speed ratios ($\omega R/V_\infty$ rated) of practical interest

are greater than 1.0 and such low values are only associated with gusts. In summary, it can be concluded that except for Kevlar composites, the design of VAWT blades fabricated from any of the materials considered in this parametric study (S2-glass, E-glass, graphite, and 2024-T6 aluminum) should be based on the normal operating conditions. The design of VAWT blades fabricated from Kevlar composites should be based on the gust conditions.

Fig. 8.1. Description of fixed blade rotor geometry ($\alpha_0 = 90^\circ$).

Fig. 8.2. Angle of attack vs. orbit angle; darrieus mode operation.

Fig. 8.3. Full range section lift coefficients for the NACA-0015 airfoil at reynolds numbers of 0.36×10^6 , 0.50×10^6 and 0.68×10^6 .

Fig. 8.4. Full range section drag coefficients for the NACA-0015 airfoil at reynolds numbers of 0.36×10^6 , 0.50×10^6 and 0.68×10^6 .

Fig. 8.5. Maximum normalized load coefficients vs. tip speed ratio.

Fig. 8.6. Blade structural analysis model.

$3\sigma = 10\%$ based on ERDA specifications

$2\sigma = \begin{cases} 10\% \text{ S2-Glass, E-Glass, and Metals} \\ 20\% \text{ Kevlar-49 and Graphite} \end{cases}$

Factor of Safety = 2.0 + Degradation due to temperature and static/dynamic fatigue environment

Environment:

Maximum Operating Temperature = 50°C

Load Cycles = 10^8 cycles

Endurance = 10^5 hours

Fig. 8.7. Design criteria.

Fig. 8.8. Blade mass/unit length and blade cost/unit length vs. blade length for S2-glass composites (normal operating conditions).

Fig. 8.9. Blade mass/unit length and blade cost/unit length vs. blade length for E-glass composites (normal operating conditions).

Fig. 8.10. Blade mass/unit length and blade cost/unit length vs. blade length for graphite composites (normal operating conditions).

Fig. 8.11. Blade mass/unit length and blade cost/unit length vs. blade length for kevlar-49 composites (normal operating conditions).

Fig. 8.12. Blade mass/unit length and blade cost/unit length vs. blade length for 2024-T6 aluminum (normal operating conditions).

Fig. 8.13. Comparison of blade mass/unit length vs. blade length for composites and aluminum with $\omega R/V_\infty = 4.0$.

Fig. 8.14. Blade mass/unit length (gust conditions) vs. blade mass/unit length (normal operating conditions).

TABLE 8-I

NOMINAL STATIC MATERIAL PROPERTIES
CANDIDATE BLADE MATERIALS

(T = 25°C, V_f = 60%)

Property	Kevlar-49 Composite	Graphite Composite	S2-Glass Composite	E-Glass Composite
Longitudinal Tensile Modulus, E ₁₁ (GPa)	75.8	124	53.8	46.2
Transverse Tensile Modulus, E ₂₂ (GPa)	4.1	6.9	20.7	12.4
Poisson's Ratio, ν ₁₂	0.34	0.27	0.25	0.25
In-Plane Shear Modulus, G ₁₂ (GPa)	2.1	5.9	6.2	6.2
Longitudinal Tensile Strength, F _{11T} (MPa)	1310	1517	1558	827
Transverse Tensile Strength, F _{22T} (MPa)	17.2	41.4	40.0	35.9
Longitudinal Compressive Strength, F _{11C} (MPa)	310	1103	593	621
Transverse Compressive Strength, F _{22C} (MPa)	96.5	124	124	138
In-Plane Shear Strength, F ₁₂ (MPa)	20.0	55.2	34.5	41.4
Composite Density (kg/m ³)	1384	1495	1993	2076

TABLE 8-II

DEGRADATION TO VARIOUS ENVIRONMENTS ⁽¹⁾

Material	(-2σ)* (%)	50°C Temp. Degradation (% of Ultimate)			Degradation Due to Static Fatigue (10 ⁵ hr) (% of Ultimate)			Degradation Due to Dynamic Fatigue (10 ⁸ cyc.) (% of Ultimate)		
		Long.	Trans.	Shear	Long.	Trans.	Shear	Long.	Trans.	Shear
Kevlar-49 Composite	20	1	7	5	50	50	50	40	-	-
Type Graphite Composite	20	0	4	4	4	20	20	55	50	-
S2-Glass and E-Glass Composite	10	3	7	5	60	60	60	80	-	-

*(-2σ) Variation includes material handling, fabrication, test data.

(1) Vicario, A. A., "Composite Material Property Data for Flywheels, Wind Blades and Pressure Vessels, September 2, 1976.

TABLE 8-III

DESIGN ALLOWABLES

Static Criteria
(2 σ low, 50°C, 10⁵ hours)

Property	Kevlar-49 Composite	Graphite Composite	S2-Glass Composite	E-Glass Composite
Longitudinal Tensile Modulus, E_{11} (GPa)	75.8	124	53.8	46.2
Transverse Tensile Modulus, E_{22} (GPa)	4.1	6.9	20.7	12.4
Poisson's Ratio, ν_{12}	0.34	0.27	0.25	0.25
In-Plane Shear Modulus, G_{12} (GPa)	2.1	5.9	6.2	6.2
Longitudinal Tensile Strength, F_{11T} (MPa)	593	1151	806	427
Transverse Tensile Strength, F_{22T} (MPa)	6.9	22.8	20.0	17.9
Longitudinal Compressive Strength, F_{11C} (MPa)	138	841	310	324
Transverse Compressive Strength, F_{22C} (MPa)	40	75.8	62.1	68.9
In-Plane Shear Strength, F_{12} (MPa)	9.0	33.8	17.2	20.7

TABLE 8-IV

DESIGN ALLOWABLES

Dynamic Criteria
(2σ low, 50°C, 10^8 cycles)

Property	Kevlar-49 Composite	Graphite Composite	S2-Glass Composite	E-Glass Composite
Longitudinal Tensile Modulus, E_{11} (GPa)	75.8	124	53.8	46.2
Transverse Tensile Modulus, E_{22} (GPa)	4.1	6.9	20.7	12.4
Poisson's Ratio, ν_{12}	0.34	0.27	0.25	0.25
In-Plane Shear Modulus, G_{12} (GPa)	2.1	5.9	6.2	6.2
Longitudinal Tensile Strength, F_{11T} (MPa)	621	552	276	138
Transverse Tensile Strength, F_{22T} (MPa)	*	14.6	*	*
Longitudinal Compressive Strength, F_{11C} (MPa)	*	*	*	*
Transverse Compressive Strength, F_{22C} (MPa)	*	*	*	*
In-Plane Shear Strength, F_{12} (MPa)	*	*	*	*

*No reliable data available.

9. COST STUDY OF CURRENT WECS

Introduction

In order to fully evaluate the practical usefulness of any wind energy conversion system (WECS), it is necessary to determine the economics involved in developing and maintaining the system. Most WECSs are in various stages of development, and relatively few have been used long enough to accurately determine the initial and operating costs. As systems develop, these costs will be more accurately determined; at the present time, however, it is useful to evaluate the economic situation in the best way possible, even though the results are preliminary. This information offers assurance that innovative systems are not going to be prohibitively costly, and at least will be priced in the same order of magnitude when compared with some of the WECSs which have been under development for several years.

Reference 9.1, produced previously in the WVU study of innovative wind machines, contains a summary of 1975 prices of some machines available at that time. The approach taken was to attempt to calculate initial installed costs for commercially available WECS without energy storage systems, and to make reasonable estimates of costs of both historical WECS and those under present development. Thus the installed cost of constructing the WVU VAWT could be compared with these costs of other machines. Some other researchers

(e.g., Quinn, ref. 9.2) have carried such evaluations further and have attempted to derive generalized cost formulas for different sizes of turbines, and have considered financing, site, and other costs as well. Even though some of these first cost estimates are preliminary, the refined cost values and identification of critical cost items in future work will be much more accurate, and will evolve from today's economic analysis efforts.

Discussion

The results of the survey made of the costs and specifications of commercially available wind energy systems, and other projects such as the Gedser and Putnam machines, are shown in Table 9-I.

Due to the wide number of "systems" available today, the capital costs were calculated as the cost of the turbine and generator only. Where component prices were not available, estimates were made as to the amount to be subtracted for towers, storage, and controls. Generally, this was about 30% of the systems cost. No allowance has been made for site preparation and other related costs. The only turbines not offered with a generator are the farm types, and this additional cost was not calculated; therefore, for these machines the price per KW would be even higher than shown if a generator were included.

In order to allow for cost inflation since 1975, the 1975 and 1976 prices were increased 13% and 8% respectively, and thus should give a more realistic comparison with the turbines for which 1977

prices were available. These percentages were those obtained from the U.S. Labor Department's Consumer Price Index for the same period. While the estimates involved in obtaining the data shown will cause some errors, it is believed that some general conclusions may be drawn from the data.

Attempts were made to correlate the data in two ways. The first, Fig. 9.1, is a plot of the estimated 1977 capital costs in dollars/KW vs. turbine rated output in KW. A meaningful comparison between the points is difficult because of the great variety of differing designs. Two trends, however, can be noticed. The first is that, generally, the costs per KW decrease with increased rated output. Secondly, there appears to be 3 "clusters" of points on the graph. The upper left hand corner generally contains the "farm" type of machine. The center group generally contains the smaller commercially available horizontal and vertical axis turbines, and the right hand group encompasses the larger horizontal axis turbines and the older projects. The first two groups contain many machines which have been produced in quantity, while the larger machines have not. Thus, the larger "experimental" machines' costs are expected to be reduced if large quantities are produced. It may also be concluded, as noted by Quinn,^{9.2} that considerable price scatter exists, which is typical of infant industries.

The second graph, Fig. 9.2, is more interesting from the standpoint of correlation. The overall efficiencies of the systems, η , were plotted against initial capital cost. The efficiency was defined

to be the ratio between the power available from the wind and the power measured at the output terminals of the system. This latter figure was taken from the manufacturer specifications whenever possible. In the case of farm machines, no commercial figures are available for electricity generated; the output power was calculated from the pumping ability of the machine with a pump having a 0.05 m (2 in) diameter cylinder and operating at the rated wind speed given by the manufacturer. The selection of the cylinder size was arbitrary, but calculations showed that the effect of considering other sizes on the efficiency was negligible.

The plot (Fig. 9.2) shows several interesting trends, the first being the separation of the points by rated wind speed. The graph breaks up into 2 distinct regions; the first of 7-9 m/s, and the second of 10^+ m/s. Two exceptions are the NASA mod 0 machine, at a rated speed of 8 m/s, and the Kedco 1600, rated at 7 m/s. These machines have fairly high efficiencies, although designed for relatively low rated speeds. It can be seen that the lowest capital costs are attained by operating at higher rated speeds and efficiencies. A feature worthy of note is the close proximity of the only VAWTs in the survey (DAF 15 and 20) to the better horizontal axis machines. While not conclusive, this comparison gives hope that VAWTs may be developed to be competitive with the more established designs.

Conclusions

Within the framework of the limited analysis of wind turbine costs which was undertaken, several conclusions may be drawn.

Initial cost per KW output is seen to decrease with increasing turbine size and efficiency. Much refinement of the costs of some of the newer models (e.g., the NASA Mod 0 family) is underway, and the operating costs of these and other new machines must yet be determined.

The initial commercially available vertical axis turbines appear to be cost competitive with the horizontal axis designs. The cost of the WVU VAWT test model (see section 7 of this report) would produce values of about \$2000/KW at 2.3 KW output. With the expected cost reduction of a developed production machine, and possible output gains with circulation control blades, this cost seems competitive when compared with the turbines surveyed (Fig. 9.1).

Continuing cost analysis is necessary to give an accurate indication of both initial and operating WECS costs.

References

- 9.1. Walters, R. E., et al, "Innovative Wind Machines: Executive Summary and Final Report," ERDA/NSF/00367-76/2, Department of Aerospace Engineering, West Virginia University, June, 1976.
- 9.2. Quinn, B., "The Consumer's Cost of Electricity from Windmills." Eleventh Intersociety Energy Conversion Engineering Conference, State Line, NV, September 12-17, 1976, Vol. II, pp. 1746-1753.

TABLE 9-1
CHARACTERISTICS OF SOME WIND TURBINES

ID#	MANUFACTURER/MODEL		DIAMETER (FT.)	RATED ² SPEED (MPH)		POWER IN (KW)	POWER ¹ OUT (KW)	EFFICIENCY	CAPITAL COST (\$/KW)	NOTES	ESTIMATED 1977 CAPITAL COST (\$/KW)
1	Amerenalt	1500	8	12	27	5.02	1.5	.295	1766	2	1996
2		2500	8	16	36	11.93	2.5	.210	1140	2	1243
3	Windworks		12	11.2	25	8.99	1.87	.207	900	3	900
4	Kedco	1200	12	9.8	22	6.11	1.2	.195	1913	4	1913
5		1600	16	7	16	1.19	1.2	.286	2413	4	2413
6		1210	12	11.2	25	8.99	2.0	.222	1298	4	1298
7		1610	16	9.8	22	12.88	2.0	.184	1598	4	1598
8	D.A.F.	15	15	10.3	23	11.43	3.2	.257	607	5	656
9		20	20	10.3	23	11.43	6.0	.308	572	5	618
10	Aeromotor	6	6	8.9	20	1.12	.08	.071	5812	5,6	6277*
11		8	8	8.9	20	1.05	.167	.084	4131	5,6	4461
12		16	16	8.9	20	1.18	.895	.109	4731	5,6	5109
13	Baker	6	6	6.7	15	.485	.075	.154	6946	5,6	7502*
14		8	8	6.7	15	.863	.157	.182	4299	5,6	4643
15		12	12	6.7	15	1.942	.408	.210	3512	5,6	3793
16	Dempster	6	6	6.7	15	.485	.078	.161	5256	5,6	5676*
17		8	8	6.7	15	.863	.165	.191	3696	5,6	3992
18		12	12	8.9	20	1.6	.394	.086	4505	5,6	4865
19	Dunlite		13	11.2	25	11.55	2.0	.190	2000	2,3	2260
20	Grunman Windstream		25	11.6	26	41.89	15	.342	1100	4,3	1100
21	Wincharger		6	10.3	23	1.75	.2	.114	2625	4,3	2625
22	Electro GMB WG50G		16.5	13.4	30	29.372	6	.204	1333	2,3	1506
23	NASA MOD 0		125	8	18	1880	100	.275	1600	4,3	1600
24	Putnam		175	13.4	30	4600	1250	.272	650	4,3	650
25	Gedser		78.2	13.4	30	659	200	.303	400	4,3	900

NOTES: Costs are for turbine and generator only.

1. Manufacturer's specification, except for #s 10-18, where power out was calculated.
2. 1975 price
3. Estimated
4. 1977 price
5. 1976 price
6. Generator not included
7. Based on 8% increase from 1975-77 and 13% increase from 1975-77.

* Not on KW-\$/KW curve, Fig. 9.1.

Fig. 9.1. Initial cost per unit output vs. turbine capacity of wind turbines.

Fig. 9.2. Initial cost per unit output vs. turbine output efficiency.

10. CONCLUSIONS

A comprehensive analysis of straight-bladed vertical axis wind turbines (VAWTs) is underway. A numerical solution of the blade aerodynamics has been extended to investigate multi-bladed turbines. Initial results indicate that turbine performance is strongly dependent upon blade drag characteristics, rotor solidity, and blade number. Parametric analysis of these effects is underway. Initial calculations show that the blade boundary layers of VAWTs may be adversely affected by the high centrifugal forces in the flow. A test model VAWT has been designed and constructed, and initial outdoor testing has been performed with low solidity blades constructed with symmetrical (NACA 0015) airfoil sections. The test results show that the blade drag coefficients differ greatly from those expected using two-dimensional test data. The motion of the blade along a curved path produces a significant change in airfoil aerodynamic properties, i.e., an airfoil with "virtual camber". Blades of the same cross-section but with higher solidity have been constructed to more fully evaluate this effect. The cost of the test model has been determined in detail during construction, and was found to be of the same order as that for horizontal axis wind turbines of the same output performance, if decreased costs resulting from volume production were realized. Initial structural and cost analysis of a VAWT blade design using composite materials for blade fabrication has been accomplished. Results of this analysis have indicated that composite material blade construction may prove to be of lower cost than conventional (aluminum) blade construction. In order to attain operating conditions of higher tip speed ratios, the composite blades may be the required construction method.

11. RECOMMENDATIONS

Based on results obtained to date, it is recommended that the comprehensive analysis of straight-bladed vertical axis wind turbines be continued. Specifically, more information is required on the theoretical aerodynamic analysis allowing performance predictions. Test data must be obtained to provide blade aerodynamic data for the theory, and to verify the predicted performance. Flow effects relating to the blade motion along a curved path must be studied further. Cost analysis and structural analysis should continue to better determine the competitiveness of the straight-bladed vertical axis wind turbine.

APPENDIX A

WVU PRESENTATIONS AND PUBLICATIONS REGARDING WIND ENERGY

PRESENTATIONS

- P1. Walters, R. E., "Innovative Vertical Axis Wind Machines," Second Workshop on Wind Energy Conversion Systems, sponsored by ERDA Solar Energy Division and NSF, Washington, DC, June 9-11, 1975.
- P2. Loth, J. L., "Wind Energy Concentrators," Second Annual UMR-MEC Conference on Energy, Rolla, MO, October 7-8, 1975.
- P3. Walters, R. E., "The Utilization of Wind Energy," Presented at the Conference on Energy, West Virginia Farm and Home Electric Energy Council, Cedar Lakes, WV, April 13, 1976.
- P4. Fanucci, J. B. and Walters, R. E., "Theoretical Performance of a Vertical Axis Wind Turbine," Vertical Axis Wind Turbine Technology Workshop, sponsored by ERDA, Sandia Laboratories, Albuquerque, NM, May 17-20, 1976.
- P5. Loth, J. L., "WVU Wind Energy Concentrators," International Symposium on Wind Energy Systems, Churchill College, Cambridge, England, September 7-9, 1976.
- P6. Migliore, P. G., Fanucci, J. B., and Squire, W., "Numerical Solution for the Unsteady Lifting Characteristics of Variable Pitch Cross-Flow Wind Turbines," Eleventh Intersociety Energy Conversion Engineering Conference, State Line, NV, September 12-17, 1976.
- P7. Huq, R. and Loth, J. L., "Vortex Kinetic Energy Concentrator," Eleventh Intersociety Energy Conversion Engineering Conference, State Line, NV, September 12-17, 1976.
- P8. Walters, R. E., Fanucci, J. B., Loth, J. L., Squire, W., Migliore, P. G., and Huq, R., "Innovative Wind Turbines," Third Annual UMR-MEC Conference on Energy, Rolla, MO, October 12-14, 1976.
- P9. Loth, J. L., "Betz Type Limits for Vortex Wind Machines", Third Biennial Conference and Workshop on Wind Energy Conversion Systems, sponsored by ERDA, Washington, DC, September 19-21, 1977.
- P10. Walters, R. E., and Migliore, P. G., "The Circulation Controlled Vertical Axis Wind Turbine", Third Biennial Conference and Workshop on Wind Energy Conversion Systems, sponsored by ERDA, Washington, DC, September 19-21, 1977.

REPORTS

- R1. Walters, R. E., Fanucci, J. B., Loth, J. L., Ness, N., Palmer, G. M., and Squire, W., "Innovative Wind Machines: Six Month Report, March 1, 1975-August 31, 1975," ERDA/NSF/00367-75/T1, Department of Aerospace Engineering, WVU, September, 1975.
- R2. Walters, R. E., Fanucci, J. B., Loth, J. L., Migliore, P. G., Ness, N., Palmer, G. M., Squire, W., and Yusuff, S., "Innovative Wind Machines: Executive Summary and Final Report," ERDA/NSF/00367-76/2, Department of Aerospace Engineering, WVU, June 1976.
- R3. Walters, R. E., Fanucci, J. B., Hill, P. W., Migliore, P. G., Squire, W., and Waltz, T. L., "Innovative Wind Turbines; The Circulation Controlled Vertical Axis Wind Turbine", Progress Report for March 1, 1976 - December 31, 1976, ORO/5135-77/1, TR-52, Department of Aerospace Engineering, WVU, January 1977.
- R4. Loth, J. L., Ness, N., and Huq, R., "Innovative Wind Turbines; The Wind Vortex Generator", Progress Report for March 1, 1976 - December 31, 1976, ORO/5135-77/2, TR-53, Department of Aerospace Engineering, WVU, January, 1977.
- R5. Loth, J. L., "Betz Type Limitations of Vortex Wind Machines", ORO/5135-77/3, TR-54, Department of Aerospace Engineering, WVU, April 1977.
- R6. Ness, N. and Yen, R.-H., "Wing Trailing Vortices", ORO/5135-77/4, TR-55, Department of Aerospace Engineering, WVU, July 1977.
- R7. Walters, R.E., Fanucci, J. B., Hill, P. W., and Migliore, P. G., "Vertical Axis Wind Turbine Development", ORO/5135-77/5, TR-58, Department of Aerospace Engineering, WVU, July 1977.
- R8. Loth, J. L., Ness, N., and Yen, R.-H., "Vortex-Type Wind Machines; Final Report", ORO/5135-77/6, TR-59, Department of Aerospace Engineering, WVU, July 1977.

THESES

- T1. Yen, R.-H., "Time Decay of a Turbulent Line Vortex", M.S. Thesis, West Virginia University, Morgantown, WV, December, 1976.
- T2. Elko, D. G., "Design, Instrumentation, and Calibration of a Vertical Axis Wind Turbine", M.S. Thesis, West Virginia University, Morgantown, WV, May, 1977.

- T3. Hoy, D. L., "Theoretical Study of Vertical Axis Wind Turbines Using Aft Tail for Blade Tracking", M.S. Thesis, West Virginia University, Morgantown, WV, December, 1977.

APPENDIX B

SYMBOL DEFINITION FOR BLADE STRUCTURAL ANALYSIS, SECTION 4

A	cross sectional area of blade, in
A_C	capture area of the rotor, ft
C	chord length of the blade, in
C_D	drag coefficient, dimensionless
C_L	lift coefficient, dimensionless
C_{F_X}	nondimensional force in X direction
C_{F_Y}	nondimensional force in Y direction
D	drag, lb
e	distance between neutral axis and outside fibers of blade, in
E	modulus of elasticity, psi
F_X	force in the X direction, lb
F_Y	force in the Y direction, lb
I_X	moment of inertia about X axis, in ⁴
L	lift, lb
l	blade half length, in
M	moment, in-lb
m	mass, slugs
R	radius of turbine, ft
R_a	center reaction, lb
S	axial component of pretension load, lb
V_∞	wind velocity, ft/sec

APPENDIX B (CONTINUED)

V_{rel}	velocity relative to blade, dimensionless
w	uniform load on the blade, lb/in
α	angle between V_{rel} and blade chord (total angle of attack), degrees
α_1	angle between relative velocity and radial axis of the machine, degrees
α_0	angle of attack between blade setting and the radial axis of the wind machine, degrees
Γ	torque, ft-lb
θ	angle between radial axis of machine and free stream, degrees
ρ	density, slugs/ft ³
σ_b	bending stress, psi
ω	angular velocity, rad/sec

APPENDIX C

COMPUTER PROGRAM FOR UNIFORM BLADE FORCE CALCULATIONS, SECTION 4

```
//ENGRAERO JOB (AS09332.C,9177,10),'ELKO'
//EXEC FORGO
//SOURCE DD *
    NAMEDLIST/DATIN/IDAL,A,B,RC,RVI,R
    .DATA DR/.017453/
10  .READ(5,DATIN)
    M=1.0
    G=32.2
    RHO=.002378
    C=R/RC
    CENT=(M/G)*WRVI**2)/(.5*RHO*C*R)
    WRITE(6,DATIN)
    WRITE(6,444)
444  FORMAT(///,' THETA      ',' ALFA1      ',' ALFA)      ',' ALFA ',
    .A ' CD      ',' CL      ',' VREL      ',' A BLADE X      ',' C BLADE X ',
    B ' T BLADE X ')
    IALFA0=90+IDAL
24  .DO 10 ITHETA=5,355,10
    IF(ITHETA.GT.90.AND.ITHETA.LT.30) IALFA0=90-.IAL
    COST=COS(DR*ITHETA)
    SINT=SIN(DR*ITHETA)
    VREL2=1.-WRVI*(2.*SINT-WRVI)
    ALFA1=ATAN2(SINT-WRVI,COST)
    IF(ALFA1.LT.0.0)ALFA1=ALFA1+6.283
    .COSA=COS(ALFA1)
    SINA=SIN(ALFA1)
    ALFA=ALFA1+FLOAT(IALFA0)*DR
    .COSB=COS(ALFA)
    SINB=SIN(ALFA)
    .IF(ALFA.GT.6.283)ALA=ALFA-6.283
    IF(ALFA.GT.3.14159)ALFA=ALFA-6.283
    IALFA1=ALFA1/DR+.5
    CL=.1*ALFA/DR
    CD=A+B*CL**2
```

APPENDIX C (CONTINUED)

```

IF (ABS (ALFA) .GT. .1607) CL=SIN(2*(ALFA-48dr00
IF (ABS (ALFA) .GT. .1607 .AND. ABS (ALFA) .; .8727) CD=SIN (ALFA)
IF (ABS (ALFA) .GT. .8727 .AND. ABS (ALFA) .lt. 2.2689)
ACD=1.8*SIN (ALFA) -.15
. IF (ABS (ALFA) .GT. 2.2689) CD=SIN (ALFA)
IATA=ALFA/DR+ .5
VREL=SQRT (VREL2)
COSC=COS (IALFA0*DR)
.SINC=SIN (IALFA0*DR)
CFXA=VREL2*(-CL*SINB+CD*COSB)
CFXC=-CENT*COSC
CFXT=CFXA+CFXC
102 FORMAT(1H , 14,3(6X,14),3(2X,F.3),3(4X,F6.2))
.WRITE(6,102) ITHETA, IAFAL, IALFA0, IATA,
A CD, CL, VREL, CFXA, CFXC, CFXT
100 CONTINUE
WRITE(c,445)
445 FORMAT(//, ' THETA ', ' A BLADE Y ', ' C BLADE Y ',
A ' T BLADE Y ', ' A MACH R ', ' C MACH R ', ' T MACH R ',
B ' A MACH T ')
DO 200 ITHETA=5,355,10
.IALFA0=90+IDAL
IF (ITHETA.GT.90.AND. ITHETA.LT.270) IALFA0=90-IDAL
COST=COS (DR*ITHETA)
SINT=SIN (DR*ITHETA)
VREL2=1.-WRVI*(2.*SINT-WRVI)
ALFAL=ATAN2 (SINT-WRVI, COST)
IF (ALFAL.LT.0.0) ALFAL=ALFAL+6.283
COSA=COS (ALFAL)
SINA=SIN (ALFAL)
.ALFA=ALFAL+FLOAT (IALFA0)*R
COSB=COS (ALFA)
SINB=SIN (ALFA)
IF (ALFA.CT.6.283) ALFA=ALFA-6.283

```

APPENDIX C (CONTINUED)

```

      IF (ALFA.GT.3.14159)ALFA=ALFA-6.283
      IALFA1=ALFA1/DR+.5
      CL=.1*ALFA/DR
      CD=A+B*CL**2
      .IF (ABS (ALFA) .GT..1607)CL=SIN (2*(ALFA-4*DR))
      IF (ABS (ALFA) .GT..1607.AND.ABS. (ALFA) .T..8727) CD=SIN (ALFA)
      .IF (ABS (ALFA) .GT..8727.AN.ABS. (ALFA) .LT.2.2689)
      .ACD+1.8*SIN (ALFA)-.15
      IF (ABS (ALFA) .GT.2.2689) CD=SIN (ALFA)
      IATA=ALFA/DR+.5
      VREL=SQRT (VREL2)
      COSC=COS (IALFA0*DR)
      SINC=SIN (IALFA0*DR)
      CFYA=VREL2*(CL*COSB+CD*SINB)
      CFYC=-CENT*SINC
      CFYT=CFYA+CFYC
      .CW2RA=VREL2.*CL*SINA-CD*COSA)
      .CW2RC=CENT
      CW2RT=CW2RA+CW2RC
      .CWRA=VREL2*(CL*COSA+CD*SINA)
103  FORMAT (1H ,14,7(7X,F7.2))
      WRITE (6,103)  ITHETA ,CFYA,CFYC,CFYT,CW2RA,W2RC,CW2RT,CWRA
200  CONTINUE
      GO TO 10
      END

```

APPENDIX D

COMPUTER PROGRAM FOR BLADE MOMENTS, STRESSES, AND DEFLECTIONS, SECTION 4

```

10.      GET LIST(w,s,e,i,1);
15.      PUT IMAGE('')(f1);
16.      f1:  IMAGE;
X SLOPE DEFLECTION MOMENT STRESS
20.      reed: DO x=0 TO 64 BY 1;
25.          ya=.53;
26.          a=1.18;
30.          k2=s/(e*i);
40.          k=sqrt(k2);
50.          ce2=(exp(2*1*k)+exp(-2*1*k))/2;
60.          se2=(exp(2*1*k)-exp(-2*1*k))/2;
70.          cel=(exp(1*k)+exp(-1*k))/2;
71.          sel=(exp(k*1)-exp(-k*1))/2;
71.1      gig=2*w*(se2/cel-se2)/k+w*1*1*k*se2;
71.2      trip=2*sel*sel-1*k*se2;
71.3      r=gig/trip;
71.4      A=r*sel/(s*k*se2);
140.      cex=(exp(k*x)+exp(-k*x))/2;
150.      sex=(exp(k*x)-exp(-k*x))/2;
155.      dy=-A*k*cex+w/(s*k2)*(-k*(sel*cex-cel*sex)/cel)+w/s*(1-x)+r/(2*s);
162.      y=-A*sex+w/(s*k2)*((cel*cex-sel*sex)/cel-1)+r*x/(2*s)+w/(2*s)*x*(2*1-x);
162.5     m=-s*A*sex+w/k**4*(k2*(cel*cex-sel*sex)/cel)-w/k2;
164.      stress=m*ya/i+s/a;
170.      PUT IMAGE(x,dy,y,m,stress)(f2);
171.      f2:  IMAGE;
-----
180.      END reed;
180.5     PUT LIST('');
181.      PUT LIST('CENTER REACTION=',r);
190.      END ;

```