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PROPAGATION OF NORMAL ZONES IN THERMALLY INSULATED SUPERCONDUCTORS*

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ABSTRACT

A comparison of measured and calculated propagation velocities for superconductors thermally insulated by silicon grease shows that the grease makes a significant contribution to the effective specific heat of the conductor and substantially reduces the propagation velocity compared with what would be expected for perfectly insulated conductors.

1. INTRODUCTION

Scherer and Turowski¹ recently measured the propagation velocity of normal zones in superconductors thermally insulated by silicon grease. They compared their results with the theory of Cherry and Gittleman² and that of Broom and Rhoderick³ and concluded, "It is difficult to find agreement with our results." The theory predicts infinite propagation velocity at the critical current, whereas "the experiments show very clearly a finite velocity." This difficulty is more apparent than real and is caused by comparing the experiments with a formula based on a sudden transition from the superconducting to the normal state. The calculations of Altov et al.,⁴ which take current sharing into account, lead to a finite propagation velocity at the critical current. Altov's results, however, substantially overestimate the propagation velocity measured by Scherer and Turowski. A plausible explanation for this

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discrepancy is that the theory of Altov et al. is based on constant thermophysical properties. Modification of Altov's theory by including the temperature dependence of the specific heat and the thermal conductivity only reduces the overestimate but does not eliminate it. The point of this paper is to show that the remaining discrepancy can be removed by making allowance for the contribution of the silicon grease to the heat capacity of the conductor, as suggested earlier by Dresner et al.⁵

2. CONSTANT THERMOPHYSICAL PROPERTIES

In 1967, Keilin et al.⁶ formulated a theory of normal zone propagation in composite superconductors based on constant thermophysical properties that took current sharing into account. However, they did not solve the problem they formulated but rather solved a simpler one in which current sharing was neglected. It remained for Altov et al.⁴ to solve the equations formulated by Keilin; this they did numerically using a high-speed computer. In 1978, I found an analytic solution to the Keilin-Altov problem,⁷ which simplifies to the following quite convenient formula for thermally insulated superconductors ($h = 0$):

$$v = \frac{c\sqrt{i}}{2} \cdot \frac{2J_{cr}}{S} \left[\frac{k\rho}{f(T_{cr} - T_b)} \right]^{1/2} \quad (1a)$$

where

$$\frac{c}{\sqrt{4 - c^2}} \arctan \left(\frac{c\sqrt{4 - c^2}}{2 - c^2} \right) + 2 \ln c = \ln \left(\frac{i}{1 - i} \right) \quad (1b)$$

As the current approaches the critical current ($i \rightarrow 1$), $c \rightarrow 2$ and v approaches a finite limit given by the second factor on the right-hand side of (1a).

To see how well this theoretical formula agrees with measurement, I compared it with results obtained at 2.17 T for the multifilamentary composite F-1900 used by Scherer and Turowski. The physical parameters of F-1900 are as follows: 1.2×0.75 mm, Cu/SC ratio = 1.95, $I_c(4.2K, 2.17 T) \approx 1000$ A. Thus $J_{cr} = 1.11 \times 10^9$ A m⁻² and $f = 0.661$. At 2.17 T, $T_{cr} = 8.14K$. The product kp we take as LT_b , where L is the Lorentz constant, 2.45×10^{-8} V² K⁻². This is the Wiedemann-Franz law, which according to Fevrier and Morize⁸ can be used in a magnetic field (though deviations have been reported lately; cf. Lottin et al.⁹ and Nick et al.¹⁰). At a field as low as 2.17 T, I presume any deviation will be small. I use my earlier data¹¹ for the specific heats of copper and NbTi, which give $S = 2.54$ mJ cm⁻³ K⁻¹ at 4.2K and 2.17 T for F-1900.

Figure 1 shows the propagation velocity as a function of dimensionless current i for F-1900. The curve marked "experiment" has been taken from the paper of Scherer and Turowski. The curve marked "constant properties" has been calculated using (1a,b) and the data given above. The constant property theory overestimates the experiments by roughly a factor of 3 over the range $0.2 < i < 1$, although it does give a finite velocity (174 m s⁻¹) at $i = 1$.

3. TEMPERATURE-DEPENDENT PROPERTIES

The obvious explanation of this discrepancy is that the specific heat S and thermal conductivity k of the conductor are not constant but vary with temperature. If we note how S and k enter (1a) and remember that S increases more strongly with temperature than k , we are led to expect that accounting for the temperature dependence of the thermophysical

properties will decrease the propagation velocity. The question is whether it will explain the entire discrepancy.

I have calculated the propagation velocity by a method described in Section 5. The physical assumptions underlying the calculations are as follows: (1) k varies directly with temperature (Wiedemann-Franz law), and (2) S varies as the 2.2-power of temperature. The latter rule is a very close approximation to S for F-1900 in the temperature range 4.2-20K (see Fig. 2). Shown in Fig. 1 is a curve marked "temperature-dependent properties" based on these calculations. It is lower than the constant property curve, as expected, but it still overestimates the experimental results by roughly a factor of 2.

4. HEAT CAPACITY OF THE POTTING MATERIAL

A plausible hypothesis that can explain the remaining discrepancy is that the silicon grease absorbs some heat as the propagation front of the normal zone passes.⁵ The silicon grease then makes a contribution to the effective heat capacity of the composite. The volume of grease involved is determined by the distance into the grease that heat can diffuse during passage of the front of the normal zone. This distance is proportional to the square root of the time it takes the wave front to pass a point. Hence, we expect the contribution of the grease to the effective heat capacity to vary inversely as the square root of the propagation velocity.

We can estimate the contribution of the grease to the heat capacity if we assume it has the same temperature dependence as the heat capacity of the conductor. The ratio of the effective heat capacity S_{eff} to the true heat capacity of the metal S is the ratio of the lower two curves

in Fig. 1. The effective heat capacity is the sum of the heat capacity of the metal S and the contribution of the grease S_g . Note that S_g is expressed as energy per unit volume of conductor per degree. Plotted in Fig. 3 is S_g vs the experimentally measured propagation velocity. The five points represent values of S_g obtained from five points roughly equally spaced on the lowest curve in Fig. 1. The line in Fig. 3 has the slope $-1/2$. The agreement of the trend of the points with the line of slope $-1/2$ strengthens the hypothesis that the silicon grease adds significantly to the effective heat capacity of the composite.

5. METHOD OF CALCULATION USED IN SECTION 3

We begin with a heat balance for a unit length of conductor

$$S \frac{\partial T}{\partial t} = \frac{\partial}{\partial x} \left(k \frac{\partial T}{\partial x} \right) + Q_J \quad (2)$$

and look for a traveling wave solution $T = T(x + vt)$, which represents a wave traveling from right to left when v is positive. Then

$$\frac{d}{dx} \left(k \frac{dT}{dx} \right) - vS \frac{dT}{dx} + Q_J = 0 \quad (3)$$

We reduce the order of this differential equation by using the substitution of Maddock et al.,¹² $s = k dT/dx$:

$$s \frac{ds}{dT} - vSs + kQ_J = 0 \quad (4)$$

If we introduce dimensionless variables, (3) becomes

$$u \frac{du}{d\tau} - \lambda u \tau^n + \tau g(\tau) = 0 \quad (5)$$

where $g(\tau)$ is the function of Keilin et al.⁶ and Maddock et al.,¹² namely,

$$g(\tau) = \begin{cases} 0, & \tau \leq \tau_{cs} = i + (1-i) \tau_{cr} \\ \frac{\tau - \tau_{cs}}{\tau_{cr} - \tau_{cs}}, & \tau_{cs} \leq \tau \leq \tau_{cr} \\ 1, & \tau_{cr} \leq \tau \end{cases} \quad (6)$$

In passing from (4) to (5) we have taken

$$S = S_b \tau^n, \quad k = k_b \tau, \quad (7)$$

that is, we have assumed that the specific heat of the conductor varies as the n th power of the temperature and that the thermal conductivity is directly proportional to the temperature.

When $\tau \leq \tau_{cs}$, $g(\tau) = 0$ and (5) is easy to solve:

$$u = \frac{\lambda}{n+1} (\tau^{n+1} - 1), \quad \tau \leq \tau_{cs} \quad (8)$$

When $\tau \geq \tau_{cr}$, $g(\tau) = 1$, but (5) is no longer soluble in simple terms.

To see how to proceed, we examine the direction field of (5) when $g(\tau) = 1$ (see Fig. 4). We see that there are two families of trajectories, those that intersect the τ -axis and those that intersect the curve C . Between these families lies a separatrix S which asymptotically approaches but

never crosses curve C . Only on the separatrix does $u \rightarrow 0$ as $\tau \rightarrow \infty$, i.e., does $dT/dx \rightarrow 0$ as $T \rightarrow \infty$. It is this last condition that we must fulfill if the solution we find here is to correspond to the limit as $h \rightarrow 0$ of the solutions found in Refs. 2-4 and 7. [In these papers, the temperature far behind the wave front becomes constant ($dT/dx = 0$) and equal to T_{as} , the temperature at which Joule heat production just balances heat transfer to the helium. As $h \rightarrow 0$, T_{as} increases without bound, so that in the limit $dT/dx \rightarrow 0$ as $T \rightarrow \infty$.] Thus, for large enough τ , $u = 1/\lambda \tau^{n-1}$.

In principle, we can find λ , the dimensionless propagation velocity, by trial and error using the following sequence of steps. First, we choose a trial value of λ . Second, we choose a starting point P on C with a very large τ . Third, we integrate (5) numerically from P to the current sharing temperature τ_{cs} . Fourth, we compare the value of u so obtained with the value calculated from (8). Fifth, we adjust λ and repeat steps 2-5. We continue until satisfactory agreement is reached in step 4.

In practice, our computations can be simplified if we do not need to take τ very large, i.e., so large that the deviation of the separatrix S from C can be neglected. We can find an asymptotic series for the separatrix S as follows. Set

$$w = \lambda \tau^{n-1} \quad (9a)$$

$$z = 1/\lambda^2 \tau^{2n} \quad (9b)$$

Then

$$2nz w \frac{dw}{dz} + (n-1)w^2 + \frac{w-1}{z} = 0 \quad (10)$$

which can be solved around $z = 0$ by the series method:

$$w = 1 - (n-1)z + 2(n-1)(2n-1)z^2 - (5n-1)(9n-5)(n-1)z^3 \\ + 2(n-1)(4n-1)(31n^2 - 30n + 7)z^4 \dots \quad (11)$$

(The leading term $w = 1$ corresponds to $u = \lambda^{-1} \tau^{-n+1}$ for large enough τ .)

Using (11), we can find a starting point P on the separatrix S at a temperature τ not too far above the critical temperature τ_{cr} , and we do not have to integrate numerically very far. The actual numerical work needed here was performed on a programmable desk calculator (Hewlett-Packard HP-97).

6. CONCLUDING REMARKS

The contribution of the silicon grease to the effective specific heat of the composite F-1900 caused a reduction in the propagation velocity of about a factor of 2 compared with what is expected for a perfectly insulated conductor. This observation should be of interest to those concerned with the protection of potted coils because the propagation velocity influences the thermal stresses in a quenching magnet by affecting the distribution of heat deposited in the winding.

Scherer and Turowski remark in their paper that for F-1900, "an extrapolation of the current to zero velocity results in a minimum propagating current of about 150 A." The implication of their remark and the use of the term "minimum propagating current" is that for

smaller currents, arbitrarily long normal zones would contract. The theory given here contradicts this notion: although the propagation velocity becomes very small for small i (as observed), it is never negative.

SYMBOLS

$\bar{c}$ = auxiliary quantity used in (1a,b)

f = volume fraction of copper in composite

g = function of Keilin et al.⁶ and Maddock et al.¹² - see (5)

i = ratio of operating current to critical current

J_{cr} = overall critical current density in composite ($A\ m^{-2}$)

k = thermal conductivity ($W\ m^{-1}\ K^{-1}$)

k_b = thermal conductivity at $T = T_b$

L = Lorentz number ($2.45 \times 10^{-8}\ V^2\ K^{-2}$)

n = exponent in temperature dependence of specific heat

Q_J = Joule power density ($W\ m^{-3}$)

$s = k(dT/dx)$

S = specific heat ($J\ m^{-3}\ K^{-1}$)

S_b = specific heat at $T = T_b$

S_g = specific heat of silicon grease

t = time (s)

T = temperature (K)

T_b = helium bath temperature (K)

T_{cr} = critical temperature (K)

$u = s/(k_b Q_J T_b)^{1/2}$

v = propagation velocity ($m\ s^{-1}$)

$w = \lambda u \tau^{n-1}$

x = distance (m)

$z = \lambda^{-2} \tau^{-2n}$

$\lambda = v S_b T_b / (k_b Q_J T_b)^{1/2}$

ρ = resistivity of copper ($\Omega\ m$)

SYMBOLS (Cont'd)

$$\tau = T/T_b$$

$$\tau_{cr} = T_{cr}/T_b$$

$$\tau_{cs} = i + (1 - i)\tau_{cr}$$

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FIGURE CAPTIONS

Fig. 1° Propagation velocity vs current for conductor F-1900 of Ref. 1 at 2.17 T.

Fig. 2 Specific heat of conductor F-1900 of Ref. 1. The specific heats of copper and NbTi used to calculate the specific heat of the composite conductor are given in Ref. 11.

Fig. 3 Contribution of the silicon grease to the specific heat of the conductor as a function of the measured velocity of propagation.

Fig. 4 A sketch of the direction field of (5) when $g = 1$.

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