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NEW SOLUTION OF THE PROBLEM OF EMP
COUPLING ON CABLES OVER GROUND OF FINITE CONDUCTIVITY

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ABSTRACT

This new solution replaces Dawson's Integral, in the old solution, by an infinite series. For typical EMP parameters, the solution is approximated by a double exponential function whose numerical constants are derived. Both forms of the new solution are convenient for the calculation of the transient.

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The frequency domain solution of this problem is described by Vance,¹ and the time domain solution is expressed in terms of Dawson's Integral. This paper describes the time domain solution of the problem in terms of an infinite series. Furthermore, for typical EMP parameter ($E_H = 50$ kV/m, $E_V = 15$ kV/m, $\alpha = 4 \times 10^6$ /s, $\beta = 4.76 \times 10^8$ /s), this solution is approximated by the sum of two exponentials whose constants are evaluated numerically by a least-squares fit algorithm.

The EMP-induced open-circuit voltage across the terminals of a line (cable) of length l at a height h , above the ground of conductivity σ , such that $\sigma \gg \epsilon_0 \omega$, is approximately¹ (these approximations are not valid for small angles ϕ and ψ):

$$V(j\omega) \approx V_i(j\omega) + V_\sigma(j\omega) \quad (1)$$

$V_i(j\omega)$ is the response of the ideal line specified by

$$V_i(j\omega) = c D G (1 - e^{-j\omega t_0}) (1 - e^{-j\omega t_l}) \quad (2)$$

$$D = \frac{E_{H0} \sin\phi + E_{V0} \sin\psi \cos\phi}{1 - \cos\phi \cos\psi} \quad (3)$$

$$G = \frac{1}{j\omega} \left(\frac{1}{\alpha + j\omega} - \frac{1}{\beta + j\omega} \right) \quad (4)$$

$$t_0 = 2h \sin\psi/c, \quad t_l = (1 - \cos\phi \cos\psi) l/c \quad (5)$$

The time domain equivalent $V_i(t)$ of $V_i(j\omega)$ is given² by

$$V_i(t) = c D \left[\frac{1}{\alpha} (1 - e^{-\alpha t}) - \frac{1}{\beta} (1 - e^{-\beta t}) \right] + \dots \quad (6)$$

where $+$. . . denotes the sum of time shifts of the same function caused by wave reflections in the line.

The term $V_\sigma(j\omega)$ accounts for the finite conductivity of the earth and can be expressed as

$$V_{\sigma}(j\omega) = 2 c \sqrt{\tau_e} D_{\sigma} e^{-j\omega t_0} (1 - e^{-j\omega t_e}) F(j\omega) \quad (7)$$

$$F(j\omega) = \frac{1}{\sqrt{j\omega}} \left(\frac{1}{\alpha + j\omega} - \frac{1}{\beta + j\omega} \right)$$

$$D_{\sigma} = \frac{E_{Ho} \sin\phi \sin\psi + E_{Vo} \cos\psi}{1 - \cos\phi \cos\psi} \quad (8)$$

The inverse Fourier transform^{3,4} of $F(j\omega)$ can be written as

$$f(t) = \frac{\sqrt{t}}{\pi} \left(e^{-\alpha t} - e^{-\beta t} \right) + \frac{2}{\pi} \sum_{v=1}^{\infty} \frac{e^{-v^2/4}}{v} \left(\frac{\sinh(v\sqrt{\alpha}t)}{\sqrt{\alpha}} e^{-\alpha t} - \frac{\sinh(v\sqrt{\beta}t)}{\sqrt{\beta}} e^{-\beta t} \right) \quad (9)$$

For typical numerical parameters of the EMP, $f(t)$ can be approximated by the following double exponential function $f_0(t)$

$$f(t) \approx f_0(t) = E_0 \left(e^{-\alpha_0 t} - e^{-\beta_0 t} \right) \quad (10)$$

$$E_0 = 3.68 \times 10^{-4}, \alpha_0 = 7.6 \times 10^5/s, \beta_0 = 1.87 \times 10^7/s$$

These values were obtained by a least-squares fit method.

Thus, the time domain equivalent, $V_{\sigma}(t)$ of $V_{\sigma}(j\omega)$, can be expressed as

$$V_{\sigma}(t) = 2 c \sqrt{\tau_e} D_{\sigma} \begin{cases} 0 & \dots \dots \dots t < t_0 \\ f(t - t_0) u(t - t_0) & \dots \dots t_0 \leq t \leq t_0 + t_e \\ f(t - t_0) u(t - t_0) & \\ - f(t - t_0 - t_e) u(t - t_0 - t_e) & t > t_0 + t_e \end{cases} \quad (11)$$

where $\tau_e = \epsilon_0/\sigma$, and $u(t - t_0)$. . . is the delayed unit step function. The total open-circuit transient voltage can be written as

$$V(t) = V_i(t) + V_o(t) \quad (12)$$

Some variations of $V(t)$ are demonstrated graphically. Figure 1 demonstrates the time dependence of $V_i(t)$, $V_o(t)$ and $V(t) = V_i(t) + V_o(t)$. Note that the rise of $V(t)$ is controlled by $V_i(t)$. $V_o(t)$ is a delayed slow rise time function. Figures 2 shown the effects of the variation of the conductivity on the waveform of $V(t)$. Similarly, Fig. 3 shows how the waveform changes as the length of the line varies. In all calculations, ψ is adjusted so that D , Eq. (3), is maximum.

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Fig. 1. Waveforms of $V_i(t)$, $V_\sigma(t)$ and $V(t) = V_i(t) + V_\sigma(t)$.

Fig. 2. Voltage versus time with conductivity σ as a parameter.

Fig. 3. Voltage versus time with line length l as parameter.