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PROPAGATION AND ABSORPTION OF ELECTROMAGNETIC WAVES IN FULLY RELATIVISTIC PLASMAS

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INTRODUCTION

Existing weakly relativistic studies of λ (ECH) (e.g., Shkarofsky,¹ Bornatici,² Chu and Hui,³ and many others) are limited to $v/c = (2T_e/m_e c^2)^{1/2} \ll 1$ or $T_e < 10$ keV. However, several fusion experiments require ECH in plasmas with $T_e \gg 10$ keV. Most notably the annuli in λ (EBT) ($100 \text{ keV} < T_e < 1 \text{ MeV}$) and thermal barriers in tandem mirrors ($T_e \sim 50$ keV in TMX Upgrade; $T_e \sim 470$ keV in MFTF-B). We have previously reported absorption calculations using a Poynting's theorem calculation.⁵

$$\frac{dP}{ds} = \frac{4\pi}{c} \mathbf{E}^* \cdot \mathbf{J}^H \cdot \mathbf{E}$$

where $\mathbf{J}^H(\Omega_e, \omega, k_r)$ is calculated relativistically and k_r and $\mathbf{E}$ are the cold plasma values. Here, we present studies of the full dispersion relation,

$$D(n_{\parallel}, n_{\perp}) = \det[(1-n^2)\mathbf{I} + \mathbf{nn} + \frac{4\pi}{\omega} \mathbf{\sigma}]$$

where $\mathbf{\sigma} = \mathbf{\sigma}^A + \mathbf{\sigma}^H$, and $\mathbf{\sigma}^A$ as well as $\mathbf{\sigma}^H$ are calculated relativistically.

Relativistic temperatures cause two important modifications to the plasma response. The cyclotron resonances $[\gamma - n_{\parallel} v_{\parallel} - i(\frac{\omega}{\Omega_e})]^{-1}$ are broadened by the variation of γ as well as $v_{\parallel}$. Finite Larmor radius effects, $k_{\perp} \rho_e$, cause higher harmonic resonances to be important. For the energies of interest here the usual expansion, $\gamma = 1 + p^2/2m^2c^2$, is invalid. We have addressed a number of questions arising from the

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relativistic effects. At what temperature do the relativistic modifications become important, and for a given temperature, how many harmonics must be included in σ^A and σ^H ? What effect does high temperature have on qualitative features of wave propagation such as cutoffs, resonances, and polarization? From simple arguments one would expect that as the relativistic electron mass increases, the plasma refraction ^{would} decrease. This is born out in calculations which show the real part of the wave refractive index, n_r , changing from the cold plasma value at low temperature to the vacuum value 1 sufficiently high temperature. These results ^{indicate} give an indication of when cold plasma ray tracing results are likely to be valid.

For the present calculations, we have restricted to $n_{||} = 0$ so that Doppler broadening is not present and relativistic effects are maximum. Also, we assume the plasma distribution function to be an isotropic relativistic Maxwellian.

Calculation of Relativistic σ [

The Relativistic conductivity tensor for an isotropic distribution function $F(p) = F(p)$ can be expressed as

$$\underline{\underline{\Sigma}} = -i \frac{2\pi e^2}{m_e n_e} \int_0^\infty \frac{p^2 dp}{T(p)} 3c^3 \frac{\partial F(T)}{\partial T} \int_0^\pi dx \sin^2 x \sum_{N=-\infty}^{\infty} \frac{J_N^2 \left(\frac{N \omega}{\omega_1} \right) \left(\frac{N \omega}{\omega_1} \right) (p, x)}{T - n_{||} p \cos x - N \left(\frac{N \omega}{\omega_1} \right)}$$

where

$$\underline{\underline{M}}_N(p, x) = \begin{bmatrix} \left(\frac{N \omega}{\omega_1} \right)^2 J_N^2 & i \frac{N \omega}{\omega_1} p_{||} J_N J'_N & \frac{N \omega}{\omega_1} p_{||} J_N^2 \\ -M_{xy} & p_{||}^2 J_N^2 & -i p_{||} p_{||} J_N J'_N \\ M_{xz} & -M_{yz} & p_{||}^2 J_N^2 \end{bmatrix}$$

The argument of the Bessel functions is $n \left(\frac{\omega}{\omega_1} \right) \sin^2 x$, q is measured in

units of m_e and $\pi(p) = (1 + p^2)^{1/2}$, $p_{||} = p \cos x$, $p_{\perp} = p \sin x$. To separate Hermitian and anti-Hermitian parts, we assume $\text{Im}(n_{||})$ to be small and positive. Then the Plemelj relation gives

$$\frac{1}{T - n_{||} p \cos x - N \frac{n_e}{\omega}} \rightarrow \text{PV} \left[\frac{1}{T - n_{||} p \cos x - N \frac{n_e}{\omega}} \right] - i\pi \delta \left(T - n_{||} p \cos x - N \frac{n_e}{\omega} \right)$$

Then we obtain

$$\sigma^H = - \frac{2\pi e^2}{m_e \omega} \int_0^\pi dx \sin^2 x \int_0^\infty \frac{dp p^2}{\pi(p)} 3C^3 \frac{\partial F}{\partial T} \sum_{n=-\infty}^{\infty} M_N(p, x) \delta \left(T - n_{||} p \cos x - N \frac{n_e}{\omega} \right);$$

$$\sigma^A = -i \frac{2\pi e^2}{m_e n_e} p \int_0^\infty \frac{dp p^2}{T} 3C^3 \frac{\partial F(T)}{\partial T} \int_{-1}^1 d\mu \sum_{n=-\infty}^{\infty} \frac{1}{T - n_{||} p \mu - N \frac{n_e}{\omega}} M(p, \mu)$$

where $\mu = \cos x$.

Assuming $n_{||} = 0$ results in a great simplification. For a relativistic Maxwellian distribution

$$F(p) = \frac{n_e}{m^3 C^3} \frac{1}{4\pi k_2(p)} \frac{[p(1+p^2)^{1/2}]}{\exp[-p(1+p^2)^{1/2}]}$$

a typical element of the conductivity tensor σ_{xx} can be expressed as

$$\sigma_{xx}^H = \frac{e^2 e^{-p}}{8K_2(p)} \frac{\omega_p^2}{\omega} \sum_{N=N_1}^{\infty} \left(N^2 \frac{\omega_p^2}{\omega^2} - 1 \right)^{1/2} e^{-p(N \frac{\omega_p}{\omega} - 1)} \frac{N^2 \omega_p^2}{n^2 \omega^2} I_N(p);$$

$$\exp[-p(N \frac{\omega_p}{\omega} - 1)]$$

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$$\sigma_{xx}^R = \frac{1}{\pi} \frac{\rho}{8K_2(\rho)} \frac{\omega_{pe}^2}{\omega} \sum_{N=-\infty}^{\infty} \text{PV} \int_0^{\infty} \frac{d\rho \rho^2 e^{-\rho T}}{\tau [T - \frac{\Omega_e}{\omega}] n^2 \omega^2} I_N(\rho) \quad \uparrow$$

where

$$I_N(\rho) = \int_{-1}^1 \mu J_N^2(n_1 \frac{\omega}{\Omega_e} \rho (1 - \mu^2)^{1/2})$$

and $p_+ = [N^2(\frac{\Omega_e^2}{\omega^2}) - 1]^{1/2}$. These expressions are integrated numerically, and solutions of the dispersion relations,

$$\epsilon_{11}(\epsilon_{22} - n^2) + \epsilon_{12}^2 = 0 \quad (\text{extraordinary mode})$$

$$n^2 = \epsilon_{33} \quad (\text{ordinary mode}) \quad \uparrow$$

where $\underline{\epsilon} = \underline{I} + 4\pi i / \omega (\underline{\sigma}^H + \underline{\sigma}^A)$, are obtained by numerical root finding.

RESULTS

Figures 1 and 2 show k_r and k_i in cm^{-1} as functions of Ω_e/ω for various electron temperatures. The plasma density $n_e = 1.2 \times 10^{12}/\text{cm}^3$ or $\omega_{pe}^2/\omega^2 = 0.3$. In Fig. 1 it is seen that the magnetic field of the right-hand cutoff ($k_r \rightarrow 0$) and the upper hybrid resonance ($k_r \rightarrow \infty$) both increase with temperature. However, the two become closer together and at sufficiently high temperature ($T_e > 77 \text{ key}$ in this example), the resonance/cutoff pair disappears. As the temperature increases, we see k_r approaching the vacuum value $\omega/c = 3.77/\text{cm}$. Above $T_e \sim 20 \text{ key}$, there are significant deviations of k_r from the cold plasma value. This difference is crucial in the vicinity of the right-hand cutoff. Although these results do not allow us to calculate paths of energy flow, they suggest that ray tracing in the range, 40 key to 400 key with cold plasma rays is not valid. For $T_e > 400-500 \text{ key}$, the ray paths should approximate vacuum rather than cold plasma results.

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In Fig. 2, we see the sharp harmonic resonance structure at low T_e broadening with increasing temperature so that successive harmonics overlap. $T_e \sim 70$ keV. Above about 100 keV the harmonic structure is essentially washed out. Notice that for fixed Ω_e/ω , σ^H has a maximum as a function of T_e . For $f = 18$ GHz, $\Omega_e/\omega = 0.5$, $\omega_{pe}^2/\omega^2 = 0.3$, and $n_i = 0$, this maximum occurs at $T_e \sim 80$ keV. Cyclotron absorption decreases with increasing T_e above this value.

An important consideration for our modeling of ring absorption in EBT is that wave absorption is calculated relatively accurately by Poynting's theorem with cold k_r and $\underline{E}$ for $T_e < 100$ keV, except in the cold plasma cutoff region and in the vicinity of the fundamental resonance. If a cold plasma component is present, the Poynting's theorem is accurate at higher temperatures.

TEMPERATURE DEPENDENCE OF k_r (Ω_e/ω)
 $\omega/2\pi = 18$ GHz $\omega_{pe}^2/\omega^2 = 0.3$ $n_i = 0$

Figure 1

TEMPERATURE DEPENDENCE OF k_i (Ω_e/ω)
 $\omega/2\pi = 18$ GHz $\omega_{pe}^2/\omega^2 = 0.3$ $n_i = 0$

Figure 2

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