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LINEAR LEAST-SQUARES ANALYSIS AND EFFECTIVE INTERACTIONS

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In the following we exhibit a few results from the application of the method of singular value analysis (SVA), a particular form of linear least-squares analysis,¹ to two sets of experimental two-body matrix elements, the 63 Chung-Wildenthal (CW) matrix elements² for the $1d5/2$, $2s1/2$, $1d3/2$ shells ($A \approx 20-40$) and the best 84 matrix elements from the work of Schiffer and True (ST).³ The latter cover the atomic weight range of $A \approx 40$ to 200 and nine shells from $1d3/2$ to $1i13/2$. Our purpose is the extraction of a first-order set of strength parameters for an effective interaction of minimum rank and simple form for each set or both sets of data. The ranges, being non-linear in their effects, are investigated separately. In this work we consider mainly local potential forms. (This is not a limitation of the approach.) The SVA method is well-suited to this problem since it

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was developed for and extensively applied to the analysis of satellite trajectories and the like,¹ where similar over-determined problems arise.

Several of the powerful features of SVA may be shown via the following over-determined problem,

$$A_{mn} X_n \approx b_m ; m > n \quad (1)$$

where A is an m by n matrix, x is a vector of n unknowns, and b is a vector of m data points. In our case A is the array of theoretical two-body matrix elements, of which there are m for each term of the potential form being studied, and b consists of the m two-body matrix elements, CW or ST. The usual solution to this classic problem is the minimization of the Euclidean norm of r, the m-dimensional residual vector, defined by

$$r \approx Ax - b \quad (2)$$

which yields the least-squares result. The SVA, while ultimately giving the standard result, also yields an ordered but non-unique sequence of solutions consisting of linear combinations of the model force components which contribute independently to the squared r-norm $\|r\|^2$. The judicious choice of the character and cut-off of this sequence will in general yield a more realistic and far different solution than the full least-squares result, as we shall show. This cut-off is essentially the same as the effective rank of the model matrix A, which is determined by the quasi-diagonal singular-value matrix.¹ The effective rank or cut-off number (c) is the best approximation for the number of independent potential parameters.

Using SVA one may determine the minimum set of decoupled potential parameters necessary to produce the "best" result, i.e. the result at cut-off. Practically, this is done by examining how $\|r\|^2$ approaches $\|r_n\|^2$, where n is the maximum possible number of independent potential parameters and $\|r_n\|^2$ is the minimum or least-squares result. In the present work we have set cut-off at the 1% level, i.e.,

$$\rho_c^2 - \rho_n^2 \approx 1\%; c \leq n \quad (3a)$$

$$\rho^2 \equiv \|r\|^2 / \|b\|^2 \quad (3b)$$

where c denotes the solution-sequence number at cut-off. The SVA method takes advantage of the non-uniqueness of the path by which ρ_n^2 is approached by arranging the initial weighting so that the coupling between potential terms is minimized or even eliminated. This has the double advantage of simplifying the form of the best result as well as making it possible to compare different models

and data sets in a meaningful way. (This will be illustrated shortly for the CW and ST matrix element sets.) These features are essential to making a judgment as to the general tenability or "universality" of an effective interaction.

In Figs. 1 and 2 we show the results of the SVA for ρ^2 in the case of the 63 CW and the (selected) 84 ST matrix elements for several different force models. (The analysis of the latter did not include the tensor force.) Each figure displays the fractional effect of the individual major components on ρ^2 . The definition for the symbols are as follows: (S,T) $\equiv$ (singlet, triplet); (O,E) $\equiv$ (odd, even); TT $\equiv$ tensor; (Y,G) $\equiv$ (Yukawa, gaussian); SD $\equiv$ surface-delta interaction. The symbols CY(4), CY(8), SD(2) mean 4-term central potential (Yukawa), 8-term central-potential (Yukawa), surface delta interaction for T = 0 and T = 1, etc.

Although our analysis is not exhaustive of all force forms certain constant features of Fig. 1 and 2 are evident and clearly independent of the potential forms employed here. For the CW analyses we note the following: First, $\Delta\rho^2$, the change in ρ^2 associated with each central term, is quite constant and ordered as follows: $\Delta\rho^2 = 70\%, 15\%, 5\%, 3.5\%$ for the TE, SE, TO, SO terms. Thus the central terms account for about 95% of the reduction in ρ^2 . Second, the improvement of an 8-parameter or two-range central force over a one-range or 4-parameter central force, even with the addition of the SDI, is well below the 1% level. Third the effect of the tensor force is quite noticeable and, we believe, denotes a real effect, because of its large fractional effect on the residual ρ^2 . For the ST matrix elements, where our analysis is not yet complete, only the first two terms, TE and SE, replicate the behavior observed in the CW analysis. This accounts for 90% of the squared r-norm. Beyond this point much more complicated force forms are required, as shown in Fig. 2 for the CY(8) plot and discussed below.

In Tables 1 and 2 we exhibit the parameter values given by the SVA. These bear out the implications of Figs. 1 and 2 as well as our earlier remarks. We discuss first the CW results of Table 1. In this case the central force parameters at cut-off are substantially the same in the SVA of the CY(2) and CY(4) models as well as the CY(4) model plus either the SDI or tensor-force additions. The same consistency pertains to the CY(8) parameters as well as the tensor parameters TTE and TTO (these are characterized by gaussian form factors). The CY(8) models require only 4 central potential terms, as can be seen from Figs. 1 and 2. Nonetheless the two-range formulation clearly shows the strongly repulsive character of the long-range TO component, as noted by Schiffer and True.³ For the CW matrix elements, at least, no significant values for an SDI interaction could be

deduced since the central components completely subsume the SDI role.

The last column of Table 1 exhibits what happens (in the case of CY(8)) if we pursue the solution to the 8-parameter or least-squares limit. The result for the parameters is not easily recognizable. However at the cut-off limit the TE and SE components are not only essentially the same in all comparable models but, in the case of the single range models, equal to the PMM parameters,⁴ within a few percent. This is significant because these parameters give the local-potential equivalent of the Kallio-Koltveit potential,⁵ a realistic interaction based on the Scott-Moszkowski prescription⁶ for treating the short-range repulsive part of the free interaction.

The results for the ST analysis (Table 2) are in good agreement with the CW results insofar as the first two terms are concerned, i.e. the TE and SE components. (The CY(2) results with range 2.0 fm are included to exhibit the insensitivity of ρ^2 to range.) The results for the odd components as well as for the SE component in the CY(4) and CY(8) models show that something more complex is required to reach the 5% level or lower. This will be our next undertaking.

In sum, the initial results of the application of SVA to the CW and ST data sets strongly support a single effective interaction which accounts for 90% to 95% of the squared-norm of the residual vector (r , Eq. (2)). The form of this potential is not unique but, at the 95% level, its rank is no more than 6 nor less than 4. Furthermore 85% to 90% is accounted for by the two central-even parameters which are produced by the Scott-Moszkowski range cut-off procedure.⁶ It is notable that this procedure makes no direct reference to nuclear data except for the postulated effect of the Pauli principle. Finally, in the case of the CW analysis, the SVA results support the utility of both tensor-force components and a long-range triplet-odd component which is strongly repulsive, the latter in conformity with the findings of Schiffer and True.³ It is presumably these components which would raise the rank of the effective interaction from 4 to 6 or possibly 7. Although it may seem from the foregoing that no more complexity is required, the burden of this conclusion is carried by the last 5-10% of the squared-norm of r . It may well be that this is due to the limitation of the experimental data to the two-body energy spectra. Other types of nuclear data (three body spectra, electromagnetic transitions, etc.) may not be so limited and will be considered in future studies.

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Table 1. SVA best results ($\rho_c^2 - \rho_n^2 \lesssim 1\%$; see text) for the CW matrix elements. The symbols are defined in text. All potential entries are in MeV and are given to two significant figures. The two-range entries (e.g., S01, S02) correspond to the ranges λ_1, λ_2 . The tensor-gaussian form factor had a range parameter of 1.0 fm. The symbol c/n refers to the SVA cut-off and rank of the potential form, as described in text. The oscillator parameter was 11.6 MeV, the same as used in reference 3.

	SD(2)	TTG(2)	CY(2)	CY(4),	CY(4),	CY(4),	CY(8),	CY(8)	CY(8)	CY(8) ^c
				SD(2)	TTG(2)	TTG(2)	SD(2)	SD(2)	SD(2)	
S01	X	X	X	150.	150.	137.	2.6	2.5	2.5	390.
S02	X	X	X	X	X	X	49.	54.	54.	190.
TE1	X	X	-120.	-120.	-117.	-116.	-6.8	-5.0	-15.	-110
TE2	X	X	X	X	X	X	-51.	-52.	-47.	16.
SE1	X	X	-75.	-75.	-74.	-78.	-3.1	-3.5	-3.8	24.
SE2	X	X	X	X	X	X	-3.6	-34.	-34.	-43.
T01	X	X	X	64.	63.	60.	1.5	1.1	1.1	-73
T02	X	X	X	X	X	X	22.	23.	23.	49.
SD0	.85	X	X	X	0.0	X	X	0.0	X	X
SD1	.67	X	X	X	0.0	X	X	0.0	X	X
TTE	X	-230.	X	X	X	-181.	-183.	X	X	X
TTO	X	22.	X	X	X	79.	78.	X	X	X
λ_1 , fm	X	X	1.0 ^a	1.0 ^a	1.0	1.0	1.0 ^b	1.0	1.0	1.0
λ_2 , fm	X	X	X	X	2.0	X	2.0	2.0	2.0	2.0
ρ^2	72%	96.4%	9%	6%	6%	3%	3%	6%	6%	5.0%
c/n	2/2	2/2	2/2	4/4	4/6	6/6	6/10	4/10	4/8	8/8

a $\lambda = 1.0$ fm is optimal for both CY(2) and CY(4).

b Very insensitive to λ ; $\lambda = 1.50$ reduces ρ^2 by 0.3% and $\lambda_T = 1.50$ by 0.1%.

c This column shows the full least-squares result for CY(8). Note large-scale changes from cols. 8-10.

Table 2. SVA best results for the 84 selected ST matrix elements. The oscillator parameters are the same as in reference 3. See caption to Table 1 for an explanation of the symbols. The tensor force was not included in this analysis.

	SD(2)	CY(2)	CY(2)	CY(4)	CY(8) ^a
S01	x	x	x	- 31	0
S02	x	x	x	x	0
TE1	x	- 110	x	- 104	0
TE2	x	x	- 49	x	- 49
SE1	x	- 69	x	- 76	- 280
SE2	x	x	- 33	x	+ 110
T01	x	x	x	20	- 660
T02	x	x	x	x	240
SD0	.58	x	x	x	x
SD1	.46	x	x	x	x
λ_1 , fm	x	1.0	x	1.0	1.0
λ_2 , fm	x	x	2.0	x	2.0
ρ^2	42%	10%	10.7%	9.4%	5.2%
c/n	2/2	2/2	2/2	4/4	4/4

^aThe SE components in this column result from the combination of two linearly-independent components. The first component is shown in column 3 under CY(2).

Fig. 1a

Fig. 1b

Plot of ρ^2 , the dimensionless squared-norm, as a function of the contribution of each term in the potential form for the CW matrix elements. The letter n denotes the full-order of the potential model and the last point on each plot is the least-squares result. The cut-off point (c) is indicated and is the effective rank of the problem. The contributions of each term in the potential are labelled with symbols defined in the text and are given in decreasing order of importance except where $\Delta\rho^2$ is $< 1\%$. The labels at the top of each plot describe the force model. The similarity of the curves arises from the decoupled character of the cut-off solutions and is reflected in the corresponding strengths of the parameters (Table 1). The plot for CY(4) is shown on both (a) and (b) for comparison.

Fig. 2 Plot of ρ^2 in the case of the SVA results for the 84 (selected) ST matrix elements. See the caption to Fig. 1. Note that the third and fourth points on the CY(5) plots refer to highly coupled solutions. See the text and Table 2.

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