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THE INVERSE FREE-ELECTRON LASER ACCELERATOR*

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1. INTRODUCTION

Over the past few years in several laboratories attention has been focused on the possible application of lasers for electron acceleration: high-power lasers, with the associated extremely high fields in the beam, potentially could provide very efficient mechanisms for acceleration.

In this paper we will not cover the whole field of laser accelerators, since the many mechanisms which have been devised to transfer energy from a laser to an electron beam have already been discussed in a previous summer school.¹ A review of the field can be also found in the Proceedings of a Workshop held at Los Alamos in 1982.²

In this Workshop Laser Accelerators have been divided in three groups: medium, near field and far field. In the first two cases a material medium, like a plasma or a grating, is used to mediate the energy transfer from the laser to the particle beam. In the last case no such material medium is needed. In this paper we will discuss only the "far-field" accelerators and among these we will only consider the "Inverse Free-Electron Laser" (IFEL), which operates on the same physical principles as the free-electron laser, and is able to transfer energy from the laser into the electron beam without making use of a physical boundary near the beam.

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We will first describe the basic physical properties of an inverse free-electron laser and make an estimate of the order of magnitude of the accelerating field obtainable with such a system; we will then apply the general ideas to the design of an actual device and through this example we will give a more accurate evaluation of the fundamental as well as the technical limitations that this acceleration scheme imposes.

2. GENERAL CHARACTERISTICS OF LASER ACCELERATORS

If we focus a gaussian laser beam propagating in the z-direction in vacuum, the laser electric field amplitude, A, changes as³

$$A = A_0 \frac{w_0}{w(z)} \exp \left\{ -\frac{x^2 + y^2}{w^2(z)} \right\} \quad (1)$$

where x and y are the transverse coordinates. The minimum spot size w_0 down to which one can focus the laser beam is related to the light wavelength λ and to the Rayleigh range R (defined as the distance over which the cross sectional area of the beam doubles its value) by the relation:³

$$\pi w_0^2 = R\lambda \quad (2)$$

The laser beam size will change according to the expression:³

$$w^2 = w_0^2 [1 + (z/R)^2] \quad (3)$$

which is similar to the relation expressing the change in size of a particle beam in a straight section of a storage ring. This relation shows that the spot size will increase quadratically with the distance from the center of the straight section and the Rayleigh range corresponds, in this analogy, to the β function of the storage ring.

If the beam has a power W , one can calculate the electric field associated with it, using the relationships

$$A = (4WZ_0/\pi W^2)^{1/2} \quad (4)$$

where Z_0 is the vacuum impedance. Therefore both λ and R are important in determining the maximum electric field attainable.

To give a numerical example, we can consider a Neodimium laser with a total power in the beam of 10^{14} W, a wavelength $\lambda = 1 \mu\text{m}$ and a Rayleigh range $R = 1 \text{ m}$; then the peak electric field that one can obtain is $E = 2.2 \times 10^{11}$ V/m, while the minimum spot size will be 10^{-6} m^2 .

The very large electric fields that are available in a laser beam constitute the main reason why we are interested in this kind of system. The main problem that one is faced with, is that the electric field in a laser beam is transverse to the direction of propagation of the beam itself, so that it cannot be used directly to accelerate particles. Many ideas on how to use a high field for acceleration have been proposed and most of them have been discussed in the Los Alamos Workshop in 1982². These ideas can be subdivided essentially into three categories:

- 1) Media accelerators: these systems use the inverse Cherenkov effect by propagating the laser beam in a gas or excite plasma waves using the laser and then utilize the plasma waves to accelerate particles.
- 2) Near field accelerators: it is possible to send a laser beam along a structure, similar to a linac, having the same characteristic dimensions of the light wavelength, such as a cavity array or a grating. This will produce a longitudinal field which can be used to accelerate particles. One important consideration in this case is that the longitudinal field can be obtained only over a distance from the structure of the order of the wavelength, so that the beam must be very small in diameter and very close to a material wall.

3) Far field accelerators: the last type of laser accelerators considered at the Los Alamos Workshop is the so-called far field accelerators, exemplified by the Inverse Free-Electron Laser which will be discussed in detail in the present lectures.

The main advantages of a system such as the IFEL are the following:

- 1) There is no need to use the near field, as it is necessary in linacs or in grating laser accelerators: the acceleration region is far removed from all material boundaries and the beam is essentially propagating in a vacuum, so that the problem of electrical breakdown at the material wall are greatly reduced, as is the problem of plasma formation when the high-intensity, high-power laser beam hits a material wall.
- 2) For the same reason, since the accelerated electron beam is not constrained to move near a material boundary, there are no strong emittance limitations, nor intensity limitations connected to the interaction of the beam with the wall surrounding it, such as coherent beam instabilities or beam breakup.
- 3) The IFEL has been proven to work: it has been shown experimentally that this system can be used to accelerate particles. Experiments with FFL in which particles were either decelerated or accelerated have been performed in several laboratories, such as Los Alamos National Laboratory, TRW, Mathematical Sciences Northwest⁴ and the results agree with the theoretical expectations. Although these experiments were performed on a small scale, with changes in the total energy of the electrons of only a few percent, nonetheless they have proven the basic physics of the system.

The lectures will be subdivided into the following parts:

- 1) The basic Physics of the IFEL will be discussed and an estimate of the order of magnitude of the accelerating field will be derived, together with a general design of the main features of the accelerator.
- 2) Two simple accelerators will be designed, one for low energy and one for high energy beams: this will allow us to identify some of the problems connected with this type of accelerator. We will give an idea of some of the accelerator parameters, such as the maximum energy attainable, the acceleration rate, the intensity and so on.
- 3) A more detailed discussion of the theory of the Free Electron Laser will follow: from it we will derive the focussing properties of the IFEL as well as the general properties of the beam in this system, such as the energy spread and the emittance.
- 4) Finally the problem of laser beam transport and focussing will be briefly discussed for a beam with peak power of the order of 10^{14} W and over a distance of the order of a few kilometers.

3. BASIC PHYSICS OF AN IFELA

In the free electron laser (FEL) the energy exchange between a plane electromagnetic wave propagating in vacuum and an electron is obtained by giving to the electron a velocity component parallel to the wave electric field.⁵ This transverse electron velocity is produced by an undulator magnet.

Let z be the wave direction of propagation, coinciding with the undulator axis, and x, y two orthogonal axis. We consider for simplicity the case of a helical undulator and a circularly polarized wave. The undulator field $\underline{B}_0$ is

$$B_{0x} = B_0 \cos \left(2\pi z / \lambda_0 \right) \quad (5)$$

$$B_{oy} = B_o \sin (2\pi z/\lambda_o) \quad (6)$$

where λ_o is the undulator period. The electric field of the laser beam is

$$E_{ox} = A_o \sin [(2\pi/\lambda)(z - ct) + \phi_o] \quad (7)$$

$$E_{oy} = -A_o \cos [(2\pi/\lambda)(z - ct) + \phi_o] \quad (8)$$

For relativistic electrons, with an energy γ , measured in rest energy units $m_o c^2$, such that $\gamma \gg 1$, the electron trajectory is determined, in order $1/\gamma^2$, by the undulator only.⁶ Assuming for the electron velocity $\underline{\beta}$ (in units of the light velocity, c):

$$\beta_z \approx 1, \beta_x, \beta_y \ll 1 \text{ and } \gamma \gg 1$$

the trajectory is a helix^{7,8} of radius

$$a = \lambda_o K / 2\pi\gamma \quad (9)$$

and velocity

$$\beta_x = \frac{K}{\gamma} \cos (2\pi z/\lambda_o) \quad (10)$$

$$\beta_y = \frac{K}{\gamma} \sin (2\pi z/\lambda_o) \quad (11)$$

where the "undulator parameter", K , is defined as

$$K = (e B_o \lambda_o) / (2\pi m_o c^2) \quad (12)$$

The electron energy change is determined by the laser electric field. Using (7), (8), (10) and (11) the equation describing this electron energy change is⁶

$$\frac{d\gamma}{dt} = \frac{e A_o K}{m_o c \gamma} \sin \phi \quad (13)$$

where ϕ , the phase of the electron oscillation relative to the wave, is

$$\phi = \frac{2\pi}{\lambda_0} z - \frac{2\pi}{\lambda} (z - ct) + \phi_0 \quad (14)$$

For a net energy exchange the phase ϕ in (13) must be constant or slowly changing. For an electron moving in the longitudinal direction with velocity β_z the change in the phase of the wave (7), (8), in one undulator period is

$$\Delta = \frac{2\pi}{\lambda} \left(\frac{1}{\beta_z} - 1 \right) \lambda_0 \quad (15)$$

If we require that Δ be equal to 2π , the change in electron oscillation phase in one period, we obtain the "synchronism condition"

$$\lambda = \lambda_0 \frac{1 - \beta_z}{\beta_z} \quad (16)$$

or also, expressing β_z in terms of γ and β_x, β_y and using (10), (11)

$$1/2$$

$$\beta_z = (1 - 1/\gamma^2 - \beta_x^2 - \beta_y^2)^{1/2} \approx 1 - (1 + K^2)/\gamma^2 \quad (17)$$

and

$$\lambda = \frac{\lambda_0}{2\gamma^2} (1 + K^2) \quad (18)$$

If (18) is satisfied the phase ϕ in (13) is a constant, ϕ_0 , and the energy will increase or decrease according to whether $\sin\phi_0 < 0$ or $\sin\phi_0 > 0$. For given λ , λ_0 and K equation (18) defines a value γ_R of the energy which we call the resonant energy.

The synchronism condition (18) requires that as a particle, having $\gamma = \gamma_R$, is accelerated either λ_0 or K have to change in order to continue to satisfy (18).⁹ In most of what follows we will consider the acceleration of a particle satisfying (18) and (13) with $\phi = \phi_0 = \text{constant}$. The effect of a deviation from the energy γ_R , as well as the effect of beam emittance and some undulator imperfections, is discussed in Section 6.

Using (13) and (12) we are now able to get a quantitative idea of the possible acceleration gradients attainable in an IFEL. Let us consider a laser with a wavelength of 10 μm , a field in the laser beam of $E_0 = 10^9 \text{ Vcm}^{-1}$, a field in the undulator magnet of 1.5 kG, a period length of 50 cm, and an electron energy of 500 MeV ($\gamma = 10^3$). The gradient that we can achieve is then of the order of 700 MeV/m.

4. SCALING LAWS FOR AN IFEL

One restriction imposed on the IFEL is that, since the electron trajectory is being wiggled in order to have an energy exchange between the laser wave and the electrons, a transverse acceleration of electrons is produced, which will lead to emission of synchrotron radiation which in turn could be a limitation when high energies are being reached. The total amount of power radiated by the electrons per unit length of the trajectory is given by the expression

$$S = \frac{8}{3} \pi^2 \frac{r_{e0}^2 c^2}{\lambda_0^2} K^2 \gamma^2 \quad (19)$$

where r_e is the classical electron radius and all other quantities have been defined in section 3. For an IFEL to accelerate particles, we have to introduce the further condition that the acceleration rate (13) should be larger the synchrotron radiation losses (19).

$$\frac{eA_o K}{\gamma} > \frac{8}{3} \pi^2 \frac{r_e}{\lambda_o^2} K^2 \gamma^2 \quad (20)$$

This condition sets a minimum value on the field in the laser beam to be used, value which is a function of energy:

$$eA_o > \frac{8}{3} \pi^2 \frac{r_e}{\lambda_o^2} K \gamma^3 \quad (21)$$

As we can see, there is a strong dependence of this value on γ , although we must remember that both K and λ can change with γ , so that the actual dependence of A_o on γ is not really γ^3 , but can be changed by adjusting either K or λ_o as γ varies.

In the case of the IFEL the synchronism condition can be written as in (18). In most cases, for high energies one can neglect the factor of 1 in Eq. (18): this gives us a scaling law for λ_o and K as we change the energy. Two simple cases can be considered: One in which we keep the field in the undulator constant (in this case λ_o must change as $\gamma^{2/3}$). In the other case λ_o is kept constant and B_o must change proportionally to γ in order to satisfy the synchronism condition. Intermediate cases, in which both parameters are changed, can be considered.

The scaling laws for the two cases, $\lambda_o = \text{constant}$ or $B_o = \text{constant}$, are in Table I. In the first case the undulator period varies as $\gamma^{2/3}$, similarly to K , the transverse velocity is decreasing as $\gamma^{-1/3}$. The electron oscillation amplitude, a , given by (9) is increasing: this could be a problem, because we want to keep this amplitude smaller than the waist of the laser beam, which in turn should be as small as possible, in order to

increase the electric field which is acting on the particles. This dependence of the oscillation amplitude on energy could impose some limitations at the high energy end of the accelerator. For this system the accelerating gradient is decreasing as $\gamma^{-1/3}$ while the synchrotron radiation losses increase as γ^2 .

TABLE I: IFEL Scaling Laws

	$B_0 = \text{constant}$	$\lambda_0 = \text{constant}$
λ_0	$\gamma^{2/3}$	constant
B_0	constant	γ
K	γ	γ
$(\beta_x^2 + \beta_y^2)^{1/2}$	$\gamma^{-1/3}$	constant
a	$\gamma^{1/3}$	constant
$d\gamma/dz$	$\gamma^{-1/3}$	constant
S	γ^2	γ^4

In the case in which the wavelength of the oscillation is kept constant, the magnetic field must increase as γ : this makes the accelerating gradient independent of energy, while the amplitude of the oscillation remains constant. On the other hand, the synchrotron radiation losses are increasing as γ^4 , like in a storage ring, which makes this second approach unattractive.

5. DESIGN OF AN IFEL

We can now summarize the five most important formulae describing the IFEL: the first is the synchronism condition, which relates the laser wavelength to the oscillation period of the electrons

$$\lambda = \frac{\lambda_o}{2\gamma^2} (1 + K^2) \quad (22)$$

Then we have the equation describing the acceleration rate

$$m_o c^2 \frac{d\gamma}{dz} = eA_o \frac{K}{\gamma} \sin \phi_o - S \quad (23)$$

The synchrotron radiation losses are given by

$$S = \frac{8}{3} \pi^2 \frac{r_e}{\lambda_o} m_o c^2 K^2 \gamma^2 \quad (24)$$

which implies the existence of a minimum acceleration rate in order to satisfy the condition

$$m_o c^2 \frac{d\gamma}{dz} > S \quad (25)$$

The transverse beam oscillation amplitude is given by

$$a = \frac{\lambda_o K}{2\pi\gamma} \quad (26)$$

A limit exists on the beam current

$$I_B < \frac{W}{m_o c^2 \gamma_f} \quad (27)$$

which is dictated by energy conservation. Here W is the total laser power, γ_f is the final energy of the particles.

5.1. Single stage IFEL

Using the five formula above we are now able to design an IFEL accelerator. We first consider a simple case in which we have a single undulator. We focus the laser beam to a minimum waist in the middle of the undulator, then we let the laser beam expand without refocussing it. This simple approach is valid only for a limited length of the undulator: if we keep increasing the length of this system without refocussing the laser beam, it will expand and the electric field available to accelerate electrons will decrease. This type of system is called a single-element, single-pass system (Fig. 1.).

Using the condition $B_0 = \text{const}$, one can obtain an expression for the final energy of the electrons as a function of their initial energy γ_0 , the laser power, W , the length of the accelerator, L , assumed to be the undulator length, the laser wavelength λ

$$(\gamma_f)^{4/3} - (\gamma_0)^{4/3} = \frac{4}{3} e \left(\frac{8WZ_0 L}{\lambda} \right)^{1/2} \sin \phi_0 \quad (28)$$

The length L , of the accelerator is in this case also the Rayleigh range, R , of the optical system used to focus the laser beam. In order to maintain the laser beam waist as constant as possible we choose the Rayleigh range equal to the undulator length.

In the case where $\lambda_0 = \text{const}$ we have a different expression

$$\gamma_f - \gamma_0 = e \left(\frac{16WZ_0 L}{\lambda} \right)^{1/2} \quad (29)$$

having also assumed $L = R$.

In (28), (29), the dependence of the final energy on the wavelength is not very strong. This implies that any kind of laser can be chosen for this purpose, no matter what the wavelength of its radiation is.

TABLE II. Single Stage IFELA

$\lambda_0 = 10 \text{ cm (constant)}$

$B_0 = 1 \text{ T (constant)}$

Laser Parameters

Power, W	$2 \cdot 10^{13} \text{ W}$
Pulse duration, τ	1 ns
Pot size, r_0	0.25 cm
Wavelength, λ	1 μm
Electric field, A_0	$2.9 \cdot 10^{10} \text{ V/m}$
Undulator Length, L	39 m

Undulator Parameters

Period, λ_0	10 cm	$3.9 \pm 23 \text{ cm}$
Magnetic field, B_0	$0.31 \rightarrow 3.8 \text{ T}$	1 T
Synchronous phase ϕ_0	$\pi/3$	$\pi/3$

Electron Beam Parameters

Initial Energy, $M_0 c^2 \gamma_0$	250 MeV	250 MeV
Final Energy, $m_0 c^2 \gamma_f$	4.2 GeV	3.8 GeV
Current, I_B	$< 5 \text{ KA}$	$< 5 \text{ KA}$
Beam radius, r_B	0.2 cm	0.2 cm
Average accelerating field	101 MeV/m	90 MeV/m
Oscillation amplitude, a	0.007 cm	10^{-2} cm
Energy spread	10^{-4}	10^{-4}
Synchrotron radiation loss at γ_f	300 KeV/m	20 KeV/m

By using equations (28), (29) and considering the single-pass system, we can derive some parameters for this type of accelerator (Table II).⁵ The laser parameters are derived from existing lasers. Assuming an initial energy of 250 MeV in both cases ($\lambda_0 = \text{const}$ and $B_0 = \text{const}$) an energy of 4.0 GeV can be reached after 40 m. The total current is limited to 5 kA, the beam radius is .2 cm, the oscillation amplitude is smaller than the spot size and the energy spread is 10^{-4} .

5.2. Multistage IFEL

The system considered above cannot produce much higher energies as few GeV. The final energy increases as the square root of the accelerator length, due to the fact that the laser beam transverse dimension increases while the available electric field decreases, so that, beyond the considered length, the final energy does not increase considerably. In order to avoid this problem it is necessary to introduce a focussing system that can maintain a large accelerating fields over much longer distances. Some idealized schemes that would accomplish this task are shown in Fig. 2 and Fig. 3. These schemes are not practical because the beam has to go through lenses or mirrors, but other solutions to this problem can in principle be found. Assuming for the moment that this is the case, we can consider a multistage IFEL, using a constant magnetic field to keep the synchrotron radiation losses low, and a focussing system with a Rayleigh range of 3 m, to produce a nearly constant electric field throughout the accelerator.

We can integrate equations (22), (23) for given initial conditions to obtain γ and λ_0 (as a function of distance) along the accelerator axis, Z , and assuming A_0 constant.

Initial and final values are given for two cases in Tables III and IV, corresponding to two different laser wavelength. The particle energy and undulator period versus Z are given in Figs. 4, 5, 6, 7.

TABLE III.

INPUT PARAMETERS

Laser Wavelength	=	1.0 μm
Laser Power	=	10^{14} W
Synchronous Phase, $\sin\phi_0$	=	0.866
Undulator Field	=	1.0 T
Initial Period	=	3.8 cm
Laser Focusing Length	=	3 m
Laser Electric Field	=	$2.24 \cdot 10^{11}$ V/m
Initial Energy	=	0.26 GeV
Waist	=	0.7 mm
Oscill. Amplit.	=	$4 \cdot 10^{-5}$ m

FINAL VALUES

Period	=	4.2 m
Oscill. Amplit.	=	0.46 mm
Undul. Param.	=	400
Accel. Length	=	3000 m
Electron Energy	=	296 GeV
Number of Periods	=	1133

TABLE IV.

INPUT PARAMETERS

Laser Wavelength	=	3 μm
Laser Power	=	10^{14} W
Synchronous Phase, $\sin\phi_0$	=	0.866
Undulator Field	=	1.3 T
Initial Period	=	10 cm
Laser Focusing Length	=	3 m
Laser Electric Field	=	$1.3 \cdot 10^{11}$ V/m
Initial Energy	=	0.8 GeV
Waist	=	1.2 mm
Oscill. Amplit.	=	0.12 mm

FINAL VALUES

Period	=	4.2 m
Oscill. Amplit.	=	0.8 mm
Undul. Param.	=	512
Accel. Length	=	2000 m
Electron Energy	=	220 GeV
Number of Periods	=	780

From the results shown in Tables III and IV we can see that the hypothesis of refocusing the laser beam has lead us to much higher energies than those obtained in the single stage case. The choice $B_0 = \text{constant}$ is also important to keep synchrotron radiation losses down, although they are not negligible.

In fact we can see from Figs. (4) and (6) that the rate of growth in electron energy starts to decrease around 200 GeV.

6. THEORY OF THE FREE-ELECTRON LASER

In this section we want to study in more detail the motion of one electron in an IFEL, to obtain some informations on focusing properties of this sytem and on the energy spread and emittance of the electron beam that it produces.

Let us consider again the FEL system with an undulator magnet, an electromagnetic wave going through the undulator, together with an electron bunch: we want to study the energy exchange between these two systems. To do this, we start by writing the equation of motion for the electrons:

$$\dot{\underline{P}} = e \underline{E} + e \underline{\beta} \times (\underline{B} + \underline{B}_0) \quad (30)$$

This equation contains two terms: one coming from the electric and magnetic fields of the wave, and the other describing the undulator magnetic field. In principle, we should add to these terms the effect of radiation reaction and of synchrotron losses, but these terms will be neglected in discussing (30). This energy loss will then be added to the equation for the energy change. The space-charge forces should be also taken into account, but they can be neglected as long as the electric field describing the space charge is smaller than A_0 / γ .⁶ If we write this condition in terms of the electron density, n_e , we obtain the relation $en_e \lambda < A_0 / \gamma$. For $\gamma = 10^3$, $\lambda = 1 \mu\text{m}$, $A_0 = 10^9 \text{ V/m}$, the electron density turns out to be $n_e < 10^{16} \text{ e}^- \text{ cm}^{-3}$. In our case the space charge term is negligible.

The equation of motion can be simplified for relativistic electrons. Assuming that the electron is essentially moving along the longitudinal direction, so that the velocity of the electron along the undulator axis is nearly c while the transverse velocity is small, then one can show that, to order γ^{-1} , the equation of motion can be separated into two equations, one of which determines the trajectory of the electron, and in which only the undulator magnetic field appears, and another one which describes the energy exchange, and in which only the electric field of the laser appears. So in this case the trajectory and the energy exchange process are decoupled:

$$\dot{\underline{\beta}} = \frac{e}{m_0 c \gamma} \underline{\beta} \times \underline{B}_0 \quad (31)$$

$$\dot{\gamma} = \frac{e}{m_0 c} \underline{\beta} \cdot \underline{E} \quad (32)$$

In order to determine the trajectory, we have to explicitly give the functional form of the magnetic field: the simplest form of a magnetic field that can be used to perform the calculations is given by a helical undulator described by the equations (5), (6), with a period λ_0 . We also assume that the plane wave propagating along the undulator is circularly polarized and is given by (7), (8).

In a helical undulator the electron trajectory is also helical.^{7,8} The transverse velocity is given by (10), (11) where the undulator parameter K is given by (12). The radius of the helical trajectory is given by (9). Notice that the transverse velocity is a constant since

$$\beta_x^2 + \beta_y^2 = \frac{K^2}{\gamma^2} \quad (33)$$

Therefore also the longitudinal velocity, β_z , is a constant

$$\beta_z = \left(1 - \frac{1}{\gamma^2} - \beta_x^2 - \beta_y^2\right)^{1/2} \quad (34)$$

and, under the assumption that $\beta_x^2 + \beta_y^2 \ll 1$, which is a condition well satisfied, the parallel velocity can be written as

$$\beta_z = 1 - \frac{1 + K^2}{2\gamma^2} \quad (35)$$

The expression for the longitudinal velocity (35) can be used to explicitly obtain from (32) the energy exchange rate, eq. (13), that we rewrite here for convenience

$$\dot{\gamma} = \frac{eA_0 K}{m_0 c \gamma} \sin \phi \quad (36)$$

where ϕ is the relative phase of the electron in its oscillation in the undulator and the electromagnetic wave and is given by (14). By taking the derivative of this equation, and using the fact that the electron is moving with a constant longitudinal velocity $dz/dt = \beta_z c$, we can obtain an expression for the derivative of the phase

$$\dot{\phi} = \frac{2\pi c}{\lambda_0} \left[\beta_z + \frac{\lambda_0}{\lambda} (\beta_z - 1) \right] \quad (37)$$

Using the expression for the parallel velocity (35), we can cast the equation (37) in the following form

$$\dot{\phi} = \frac{2\pi c}{\lambda_0} \left(1 - \frac{\lambda_0}{\lambda} \frac{1 + K^2}{2\gamma^2} \right) \quad (38)$$

This quantity become zero when the condition

$$\lambda = \frac{\lambda_0}{2\gamma^2} (1 + K^2) \quad (39)$$

connecting the electron energy, the undulator period, the light wavelength and the parameter K , is satisfied. This condition can only be satisfied for a particular energy, γ_R , called the resonant energy, for which the phase remains constant. In this case the energy exchange can be written as

$$\dot{\gamma}_R = \frac{eA_0 K}{m_0 c \gamma_R} \sin \phi_R \quad (40)$$

with $\phi_R = \text{constant}$ and

$$\gamma_R^2 = \frac{\lambda_0}{2\lambda} (1 + K^2) \quad (41)$$

In general, for energies different from the resonant energy, ϕ will change and we will have to consider the two equations

$$\dot{\gamma} = \frac{eA_0 K}{m_0 c \gamma} \sin \phi \quad (42)$$

$$\dot{\phi} = \frac{2\pi c}{\lambda_0} \left(1 - \frac{\gamma_R^2}{\gamma^2}\right) \quad (43)$$

As the electrons are being accelerated, the resonant energy will change so that λ_0 and K will have to change accordingly along the undulator, to satisfy the condition (41).

6.1 Small Oscillation Regime

We now consider the case in which the electron does not have exactly the resonant energy, so that its phase will be changing along the undulator. The equations describing this system are nonlinear, but they can be considerably simplified if we limit our analysis to the case of small oscillations. We consider a particle with an energy only slightly different from the resonant energy. We introduce the quantity

$$\eta = \frac{\gamma - \gamma_R}{\gamma_R} \ll 1 \quad (44)$$

and we linearize the equation with respect to η . We assume that the synchronism condition (41) is always satisfied and we can relate the change in γ_R to the change in the parameters λ and K using the synchronism condition and Eq. (40). From the two equations (40), (42) we can obtain the expression

$$\frac{d}{dt} (\gamma^2 - \gamma_R^2) = \frac{2eA_o K}{m_o c} (\sin \phi - \sin \phi_R) \quad (45)$$

Assuming also that γ_R is slowly varying, i.e.

$$\frac{\lambda_o}{\gamma_R^2} \frac{d\gamma_R^2}{dz} \ll 1 \quad (46)$$

we can linearize equation (45) and (43) and obtain, using (44)

$$\dot{\eta} = -\frac{\omega_o}{2} \left(\frac{\Omega}{\omega_o} \right)^2 (\sin \phi - \sin \phi_R) \quad (47)$$

$$\dot{\phi} = 2 \omega_o \eta \quad (48)$$

where

$$\omega_o = \frac{2\pi c}{\lambda_o} \quad (49)$$

and

$$\left(\frac{\Omega}{\omega_o} \right)^2 = \frac{eA_o \lambda_o K}{\pi m_o c^2 \gamma_R^2} \quad (50)$$

Ω represents the small oscillation frequency, which is related to the electric field, the undulator period, the undulator parameter, and the resonant energy. Equations (47), (48) are similar to the ones describing synchrotron oscillations in a linac or in a circular accelerator. Equation (50) can also be written in a slightly different form, by introducing the laser wavelength λ

$$\left(\frac{\Omega}{\omega_o}\right)^2 = \frac{2eA_o \lambda}{\pi m_o c^2} \frac{K}{1 + K^2} \quad (51)$$

From this equation we can see that, as we move along the accelerator, λ and A_o are constant, but the small oscillation amplitude is not constant, since it changes as $1/K$.

It is convenient to rewrite the equation of η , ϕ using the variable

$$\tau = \omega_o \tau \quad (52)$$

which changes by 2π in one undulator period.

With the notation $x' = d x/dt$ we have

$$\eta' = -\frac{1}{2} \left(\frac{\Omega}{\omega_o}\right)^2 (\sin \phi - \sin \phi_R) \quad (53)$$

$$\phi' = 2\eta \quad (54)$$

The equations describing the synchrotron oscillations can be derived from an Hamiltonian of the form

$$H = \eta^2 + V(\phi) \quad (55)$$

$$V(\phi) = -\frac{1}{2} \left(\frac{\Omega}{\omega_o}\right)^2 (\cos \phi + \phi \sin \phi_R) \quad (56)$$

The potential decreases on the average as ϕ increases (Fig. 8), but the oscillations around the straight line allow particles to be captured in the potential well, the "bucket", if their kinetic energy is small enough. The phase acceptance of the bucket is given by⁹

$$\Delta\phi = \pi - \phi_R - \phi^* \quad (57)$$

where ϕ^* is defined by the relationship

$$V(\phi^*) = V(\pi - \phi_R) \quad (58)$$

The potential well depth is given by

$$\eta_{\text{MAX}} = \left(\frac{\Omega}{\omega_o}\right) (\cos \phi_R - (\phi_R - \pi/2) \sin \phi_R)^{1/2} \quad (59)$$

which is proportional to the small amplitude oscillation frequency. Both phase acceptance and potential well depth are decreasing as we move along the accelerator.

The potential well depth also defines the maximum energy spread that can be accepted in the beam for stable operation. For the example given before in Table III, the energy spread that can be accepted is in the order of

$$\frac{\Delta E}{E} \lesssim \frac{\Omega}{\omega_o} = \frac{2eA_o \lambda}{\pi m c^2} \cdot \frac{K}{1+K^2} \approx 6 \times 10^{-4} \quad (60)$$

at the maximum energy of 300 GeV, while it is much larger at the low energy end of the accelerator.

6.2 The Maxwell-Vlasov Equations

In this description of the electron motion in the accelerator we have assumed that the electric field of the laser beam remains constant. This is not the case in reality, because there is an energy exchange between the electrons and the laser beam, so that in a complete description of the system

one should consider not only the equation describing the electron motion, but also the equation describing the evolution of the laser field and of the radiation produced by the electrons: this can be achieved by using Maxwell's equations, together with the equation of motion of the electrons and Vlasov's equation, which describes the evolution of the electron distribution function along the accelerator.¹⁰

A vector potential, $\underline{A}$, can be introduced such that

$$\nabla \cdot \underline{A} = - \frac{4\pi}{c} \underline{J} \quad (61)$$

where we use the Coulomb gauge and the transverse current is defined as a sum over the electrons having position $\underline{r}_i$ and velocity $\underline{\beta}_i$ by

$$J_{\perp x} = e \sum_i \beta_{xi} \delta(\underline{r} - \underline{r}_i) \quad (62)$$

$$J_{\perp y} = e \sum_i \beta_{yi} \delta(\underline{r} - \underline{r}_i) \quad (63)$$

Equation (60) can be satisfied by a potential of the form

$$\underline{A} = \frac{\lambda}{2\pi} A_0(z, t) \left\{ -\hat{x} \cos \left[\frac{2\pi}{\lambda} (z-t) + \phi_0(z, t) \right] + \hat{y} \sin \left[\frac{2\pi}{\lambda} (z-t) + \phi_0(z, t) \right] \right\} \quad (64)$$

in which both the amplitude, A_0 , and the phase, ϕ_0 , are allowed to vary.

As a result of the interaction, both the amplitude and the phase of the laser beam will change with z and t . Assuming that the dependence is a slowly varying one, then the equation for the vector potential can be written as two equations for the amplitude and phase

$$\frac{1}{c} \frac{\partial A_0}{\partial t} + \frac{\partial A_0}{\partial z} = - \frac{2\pi}{c} J_{\perp} \quad (65)$$

$$A_0 \left(\frac{1}{c} \frac{\partial \phi_0}{\partial t} + \frac{\partial \phi_0}{\partial z} \right) = + \frac{2\pi}{c} J_2 \quad (66)$$

which are coupled to the currents J_1, J_2 related to $J_{\perp x}, J_{\perp y}$ by

$$J_1 = J_{\perp x} \sin \left(\frac{2\pi}{\lambda} (z-ct) \right) + J_{\perp y} \cos \left(\frac{2\pi}{\lambda} (z-ct) \right) \quad (67)$$

$$J_2 = - J_{\perp x} \cos \left(\frac{2\pi}{\lambda} (z-ct) \right) + J_{\perp y} \sin \left(\frac{2\pi}{\lambda} (z-ct) \right) \quad (68)$$

It is usually a good enough approximation to describe the electron beam as a continuum by introducing a distribution function $f = f(\eta, \phi, t)$ which satisfies Vlasov's equation,

$$\frac{\partial f}{\partial t} + \dot{\phi} \frac{\partial f}{\partial \phi} + \dot{\eta} \frac{\partial f}{\partial \eta} = 0 \quad (69)$$

then the currents can be written as averages $\langle \rangle$, over the particle distribution function, f .

$$J_1 = \frac{ecK}{\gamma_R} \rho(\underline{r}) \langle \sin \phi \rangle \quad (70)$$

$$J_2 = \frac{ecK}{\gamma_R} \rho(\underline{r}) \langle \cos \phi \rangle \quad (71)$$

where $\rho(\underline{r})$ is the spatial density of the electron beam before its interaction with the laser field. We finally obtain a complete set of equations describing our system, including the evolution of the laser beam, combining (65), (66), (69) and (53), (54). We will not solve these equations in detail: the main conclusions that we have derived would not be changed. A complete discussion of this system is however needed for the case in which so much energy is extracted from the laser beam that the amplitude of the electric field could not be considered constant.

6.3 Transverse Stability

We will now discuss the condition of transverse focusing. in order to do this, we must introduce a more complete description of the magnetic field. Up to now we have used the undulator magnetic field as given in (5),

(6). It is easy to verify that this type of magnetic field does not satisfy Maxwell's equations: in particular the equation $\nabla \times B = 0$ is not satisfied. To avoid this problem, we must add a longitudinal component¹¹ which must be zero on the axis but which can be different from zero off axis

$$B_z = - (2\pi/\lambda_0) B_0 \left\{ x \cos(2\pi z/\lambda_0) + y \sin(2\pi z/\lambda_0) \right\} \quad (72)$$

this component has the effect of introducing a transverse focusing which can give stability to the electron motion. If we consider again the equations of motion (31), or

$$\dot{\underline{\beta}} = \underline{\beta} \times \underline{\omega} \quad (73)$$

where

$$\underline{\omega} = \frac{eB}{m_0 \gamma c} \quad (74)$$

we obtain the following equations

$$\dot{\beta}_x = -\beta_z \omega_y + \beta_y \omega_z \quad (75)$$

$$\dot{\beta}_y = \beta_z \omega_x - \beta_x \omega_z \quad (76)$$

$$\dot{\beta}_z = \beta_x \omega_y - \beta_y \omega_x \quad (77)$$

Assuming that $\beta_x, \beta_y \ll 1$ and

$$\frac{x}{\lambda_0} \ll 1, \frac{y}{\lambda_0} \ll 1 \quad (78)$$

the components $\beta_y \omega_z, \beta_x \omega_z$ are of second order, because the transverse velocity is much smaller than β_z and the longitudinal component of the magnetic field is different from zero only off axis and it is proportional to

the displacement. In this approximation the problem can be solved by an iterative process: we first derive the first order solution from the equations

$$\dot{\beta}_x = -\beta_z \omega_y \quad (79)$$

$$\dot{\beta}_y = \beta_z \omega_x \quad (80)$$

which gives

$$\beta_z = \text{constant} \quad (81)$$

$$\beta_x = \frac{K}{\gamma} \cos (2\pi z/\lambda_o) \quad (82)$$

$$\beta_y = \frac{K}{\gamma} \sin (2\pi z/\lambda_o) \quad (83)$$

The transverse velocities are rotating along the axis of the undulator, as we have already seen. These solutions can now be used to calculate the second order terms. For instance for the x component we obtain

$$\begin{aligned} \dot{\beta}_x = & -\frac{eB_o \beta_z}{m_o \gamma c} \cos (2\pi z/\lambda_o) \\ & -\left(\frac{2\pi K}{\lambda_o \gamma}\right)^2 \cos (2\pi z/\lambda_o) [x \cos (2\pi z/\lambda_o) + y \sin (2\pi z/\lambda_o)] \end{aligned} \quad (84)$$

By averaging the last term over a number of undulator periods, the equations for β_x , β_y are reduced to the form

$$\dot{\beta}_x = -\frac{2\pi \beta_z cK}{\lambda_o \gamma} \cos (2\pi z/\lambda_o) - 2\left(\frac{2\pi cK}{\lambda_o \gamma}\right)^2 x \quad (85)$$

$$\dot{\beta}_y = -\frac{2\pi \beta_z cK}{\lambda_o \gamma} \sin (2\pi z/\lambda_o) - 2\left(\frac{2\pi cK}{\lambda_o \gamma}\right)^2 y \quad (86)$$

The last terms in the above equations are proportional to the displacement from the axis and provide focusing at a frequency

$$\Omega_f^2 = \frac{1}{2} \left(\frac{2\pi c K}{\lambda_o \gamma} \right)^2 \quad (87)$$

which is equivalent to the betatron oscillation frequency in a synchrotron. We should notice that, since this frequency is proportional to K/γ , it is not constant throughout the accelerator. The wavelength associated with these oscillations will be given by

$$\lambda_f = 2^{1/2} \frac{\lambda_o K}{\gamma} \quad (88)$$

Although the focusing properties do change as the particles move along the accelerator and they gain energy, both longitudinal and transverse focusing are present, so that a stable beam exists throughout the accelerator.

6.4 Effects of Errors and Emittance Limitation

Although stability exists, we have not yet considered the necessary tolerances in the construction of the undulator and the constraint on the transverse emittance. We have seen that the accepted energy spread at an energy of 300 GeV is of the order of 10^{-3} . We can now consider the problem of transverse emittance by looking at the effect of an error either in the undulator period, or the magnetic field, or of a change in the particle energy on the phase of oscillation. The phase change of one electron in going through one undulator period can be calculated using Eq. (14) and we can write it as

$$\phi(z + \lambda_o) - \phi(z) = 2\pi \left(1 + \frac{\lambda_o}{\lambda} \frac{\beta_z - 1}{\beta_z} \right) \quad (89)$$

For a resonant particle λ , λ_0 and β_z are related by equation (16) or (17) in which case $\phi(z + \lambda_0) - \phi(z) = 0$.

If λ_0 or β_z are slightly changed from their ideal values

$$\lambda_0 \rightarrow \lambda_0 + \Delta\lambda_0, \quad \beta_z \rightarrow \beta_z + \Delta\beta_z$$

then the phase change in one undulator period is different from zero and is given by

$$\frac{\Delta\phi}{2\pi} = -\frac{\Delta\lambda_0}{\lambda_0} + \frac{1}{1 - \beta_z} \frac{\Delta\beta_z}{\beta_z} \quad (90)$$

The longitudinal velocity, β_z , is related to the particle energy and transverse velocity by (17). A change in β_z can be produced by a change in energy, $\Delta\gamma$, or by a change in transverse velocity, $\Delta(\beta_x^2 + \beta_y^2)$. Using (10), (11) this last term can be written as

$$\Delta(\beta_x^2 + \beta_y^2) = \Delta\left(\frac{K^2}{\gamma^2}\right) + \theta^2 \quad (91)$$

where θ^2 describes the uncertainty in the electron transverse velocity at the undulator entrance, due to the angular spread in the electron beam. This angular spread is related also to the transverse beam dimension "a", since in a magnetic transport channel and at constant electron energy the quantity

$$\epsilon = \pi\theta a \quad (92)$$

the beam emittance, is an invariant.

Using (17) and (91) we can now evaluate the variation in β_z

$$\Delta\beta_z = 2 \frac{1 + K^2}{\gamma_R^2} \frac{\Delta\gamma}{\gamma} - 2 \frac{K^2}{\gamma_R^2} \frac{\Delta K}{K} - \theta^2 \quad (93)$$

The undulator parameter, K , can change because of a variation in the undulator period, λ_o , or magnetic field, B_o . Using (12) we have

$$\frac{\Delta K}{K} = \frac{\Delta \lambda_o}{\lambda_o} + \frac{\Delta B_o}{B_o} \quad (94)$$

From (93), (94), we have

$$\frac{\Delta \phi}{2\pi} = 4 \frac{\Delta \gamma}{\gamma} - \frac{1 + 4K^2}{1 + K^2} \frac{\Delta \lambda_o}{\lambda_o} - \frac{4K^2}{1 + K^2} \frac{\Delta B_o}{B_o} - 2 \frac{\gamma_R^2}{1 + K^2} \theta^2 \quad (95)$$

The magnetic field in the undulator can change from its ideal value because of an error in the magnet construction, $\left(\frac{\Delta B}{B}\right)_e$, or because of its variation transverse to the undulator axis, as discussed in section 6.3. This last term can be evaluated using (72) and averaging over one undulator period, so that for the total magnetic field variation we have

$$\frac{\Delta B_o}{B_o} = \left(\frac{\Delta B}{B}\right)_e + \left(\frac{\pi a}{\lambda_o}\right)^2 \quad (96)$$

From (96), (95) we can now write the phase variations as

$$\frac{\Delta \phi}{2\pi} = - \left[\frac{1 + 4K^2}{1 + 4K^2} \frac{\Delta \lambda_o}{\lambda_o} + \frac{4K^2}{1 + 4K^2} \left(\frac{\Delta B}{B}\right)_e \right] - \frac{4}{1 + K^2} \left[K^2 \left(\frac{\pi a}{\lambda_o}\right)^2 + \frac{\gamma_R^2}{2} \theta^2 \right] - \frac{4\Delta \gamma}{\gamma} \quad (97)$$

The possible error in phase can finally be considered as the sum of three terms: the first is due to the errors in construction of the undulator, the second is due to the non-zero beam emittance, and the last term describes the effect of the energy spread. We see that the effect of undulator errors, and of the beam emittance is equivalent to that of an energy error. Since the energy spread must be limited, for beam stability, to a value smaller than the potential well depth, $\left(\frac{\Delta \gamma}{\gamma}\right)_{MAX}$, given by (59), we must require that each of the terms in (97) be smaller than $(\Delta \gamma / \gamma)_{MAX}$. The conditions can be summarized as

$$\frac{\Delta\gamma}{\gamma} < \left(\frac{\Delta\gamma}{\gamma}\right)_{\text{MAX}} \quad (98)$$

$$\frac{1+4K^2}{1+K^2} \frac{\Delta\lambda_o}{\lambda_o} + \frac{4K^2}{1+K^2} \left(\frac{\Delta B_o}{B_o}\right)_e < \left(\frac{\Delta\gamma}{\gamma}\right)_{\text{MAX}} \quad (99)$$

$$\frac{4}{1+K^2} \left[K^2 \left(\frac{\pi a}{\lambda_o}\right)^2 + \frac{\gamma_R^2}{2} \theta^2 \right] < \left(\frac{\Delta\gamma}{\gamma}\right)_{\text{MAX}} \quad (100)$$

which must be independently satisfied.

Let us consider in more detail the second term of equation (97), which is due to the finite transverse dimension of the beam and to the angular spread. These two terms are related to each other. As discussed in section 6.3 the undulator magnet provides transverse focusing, so that the electrons execute oscillations in the x,y plane with an oscillation wavelength given by (88). The amplitude and velocity of this oscillation are related by

$$a = \Lambda_f \theta / 2\pi \quad (101)$$

By using the emittance and Λ_f , (92), (88) we can rewrite equation (100) as

$$\frac{4K^2}{1+K^2} \left[\frac{\Lambda_f}{2\lambda_o^2} + \frac{\gamma_R^2}{K^2 \Lambda_f} \right] \epsilon < \left(\frac{\Delta\gamma}{\gamma}\right)_{\text{MAX}} \quad (102)$$

We can minimize the l.h.s. of (102) respect to Λ_f . This will determine the maximum possible emittance in the beam which will still be accelerated. This minimum occurs for

$$\Lambda_f = 2^{1/2} \frac{\lambda_o \gamma_R}{K} \quad (103)$$

this expression is exactly the focusing wavelength in the undulator due to the presence of the longitudinal magnetic field, as given in (88). For the optimum choice of λ_f the condition (102) the emittance can be written as

$$\epsilon < \frac{\lambda_o^2 (1+K^2)}{6K^2 \lambda_f} \left(\frac{\Delta Y}{Y} \right)_{\text{MAX}} \quad (104)$$

or

$$\epsilon < \frac{2^{1/2}}{6} \frac{\lambda_Y R}{K} \left(\frac{\Delta Y}{Y} \right)_{\text{MAX}} \quad (105)$$

where the emittance is directly related to the wavelength of the laser used to produce the acceleration, to the maximum depth of the potential well and to the inverse of the transverse velocity. This relationship shows that using a laser with a shorter wavelength limits the acceptable emittance.

Using the results of this section we can evaluate the limitations on emittance, energy spread, undulator period for the IFEL given in Table III. These are summarized in Table V, where we consider the beam at injection and at a final energy of 300 GeV.

TABLE V

E GeV	K	λ_o m	$\left(\frac{\Delta E}{E} \right)$ MAX	$\frac{\Delta \lambda_o}{\lambda_o}$	$\frac{\Delta \lambda}{\text{mm}}$	$\frac{K}{Y_R}$	ϵ m rad
0.25	3.5	0.038	7.7%	5%	2	7×10^{-3}	1.5×10^{-5}
300	400	4.3	7×10^{-4}	5×10^{-4}	2	7×10^{-4}	1.5×10^{-6}

It is interesting to notice that the absolute error in λ_o remains constant at all energies so that the tolerances on λ_o itself is not very tight. The beam emittance acceptable at injection is quite large and is comparable with

what can be usually obtained from a 250 MeV linac, and it decreases by an order of magnitude at the high energy end of the accelerator. But we must remember that as the energy increases, the beam emittance decreases naturally due to adiabatic damping (it decreases by a factor of 12,000) so that the values of the maximum acceptable beam emittance do not constitute real restrictions.

7. DESIGN PARAMETERS OF AN IFEL $e^+ - e^-$ COLLIDER

In this section we want to evaluate the characteristics of a 300 GeV e^+e^- collider based on an IFEL.

If we consider the IFEL as a part of a e^+e^- linear collider, two parameters are important for the analysis of the system, the luminosity and the disruption parameter.¹² The luminosity, for an axially symmetric beam is

$$L = h \frac{f N^2}{4\pi\sigma^2} \quad (106)$$

where f is the system repetition rate, N the number, assumed equal, of electrons or positrons per bunch, $\sigma_{\perp}$ the r.m.s. transverse radius of the bunch and h is an enhancement factor describing the self-focusing effect of electron and positron during their interaction. The disruption parameter, D , is

$$D = \frac{r_e \sigma_z N}{\gamma \sigma_{\perp}^2} \quad (107)$$

where σ_z is the r.m.s. bunch length. The disruption parameter D is the equivalent of the betatron tune-shift in a circular machine, produced by the beam-beam interaction, and must be limited to a quantity of the order of 1. At the same time the forces between the two beams are focusing forces: when

the two bunches e^+e^- collide, the transverse cross section of the bunches decreases, which in turn increases the luminosity: this will happen for a range of values of the disruption parameter between 2 and 10.¹² If D is smaller than 1, this focusing effect does not take place so that no gain in luminosity is achieved from the beam-beam interaction. If D is larger than 10, the interaction is so strong that instabilities set up which decrease the effective luminosity. In this case the enhancement factor is less than 1 and decreases with D .

In order to estimate the parameters considered above we assume that $D = 10$ and that the emittance at 300 GeV is 10^{-8} m rad which is consistent with the value used at SLAC in the design of the linear collider. for a final energy of 300 GeV and for a focusing function $\beta^* = 1$ cm, at the interaction point, the transverse cross section radius of the beam at the crossing point is of the order of $\sigma = 5.8 \times 10^{-5}$ cm. This quantity is four times smaller than what estimated for the linear collider, the reason being that at higher energies the adiabatic damping acts more effectively in reducing the beam transverse cross section.

We now assume that the laser pulse length is long enough to accelerate all the electrons in the bunch. From energy conservation, to accelerate N electron to a final energy E_f , using a laser pulse of peak power W , we must have

$$eNE_f = \eta \, 2\pi^{1/2} W \frac{\sigma_z}{c} \quad (108)$$

with an r.m.s. length equal to $2^{1/2}\sigma_z$ and where η gives the fraction of the total energy in the laser pulse which is transferred to the electron

bunch. Using the equation (108) and solving it for N, we can express the disruption parameter in terms of η .

$$D = \frac{2\pi^{1/2} r_e \sigma_z^2 \eta W}{\gamma \sigma_{\perp}^2 e E_f c} \quad (109)$$

Assuming $n = .2$, $D = 10$, $W = 10^{14}$ W, $E_f = 300$ GeV, $\sigma = 5.8 \times 10^{-5}$ cm, we have $\sigma_z = 1.2$ cm. From the calculated bunch length and the given value of D, it is possible to derive the number of particles in the bunch from energy conservation considerations, Eq. (40). This number in the case considered above is $6 \cdot 10^{10}$ e⁻/bunch which, together with the emittance considered before and assuming $h = 1.5$, gives a luminosity of

$$L = 1.2 \times 10^{29} f \text{ cm}^{-2} \quad (110)$$

In order to obtain a luminosity of $10^{32} \text{ cm}^{-2} \text{ s}^{-1}$, a repetition rate $f = 800 \text{ s}^{-1}$ is needed. From these quantities it is possible to calculate the average power that the laser must deliver. Since each laser pulse has a peak power of 10^{14} W and the r.m.s. duration of the pulse is 56 ps, then the total laser energy in the pulse is

$$J = 2\pi^{1/2} \sigma_z \frac{W}{c} = 1.4 \times 10^4 \text{ J} \quad (111)$$

and the average laser power, for a repetition rate of 800 Hz, is 11 MW. Assuming that the laser efficiency is of the order of 10%, a DC power of 110 MW per beam is needed to operate the system: this quantity is smaller than what would be needed in order to operate a conventional linac at the same luminosity and the same final energy. The main physical reason for the higher efficiency of this type of system is that the electron bunch and the laser pulse length are of the same approximate size: in this way the energy

is localized only where it is needed to accelerate particles, unlike in a conventional RF linac in which the energy is spread over lengths much longer than the particle bunch, fact which considerably decreased the efficiency of the system.

8. LASER BEAM FOCUSING

One of the main problems of this system, which was mentioned earlier, consists in being able to propagate and periodically refocus the beam down to a small waist. Lenses or mirrors might not be readily practical because they would interfere with the beam propagation. The use of either dielectric or metal waveguides has been proposed.¹³

In this section we want to give a very simple discussion of one possible focusing system based on a metallic waveguide. We assume this guide to be formed by two planes parallel to the z-y plane and separated by a distance "a".

We also assume that we are using a transverse undulator so that the electron trajectory is planar and in the z-y plane (the magnetic field of the undulator is in the x-direction).

The waveguide can be formed with two metallic strips having a width "w" larger than the separation "a" between them and a length in the x-direction equal to the accelerator length. The guide can transmit both $TE_{m,0}$ and $TM_{m,0}$ modes. The attenuation of the TM modes is much larger than for the TE modes and we will consider only the transmission of the laser beam in the TE configuration.

The $TE_{m,0}$ mode electric field can be written as:¹⁴

$$\underline{E} = A_0 \hat{y} \sin((\pi m/2)(1 - 2x/a)) \cdot \exp(-i\beta_m z) \quad (112)$$

$$\beta_m = [(2\pi/\lambda)^2 - (m\pi/a)^2]^{1/2} \quad (113)$$

The mode m can be thought of as the superposition of two plane waves propagating at an angle θ respect to the z axis with

$$\theta_m = m\lambda/2a \quad (114)$$

The absorption loss in one reflection on the metallic plan depends on the angle θ_m and on the metal refraction index, n ,

$$A = 4\theta_m \operatorname{Re}(1/n) \quad (115)$$

Since the number of reflections per unit length is θ/a the loss per unit length can be written.¹⁴

$$a = (m^2\lambda^2/a^2) \operatorname{Re}(1/n) \quad (116)$$

The ratio of the field on the wall to the field on axis is:

$$E_{\text{wall}}/E_{\text{axis}} = (m\lambda/a) \operatorname{Re}(1/n) \quad (117)$$

To estimate how much of an input beam one can couple into the guide we can assume the input beam to be gaussian $A \propto \exp(-x_0^2/w_0^2)$, with w_0 given by (2), and to expand it in plane waves. The fraction of the input power and of the input field coupled into the m -th guide mode is:¹⁴

$$f_m(\eta) = \left(\frac{2}{\pi}\right)^{5/2} \frac{1}{\eta} \left[\int_{-\pi/2}^{\pi/2} \cos(mu) \exp(-4u^2/\pi^2\eta^2) du \right] \quad (118)$$

with

$$\eta = 2w_0/a \quad (119)$$

and

$$A_m = (\pi/2)^{1/4} \eta^{1/2} f_m(\eta) \quad (120)$$

The power fraction f_m has a maximum for the mode $m = 1$ when $2w_0/a = .707$. The value of this maximum is $f_1 = 98.9\%$. For the same value of $2w_0/a$ one also has $A_m \approx 1$.

As an example let us consider the case $a = 4 \text{ mm}$, $2W_0/a = .707$, $m = 1$,
 $\lambda = 1 \cdot 10^{-6} \text{ m}$, $P = 5 \cdot 10^{13} \text{ W}$; we then have

$$\theta_1 = 1.25 \cdot 10^{-4}$$

$$\alpha = 3 \cdot 10^{-7}$$

$$E_{\text{wall}}/E_{\text{axis}} = 5 \cdot 10^{-6}$$

$$E_{\text{wall}} = 10^6 \text{ V/m}$$

We see from these numbers that the attenuation is rather small and the field on the wall is below the known damage threshold.¹³

As one can see from (116) the attenuation increases rapidly with the mode number. To obtain the small attenuation value given before it is very important to avoid any mode coupling as it might, for instance, be produced by surface irregularities. This means that the planes should be worked to optical quality.

Although the coupling efficiency to the guide can be very high the coupling region remains a critical area of this system and one should consider how to optimize it by using properly shaped surfaces.¹³

A conceptual design of this type of accelerator is now possible: the cross section of it is schematically shown in Fig. 9. In this case the undulator or magnets could be built out of permanent magnets and with a period variable along the accelerator. A system of this sort would be very compact and would need no additional power requirement beside the one necessary to operate the laser. A slightly different structure can be conceived for a helical undulator. In this case superconducting magnets might be required, which would place an additional load on the power consumption.

The overall system for the IFEL accelerator would have a structure similar to the one illustrated in Fig. 10. A damping ring would provide an electron bunch with the proper emittance and the proper beam energy spread. A FEL could be used to provide a high power laser beam which would then be emitted into the IFEL. This system can be conceived as an energy transformer: a low energy electron beam (100 MeV) with very high intensity releases energy into a laser pulse which in turn is used to accelerate another electron beam to very high energies. In this case the laser field acts as the mediator of the energy exchange.

9. CONCLUSIONS

A number of problems have surfaced in the design of this type of accelerator: we now summarize the main ones.

1. Problems of the laser beam transport and focusing system are still the most important to be solved before this type of accelerator becomes a reality, although some ideas have been proposed to avoid interaction of the beam with the focusing system.
2. The efficiency of the system could be increased by increasing the ratio of the transverse to the longitudinal velocity, which, in our system, is of the order of 10^{-3} . If this ratio could be increased, the overall length of the accelerator and the laser peak power could both be reduced. A possible solution to this problem could be a thorough optimization of the system. The electron dynamics could be modified to increase β but, on the other hand, an excessive increase in it would lead to intolerable losses through synchrotron radiation.

3. The overall efficiency of the IFEL accelerator could be considerably increased if the energy left in the laser beam at the end of the accelerator could be at least in part reused: in fact, somewhere between 80 and 80% of the energy is still available, having not been transferred to the particles, and, by proper means, could be reflected back to the input of the accelerator.

4. At the moment no laser exists that would be practically usable in this type of accelerator, which requires high peak power, high average power, and high repetition rate. Although systems exist with the required peak power, the optical quality of the beam is not what is needed for this type of system, and, most of all, the repetition rate is too small by several orders of magnitude. The FEL would have a net advantage over conventional lasers because the excess energy created by the low efficiency of the laser must be removed as heat, while for an FEL the excess energy remains in the electron beam which can be easily removed from the system and conveniently dumped.

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Figure Captions

- Fig. 1. Schematic representation of a single pass IFEL.
- Figs. 2,3. Multistage IFEL idealized scheme.
- Fig. 4. Electron energy versus distance in a multistage IFEL; accelerator parameter as given in Table III.
- Fig. 5. Undulator period versus distance; parameters as given in Table III.
- Fig. 6. Electron energy versus distance; parameters of Table IV.
- Fig. 7. Undulator period versus distance; parameters of Table IV.
- Fig. 8. Longitudinal phase space potential.
- Fig. 9. Schematic cross section of undulator - light guide system.
- Fig. 10. Schematic representation of a system for a high energy IFEL; Two such systems would be needed for a $e^+ - e^-$ collider.

Fig. 1.

Fig. 2.

Fig. 3.

ELECTRON ENERGY, GeV

296

DISTANCE, m

3000

Fig. 4.

FIG. 5.

Fig. 6.

FIG. 7.

Fig. 8.

Fig. 9.

Fig. 10.